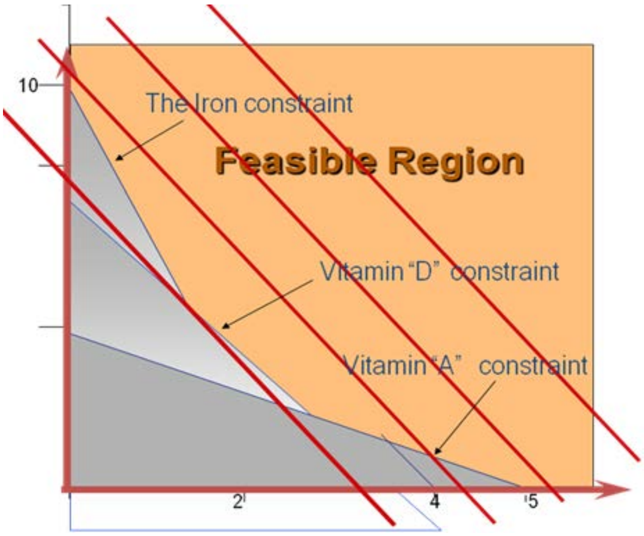
School of Engineering

E210 – Operations Planning

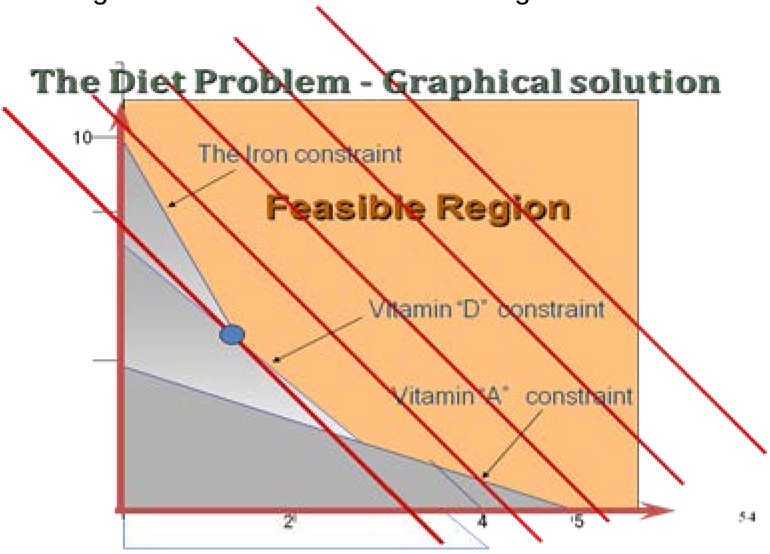
ESE Workshop Questions for Operations Planning (23 Aug 2018)

Please remember to review workshop questions for MSA

Question 1



a) If the above problem is a minimization problem, identify the optimal solution. Which constraints are binding? Which constraint is not binding?

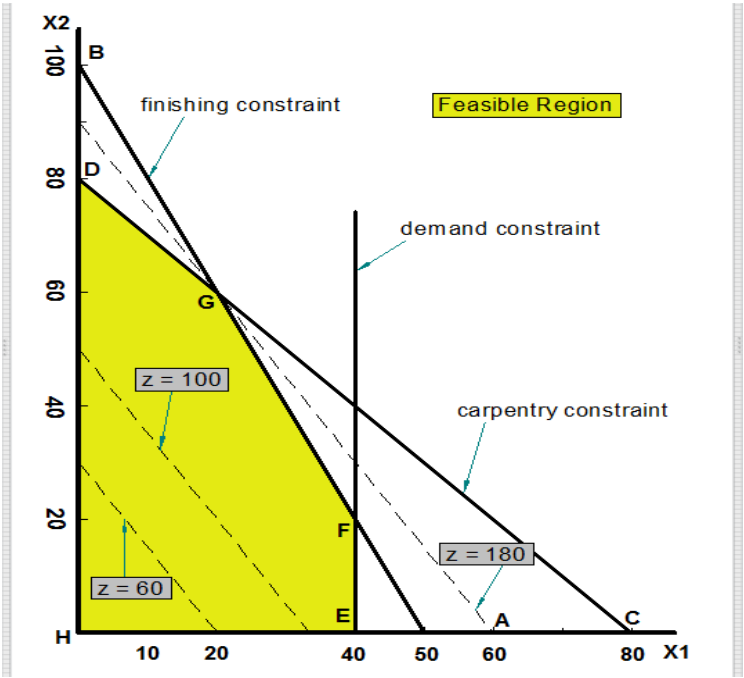


The optimal solution is the corner point determined by the Iron and Vitamin D constraints (shown in the above figure).

The Iron and Vitamin D constraints are binding; the Vitamin A constraint is not binding.

b) If the above problem is a maximization problem, is there an optimal solution?
No. The feasible region is unbounded.

c) Consider the following graph with the objective function line and the constraints shown below.



Max Profit $z = 3x_1 + 2x_2$ (objective function)

Subject to: (s.t.)

$2x_1 + x_2 \le 100$ (finishing constraint)

$x_1 + x_2 \le 80$ (carpentry constraint)

$x_1 \le 40$ (constraint on demand for soldiers)

$x_1 \ge 0$ (sign restriction)

$x_2 \ge 0$ (sign restriction)

According to the LP formulation given above, identify the optimal solution and the corresponding objective function value. Identify the constraints that are binding and the constraint that is not binding.

Get the optimal X1 and X2 values by observation; if difficult (for example, values are not integers), can solve simultaneous equations using the two constraint lines (finishing and carpentry constraints), as the intersection of the two lines is the optimal solution.

X1 =20; X2 = 60; Z = 3*20+2*60 = 180

The Finishing and Carpentry constraints are binding because the optimal solution is on the two constraint lines. The Demand constraint is not binding.

One more practice on graphical method: Quiz of P03:

Decision Variables

Let X_1 be number of pans of coffee cakes to make

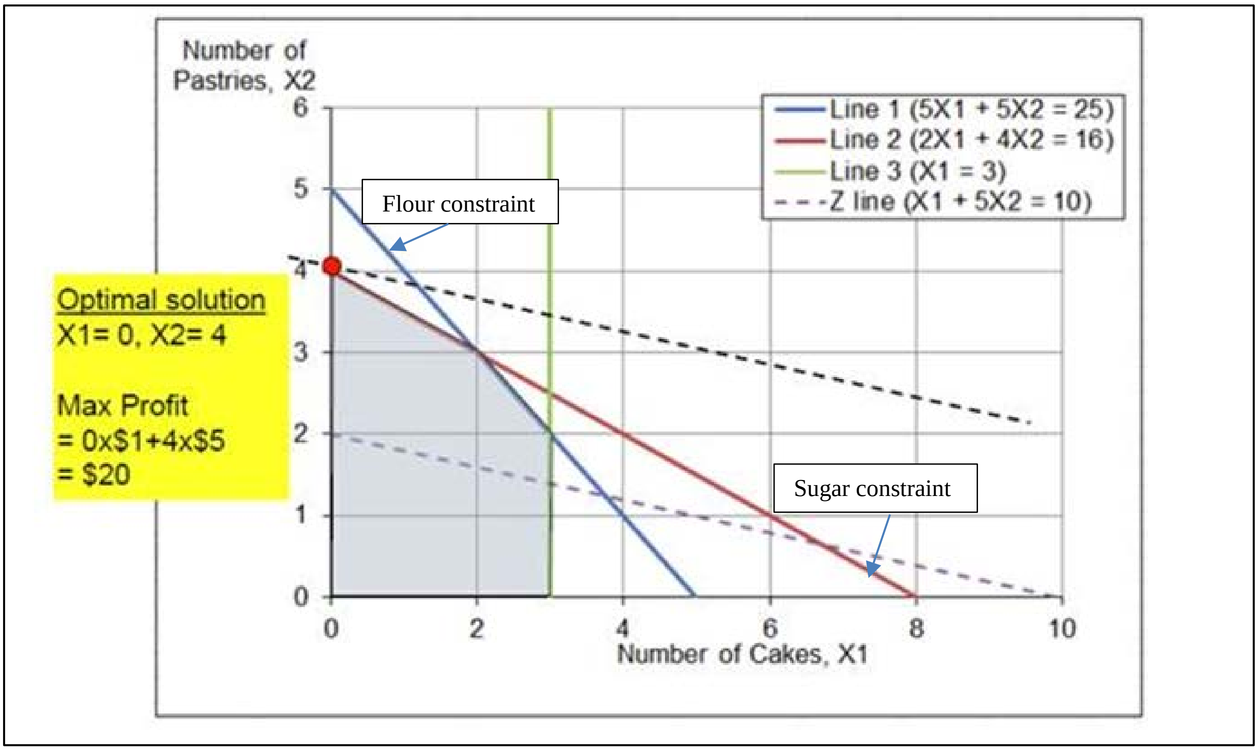
Let X_2 be number of pans of Danish pastries to make

Objective Function

Maximize $Z = X_1 + 5X_2$

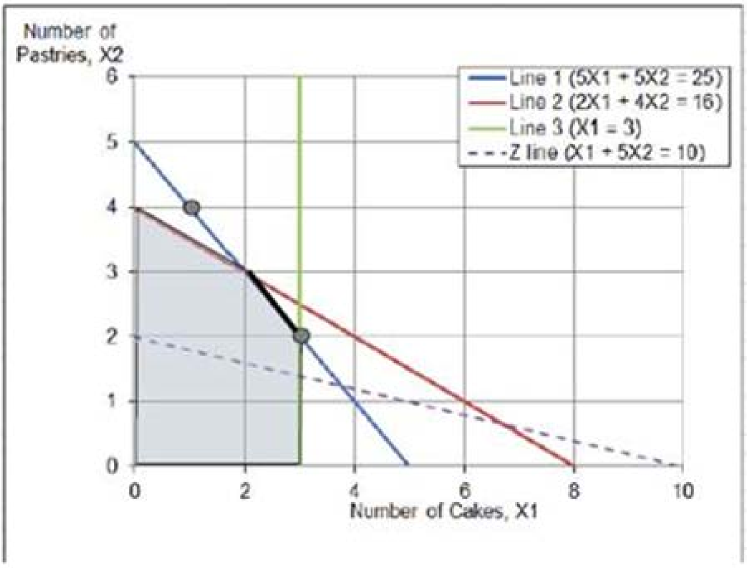
Constraints

$5X_1 + 5X_2$	≤	25	(Flour constraint)
$2X_1 + 4X_2$	≤	16	(Sugar constraint)
X_1	≤	3	(Non-negative X_1)
X_1	≥	0	(Non-negative X_2)
X_2	≥	0	



Q: Possible to make 1 cake and 4 pastries?
A: No, out of feasible region. Point is on line 1 (all flour will be used) but above line 2 (will exceed amount of sugar available)

Q: Possible to make 3 cakes and 2 pastries?
A: Yes, solution point is within feasible region. Point is on line 1 (all flour will be used – binding constraint) but below line 2 (sugar is non-binding constraint).
Leftover amount of sugar = 16 – (2x3+4x2) = 2 pounds



Q What happens when Z = 3X1 + 3X2?
A: Z-line becomes parallel to Line 1. There will be more than one optimal solution.

Question 2

A factory manufactures three models of a product, which requires three resources – labor, material, and administration. The unit profits of these products are \$1000, \$600, and \$400 respectively. There are 1200 hour of labor, 6000 kg of material and 3000 hour of administration available per month. The resource requirements for the products to manufacture are given in the table below.

Products	Labor (hour)	Material (kg)	Administration (hour)	Unit Profit (\$)
Model 1	5	50	10	1000
Model 2	5	20	10	600
Model 3	5	25	30	400
Available Resource	1200	6000	3000	-

Formulate a linear programming model for this problem

Decision Variables

Let X_1 be the quantity of model 1 to make

Let X_2 be the quantity of model 2 to make

Let X_3 be the quantity of model 3 to make

Objective Function

maximize Profit $Z = 1000 X_1 + 600 X_2 + 400 X_3$

Constraints

$5X_1 + 5X_2 + 5X_3$	$\leq$	1200	(Labour)
$50X_1 + 20X_2 + 25X_3$	$\leq$	6000	(Material)
$10X_1 + 10X_2 + 30X_3$	$\leq$	3000	(Administration)
X_1 , X_2 , X_3	$\geq$	0	(Non-negative variables)

Question 3

You have three investment options. The returns from the investment under different economic conditions are given in the following payoff table.

Investment Options	Economic Condition		
	Favorable (40%)	Stable (40%)	Unfavorable (20%)
Option 1	8000	3000	-500
Option 2	3000	1800	1000
Option 3	3800	2200	600

a) Which investment option should you go for based on the EMV method?

EMV of Option 1: 8000*0.4 + 3000*0.4 + (-500)* 0.2 = $4300

EMV of Option 2: 3000*0.4 + 1800*0.4 + 1000* 0.2 = $2120

EMV of Option 3: 3800*0.4 + 2200*0.4 + 600* 0.2 = $2520

Option 1 has the highest EMV, so the best investment option is Option 1.

b) What is the EVPI?

EVwithPI = 8000*0.4 + 3000 *0.4 + 1000*0.2 = $4600

EVPI = EVwithPI – EMV of the best alternative = $4600 – $4300 = $300

c) If you want to choose the investment option based on a utility function:

$U(x) = 1 - e^{-x/R}$ with R = 500

i. Construct the utility table.

Utility Table

Investment Options	Economic Condition		
	Favorable (40%)	Stable (40%)	Unfavorable (20%)
Option 1	0.999999887	0.997521248	-1.718281828
Option 2	0.997521248	0.972676278	0.864664717
Option 3	0.999499549	0.98772266	0.698805788

U(1000)=1-e^(-1000/500) = 1-e^(-2) = 1- 0.135335 = 0.86466

ii. What will be your investment option based on the expected utility?

Calculate the expected utility for each option and determine the best action: Option 2;

Expected Utility:

Option 1: … = 0.4554

Option 2: … = 0.9610

Option 3: … = 0.9347

iii. Compare your decisions made based on the EMV and the expected utility, are they the same? What is the advantage of using expected utility in decision making?

No. The advantage of using the expected utility in decision making is that decision maker's risk attitude (profile) is also taken into consideration in the decision making.

iv. With the utility function $U(x) = 1 – e^{-x/R}$, what is your risk attitude? What do you call R? If your friend also uses the utility function $U(x) = 1 – e^{-x/R}$ with R = 100, comment on his risk attitude in comparison with yours.

Risk attitude: risk averse

R: risk tolerance

As risk tolerance R decreases, the decision maker becomes more risk averse. With lower value of R, your friend can tolerate lower level of risk than you.

Question 4

You are given two alternatives in a game show.

Alternative 1: receive $100 and leave the game

Alternative 2: play the game where if you win, you receive $1000; if you lose, you pay $400. The probability that you win is p.

a) What is the certainty equivalent to alternative 2?

CE = $100

b) If you think that the two alternatives are indifferent when p is 0.4, what is your risk premium? What is your risk attitude? Explain.

Risk premium = EMV of alternative 2 – CE

=1000*0.4+(-400)*(1-0.4) - 100
=$60

Risk attitude: risk averse. Because risk premium is positive.

c) If your friend Amy thinks that the two alternatives are indifferent when p = 0.3, what is her risk premium? What is her risk attitude? Explain.

Risk premium = EMV of alternative 2 – CE
=1000*0.3+(-400)*(1-0.3) - 100
=-80

Risk attitude: risk seeking. Because risk premium is negative.

d) Determine p for a person who is risk neutral.

Risk premium = EMV of alternative 2 – CE = 0
1000*p+(-400)*(1-p) – 100 =0
p=5/14 =0.357

Question 5

A company has two manufacturing plants A and B. The plants can supply the following numbers of products to the company's distributors each month:

Plant	Monthly Supply (unit of products)
A.	5800
B.	5400
Total	11200

The distributors which are spread throughout five countries have the following total monthly demand:

Distributor	Monthly Demand (units of product)
1	2600
2	3050
3	2300
4	1800
5	1450
Total	11200

The company must pay the following shipping cost per unit of the product:

From	To (cost, $)				
	1	2	3	4	5
A	12	15	17	14	16
B	9	7	10	6	18

a) What is the objective of the above product distribution problem?

To minimize the total distribution cost

b) Formulate a Linear Programming (LP) model for the product distribution problem. When using Excel Solver to find the optimal solution based on the LP method, do you need to add in the integer solution requirement on the decision variables? Justify your answer.

LP formulation:

Decision variables:
Let Xij be the shipment amount from Plant i to Distributor j, where i = A, B and j = 1, 2,…, 5

Objective function:

Minimize the total transportation cost:
Z = 12XA1+15XA2+17XA3+14XA4+16XA5+9XB1+7XB2+10XB3+6XB4+18XB5

Constraints:

Supply constraints: 2 constraints, one for each supply node.

Plant A: XA1+XA2+XA3+XA4+XA5 = 5800
Plant B: XB1+XB2+XB3+XB4+XB5 = 5400

Demand constraints: 5 constraints, one for each demand node.

Distributor 1: XA1+XB1 = 2600
Distributor 2: XA2+XB2 = 3050
Distributor 3: XA3+XB3 = 2300
Distributor 4: XA4+XB4 = 1800
Distributor 5: XA5+XB5 = 1450

Non-negativity constraints:
Xij>=0

No. because the transportation problem has the integer solution property: as long as demand and supply have integer values, the optimal solution is guaranteed to have integer values.

Additional practice:

What if demand from distributor 1 increases from 2600 to 3000?

The transportation problem will not be balanced: demand more than supply. Sign of the demand constraints will be changed to <=, because demand cannot be all fulfilled.

Constraints:

Supply constraints:

Plant A: XA1+XA2+XA3+XA4+XA5 = 5800
Plant B: XB1+XB2+XB3+XB4+XB5 = 5400

Demand constraints:

Distributor 1: XA1+XB1 <= 3000
Distributor 2: XA2+XB2 <= 3050
Distributor 3: XA3+XB3 <= 2300
Distributor 4: XA4+XB4 <= 1800
Distributor 5: XA5+XB5 <= 1450

What if supply from Plant A increases from 5800 to 6000?

The transportation problem will not be balanced: supply more than demand. Sign of the supply constraints will be changed to <=, because supply cannot be all used up.

Constraints:

Supply constraints:

Plant A: XA1+XA2+XA3+XA4+XA5 <= 6000
Plant B: XB1+XB2+XB3+XB4+XB5 <= 5400

Demand constraints:

Distributor 1: XA1+XB1 = 2600
Distributor 2: XA2+XB2 = 3050
Distributor 3: XA3+XB3 = 2300
Distributor 4: XA4+XB4 = 1800
Distributor 5: XA5+XB5 = 1450

Question 6

A. Table below shows the hobby codes of your friends.

Friend	Hobbies
A	3, 8, 10, 14, 15
B	2, 3, 7, 8, 9, 12, 13, 15
C	3, 5, 10, 14
D	2, 7, 8, 11, 12, 13
E	1, 4, 5, 9, 10
F	2, 5, 7, 8, 9, 10
G	3, 4, 15
H	4, 10
I	1, 6

a) Determine the dissimilarity index between your fiends A and B

D_{AB}= 1-3/(3+7) = 0.7

b) Determine the dissimilarity index between your fiends A and C

1-3/(3+3) = 0.5

Note:

When dij is 0, means objects i and j have exactly the same traits

When dij is 1, means objects i and j have no common traits

B. Based on the table given, you want to plan a seating arrangement using the minimum spanning tree model.

a) What are the arcs in the minimum spanning tree model?

Dissimilarity indices

b) What are the nodes in the minimum spanning tree model?

Your friends

c) What is your objective when linking your friends based on the minimum spanning tree model?

Minimize the dissimilarity among your friends in terms of their hobbies.

C. You are given the following dissimilarity index table:

Friends	D	E	F	G	H	I
D	-	1	0.666667	1	1	1
E	1	-	0.625	0.857143	0.6	0.833333
F	0.666667	0.625	-	1	0.857143	1
G	1	0.857143	1	-	0.75	1
H	1	0.6	0.857143	0.75	-	1
I	1	0.833333	1	1	1	-

a) To construct the minimum spanning tree, you started with node E, and linked E with H as 'H' is the 'closest' to E in terms of the dissimilarity index.

i. Which method are you using to construct the minimum spanning tree?

Prim's algorithm (method)

ii. What is the next node to be linked to the partial minimum spanning tree you constructed? How do you link the node to the partial minimum spanning tree? Justify your answer.

F. the closest to E is F with dissimilarity index of 0.625; the closest to H is G with dissimilarity index of 0.75; so the next node to be linked to the partial minimum spanning tree is F

Link F to E.

b) Write down the solution in pairs of connections and in order of sequence if you apply the Kruskal's algorithm to construct the minimum spanning tree.

E-H, E-F, F-D, H-G and E-I.

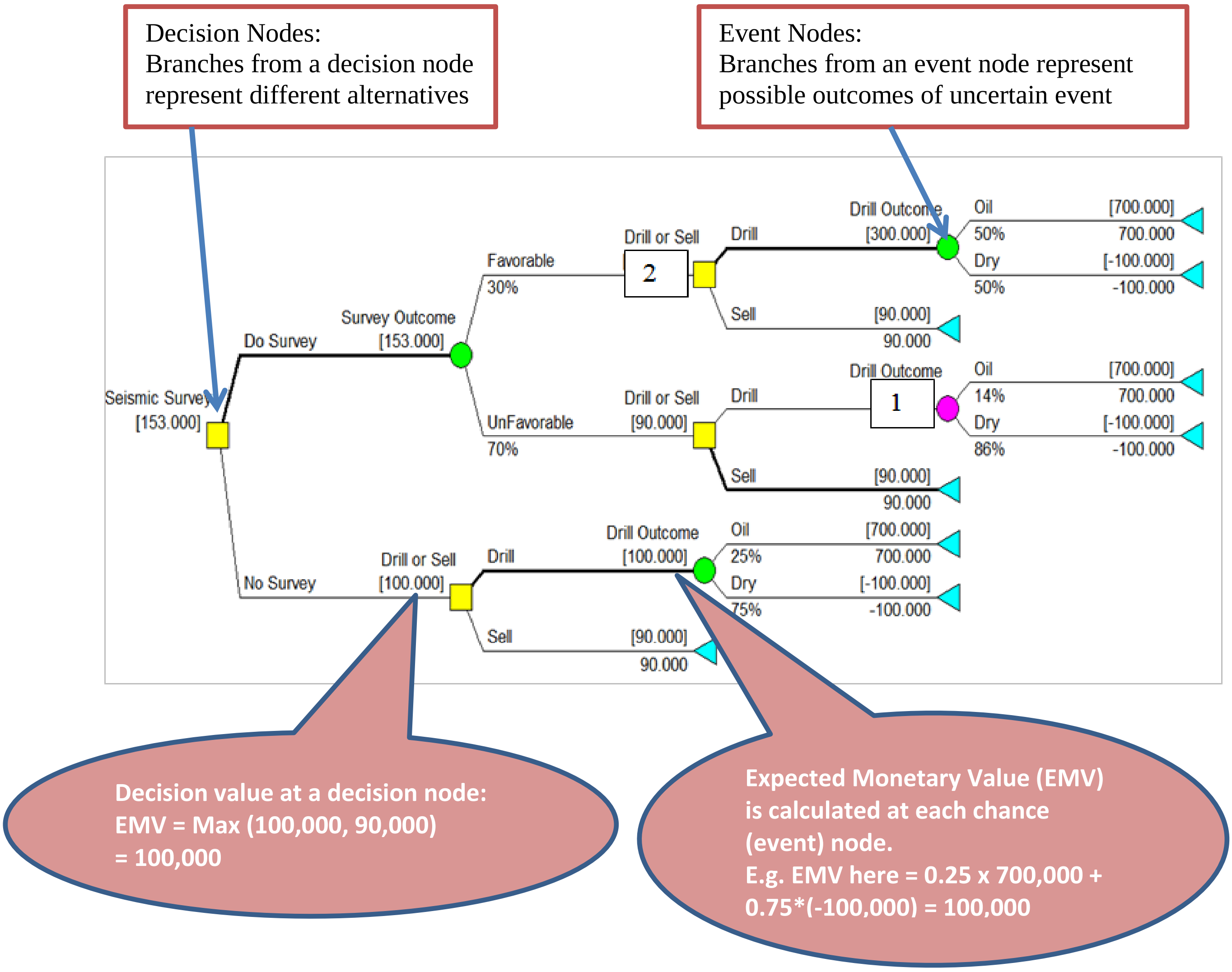
c) List two criteria you can use to put your friends into different tables.

- Pre-specified number of tables
- Maximum allowable dissimilarity in each table
  - ✓ For example, Table dissimilarity ≤ 2
- Minimum number of friends in each table
  - ✓ For example, at least 3 friends in each table

Question 7

An oil company, OIL, owns a tract of land that may contain oil. The chance that the land contains oil is 0.25. Because of this prospect, another oil company has offered to purchase the land for $90,000. However OIL is considering holding the land in order to drill for oil itself. If oil is found, the expected profit is $700,000. However, if the land is dry, a loss of $100,000 will be incurred. Another option prior to making a decision is to conduct a detailed seismic survey to obtain a better estimation of the probability of finding oil at the cost of $30,000.

Figure below is the decision tree generated by DPL for the above decision making problem.



a) Calculate the EMV value when Seismic survey outcome is ‘UnFavorable’ and the decision is to ‘Drill’ labeled as ‘1’. Show your workings.

   700,000*0.14+(-100,000)*0.86 = $12,000

b) When seismic survey outcome is 'Favorable', what is your recommendation for the 'Drill or Sell' decision? What is the decision value associated with this decision? Show your workings clearly.

Drill. Decision value = max (300,000, 90,000) = $300,000

c) Should the company conduct the seismic survey at the cost of $30,000? Justify your answer with workings.

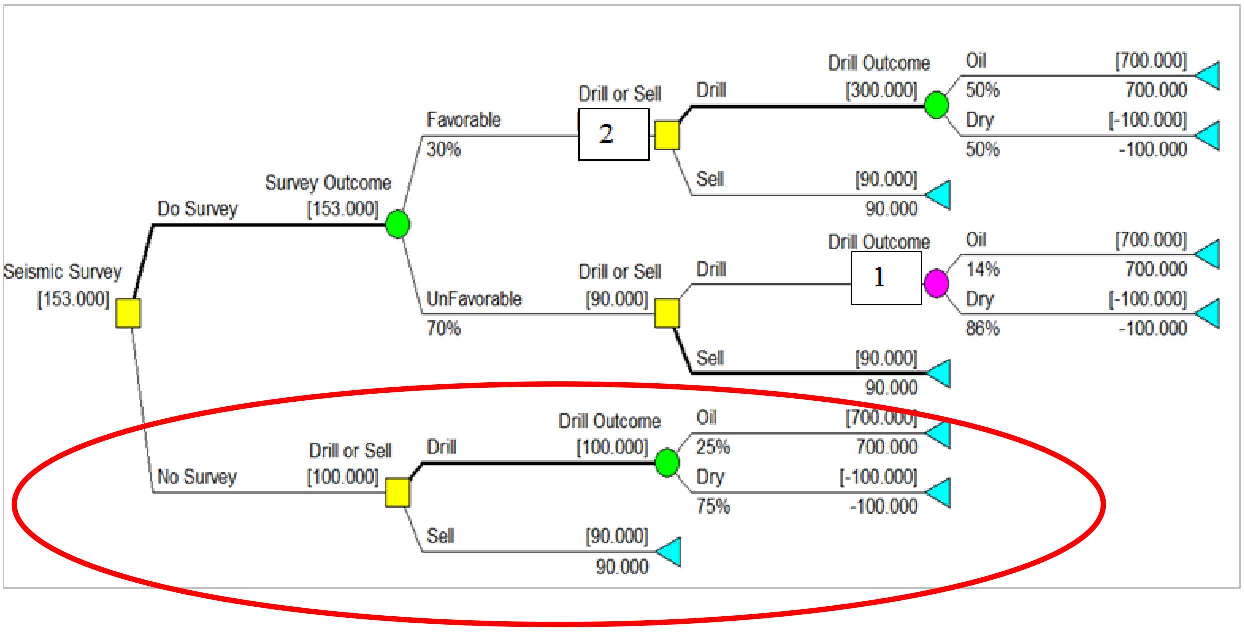
Compare the cost of seismic survey with EVSI. So find EVSI first

EVSI = Expected payoff **with** sample (additional) information – Expected payoff **without** sample (additional) information

=153,000 – 100,000 = $53,000

Since $53,000 > $30,000, the company should do the seismic survey.

d) Calculate EVPI and the efficiency of sample information. Show your workings clearly.



Alternatives	Drill Outcome	
	Oil	Dry
	25%	75%
Drill	700,000	-100,000
Sell	90,000	90,000

EVwithPI = 0.25*700,000 + 0.75 *90,000 =$242,500

EVPI = $242,500 – $100,000 = $142,500

Efficiency of sample information = EVSI/EVPI = 53,000/142,500 = 37.19%

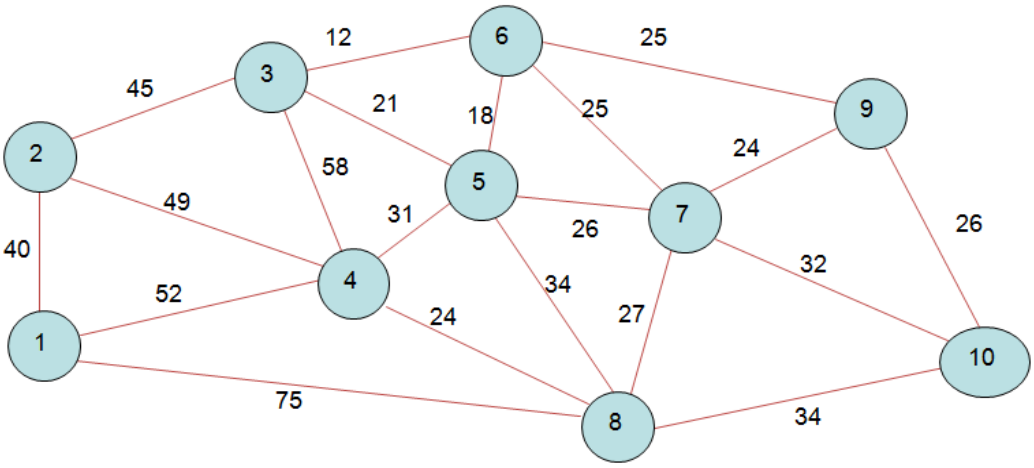
e) State the overall best course of action the company should take. What is the total expected profit associated with the best course of action?

Do the seismic survey, when the survey outcome is favorable, drill for oil; when the survey outcome is unfavorable, sell the land.

The total expected profit: \$153,000 – \$30,000 = \$123,000.

Question 8

Several oil companies are jointly planning to build an oil pipeline to connect several south-western, south-eastern, and Midwestern cities, as shown in the following network:



The distances (in kilometer) between cities are shown on each branch. Determine a pipeline system using the following algorithms that will connect all 10 cities, using the minimum number of miles of pipe, and indicate how many miles of pipe will be used.

a) Prim's Algorithm (starting from node 9)

Node 9 →Node 7
Node 7 →Node 6 (or Node 9 →Node 6)
Node 6 →Node 3
Node 6 →Node 5
Node 9 →Node 10
Node 7 →Node 8
Node 8 →Node 4
Node 3 →Node 2
Node 2→ Node 1

Total Miles of pipe = 241KM

b) Kruskal's Algorithm

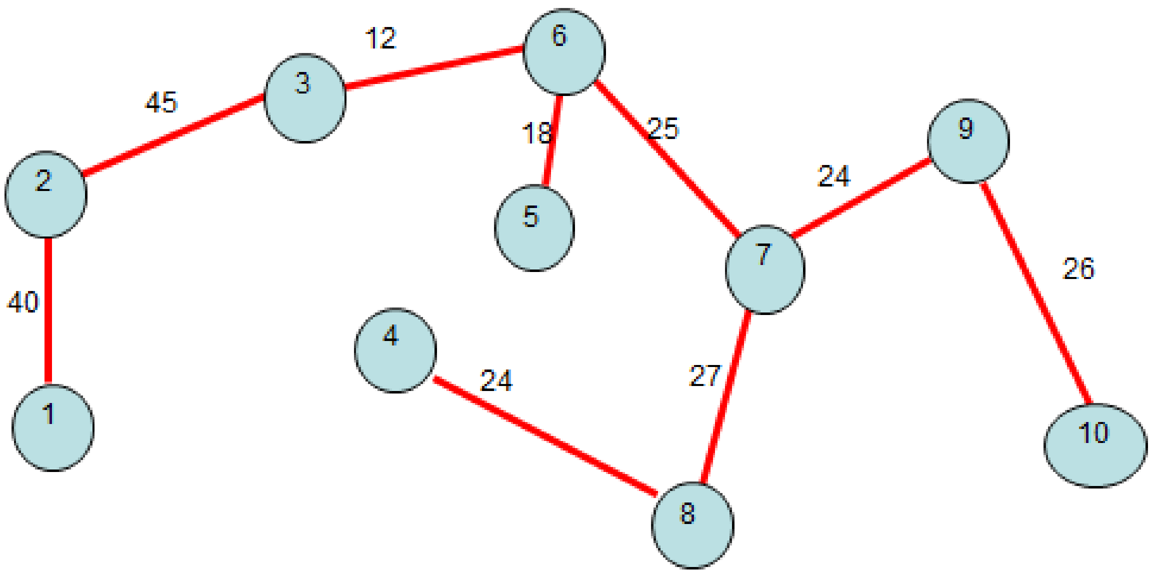
i. Arc 3 - 6
Arc 6 –5
Arc 4 – 8
Arc 7 – 9

Arc 6 – 7
Arc 9 – 10
Arc 7 – 8
Arc 2 – 1
Arc 2 –3

Total Miles of pipe = 241KM

ii. If you are required to list out the first three pair of arcs to connect in the right sequence, you should answer:
Arc 3 - 6
Arc 6 –5
Arc 4 – 8

c) Draw the Minimum Spanning Tree



d) If the cost of building the pipe network is $80 thousand per kilometer and the oil companies only catered 20 million for this project, will the oil companies be able to proceed with the project? Explain.

Total cost = 241 * 0.08 = 19.28 million.
Yes. The oil companies will be able to proceed with the project as the cost is lower than the allowable budget of 20 million.

Question 9

A large bank is going to make one of three investments. The economy will have one of three possible states during the life of the investment: improve, remain stable, or worsen. The trust officer believes the respective probabilities are 0.1, 0.5, and 0.4. The estimated payoff table is as follows:

Investment	Improve	Remain stable	Worsen
A1	30	5	-10
A2	40	10	-30
A3	-10	0	15

a. Determine the optimal action based on the EMV method

b. Calculate the EMV with perfect information and EVPI.

c. If the trust officer makes his decision based on the following utility functions, determine the optimal action to take under each scenario based on the expected utility.

- ✓ $U(x) = x^{1/3}$
- ✓ $U(x) = 2x + 6$

Investment	Improve	Remain stable	Worsen	EMV
	0.1	0.5	0.4	
A1	30	5	-10	1.5
A2	40	10	-30	-3
A3	-10	0	15	5
Max	40	10	15	

EVwithPI = 40*0.1 + 10*0.5 + 15*0.4 = 15

EVPI = EVwithPI – EMV of the best alternative = 15 – 5 = 10

$U(x) = 2x + 6$

Investment	Improve	Remain stable	Worsen	
	0.1	0.5	0.4	
A1	66	16	-14	9
A2	86	26	-54	0
A3	-14	6	36	16

$U(x) = x^{1/3}$

Investment	Improve	Remain stable	Worsen	
	0.1	0.5	0.4	
A1	3.107233	1.709975947	-2.15443	0.303937
A2	3.419952	2.15443469	-3.10723	0.17632
A3	-2.15443	0	2.466212	0.771041

<u>Question 10</u>

A form teacher of primary six wants to assign his 8 students to 3 interest groups taking into account student attributes and their preferences. There are certain constraints on the composition of each group. Each group must contain a certain minimum number of female students and certain minimum numbers of students who are good at the following subjects: Math, Science, and English. Here is a table with the 3 groups and their student requirements:

Table 1: Group requirements

Group Number	Size of Group	Minimum Number of Students Good at Math	Minimum Number of Students Good at Science	Minimum Number of Students Good at English
#1	2	1	-	1
#2	3	2	1	1
#3	3	1	-	2

Here is a table listing students' preferences and their attributes:

Table 2: Student Attributes and their preference

Student Name	Group Preference			Subjects good at
	#1	#2	#3	
1	3	2	1	English
2	2	3	1	Math, English
3	1	3	2	Science
4	1	3	2	Math, English
5	3	1	2	Math, Science
6	2	3	1	English
7	2	1	3	Math, Science
8	3	2	1	English

'1'- Preferred; '2'- preferred; '3'-Not Preferred

The form teacher also understands that students 1 and 5 must be in the same group; either student 4 or 6 must be in group 2 and; students 6 and 8 cannot be together.

Formulate a binary integer programming (BIP) model for the above problem.

<u>Decision Variables:</u>
Let Xij be the assignment of student i to group j, where i = 1,2,…,8 and j = 1,2,3. Xij = 1 represents that student i is assigned to group j; Xij = 0 represents that student i is not assigned to group j.

The decision variables are called binary decision variables, because each decision variable can only take two values, 0 and 1.

Objective Function:
Minimize total preference score,
$Z = 3X_{11}+2X_{12}+1X_{13}+2X_{21}+.....+X_{83}$

Group Size:
Group 1: $X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+X_{61}+X_{71}+X_{81} = 2$
Group 2: $X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+X_{62}+X_{72}+X_{82} = 3$
Group 3: $X_{13}+X_{23}+X_{33}+X_{43}+X_{53}+X_{63}+X_{73}+X_{83} = 3$

Each student must be assigned to one group:
Student 1: $X_{11}+X_{12}+X_{13} = 1$
Student 2: $X_{21}+X_{22}+X_{23} = 1$
Student 3: $X_{31}+X_{32}+X_{33} = 1$
Student 4: $X_{41}+X_{42}+X_{43} = 1$
Student 5: $X_{51}+X_{52}+X_{53} = 1$
Student 6: $X_{61}+X_{62}+X_{63} = 1$
Student 7: $X_{71}+X_{72}+X_{73} = 1$
Student 8: $X_{81}+X_{82}+X_{83} = 1$

Math Requirement
Group 1: $X_{21}+X_{41}+X_{51}+X_{71} >= 1$
Group 2: $X_{22}+X_{42}+X_{52}+X_{72} >= 2$
Group 3: $X_{23}+X_{43}+X_{53}+X_{73} >= 1$

Science Requirement
Group 2: $X_{32}+X_{52}+X_{72} >= 1$

English Requirement
Group 1: $X_{11}+X_{21}+X_{41}+X_{61}+X_{81} >= 1$
Group 2: $X_{12}+X_{22}+X_{42}+X_{62}+X_{82} >= 1$
Group 3: $X_{13}+X_{23}+X_{43}+X_{63}+X_{83} >= 2$

Students 1 and 5 must be in the same group
Group 1: $X_{11} - X_{51} = 0$
Group 2: $X_{12} - X_{52} = 0$
Group 3: $X_{13} - X_{53} = 0$

Either Student 4 or 6 must be in Group 2
$X_{42} + X_{62} = 1$

Students 6 and 8 cannot be together in the same group
Group 1: $X_{61} + X_{81} <= 1$
Group 2: $X_{62} + X_{82} <= 1$
Group 3: $X_{63} + X_{83} <= 1$

Each Xij is binary.

Take note, if decision variables are given, you must use the defined decision variables to do the formulation.