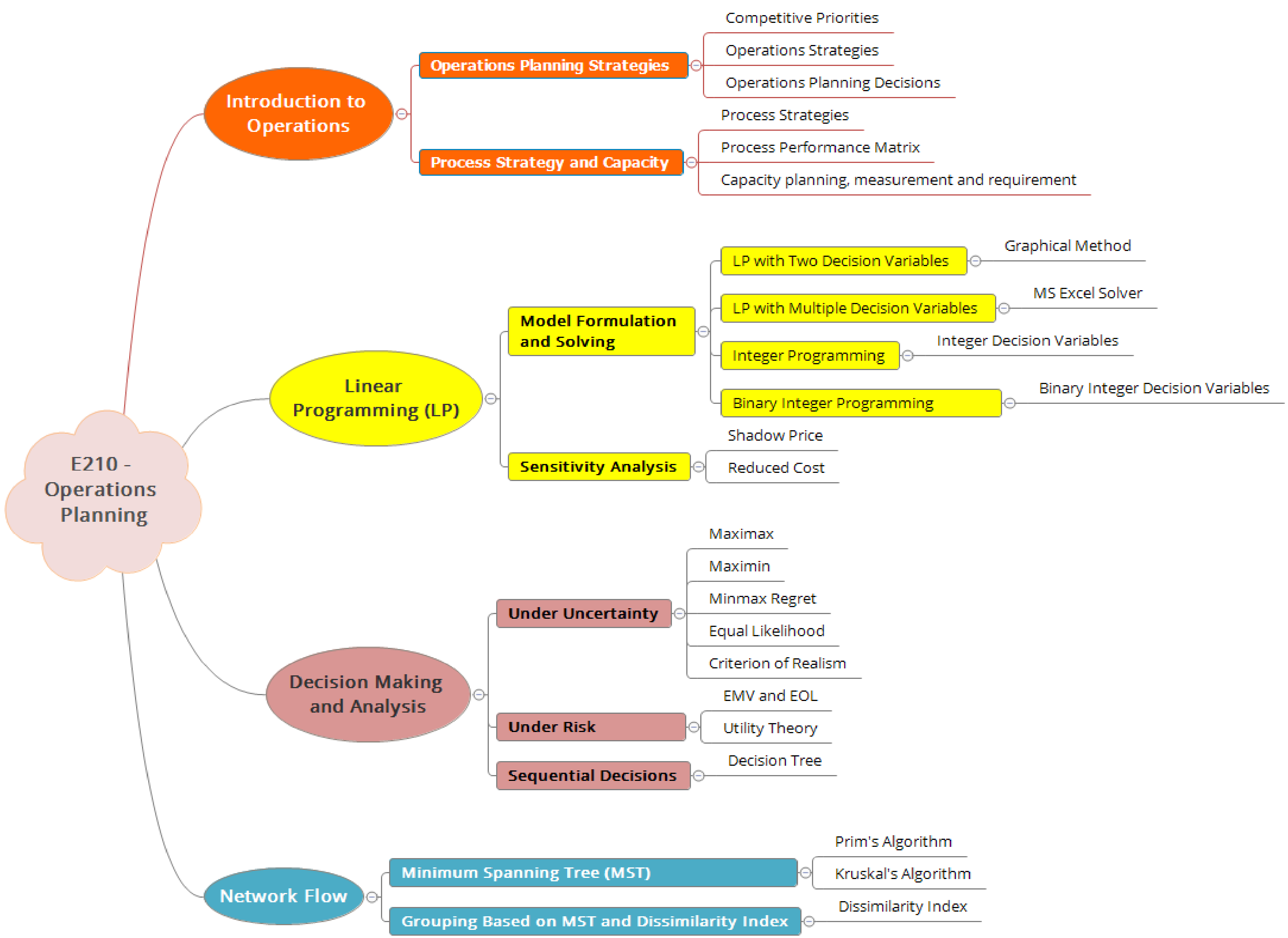
Problem 05

The Transportation Model

E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree

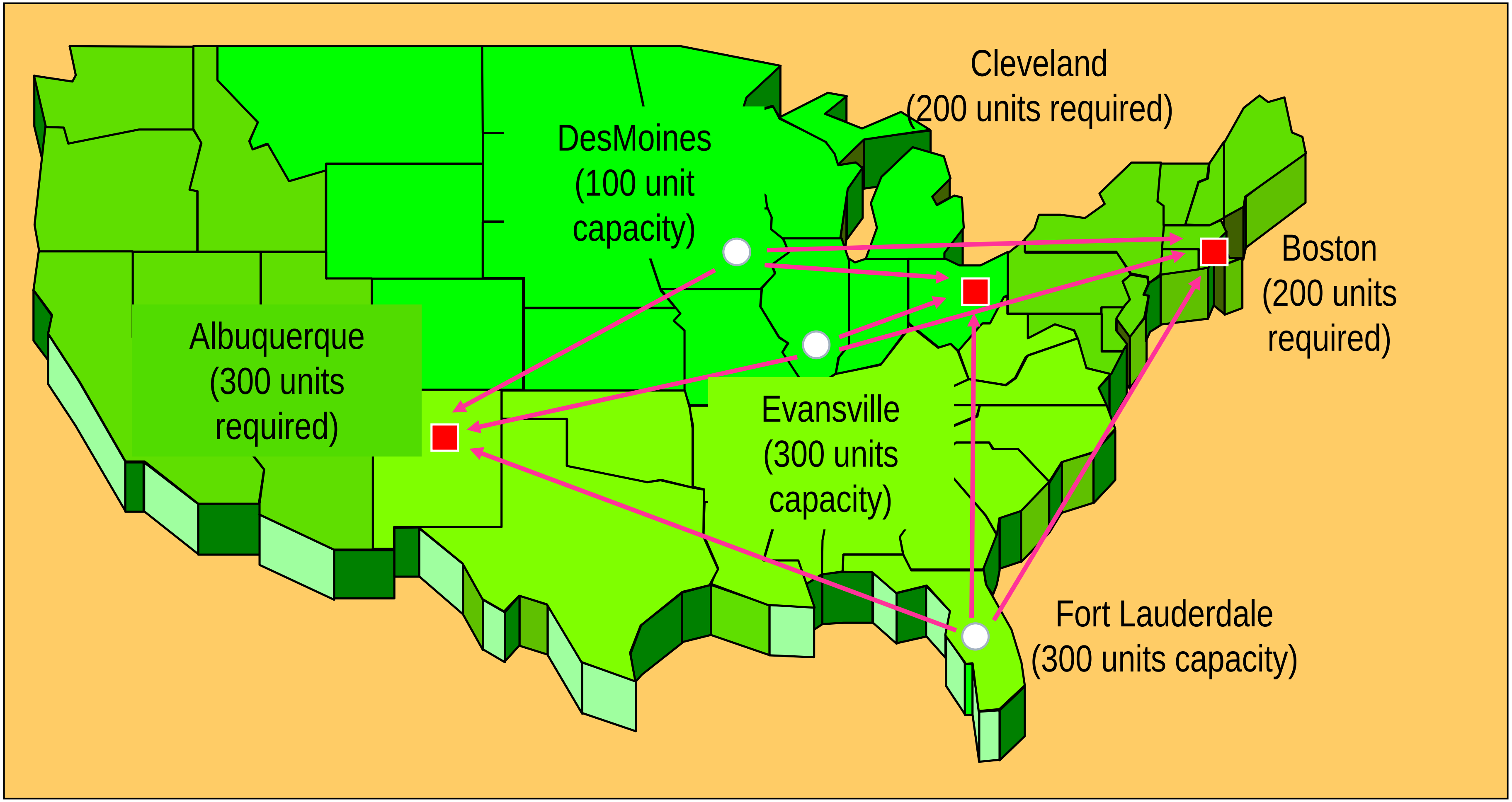


Transportation Problem

- How much should be shipped from several sources to several destinations
  - Sources: supply centers such as factories, warehouses, etc.
  - Destinations: receiving centers such as warehouses, retail stores, customers, etc.

- Transportation model
  - The objective is to find the shipping arrangement with the lowest cost
  - The Cost Assumption
    - ✓ The cost of distributing units from any particular source to any particular destination is directly proportional to the number of units distributed.
    - ✓ This cost is just the unit cost of distribution times the number of units distributed

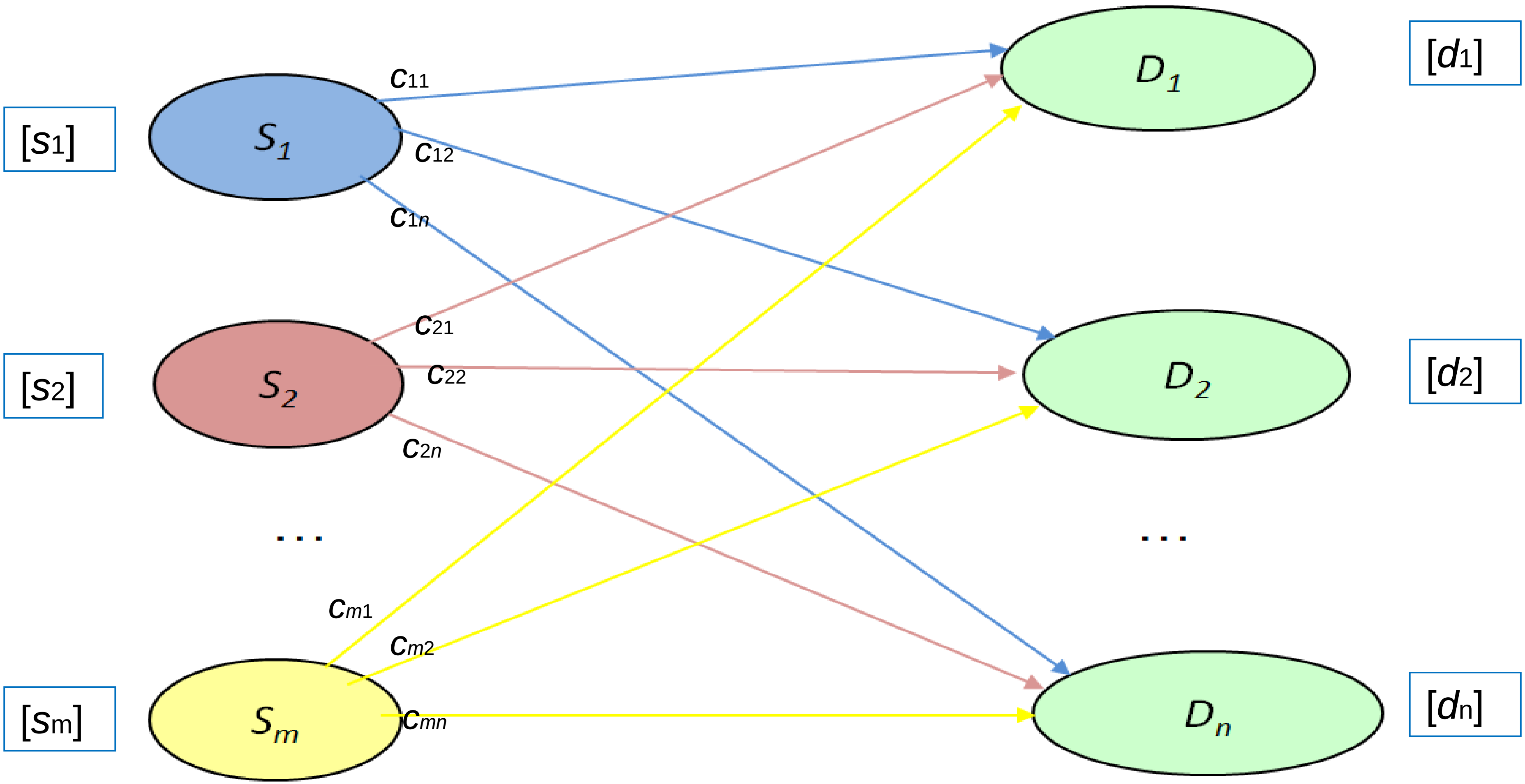
Transportation Problem (Example)



Transportation Table

	Cost per Unit Distributed				
	Destination				
	1	2	…	n	Supply
1	c_{11}	c_{12}	…	c_{1n}	S_1
2	c_{21}			c_{2n}	S_2
Source **…**	…	…	…	…	…
m	c_{m1}	c_{m2}		c_{mn}	S_m
Demand	d_1	d_2	…	d_n	

Network Representation



The Transportation Model

- Any problem (whether involving transportation or not) fits the model for a transportation problem if

  - It can be described completely in terms of a transportation table that identifies all the sources, destinations, supplies, demands, and unit costs, and

  - satisfies both the requirements assumption and the cost assumption.

  - ***The objective is to minimize the total cost of distributing the units.***

Transportation Problem Solution Steps

- Define problem
- Set up transportation table (matrix) or network diagram which
  - Summarizes all data
  - Keeps track of computations

Solution:

- Develop a heuristic solution
  - ➢ North-West Corner Rule or Lowest Cost Method
- Find an optimal solution
  - ➢ Solve LP model using Solver

Heuristic Solution – Lowest/Least Cost Method

- Identify the cell with the lowest cost. Arbitrarily break any ties for the lowest cost.
- Allocate as many units as possible to that cell without exceeding the supply or demand. Then cross out that row or column (or both) that is exhausted by this assignment.
- Find the cell with the lowest cost from the remaining cells.
- Repeat steps 2 & 3 until all units have been allocated.

Heuristic Solution – North-West Corner Rule

- Identify the cell which is at the North West corner.
- Allocate as many units as possible to that cell without exceeding the supply or demand. Then cross out that row or column (or both) that is exhausted by this assignment.
- Move to the next cell to the right or below, depending on whether the column or row has been crossed out.
- Repeat steps 2 & 3 until all units have been allocated.

Problem 05
Suggested Solution

Today's Problem: Network Representation

Today's Problem: Transportation Table (Data Setup)

	Shipping Cost per Box	Destinations				
	To Customers at / From Warehouse at	Hanoi (1)	Taipei (2)	Jakarta (3)	Manila (4)	*Warehouse Supply*
Sources	Hong Kong (1)	8	11	39	17	180
	Bangkok (2)	23	40	34	38	130
	Singapore (3)	28	43	14	32	110
	Customer Demand	115	85	125	95	420

Today's Problem:
Heuristic Solution using Lowest (Least) Cost Method

Cost per box

To (Customer at) / ***From (Warehouse at)***	**Hanoi (1)**	**Taipei (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	8	11	39	17	**180**
Bangkok (2)	23	40	34	38	**130**
Singapore (3)	28	43	14	32	**110**
Customer Demand	**115**	**85**	**125**	**95**	**420**

Lowest / Least Cost Method	**Hanoi (1)**	**Taipei (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	115	65			**180**
Bangkok (2)		20	15	95	**130**
Singapore (3)			110		**110**
Customer Demand	**115**	**85**	**125**	**95**	**420**

Total cost = \$8*115 + \$11*65 + \$40*20 + \$34*15 + \$14*110 + \$38*95 = \$8,095

Today's Problem:
Heuristic Solution using North-West Corner Rule

Cost per box

To (Customer at) / *From (Warehouse at)*	**Hanoi (1)**	**Taipei (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	8	11	39	17	**180**
Bangkok (2)	23	40	34	38	**130**
Singapore (3)	28	43	14	32	**110**
Customer Demand	**115**	**85**	**125**	**95**	**420**

NorthWest Corner Rule	**Hanoi (1)**	**Taipei (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	115	65			**180**
Bangkok (2)		20	110		**130**
Singapore (3)			15	95	**110**
Customer Demand	**115**	**85**	**125**	**95**	**420**

Total cost = \$8*115 + \$11*65 + \$40*20 + \$34*110 + \$14*15 + \$32*95 = \$9,425

How about the transportation table is re-arranged? Will the North-West Corner Rule give the same solution?

Today's Problem:
Heuristic Solution using North-West Corner Rule

If the original transportation table is arranged as follows, apply the North-West Corner to check the new distribution plan.

To (Customers at) / *From (Plant at)*	**Taipei (1)**	**Hanoi (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	11	8	39	17	**180**
Bangkok (2)	40	23	34	38	**130**
Singapore (3)	43	28	14	32	**110**
Customer Demand	**85**	**115**	**125**	**95**	**420**

NorthWest Corner Rule	**Taipei (1)**	**Hanoi (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	85	95			**180**
Bangkok (2)		20	110		**130**
Singapore (3)			15	95	**110**
Customer Demand	**85**	**115**	**125**	**95**	**420**

Total cost = $11*85 + $8*95 + $23*20 + $34*110 + $14*15 + $32*95 = $9,145

The solution from the North-West Corner incurs higher cost than the Lowest Cost method. But is Lowest Cost method the **optimal solution**?

Today's Problem: LP Formulation

Decision variables

Let X_{ij} be the number of boxes of the products to be shipped from warehouse *i* to customers at location *j* (i = 1,2,3; j = 1,2,3,4)

(E.g. X_{11} is the number of boxes of the products to be shipped from warehouse at Hong Kong to customers at Hanoi)

Objective function

Minimize Z cost = $8X_{11} + 11X_{12} + 39X_{13} + 17X_{14} + 23X_{21} + 40X_{22} + 34X_{23} + 38X_{24} + 28X_{31} + 43X_{32} + 14X_{33} + 32X_{34}$

Constraints

Subject to:

Warehouse at Hong Kong supply:	$X_{11} + X_{12} + X_{13} + X_{14} = 180$
Warehouse at Bangkok supply:	$X_{21} + X_{22} + X_{23} + X_{24} = 130$
Warehouse at Singapore supply:	$X_{31} + X_{32} + X_{33} + X_{34} = 110$
Customers at Hanoi demand:	$X_{11} + X_{21} + X_{31} = 115$
Customers at Taipei demand:	$X_{12} + X_{22} + X_{32} = 85$
Customers at Jakarta demand:	$X_{13} + X_{23} + X_{33} = 125$
Customers at Manila demand:	$X_{14} + X_{24} + X_{34} = 95$

Non-negativity $X_{ij} \geq 0$ (i = 1,2,3; j = 1,2,3,4)

Integer Solutions Property

Take note that in the LP formulation, we did not include the integer requirement constraints in our formulation.

- As long as all its supplies and demands have integer values, any transportation problem with feasible solutions is guaranteed to have an optimal solution with integer values for all its decision variables.

- Therefore, it is not necessary to add constraints to the model that restrict these variables to only have integer values.

Today's Problem:
Optimal Solution using Excel Solver

Shipment Quantity

Allocated Units

To (Customer at) / ***From (Warehouse at)***	**Hanoi (1)**	**Taipei (2)**	**Jakarta (3)**	**Manila (4)**	***Warehouse Supply***
Hong Kong (1)	0	85	0	95	**180**
Bangkok (2)	115	0	15	0	**130**
Singapore (3)	0	0	110	0	**110**
Customer Demand	**115**	**85**	**125**	**95**	**420**

LP Total cost = $7,245

Compared to North-West Corner Rule ($9,425), difference of $2,180

Compared to Minimum Cost Method ($8,095), difference of $850

Today's Problem:
Unbalanced Supply-Demand Formulation

- If supply at Hong Kong increases from 180 to 220 boxes of the products while supply at the rest of the warehouses remains unchanged
- **Total Supplies (460 boxes of the products) > Total Demands (420 units of the products)**
- **If total supply > total demand, all or some supplies will not be distributed.**

Warehouse at Hong Kong supply: $X_{11} + X_{12} + X_{13} + X_{14} \le 220$

Warehouse at Bangkok supply: $X_{21} + X_{22} + X_{23} + X_{24} \le 130$

Warehouse at Singapore supply: $X_{31} + X_{32} + X_{33} + X_{34} \le 110$

- All demands will be fulfilled. No change to the demands' constraints.

Today’s Problem:
Unbalanced Supply-Demand Formulation

- Demand at Taipei increases from 85 to 100 boxes of the products while demand from other customers remains unchanged.
- **Total Demands (435 boxes of the products) > Total Supplies (420 boxes of the products)**
- **If total demand > total supply, all or some demands will not be fulfilled.**

Customers at Hanoi demand: $X_{11} + X_{21} + X_{31} \quad <= 115$

Customers at Taipei demand: $X_{12} + X_{22} + X_{32} \quad <= 100$

Customers at Jakarta demand: $X_{13} + X_{23} + X_{33} \quad <= 125$

Customers at Manila demand: $X_{14} + X_{24} + X_{34} \quad <= 95$

- All supplies will be depleted. No change to the supplies’ constraints.

Today's Problem: Sensitivity Analysis

- Unit shipping cost of Bangkok warehouse is varied +/- 20%

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B6	Bangkok (2) to Hanoi (1)	115	0	23	6	20
B7	Bangkok (2) to Taipei (2)	0	14	40	1E+30	14
B8	Bangkok (2) to Jakarta (3)	15	0	34	20	20
B9	Bangkok (2) to Manila (4)	0	6	38	1E+30	6

Within the Allowable Increase/Decrease range

Optimal solution does not change, but objective function value changes

Bangkok to	Original Unit Cost	Reduce Cost	20% of Unit Cost	Increase 20%	Revised Shipping Cost	Decrease 20%	Revised Shipping Cost
Hanoi	23	0	4.6	27.6	7,774	18.4	6,716
Taipei	40	14	8	48	No Impact	32	No Impact
Jakarta	34	0	6.8	40.8	7,347	27.2	7,143
Manila	38	6	7.6	45.6	No Impact	30.4	7093

Exceed Allowable Decrease

Both optimal solution and objective function value change

Today's problem:
Impossible Supply Routes Formulation

Example: Suppose that customers at Taipei cannot get the products from the warehouse at Hong Kong.

Possible ways to formulate this:

1. The big M method in LP formulation. Can set the unit cost of shipping from warehouse at Hong Kong to customers at Taipei to be very high (in comparison to the rest of the shipping cost). For example: 99999$/box. How big is big enough? 10000 times, for example.
2. Can remove the corresponding decision variable, representing removal of the link between the supply and the demand points.
3. Can also add in one more constraint which restricts the decision variable to be zero, i.e., $X_{12} = 0$.

Conclusion

- Transportation problems deal with the distribution of goods from several sources of supply to several destinations having the demand.

- The objective is to schedule the shipment in a way to minimize total transportation costs.

- A transportation model can also be applied to other areas including facility location, production-inventory control, personnel assignment and equipment maintenance.

Learning Objectives

At the end of the lesson, students should be able to:

- ✓ Identify a transportation problem that deals with the distribution of goods from multiple supply sources to multiple demand destinations. Set up the transportation table.
- ✓ Develop heuristic solution using North West Corner Rule and Lowest Cost Method.
- ✓ Formulate the transportation problem that involves minimizing the cost of shipping goods from a set of origins to a set of destinations as a linear programming model.
- ✓ Apply Excel Solver to find an optimal shipment allocation to minimize total transportation cost.
- ✓ Compare and contrast the solutions from heuristic methods with that of linear programming method.

Overview of E210 Operation Planning Module