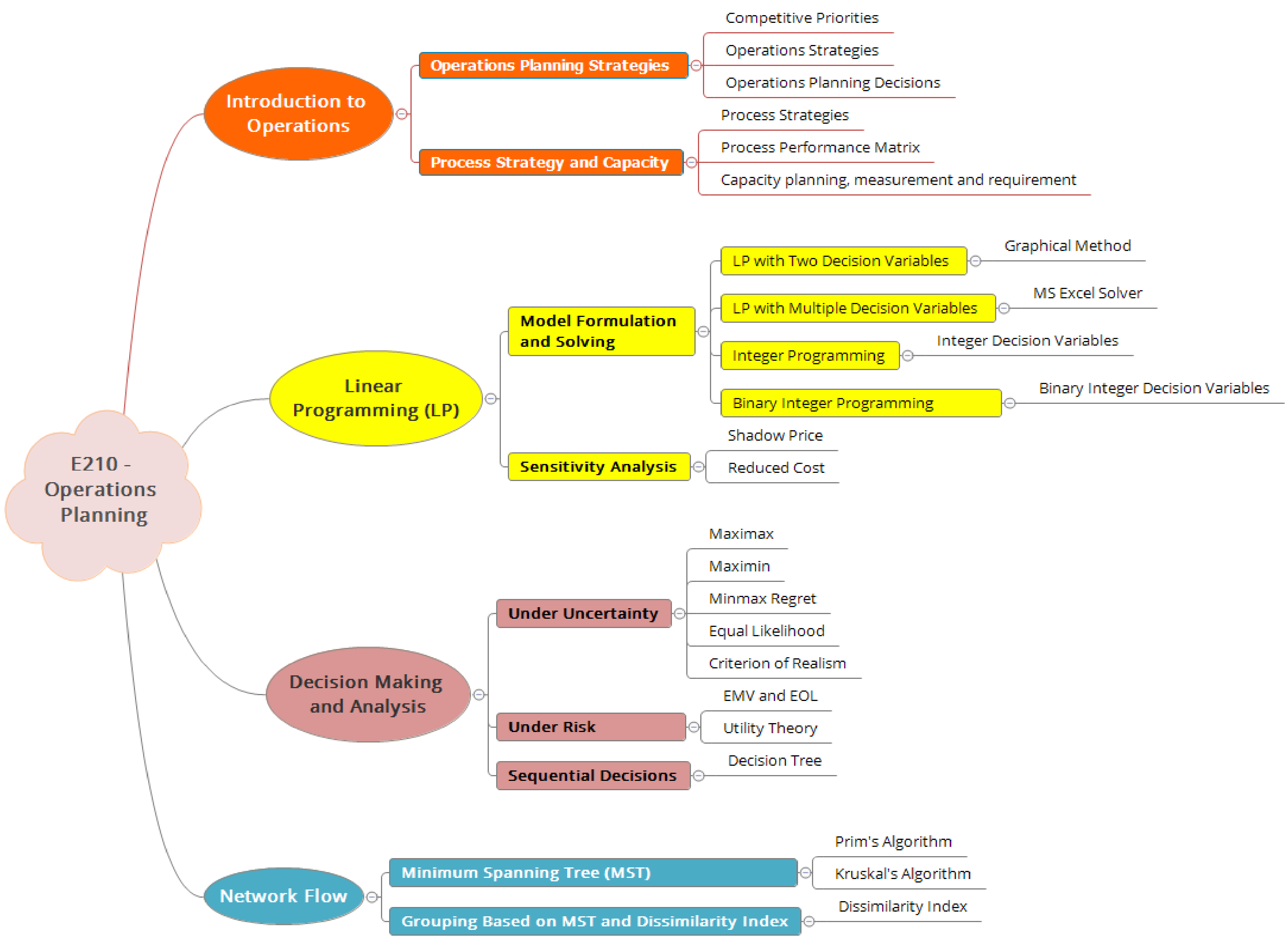
Lesson 03

The Graphical Method

E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree



Scenario

- Company XYZ is specialized in manufacturing of two models of products, Model A and Model B.
- Table below summarizes the amount of machine hours and labour hours needed to produce one unit of the products, with expected profit earned from each of the products sold.

Product	Machine (Hours)	Labor (Hours)	Profit ($)
Model A	0.4	0.9	20
Model B	0.8	0.6	25

- In the coming week, Company XYZ has a maximum of 80 machine hours and 72 labour hours available, respectively.
- Help Company XYZ determine the optimum quantity to make for each model of the products in order to maximize the profit.
- Determine what is limiting the company from making more profit and how the profit will be affected if more resources are available.

Suggested Solution – LP Model Formulation

LP Formulation

Decision Variables

Let X_1 be the number of Model A to produce weekly

Let X_2 be the number of Model B to produce weekly

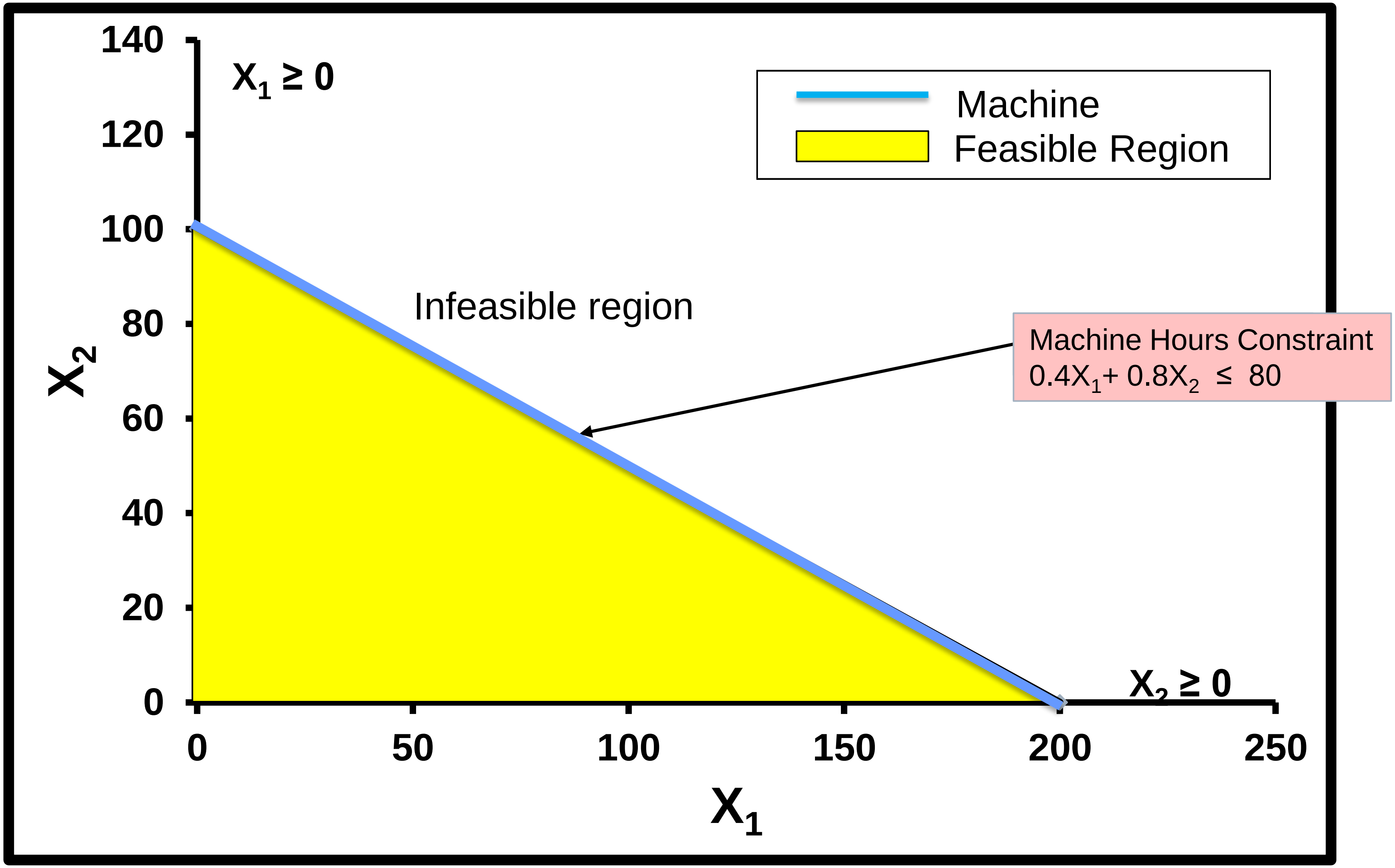
Objective Function

Maximize $Z = 20X_1 + 25X_2$

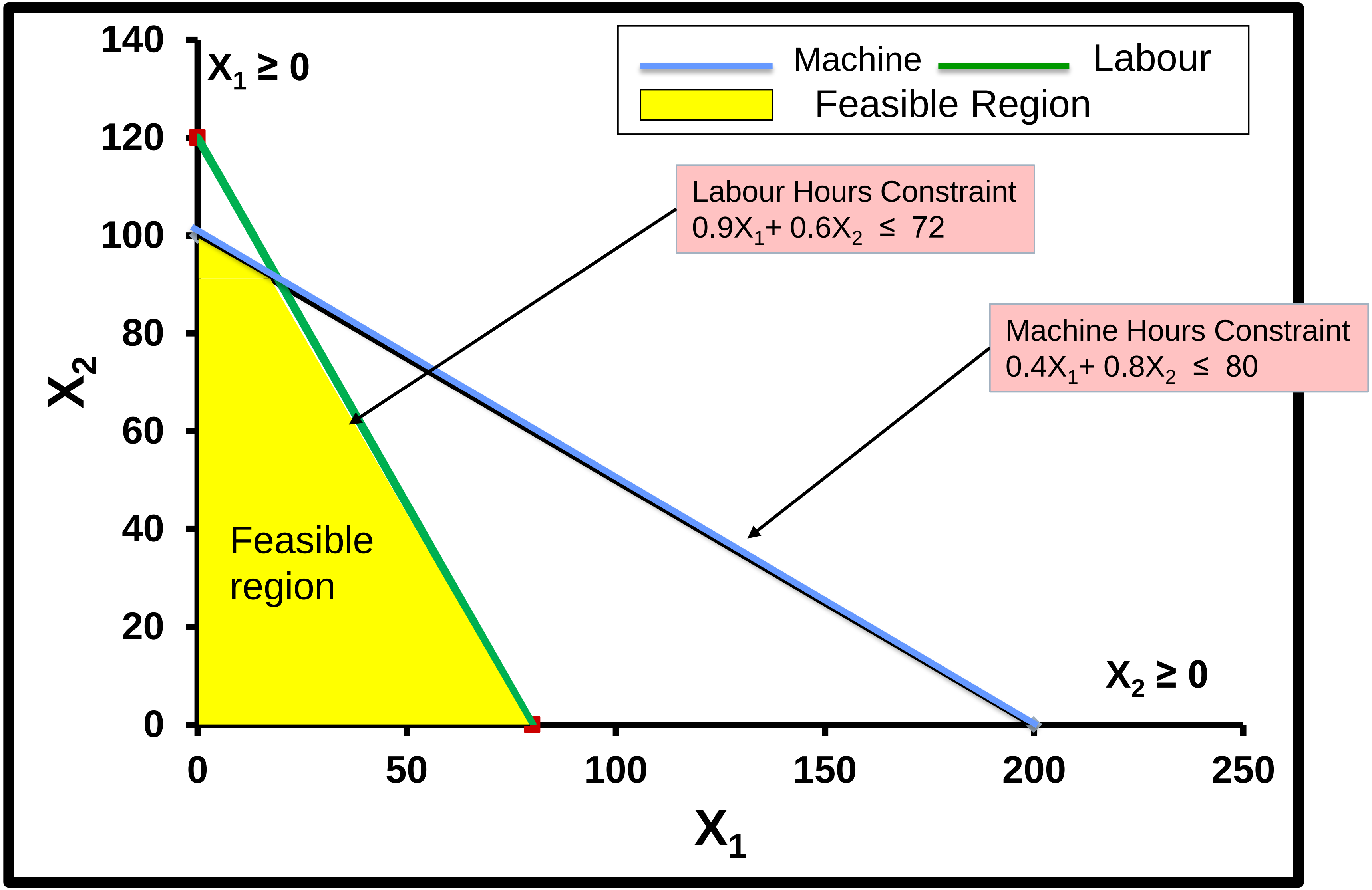
Constraints

$0.4 X_1 + 0.8 X_2$	$\leq$	80	(Machine hours constraint)
$0.9 X_1 + 0.6 X_2$	$\leq$	72	(Labour hours constraint)
X_1	$\geq$	0	(Non-negative X_1)
X_2	$\geq$	0	(Non-negative X_2)

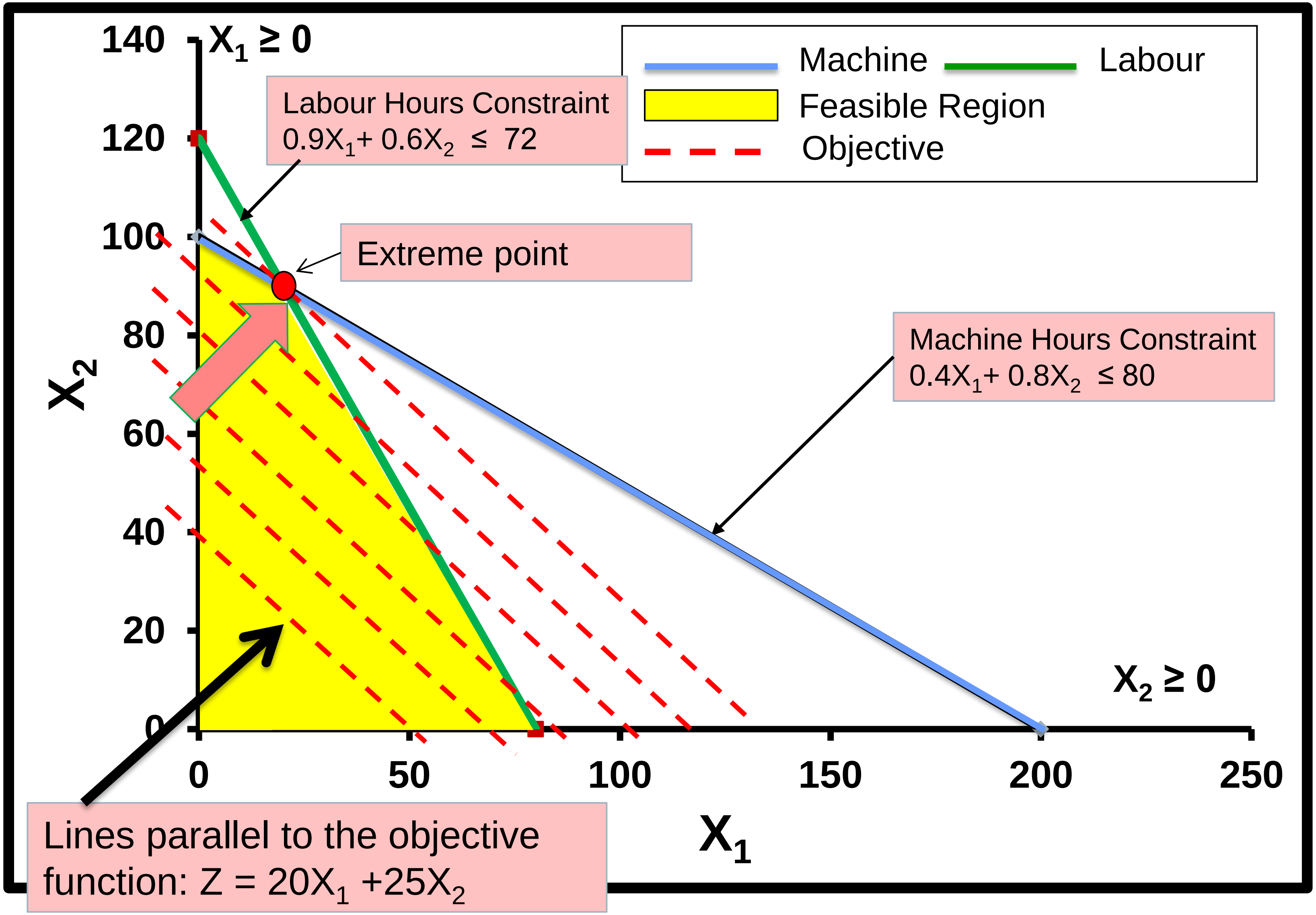
Graphical Solution: Machine Hours Constraint ($0.4X_1 + 0.8X_2 \leq 80$)



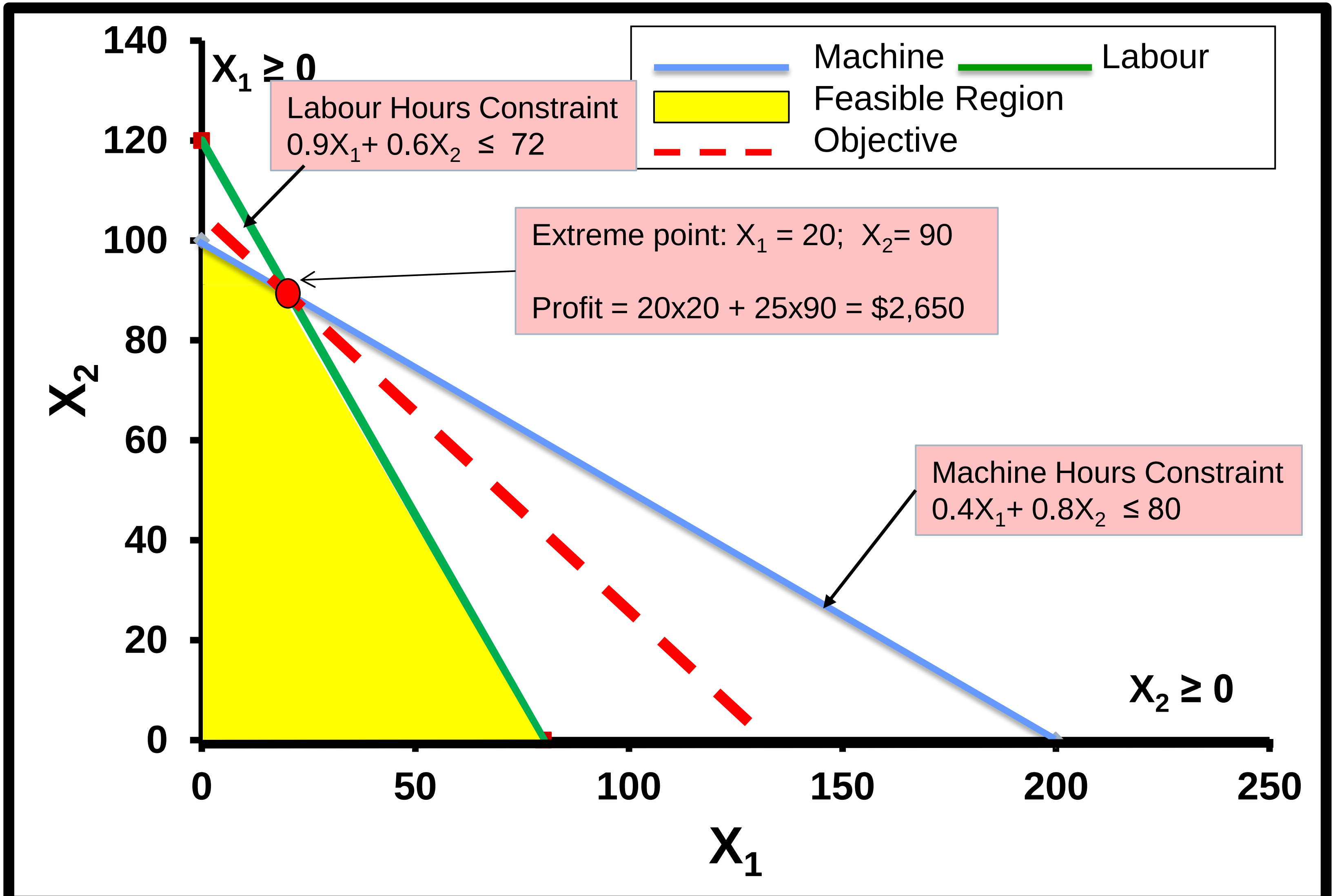
Graphical Solution: Labour Hours Constraint ($0.9X_1 + 0.6X_2 \leq 72$)



Graphical Solution: Possible Solutions for the Objective Function ($Z=20X_1+25X_2$)

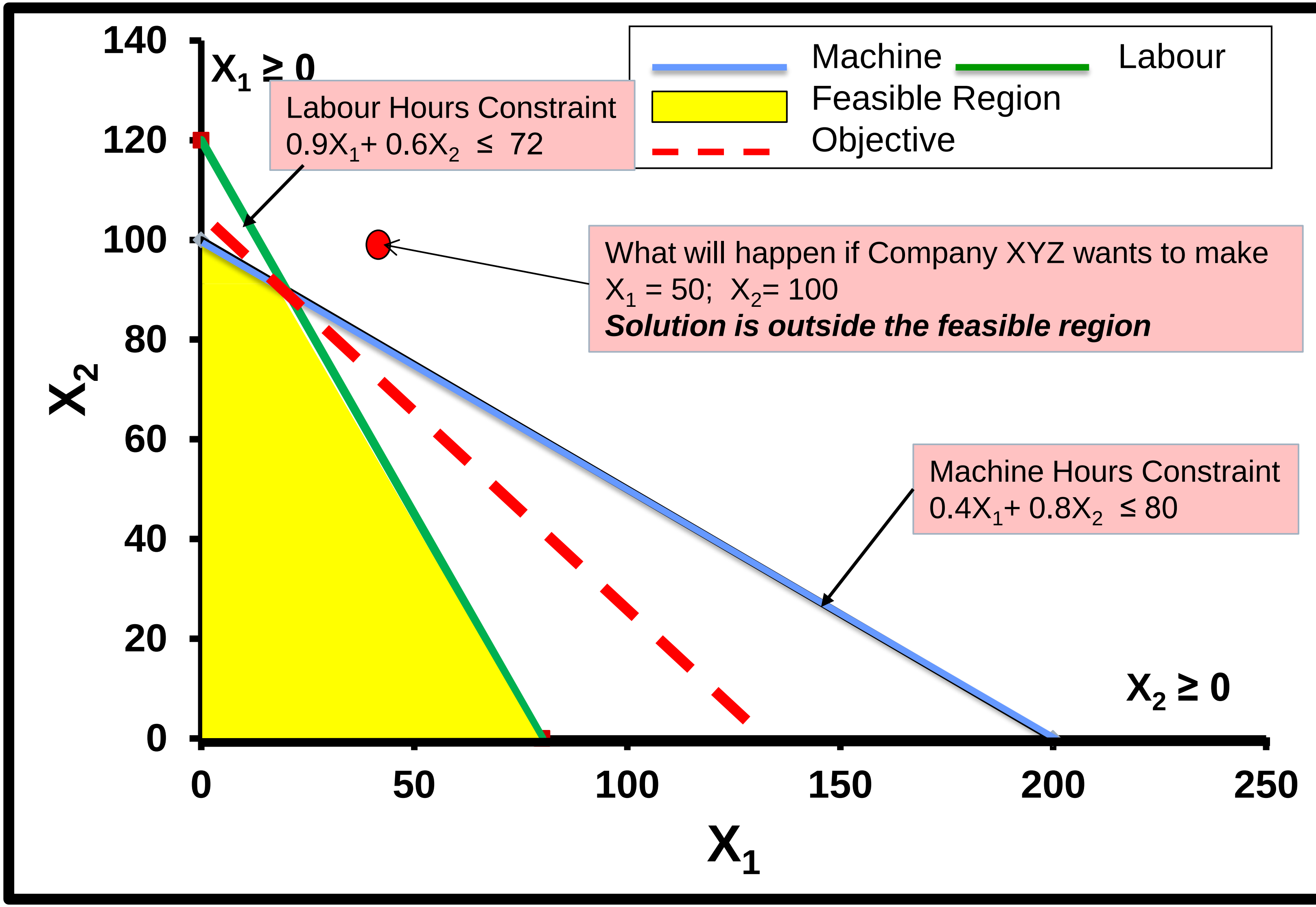


Graphical Solution: Optimized Objective Function



Note: If a <u>unique</u> optimal solution exists, it occurs at one of the extreme points of the feasible region. The objective function value of this optimal solution: $2,650

Graphical Solution: Outside Feasible Region



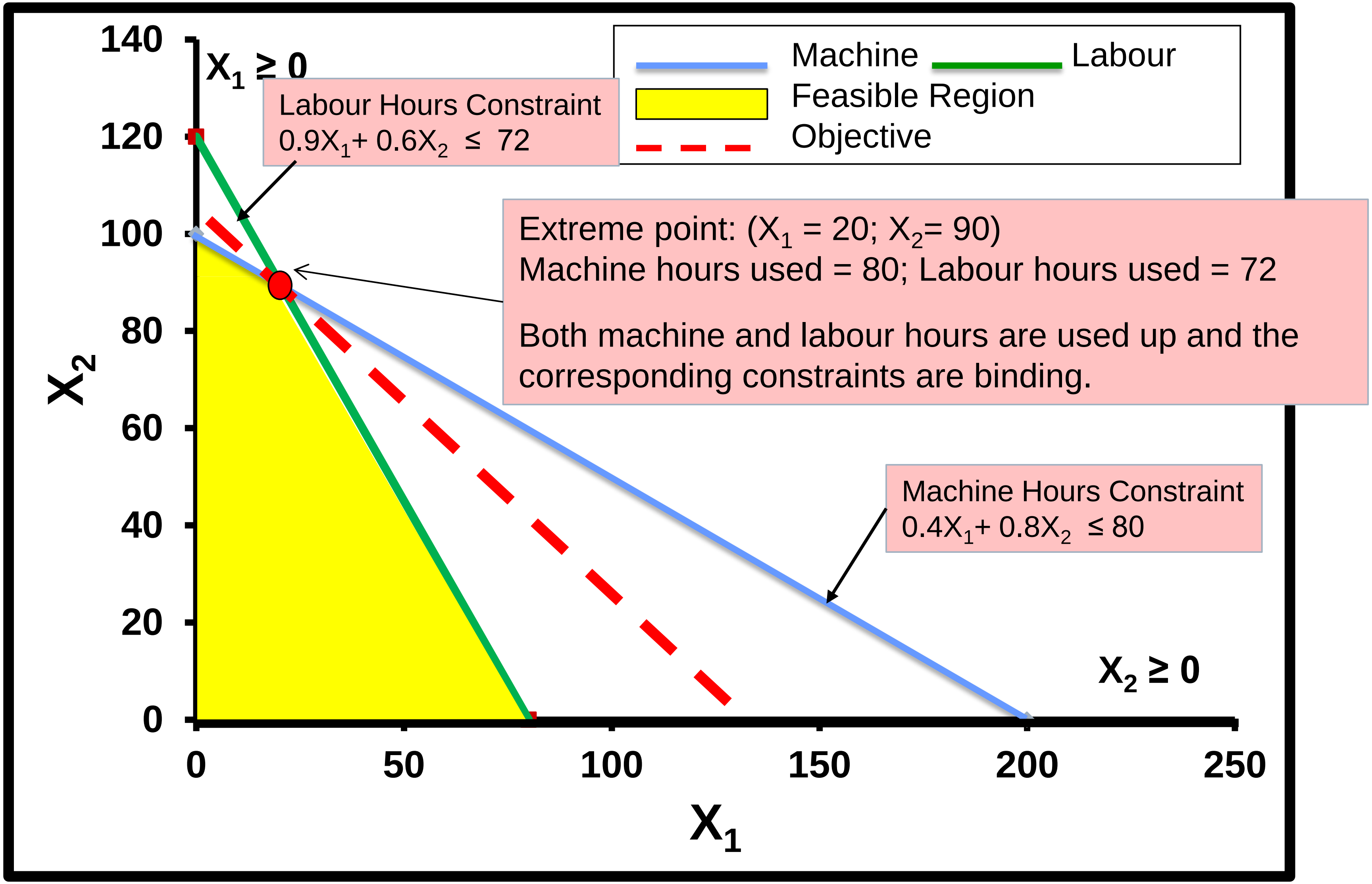
Solutions

- The optimal solution at which profit is maximized is $X_1 = 20$, $X_2 = 90$.
- In other words, we recommend to make 20 units of Model A and 90 units of Model B per week which will yield a maximum profit of $2,650.

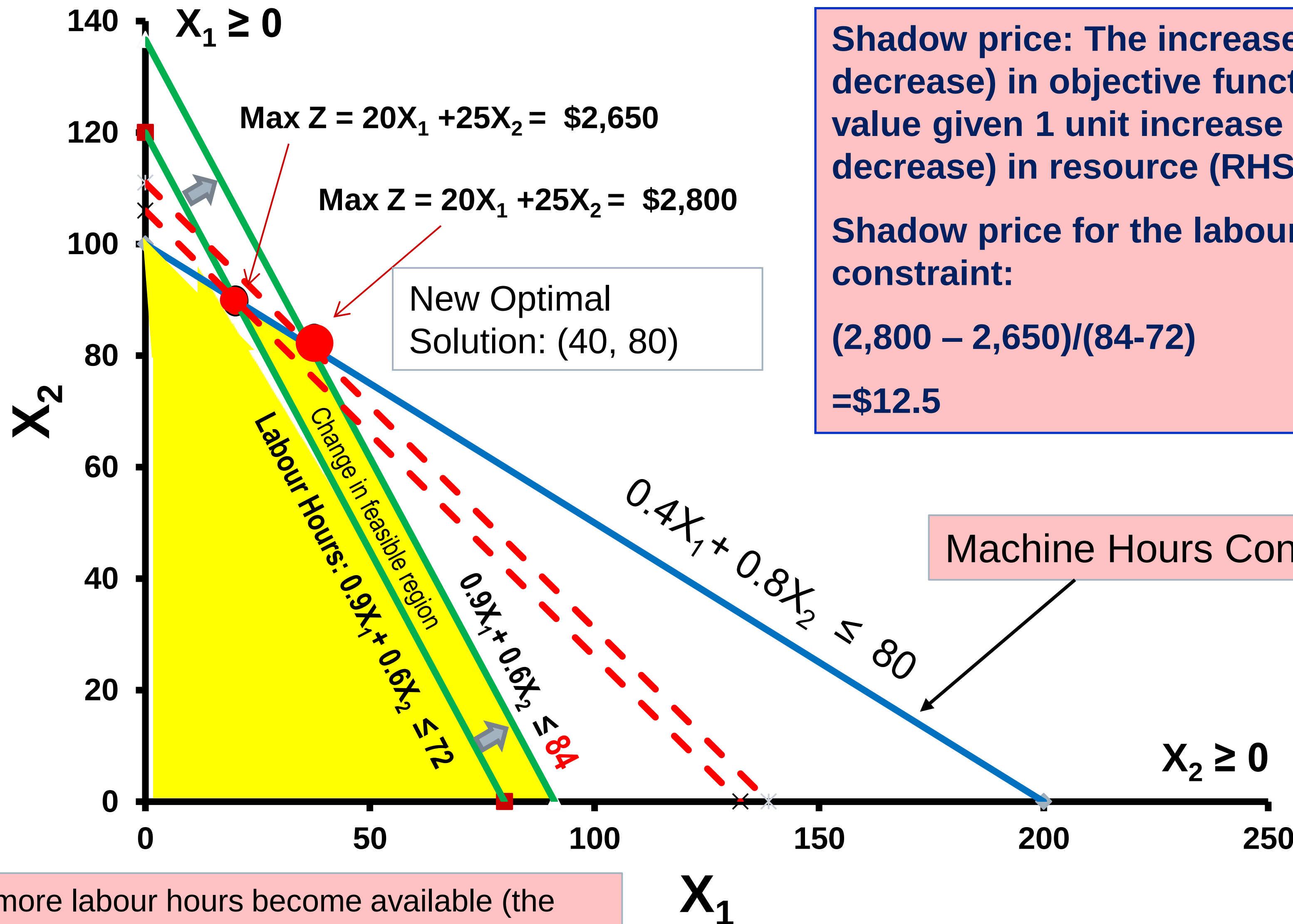
<u>Profit = 20x20 + 25x90 = $2,650</u>

- The assumptions are that the coefficient and right-hand side values in the constraints are constant, and that the linearity assumption holds for all values of the decision variables.

Sensitivity Analysis: Binding/Nonbinding Constraints



Sensitivity Analysis of Right-hand Side Value of Constraints



New Optimal Solution: (40, 80)

Shadow price: The increase (or decrease) in objective function value given 1 unit increase (or decrease) in resource (RHS value)

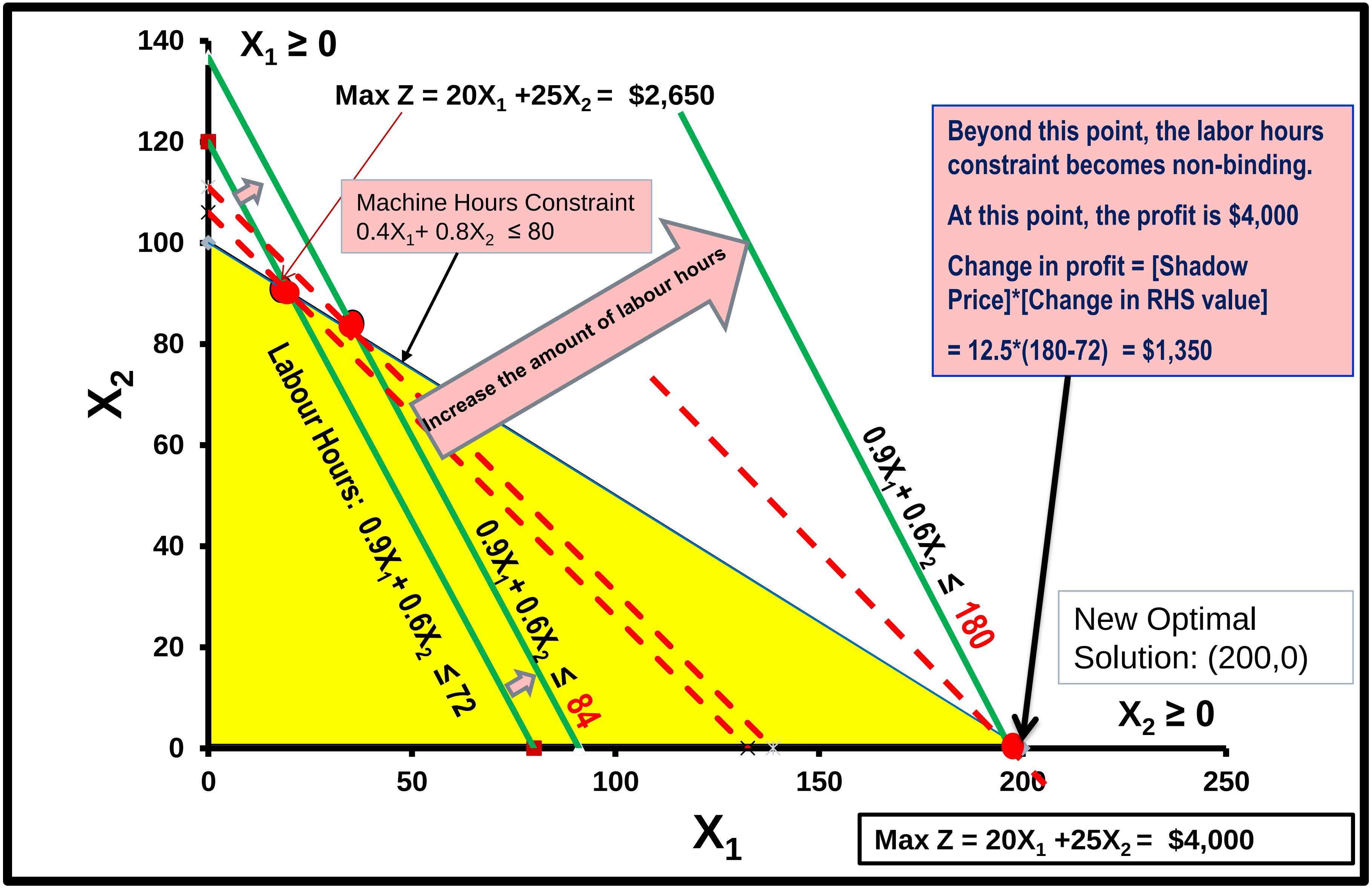
Shadow price for the labour hours constraint:

(2,800 – 2,650)/(84-72)

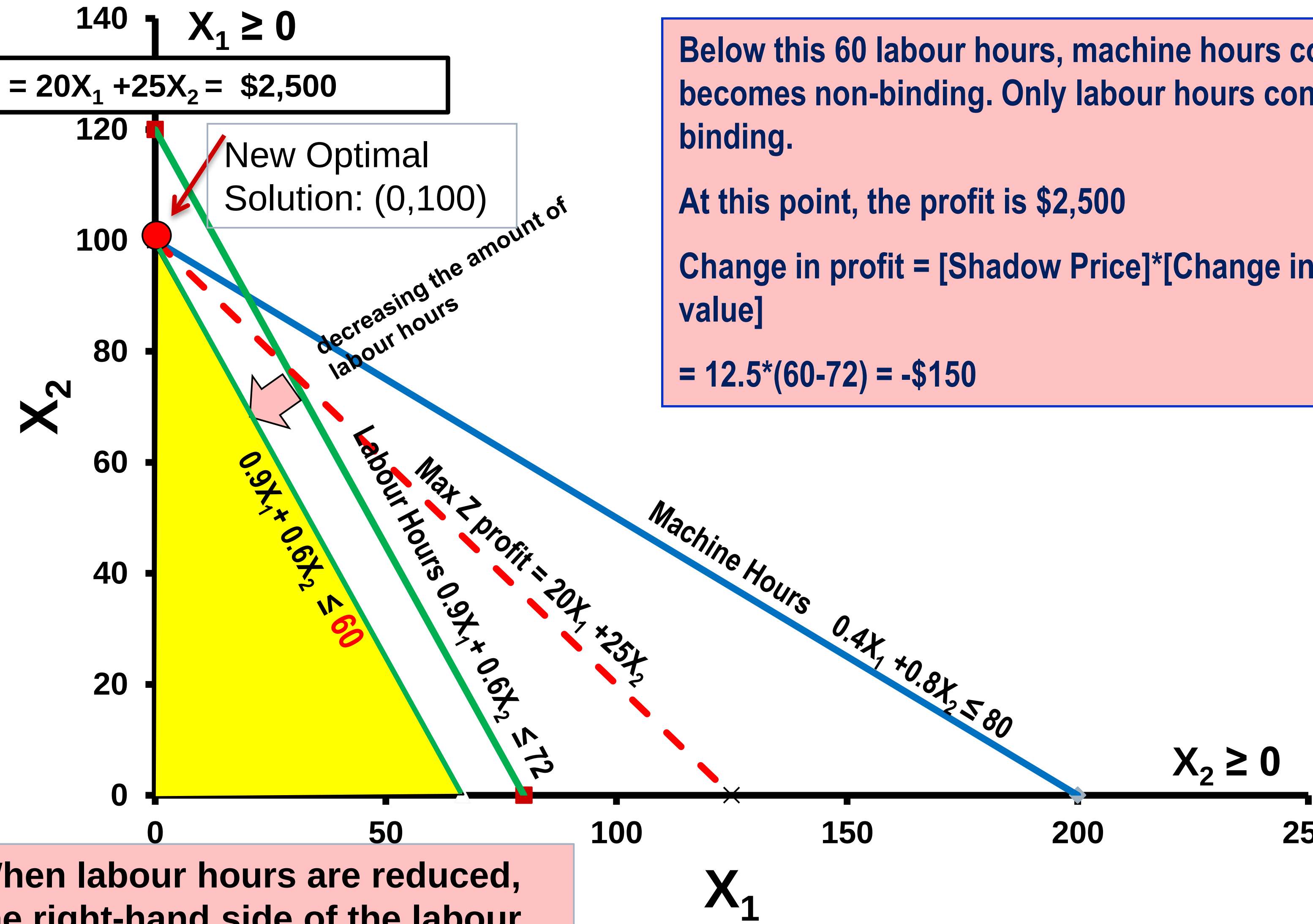
=$12.5

When more labour hours become available (the labour hours constraint is relaxed), the right-hand side of the labour hours constraint increases.

Sensitivity Analysis of Right-hand Side Value of Constraints



Sensitivity Analysis of Right-hand Side Value of Constraints



Below this 60 labour hours, machine hours constraint becomes non-binding. Only labour hours constraint is binding.

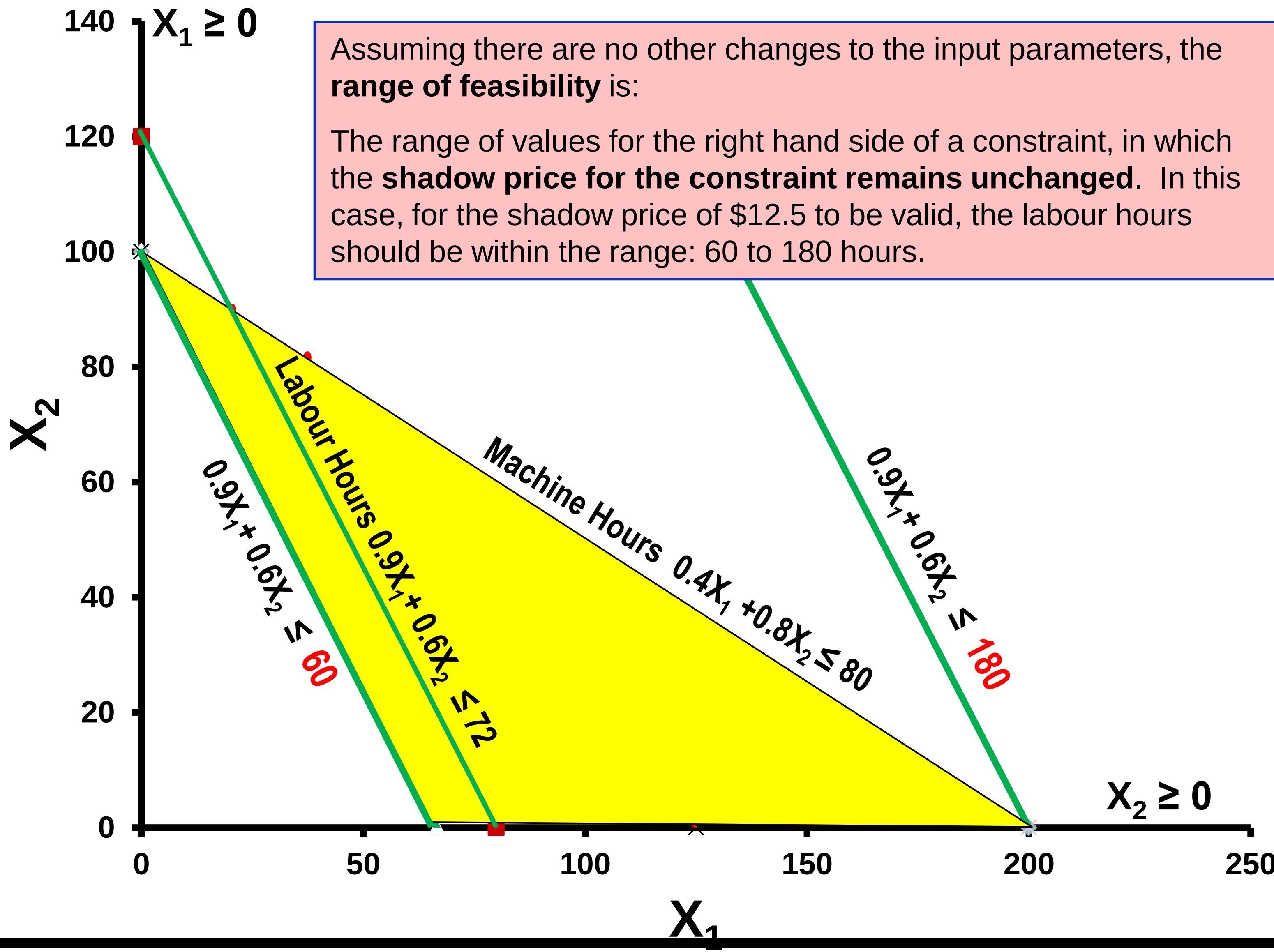
At this point, the profit is $2,500

Change in profit = [Shadow Price]*[Change in RHS value]

= 12.5*(60-72) = -$150

When labour hours are reduced, the right-hand side of the labour hours constraint decreases.

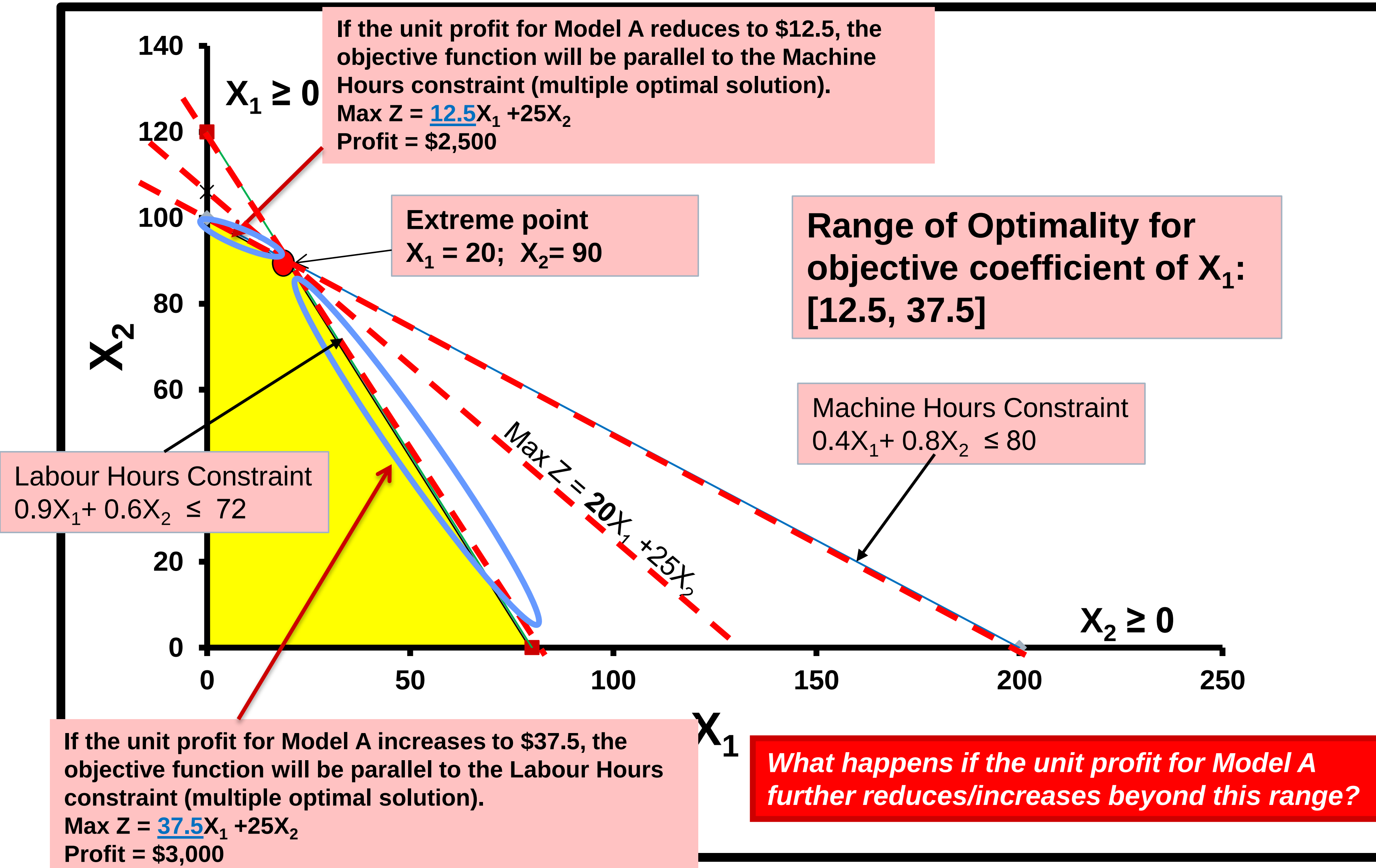
Range of Feasibility – Labour Hours



Assuming there are no other changes to the input parameters, the **range of feasibility** is:

The range of values for the right hand side of a constraint, in which the **shadow price for the constraint remains unchanged**. In this case, for the shadow price of \$12.5 to be valid, the labour hours should be within the range: 60 to 180 hours.

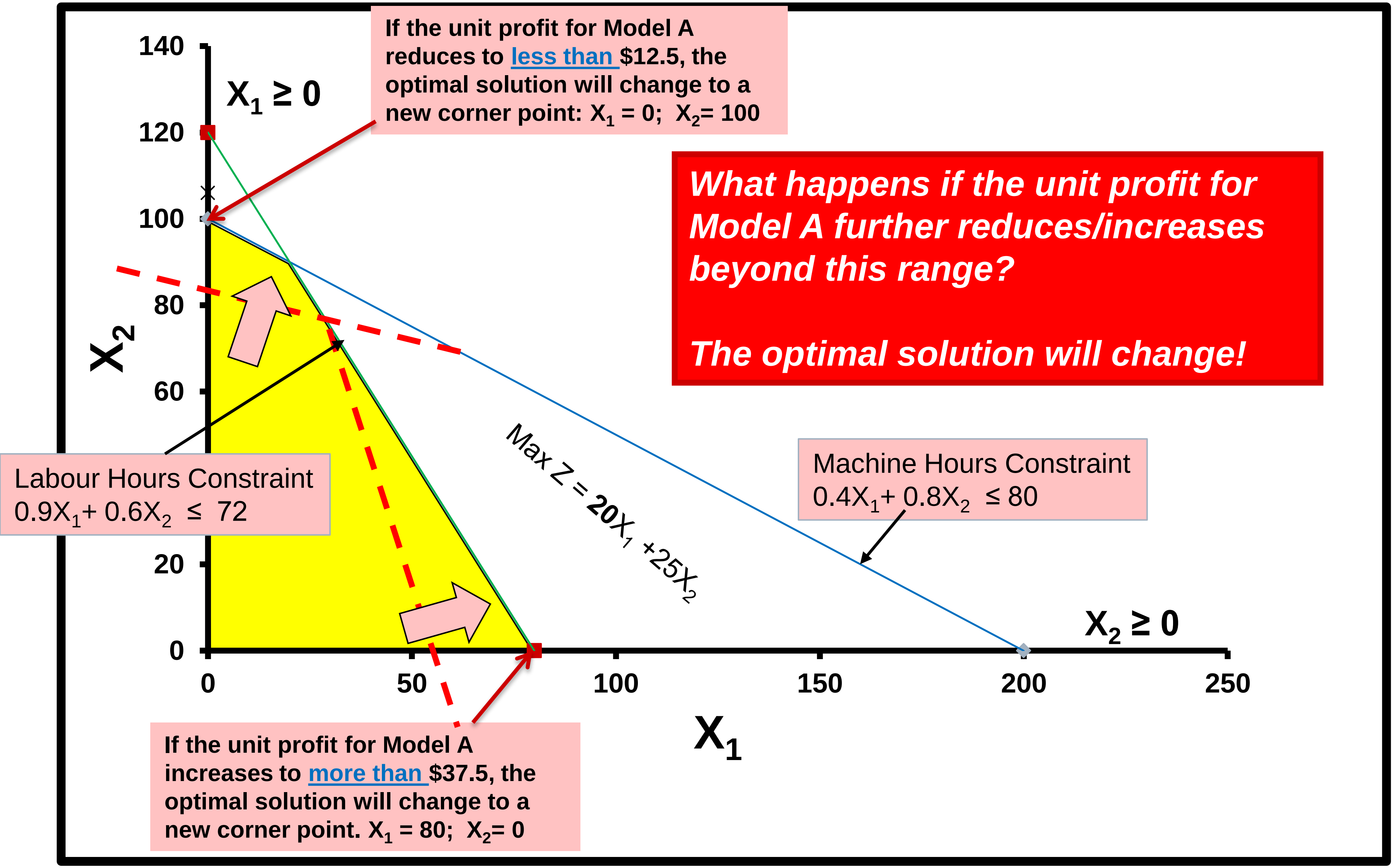
Sensitivity Analysis of Objective Function Coefficients - Range of Optimality



What happens if the unit profit for Model A further reduces/increases beyond this range?

Sensitivity Analysis of Objective Function Coefficients

- What happens if the unit profit further reduces/increases?



Learning Objectives

At the end of the lesson, students should be able to:

- Identify problems that linear programming can handle.
- Identify the assumptions and elements of a linear programming problem. (Objective function, Decision variables and Constraints).
- Formulate and solve Linear Programming (LP) problems with two decision variables using graphical method.
- Perform sensitivity analysis on the objective function coefficients and right-hand side values of the constraints using graphical method.

Overview of E210 Operation Planning Module