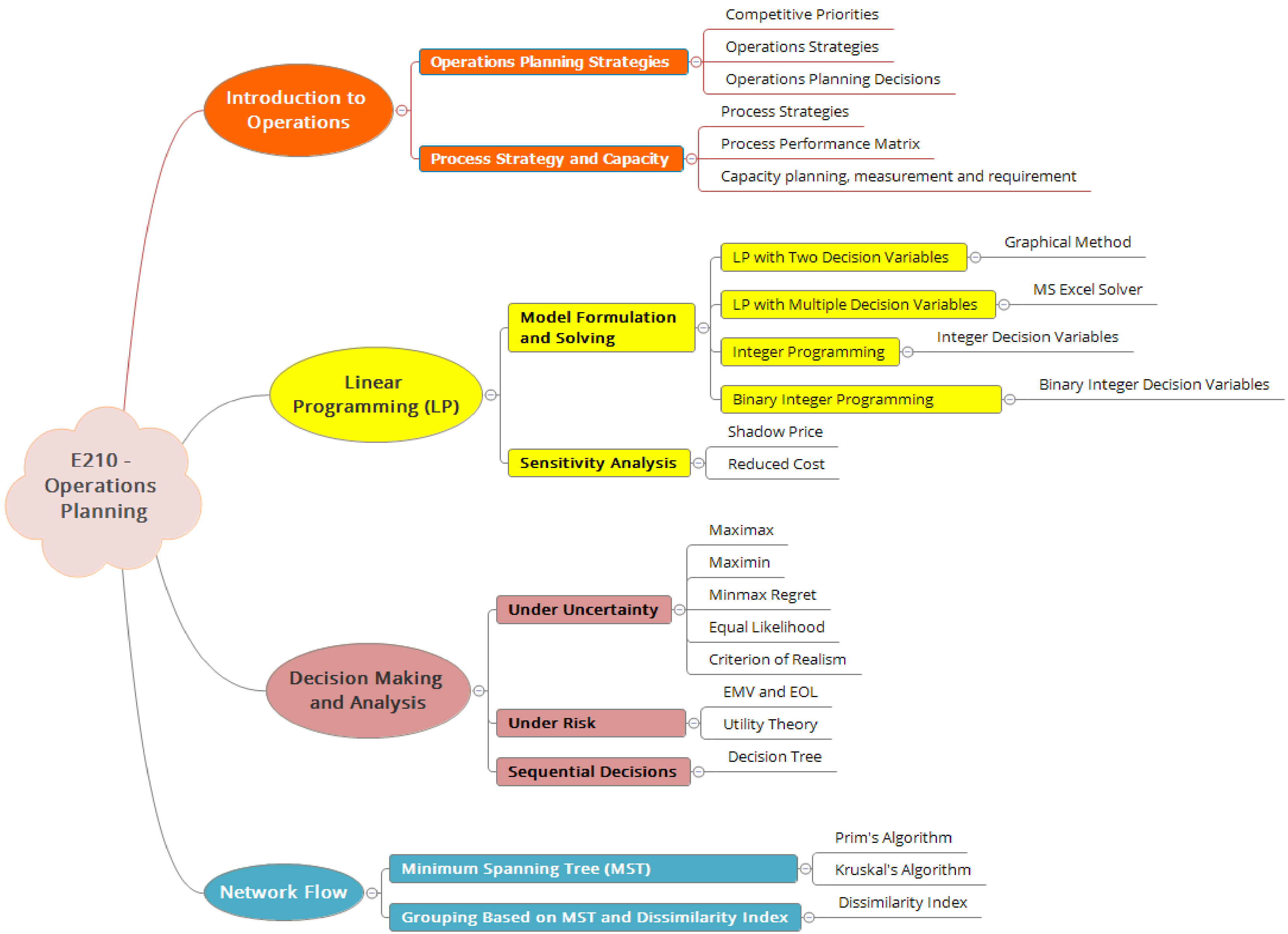
Problem 07

Crane Operators Scheduling– Part II

E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree



Staff Assignment Problem

- The staff assignment problem considers how to assign staff to shifts according to the preferences of staff and requirements of the shifts.

- Shift requirements
  - The number of staff required in each shift, the required staff seniority level and skills, are <u>hard rules</u> which cannot be ignored.
  - In the model, they are translated into ***constraints*** to be met.

- Staff preferences
  - Staff preferences are <u>soft rules</u> which should be met as much as possible to keep staff spirits up.
  - In the model, they are translated into ***objective*** to be optimized.

Manpower Scheduling Problem

- The staff assignment problem is a typical Manpower Scheduling Problem commonly seen in many organizations.
- The problem is to optimally match available labour resource to the needs for labour of an organization considering all applicable constraints.
- Other applications of the Manpower Scheduling Problem include:
  - Nurses in hospitals
  - Operators in telephone companies
  - Aircrew at airline stations
  - Patrol persons in police departments
  - Workforce at fast food restaurants, and
  - Labour in manufacturing systems.

Common Constraints in the Manpower Scheduling Problem

- The Manpower Scheduling Problem is a complex problem due to its many constraints to be considered.

- Some constraints are:
  - **Workforce requirement**: determines the needed number of labour for each day or the planning period.
  - **Conflict**: an employee cannot be assigned to
    - more than one task in the same shift.
    - two shifts that are in conflict with each other.
  - **Ability**: each employee has his qualifications and personal skills that enable him to do or not to do certain types of tasks.
  - **Sequence of shifts**: some sequences of shifts are not allowed: for example, working in two shifts, the rest period between them is less than certain limit (for example, 16 hours).
  - **Work and rest periods**: common rest period is two days per week that may be consecutive or not.

Shift Personnel Scheduling

- In shift personnel scheduling, it is often important to ensure balanced workload and fairness, in terms of the number of least preferred shift (e.g. night shift) that a staff has to work, and the rest days of the week.

- A typical scheduling problem might be to plan a schedule for a repeatable period of time (e.g. an entire month), taking into account the hard constraints (shift hours and rest period requirements) and soft constraints (equitable workload and personal preferences)

- The LP model for such a scheduling problem would contain many decision variables as each decision variable corresponds to one possible assignment of a personnel to a particular shift period. A one-week scheduling model may contain 7 days x 3 shifts = 21 decision variables for one employee!

Stages of the Manpower Scheduling Problem

- Determine the quantity of work to be done and labour skill levels required.

- Determine the staffing required to do the work for each time period in terms of the number and type of labour needed to meet the demand throughout a planning cycle. (P06 addresses a Manpower Scheduling Problem at this stage.)

- Develop a workforce schedule. The schedule should assign individual staff to meet requirements in all work periods while accounting for employee constraints. (The staff assignment problem today is at this stage.)

Problem 07
Suggested Solution

Today's Problem: Crane Operator Assignment

Quantify the information needed in the formulation

- Before we can mathematically formulate the problem, all information should be in a **quantitative form**.

Operator	Shift Preference			Seniority Level	Skills for Crane Lifting Operations		
	Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)		Single Lift (S)	Twin Lift (T)	Multiple Lift (M)
Alan	2	3	1	0	0	0	1
Benny	2	3	1	0	0	1	1
Calvin	1	3	2	1	1	0	0
David	3	1	2	1	1	1	1
Eric	2	3	1	1	1	1	0
Freddy	1	3	2	1	0	1	0
Gary	1	2	3	0	1	0	0
Hayden	1	2	3	0	1	0	1
Ivan	2	1	3	1	1	1	0
Jacky	3	1	2	0	1	0	1
Kellan	3	1	2	0	1	0	0
Lucas	2	1	3	1	1	0	0
Martin	2	3	1	1	1	1	1
Nolan	1	2	3	1	0	1	0
Orlando	1	2	3	1	1	0	0
Parker	3	2	1	1	1	1	0
Quinton	2	3	1	0	0	0	1

Preference:
1: Most Preferred; 2: Neutral
3: Least Preferred

Seniority:
1: Senior
0: Not senior

Speciality skills:
1: Equipped
0: Not equipped

Binary Integer Programming Model Formulation

- **Decision Variables**:

Let X_{ij} represent the decision whether operator *i* (*i*=A, B, …, Q) is to be assigned to shift *j* (*j*=1, 2, 3). In total there are 51 decision variables.

X_{ij} is a **binary variable**: $$X_{ij} = \begin{cases} 1 \text{ operater } i \text{ is assigned to shift } j, \\ 0 \text{ otherwise.} \end{cases}$$

This special case of Integer Programming where decision variables are required to be 0 or 1 is called Binary Integer Programming or 0 - 1 Integer Programming.

- **Objective Function:**
  - The objective is to minimize the total preference score.
  - Minimize total preference score, Z =

$$\boxed{2X_{A1}+3X_{A2}+X_{A3}+2X_{B1}+3X_{B2}+X_{B3}+\ldots+2X_{Q1}+3X_{Q2}+X_{Q3}}$$

A sum product of decision variables and the preference score

= **SUMPRODUCT(X_{ij} ,P_{ij})** where P_{ij} is the preference score of operator i if assigned to shift j

Construct Objective Function in Excel

SUM | =SUMPRODUCT(C3:E19,F3:H19)

	B	C	D	E	F	G	H
1	Operator	Decision Variables			Shift Preference		
2		Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)	Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)
3	Alan				2	3	1
4	Benny				2	3	1
5	Calvin				1	3	2
6	David				3	1	2
7	Eric				2	3	1
8	Freddy				1	3	2
9	Gary				1	2	3
10	Hayden				1	2	3
11	Ivan				2	1	3
12	Jacky				3	1	2
13	Kellan				3	1	2
14	Lucas				2	1	3
15	Martin				2	3	1
16	Nolan				1	2	3
17	Orlando				1	2	3
18	Parker				3	2	1
19	Quinton				2	3	1
20							
21	Objective: Minimize total preference score			=SUMPRODUCT(C3:E19,F3:H19)			

SUMPRODUCT(array1, [array2], [array3], [array4], ...)

Sheet1 | P07 Solution

A sum product of decision variables and the preference score

Binary Integer Programming Model Formulation

- **Constraints**:

➢ **Number of operators required:** one constraint for each shift (Total = 3 constraints). For example,

7 operators are needed in Shift (0700-1500): $\boxed{X_{A1}+X_{B1}+X_{C1}+\ldots+X_{Q1} = 7}$

➢ **Assignment:** Each operator can only be assigned to one shift. One constraint for each operator. (Total = 17 constraints). For example:

Operator Alan: $\boxed{X_{A1}+X_{A2}+X_{A3} = 1}$

➢ **Seniority level:** one constraint for each shift. (Total = 3 constraints)

E.g. 4 Senior operators are required in Shift (0700-1500):

$$\boxed{X_{C1}+ X_{D1}+ X_{E1}+ X_{F1}+ X_{I1}+ X_{L1}+ X_{M1}+ X_{N1}+X_{O1}+ X_{P1} \geq 4}$$

Binary Integer Programming Model Formulation

- **Other Constraints**:

- **Specialty skills:** one constraint for each skill and each shift (Total = 3 x 3 = 9 constraints)

  E.g. At least 2 operators equipped with Twin Lift skills in Shift (1500-2300):

$$\boxed{X_{B2} + X_{D2} + X_{E2} + X_{F2} + X_{I2} + X_{M2} + X_{N2} + X_{P2} \geq 2}$$

- **Two operators (Gary and Orlando) must be in the same shift:** one constraint for each operator (Total = 3 constraints). For example:

  Same assignment for Shift (2300-0700): $\boxed{X_{G3} - X_{O3} = 0}$

- **Three operators (Eric, Lucas and Parker) must be in different shifts:** one constraint for each shift (Total = 3 constraints). For example:

  Only 1 of the 3 operators can be in Shift (1500-2300): $\boxed{X_{E2} + X_{L2} + X_{P2} = 1}$

- **All** X_{ij} **are binary**

Binary Integer Programming Model Formulation

Your practice:

- At least 5 operators equipped with Single Lift skills in Shift (0700-1500).
- At least 1 operator equipped with Multiple Lift skills in Shift (2300-0700).

Binary Integer Programming Model Formulation

Solutions:

- At least 5 operators equipped with Single Lift skills in Shift (0700-1500).

$$X_{C1} + X_{D1} + X_{E1} + X_{G1} + X_{H1} + X_{I1} + X_{J1} + X_{K1} + X_{L1} + X_{M1} + X_{O1} + X_{P1} >= 5$$

- At least 1 operator equipped with Multiple Lift skills in Shift (2300-0700).

$$X_{A3} + X_{B3}\ X_{D3} + X_{H3} + X_{J3} + X_{M3} + X_{Q3} >= 1$$

Assignment Restriction Constraints in Excel

Operator	Decision Variables		
	Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)
Alan			
Benny			
Calvin			
David			
Eric			
Freddy			
Gary			
Hayden			
Ivan			
Jacky			
Kellan			
Lucas			
Martin			
Nolan			
Orlando			
Parker			
Quinton			

Operator	Assignment to one shift only		
		Sign	Right-hand side
Alan	=SUM(C3:E3)	=	1
Benny	SUM(number1, [number2], ...)		1
Calvin	0	=	1
David	0	=	1
Eric	0	=	1
Freddy	0	=	1
Gary	0	=	1
Hayden	0	=	1
Ivan	0	=	1
Jacky	0	=	1
Kellan	0	=	1
Lucas	0	=	1
Martin	0	=	1
Nolan	0	=	1
Orlando	0	=	1
Parker	0	=	1
Quinton	0	=	1

Assignment restriction – one operator to one shift: 17 constraints

Solver Parameters

Set Objective: $F$21

To: Max Min Value Of: 0

By Changing Variable Cells:
$C$3:$E$19

Subject to the Constraints:
$C$3:$E$19 = binary
$D$24:$F$24 = $D$26:$F$26
$D$28:$F$28 >= $D$30:$F$30

Add

Set decision variables to be binary values (0 or 1)

Shift Requirement Constraints in Excel

	Shift	Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)
No. of Operators Required	Left-hand side	0	0	0
	Sign	=	=	=
	Right-hand side	7	6	4
No. of Senior Operators Required	Left-hand side	0	0	0
	Sign	>=	>=	>=
	Right-hand side	4	3	2
No. of Operators Proficient in Single Lift Skills Required	Left-hand side	0	0	0
	Sign	>=	>=	>=
	Right-hand side	5	4	3
No. of Operators Proficient in Twin Lift skills Required	Left-hand side	0	0	0
	Sign	>=	>=	>=
	Right-hand side	4	2	2

No. of Operators Proficient in Multiple Lift skills Required	Left-hand side	0	0	0
	Sign	>=	>=	>=
	Right-hand side	3	2	1
Gary and Orlando must be in the same shift	Left-hand side	0	0	0
	Sign	=	=	=
	Right-hand side	0	0	0
Eric, Lucas and Parker each must be in a different shift	Left-hand side	0	0	0
	Sign	=	=	=
	Right-hand side	1	1	1

7 types of shift requirements: Total = 7x3 = 21 constraints

Excel Solution in Summary

Operator	Decision Variables		
	Shift (0700-1500)	Shift (1500-2300)	Shift (2300-0700)
Alan	0	1	0
Benny	1	0	0
Calvin	0	0	1
David	0	0	1
Eric	1	0	0
Freddy	1	0	0
Gary	1	0	0
Hayden	1	0	0
Ivan	0	1	0
Jacky	0	1	0
Kellan	0	1	0
Lucas	0	1	0
Martin	1	0	0
Nolan	0	1	0
Orlando	1	0	0
Parker	0	0	1
Quinton	0	0	1

Shift (0700-1500):
- Benny, Eric, Freddy, Gary, Hayden, Martin and Orlando.
- 4 senior operators
- 5 (S), 4 (T), and 3 (M)

Shift (1500-2300):
- Allan, Ivan, Jacky, Kellan, Lucas and Nolan.
- 3 senior operators
- 4 (S), 2 (T), 2 (M)

Shift (2300-0700):
- Calvin, David, Parker and Quinton.
- 3 senior operators
- 3 (S), 2 (T), 2 (M)

Total Preference Score = 25

Conclusion

- The operator assignment problem considers how to assign crane operators to shifts while satisfying various shift requirements.
- It is a typical Manpower Scheduling Problem which is to optimally match available resource to the labour requirement of an organization considering all applicable constraints.
- The Manpower Scheduling Problem can be very complex as the number of decision variables and the number of applicable constraints are generally very large.
- Binary Integer Programming is one method that can be applied to formulate and solve the Manpower Scheduling Problem.

Learning Objectives

At the end of the lesson, students should be able to:

- Solve a typical manpower scheduling problem with multiple decision variables in a service company
- Differentiate the type of decision variables as Binary Integers.
- Identify objectives of manpower scheduling – meet service level, minimize cost and/or workload balancing.
- Formulate and relate constraints to optimally match available resource to the needs for labor of an organization and reflect conditions to be met in real life working environment.

Overview of E210 Operation Planning Module