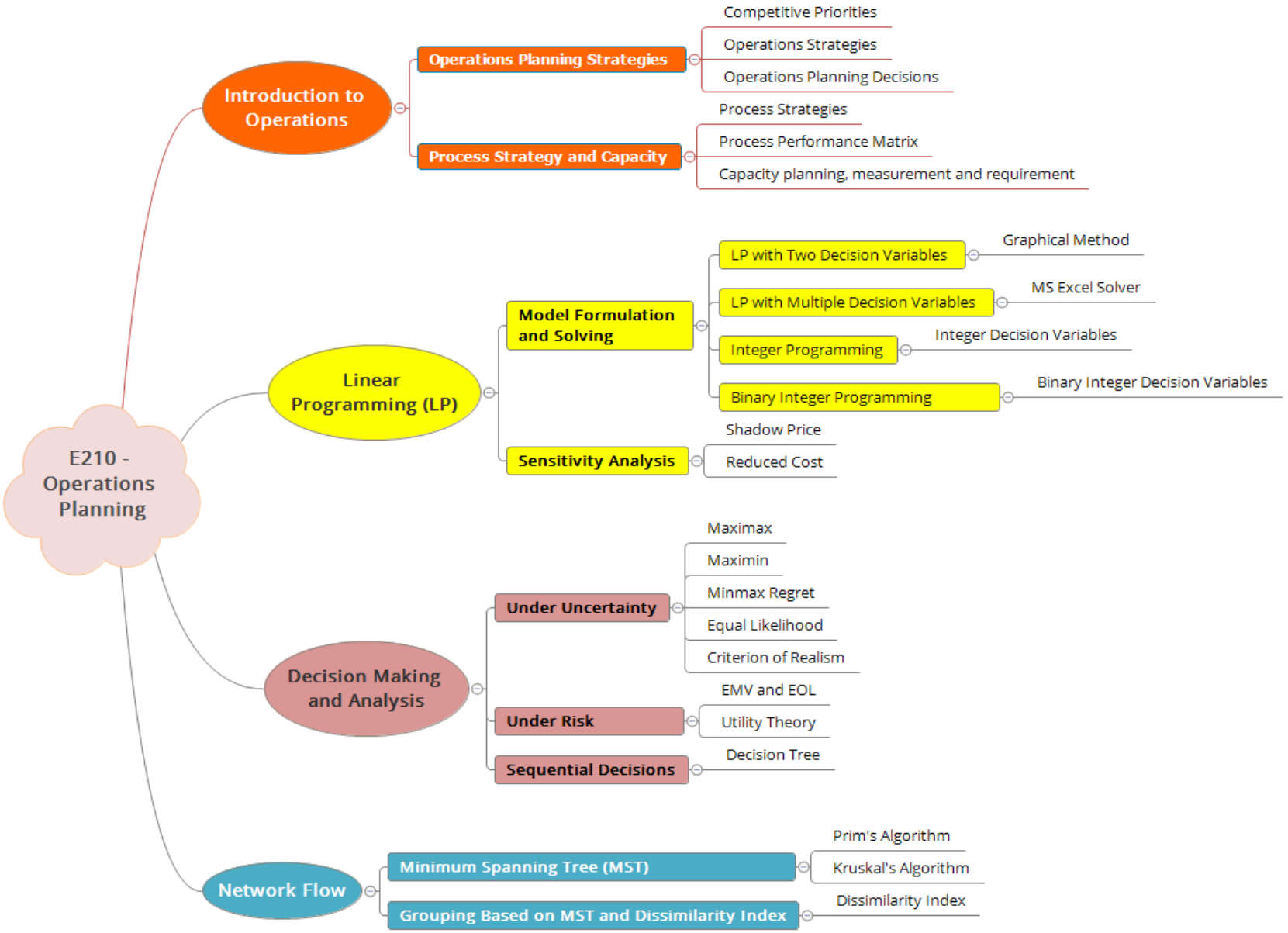
Lesson 09

The Utility Theory

E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree



Scenario – Investment Plans

- Mrs. Lim works as a sales director in a retail outlet of a large chain store. She recently received a special year-end bonus and was trying to find an investment plan for it.
- After some initial analysis, Mrs. Lim found 3 choices she can invest her bonus in. The following table summarizes the three investment plans and the respective payoffs per year under different market conditions.

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

- Mrs. Lim is discussing the investment plans with her family.
- Her son, Alan, advises her to choose the High-risk Stock as he is willing to try his luck and is obviously drawn to the possibility of high payoff.

- Her daughter, Amy, recommends the Savings Account as she always tries to play safe and avoid losses whenever possible.
- Mr. Lim, on the other hand, advises her to invest in the low-risk Stock because he neither agrees with his son's optimism nor his daughter's pessimism.
- Is there a systematic method for her to decide which investment plan to go for, in view of her risk attitude?

Suggested Soution

- **Decision Maker**: Mrs. Lim
- **Alternatives***: Investments*
  - *High-risk Stock*
  - *Low-risk Stock*
  - *Savings Account*
- **Criteria considered in the Decision Making**:
  - *Profit/payoff*
- **States of nature**: *Market Performance*
  - *Poor*
  - *Average*
  - *Good*

Suggested Solution

- Payoffs

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

- Mrs. Lim's family members have different opinions about the investment plans. Who do you think is really taking the risk?
- Which decision would you advise Mrs. Lim?
- Research and help Mrs. Lim to make decision based on her willingness to take risk. Incorporate her attitude towards risk into the decision.

Payoff Table with EMV

Investment / Annual Profit/Loss in $1000	Market Performance			EMV
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	-24	8	24	6.4
Low-risk Stock	-12	12	16	8.4
Savings Account	3	3	3	3.0

Applying what we learned in P08, based on the highest EMV, Mrs. Lim should choose “Low-risk Stock”.

However, depending on an individual’s willingness to take the risk of losing money, the decision could be different.

- Alan is always willing to try his luck and is drawn to the possibility of high payoff ($24K). He will go for the “High-risk Stock”.
- Amy, on the other hand, is more unwilling to risk losing money. She will decide that “Savings Account” is the best alternative.
- Mr. Lim, on the other hand, neither agrees with his son’s optimism nor his daughter’s pessimism. He will go for the “Low-risk Stock”.

Utility Function Assessment – Alan

- To determine the utility for $12K payoff, we first assign a utility value of 0 to the worst outcome (-$24K) and a utility value of 1 to the best outcome ($24K).

  i.e. U(-$24K) = 0; U($24K) = 1

- Then we must identify the probability *p* at which Alan is indifferent between the following 2 alternatives:

  - Alternative1: Receive $12K with certainty
  - Alternative 2: Receive $24K with probability

    p and lose $24K with probability (1-*p*)

Finding the Utility Value

- Assume that, to be indifferent between the alternatives, the *p* for $24K has to be at least 0.45.
  - If the actual probability is less than *0.45*, then Alternative 1 is preferred.
  - If the probability is greater than *0.45*, Alternative 2 is preferred.
- Utility of $12K payoff is calculated as follows:

Expected utility of Alternative 1 = Expected utility of Alternative 2

U($12K) = U($-24K)*(1-*p)* + U($24K)(p)

= (0)(1-*p)* + (1)(*p*) = p

= 0.45

- Repeat the steps to determine

Alan's utility values for other payoffs.

Alan's Utility Function	
Amount of Payoff ($'000)	Utility Value U(X)
-24	0
-12	
3	
8	
12	0.45
16	
24	1

Comparison of Utility Functions

- Utility values for Alan, Amy, Mrs. Lim and Mr. Lim were given in tables below.

Alan's Utility Function	
Amount of Payoff ($'000)	Utility Value U(X)
-24	0
-12	0.05
3	0.2
8	0.32
12	0.45
16	0.65
24	1

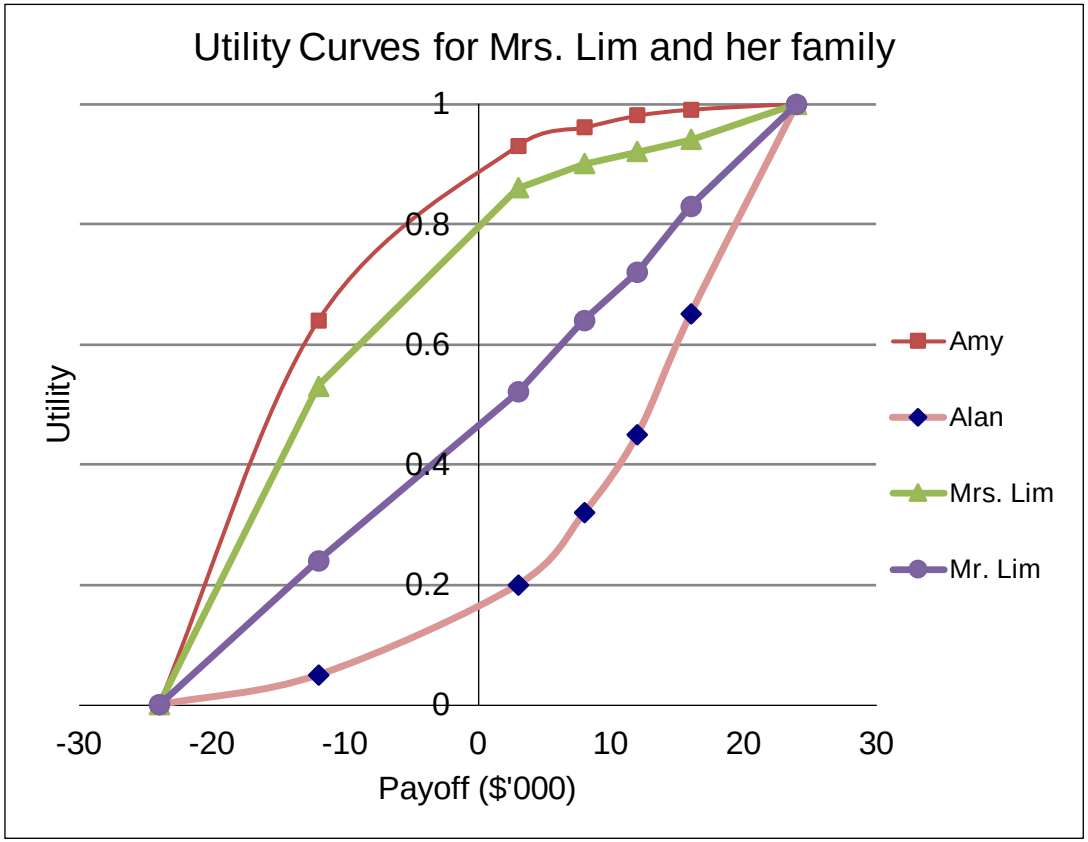
Amy's Utility Function	
Amount of Payoff ($'000)	Utility Value U(X)
-24	0
-12	0.64
3	0.93
8	0.96
12	0.98
16	0.99
24	1

Mrs. Lim's Utility Function	
Amount of Payoff ($'000)	Utility Value U(X)
-24	0
-12	0.53
3	0.86
8	0.9
12	0.92
16	0.94
24	1

Mr. Lim's Utility Function	
Amount of Payoff ($'000)	Utility Value U(X)
-24	0
-12	0.24
3	0.52
8	0.64
12	0.72
16	0.83
24	1

Utility Curves

- Once we determined the utility of points between losing $24K to a profit of $24K for Mrs. Lim and her family members, we can establish the general shape of the utility functions.



From the four curves, we can see that Alan is risk seeking, Mrs. Lim and Amy are risk averse, and Mr. Lim is risk neutral.

Maximizing Expected Utility – Alan

Original Payoff Table

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

Utility Function determined via Certainty Equivalence

Investment / Utility Value	Market Performance			Expected Utility
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	0	0.32	1	0.46
Low-risk Stock	0.05	0.45	0.65	0.43
Savings Account	0.2	0.2	0.2	0.20

To calculate the Expected Utility (EU), apply the same approach as finding the EMV.

E.g. High-risk Stock
=0.2*0+0.5
*0.32+0.3*1
=0.46

The best decision would be to select "High-risk Stock" as it has the highest expected utility.

Maximizing Expected Utility – Amy

Original Payoff Table

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

Utility Function determined via Certainty Equivalence

Investment / Utility Value	Market Performance			Expected Utility
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	0	0.96	1	0.78
Low-risk Stock	0.64	0.98	0.99	0.92
Savings Account	0.93	0.93	0.93	0.93

To calculate the Expected Utility (EU), apply the same approach as finding the EMV.

Eg. Low-risk Stock =0.2*0.64+0.5 *0.98+0.3*0.99 =0.92

The best decision would be to select "Savings Account" as it has the highest expected utility.

Using Exponential Utility Function – Amy

- Assume that Amy has a Risk Tolerance (R) of $5K. Determine the best decision based on the Exponential Utility Function.

Investment / Utility Value	Market Performance			Expected Utility
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	-120.51	0.80	0.99	-23.41
Low-risk Stock	-10.02	0.91	0.96	-1.26
Savings Account	0.45	0.45	0.45	0.45

The best decision here would be “Savings Account” as it has the highest expected utility.

Maximizing Expected Utility – Mrs. Lim

Original Payoff Table

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

Utility Function determined via Certainty Equivalence

Investment / Utility Value	Market Performance			Expected Utility
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	0	0.9	1	0.75
Low-risk Stock	0.53	0.92	0.94	0.85
Savings Account	0.86	0.86	0.86	0.86

To calculate the Expected Utility (EU), apply the same approach as finding the EMV.

E.g. Low-risk Stock
=0.2*0.53+0.5
*0.92+0.3*0.94
=0.85

The best decision would be to select "Savings Account" as it has the highest expected utility.

Maximizing Expected Utility – Mr. Lim

Original Payoff Table

Investment / Annual Profit/Loss in $1000	Market Performance		
	Poor	Average	Good
	20%	50%	30%
High-risk Stock	-24	8	24
Low-risk Stock	-12	12	16
Savings Account	3	3	3

Utility Function determined via Certainty Equivalence

Investment / Utility Value	Market Performance			Expected Utility
	Poor	Average	Good	
	20%	50%	30%	
High-risk Stock	0	0.64	1	0.62
Low-risk Stock	0.24	0.72	0.83	0.66
Savings Account	0.52	0.52	0.52	0.52

To calculate the Expected Utility (EU), apply the same approach as finding the EMV.

Eg. Low-risk Stock =0.2*0.24+0.5 *0.72+0.3*0.83 =0.66

The best decision would be to select "Low-risk Stock" as it has the highest expected utility.

Calculation of Risk Premium

- Assume that Alan is **indifferent** between the following 2 alternatives:

  Alternative 1: Certainty Equivalent is $2500 and

  Alternative 2: 10% chance of losing $5000, 30% chance of earning $2500, and 60% chance of earning $3000.

- The expected monetary value for Alternative 2 is:

  EMV = 0.1*(-$5000) + 0.3*($2500) + 0.6*($3000) = $2050

- The Risk premium = $2050 – $2500= **-$450**
- Alan is willing to bear higher risk.
  - Though the CE is $450 higher than the EMV of the risky Alternative 2, he is willing to risk losing this $450 to forgo $2500 and entering the risk to gain $3000.
  - He is not willing to accept a Certainty Equivalent of $2500 but prefers the risk associated with a decision that has a lower EMV of $2050.
  - Hence, Alan is a **Risk Seeker (Risk Premium = negative).**

Calculation of Risk Premium

- Assume that Amy is **indifferent** between the following 2 alternatives:

  Alternative 1: Certainty Equivalent is $1500 and

  Alternative 2: 10% chance of losing $5000, 30% chance of earning $2500, and 60% chance of earning $3000.

- The expected monetary value for Alternative 2 is:

  EMV = 0.1*(-$5000) + 0.3*($2500) + 0.6*($3000) = $2050

- The Risk premium = $2050 – $1500= **$550**

- Amy prefers lower risk:

  - She is willing to accept a Certainty Equivalent of $1500 to avoid the risk associated with a decision that has a higher EMV of $2050.

  - She is willing to forgo $550 out of the expected value of $2050 to avoid the risk of losing $5000. Hence, Amy is **risk averse (Risk Premium is positive).**

- Similar to Amy, Mrs. Lim is also risk averse. She will have a positive Risk Premium as well.

Calculation of Risk Premium

- Assume that Mr. Lim is **indifferent** between the following 2 alternatives:

  Alternative 1: Certainty Equivalent is $2050 and

  Alternative 2: 10% chance of losing $5000, 30% chance of earning $2500, and 60% chance of earning $3000.

- The expected monetary value for Alternative 2 is:

  EMV = 0.1*(-$5000) + 0.3*($2500) + 0.6*($3000)

  = $2050

- The Risk premium = $2050 – $2050= **$0**
- Mr. Lim is **risk neutral (Risk Premium is 0).**

Recommendations

- As Mrs. Lim is risk averse, she should consider Amy's recommendation of "Savings Account" rather than Alan's recommendation of "High Risk Stock" or Mr. Lim's recommendation of "Low Risk Stock" as her investment plan.
- Other considerations, such as Mrs. Lim's long-term investment strategies, market performance, etc., should also be included.

Learning Objectives

- Make decision based on decision maker's attitude towards risks
  - Use Certainty Equivalent method to construct the utility function of any decision maker (risk-averse, risk-neutral, risk-seeker).
  - Calculate Risk Premium.
  - Use Exponential Utility Function to calculate the decision maker's utility
  - Calculate the Expected Utility to determine the appropriate decision for the decision maker.

Overview of E210 Operation Planning Module