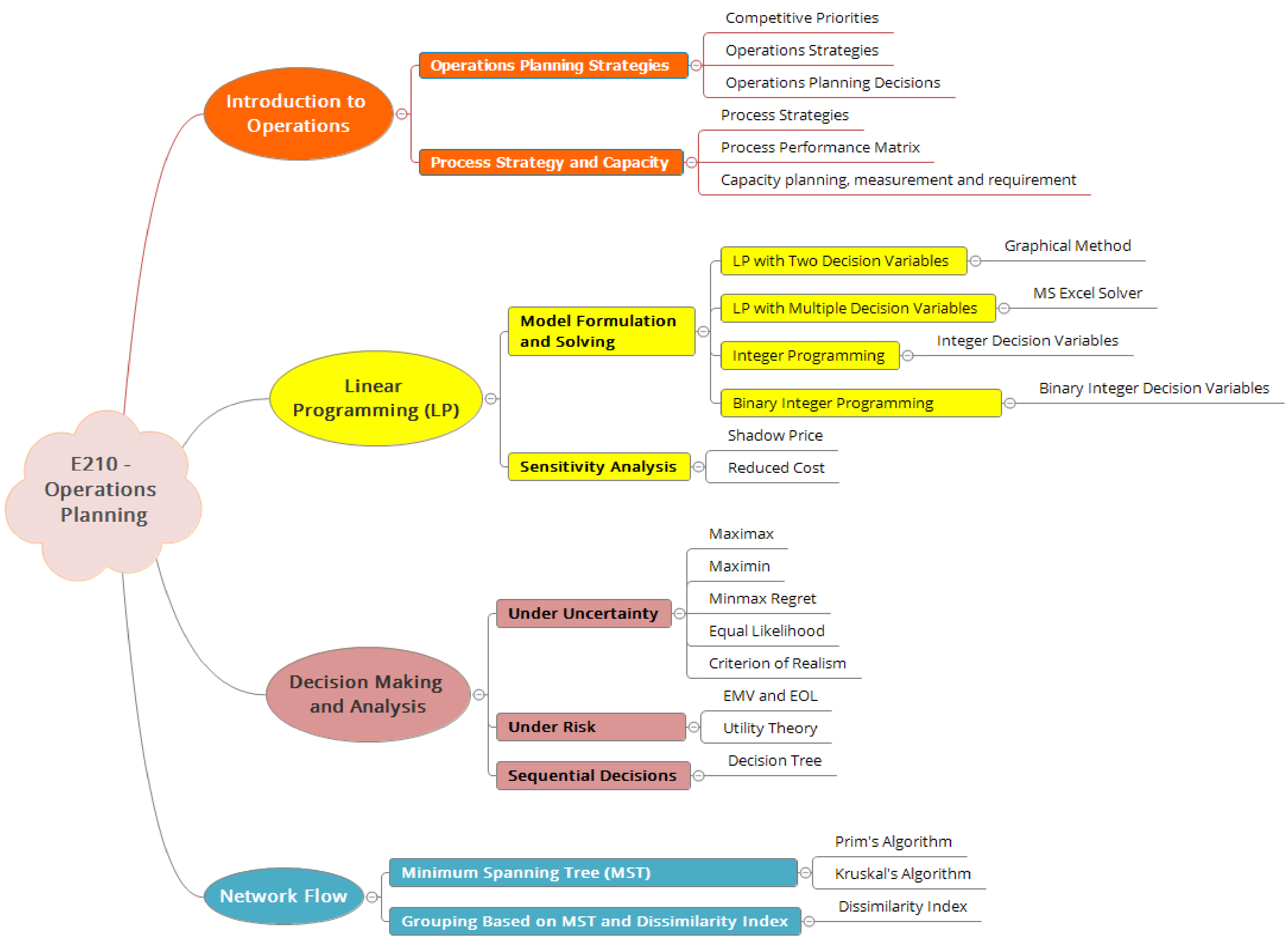
Problem 04

The Power of Excel Solver

E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree



Recap: LP Structure

Define decision variables x_1, x_2 and x_3

Maximize (profit/ sales/ results) or
Minimize (cost/ time/ resources):

$Z = c_1x_1 + c_2x_2 + c_3x_3$ (objective function)

Objective function coefficients

Subject to:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \; <= b_1$ (limited resources)

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \; = b_2$ (exact requirement)

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \; >= b_3$ (minimum requirement)

$x_1, \; x_2, \; x_3 \; >= 0$ (non-negativity)

Constraint coefficients

Recap: Assumptions of Linear Programming (LP)

- The ***Certainty*** assumption:
  - Each model parameter is known with certainty.

    Is this always true???
  - Model parameters here refer to objective function coefficients, right-hand side values of model constraints, and constraint coefficients.
- The ***Proportionality*** assumption:
  - The contribution to the objective function (left-hand side of each constraint) from each decision variable is proportional to the value of the decision variable.
- The ***Additivity*** assumption:
  - Total contributions to the objective function (left-hand side of each constraint) is the sum of individual contributions from each variable.
- The ***Continuity*** assumption:
  - Variables can take on any value within a given feasible range

Recap: Role of Sensitivity Analysis of Optimal solution

- When formulating LP models, the model parameters are assumed to be **known with certainty**
- In reality, model parameters are simply estimates (best guesses) that are subject to change.
- Sensitivity analysis is the analysis of parameter changes and their effects on the model solution.
- Sensitivity analysis provides a better picture of how the solution to a problem will change if different parameters in the model change.

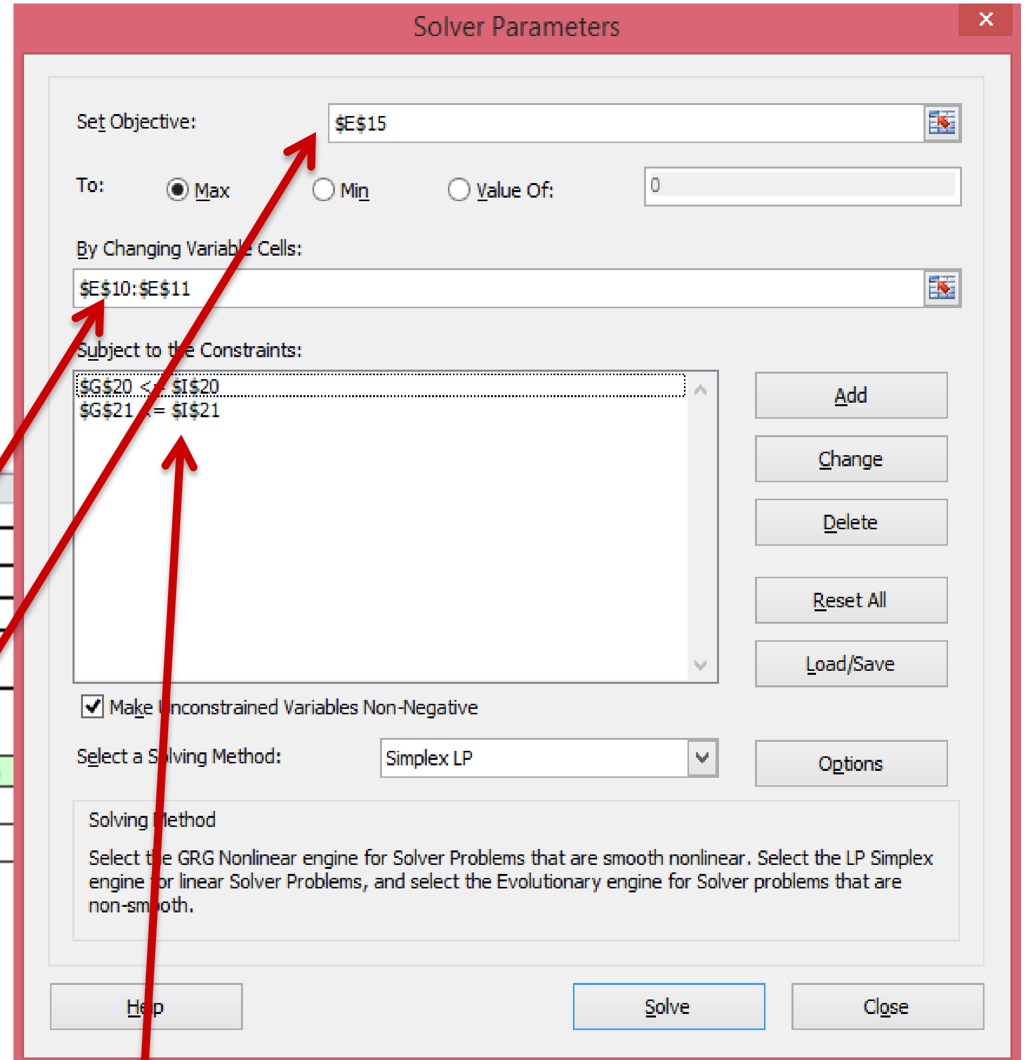
Recap: Sensitivity Analysis

- Sensitivity analysis aids in answering questions such as:
  - **Range of optimality** - The range of values that the **objective function coefficients** can assume without changing the optimal solution.
  - **Range of feasibility** - The range of values for a right hand side of a constraint, in which the shadow prices for the constraints remain unchanged - the valid range of **shadow prices.**
  - **Reduced Cost** - The impact of forcing a variable which is currently zero to be non-zero on objective function value.
  - **Shadow price** - The impact of increasing or decreasing the right-hand side value of various constraints on the optimal objective value.
  - The impact of changing the **constraint coefficients** on the optimal solution to the problem.

Recap: Sensitivity Analysis

- The information and insights obtained through sensitivity analysis are valuable to management
  - They provide an indication of the degree of flexibility that is inherent in an operating environment.
- Almost all commercial software for linear programming provides
  - Right-hand-side ranging information
  - Objective coefficient ranging information.
  - Other sensitivity analysis, such as adding new variable and constraint, is dealt by solving the modified linear programming problem.

Examples of Using EXCEL SOLVER



	Leather (m^2)	Labour (Hours)	Profit ($)
Wallet	1	4	50
Pouch	2	3	40
Available	300	540	

Decision Variables

Variables	Name	Amount (Units)	Profit ($)
X_1	Wallets		50.00
X_2	Pouches		40.00

Objective Function

Z Total Profit

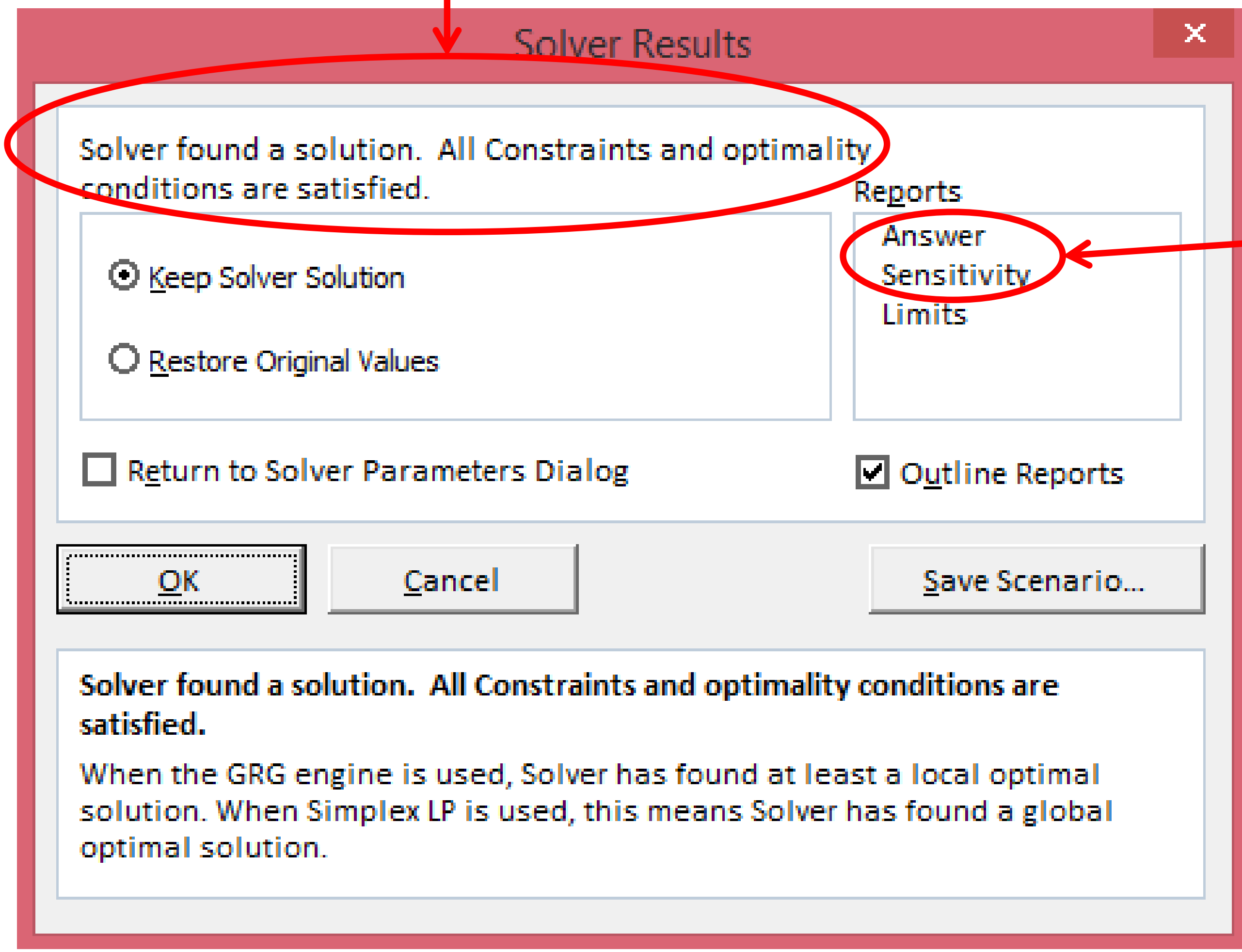
Constraints

X_1	X_2		Constraint	LHS	Sign	RHS
1	2		Leather		≤	300.00
4	3		Labour		≤	540.00

Constraint Equation
$1*X_1 + 2*X_2 <= 300$
$4*X_1 + 3*X_2 <= 540$

Examples of Using EXCEL SOLVER

Optimal Solution found !



Select to generate Reports

Examples of Using EXCEL SOLVER

EXCEL SOLVER SOLUTION:

	Leather (m^2)	Labour (Hours)	Profit ($)
Wallet	1	4	50
Pouch	2	3	40
Available	300	540	

Decision Variables

Variables	Name	Amount (Units)	Profit ($)
X_1	Wallets	36.00	50.00
X_2	Pouches	132.00	40.00

Optimal Solution

Objective Function

Z	Total Profit	$7,080.00

Optimal Objective Function Value

Objective Function
Max $Z = 50{*}X_1 + 40{*}X_2$

Constraints

X_1	X_2		Constraint	LHS	Sign	RHS
1	2		Leather	300.00	≤	300.00
4	3		Labour	540.00	≤	540.00

Constraint Equation
$1{*}X_1 + 2{*}X_2 <= 300$
$4{*}X_1 + 3{*}X_2 <= 540$

Examples of Answer Report in EXCEL SOLVER

Microsoft Excel 14.0 Answer Report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$15	Total Profit Amount (Units)	$7,080.00	$7,080.00

- **Optimal objective function Value – Max Profit**

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$E$10	Wallets Amount (Units)	36.00	36.00	Contin
$E$11	Pouches Amount (Units)	132.00	132.00	Contin

- **Optimal number of Wallets and Pouches to make**

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$20	Leather LHS	300.00	$G$20<=$I$20	Binding	0
$G$21	Labour LHS	540.00	$G$21<=$I$21	Binding	0

- **All constraints meet and used up**

Examples of Sensitivity Report in EXCEL SOLVER

Sensitivity Report

Microsoft Excel 14.0 Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$10	Wallets Amount (Units)	36	0	50	3.333333333	30
$E$11	Pouches Amount (Units)	132	0	40	60	2.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$20	Leather LHS	300	2	300	60	165
$G$21	Labour LHS	540	12	540	660	90

Reduced Cost *The minimum amount by which the OFC of a variable should change to cause that variable to become non-zero.*

Range of Optimality

Range of profit contributions for Wallets that will retain the current solution of (36 wallets and 132 pouches) optimal.

Shadow Price

The change in the objective function value per unit increase in the RHS of the constraint.

Range of feasibility

-Range of Shadow Price Validity.

-For example, if the increase in the RHS of labour hours constraint is beyond the allowable increase of 660, then the shadow price will change.

Problem 04
Suggested Solution

Problem Formulation

LP Formulation

Decision Variables

Let X_1 be the number of decorative vases to produce per week

Let X_2 be the number of decorative jars to produce per week

Let X_3 be the number of tea sets to produce per week

Objective Function

Maximize Total Profit = $75X_1 + 60X_2 + 90X_3$

Constraints

$1.2X_1 + 0.6X_2 + 1.8X_3$	$\leq$	240	(Availability of Clay Constraint)
$4X_1 + 6X_2 + 8X_3$	$\leq$	1260	(Availability of Labour Hours Constraint)
$3.6X_1 + 3X_2 + 4X_3$	$\leq$	600	(Availability of Machine Hours Constraint)
X_2	$\geq$	64	(Units of Decorative Jar Pre-ordered)
X_3	$\geq$	30	(Units of Tea Set Pre-ordered)
X_1, X_2, X_3	$\geq$	0	(Non-negativity)

Spreadsheet Formulation (Excel Solver)

Decision Variables

Variables	Name	Quantity	Profit ($)
X_1	Decorative Vase	0.00	75.00
X_2	Decorative Jar	64.00	60.00
X_3	Tea Set	102.00	90.00

Production Plan

Objective Function

Z	Total Profit	$13,020.00

Constraints

X_1	X_2	X_3	Constraint	LHS	Sign	RHS
1.2	0.6	1.8	Availability of Clay	222.00	≤	240.00
4	6	8	Availability of Labour Hours	1200.00	≤	1260.00
3.6	3	4	Availability of Machine Hours	600.00	≤	600.00
0	1	0	Units of Decorative Jar Pre-ordered	64.00	≥	64.00
0	0	1	Units of Tea Set Pre-ordered	102.00	≥	30.00

The Answer Report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$E$12	Total Profit Quantity	$0.00	$13,020.00

Maximize total Profit

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$E$6	Decorative Vase Quantity	0.00	0.00	Contin
$E$7	Decorative Jar Quantity	0.00	64.00	Contin
$E$8	Tea Set Quantity	0.00	102.00	Contin

Optimal solution

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$17	Availability of Clay LHS	222.00	$G$17<=$I$17	Not Binding	18
$G$18	Availability of Labour Hours LHS	1200.00	$G$18<=$I$18	Not Binding	60
$G$19	Availability of Machine Hours LHS	600.00	$G$19<=$I$19	Binding	0
$G$20	Units of Decorative Jar Pre-ordered LHS	64.00	$G$20>=$I$20	Binding	0.00
$G$21	Units of Tea Set Pre-ordered LHS	102.00	$G$21>=$I$21	Not Binding	72.00

Binding, Non-binding Constraints and Slacks

Non-binding Constraints

Eg. Availability of Clay: 18 kg of unused Clay (non-binding constraint; slack = 240-222 = 18 kg)

Labour Hours: 60 hours of unused Labour Hours (non-binding constraint; slack = 1260-1200 = 60 hours)

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$17	Availability of Clay LHS	222.00	$G$17<=$I$17	Not Binding	18
$G$18	Availability of Labour Hours LHS	1200.00	$G$18<=$I$18	Not Binding	60
$G$19	Availability of Machine Hours LHS	600.00	$G$19<=$I$19	Binding	0
$G$20	Units of Decorative Jar Pre-ordered LHS	64.00	$G$20>=$I$20	Binding	0.00
$G$21	Units of Tea Set Pre-ordered LHS	102.00	$G$21>=$I$21	Not Binding	72.00

Binding Constraint

Eg. Availability of Machine Hours is binding (slack = 0).

Increase in Machine Hours (currently at 600 hours) would improve the optimal solution (i.e. increase the total profit).

Shadow Price of Constraints

How much is the worth of the additional resource?

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$17	Availability of Clay LHS	222	0	240	1E+30	18
$G$18	Availability of Labour Hours LHS	1200	0	1260	1E+30	60
$G$19	Availability of Machine Hours LHS	600	22.5	600	30	288
$G$20	Units of Decorative Jar Pre-ordered LHS	64	-7.5	64	96	24
$G$21	Units of Tea Set Pre-ordered LHS	102	0	30	72	1E+30

- Shadow price: The increase (or decrease) in objective function given 1 unit increase (or decrease) in resource (RHS value)
- The shadow price remains valid if and only if the right-hand-side (RHS) value stays <u>within</u> the range of allowable increase & decrease:

  (R. H. Side - Allowable Decrease, R. H. Side + Allowable Increase).
- Shadow price is <u>non-zero only for binding constraints</u>. For non-binding constraints (surplus resources), additional resource is not valuable.
- For example, increasing the amount of Clay and Labour Hours does not change the total profit.

Shadow Price – Contribution to Objective Function Value

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$17	Availability of Clay LHS	222	0	240	1E+30	18
$G$18	Availability of Labour Hours LHS	1200	0	1260	1E+30	60
$G$19	Availability of Machine Hours LHS	600	22.5	600	30	288
$G$20	Units of Decorative Jar Pre-ordered LHS	64	-7.5	64	96	24
$G$21	Units of Tea Set Pre-ordered LHS	102	0	30	72	1E+30

- Suppose the Machine Hours availability is reduced to 500 due to maintenance. With the decrease of 100 hours (allowable) in Machine Hours, the total profit (objective function) will decrease to:

  $13,020 - (($22.5) x 100) = $10,770

- The shadow price of Machine Hours ($22.5) is valid when it is between 312 and 630 hours. ***(Range of feasibility)***
- Note that the optimal solution changes when the RHS value of a binding constraint changes. *(i.e. Production of Decorative Jar and Tea Set would no longer be 64 and 102 sets.)*

Range of Optimality for Objective Function Coefficients

For the optimal solution to remain unchanged, how much can the objective function coefficient change?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Decorative Vase Quantity	0	-6	75	6	1E+30
$E$7	Decorative Jar Quantity	64	0	60	7.5	1E+30
$E$8	Tea Set Quantity	102	0	90	1E+30	6.666666667

- For the solution (production plan) to stay optimal, the objective function coefficient can assume the range ('Coefficient – Allowable Decrease', 'Coefficient + Allowable Increase')
- In other words, the **optimal solution (X_1= 0; X_2=64,X_3=102)** is valid when the unit profit stays within the following range:

Product	Unit Profit
Decorative Vase	$P_1 \leq \$81$
Decorative Jar	$P_2 \leq \$67.5$
Tea Set	$P_3 \geq \$83.33$

Range of Optimality for Objective Function Coefficients

NOTE

- For multiple optimal solutions, the allowable increase or allowable decrease for some objective function coefficients will be zero. See an example below:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$1	x14	0	1	5	1E+30	1
$B$2	x15	0	2	10	1E+30	2
$B$3	x16	0	6	12	1E+30	6
$B$4	x18	50	0	7	0	1
$B$5	x25	0	0	8	1E+30	0
$B$6	x26	250	0	6	2	6
$B$7	x27	0	6	13	1E+30	6
$B$8	x34	300	0	3	0	2

- To find an alternate optimal solution:
  - Add a constraint that holds the objective function at the current optimal value, then try to optimize other possible objectives.

Reduced Cost of a Zero-Value Decision Variable

What does it take to produce one unit of Decorative Vase?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Decorative Vase Quantity	0	-6	75	6	1E+30
$E$7	Decorative Jar Quantity	64	0	60	7.5	1E+30
$E$8	Tea Set Quantity	102	0	90	1E+30	6.666666667

Reduced Cost: An estimate of how much the objective function will change if a decision variable which is currently 0 is forced to be non-zero. For example,

- Decorative Vase (X_1) has a final value of 0 unit to produce. To produce 1 unit of Decorative Vase, i.e., X_1= 1, then unit profit must be reduced by at least (-$6) (meaning the unit profit has to be increased by more than $6). So, in order to be worthwhile to produce 1 unit of Decorative Vase, its unit profit should be at least: $75 + $6 = $81.

How to Calculate the Reduced Cost?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Decorative Vase Quantity	0	-6	75	6	1E+30
$E$7	Decorative Jar Quantity	64	0	60	7.5	1E+30
$E$8	Tea Set Quantity	102	0	90	1E+30	6.666666667

Reduced cost is the amount per unit of the product that contributes to overall objective function minus the value (shadow price) of the resources it consumes.

Objective coeff.

Shadow price

Reduced cost for Decorative Jar, X_2

= 60 – (0.6 x 0) – (6 x 0) – (3 x 22.5) – (1 x (-7.5)) – (0 x 0)

= **0**

Cell	Name	Final Value	Shadow Price
$G$17	Availability of Clay LHS	222	0
$G$18	Availability of Labour Hours LHS	1200	0
$G$19	Availability of Machine Hours LHS	600	22.5
$G$20	Units of Decorative Jar Pre-ordered LHS	64	-7.5
$G$21	Units of Tea Set Pre-ordered LHS	102	0

Constraints

Constraint coefficient for X_2

X_1	X_2	X_3	Constraint
1.2	0.6	1.8	Availability of Clay
4	6	8	Availability of Labour Hours
3.6	3	4	Availability of Machine Hours
0	1	0	Units of Decorative Jar Pre-ordered
0	0	1	Units of Tea Set Pre-ordered

How to Calculate the Reduced Cost?

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Decorative Vase Quantity	0	-6	75	6	1E+30
$E$7	Decorative Jar Quantity	64	0	60	7.5	1E+30
$E$8	Tea Set Quantity	102	0	90	1E+30	6.666666667

ints

Name	Final Value	Shadow Price
Availability of Clay LHS	222	0
Availability of Labour Hours LHS	1200	0
Availability of Machine Hours LHS	600	22.5
Units of Decorative Jar Pre-ordered LHS	64	-7.5
Units of Tea Set Pre-ordered LHS	102	0

Likewise,
Reduced cost for Tea Set:
= 90 – (1.8 x 0) – (8 x 0) – (4 x 22.5) – (0 x (-7.5)) – (1 x 0) = $ 0

Constraints

X_1	X_2	X_3	Constraint
1.2	0.6	1.8	Availability of Clay
4	6	8	Availability of Labour Hours
3.6	3	4	Availability of Machine Hours
0	1	0	Units of Decorative Jar Pre-ordered
0	0	1	Units of Tea Set Pre-ordered

Determine the Requirement to Change Decision Variable from Zero Value to Non-Zero

Reduction in Machine Hours needed to produce 1 unit of Decorative Vase so that it is worthwhile to produce it.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Decorative Vase Quantity	0	-6	75	6	1E+30
$E$7	Decorative Jar Quantity	64	0	60	7.5	1E+30
$E$8	Tea Set Quantity	102	0	90	1E+30	6.666666667

- Let *T* be the amount of "revised" Machine Hours needed for producing 1 unit of Decorative Vase

Let reduced cost of Decorative Vase = 0

75 - (1.2*0) - (4*0) - (T*22.5) - (0*(-7.5) - (0*0) = 0

T = 75/22.5 hours = 200 minutes

Cell	Name	Final Value	Shadow Price
$G$17	Availability of Clay LHS	222	0
$G$18	Availability of Labour Hours LHS	1200	0
$G$19	Availability of Machine Hours LHS	600	22.5
$G$20	Units of Decorative Jar Pre-ordered LHS	64	-7.5
$G$21	Units of Tea Set Pre-ordered LHS	102	0

- From the computation, to produce Decorative Vase, the amount of Machine Hours required has to reduce by more than 16 minutes (3.6*60 - 200).

X_1	X_2	X_3	Constraint
1.2	0.6	1.8	Availability of Clay
4	6	8	Availability of Labour Hours
3.6	3	4	Availability of Machine Hours
0	1	0	Units of Decorative Jar Pre-ordered
0	0	1	Units of Tea Set Pre-ordered

Conclusion – Optimal Solution & Critical Constraint

- The maximum profit for the production is $13,020.
- For the current availability of the resources and customer demands, Andy will recommend to produce 64 units of Decorative Jar and 102 units of Tea Set **(optimal solution).**
- Current critical constraint is Machine Hours
  - An increase of 1 unit of Machine Hours increases the total profit by $22.5 (shadow price).
  - This shadow price is valid so long as the Machine Hours is between 312 to 630 hours.
  - There is an excess of 18 kg of Clay (240-222 = 18 kg) and excess of 60 Labour Hours (1260 - 1200 = 60 hours).

Conclusion – Range of Optimality & Reduced Cost

- The current optimal solution remains optimal at the following unit profit range:

(for example: P_1 represents unit profit for Decorative Vase, etc.)

Product	Unit Profit
Decorative Vase	$P_1 \leq \$81$
Decorative Jar	$P_2 \leq \$67.5$
Tea Set	$P_3 \geq \$83.33$

- Currently it is not viable to produce Decorative Vase. It is only profitable to produce when:
  - The unit profit increases to more than $81, or
  - The required Machine Hours is less than 75/22.5 = 3.33 hours

Learning Objectives

At the end of the lesson, students should be able to:

- Formulate and solve the LP model with more than two variables using MS Excel Solver.
- Generate reports & perform sensitivity analysis using reports from MS Excel Solver:
  - Identify binding and non-binding constraints and explain their significance.
  - Explain and relate the concept of Shadow Price to the right-hand side value of various constraints on the optimal objective value.
  - Identify the range of values that the objective function coefficients can assume without changing the optimal solution.
  - Explain and relate concept of Reduced Cost and determine the impact of forcing a decision variable which has zero value to be non-zero on the objective function value.
  - Calculate and relate the application of Reduced Cost.

E-learning Video

- You may wish to access the e-learning video from : **https://docs.google.com/file/d/0Bz-Yttbj61bxTXAtQXplQ2RxN2c/edit?usp=sharing**

Overview of E210 Operation Planning Module