E210 – Operations Planning

Practice Questions for Operations Planning (6 Jun 2018)

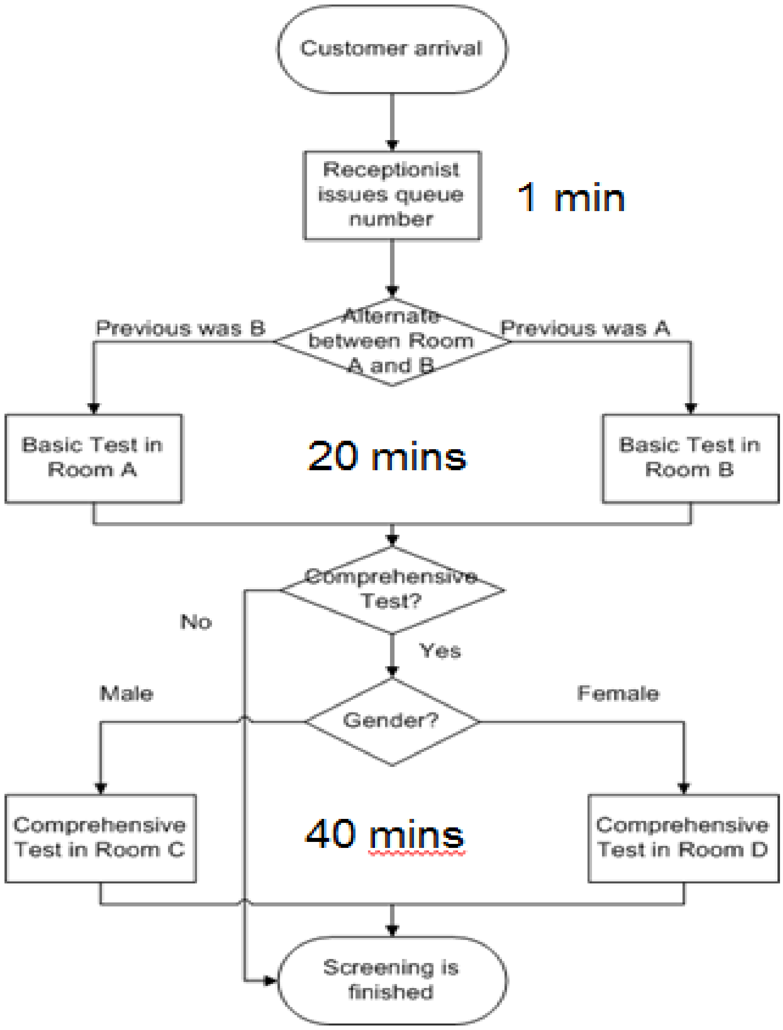
1. If a pizza shop aims to provide low cost pizza with a delivery time promised to be within 30 mins upon order placement, which competitive priorities do you think they are considering? Explain.

   Cost - They aim to provide low cost pizza.
   Time (Delivery speed) – They promise to deliver the pizza within 30- mins upon placement of order.

2. First Healthcare is a clinic that provides health screening services. They provide three different health screening packages: Basic, Comprehensive for Men and Comprehensive for Women. Currently, there are four testing rooms in the clinic: two (Testing Rooms A & B) are used for the Basic package, one (Testing Room C) for the Comprehensive for Men package and the other (Testing Room D) for the Comprehensive for Women package.

   When customers arrive for the check-up, a receptionist will check the customer details after which a queue number will be given. The receptionist alternates queue numbers between Testing Rooms A and B to balance the work load of both rooms. Customers who sign up for the Basic package only need to go to either one of the two rooms (Testing Room A or B), while those who sign up for the Comprehensive packages will need to go through the Basic package at Testing Room A or B, after which they will need to go to Testing Room C or D depending on their gender.

   The figure below shows the process map for the above scenario. Assume that the average waiting time for the Basic Test is 5 minutes and the average waiting time for the Comprehensive Test is 10 minutes.



a) Explain what flowtime is and calculate the flow time for:
   i) Basic Test Package
   ii) Comprehensive Test Package

Flow time is ….

Flow time calculation
Assume that timings given are average timings.

i) Flow time for basic test package
   = … = 26 mins
ii) Flow time for comprehensive test package = … = 76 mins

b) Explain what cycle time is. Determine the cycle time and calculate the throughput of a female customer taking up a comprehensive test.

Cycle time is ….

Cycle time = 40 mins

Throughput = … = 0.025 customers per minute

3. A copy center prepares bound reports for three clients. The processing time to run, collate and bind each copy varies and depends on the number of pages and types of print. The center operates 280 days a year with one 10-hour shift. Management wants to prescribe a target utilization level of 90%.

a) Based on the table of information below, determine how many machines are needed at the copy center.

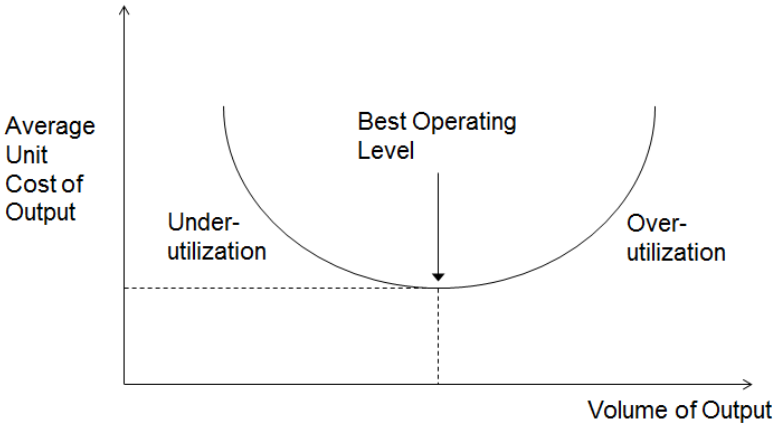
	Client X	Client Y	Client Z
Annual Demand forecast (copies)	2000	5000	3000
Standard processing time (hour/copy)	0.3	0.4	0.5

Number of machines required = total processing hours needed / hours available
= …
= 1.63 (round up to 2)
2 machines are required.

b) Assume that the copy centre prefers to improve productivity through higher utilization of the machines and there are possible emergences of new copy machine models. Recommend a capacity change strategy to the copy center. State one advantage and one disadvantage of the proposed capacity change strategy.

Wait-and-see or lag strategy
Advantage: …
Disadvantage: …

4. The best operating level is:



a) the maximum point of the cost curve
b) the level of capacity for which average unit cost is minimized
c) maximum capacity
d) the level of capacity for which total cost is minimized
e) the level of capacity for which average unit cost is maximized

5. Consider the case where a process utilizes a machine to produce an item.

a) If the number of hours in each shift is 10 hours, and the machine operates on average 8 hours, what is the capacity utilization of the machine?

b) If the process is able to produce 1000 items a month, and during the month of January, 950 items are produced, what is the capacity utilization of the process?

State the type of measure of capacity used in each scenario.

a) Capacity utilization of the machine = … = 80%, Input measure
b) Capacity utilization of the process = … = 95%, Output measure

6. You are an investment banker who is helping a client to invest up to $20,000. There are 2 choices of funds – Fund A yields 20% return and Fund B yields 10% return. Due to risk of currency depreciation in the coming months, it is possible that Fund A will lose 40% of its principle value and Fund B 20%. You wish to limit the potential loss of the investment to less than $6,000 should it happens.

Furthermore, to maintain the balance between risk and return of the 2 funds, your research shows that no more than $16,000 should be invested in Fund A (high risk) and no more than $7,000 in Fund B (low return).

In order to determine how much to invest in each fund, we formulate it as a Linear Programing model.

a) Define the decision variables.

Let X_1 = amount to be invested in Fund A
X_2 = amount to be invested in Fund B

b) Write down the objective function.

Maximize investment return: $Z = 0.2 X_1 + 0.1 X_2$

c) Formulate the constraints that you need to satisfy.

Subject to:

Investment limit	$X_1 + X_2$	$\leq$ 20,000
Loss limit	$0.4 X_1 + 0.2 X_2$	$\leq$ 6,000
Fund A limit	X_1	$\leq$ 16,000
Fund B limit	X_2	$\leq$ 7,000
Non-negativity	X_1, X_2	$\geq$ 0

7. Company ABC produces four types of products (A1, A2, A3 and A4) with different profitability. All four types of products consume three types of raw materials. Table below shows the profit from each product, raw material consumption, and daily amount available for each type of raw material.

Product	**Resources (kg)**			**Unit Profit ($)**
	Raw Material 1	**Raw Material 2**	**Raw Material 3**	
A1	4	8	2	65
A2	10	12	4	110
A3	16	10	8	128
A4	6	18	5	85
Resource Available Daily	1000	1010	510	-

To plan the optimal amount of each product to produce, you are to formulate the problem as a Linear Programming (LP) model. Assume that the decision variables have been defined as:

X1: Number of units of Product A1 to produce;
X2: Number of units of Product A2 to produce;
X3: Number of units of Product A3 to produce;
X4: Number of units of Product A4 to produce.

When decision variables are given, you must use these decision variables

a) Construct the objective function for the LP model.

Maximize profit Z = 65*X1+110*X2+128*X3+85*X4

b) Formulate the following constraints for the LP model.

i) The total amount of raw material 1 consumed cannot exceed the amount available.

4X1+10X2+16X3 +6X4 <= 1000

ii) The total amount of raw material 2 consumed cannot exceed the amount available.

8X1+12X2+10X3+18X4 <= 1010

iii) The total amount of raw material 3 consumed cannot exceed the amount available.

2X1+4X2+8X3+5X3 <= 510

c) Examine the answer and sensitivity reports generated and answer the following questions.

Decision variables, optimal amount

Variable Cells

Cell	Name	Original Value	Final Value
$E$6	Product A1 Quantity	0.00	70.00
$E$7	Product A2 Quantity	0.00	0.00
$E$8	Product A3 Quantity	0.00	45.00
$E$9	Product A4 Quantity	0.00	0.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$18	Raw Material 1	1000.00	$G$18<=$I$18	Binding	0
$G$19	Raw Material 2	1010.00	$G$19<=$I$19	Binding	0
$G$20	Raw Material 3	500.00	$G$20<=$I$20	Not Binding	10

Reduced cost showing how much the objective function will change if a decision variable which is currently 0 is forced to be non-zero.

Range for the objective coefficient for the current optimal solution to remain optimal

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$6	Product A1 Quantity	70	0	65	37.4	4.304347826
$E$7	Product A2 Quantity	0	-4.5	110	4.5	1E+30
$E$8	Product A3 Quantity	45	0	128	132	12.375
$E$9	Product A4 Quantity	0	-48.5	85	48.5	1E+30

Shadow price showing the increase (or decrease) in objective function given 1 unit increase (or decrease) in resource (RHS

Range for constraint RHS for the shadow price to remain valid

Constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$18	Raw Material 1	1000	4.25	1000	20	495
$G$19	Raw Material 2	1010	6	1010	990	385
$G$20	Raw Material 3	500	0	510	1E+30	10

i) What is the optimal solution? Calculate the corresponding maximum profit.

Product A1 Quantity 70
Product A2 Quantity 0
Product A3 Quantity 45
Product A4 Quantity 0

Maximum profit = … = $10,310

ii) Based on the sensitivity report above, is this a unique optimal solution? Explain.

Yes. There are alternate optimal solutions if any of the allowable increase or decrease values for the objective coefficients are zero. There is no zero allowable increase or decrease in the above report, thus no alternate optimal solution to this problem.

iii) What is the range for the 'Raw Material 1' constraint RHS for the shadow price to remain valid?

The shadow price remains valid if and only if the right-hand-side (RHS) value stays within the range of allowable increase & decrease,

… to … -→ 505kg to 1020kg

iv) Is it possible to reduce the available amount of Raw Material 2 from 1010kg to a lower value while maintaining the current profit? Explain.

The sensitivity report shows that the 'Raw Material 2' constraint is binding. So reducing the available amount of Raw Material 2 would reduce profit. Therefore, it is not possible to reduce the available amount of Raw Material 2 from 1010kg to a lower value without affecting the expected profit.

v) If you would like to reduce the available amount of Raw Material 3 to a lower value but would still like to gain the same amount of maximum profit, how much can you reduce the available amount to? Explain.

You can reduce till 500kg as the allowable decrease is 10kg from 510kg before the shadow price of $0 becomes invalid.

vi) Assume that you are allowed to decrease the available amount of Raw Material 1 from 1000kg to 900kg. Explain how this will affect the optimal profit. Will the optimal solution change?

Since the decrease in the available amount of Raw Material 1 is 100kg which is within the allowable decrease, the shadow price will be $4.25. So the optimal profit will decrease by … = $425.
The optimal solution will change.

vii) If 90 kg of 'Raw Material 2' is available at the cost of $500, is it worthwhile to get the additional raw material?

Shadow price of Raw Material 2 = $6
Cost of additional 1 kg of Raw Material 2 = … = $5.56 < $6 (shadow price of Raw Material 2); also the allowable increase for Raw Material 2 is 990 kg.
So it is worthwhile to get the additional 90 kg of Raw Material 2.

viii) If the company is to produce **one** unit of 'Product A2', how will this affect the total profit?

Total profit will decrease by $4.5, which is the reduced cost for Product A2.

ix) For ‘Product A2’, if the unit profit, unit consumption of ‘Raw Material 1’, and unit consumption of ‘Raw Material 3’ all remain unchanged, what is the minimum decrease in the ‘Raw Material 2’ consumed per unit of ‘Product A2’ so that producing ‘Product A2’ becomes profitable?

Reduced cost

The amount per unit of the product that contributes to overall objective function (unit profit) - the value (shadow price) of the resources it consumes.

Objective coeff. of A2

Shadow price

110 – 10*4.25- M*6 – 4*0 = 0

Constraint coefficient for Product A2

6M = 110 – 42.5

M = 67.5/6 = 11.25kg

Minimum decrease in ‘Raw Material 2’ consumed per unit of ‘Product A2:
12 – 11.25 = 0.75kg

x) Determine the minimum increase in the unit profit of ‘Product A4’ so that producing product A4 will become worthwhile.

Current unit profit from Product A4 (objective coefficient of Product A4) is $85. Since the reduced cost for Product A4 is $-48.5, the minimum unit profit from Product A4 should be … =$133.5.

xi) In order for the current optimal solution to remain optimal, what is the range of unit profit of Product A2 allowed?

Product A2 current unit profit (X2 objective coefficient): $110
X2 objective coefficient allowable increase: $4.5
X2 objective coefficient allowable decrease: infinity
So the range for Product A2 unit profit (X2 objective coefficient):

0 <= Unit profit of A2 <= …

0<= Unit profit of A2<=$114.5

8. A Department Store opens on a 24-hour basis. The store manager has divided the 24-hour day into 5 full-time shifts. The store can also hire part-time employee at certain time periods. The time periods, manpower requirements, full-time and part-time shifts, and full-time and part-time employee daily cost are given in the following Table.

Time period	Time Period Covered by					Time Period Covered by				Minimum No. of Employee Needed
	Full-time Shift					Part-time Shift				
	1	2	3	4	5	1	2	3	4	
6:00 am – 8:00 am	√									48
8:00 am – 10:00 am	√	√				√				79
10:00 am – 12:00 pm	√	√				√				65
12:00 pm – 2:00 pm	√	√	√				√			87
2:00 pm – 4:00 pm		√	√				√			64
4:00 pm – 6:00 pm			√	√				√		73
6:00 pm – 8:00 pm			√	√				√		82
8:00 pm – 10:00 pm				√					√	43
10:00 pm – 12:00 am				√	√				√	52
12:00 am – 6:00 am					√					15
Daily cost per employee	$170	$160	$175	$180	$195	$70	$60	$75	$80	

The store employee must report for work at the beginning of one of these shifts. Meanwhile, during peak-hours (10:00am to 2:00pm; 6:00pm to 8:00pm), the number of full-time employee must meet certain conditions specified. The store manager wants to know how to schedule the full-time and part-time employees so that the total daily personnel cost is a minimum.

a) Define the Decision Variables.

Let X_i (i=1,2,3,4,5) be the number of full-time staff on full-time shift i and Y_j (j=1,2,3,4) be the number of part-time staff on part-time shift j.

b) State the Objective Function.

Minimize daily staff cost, Z =
$\$170*X_1+\$160*X_2+\$175*X_3+\$180*X_4+\$195*X_5+\$70*Y_1+\$60*Y_2+\$75Y_3+\$80Y_4$

c) Formulate the following constraints:

i) The number of full-time employee must be at least 7/4 that of part-time employee during 10am to 11am.

4(X1+X2) -7Y1 >=0

ii) The number of part-time employee must be at most 3/5 that of full-time employee during 1pm to 2pm.

3(X1+X2+X3)-5Y2>=0

iii) The number of part-time employee must be at most 4/9 that of total employee during 6pm to 8pm.

4(X3+X4)-5Y3>=0

iv) Meet the number of staff requirement for the period between 7am – 9am

7am to 8am: X1>=48

8am to 9am: X1+X2+Y1>=79

v) Meet the number of staff requirement for the period between 3pm – 5pm

3pm to 4pm: X2+X3+Y2>=64

4pm to 5pm: X3+X4+Y3>=73

vi) If the store wants to limit the total daily personnel cost for part-time staff to be within $2000, write out the constraint required.

70*Y1+60*Y2+75*Y3+80*Y4<= 2000

d) Do you need the integer requirement on the decision variables? Why cannot use the ‘round-off’ method instead to find the optimal solution? Explain.

Yes.

The round-off solution may not be optimal; may not be feasible.

9. A production engineer is to assign 12 machines to 3 locations. Table below shows the material handling cost (based on locations of the machines) and the number of machines required at each of the locations. He is considering how to optimally assign the machines to the locations to achieve the least material handling cost.

Machines	**Location**		
	A	**B**	C
1	$45	$65	$ 80
2	$50	$40	$ 70
3	$62	$49	$ 90
4	$56	$45	$ 50
5	$48	$61	$ 46
6	$53	$64	$ 70
7	$45	$65	$ 80
8	$50	$40	$ 60
9	$62	$49	$ 47
10	$56	$45	$ 54
11	$48	$61	$ 23
12	$53	$64	$ 67
No. of Machines required	4	6	2

a) Define the decision variables and state the meaning of the decision variables. In total how many decision variables do you have? How are these decision variables different from those in Q7? Explain.

Decision variables: Xij =1 or 0 with Xij = 1 representing machine i (1, 2, …, 12) is to be assigned to location j (A, B, C); Xij = 0 representing machine i (1, 2, …, 12) is **not** to be assigned to location j (A, B, C).

In total we have 12*3 = 36 decision variables.
These decision variables are called **binary** decision variables in the sense that it can only take on 2 possible values: 0 or 1.

b) Formulate the following constraints:

i) Location A, B, C each requires 4, 6, 2 machines.

Location A: X1A+X2A+…+X12A = 4
Location B: X1B+X2B+…+X12B = 6
Location C: X1C+X2C+…+X12C = 2

ii) Each machine needs to be assigned to one location. In total how many such constraints do you have?

Machine 1: X1A+X1B+X1C = 1
Machine 2: X2A+X2B+X2C = 1
……
Machine 12: X12A+X12B+X12C = 1

In total, there are 12 such constraints, one for each machine.

iii) Machine '#1' and '#2' cannot be both assigned to 'location C'.

X1C+ X2C<=1

iv) Either machine '#1' or '#9' must be assigned to location B.

X1B + X9B =1

10. A publisher has two warehouses A and B. The warehouses can supply the following numbers of books to the bookstores each month:

Warehouse	Monthly Supply (number of books)
A	4275
B	5370
Total	9645

The five bookstores have the following total monthly demand:

Bookstore	Monthly Demand (number of books)
1	1950
2	2250
3	1725
4	1350
5	1050
Total	8325

The publisher must pay the following shipping cost per book:

From	**To (cost, $)**				
	1	**2**	**3**	**4**	**5**
A	1.2	1.5	1.7	1.4	1.6

B	0.9	0.7	1.1	0.6	1.8

a) Use the North-West Corner Rule, the Lowest Cost Method and the LP method to determine the distribution plan to minimize the total distribution cost.

North West Corner Rule	**Bookstore (1)**	**Bookstore (2)**	**Bookstore (3)**	**Bookstore (4)**	**Bookstore (5)**	***Supply***
Warehouse A	1950	2250	75			**4275**
Warehouse B			1650	1350	1050	**5370**
Demand	**1950**	**2250**	**1725**	**1350**	**1050**	

North-West Corner Rule
= … = $10,357.5

Lowest / Least Cost Method	**Bookstore (1)**	**Bookstore (2)**	**Bookstore (3)**	**Bookstore (4)**	**Bookstore (5)**	***Supply***
Warehouse A	180		1725		1050	**4275**
Warehouse B	1770	2250		1350		**5370**
Demand	**1950**	**2250**	**1725**	**1350**	**1050**	

Lowest Cost Method
= … = $8,806.5

LP method
=$8,289

– Decision variables
Let X_{ij} be the number of books to be shipped from warehouse *i* to bookstore *j* (i = A,B; j = 1,2,3,4,5) – Decision variables

(E.g. X_{A1} be the number books to be shipped from warehouse A to bookstore 1)

– Objective function
Minimize Z cost = $1.2X_{A1} + 1.5X_{A2} + 1.7X_{A3} + 1.4X_{A4} + 1.6X_{A5} + 0.9X_{B1} + 0.7X_{B2} + 1.1X_{B3} + 0.6X_{B4} + 1.8X_{B5}$

- Constraints
Subject to:

Warehouse A supply:	$X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5}$	**<=** 4275
Warehouse B supply:	$X_{B1} + X_{B2} + X_{B3} + X_{B4} + X_{B5}$	**<=** 5370
Bookstore 1 demand:	$X_{A1} + X_{B1}$	= 1950
Bookstore 2 demand:	$X_{A2} + X_{B2}$	= 2250
Bookstore 3 demand:	$X_{A3} + X_{B3}$	= 1725
Bookstore 4 demand:	$X_{A4} + X_{B4}$	= 1350
Bookstore 5 demand:	$X_{A5} + X_{B5}$	= 1050
– Non-negativity	Xij >= 0 (i = A,B; j = 1,2,3,4,5)	

b) If it is impossible to ship from warehouse A to bookstore 5, how would you modify the shipping cost table to represent this impossibility?

Set the unit cost of shipping the books from Warehouse A to Bookstore 5 to be very high (in comparison with the rest of the shipping cost). For example: $99999/book. Also known as the big M method in LP formulation.