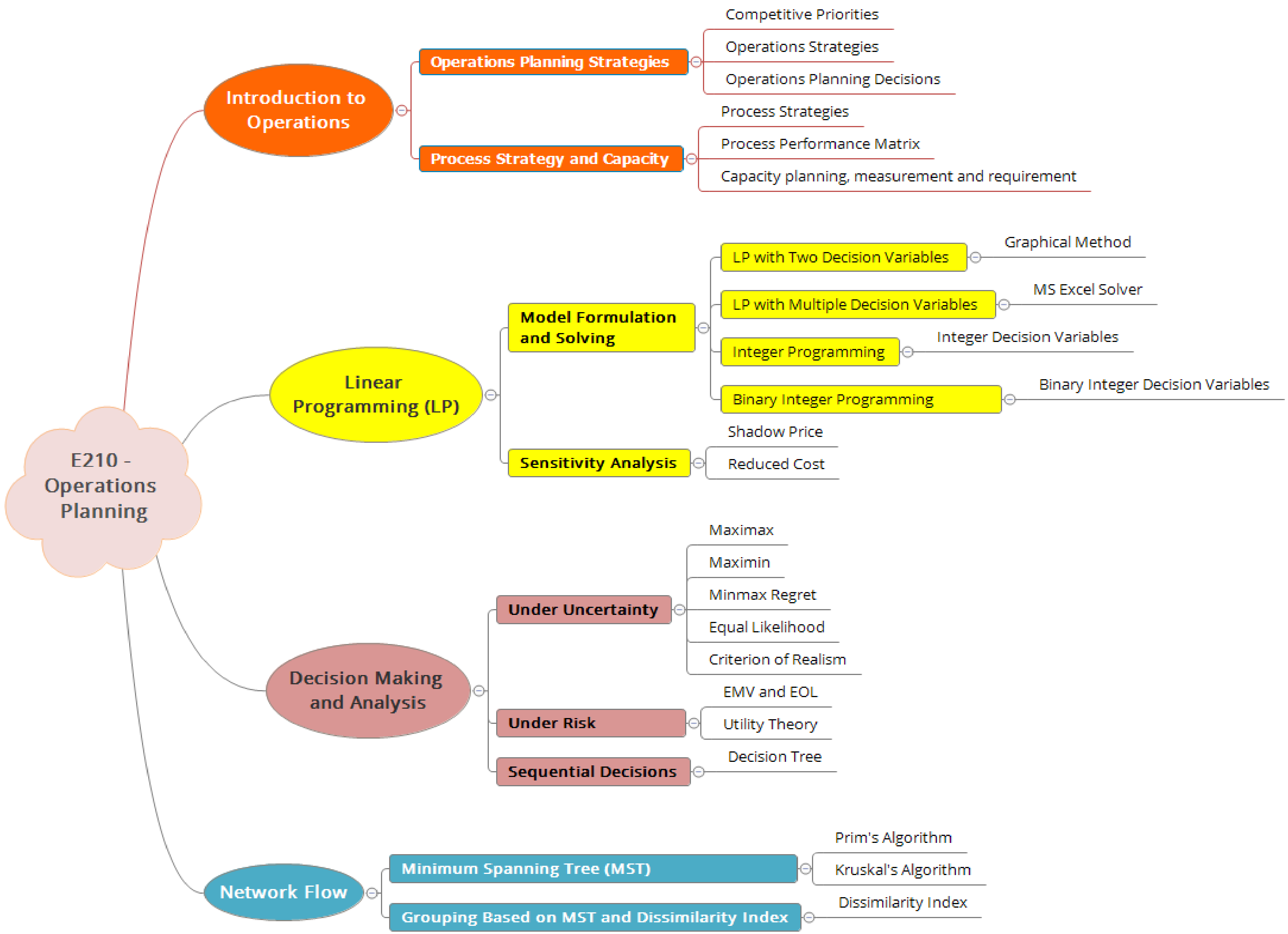
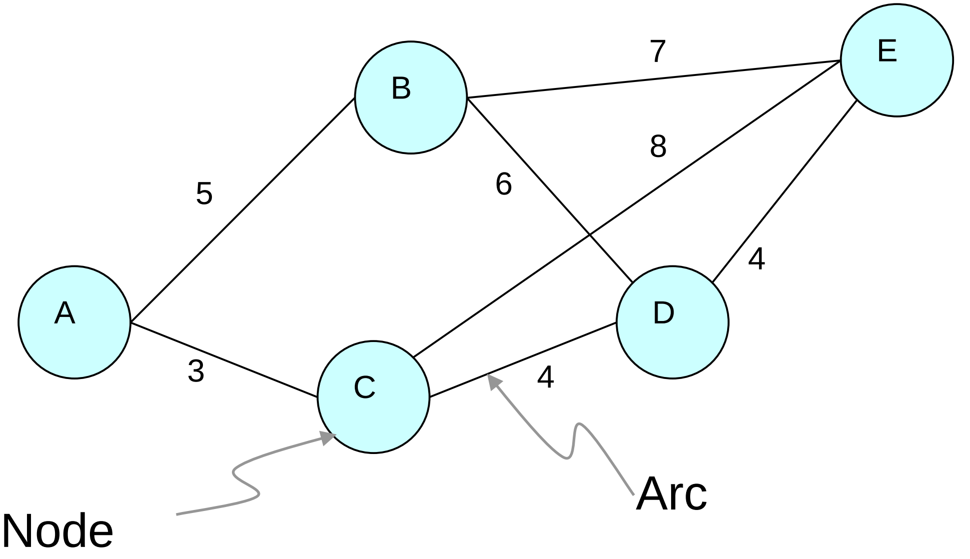
Problem 12
Network of Lights
E210 – Operations Planning

SCHOOL OF ENGINEERING

E210 Operations Planning Topic Tree



Network Definitions



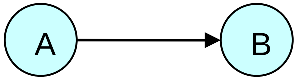
A network consists of:

- A set of points - nodes (or vertices) and
- A set of lines connecting certain pairs of the nodes – arcs (or links, branches, edges).

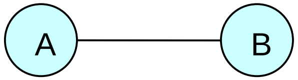
Network Definitions - Arcs

- Arcs are labeled by naming the nodes at either end, e.g. AB

- The arcs of a network may have a flow of some type through them
- If flow through an arc is allowed in only one direction (e.g. one-way street), the arc is said to be a directed arc.

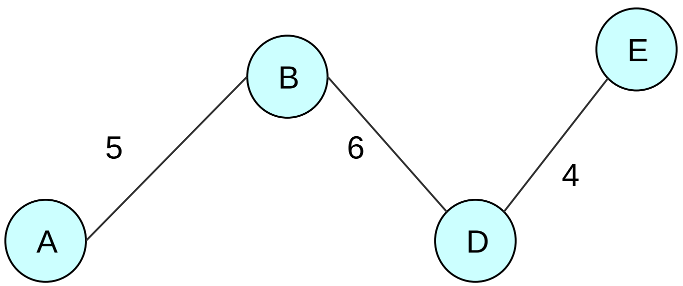


- If flow through an arc is allowed in either direction, the arc is said to be undirected.

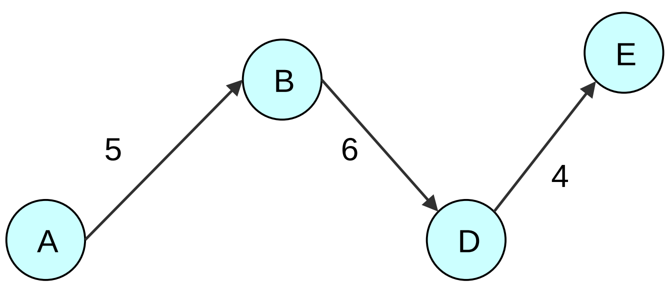


Network Definition - Path

- A path between two nodes is a sequence of distinct arcs connecting nodes that are not connected by an arc. E.g. path between nodes A and E, AB $\rightarrow$ BD$\rightarrow$DE

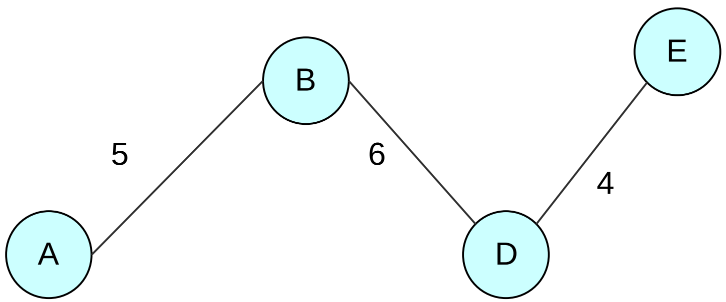


- A directed path from node *i* to node *j* is a sequence of connecting arcs whose direction (if any) is towards node *j*, so that flow from node *i* to node *j* along this path is feasible.

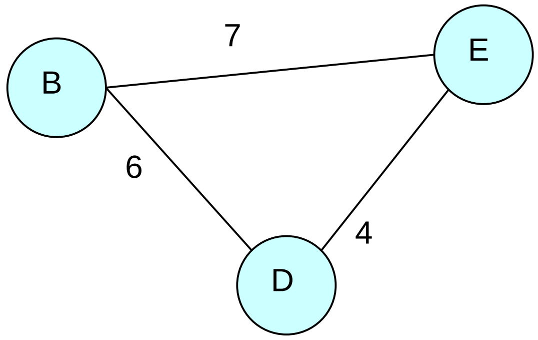


Network Definition - Path

- An undirected path from node *i* to node *j* is a sequence of connecting arcs whose direction (if any) can be either toward or away from node *j*.



- A path that begins and ends at the same node is called a cycle.



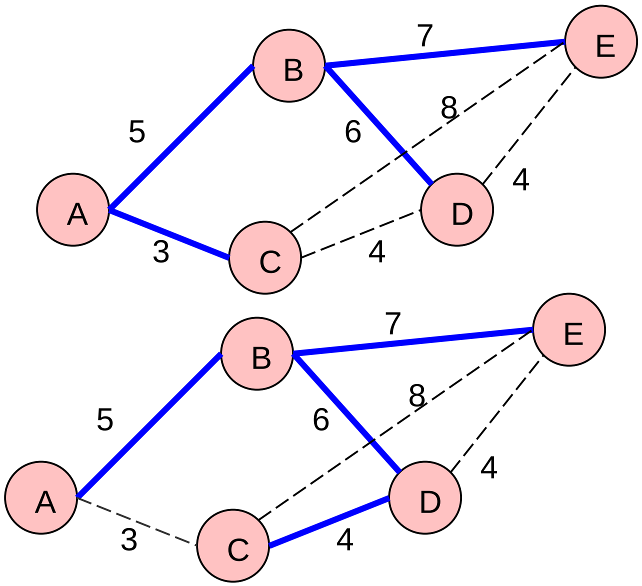
Network Definition

- Two nodes that are connected by an arc are called adjacent nodes.
- Two nodes are said to be connected if the network contains at least one undirected path between them.
- A connected network is a network where every pair of nodes is connected.

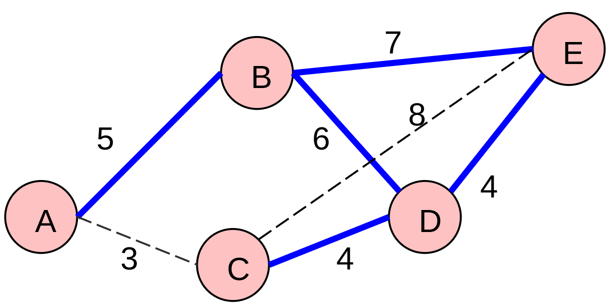
Components of Typical Networks

Nodes	**Arcs**	**Flow**
Intersections	Roads	Vehicles
Airports	Air lanes	Aircraft
Switching points	Wires, Channels	Messages
Pumping stations	Pipes	Fluids
Work Centres	Materials-handling route	Jobs

Spanning Tree



Examples of spanning tree (in blue)



Not a spanning tree, has cycle BDE

A spanning tree is a tree composed of all the nodes and some of the arcs in the network. Two nodes are connected by exactly one path, **<u>no cycles</u>** are present in a spanning tree.

Minimum Spanning Tree

- The Minimum Spanning Tree (MST) problem is to connect all nodes in a network so that the total branch lengths are minimized.
- There are two algorithms commonly used, Prim’s algorithm and Kruskal’s algorithm, both of which are greedy algorithms.

Prim’s Algorithm

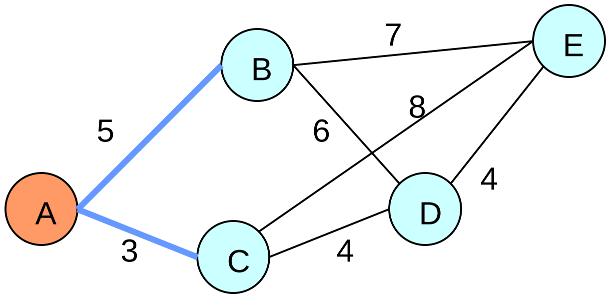
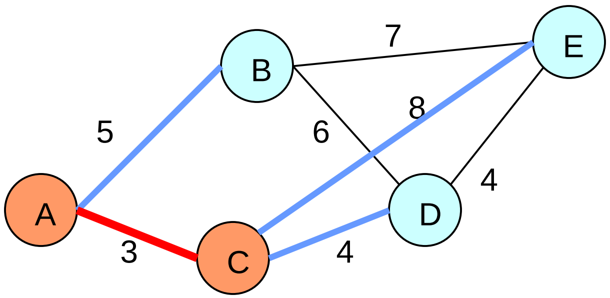
Step 1: Select any starting node

Step 2: Select the node closest to the starting node to join the spanning tree

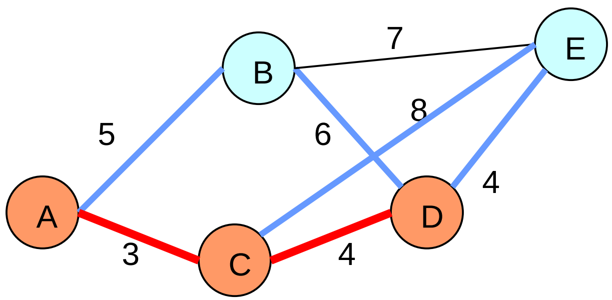
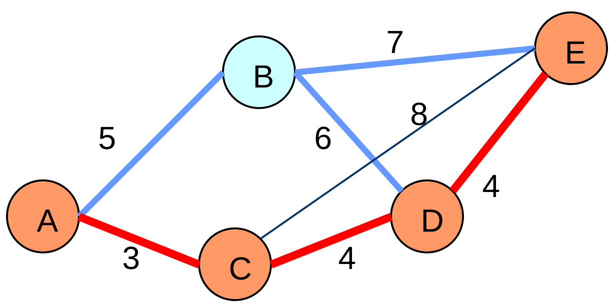
Step 3: Select the closest node not presently in the spanning tree

Step 4: Repeat Step 3 until all nodes have joined the spanning tree

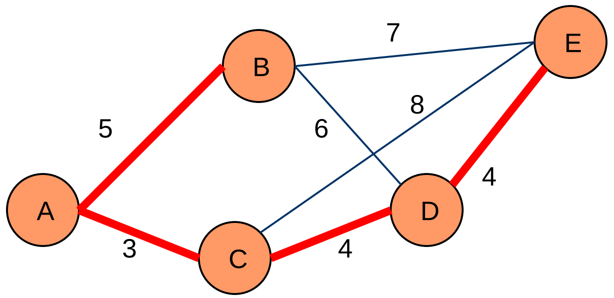
Prim’s Algorithm - Example

Network	Description	Solution Set	Fringe
	Node A is selected as a starting node for convenience	A	B, C
	Node C is added to the solution set as it is the closest to the starting node, node A. Fringe nodes are nodes not connected to nodes in the solution set.	AC	B, D, E

Prim’s Algorithm - Example

Network	Description	Solution Set	Fringe
	Node D is added to the solution set as it is closest to nodes in the solution set (A and C).	AC, CD	B, E
	Node E is added to the solution set as it is closest to nodes in the solution set (A, C, D) Arc CE becomes redundant as node C is connected to node E via path CD -> DE	AC, CD, DE	B

Prim’s Algorithm - Example

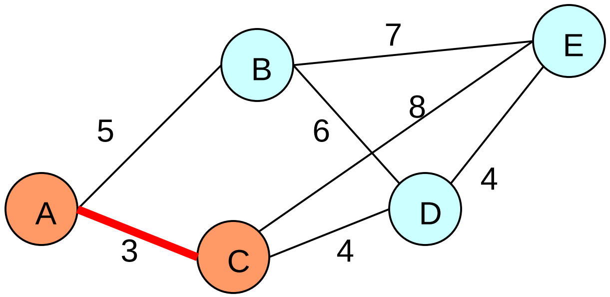
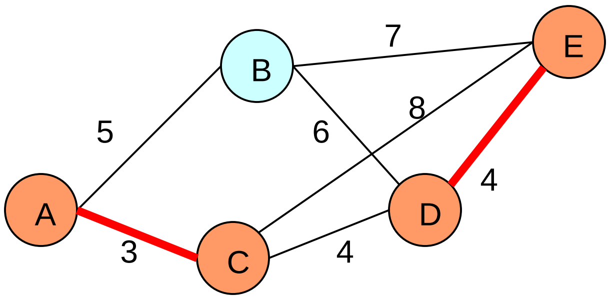
Network	Description	Solution Set	Fringe
	Node B is added to the solution set as it is closest to nodes in the solution set (A, C, D, E). Since all the nodes have joined the solution set, stop the algorithm. The minimum spanning tree is highlighted in red.	AC, CD, DE, AB	

Minimum length:
3(AC)+4(CD)+4(DE)+5(AB) = 16

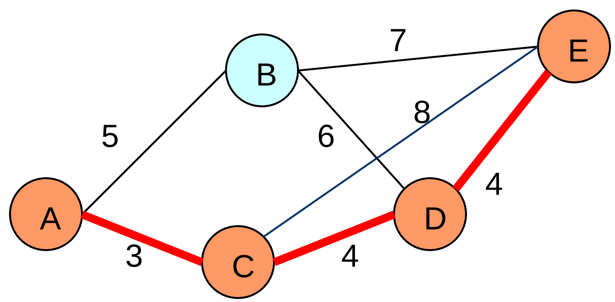
Kruskal’s Algorithm

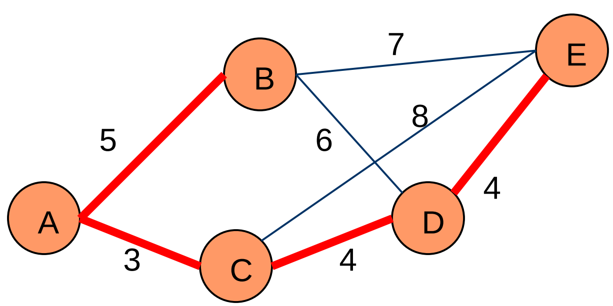
- Kruskal’s algorithm is to find the set of arcs that results in the minimum cost.
- At each stage, add the shortest arc that connects two nodes that are not originally connected into the solution set.

Kruskal’s Algorithm - Example

Network	Description	Solution Set
	Shortest arc AC is added into the solution set.	AC
	CD and DE are the shortest arcs, with lengths 4, and arc DE has been arbitrarily chosen to be added to the solution set.	AC, DE

Kruskal’s Algorithm - Example

<table>
<tr><th>Network</th><th>Description</th><th>Solution Set</th></tr>
<tr><td>
</td><td>Shortest arc CD is added to the solution set.

Arc CE is marked as a redundant arc as it would form a cycle CD -> DE -> EC</td><td>AC, DE, CD</td></tr>
<tr><td>
</td><td>Shortest arc AB is added to solution set.

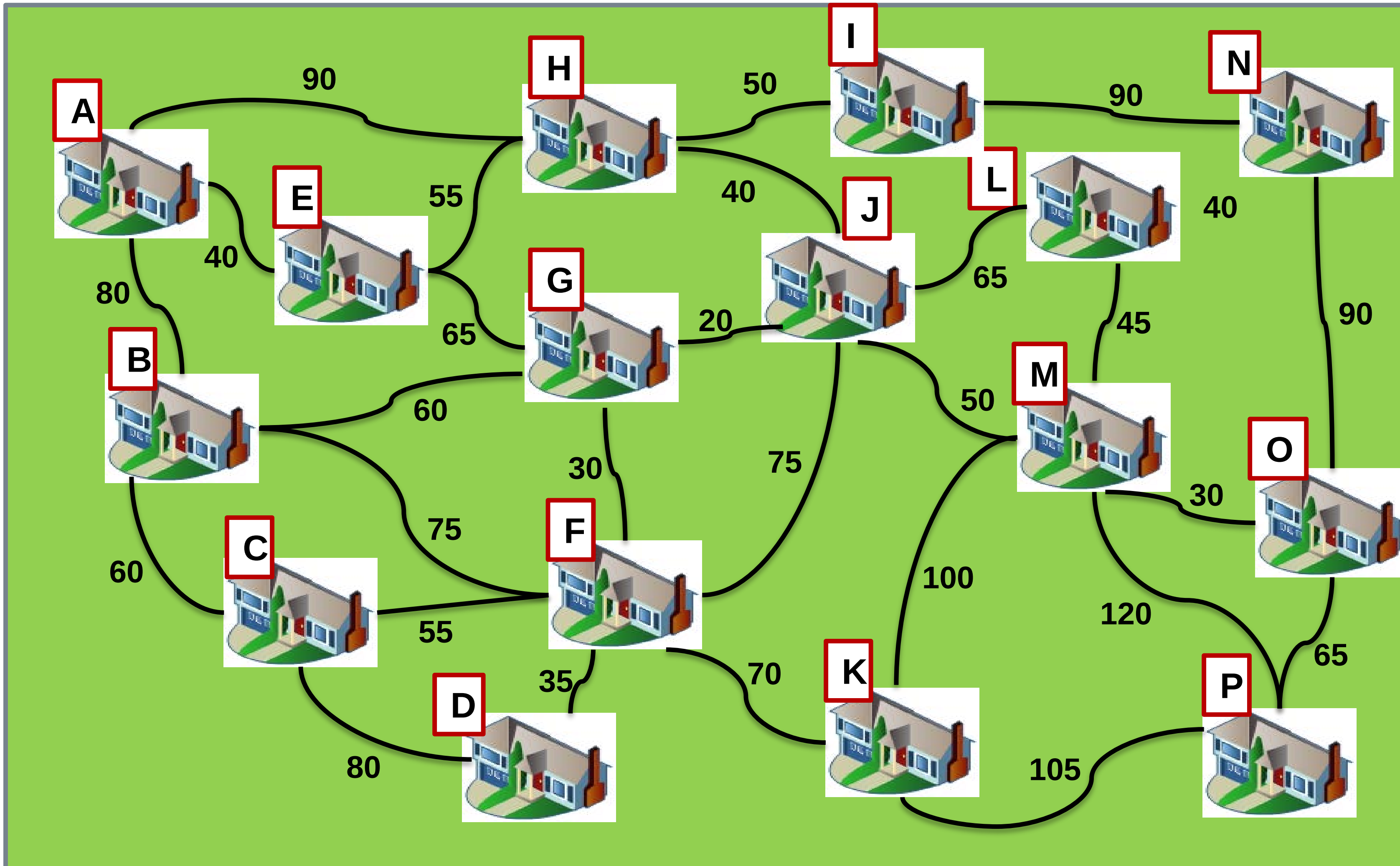
Minimum spanning tree is found. Compare this tree with the tree found using Prim’s algorithm.</td><td>AC, DE, CD, AB</td></tr>
</table>

Minimum length:
3(AC)+4(DE)+4(CD)+5(AB) = 16

Problem 12
Suggested Solution

Today's Problem: Problem Discussion

Given topographical map:

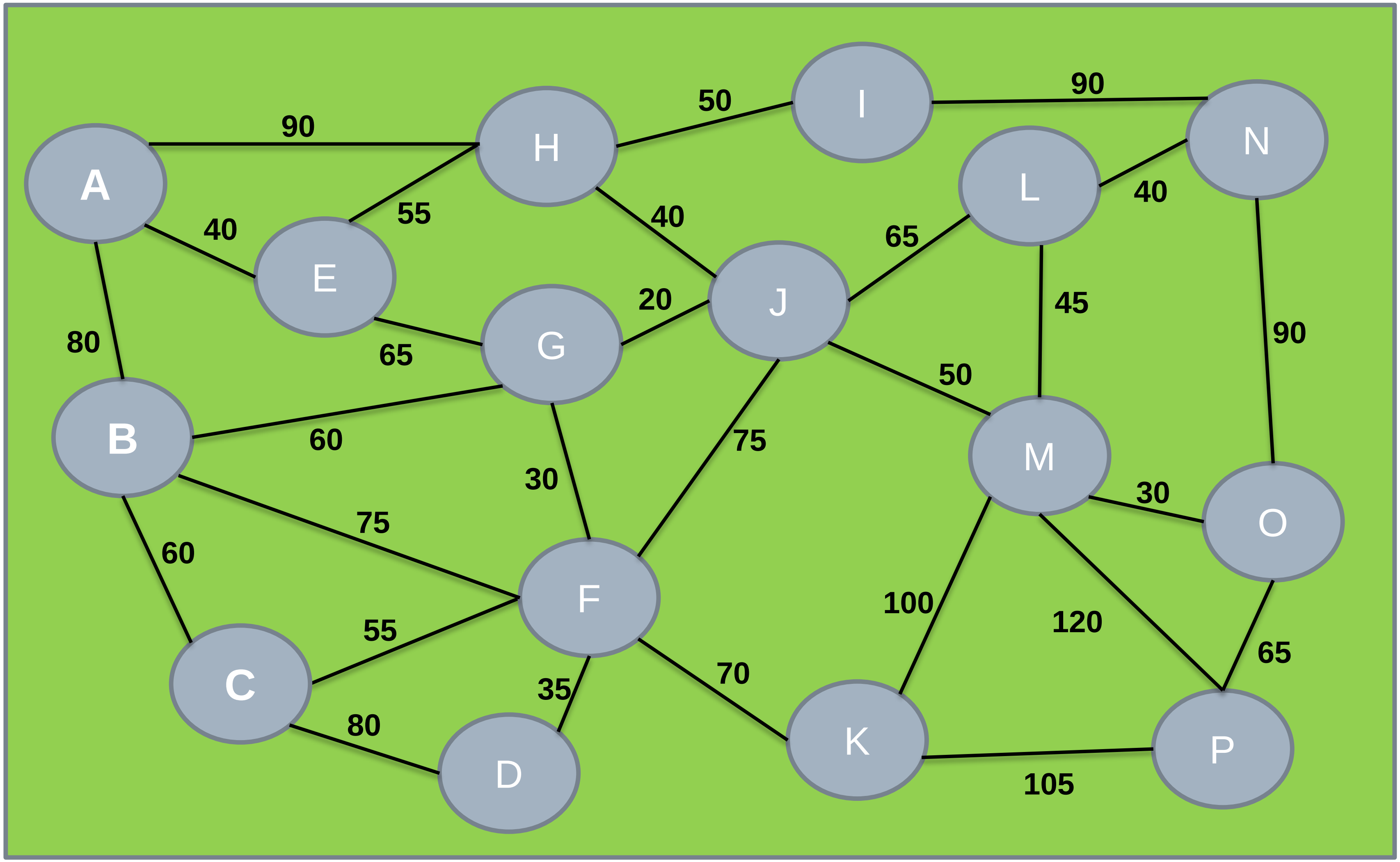


Today’s Problem: Network Modelling

- The LED lights connection between buildings in Orchard Road can be represented as a network model.
- Each building is represented by a node.
- The LED rope lights routes are represented by arcs.
- The values on the arc represent the length of LED rope lights needed.
- Constraint:
  - Supplier can only supply a maximum of 700m of LED rope lights at a cost of $18 per meter.

Network Model Representation

Translated Network Model:



Minimum Spanning Tree

- The problem can be solved as a minimum spanning tree problem using the Prim's Algorithm or the Kruskal's Algorithm.
- The resulting minimum spanning tree is connected by the **arcs in bold**.

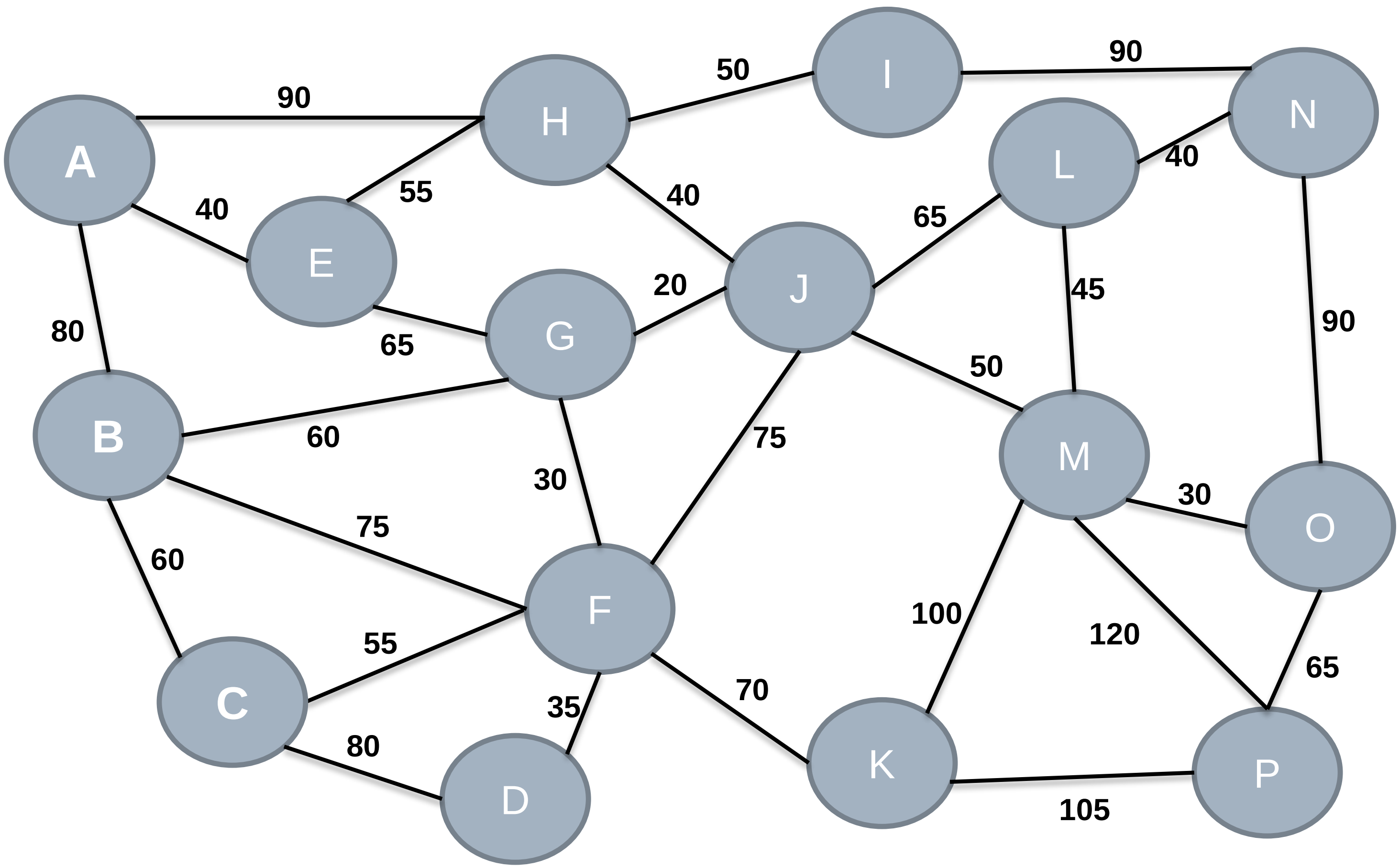
Note:

- For a network with *n* nodes, we need *n-1* arcs to create the minimum spanning tree
- There are multiple minimum spanning trees in this problem. In general, when the weightage of arcs are not unique, multiple optimal solutions may exist.

Today's Solution: Prim's Algorithm

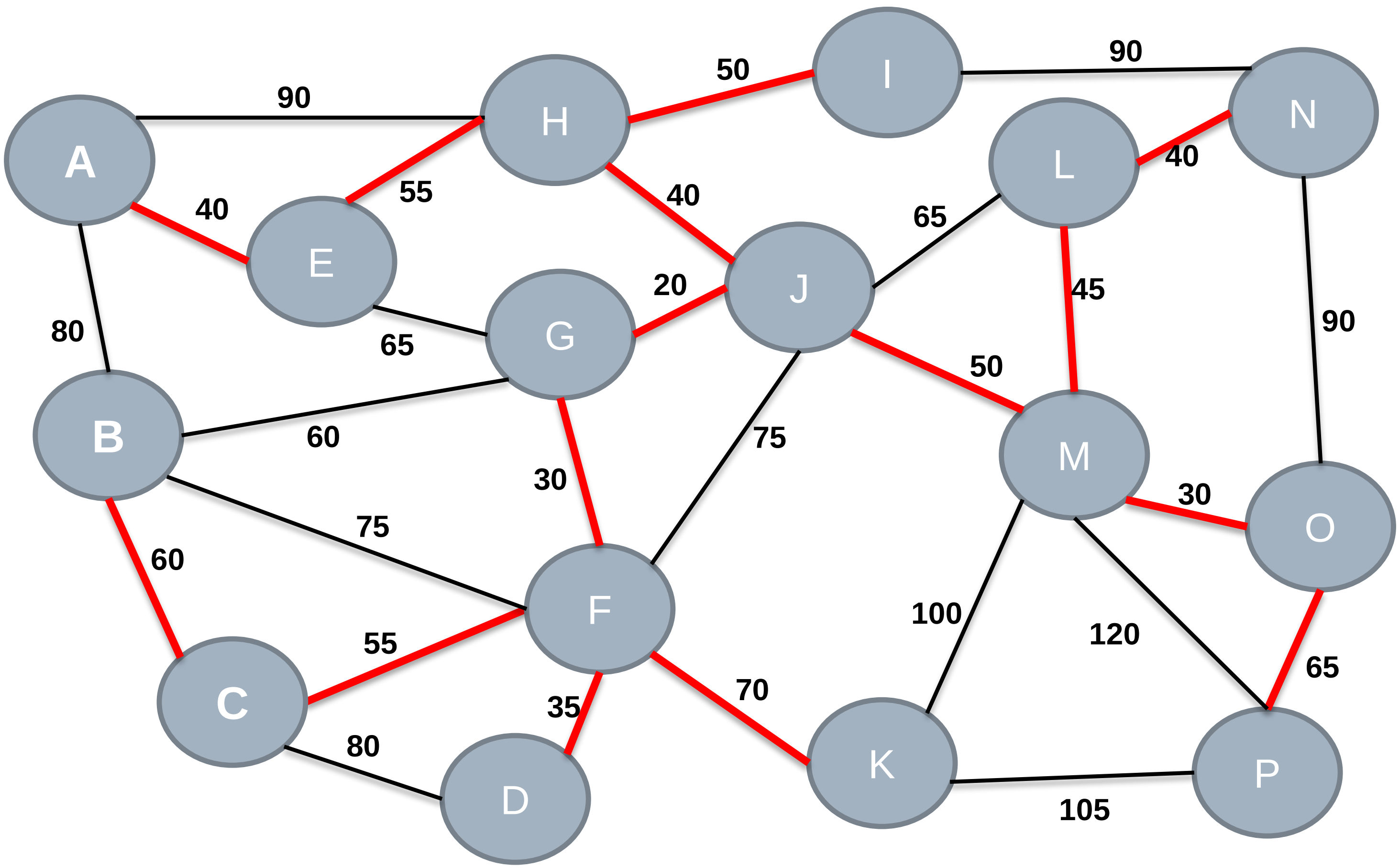
Starting from Node A,
<u>Sequence</u>
Nodes A – E

.
.
.
.
.
.
.

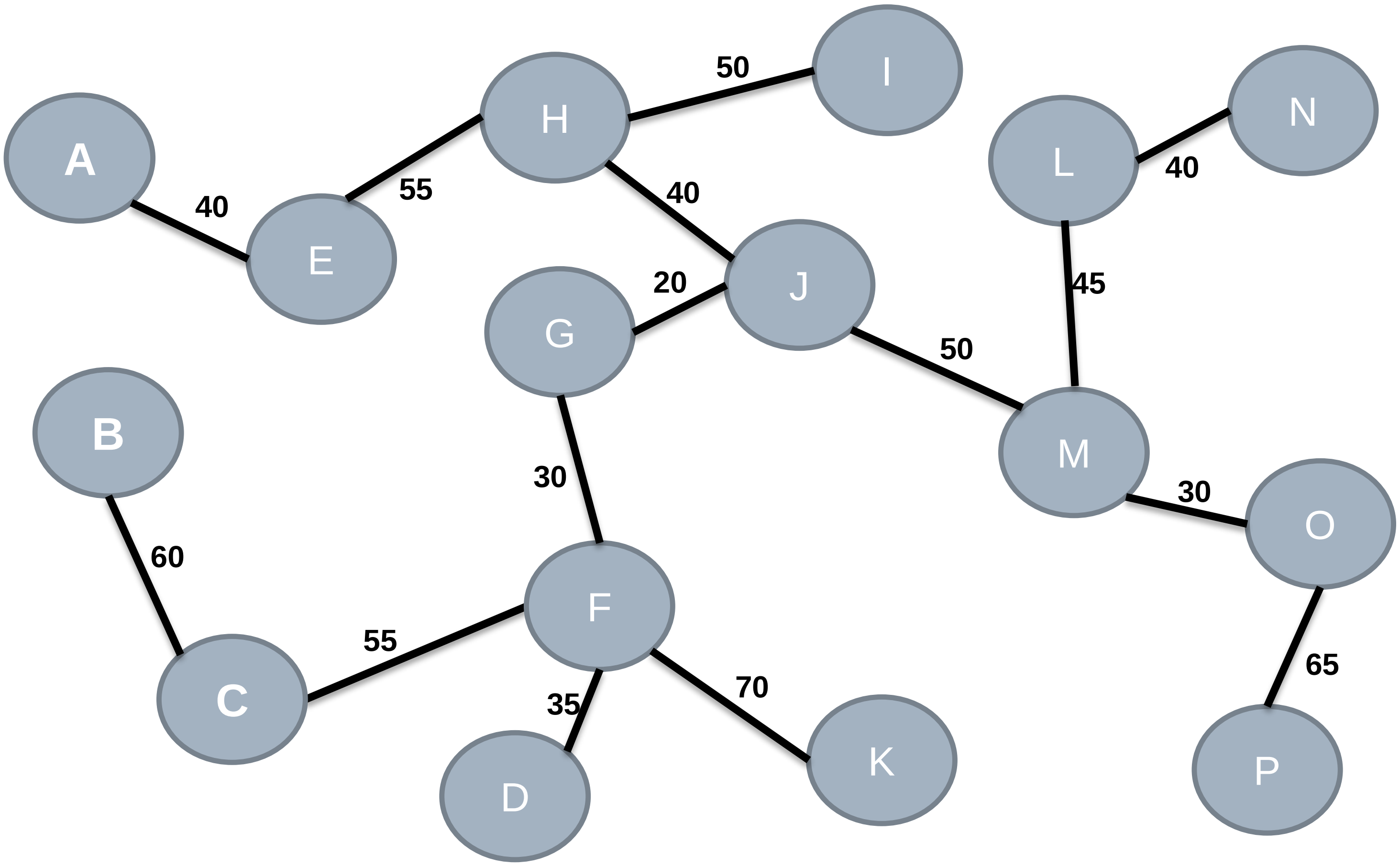


Today's Solution: Prim's Algorithm

Starting from Node A,
<u>Sequence</u>
Nodes A – E
Nodes E – H
Nodes H – J
Nodes J – G
Nodes G – F
Nodes F – D
Nodes H – I
Nodes J – M
Nodes M – O
Nodes M – L
Nodes L – N
Nodes F – C
Nodes C – B
(or Nodes G – B)
Nodes O – P
Nodes F – K



Today’s Solution: Prim’s Algorithm

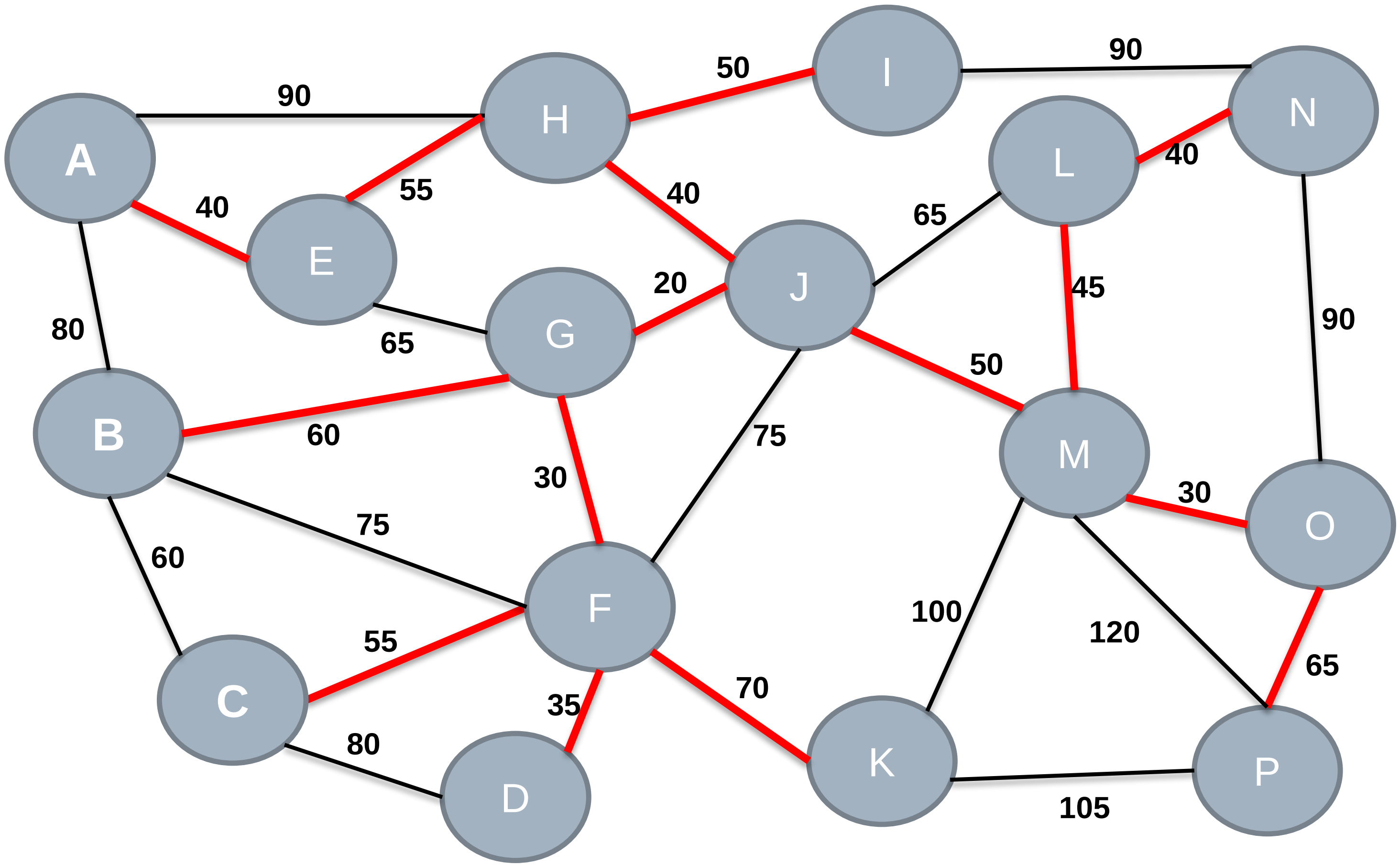


Network length = 685m

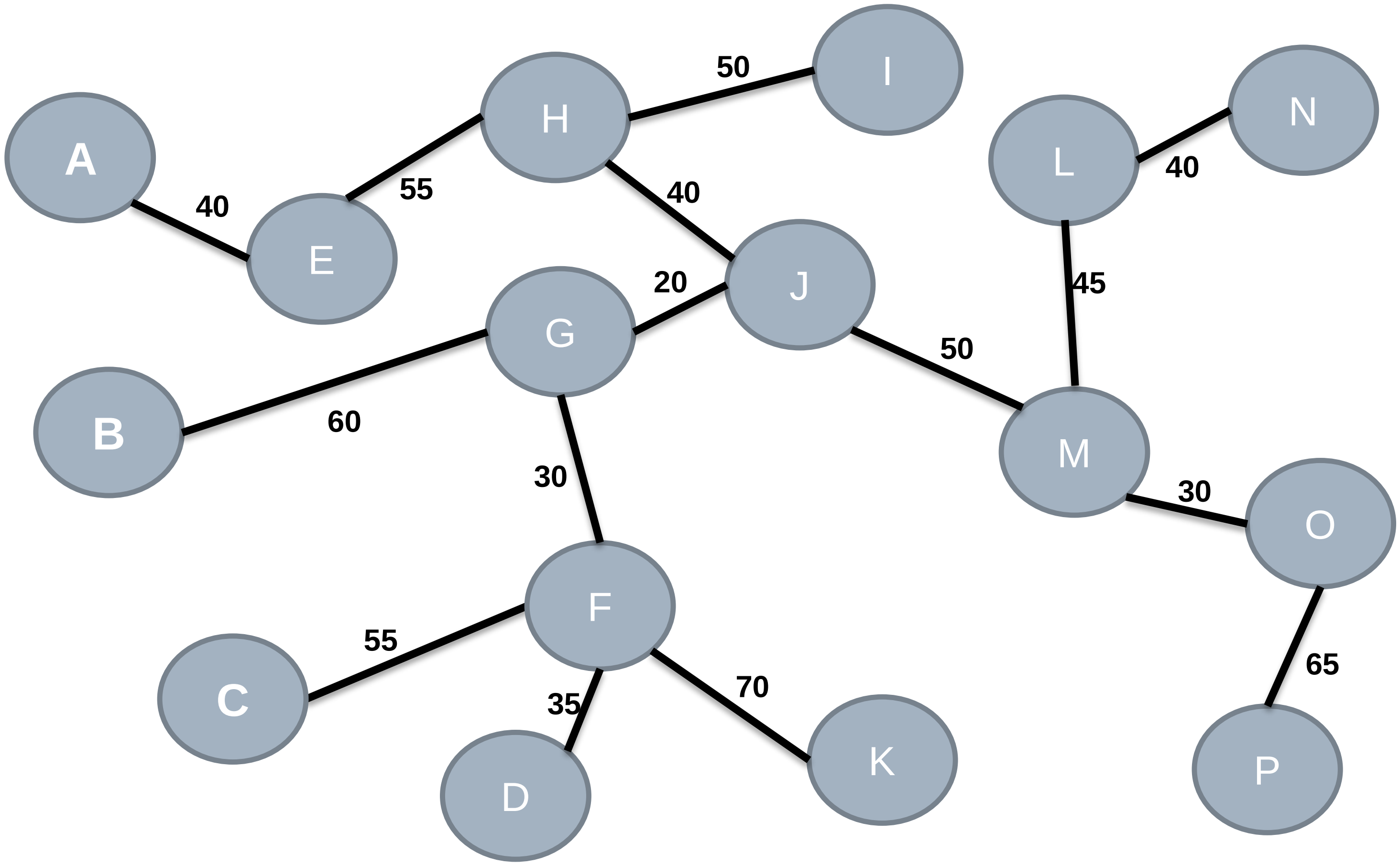
Today's Solution: Kruskal's Algorithm

Sequence

Nodes G – J
Nodes G – F
Nodes M – O
Nodes F – D
Nodes H – J
Nodes A – E
Nodes L – N
Nodes L – M
Nodes J – M
Nodes H – I
Nodes H – E
Nodes F – C
Nodes G – B
(or Nodes C – B)
Nodes O – P
Nodes F – K



Today's Solution: Kruskal's Algorithm



Network length = 685m

Note that there is more than 1 optimal solution for this problem.

LED Roping Lights Length and Cost

- The minimum length of LED rope lights to connect all the buildings is 685 meters
  - It is the total length obtained from summing the length of the arcs of the minimum spanning tree.
- There is a stock of 700 meters available.
- The lowest cost of the LED lights installation is $18/m x 685m = $12,330.

Conclusion

- The aim of the Minimum Spanning Tree problem is to connect all nodes in a network so that the total branch lengths are minimized.
- Two algorithms can be used to solve the Minimum Spanning Tree problems
  - Prim's Algorithm: start with a starting node;
  - Kruskal's Algorithm: start with the shortest arc;
- Both methods will give the same optimal value, but there may be multiple optimal solutions if the arc weightages are not unique.

Learning Objectives

- Define a network using nodes and arcs
- Identify the Minimum Spanning Tree problem and its applications
- Apply Prim’s and Kruskal’s algorithms to solve typical minimum spanning tree problems.

Overview of E210 Operation Planning Module