The GraphBLAS C API Specification $^{\dagger}:$

Version 2.1

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- ³ [Scott: THIS IS A DRAFT VERION. Update acks and remove DRAFT before release.]
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 $^{^{\}dagger} \textsc{Based}$ on GraphBLAS Mathematics by Jeremy Kepner

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$_{\scriptscriptstyle{173}}$ Chapter 1

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4 Introduction

The GraphBLAS standard defines a set of matrix and vector operations based on semiring algebraic structures. These operations can be used to express a wide range of graph algorithms. This document defines the C binding to the GraphBLAS standard. We refer to this as the GraphBLAS CAPI (Application Programming Interface).

The GraphBLAS C API is built on a collection of objects exposed to the C programmer as opaque data types. Functions that manipulate these objects are referred to as *methods*. These methods fully define the interface to GraphBLAS objects to create or destroy them, modify their contents, and copy the contents of opaque objects into non-opaque objects; the contents of which are under direct control of the programmer.

The GraphBLAS C API is designed to work with C99 (ISO/IEC 9899:199) extended with *static* type-based and number of parameters-based function polymorphism, and language extensions on par with the _Generic construct from C11 (ISO/IEC 9899:2011). Furthermore, the standard assumes programs using the GraphBLAS C API will execute on hardware that supports floating point arithmetic such as that defined by the IEEE 754 (IEEE 754-2008) standard.

The GraphBLAS C API assumes programs will run on a system that supports acquire-release memory orders. This is needed to support the memory models required for multithreaded execution as described in section 2.5.2.

Implementations of the GraphBLAS C API will target a wide range of platforms. We expect cases will arise where it will be prohibitive for a platform to support a particular type or a specific parameter for a method defined by the GraphBLAS C API. We want to encourage implementors to support the GraphBLAS C API even when such cases arise. Hence, an implementation may still call itself "conformant" as long as the following conditions hold.

- Every method and operation from chapter 4 is supported for the vast majority of cases.
- Any cases not supported must be documented as an implementation-defined feature of the GraphBLAS implementation. Unsupported cases must be caught as an API error (section 2.6) with the parameter GrB_NOT_IMPLEMENTED returned by the associated method call.
- It is permissible to omit the corresponding nonpolymorphic methods from chapter 5 when it

is not possible to express the signature of that method.

The number of allowed omitted cases is vague by design. We cannot anticipate the features of target platforms, on the market today or in the future, that might cause problems for the GraphBLAS specification. It is our expectation, however, that such omitted cases would be a minuscule fraction of the total combination of methods, types, and parameters defined by the GraphBLAS C API specification.

The remainder of this document is organized as follows:

- Chapter 2: Basic Concepts
- Chapter 3: Objects

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- Chapter 4: Methods
- Chapter 5: Nonpolymorphic interface
- Appendix A: Revision history
- Appendix B: Non-opaque data format definitions
- Appendix C: Examples

Chapter 2

Basic concepts

- The GraphBLAS C API is used to construct graph algorithms expressed "in the language of linear algebra." Graphs are expressed as matrices, and the operations over these matrices are generalized through the use of a semiring algebraic structure.
- In this chapter, we will define the basic concepts used to define the GraphBLAS C API. We provide the following elements:
- Glossary of terms and notation used in this document.
- Algebraic structures and associated arithmetic foundations of the API.
- Functions that appear in the GraphBLAS algebraic structures and how they are managed.
- Domains of elements in the GraphBLAS.
- Indices, index arrays, scalar arrays, and external matrix formats used to expose the contents of GraphBLAS objects.
- The GraphBLAS opaque objects.
- The execution and error models implied by the GraphBLAS C specification.
- Enumerations used by the API and their values.

$_{332}$ 2.1 Glossary

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333 2.1.1 GraphBLAS API basic definitions

- application: A program that calls methods from the GraphBLAS C API to solve a problem.
- GraphBLAS C API: The application programming interface that fully defines the types, objects, literals, and other elements of the C binding to the GraphBLAS.

- function: Refers to a named group of statements in the C programming language. Methods, operators, and user-defined functions are typically implemented as C functions. When referring to the code programmers write, as opposed to the role of functions as an element of the GraphBLAS, they may be referred to as such.
- method: A function defined in the GraphBLAS C API that manipulates GraphBLAS objects or other opaque features of the implementation of the GraphBLAS API.
- operator: A function that performs an operation on the elements stored in GraphBLAS matrices and vectors.
 - GraphBLAS operation: A mathematical operation defined in the GraphBLAS mathematical specification. These operations (not to be confused with operators) typically act on matrices and vectors with elements defined in terms of an algebraic semiring.

348 2.1.2 GraphBLAS objects and their structure

- non-opaque datatype: Any datatype that exposes its internal structure and can be manipulated directly by the user.
- opaque datatype: Any datatype that hides its internal structure and can be manipulated only through an API.
 - GraphBLAS object: An instance of an opaque datatype defined by the GraphBLAS C API that is manipulated only through the GraphBLAS API. There are four kinds of GraphBLAS opaque objects: domains (i.e., types), algebraic objects (operators, monoids and semirings), collections (scalars, vectors, matrices and masks), and descriptors.
 - handle: A variable that holds a reference to an instance of one of the GraphBLAS opaque objects. The value of this variable holds a reference to a GraphBLAS object but not the contents of the object itself. Hence, assigning a value to another variable copies the reference to the GraphBLAS object of one handle but not the contents of the object.
 - domain: The set of valid values for the elements stored in a GraphBLAS collection or operated on by a GraphBLAS operator. Note that some GraphBLAS objects involve functions that map values from one or more input domains onto values in an output domain. These GraphBLAS objects would have multiple domains.
 - collection: An opaque GraphBLAS object that holds a number of elements from a specified domain. Because these objects are based on an opaque datatype, an implementation of the GraphBLAS C API has the flexibility to optimize the data structures for a particular platform. GraphBLAS objects are often implemented as sparse data structures, meaning only the subset of the elements that have values are stored.
 - *implied zero*: Any element that has a valid index (or indices) in a GraphBLAS vector or matrix but is not explicitly identified in the list of elements of that vector or matrix. From a mathematical perspective, an *implied zero* is treated as having the value of the zero element of the relevant monoid or semiring. However, GraphBLAS operations are purposefully defined

- using set notation in such a way that it makes it unnecessary to reason about implied zeros.

 Therefore, this concept is not used in the definition of GraphBLAS methods and operators.
 - mask: An internal GraphBLAS object used to control how values are stored in a method's output object. The mask exists only inside a method; hence, it is called an *internal opaque object*. A mask is formed from the elements of a collection object (vector or matrix) input as a mask parameter to a method. GraphBLAS allows two types of masks:
 - 1. In the default case, an element of the mask exists for each element that exists in the input collection object when the value of that element, when cast to a Boolean type, evaluates to true.
 - 2. In the *structure only* case, masks have structure but no values. The input collection describes a structure whereby an element of the mask exists for each element stored in the input collection regardless of its value.
 - complement: The complement of a GraphBLAS mask, M, is another mask, M', where the elements of M' are those elements from M that do not exist.

2.1.3 Algebraic structures used in the GraphBLAS

- associative operator: In an expression where a binary operator is used two or more times consecutively, that operator is associative if the result does not change regardless of the way operations are grouped (without changing their order). In other words, in a sequence of binary operations using the same associative operator, the legal placement of parenthesis does not change the value resulting from the sequence operations. Operators that are associative over infinitely precise numbers (e.g., real numbers) are not strictly associative when applied to numbers with finite precision (e.g., floating point numbers). Such non-associativity results, for example, from roundoff errors or from the fact some numbers can not be represented exactly as floating point numbers. In the GraphBLAS specification, as is common practice in computing, we refer to operators as associative when their mathematical definition over infinitely precise numbers is associative even when they are only approximately associative when applied to finite precision numbers.
 - No GraphBLAS method will imply a predefined grouping over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.
- commutative operator: In an expression where a binary operator is used (usually two or more times consecutively), that operator is commutative if the result does not change regardless of the order the inputs are operated on.
- No GraphBLAS method will imply a predefined ordering over any commutative operators. Implementations of the GraphBLAS are encouraged to exploit commutativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the commutative operations.

• GraphBLAS operators: Binary or unary operators that act on elements of GraphBLAS objects. GraphBLAS operators are used to express algebraic structures used in the GraphBLAS such as monoids and semirings. They are also used as arguments to several GraphBLAS methods. There are two types of GraphBLAS operators: (1) predefined operators found in Table 3.5 and (2) user-defined operators created using GrB_UnaryOp_new() or GrB_BinaryOp_new() (see Section 4.2.2).

- monoid: An algebraic structure consisting of one domain, an associative binary operator, and the identity of that operator. There are two types of GraphBLAS monoids: (1) predefined monoids found in Table 3.7 and (2) user-defined monoids created using GrB_Monoid_new() (see Section 4.2.2).
 - semiring: An algebraic structure consisting of a set of allowed values (the domain), a commutative and associative binary operator called addition, a binary operator called multiplication (where multiplication distributes over addition), and identities over addition (θ) and multiplication (1). The additive identity is an annihilator over multiplication.
 - GraphBLAS semiring: is allowed to diverge from the mathematically rigorous definition of a semiring since certain combinations of domains, operators, and identity elements are useful in graph algorithms even when they do not strictly match the mathematical definition of a semiring. There are two types of GraphBLAS semirings: (1) predefined semirings found in Tables 3.8 and 3.9, and (2) user-defined semirings created using GrB_Semiring_new() (see Section 4.2.2).
- index unary operator: A variation of the unary operator that operates on elements of GraphBLAS vectors and matrices along with the index values representing their location in the objects. There are predefined index unary operators found in Table 3.6), and user-defined operators created using GrB_IndexUnaryOp_new (see Section 4.2.2).

5 2.1.4 The execution of an application using the GraphBLAS C API

- program order: The order of the GraphBLAS method calls in a thread, as defined by the text of the program.
 - host programming environment: The GraphBLAS specification defines an API. The functions from the API appear in a program. This program is written using a programming language and execution environment defined outside of the GraphBLAS. We refer to this programming environment as the "host programming environment".
- execution time: time expended while executing instructions defined by a program. This term is specifically used in this specification in the context of computations carried out on behalf of a call to a GraphBLAS method.
 - sequence: A GraphBLAS application uniquely defines a directed acyclic graph (DAG) of GraphBLAS method calls based on their program order. At any point in a program, the state of any GraphBLAS object is defined by a subgraph of that DAG. An ordered collection of GraphBLAS method calls in program order that defines that subgraph for a particular object is the sequence for that object.

• complete: A GraphBLAS object is complete when it can be used in a happens-before relationship with a method call that reads the variable on another thread. This concept is used when reasoning about memory orders in multithreaded programs. A GraphBLAS object defined on one thread that is complete can be safely used as an IN or INOUT argument in a method-call on a second thread assuming the method calls are correctly synchronized so the definition on the first thread happens-before it is used on the second thread. In blocking-mode, an object is complete after a GraphBLAS method call that writes to that object returns. In nonblocking-mode, an object is complete after a call to the GrB_wait() method with the GrB_COMPLETE parameter.

- materialize: A GraphBLAS object is materialized when it is (1) complete, (2) the computations defined by the sequence that define the object have finished (either fully or stopped at an error) and will not consume any additional computational resources, and (3) any errors associated with that sequence are available to be read according to the GraphBLAS error model. A GraphBLAS object that is never loaded into a non-opaque data structure may potentially never be materialized. This might happen, for example, if the operations associated with the object are fused or otherwise changed by the runtime system that supports the implementation of the GraphBLAS C API. An object can be materialized by a call to the materialize mode of the GrB_wait() method.
- context: An instance of the GraphBLAS C API implementation as seen by an application. An application can have only one context between the start and end of the application. A context begins with the first thread that calls GrB_init() and ends with the first thread to call GrB_finalize(). It is an error for GrB_init() or GrB_finalize() to be called more than one time within an application. The context is used to constrain the behavior of an instance of the GraphBLAS C API implementation and support various execution strategies. Currently, the only supported constraints on a context pertain to the mode of program execution.
- program execution mode: Defines how a GraphBLAS sequence executes, and is associated with the context of a GraphBLAS C API implementation. It is set by an application with its call to GrB_init() to one of two possible states. In blocking mode, GraphBLAS methods return after the computations complete and any output objects have been materialized. In nonblocking mode, a method may return once the arguments are tested as consistent with the method (i.e., there are no API errors), and potentially before any computation has taken place.

2.1.5 GraphBLAS methods: behaviors and error conditions

- implementation-defined behavior: Behavior that must be documented by the implementation and is allowed to vary among different compliant implementations.
 - undefined behavior: Behavior that is not specified by the GraphBLAS C API. A conforming implementation is free to choose results delivered from a method whose behavior is undefined.
 - thread-safe: Consider a function called from multiple threads with arguments that do not overlap in memory (i.e. the argument lists do not share memory). If the function is thread-safe

then it will behave the same when executed concurrently by multiple threads or sequentially on a single thread.

- dimension compatible: GraphBLAS objects (matrices and vectors) that are passed as parameters to a GraphBLAS method are dimension (or shape) compatible if they have the correct number of dimensions and sizes for each dimension to satisfy the rules of the mathematical definition of the operation associated with the method. If any dimension compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB_DIMENSION_MISMATCH error is returned.
- domain compatible: Two domains for which values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other, and a domain from a user-defined type is only compatible with itself. If any domain compatibility rule above is violated, execution of the GraphBLAS method ends and the GrB_DOMAIN_MISMATCH error is returned.

503 2.2 Notation

	Notation	Description
	$\overline{D_{out}, D_{in}, D_{in_1}, D_{in_2}}$	Refers to output and input domains of various GraphBLAS operators.
	$\mathbf{D}_{out}(*), \mathbf{D}_{in}(*),$	Evaluates to output and input domains of GraphBLAS operators (usually
	$\mathbf{D}_{in_1}(*), \mathbf{D}_{in_2}(*)$	a unary or binary operator, or semiring).
	$\mathbf{D}(*)$	Evaluates to the (only) domain of a GraphBLAS object (usually a monoid,
		vector, or matrix).
	f	An arbitrary unary function, usually a component of a unary operator.
	$\mathbf{f}(F_u)$	Evaluates to the unary function contained in the unary operator given as
	· · · · · ·	the argument.
	\odot	An arbitrary binary function, usually a component of a binary operator.
	$\bigcirc(*)$	Evaluates to the binary function contained in the binary operator or monoid
		given as the argument.
	\otimes	Multiplicative binary operator of a semiring.
	\oplus	Additive binary operator of a semiring.
	$\bigotimes(S)$	Evaluates to the multiplicative binary operator of the semiring given as the
	O (%)	argument.
	$\bigoplus(S)$	Evaluates to the additive binary operator of the semiring given as the argu-
	$\Phi(\sim)$	ment.
	0 (*)	The identity of a monoid, or the additive identity of a GraphBLAS semiring.
	$\mathbf{L}(*)$	The contents (all stored values) of the vector or matrix GraphBLAS objects.
	2(1)	For a vector, it is the set of (index, value) pairs, and for a matrix it is the
		set of (row, col, value) triples.
	$\mathbf{v}(i)$ or v_i	The i^{th} element of the vector \mathbf{v} .
,	$\mathbf{size}(\mathbf{v})$	The size of the vector \mathbf{v} .
	$\mathbf{ind}(\mathbf{v})$	The set of indices corresponding to the stored values of the vector \mathbf{v} .
	$\mathbf{nrows}(\mathbf{A})$	The number of rows in the A .
	$\mathbf{ncols}(\mathbf{A})$	The number of columns in the A .
	indrow(A)	The set of row indices corresponding to rows in A that have stored values.
	indcol(A)	The set of column indices corresponding to columns in A that have stored
	macor(11)	values.
	$\mathbf{ind}(\mathbf{A})$	The set of (i, j) indices corresponding to the stored values of the matrix.
	$\mathbf{A}(i,j)$ or A_{ij}	The element of A with row index i and column index j .
	$\mathbf{A}(:,j)$ or n_{ij} $\mathbf{A}(:,j)$	The element of \mathbf{A} with row index t and column index f . The j^{th} column of matrix \mathbf{A} .
	$\mathbf{A}(i,j)$	The i^{th} row of matrix \mathbf{A} .
	$\mathbf{A}^{(t,\cdot)}$	The transpose of matrix \mathbf{A} .
	$ eg \mathbf{M}$	The complement of M.
		The structure of M.
	$rac{\mathrm{s}(\mathbf{M})}{\mathbf{\widetilde{t}}}$	A temporary object created by the GraphBLAS implementation.
		A method argument type that is void * or one of the types from Table 3.2.
	< type >	
	GrB_ALL	A method argument literal to indicate that all indices of an input array should be used.
	CrD Tuno	
	GrB_Type	A method argument type that is either a user defined type or one of the
	CuD Object	types from Table 3.2.
	GrB_Object	A method argument type referencing any of the GraphBLAS object types.
	GrB_NULL	The GraphBLAS NULL.

2.3 Mathematical foundations

Graphs can be represented in terms of matrices. The values stored in these matrices correspond to attributes (often weights) of edges in the graph. Likewise, information about vertices in a graph are stored in vectors. The set of valid values that can be stored in either matrices or vectors is referred to as their domain. Matrices are usually sparse because the lack of an edge between two vertices means that nothing is stored at the corresponding location in the matrix. Vectors may be sparse or dense, or they may start out sparse and become dense as algorithms traverse the graphs.

Operations defined by the GraphBLAS C API specification operate on these matrices and vectors to carry out graph algorithms. These GraphBLAS operations are defined in terms of GraphBLAS semiring algebraic structures. Modifying the underlying semiring changes the result of an operation to support a wide range of graph algorithms. Inside a given algorithm, it is often beneficial to change the GraphBLAS semiring that applies to an operation on a matrix. This has two implications for the C binding of the GraphBLAS API.

First, it means that we define a separate object for the semiring to pass into methods. Since in many cases the full semiring is not required, we also support passing monoids or even binary operators, which means the semiring is implied rather than explicitly stated.

Second, the ability to change semirings impacts the meaning of the *implied zero* in a sparse rep-resentation of a matrix or vector. This element in real arithmetic is zero, which is the identity of the addition operator and the annihilator of the multiplication operator. As the semiring changes, this implied zero changes to the identity of the addition operator and the annihilator (if present) of the *multiplication* operator for the new semiring. Nothing changes regarding what is stored in the sparse matrix or vector, but the implied zeros within them change with respect to a particular operation. In all cases, the nature of the implied zero does not matter since the GraphBLAS C API requires that implementations treat them as nonexistent elements of the matrix or vector.

As with matrices and vectors, GraphBLAS semirings have domains associated with their inputs and outputs. The semirings in the GraphBLAS C API are defined with two domains associated with the input operands and one domain associated with output. When used in the GraphBLAS C API these domains may not match the domains of the matrices and vectors supplied in the operations.

In this case, only valid domain compatible casting is supported by the API.

The mathematical formalism for graph operations in the language of linear algebra often assumes that we can operate in the field of real numbers. However, the GraphBLAS C binding is designed for implementation on computers, which by necessity have a finite number of bits to represent numbers. Therefore, we require a conforming implementation to use floating point numbers such as those defined by the IEEE-754 standard (both single- and double-precision) wherever real numbers need to be represented. The practical implications of these finite precision numbers is that the result of a sequence of computations may vary from one execution to the next as the grouping of operands (because of associativity) within the operations changes. While techniques are known to reduce these effects, we do not require or even expect an implementation to use them as they may add

¹More information on the mathematical foundations can be found in the following paper: J. Kepner, P. Aaltonen, D. Bader, A. Buluç, F. Franchetti, J. Gilbert, D. Hutchison, M. Kumar, A. Lumsdaine, H. Meyerhenke, S. McMillan, J. Moreira, J. Owens, C. Yang, M. Zalewski, and T. Mattson. 2016, September. Mathematical foundations of the GraphBLAS. In 2016 IEEE High Performance Extreme Computing Conference (HPEC) (pp. 1-9). IEEE.

Table 2.1:	Types of	GraphBLAS	opaque	objects.

GrB_Object types	Description
GrB_Type	Scalar type.
GrB_UnaryOp	Unary operator.
$GrB_IndexUnaryOp$	Unary operator, that operates on a single value and its location index values.
GrB_BinaryOp	Binary operator.
GrB_Monoid	Monoid algebraic structure.
GrB_Semiring	A GraphBLAS semiring algebraic structure.
GrB_Scalar	One element; could be empty.
GrB_Vector	One-dimensional collection of elements; can be sparse.
GrB_Matrix	Two-dimensional collection of elements; typically sparse.
GrB_Descriptor	Descriptor object, used to modify behavior of methods (specifically
	GraphBLAS operations).

considerable overhead. In most cases, these roundoff errors are not significant. When they are significant, the problem itself is ill-conditioned and needs to be reformulated.

545 2.4 GraphBLAS opaque objects

Objects defined in the GraphBLAS standard include types (the domains of elements), collections of elements (matrices, vectors, and scalars), operators on those elements (unary, index unary, and binary operators), algebraic structures (semirings and monoids), and descriptors. GraphBLAS objects are defined as opaque types; that is, they are managed, manipulated, and accessed solely through the GraphBLAS application programming interface. This gives an implementation of the GraphBLAS C specification flexibility to optimize objects for different scenarios or to meet the needs of different hardware platforms.

A GraphBLAS opaque object is accessed through its *handle*. A handle is a variable that references an instance of one of the types from Table 2.1. An implementation of the GraphBLAS specification has a great deal of flexibility in how these handles are implemented. All that is required is that the handle corresponds to a type defined in the C language that supports assignment and comparison for equality. The GraphBLAS specification defines a literal GrB_INVALID_HANDLE that is valid for each type. Using the logical equality operator from C, it must be possible to compare a handle to GrB_INVALID_HANDLE to verify that a handle is valid.

Every GraphBLAS object has a *lifetime*, which consists of the sequence of instructions executed in program order between the *creation* and the *destruction* of the object. The GraphBLAS C API predefines a number of these objects which are created when the GraphBLAS context is initialized by a call to GrB_init and are destroyed when the GraphBLAS context is terminated by a call to GrB_finalize.

An application using the GraphBLAS API can create additional objects by declaring variables of the appropriate type from Table 2.1 for the objects it will use. Before use, the object must be initialized

with a call call to one of the object's respective *constructor* methods. Each kind of object has at least one explicit constructor method of the form GrB_*_new where '*' is replaced with the type of object (e.g., GrB_Semiring_new). Note that some objects, especially collections, have additional constructor methods such as duplication, import, or descrialization. Objects explicitly created by a call to a constructor should be destroyed by a call to GrB_free. The behavior of a program that calls GrB free on a pre-defined object is undefined.

These constructor and destructor methods are the only methods that change the value of a handle.

Hence, objects changed by these methods are passed into the method as pointers. In all other
cases, handles are not changed by the method and are passed by value. For example, even when
multiplying matrices, while the contents of the output product matrix changes, the handle for that
matrix is unchanged.

Several GraphBLAS constructor methods take other objects as input arguments and use these objects to create a new object. For all these methods, the lifetime of the created object must end strictly before the lifetime of any dependent input objects. For example, a vector constructor GrB_Vector_new takes a GrB_Type object as input. That type object must not be destroyed until after the created vector is destroyed. Similarly, a GrB_Semiring_new method takes a monoid and a binary operator as inputs. Neither of these can be destroyed until after the created semiring is destroyed.

Note that some constructor methods like GrB_Vector_dup and GrB_Matrix_dup behave differently.
In these cases, the input vector or matrix can be destroyed as soon as the call returns. However,
the original type object used to create the input vector or matrix cannot be destroyed until after
the vector or matrix created by GrB_Vector_dup or GrB_Matrix_dup is destroyed. This behavior
must hold for any chain of duplicating constructors.

Programmers using GraphBLAS handles must be careful to distinguish between a handle and the object manipulated through a handle. For example, a program may declare two GraphBLAS objects of the same type, initialize one, and then assign it to the other variable. That assignment, however, only assigns the handle to the variable. It does not create a copy of that variable (to do that, one would need to use the appropriate duplication method). If later the object is freed by calling GrB_free with the first variable, the object is destroyed and the second variable is left referencing an object that no longer exists (a so-called "dangling handle").

In addition to opaque objects manipulated through handles, the GraphBLAS C API defines an additional opaque object as an internal object; that is, the object is never exposed as a variable within an application. This opaque object is the mask used to control which computed values can be stored in the output operand of a *GraphBLAS operation*. Masks are described in Section 3.5.4.

2.5 Execution model

A program using the GraphBLAS C API is called a GraphBLAS application. The application constructs GraphBLAS objects, manipulates them to implement a graph algorithm, and then extracts values from the GraphBLAS objects to produce the results for that algorithm. Functions defined within the GraphBLAS C API that manipulate GraphBLAS objects are called *methods*. If the method corresponds to one of the operations defined in the GraphBLAS mathematical specifica-

tion, we refer to the method as an operation.

The GraphBLAS application specifies an ordered collection of GraphBLAS method calls defined by the order they appear in the text of the program (the *program order*). These define a directed acyclic graph (DAG) where nodes are GraphBLAS method calls and edges are dependencies between method calls.

Each method call in the DAG uniquely and unambiguously defines the output GraphBLAS objects as long as there are no execution errors that put objects in an invalid state (see Section 2.6). An ordered collection of method calls, a subgraph of the overall DAG for an application, defines the state of a GraphBLAS object at any point in a program. This ordered collection is the *sequence* for that object.

Since the GraphBLAS execution is defined in terms of a DAG and the GraphBLAS objects are opaque, the semantics of the GraphBLAS specification affords an implementation considerable flexibility to optimize performance. A GraphBLAS implementation can defer execution of nodes in the DAG, fuse nodes, or even replace whole subgraphs within the DAG to optimize performance. We discuss this topic further in section 2.5.1 when we describe *blocking* and *non-blocking* execution modes.

A correct GraphBLAS application must be *race-free*. This means that the DAG produced by an application and the results produced by execution of that DAG must be the same regardless of how the threads are scheduled for execution. It is the application programmer's responsibility to control memory orders and establish the required synchronized-with relationships to assure race-free execution of a multi-threaded GraphBLAS application. Writing race-free GraphBLAS applications is discussed further in Section 2.5.2.

629 2.5.1 Execution modes

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The execution of the DAG defined by a GraphBLAS application depends on the execution mode of the GraphBLAS program. There are two modes: blocking and nonblocking.

- blocking: In blocking mode, each method finishes the GraphBLAS operation defined by the method and all output GraphBLAS objects are materialized before proceeding to the next statement. Even mechanisms that break the opaqueness of the GraphBLAS objects (e.g., performance monitors, debuggers, memory dumps) will observe that the operation has finished.
- nonblocking: In nonblocking mode, each method may return once the input arguments have been inspected and verified to define a well formed GraphBLAS operation. (That is, there are no API errors; see Section 2.6.) The GraphBLAS method may not have finished, but the output object is ready to be used by the next GraphBLAS method call. If needed, a call to GrB_wait with GrB_COMPLETE or GrB_MATERIALIZE can be used to force the sequence for a GraphBLAS object (obj) to finish its execution.

The execution mode is defined in the GraphBLAS C API when the context of the library invocation is defined. This occurs once before any GraphBLAS methods are called with a call to the

GrB_init() function. This function takes a single argument of type GrB_Mode with values shown in Table 3.1(a).

An application executing in nonblocking mode is not required to return immediately after input arguments have been verified. A conforming implementation of the GraphBLAS C API running in nonblocking mode may choose to execute as if in blocking mode. A sequence of operations in nonblocking mode where every GraphBLAS operation with output object obj is followed by a GrB_wait(obj, GrB_MATERIALIZE) call is equivalent to the same sequence in blocking mode with GrB_wait(obj, GrB_MATERIALIZE) calls removed.

Nonblocking mode allows for any execution strategy that satisfies the mathematical definition of the sequence. The methods can be placed into a queue and deferred. They can be chained together and fused (e.g., replacing a chained pair of matrix products with a matrix triple product). Lazy evaluation, greedy evaluation, and asynchronous execution are all valid as long as the final result agrees with the mathematical definition provided by the sequence of GraphBLAS method calls appearing in program order.

Blocking mode forces an implementation to carry out precisely the GraphBLAS operations defined by the methods and to complete each and every method call individually. It is valuable for debugging or in cases where an external tool such as a debugger needs to evaluate the state of memory during a sequence of operations.

In a sequence of operations free of execution errors, and with input objects that are well-conditioned, the results from blocking and nonblocking modes should be identical outside of effects due to roundoff errors associated with floating point arithmetic. Due to the great flexibility afforded to an implementation when using nonblocking mode, we expect execution of a sequence in nonblocking mode to potentially complete execution in less time.

It is important to note that, processing of nonopaque objects is never deferred in GraphBLAS.
That is, methods that consume nonopaque objects (e.g., GrB_Matrix_build(), Section 4.2.5.9) and
methods that produce nonopaque objects (e.g., GrB_Matrix_extractTuples(), Section 4.2.5.13) always finish consuming or producing those nonopaque objects before returning regardless of the
execution mode.

Finally, after all GraphBLAS method calls have been made, the context is terminated with a call to GrB_finalize(). In the current version of the GraphBLAS C API, the context can be set only once in the execution of a program. That is, after GrB_finalize() is called, a subsequent call to GrB_init() is not allowed.

$_{577}$ 2.5.2 Multi-threaded execution

The GraphBLAS C API is designed to work with applications that utilize multiple threads executing within a shared address space. This specification does not define how threads are created, managed and synchronized. We expect the host programming environment to provide those services.

A conformant implementation of the GraphBLAS must be *thread safe*. A GraphBLAS library is thread safe when independent method calls (i.e., GraphBLAS objects are not shared between method calls) from multiple threads in a race-free program return the same results as would follow

from their sequential execution in some interleaved order. This is a common requirement in software libraries.

Thread safety applies to the behavior of multiple independent threads. In the more general case for multithreading, threads are not independent; they share variables and mix read and write operations to those variables across threads. A memory consistency model defines which values can be returned when reading an object shared between two or more threads. The GraphBLAS specification does not define its own memory consistency model. Instead the specification defines what must be done by a programmer calling GraphBLAS methods and by the implementor of a GraphBLAS library so an implementation of the GraphBLAS specification can work correctly with the memory consistency model for the host environment.

A memory consistency model is defined in terms of happens-before relations between methods in different threads. The defining case is a method that writes to an object on one thread that is read (i.e., used as an IN or INOUT argument) in a GraphBLAS method on a different thread. The following steps must occur between the different threads.

• A sequence of GraphBLAS methods results in the definition of the GraphBLAS object.

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- The GraphBLAS object is put into a state of completion by a call to GrB_wait() with the GrB_COMPLETE parameter (see Table 3.1(b)). A GraphBLAS object is said to be *complete* when it can be safely used as an IN or INOUT argument in a GraphBLAS method call from a different thread.
- Completion happens before a synchronized-with relation that executes with at least a release memory order.
- A synchronized-with relation on the other thread executes with at least an acquire memory order.
- This synchronized-with relation happens-before the GraphBLAS method that reads the graph-BLAS object.

We use the phrase at least when talking about the memory orders to indicate that a stronger memory order such as sequential consistency can be used in place of the acquire-release order.

A program that violates these rules contains a data race. That is, its reads and writes are unordered across threads making the final value of a variable undefined. A program that contains a data race is invalid and the results of that program are undefined. We note that multi-threaded execution is compatible with both blocking and non-blocking modes of execution.

Completion is the central concept that allows GraphBLAS objects to be used in happens-before relations between threads. In earlier versions of GraphBLAS (1.X) completion was implied by any operation that produced non-opaque values from a GraphBLAS object. These operations are summarized in Table 2.2). In GraphBLAS 2.0, these methods no longer imply completion. This change was made since there are cases where the non-opaque value is needed but the object from which it is computed is not. We want implementations of the GraphBLAS to be able to exploit this case and not form the opaque object when that object is not needed.

Table 2.2: Methods that extract values from a GraphBLAS object that forcing completion of the operations contributing to that particular object in GraphBLAS 1.X. In GraphBLAS 2.0, these methods *do not* force completion.

Method	Section
GrB_Vector_nvals	4.2.4.6
GrB_Vector_extractElement	4.2.4.10
GrB_Vector_extractTuples	4.2.4.11
GrB_Matrix_nvals	4.2.5.8
$GrB_Matrix_extractElement$	4.2.5.12
GrB _ $Matrix$ _ $extract$ $Tuples$	4.2.5.13
GrB_reduce (vector-scalar value variant)	4.3.10.2
GrB_reduce (matrix-scalar value variant)	4.3.10.3

$_{2}$ 2.6 Error model

All GraphBLAS methods return a value of type GrB_Info (an enum) to provide information available to the system at the time the method returns. The returned value will be one of the defined values shown in Table 3.16. The return values fall into three groups: informational, API errors, and execution errors. While API and execution errors take on negative values, informational return values listed in Table 3.16(a) are non-negative and include GrB_SUCCESS (a value of 0) and GrB_NO_VALUE.

An API error (listed in Table 3.16(b)) means that a GraphBLAS method was called with parameters that violate the rules for that method. These errors are restricted to those that can be determined by inspecting the dimensions and domains of GraphBLAS objects, GraphBLAS operators, or the values of scalar parameters fixed at the time a method is called. API errors are deterministic and consistent across platforms and implementations. API errors are never deferred, even in nonblocking mode. That is, if a method is called in a manner that would generate an API error, it always returns with the appropriate API error value. If a GraphBLAS method returns with an API error, it is guaranteed that none of the arguments to the method (or any other program data) have been modified. The informational return value, GrB_NO_VALUE, is also deterministic and never deferred in nonblocking mode.

Execution errors (listed in Table 3.16(c)) indicate that something went wrong during the execution of a legal GraphBLAS method invocation. Their occurrence may depend on specifics of the execution environment and data values being manipulated. This does not mean that execution errors are the fault of the GraphBLAS implementation. For example, a memory leak could arise from an error in an application's source code (a "program error"), but it may manifest itself in different points of a program's execution (or not at all) depending on the platform, problem size, or what else is running at that time. Index out-of-bounds errors, for example, always indicate a program error.

If a GraphBLAS method returns with any execution error other than GrB_PANIC, it is guaranteed that the state of any argument used as input-only is unmodified. Output arguments may be left in an invalid state, and their use downstream in the program flow may cause additional errors. If a

GraphBLAS method returns with a GrB_PANIC execution error, no guarantees can be made about the state of any program data.

In nonblocking mode, execution errors can be deferred. A return value of GrB_SUCCESS only guarantees that there are no API errors in the method invocation. If an execution error value is returned by a method with output object obj in nonblocking mode, it indicates that an error was found during execution of any of the pending operations on obj, up to and including the GrB_wait() method (Section 4.2.8) call that completes those pending operations. When possible, that return value will provide information concerning the cause of the error.

As discussed in Section 4.2.8, a GrB_wait(obj) on a specific GraphBLAS object obj completes all pending operations on that object. No additional errors on the methods that precede the call to GrB_wait and have obj as an OUT or INOUT argument can be reported. From a GraphBLAS perspective, those methods are *complete*. Details on the guaranteed state of objects after a call to GrB_wait can be found in Section 4.2.8.

After a call to any GraphBLAS method that modifies an opaque object, the program can re-763 trieve additional error information (beyond the error code returned by the method) though a call 764 to the function GrB_error(), passing the method's output object as described in Section 4.2.9. 765 The function returns a pointer to a NULL-terminated string, and the contents of that string are 766 implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error 767 string. GrB error() is a thread-safe function, in the sense that multiple threads can call it simul-768 taneously and each will get its own error string back, referring to the object passed as an input 769 argument. 770

$_{m}$ Chapter 3

Objects

In this chapter, all of the enumerations, literals, data types, and predefined opaque objects defined in the GraphBLAS API are presented. Enumeration literals in GraphBLAS are assigned specific 774 values to ensure compatibility between different runtime library implementations. The chapter starts by defining the enumerations that are used by the init() and wait() methods. Then a num-776 ber of transparent (i.e., non-opaque) types that are used for interfacing with external data are 777 defined. Sections that follow describe the various types of opaque objects in GraphBLAS: types 778 (or domains), algebraic objects, collections and descriptors. Each of these sections also lists the 770 predefined instances of each opaque type that are required by the API. This chapter concludes with 780 a section on the definition for GrB Info enumeration that is used as the return type of all methods. 781

$_{\scriptscriptstyle{782}}$ 3.1 Enumerations for init() and wait()

Table 3.1 lists the enumerations and the corresponding values used in the GrB_init() method to set the execution mode and in the GrB_wait() method for completing or materializing opaque objects.

⁷⁸⁵ 3.2 Indices, index arrays, and scalar arrays

In order to interface with third-party software (i.e., software other than an implementation of the GraphBLAS), operations such as GrB_Matrix_build (Section 4.2.5.9) and GrB_Matrix_extractTuples (Section 4.2.5.13) must specify how the data should be laid out in non-opaque data structures. To this end we explicitly define the types for indices and the arrays used by these operations.

For indices a typedef is used to give a GraphBLAS name to a concrete type. We define it as follows:

typedef uint64 t GrB Index;

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The range of valid values for a variable of type GrB_Index is [0, GrB_INDEX_MAX] where the largest index value permissible is defined with a macro, GrB_INDEX_MAX. For example:

#define GrB_INDEX_MAX ((GrB_Index) 0x0ffffffffffffffffff;);

An implementation is required to define and document this value.

An index array is a pointer to a set of GrB Index values that are stored in a contiguous block of 796 memory (i.e., GrB_Index*). Likewise, a scalar array is a pointer to a contiguous block of memory 797 storing a number of scalar values as specified by the user. Some GraphBLAS operations (e.g., 798 GrB assign) include an input parameter with the type of an index array. This input index array 799 selects a subset of elements from a GraphBLAS vector or matrix object to be used in the operation. 800 In these cases, the literal GrB_ALL can be used in place of the index array input parameter to 801 indicate that all indices of the associated GraphBLAS vector or matrix object should be used. An 802 implementation of the GraphBLAS C API has considerable freedom in terms of how GrB_ALL 803 is defined. Since GrB_ALL is used as an argument for an array parameter, it must use a type 804 consistent with a pointer. GrB_ALL must also have a non-null value to distinguish it from the 805 erroneous case of passing a NULL pointer as an array. 806

3.3 Types (domains)

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In GraphBLAS, domains correspond to the valid values for types from the host language (in our case, the C programming language). GraphBLAS defines a number of operators that take elements from one or more domains and produce elements of a (possibly) different domain. GraphBLAS also defines three kinds of collections: matrices, vectors and scalars. For any given collection, the elements of the collection belong to a *domain*, which is the set of valid values for the elements. For any variable or object V in GraphBLAS we denote as $\mathbf{D}(V)$ the domain of V, that is, the set of possible values that elements of V can take.

Table 3.1: Enumeration literals and corresponding values input to various GraphBLAS methods.

(a) GrB_Mode execution modes for the GrB_init method.

Symbol	Value	Description
GrB_NONBLOCKING	0	Specifies the nonblocking mode context.
GrB_BLOCKING	1	Specifies the blocking mode context.

(b) GrB_WaitMode wait modes for the GrB_wait method.

Symbol	Value	Description
GrB_COMPLETE	0	The object is in a state where it can be used in a happens-
		before relation so that multithreaded programs can be prop-
		erly synchronized.
GrB_MATERIALIZE	1	The object is <i>complete</i> , and in addition, all computation of
		the object is finished and any error information is available.
	•	•

Table 3.2: Predefined GrB_Type values, and the corresponding GraphBLAS domain suffixes, C type (for scalar parameters), and domains for GraphBLAS. The domain suffixes are used in place of I, F, and T in Tables 3.5, 3.6, 3.7, 3.8, and 3.9).

GrB_Type	GrB_Type_Code	Suffix	C type	Domain
-	GrB_UDT_CODE=0	UDT	-	-
GrB_BOOL	GrB_BOOL_CODE=1	BOOL	bool	$\{ \mathtt{false}, \mathtt{true} \}$
GrB_INT8	GrB_INT8_CODE=2	INT8	int8_t	$\mathbb{Z}\cap[-2^7,2^7)$
GrB_UINT8	GrB_UINT8_CODE=3	UINT8	uint8_t	$\mathbb{Z}\cap[0,2^8)$
GrB_INT16	$GrB_{INT16}_{CODE=4}$	INT16	int16_t	$\mathbb{Z} \cap [-2^{15}, 2^{15})$
GrB_UINT16	GrB_UINT16_CODE=5	UINT16	uint16_t	$\mathbb{Z}\cap[0,2^{16})$
GrB_INT32	GrB_INT32_CODE=6	INT32	int32_t	$\mathbb{Z} \cap [-2^{31}, 2^{31})$
GrB_UINT32	GrB_UINT32_CODE=7	UINT32	uint32_t	$\mathbb{Z} \cap [0, 2^{32})$
GrB_INT64	GrB_INT64_CODE=8	INT64	int64_t	$\mathbb{Z} \cap [-2^{63}, 2^{63})$
GrB_UINT64	GrB_UINT64_CODE=9	UINT64	uint64_t	$\mathbb{Z} \cap [0, 2^{64})$
GrB_FP32	GrB_FP32_CODE=10	FP32	float	IEEE 754 binary32
GrB_FP64	GrB_FP64_CODE=11	FP64	double	IEEE 754 binary64

The domains for elements that can be stored in collections and operated on through GraphBLAS methods are defined by GraphBLAS objects called GrB_Type. The predefined types and corresponding domains used in the GraphBLAS C API are shown in Table 3.2. The Boolean type (bool) is defined in stdbool.h, the integral types (int8_t, uint8_t, int16_t, uint16_t, int32_t, uint32_t, int64_t, uint64_t) are defined in stdint.h, and the floating-point types (float, double) are native to the language and platform and in most cases defined by the IEEE-754 standard. UDT stands for user-defined type and is the type code returned for all objects which use a non-predefined type. Implementations which add new types should start their GrB_Type_Codes at 100 to avoid possible conflicts with built-in types which may be added in the future.

3.4 Algebraic objects, operators and associated functions

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GraphBLAS operators operate on elements stored in GraphBLAS collections. A binary operator is a function that maps two input values to one output value. A unary operator is a function that maps one input value to one output value. Binary operators are defined over two input domains and produce an output from a (possibly different) third domain. Unary operators are specified over one input domain and produce an output from a (possibly different) second domain.

In addition to the operators that operate on stored values, GraphBLAS also supports *index unary operators* that maps a stored value and the indices of its position in the matrix or vector to an output value. That output value can be used in the index unary operator variants of apply (§ 4.3.8) to compute a new stored value, or be used in the select operation (§ 4.3.9) to determine if the stored input value should be kept or annihilated.

so Some GraphBLAS operations require a monoid or semiring. A monoid contains an associative

Table 3.3: Operator input for relevant GraphBLAS operations. The semiring add and times are shown if applicable.

Operation	Operator input
mxm, mxv, vxm	semiring
eWiseAdd	binary operator
	monoid
	semiring (add)
eWiseMult	binary operator
	monoid
	semiring (times)
reduce (to vector or GrB_Scalar)	binary operator
	monoid
reduce (to scalar value)	monoid
apply	unary operator
	binary operator with scalar
	index unary operator
select	index unary operator
kronecker	binary operator
	monoid
	semiring
dup argument (build methods)	binary operator
accum argument (various methods)	binary operator

binary operator where the input and output domains are the same. The monoid also includes an identity value of the operator. The semiring consists of a binary operator – referred to as the "times" operator – with up to three different domains (two inputs and one output) and a monoid – referred to as the "plus" operator – that is also commutative. Furthermore, the domain of the monoid must be the same as the output domain of the "times" operator.

The GraphBLAS algebraic objects operators, monoids, and semirings are presented in this section.
These objects can be used as input arguments to various GraphBLAS operations, as shown in
Table 3.3. The specific rules for each algebraic object are explained in the respective sections of
those objects. A summary of the properties and recipes for building these GraphBLAS algebraic
objects is presented in Table 3.4.

A number of predefined operators are specified by the GraphBLAS C API. They are presented in tables in their respective subsections below. Each of these operators is defined to operate on specific GraphBLAS types and therefore, this type is built into the name of the object as a suffix. These suffixes and the corresponding predefined GrB_Type objects that are listed in Table 3.2.

$_{\circ}$ 3.4.1 Operators

A GraphBLAS unary operator $F_u = \langle D_{out}, D_{in}, f \rangle$ is defined by two domains, D_{out} and D_{in} , and an operation $f: D_{in} \to D_{out}$. For a given GraphBLAS unary operator $F_u = \langle D_{out}, D_{in}, f \rangle$, we

Table 3.4: Properties and recipes for building GraphBLAS algebraic objects: unary operator, binary operator, monoid, and semiring (composed of operations *add* and *times*).

(a) Properties of algebraic objects.

Object	Must be	Must be	Identity	Number
	commutative	associative	must exist	of domains
Unary operator	n/a	n/a	n/a	2
Binary operator	no	no	no	3
Monoid	no	yes	yes	1
Reduction add	yes	yes	yes (see Note 1)	1
Semiring add	yes	yes	yes	1
Semiring times	no	no	no	3 (see Note 2)

(b) Recipes for algebraic objects.

Object	Recipe	Number of domains
Unary operator	Function pointer	2
Binary operator	Function pointer	3
Monoid	Associative binary operator with identity	1
Semiring	Commutative monoid + binary operator	3

Note 1: Some high-performance GraphBLAS implementations may require an identity to perform reductions to sparse objects like GraphBLAS vectors and scalars. According to the descriptions of the corresponding GraphBLAS operations, however, this identity is mathematically not necessary. There are API signatures to support both. Note 2: The output domain of the semiring times must be same as the domain of the semiring's add monoid. This

ensures three domains for a semiring rather than four.

```
define \mathbf{D}_{out}(F_u) = D_{out}, \mathbf{D}_{in}(F_u) = D_{in}, and \mathbf{f}(F_u) = f.
     A GraphBLAS binary operator F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle is defined by three domains, D_{out}, D_{in_1}, D_{in_2}, \odot \rangle
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     D_{in_2}, and an operation \odot: D_{in_1} \times D_{in_2} \to D_{out}. For a given GraphBLAS binary operator F_b =
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     \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle, we define \mathbf{D}_{out}(F_b) = D_{out}, \mathbf{D}_{in_1}(F_b) = D_{in_1}, \mathbf{D}_{in_2}(F_b) = D_{in_2}, and \mathbf{O}(F_b) = D_{out}
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     \odot. Note that \odot could be used in place of either \oplus or \otimes in other methods and operations.
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     A GraphBLAS index unary operator F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle is defined by three
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     domains, D_{out}, D_{in_1}, D_{in_2}, the domain of GraphBLAS indices, and an operation f_i: D_{in_1} \times I_{U64}^2 \times I_{U64}
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     D_{in_2} \to D_{out} (where I_{U64} corresponds to the domain of a GrB_Index). For a given GraphBLAS
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     index operator F_i, we define \mathbf{D}_{out}(F_i) = D_{out}, \mathbf{D}_{in_1}(F_i) = D_{in_1}, \mathbf{D}_{in_2}(F_i) = D_{in_2}, and \mathbf{f}(F_i) = f_i.
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     User-defined operators can be created with calls to GrB_UnaryOp_new, GrB_BinaryOp_new, and
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     GrB_IndexUnaryOp_new, respectively. See Section 4.2.2 for information on these methods. The
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     GraphBLAS C API predefines a number of these operators. These are listed in Tables 3.5 and 3.6.
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     Note that most entries in these tables represent a "family" of predefined operators for a set of
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     different types represented by the T, I, or F in their names. For example, the multiplicative
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     inverse (GrB_MINV_F) function is only defined for floating-point types (F = FP32 or FP64). The
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division (GrB_DIV_T) function is defined for all types, but only if $y \neq 0$ for integral and floating

point types and $y \neq false$ for the Boolean type.

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Table 3.5: Predefined unary and binary operators for GraphBLAS in C. The T can be any suffix from Table 3.2, I can be any integer suffix from Table 3.2, and F can be any floating-point suffix from Table 3.2.

Operator	GraphBLAS			
type	identifier	Domains	Description	
GrB_UnaryOp	$GrB_IDENTITY_T$	$T \to T$	f(x) = x	identity
GrB_UnaryOp	GrB_ABS_T	$T \to T$	f(x) = x ,	absolute value
GrB_UnaryOp	GrB_AINV_T	$T \to T$	f(x) = -x,	additive inverse
GrB_UnaryOp	GrB_MINV_F	$F \to F$	$f(x) = \frac{1}{x}$	multiplicative inverse
GrB_UnaryOp	GrB_LNOT	$ exttt{bool} o exttt{bool}$	$f(x) = \neg x,$	logical inverse
GrB_UnaryOp	GrB_BNOT_I	$I \rightarrow I$	$\int f(x) = \tilde{x},$	bitwise complement
CoD Discos O	C.D. LOD			1 : 1 OD
GrB_BinaryOp	GrB_LOR	$bool \times bool \rightarrow bool$	$f(x,y) = x \vee y,$	logical OR
GrB_BinaryOp	GrB_LAND	$bool \times bool \rightarrow bool$	$\int f(x,y) = x \wedge y,$	logical AND
GrB_BinaryOp	GrB_LXOR	$bool \times bool \rightarrow bool$	$f(x,y) = \underbrace{x \oplus y}_{f},$	logical XOR
GrB_BinaryOp	GrB_LXNOR	$bool \times bool \rightarrow bool$	$f(x,y) = \overline{x \oplus y},$	logical XNOR
GrB_BinaryOp	GrB_BOR_I	$I \times I \to I$	$\int_{0}^{\infty} f(x,y) = x \mid y,$	bitwise OR
GrB_BinaryOp	GrB_BAND_I	$I \times I \to I$	$\int_{C} f(x,y) = x \& y,$	bitwise AND
GrB_BinaryOp	GrB_BXOR_I	$I \times I \to I$	$f(x,y) = \underline{x \cdot y},$	bitwise XOR
GrB_BinaryOp	GrB_BXNOR_I	$I \times I \to I$	$f(x,y) = \overline{x \cdot y},$	bitwise XNOR
GrB_BinaryOp	GrB_EQ_T	$T \times T \rightarrow \text{bool}$	f(x,y) = (x == y)	equal
GrB_BinaryOp	GrB_NE_T	$T \times T \to \text{bool}$	$f(x,y) = (x \neq y)$	not equal
GrB_BinaryOp	GrB_GT_T	$T \times T \rightarrow \texttt{bool}$	f(x,y) = (x > y)	greater than
GrB_BinaryOp	GrB_LT_T	$T \times T \to \texttt{bool}$	f(x,y) = (x < y)	less than
GrB_BinaryOp	GrB_GE_T	$T \times T \to \texttt{bool}$	$f(x,y) = (x \ge y)$	greater than or equal
GrB_BinaryOp	GrB_LE_T	$T \times T \to \texttt{bool}$	$f(x,y) = (x \le y)$	less than or equal
GrB_BinaryOp	GrB_ONEB_T	$T \times T \to T$	$\int_{C} f(x,y) = 1,$	1 (cast to T)
GrB_BinaryOp	GrB_FIRST_T	$T \times T \to T$	$\int f(x,y) = x,$	first argument
GrB_BinaryOp	GrB_SECOND_T	$T \times T \to T$	$\int f(x,y) = y,$	second argument
GrB_BinaryOp	GrB_MIN_T	$T \times T \to T$	f(x,y) = (x < y) ? x : y,	minimum
GrB_BinaryOp	GrB_MAX_T	$ T \times T \to T $	f(x,y) = (x > y) ? x : y,	maximum
GrB_BinaryOp	GrB_PLUS_T	$T \times T \to T$	$\int f(x,y) = x + y,$	addition
GrB_BinaryOp	GrB_MINUS_T	$T \times T \to T$	$\int_{C} f(x,y) = x - y,$	subtraction
GrB_BinaryOp	GrB_TIMES_T	$T \times T \to T$	$\int_{C} f(x,y) = xy,$	multiplication
GrB_BinaryOp	GrB_DIV_T	$T \times T \to T$	$f(x,y) = \frac{x}{y},$	division

Table 3.6: Predefined index unary operators for GraphBLAS in C. The T can be any suffix from Table 3.2. I_{U64} refers to the unsigned 64-bit, GrB_Index, integer type, I_{32} refers to the signed, 32-bit integer type, and I_{64} refers to signed, 64-bit integer type. The parameters, u_i or A_{ij} , are the stored values from the containers where the i and j parameters are set to the row and column indices corresponding to the location of the stored value. When operating on vectors, j will be passed with a zero value. Finally, s is an additional scalar value used in the operators. The expressions in the "Description" column are to be treated as mathematical specifications. That is, for the index arithmetic functions in the first two groups below, each one of i, j, and s is interpreted as an integer number in the set \mathbb{Z} . Functions are evaluated using arithmetic in \mathbb{Z} , producing a result value that is also in \mathbb{Z} . The result value is converted to the output type according to the rules of the C language. In particular, if the value cannot be represented as a signed 32- or 64-bit integer type, the output is implementation defined. Any deviations from this ideal behavior, including limitations on the values of i, j, and s, or possible overflow and underflow conditions, must be defined by the implementation.

Operator type	GraphBLAS	Don	nains (-	is don'	t care)	Description			
Type	identifier	A, u	i, j	s	result				
GrB_IndexUnaryOp	GrB_ROWINDEX_ $I_{32/64}$	_	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	(i+s),	replace with its row index (+ s)
	,	_	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(u_i, i, 0, s)$	=	(i+s)	
$GrB_IndexUnaryOp$	GrB_COLINDEX $I_{32/64}$	_	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij},i,j,s)$	=	(j+s)	replace with its column index $(+ s)$
$GrB_IndexUnaryOp$	GrB_DIAGINDEX $I_{32/64}$	_	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij},i,j,s)$	=	(j-i+s)	replace with its diagonal index $(+ s)$
GrB_IndexUnaryOp	GrB_TRIL	_	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j \le i + s)$	triangle on or below diagonal s
$\underline{\leftarrow}$ GrB_IndexUnaryOp	GrB_TRIU	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	$(j \ge i + s)$	triangle on or above diagonal s
$^{\odot}$ GrB_IndexUnaryOp	GrB_DIAG	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	(j == i + s)	diagonal s
$GrB_IndexUnaryOp$	GrB_OFFDIAG	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	$(j \neq i + s)$	all but diagonal s
$GrB_IndexUnaryOp$	GrB_COLLE	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	$(j \le s)$	columns less or equal to s
$GrB_IndexUnaryOp$	GrB_COLGT	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	(j>s)	columns greater than s
$GrB_IndexUnaryOp$	GrB_ROWLE	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	$(i \leq s),$	rows less or equal to s
		_	I_{U64}	I_{64}	bool	$f(u_i, i, 0, s)$	=	$(i \le s)$	
$GrB_IndexUnaryOp$	GrB_ROWGT	_	I_{U64}	I_{64}	bool	$f(A_{ij},i,j,s)$	=	(i>s),	rows greater than s
		_	I_{U64}	I_{64}	bool	$f(u_i, i, 0, s)$	=	(i > s)	
GrB_IndexUnaryOp	$GrB_VALUEEQ_T$	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} == s),$	elements equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i == s)$	
$GrB_IndexUnaryOp$	$GrB_VALUENE_T$	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} \neq s),$	elements not equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \neq s)$	
$GrB_IndexUnaryOp$	GrB_VALUELT_T	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} < s),$	elements less than value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i < s)$	
$GrB_IndexUnaryOp$	GrB_VALUELE_T	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} \leq s),$	elements less or equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \le s)$	
$GrB_IndexUnaryOp$	$GrB_VALUEGT_T$	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} > s),$	elements greater than value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i > s)$	
$GrB_IndexUnaryOp$	$GrB_VALUEGE_T$	T	_	T	bool	$f(A_{ij},i,j,s)$	=	$(A_{ij} \geq s),$	elements greater or equal to value s
		T	_	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \ge s)$	

3.4.2 Monoids

- A GraphBLAS monoid $M = \langle D, \odot, 0 \rangle$ is defined by a single domain D, an associative¹ operation $0 \in D \times D \to D$, and an identity element $0 \in D$. For a given GraphBLAS monoid $M = \langle D, \odot, 0 \rangle$ we define $\mathbf{D}(M) = D$, $\mathbf{O}(M) = \mathbf{O}$, and $\mathbf{O}(M) = 0$. A GraphBLAS monoid is equivalent to the conventional monoid algebraic structure.
- Let $F = \langle D, D, D, \odot \rangle$ be an associative GraphBLAS binary operator with identity element $0 \in D$. Then $M = \langle F, 0 \rangle = \langle D, \odot, 0 \rangle$ is a GraphBLAS monoid. If \odot is commutative, then M is said to be a commutative monoid. If a monoid M is created using an operator \odot that is not associative, the outcome of GraphBLAS operations using such a monoid is undefined.
- User-defined monoids can be created with calls to GrB_Monoid_new (see Section 4.2.2). The GraphBLAS C API predefines a number of monoids that are listed in Table 3.7. Predefined monoids are named $GrB_op_MONOID_T$, where op is the name of the predefined GraphBLAS operator used as the associative binary operation of the monoid and T is the domain (type) of the monoid.

3.4.3 Semirings

- A GraphBLAS semiring $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$ is defined by three domains D_{out}, D_{in_1} , and D_{in_2} ; an associative¹ and commutative additive operation $\oplus : D_{out} \times D_{out} \to D_{out}$; a multiplicative operation $\otimes : D_{in_1} \times D_{in_2} \to D_{out}$; and an identity element $0 \in D_{out}$. For a given GraphBLAS semiring $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$ we define $\mathbf{D}_{in_1}(S) = D_{in_1}$, $\mathbf{D}_{in_2}(S) = D_{in_2}$, $\mathbf{D}_{out}(S) = D_{out}$, $\mathbf{D}_{out}(S) = D_{out}(S)$
- Let $F = \langle D_{out}, D_{in_1}, D_{in_2}, \otimes \rangle$ be an operator and let $A = \langle D_{out}, \oplus, 0 \rangle$ be a commutative monoid, then $S = \langle A, F \rangle = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0 \rangle$ is a semiring.
- In a GraphBLAS semiring, the multiplicative operator does not have to distribute over the additive operator. This is unlike the conventional *semiring* algebraic structure.
- Note: There must be one GraphBLAS monoid in every semiring which serves as the semiring's additive operator and specifies the same domain for its inputs and output parameters. If this monoid is not a commutative monoid, the outcome of GraphBLAS operations using the semiring is undefined.
- A UML diagram of the conceptual hierarchy of object classes in GraphBLAS algebra (binary operators, monoids, and semirings) is shown in Figure 3.1.
- User-defined semirings can be created with calls to GrB_Semiring_new (see Section 4.2.2). A list of predefined true semirings and convenience semirings can be found in Tables 3.8 and 3.9, respectively.

 Predefined semirings are named GrB_add_mul_SEMIRING_T, where add is the semiring additive operation, mul is the semiring multiplicative operation and T is the domain (type) of the semiring.

¹It is expected that implementations of the GraphBLAS will utilize floating point arithmetic such as that defined in the IEEE-754 standard even though floating point arithmetic is not strictly associative.

Table 3.7: Predefined monoids for GraphBLAS in C. Maximum and minimum values for the various integral types are defined in $\mathtt{stdint.h.}$ Floating-point infinities are defined in $\mathtt{math.h.}$ The x in $\mathsf{UINT}x$ or $\mathsf{INT}x$ can be one of 8, 16, 32, or 64; whereas in $\mathsf{FP}x$, it can be 32 or 64.

$\operatorname{GraphBLAS}$	Domains, T		
identifier	$(T \times T \to T)$	Identity	Description
GrB_PLUS_MONOID_T	UINTx	0	addition
	INTx	0	
	FPx	0	
$GrB_TIMES_MONOID_T$	UINTx	1	multiplication
	INTx	1	
	FPx	1	
$GrB _MIN _MONOID _T$	UINTx	$UINTx_\mathtt{MAX}$	minimum
	INTx	$INTx_{MAX}$	
	FPx	INFINITY	
$GrB_MAX_MONOID_T$	UINTx	0	maximum
	INTx	$ $ INT x _MIN	
	FPx	-INFINITY	
GrB_LOR_MONOID_BOOL	BOOL	false	logical OR
GrB_LAND_MONOID_BOOL	BOOL	true	logical AND
GrB_LXOR_MONOID_BOOL	BOOL	false	logical XOR (not equal)
GrB_LXNOR_MONOID_BOOL	BOOL	true	logical XNOR (equal)

Table 3.8: Predefined true semirings for GraphBLAS in C where the additive identity is the multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

	Domains, T	+ identity	
GraphBLAS identifier	$ (T \times T \to T) $	\times annihilator	Description
$GrB_PLUS_TIMES_SEMIRING_T$	UINTx	0	arithmetic semiring
	INTx	0	
	$\mid FPx$	0	
$GrB _MIN _PLUS _SEMIRING _T$	$\bigcup UINT x$	$\mathtt{UINT}x_\mathtt{MAX}$	min-plus semiring
	INTx	$\mathtt{INT}x\mathtt{_MAX}$	
	FPx	INFINITY	
$GrB_MAX_PLUS_SEMIRING_T$	INTx	$\mathtt{INT}x\mathtt{_MIN}$	max-plus semiring
	FPx	-INFINITY	
$GrB _MIN _TIMES _SEMIRING _T$	$\bigcup UINT x$	$\mathtt{UINT}x_\mathtt{MAX}$	min-times semiring
$GrB _MIN _MAX _SEMIRING _T$	$\bigcup UINT x$	$\mathtt{UINT}x_\mathtt{MAX}$	min-max semiring
	INTx	$\mathtt{INT}x\mathtt{_MAX}$	
	FPx	INFINITY	
$GrB_MAX_MIN_SEMIRING_T$	$\bigcup UINT x$	0	max-min semiring
	INTx	$\mathtt{INT}x\mathtt{_MIN}$	
	FPx	-INFINITY	
$GrB_MAX_TIMES_SEMIRING_T$	$\bigcup UINT x$	0	max-times semiring
$GrB_PLUS_MIN_SEMIRING_T$	$\bigcup UINT x$	0	plus-min semiring
GrB_LOR_LAND_SEMIRING_BOOL	BOOL	false	Logical semiring
GrB_LAND_LOR_SEMIRING_BOOL	BOOL	true	"and-or" semiring
GrB_LXOR_LAND_SEMIRING_BOOL	BOOL	false	same as NE_LAND
GrB_LXNOR_LOR_SEMIRING_BOOL	BOOL	true	same as EQ_LOR

Table 3.9: Other useful predefined semirings for GraphBLAS in C that don't have a multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

	Domains, T		
GraphBLAS identifier	$(T \times T \to T)$	+ identity	Description
GrB_MAX_PLUS_SEMIRING_T	UINTx	0	max-plus semiring
$GrB _MIN _TIMES _SEMIRING _T$	INTx	$\mathtt{INT}x\mathtt{_MAX}$	min-times semiring
	FPx	INFINITY	
$GrB_MAX_TIMES_SEMIRING_T$	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	max-times semiring
	FPx	-INFINITY	
$GrB_PLUS_MIN_SEMIRING_T$	INTx	0	plus-min semiring
	FPx	0	
$GrB _MIN _FIRST _SEMIRING _T$	UINTx	$\mathtt{UINT}x_\mathtt{MAX}$	min-select first semiring
	INTx	$\mathtt{INT}x\mathtt{_MAX}$	
	FPx	INFINITY	
$GrB _MIN _SECOND _SEMIRING _T$	UINTx	$\mathtt{UINT}x_\mathtt{MAX}$	min-select second semiring
	INTx	$\mathtt{INT}x\mathtt{_MAX}$	
	FPx	INFINITY	
$GrB_MAX_FIRST_SEMIRING_T$	UINTx	0	max-select first semiring
	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	
	FPx	-INFINITY	
$GrB_MAX_SECOND_SEMIRING_T$	UINTx	0	max-select second semiring
	INTx	$\mathtt{INT}x_{\mathtt{MIN}}$	
	FPx	-INFINITY	

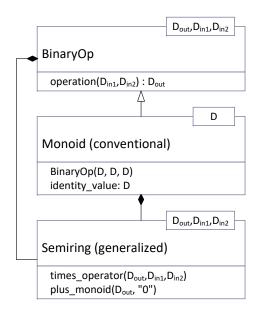


Figure 3.1: Hierarchy of algebraic object classes in GraphBLAS. GraphBLAS semirings consist of a conventional monoid with one domain for the addition function, and a binary operator with three domains for the multiplication function.

904 3.5 Collections

905 **3.5.1** Scalars

A GraphBLAS scalar, $s = \langle D, \{\sigma\} \rangle$, is defined by a domain D, and a set of zero or one scalar value, σ , where $\sigma \in D$. We define $\mathbf{size}(s) = 1$ (constant), and $\mathbf{L}(s) = \{\sigma\}$. The set $\mathbf{L}(s)$ is called the contents of the GraphBLAS scalar s. We also define $\mathbf{D}(s) = D$. Finally, $\mathbf{val}(s)$ is a reference to the scalar value, σ , if the GraphBLAS scalar is not empty, and is undefined otherwise.

910 **3.5.2** Vectors

A vector $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$ is defined by a domain D, a size N > 0, and a set of tuples (i, v_i) where $0 \le i < N$ and $v_i \in D$. A particular value of i can appear at most once in \mathbf{v} . We define size(\mathbf{v}) = N and $\mathbf{L}(\mathbf{v}) = \{(i, v_i)\}$. The set $\mathbf{L}(\mathbf{v})$ is called the *content* of vector \mathbf{v} . We also define the set $\mathbf{ind}(\mathbf{v}) = \{i : (i, v_i) \in \mathbf{L}(\mathbf{v})\}$ (called the *structure* of \mathbf{v}), and $\mathbf{D}(\mathbf{v}) = D$. For a vector \mathbf{v} , $\mathbf{v}(i)$ is a reference to v_i if $(i, v_i) \in \mathbf{L}(\mathbf{v})$ and is undefined otherwise.

916 3.5.3 Matrices

A matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$ is defined by a domain D, its number of rows M > 0, its 917 number of columns N > 0, and a set of tuples (i, j, A_{ij}) where $0 \le i < M$, $0 \le j < N$, and 918 $A_{ij} \in D$. A particular pair of values i, j can appear at most once in **A**. We define $\mathbf{ncols}(\mathbf{A}) = N$, 919 $\mathbf{nrows}(\mathbf{A}) = M$, and $\mathbf{L}(\mathbf{A}) = \{(i, j, A_{ij})\}$. The set $\mathbf{L}(\mathbf{A})$ is called the *content* of matrix \mathbf{A} . We also 920 define the sets $indrow(\mathbf{A}) = \{i : \exists (i, j, A_{ij}) \in \mathbf{A}\}$ and $indcol(\mathbf{A}) = \{j : \exists (i, j, A_{ij}) \in \mathbf{A}\}$. (These 921 are the sets of nonempty rows and columns of A, respectively.) The structure of matrix A is the 922 set $ind(A) = \{(i,j) : (i,j,A_{ij}) \in L(A)\}, \text{ and } D(A) = D.$ For a matrix A, A(i,j) is a reference to A_{ij} if $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})$ and is undefined otherwise. 924 If **A** is a matrix and $0 \leq j < N$, then $\mathbf{A}(:,j) = \langle D, M, \{(i,A_{ij}) : (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$ is a 925 vector called the j-th column of A. Correspondingly, if A is a matrix and $0 \le i < M$, then 926 $\mathbf{A}(i,:) = \langle D, N, \{(j,A_{ij}): (i,j,A_{ij}) \in \mathbf{L}(\mathbf{A})\} \rangle$ is a vector called the *i*-th row of \mathbf{A} . Given a matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$, its transpose is another matrix $\mathbf{A}^T = \langle D, N, M, \{(j, i, A_{ij}) : A_{ij} :$ 928 $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A}) \} \rangle$.

3.5.3.1 External matrix formats

The specification also supports the export and import of matrices to/from a number of commonly used formats, such as COO, CSR, and CSC formats. When importing or exporting a matrix to or from a GraphBLAS object using GrB_Matrix_import (§ 4.2.5.17) or GrB_Matrix_export (§ 4.2.5.16), it is necessary to specify the data format for the matrix data external to GraphBLAS, which is being imported from or exported to. This non-opaque data format is specified using an argument of enumeration type GrB_Format that is used to indicate one of a number of predefined formats. The predefined values of GrB_Format are specified in Table 3.10. A precise definition of the non-opaque data formats can be found in Appendix B.

Table 3.10: GrB_Format enumeration literals and corresponding values for matrix import and export methods.

Symbol	Value	Description
GrB_CSR_FORMAT	0	Specifies the compressed sparse row matrix format.
GrB_CSC_FORMAT	1	Specifies the compressed sparse column matrix format.
GrB_COO_FORMAT	2	Specifies the sparse coordinate matrix format.

939 **3.5.4** Masks

The GraphBLAS C API defines an opaque object called a *mask*. The mask is used to control how computed values are stored in the output from a method. The mask is an *internal* opaque object; that is, it is never exposed as a variable within an application.

The mask is formed from input objects to the method that uses the mask. For example, a Graph-BLAS method may be called with a matrix as the mask parameter. The internal mask object is constructed from the input matrix in one of two ways. In the default case, an element of the mask is created for each tuple that exists in the matrix for which the value of the tuple cast to Boolean evaluates to true. Alternatively, the user can specify *structure*-only behavior where an element of the mask is created for each tuple that exists in the matrix *regardless* of the value stored in the input matrix.

The internal mask object can be either a one- or a two-dimensional construct. One- and twodimensional masks, described more formally below, are similar to vectors and matrices, respectively, except that they have structure (indices) but no values. When needed, a value is implied for the elements of a mask with an implied value of true for elements that exist and an implied value of false for elements that do not exist (i.e., the locations of the mask that do not have a stored value imply a value of false). Hence, even though a mask does not contain any values, it can be considered to imply values from a Boolean domain.

A one-dimensional mask $\mathbf{m} = \langle N, \{i\} \rangle$ is defined by its number of elements N > 0, and a set $\mathbf{ind}(\mathbf{m})$ of indices $\{i\}$ where $0 \le i < N$. A particular value of i can appear at most once in \mathbf{m} . We define $\mathbf{size}(\mathbf{m}) = N$. The set $\mathbf{ind}(\mathbf{m})$ is called the *structure* of mask \mathbf{m} .

A two-dimensional mask $\mathbf{M} = \langle M, N, \{(i,j)\} \rangle$ is defined by its number of rows M > 0, its number of columns N > 0, and a set $\mathbf{ind}(\mathbf{M})$ of tuples (i,j) where $0 \le i < M$, $0 \le j < N$. A particular pair of values i,j can appear at most once in \mathbf{M} . We define $\mathbf{ncols}(\mathbf{M}) = N$, and $\mathbf{nrows}(\mathbf{M}) = M$. We also define the sets $\mathbf{indrow}(\mathbf{M}) = \{i : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$ and $\mathbf{indcol}(\mathbf{M}) = \{j : \exists (i,j) \in \mathbf{ind}(\mathbf{M})\}$. These are the sets of nonempty rows and columns of \mathbf{M} , respectively. The set $\mathbf{ind}(\mathbf{M})$ is called the structure of mask \mathbf{M} .

One common operation on masks is the *complement*. For a one-dimensional mask \mathbf{m} this is denoted as $\neg \mathbf{m}$. For a two-dimensional mask \mathbf{M} , this is denoted as $\neg \mathbf{M}$. The complement of a one-dimensional mask \mathbf{m} is defined as $\operatorname{ind}(\neg \mathbf{m}) = \{i : 0 \le i < N, i \notin \operatorname{ind}(\mathbf{m})\}$. It is the set of all possible indices that do not appear in \mathbf{m} . The complement of a two-dimensional mask \mathbf{M} is defined as the set $\operatorname{ind}(\neg \mathbf{M}) = \{(i,j) : 0 \le i < M, 0 \le j < N, (i,j) \notin \operatorname{ind}(\mathbf{M})\}$. It is the set of all possible indices that do not appear in \mathbf{M} .

972 3.6 Descriptors

Descriptors are used to modify the behavior of a GraphBLAS method. When present in the signature of a method, they appear as the last argument in the method. Descriptors specify how the other input arguments corresponding to GraphBLAS collections – vectors, matrices, and masks – should be processed (modified) before the main operation of a method is performed. A complete list of what descriptors are capable of are presented in this section.

The descriptor is a lightweight object. It is composed of (*field*, *value*) pairs where the *field* selects one of the GraphBLAS objects from the argument list of a method and the *value* defines the indicated modification associated with that object. For example, a descriptor may specify that a particular input matrix needs to be transposed or that a mask needs to be complemented (defined in Section 3.5.4) before using it in the operation.

For the purpose of constructing descriptors, the arguments of a method that can be modified

are identified by specific field names. The output parameter (typically the first parameter in a GraphBLAS method) is indicated by the field name, GrB_OUTP. The mask is indicated by the 985 GrB_MASK field name. The input parameters corresponding to the input vectors and matrices are 986 indicated by GrB INP0 and GrB INP1 in the order they appear in the signature of the GraphBLAS 987 method. The descriptor is an opaque object and hence we do not define how objects of this type 988 should be implemented. When referring to (field, value) pairs for a descriptor, however, we often use 989 the informal notation desc[GrB Desc Field]. GrB Desc Value without implying that a descriptor is 990 to be implemented as an array of structures (in fact, field values can be used in conjunction with 991 multiple values that are composable). We summarize all types, field names, and values used with 992 descriptors in Table 3.11. 993

In the definitions of the GraphBLAS methods, we often refer to the *default behavior* of a method with respect to the action of a descriptor. If a descriptor is not provided or if the value associated with a particular field in a descriptor is not set, the default behavior of a GraphBLAS method is defined as follows:

- Input matrices are not transposed.
- The mask is used, as is, without complementing, and stored values are examined to determine whether they evaluate to true or false.
 - Values of the output object that are not directly modified by the operation are preserved.

GraphBLAS specifies all of the valid combinations of (field, value) pairs as predefined descriptors.

Their identifiers and the corresponding set of (field, value) pairs for that identifier are shown in

Table 3.12.

1005 3.7 Fields

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All GraphBLAS objects and implementations contain fields like those in the descriptor, which provide information to users and allow setting runtime parameters and hints. All GraphBLAS objects are required to implement the GrB_get and GrB_set methods required to query and set these fields. The library itself also contains several (*field*, *value*) pairs, which provide defaults to object level fields, and implementation information such as the version number or implementation name.

The required value, field pairs available for each object are defined in 3.13. Implementations may add their own custom GrB_Field enum values to extend the behavior of objects and methods. A field must always be readable, but in many cases may not be writable. Such read-only fields might contain static, compile-time information such as GrB_API_VER, while others are determined by other operations, such as GrB_BLOCKING_MODE which is determined by GrB_Init.

GrB_INVALID_VALUE must be returned when attempting to write to fields which are read only.

The GrB_Field enumeration is defined by the values in Table 3.13, and selected values are described in Table 3.14.

Table 3.11: Descriptors are GraphBLAS objects passed as arguments to GraphBLAS operations to modify other GraphBLAS objects in the operation's argument list. A descriptor, desc, has one or more (*field*, *value*) pairs indicated as desc[GrB_Desc_Field].GrB_Desc_Value. In this table, we define all types and literals used with descriptors.

(a) Types used with GraphBLAS descriptors.

Type	Description
GrB_Descriptor	Type of a GraphBLAS descriptor object.
GrB_Desc_Field	The descriptor field enumeration.
GrB_Desc_Value	The descriptor value enumeration.

(b) Descriptor field names of type GrB_Desc_Field enumeration and corresponding values.

Field Name	Value	Description
GrB_OUTP	0	Field name for the output GraphBLAS object.
GrB_MASK	1	Field name for the mask GraphBLAS object.
GrB_INP0	2	Field name for the first input GraphBLAS object.
GrB_INP1	3	Field name for the second input GraphBLAS object.

(c) Descriptor field values of type $\mathsf{GrB_Desc_Value}$ enumeration and corresponding values.

Value Name	Value	Description
GrB_DEFAULT	0	The default (unset) value for each field.
GrB_REPLACE	1	Clear the output object before assigning computed values.
GrB_COMP	2	Use the complement of the associated object. When combined
		with GrB_STRUCTURE, the complement of the structure of the
		associated object is used without evaluating the values stored.
GrB_TRAN	3	Use the transpose of the associated object.
GrB_STRUCTURE	4	The write mask is constructed from the structure (pattern of
		stored values) of the associated object. The stored values are
		not examined.
GrB_COMP_STRUCTURE	6	Shorthand for both GrB_COMP and GrB_STRUCTURE.
	'	'

Table 3.12: Predefined GraphBLAS descriptors. The list includes all possible descriptors, according to the current standard. Columns list the possible fields and entries list the value(s) associated with those fields for a given descriptor.

Identifier	GrB_OUTP	GrB_MASK	GrB_INP0	GrB_INP1
GrB_NULL	_	_	_	_
GrB_DESC_T1	_	_	_	GrB_TRAN
GrB_DESC_T0	_	_	GrB_TRAN	_
GrB_DESC_T0T1	_	_	GrB_TRAN	GrB_TRAN
GrB_DESC_C	_	GrB_COMP	_	_
GrB_DESC_S	_	GrB_STRUCTURE	_	_
GrB_DESC_CT1	_	GrB_COMP	_	GrB_TRAN
GrB_DESC_ST1	_	GrB_STRUCTURE	_	GrB_TRAN
GrB_DESC_CT0	_	GrB_COMP	GrB_TRAN	_
GrB_DESC_ST0	_	GrB_STRUCTURE	GrB_TRAN	_
GrB_DESC_CT0T1	_	GrB_COMP	$GrB _TRAN$	GrB_TRAN
GrB_DESC_ST0T1	_	GrB_STRUCTURE	GrB_TRAN	GrB_TRAN
GrB_DESC_SC	_	GrB_STRUCTURE, GrB_COMP	_	_
GrB_DESC_SCT1	_	GrB_STRUCTURE, GrB_COMP	_	GrB_TRAN
GrB_DESC_SCT0	_	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	_
GrB_DESC_SCT0T1	_	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_R	GrB_REPLACE	_	_	_
GrB_DESC_RT1	GrB_REPLACE	_	_	GrB_TRAN
GrB_DESC_RT0	GrB_REPLACE	_	GrB_TRAN	_
GrB_DESC_RT0T1	GrB_REPLACE	_	$GrB _TRAN$	GrB_TRAN
GrB_DESC_RC	GrB_REPLACE	GrB_COMP	_	_
GrB_DESC_RS	GrB_REPLACE	GrB_STRUCTURE	_	_
GrB_DESC_RCT1	GrB_REPLACE	GrB_COMP	_	GrB_TRAN
GrB_DESC_RST1	GrB_REPLACE	GrB_STRUCTURE	_	GrB_TRAN
GrB_DESC_RCT0	GrB_REPLACE	GrB_COMP	$GrB _TRAN$	_
GrB_DESC_RST0	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	_
GrB_DESC_RCT0T1	GrB_REPLACE	GrB_COMP	$GrB _TRAN$	GrB_TRAN
GrB_DESC_RST0T1	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	GrB_TRAN
GrB_DESC_RSC	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	_	_
GrB_DESC_RSCT1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	_	GrB_TRAN
GrB_DESC_RSCT0	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	_
GrB_DESC_RSCT0T1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	GrB_TRAN
	,			

1020 **3.7.1** Input Types

Allowable types used in GrB_get and GrB_set are INT32, GrB_Scalar, char*, void*, and SIZE. Each GrB_Field is associated with exactly one of these types as defined in Table 3.13. Implementations that add additional GrB_Fields must document the type associated with each GrB_Field.

1024 3.7.1.1 INT32 Handling

1025 INT32 types use a 32-bit signed integer type. This can be used both for numeric values as well as
1026 enumerated C types. Enumerated types must specify the numeric value for each enum, and the
1027 value specified must fit within the allowable 32-bit signed integer range.

1028 3.7.1.2 GrB_Scalar Handling

When calling GrB_get, the user must provide an already initialized GrB_Scalar object to which the implementation will write a value of the correct element type. When calling GrB_set, the GrB_Scalar must not be empty, otherwise a GrB_EMPTY_OBJECT error is raised.

3.7.1.3 String (char*) Handling

When the input to GrB_set is a char* the input array is null terminated. The GraphBLAS implementation must copy this array into internal data structures. Using GrB_get for strings requires two calls. First, call GrB_get with the field and object, but pass size_t* as the value argument. The implementation will return the size of the string buffer that the user must create. Second, call GrB_get with the field and object, this time passing a pointer to the newly created string buffer. The GraphBLAS implementation will write to this buffer, including a trailing null terminator. The size returned in the first call will include enough bytes for the null terminator.

1040 **3.7.1.4** void* Handling

When the input to GrB set is a void*, an extra size t argument is passed to indicate the size of the 1041 buffer. The GraphBLAS implementation must copy this many bytes from the buffer into internal 1042 data structures. Similar to reading strings, GrB get must be called twice for void*. The first call 1043 passes size t* to find the required buffer size. The user must create a buffer and then pass the 1044 pointer to GrB_get. The implementation will write to this buffer. No standard specification or 1045 protocol is required for the contents of void*. It is meant to be a mechanism to allow full freedom 1046 for GraphBLAS implementations with needs that cannot be handled using INT32, GrB_Scalar, or 1047 Strings. 1048

1049 3.7.1.5 SIZE Handling

SIZE types use a size_t type. Normally, SIZE is used in conjunction with char* and void* to indicate the buffer size. However, it can also be used when the actual return type is size_t, as is the case

1052 for the size of a Type.

1053 3.7.2 Hints

Several fields are *hints* (marked H in Table 3.13). Hints are used to represent intended use cases or best guesses, but do not determine strict behavior. When GrB_set is called with a hint, the GraphBLAS implementation should return GrB_SUCCESS, but is free to use or ignore the hint.

When GrB_get is called, the implementation should return a best guess on the correct answer. If there is no clear answer, the implementation should return GrB_UNKNOWN.

1059 **3.7.3** GrB_NAME

The GrB_NAME field is a special case regarding writability. All user-defined objects have a GrB_NAME field which defaults to an empty string. Collections and GrB_Descriptors may have their GrB_NAME set at any time. User-defined algebraic objects and GrB_Types may only have their GrB_NAME set once to a globally unique value. Attempting to set this field after it has already been set will return a GrB_ALREADY_SET_error code.

Built-in algebraic objects and GrB_Types have names which can be read, but not written to. The name returned will be the string form of the GrB_Type listed in Table 3.2 or the GraphBLAS identifier listed in Tables 3.5, 3.6, 3.7, 3.8, and 3.9. For example, the name of GrB_BOOL type is "GrB_BOOL" (8 characters) and the name of GrB_MIN_FP64 binary op is "GrB_MIN_FP64" (12 characters).

The GrB_NAME of the global context is read-only and returns the name of the library implementation.

Field Name	W H	Value	Implementing Objects	Type
GrB_OUTP_FIELD	W	0	GrB_Descriptor	INT32 (GrB_Desc_Value)
GrB_MASK_FIELD	W —	1	GrB_Descriptor	INT32 (GrB_Desc_Value)
GrB_INP0_FIELD	W —	2	GrB_Descriptor	INT32 (GrB_Desc_Value)
GrB_INP1_FIELD	W	3	GrB_Descriptor	INT32 (GrB_Desc_Value)
GrB_NAME	*	10	Global, Collection, Algebraic, Type	Null terminated char*
GrB_LIBRARY_VER_MAJOR		11	Global	INT32
GrB_LIBRARY_VER_MINOR	_ —	12	Global	INT32
GrB_LIBRARY_VER_PATCH		13	Global	INT32
GrB_API_VER_MAJOR		14	Global	INT32
GrB_API_VER_MINOR		15	Global	INT32
GrB_API_VER_PATCH		16	Global	INT32
GrB_BLOCKING_MODE		17	Global	INT32 (GrB_Mode)
GrB_STORAGE_ORIENTATION_HINT	W H	100	Global, Collection	INT32 (GrB_Orientation)
GrB_ELTYPE_CODE		102	Collection, Type	INT32 (GrB_Type_Code)
GrB_INPUT1TYPE_CODE	- -	103	Algebraic	INT32 (GrB_Type_Code)
GrB_INPUT2TYPE_CODE		104	Algebraic	INT32 (GrB_Type_Code)
GrB_OUTPUTTYPE_CODE		105	Algebraic	INT32 (GrB_Type_Code)
GrB_ELTYPE_STRING		106	Collection, Type	Null terminated char*
GrB_INPUT1TYPE_STRING		107	Algebraic	Null terminated char*
GrB_INPUT2TYPE_STRING		108	Algebraic	Null terminated char*
GrB_OUTPUTTYPE_STRING		109	Algebraic	Null terminated char*
GrB_SIZE	_ _	110	Туре	SIZE

Table 3.14: Descriptions	of select field, value pairs listed in 3.13
Field Name	Description
GrB_NAME	The name of any GraphBLAS object,
	or the name of the library implementation.
GrB_BLOCKING_MODE	The blocking mode as set by GrB_init
GrB_STORAGE_ORIENTATION_HINT	Hint to the library that a collection is best stored in
	a row (lexicographic) or column (colexicographic) ma-
	jor format.
GrB_ELTYPE_(CODE/STRING)	The element type of a collection.
GrB_INPUT1TYPE_(CODE/STRING)	The type of the first argument to an operator.
	Returns GrB_NO_VALUE for Semirings and
	IndexUnaryOps which depend only on the index.
GrB_INPUT2TYPE_(CODE/STRING)	The type of the second argument to an operator.
	Returns GrB_NO_VALUE for Semirings, UnaryOps,
	and IndexUnaryOps which depend only on the index.
GrB_OUTPUTTYPE_(CODE/STRING)	The type of the output of an operator.
GrB_SIZE	The size of the GrB_Type.

3.8 GrB_Info return values

All GraphBLAS methods return a GrB_Info enumeration value. The three types of return codes (informational, API error, and execution error) and their corresponding values are listed in Table 3.16.

Table 3.15: Enumerations not defined elsewhere in the documents and used when getting or setting fields are defined in the following tables.

(a) Field values of type GrB_Orientation.

Value Name	Value	Description
GrB_ROWMAJOR	0	The majority of iteration over the object will be row-wise.
GrB_COLMAJOR	1	The majority of iteration over the object will be column-wise.
GrB_BOTH	2	Iteration may occur with equal frequency in both directions.
GrB_UNKNOWN	3	No indication is given or is unknown.
	ļi	

Table 3.16: Enumeration literals and corresponding values returned by GraphBLAS methods and operations.

(a) Informational return values

Symbol	Value	Description
GrB_SUCCESS	0	The method/operation completed successfully (blocking mode), or
		encountered no API errors (non-blocking mode).
GrB_NO_VALUE	1	A location in a matrix or vector is being accessed that has no stored
		value at the specified location.

(b) API errors

Symbol	Value	Description
GrB_UNINITIALIZED_OBJECT	-1	A GraphBLAS object is passed to a method before
		new was called on it.
GrB_NULL_POINTER	-2	A NULL is passed for a pointer parameter.
GrB_INVALID_VALUE	-3	Miscellaneous incorrect values.
GrB_INVALID_INDEX	-4	Indices passed are larger than dimensions of the ma-
		trix or vector being accessed.
GrB_DOMAIN_MISMATCH	-5	A mismatch between domains of collections and op-
		erations when user-defined domains are in use.
GrB_DIMENSION_MISMATCH	-6	Operations on matrices and vectors with incompati-
		ble dimensions.
GrB_OUTPUT_NOT_EMPTY	-7	An attempt was made to build a matrix or vector
		using an output object that already contains valid
		tuples (elements).
GrB_NOT_IMPLEMENTED	-8	An attempt was made to call a GraphBLAS method
		for a combination of input parameters that is not
		supported by a particular implementation.
GrB_ALREADY_SET	-9	An attempt was made to write to a field which may
		only be written to once.

(c) Execution errors

Symbol	Value	Description
GrB_PANIC	-101	Unknown internal error.
GrB_OUT_OF_MEMORY	-102	Not enough memory for operations.
GrB_INSUFFICIENT_SPACE	-103	The array provided is not large enough to hold out-
GrB_INVALID_OBJECT	-104	put. One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous
		execution error.
GrB_INDEX_OUT_OF_BOUNDS	-105	Reference to a vector or matrix element that is outside the defined dimensions of the object.
GrB_EMPTY_OBJECT	-106	One of the opaque GraphBLAS objects does not
		have a stored value.

Chapter 4

$_{\scriptscriptstyle{077}}$ Methods

This chapter defines the behavior of all the methods in the GraphBLAS C API. All methods can be declared for use in programs by including the GraphBLAS.h header file.

We would like to emphasize that no GraphBLAS method will imply a predefined order over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.

.084 4.1 Context methods

The methods in this section set up and tear down the GraphBLAS context within which all Graph-BLAS methods must be executed. The initialization of this context also includes the specification of which execution mode is to be used.

1088 4.1.1 init: Initialize a GraphBLAS context

 $_{1089}$ Creates and initializes a GraphBLAS C API context.

1090 C Syntax

GrB_Info GrB_init(GrB_Mode mode);

Parameters

1092

1093

mode Mode for the GraphBLAS context. Must be either GrB_BLOCKING or GrB_NONBLOCKING.

1094 Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_INVALID_VALUE invalid mode specified, or method called multiple times.

1098 Description

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The init method creates and initializes a GraphBLAS C API context. The argument to GrB_init defines the mode for the context. The two available modes are:

- GrB_BLOCKING: In this mode, each method in a sequence returns after its computations have completed and output arguments are available to subsequent statements in an application. When executing in GrB_BLOCKING mode, the methods execute in program order.
- GrB_NONBLOCKING: In this mode, methods in a sequence may return after arguments in the method have been tested for dimension and domain compatibility within the method but potentially before their computations complete. Output arguments are available to subsequent GraphBLAS methods in an application. When executing in GrB_NONBLOCKING mode, the methods in a sequence may execute in any order that preserves the mathematical result defined by the sequence.

An application can only create one context per execution instance. An application may only call GrB_Init once. Calling GrB_Init more than once results in undefined behavior.

1112 4.1.2 finalize: Finalize a GraphBLAS context

Terminates and frees any internal resources created to support the GraphBLAS C API context.

1114 C Syntax

```
GrB_Info GrB_finalize();
```

116 Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

1119 Description

The finalize method terminates and frees any internal resources created to support the GraphBLAS C API context. GrB_finalize may only be called after a context has been initialized by calling GrB_init, or else undefined behavior occurs. After GrB_finalize has been called to finalize a Graph-BLAS context, calls to any GraphBLAS methods, including GrB_finalize, will result in undefined behavior.

1125 4.1.3 getVersion: Get the version number of the standard.

1126 Query the library for the version number of the standard that this library implements.

1127 C Syntax

```
GrB_Info GrB_getVersion(unsigned int *version, unsigned int *subversion);
```

1130 Parameters

version (OUT) On successful return will hold the value of the major version number.

version (OUT) On successful return will hold the value of the subversion number.

1133 Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

1136 Description

1142

The getVersion method is used to query the major and minor version number of the GraphBLAS C API specification that the library implements at runtime. To support compile time queries the following two macros shall also be defined by the library.

```
#define GRB_VERSION 2
#define GRB_SUBVERSION 0
```

4.2 Object methods

This section describes methods that setup and operate on GraphBLAS opaque objects but are not part of the the GraphBLAS math specification.

1145 4.2.1 Get and Set methods

The methods in this section query and, optionally, set internal fields of GraphBLAS objects.

1147 4.2.1.1 get: Query the value of an object

1148 C Syntax

GrB Info GrB get(GrB < OBJ > o, <type > value, GrB Field field);

1150 Parameters

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1163

OBJ (IN) An existing, valid GraphBLAS object (collection, operation, type) which is being queried. To indicate the global context, the constant GrB_Global is used.

value (OUT) A pointer to or GrB_Scalar containing a value whose type is dependent on field which will be filled with the current value of the field. type may be int32_t*, size t*, GrB Scalar, char* or void*.

field (IN) The field being queried.

1157 Return Value

GrB_SUCCESS The method completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT the value parameter is GrB_Scalar and has not been initialized by a call to new.

GrB_INVALID_VALUE invalid value type provided for the field or invalid field.

1164 Description

Queries a field of an existing GraphBLAS object. The type of the argument is uniquely determined by field. For the case of char* and void*, the value can be replaced with size_t* to get the required buffer size to hold the response. Fields marked as hints in Table 3.13 will return a hint on how best to use the object.

1169 4.2.1.2 set: Set field of an object

1170 Set the content for a field for an existing GraphBLAS object.

1171 C Syntax

```
GrB_Info GrB_set(GrB_<OBJ> o, <type> value, GrB_Field field);
GrB_Info GrB_set(GrB_<OBJ> o, void *value, GrB_Field field, size_t voidSize);
```

1174 Parameters

OBJ (IN) The GraphBLAS object which is having field set. To indicate the global context, the constant GrB_Global is used.

value (IN) A value whose type is dependent on field. type may be a int32_t, GrB_Scalar, char* or void*.

field (IN) The field being set.

voidSize (IN) The size of the void* buffer. Note that a size is not needed for char* because the string is assumed null-terminated.

1182 Return Values

1179

GrB_SUCCESS The method completed successfully.

GrB_PANIC unknown internal error.

Grb Out Of Memory not enough memory available for operation.

1186 GrB_UNINITIALIZED_OBJECT the GrB_Scalar parameter has not been initialized by a call to new.

GrB_INVALID_VALUE invalid value set on the field, invalid field, or field is read-only.

GrB_ALREADY_SET this field has already been set, and may only be set once.

1189 Description

1190 Set a field of OBJ or the Global context to a new value.

1191 4.2.2 Algebra methods

1192 4.2.2.1 Type_new: Construct a new GraphBLAS (user-defined) type

Creates a new user-defined GraphBLAS type. This type can then be used to create new operators, monoids, semirings, vectors and matrices.

1195 C Syntax

```
GrB_Info GrB_Type_new(GrB_Type *utype,
size_t sizeof(ctype));
```

1198 Parameters

1201

1202

1205

1206

utype (INOUT) On successful return, contains a handle to the newly created user-defined GraphBLAS type object.

ctype (IN) A C type that defines the new GraphBLAS user-defined type.

Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_NULL_POINTER utype pointer is NULL.

1207 Description

Given a C type ctype, the Type_new method returns in utype a handle to a new GraphBLAS type that is equivalent to the C type. Variables of this ctype must be a struct, union, or fixed-size array.

In particular, given two variables, src and dst, of type ctype, the following operation must be a valid way to copy the contents of src to dst:

```
memcpy(&dst, &src, sizeof(ctype))
```

A new, user-defined type utype should be destroyed with a call to GrB_free(utype) when no longer needed.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

1217 4.2.2.2 UnaryOp_new: Construct a new GraphBLAS unary operator

Initializes a new GraphBLAS unary operator with a specified user-defined function and its types (domains).

1220 C Syntax

```
GrB_Info GrB_UnaryOp_new(GrB_UnaryOp *unary_op,
void (*unary_func)(void*, const void*),
GrB_Type d_out,
GrB_Type d_in);
```

1225 Parameters

unary_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS unary operator object.

unary_func (IN) a pointer to a user-defined function that takes one input parameter of d_in's type and returns a value of d_out's type, both passed as void pointers. Specifically the signature of the function is expected to be of the form:

void func(void *out, const void *in);

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d_out (IN) The GrB_Type of the return value of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined Graph-BLAS type.

d_in (IN) The GrB_Type of the input argument of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

1239 Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT any GrB_Type parameter (for user-defined types) has not been initialized by a call to GrB_Type_new.

GrB_NULL_POINTER unary_op or unary_func pointers are NULL.

1246 Description

The UnaryOp_new method creates a new GraphBLAS unary operator

$$f_u = \langle \mathbf{D}(\mathsf{d_out}), \mathbf{D}(\mathsf{d_in}), \mathsf{unary_func} \rangle$$

```
and returns a handle to it in unary_op.
```

The implementation of unary_func must be such that it works even if the d_out and d_in arguments are aliased. In other words, for all invocations of the function:

```
unary_func(out,in);
```

the value of out must be the same as if the following code was executed:

```
 D(d_{in}) *tmp = malloc(sizeof(D(d_{in}))); 
 memcpy(tmp,in,sizeof(D(d_{in}))); 
 unary_func(out,tmp); 
 free(tmp);
```

1258 It is not an error to call this method more than once on the same variable; however, the handle to 1259 the previously created object will be overwritten.

4.2.2.3 BinaryOp_new: Construct a new GraphBLAS binary operator

Initializes a new GraphBLAS binary operator with a specified user-defined function and its types (domains).

1263 C Syntax

```
1264
             GrB_Info GrB_BinaryOp_new(GrB_BinaryOp *binary_op,
                                            void
                                                          (*binary_func)(void*,
1265
                                                                           const void*,
1266
                                                                           const void*),
1267
                                            GrB_Type
                                                            d_out,
1268
                                            GrB_Type
                                                            d_in1,
1269
                                            GrB_Type
                                                            d_in2);
1270
```

Parameters

1271

1278

binary_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS binary operator object.

binary_func (IN) A pointer to a user-defined function that takes two input parameters of types
d_in1 and d_in2 and returns a value of type d_out, all passed as void pointers.

Specifically the signature of the function is expected to be of the form:

```
void func(void *out, const void *in1, const void *in2);
```

```
d_out (IN) The GrB_Type of the return value of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
```

- d_in1 (IN) The GrB_Type of the left hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- d_in2 (IN) The GrB_Type of the right hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

1288 Return Values

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1283

1284

1285

1286

1287

1294

```
GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a
```

call to GrB_Type_new.

GrB_NULL_POINTER binary_op or binary_func pointer is NULL.

1295 Description

The BinaryOp_new methods creates a new GraphBLAS binary operator

```
f_b = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{binary\_func} \rangle
```

and returns a handle to it in binary_op.

The implementation of binary_func must be such that it works even if any of the d_out, d_in1, and d_in2 arguments are aliased to each other. In other words, for all invocations of the function:

```
binary_func(out,in1,in2);
```

the value of out must be the same as if the following code was executed:

```
\begin{array}{lll} & \mathbf{D}(\texttt{d\_in1}) * \texttt{tmp1} = \texttt{malloc(sizeof}(\mathbf{D}(\texttt{d\_in1}))); \\ & \mathbf{D}(\texttt{d\_in2}) * \texttt{tmp2} = \texttt{malloc(sizeof}(\mathbf{D}(\texttt{d\_in2}))); \\ & \texttt{1305} & \texttt{memcpy}(\texttt{tmp1,in1,sizeof}(\mathbf{D}(\texttt{d\_in1}))); \\ & \texttt{1306} & \texttt{memcpy}(\texttt{tmp2,in2,sizeof}(\mathbf{D}(\texttt{d\_in2}))); \\ & \texttt{1307} & \texttt{binary\_func(out,tmp1,tmp2)}; \\ & \texttt{1308} & \texttt{free(tmp2)}; \\ & \texttt{1309} & \texttt{free(tmp1)}; \\ \end{array}
```

1310 It is not an error to call this method more than once on the same variable; however, the handle to 1311 the previously created object will be overwritten.

1312 4.2.2.4 Monoid_new: Construct a new GraphBLAS monoid

1313 Creates a new monoid with specified binary operator and identity value.

1314 C Syntax

```
GrB_Info GrB_Monoid_new(GrB_Monoid *monoid,
GrB_BinaryOp binary_op,
<type> identity);
```

1318 Parameters

- monoid (INOUT) On successful return, contains a handle to the newly created GraphBLAS monoid object.
- binary_op (IN) An existing GraphBLAS associative binary operator whose input and output types are the same.
- identity (IN) The value of the identity element of the monoid. Must be the same type as
 the type used by the binary_op operator.

1325 Return Values

- GrB_SUCCESS operation completed successfully.
- GrB_PANIC unknown internal error.
- GrB_OUT_OF_MEMORY not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT the GrB_BinaryOp (for user-defined operators) has not been initialized by a call to GrB_BinaryOp_new.
- 1331 GrB NULL POINTER monoid pointer is NULL.
- GrB_DOMAIN_MISMATCH all three argument types of the binary operator and the type of the identity value are not the same.

1334 Description

The Monoid_new method creates a new monoid $M = \langle \mathbf{D}(\mathsf{binary_op}), \mathsf{binary_op}, \mathsf{identity} \rangle$ and returns a handle to it in monoid.

If binary_op is not associative, the results of GraphBLAS operations that require associativity of this monoid will be undefined.

1339 It is not an error to call this method more than once on the same variable; however, the handle to 1340 the previously created object will be overwritten.

1341 4.2.2.5 Semiring_new: Construct a new GraphBLAS semiring

1342 Creates a new semiring with specified domain, operators, and elements.

1343 C Syntax

```
GrB_Info GrB_Semiring_new(GrB_Semiring *semiring,

GrB_Monoid add_op,

GrB_BinaryOp mul_op);
```

347 Parameters

1352

1353

1354

semiring (INOUT) On successful return, contains a handle to the newly created GraphBLAS semiring.

add_op (IN) An existing GraphBLAS commutative monoid that specifies the addition operator and its identity.

mul_op (IN) An existing GraphBLAS binary operator that specifies the semiring's multiplication operator. In addition, mul_op's output domain, $\mathbf{D}_{out}(\mathsf{mul_op})$, must be the same as the add_op's domain $\mathbf{D}(\mathsf{add_op})$.

1355 Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

1358 GrB_OUT_OF_MEMORY not enough memory available for this method to complete.

GrB_UNINITIALIZED_OBJECT the add_op (for user-define monoids) object has not been initialized with a call to GrB_Monoid_new or the mul_op (for user-defined operators) object has not been initialized by a call to GrB_BinaryOp new.

GrB_NULL_POINTER semiring pointer is NULL.

GrB_DOMAIN_MISMATCH the output domain of mul_op does not match the domain of the add_op monoid.

1366 Description

1367 The Semiring_new method creates a new semiring:

```
S = \langle \mathbf{D}_{out}(\mathsf{mul\_op}), \mathbf{D}_{in_1}(\mathsf{mul\_op}), \mathbf{D}_{in_2}(\mathsf{mul\_op}), \mathsf{add\_op}, \mathsf{mul\_op}, \mathbf{0}(\mathsf{add\_op}) \rangle
```

and returns a handle to it in semiring. Note that $\mathbf{D}_{out}(\mathsf{mul} \ \mathsf{op})$ must be the same as $\mathbf{D}(\mathsf{add} \ \mathsf{op})$.

1370 If add_op is not commutative, then GraphBLAS operations using this semiring will be undefined.

1371 It is not an error to call this method more than once on the same variable; however, the handle to 1372 the previously created object will be overwritten.

1373 **4.2.2.6** IndexUnaryOp_new: Construct a new GraphBLAS index unary operator [Scott: NEW CONTENT]

Initializes a new GraphBLAS index unary operator with a specified user-defined function and its types (domains).

1377 C Syntax

```
GrB_Info GrB_IndexUnaryOp_new(GrB_IndexUnaryOp
                                                                  *index_unary_op,
1378
                                            void (*index unary func)(void*,
1379
                                                                          const void*,
1380
                                                                         GrB Index,
1381
                                                                         GrB_Index,
1382
                                                                         const void*),
1383
                                            GrB_Type
                                                                    d_out,
1384
                                            GrB_Type
                                                                    d_in1,
1385
                                            GrB_Type
                                                                    d_in2);
1386
```

1387 Parameters

index_unary_op (INOUT) On successful return, contains a handle to the newly created Graph-BLAS index unary operator object.

index_unary_func (IN) A pointer to a user-defined function that takes input parameters of types d_in1, GrB_Index, GrB_Index and d_in2 and returns a value of type d_out. Except for the GrB_Index parameters, all are passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void *out,
const void *in1,
GrB_Index row_index,
GrB_Index col_index,
```

```
const void *in2);
1398
1399
                d_out (IN) The GrB_Type of the return value of the index unary operator being created.
1400
                       Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined
1401
                       GraphBLAS type.
1402
                d in (IN) The GrB Type of the first input argument of the index unary operator being
1403
                       created and corresponds to the stored values of the GrB_Vector or GrB_Matrix
1404
                       being operated on. Should be one of the predefined GraphBLAS types in Ta-
1405
                       ble 3.2, or a user-defined GraphBLAS type.
1406
                d_in2 (IN) The GrB_Type of the last input argument of the index unary operator be-
1407
                       ing created and corresponds to a scalar provided by the GraphBLAS operation
1408
                       that uses this operator. Should be one of the predefined GraphBLAS types in
1409
                       Table 3.2, or a user-defined GraphBLAS type.
1410
    Return Values
1411
                     GrB_SUCCESS operation completed successfully.
1412
                        GrB_PANIC unknown internal error.
1413
         GrB OUT OF MEMORY not enough memory available for operation.
1414
   GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a
1415
                                     call to GrB Type new.
1416
             GrB_NULL_POINTER index_unary_op or index_unary_func pointer is NULL.
1417
    Description
1418
     The IndexUnaryOp_new methods creates a new GraphBLAS index unary operator
1410
          f_i = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{index\_unary\_func} \rangle
1420
    and returns a handle to it in index unary op.
1421
    The implementation of index unary func must be such that it works even if any of the d out,
    d in1, and d in2 arguments are aliased to each other. In other words, for all invocations of the
1423
    function:
1424
           index_unary_func(out,in1,row_index,col_index,n,in2);
1425
    the value of out must be the same as if the following code was executed (shown here for matrices):
1426
```

```
GrB_Index row_index = ...;
1427
          GrB_Index col_index = ...;
1428
         D(d_{in1}) *tmp1 = malloc(sizeof(D(d_{in1})));
1429
         D(d_{in2}) *tmp2 = malloc(sizeof(D(d_{in2})));
1430
         memcpy(tmp1,in1,sizeof(D(d in1)));
1431
         memcpy(tmp2,in2,sizeof(D(d_in2)));
1432
          index unary func(out,tmp1,row index,col index,tmp2);
1433
         free(tmp2);
1434
         free(tmp1);
1435
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

1438 4.2.3 Scalar methods

1439 4.2.3.1 Scalar_new: Construct a new scalar

1440 Creates a new empty scalar with specified domain.

1441 C Syntax

```
GrB_Info GrB_Scalar_new(GrB_Scalar *s,

GrB_Type d);
```

Parameters Parameters

- s (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.
- d (IN) The type corresponding to the domain of the scalar being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

1450 Return Values

1455

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar s is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB_error() to access any error 1458 messages generated by the implementation. 1459 GrB_OUT_OF_MEMORY Not enough memory available for operation. 1460 GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new 1461 (needed for user-defined types). 1462 GrB_NULL_POINTER The s pointer is NULL. 1463

Description 1464

Creates a new GraphBLAS scalar s of domain $\mathbf{D}(\mathsf{d})$ and empty $\mathbf{L}(s)$. The method returns a handle 1465 to the new scalar in s.

It is not an error to call this method more than once on the same variable; however, the handle to 1467 the previously created object will be overwritten. 1468

4.2.3.2Scalar_dup: Construct a copy of a GraphBLAS scalar 1469

Creates a new scalar with the same domain and contents as another scalar.

C Syntax 1471

```
GrB_Info GrB_Scalar_dup(GrB_Scalar
                                                           *t,
1472
                                        const GrB_Scalar s);
1473
```

Parameters 1474

- t (INOUT) On successful return, contains a handle to the newly created GraphBLAS 1475 scalar. 1476
- s (IN) The GraphBLAS scalar to be duplicated. 1477

Return Values 1478

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-1479 blocking mode, this indicates that the API checks for the input 1480 arguments passed successfully. Either way, output scalar t is ready 1481 to be used in the next method of the sequence. 1482 1483

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque 1484 GraphBLAS objects (input or output) is in an invalid state caused 1485

by a previous execution error. Call GrB_error() to access any error 1486 messages generated by the implementation. 1487 GrB_OUT_OF_MEMORY Not enough memory available for operation. 1488 GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup. 1490 GrB_NULL_POINTER The t pointer is NULL. 1491 Description 1492 Creates a new scalar t of domain $\mathbf{D}(s)$ and contents $\mathbf{L}(s)$. The method returns a handle to the new 1493 scalar in t. It is not an error to call this method more than once with the same output variable; however, the 1495 handle to the previously created object will be overwritten. 4.2.3.3Scalar_clear: Clear/remove a stored value from a scalar 1497 Removes the stored value from a scalar. 1498 C Syntax 1499 GrB_Info GrB_Scalar_clear(GrB_Scalar s); 1500 **Parameters** s (INOUT) An existing GraphBLAS scalar to clear. 1502 Return Values 1503 GrB_SUCCESS In blocking mode, the operation completed successfully. In non-1504 blocking mode, this indicates that the API checks for the input 1505 arguments passed successfully. Either way, output scalar s is ready 1506 to be used in the next method of the sequence. 1507 GrB_PANIC Unknown internal error. 1508 GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque 1509 GraphBLAS objects (input or output) is in an invalid state caused 1510 by a previous execution error. Call GrB_error() to access any error 1511 messages generated by the implementation. 1512

GrB_OUT_OF_MEMORY Not enough memory available for operation.

1513

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

1516 Description

Removes the stored value from an existing scalar. After the call, L(s) is empty. The size of the scalar does not change.

1519 4.2.3.4 Scalar_nvals: Number of stored elements in a scalar

Retrieve the number of stored elements in a scalar (either zero or one).

1521 C Syntax

```
GrB_Info GrB_Scalar_nvals(GrB_Index *nvals, const GrB_Scalar s);
```

1524 Parameters

1527

1536

nvals (OUT) On successful return, this is set to the number of stored elements in the scalar (zero or one).

s (IN) An existing GraphBLAS scalar being queried.

1528 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

GrB_NULL_POINTER The nvals pointer is NULL.

Return nvals(s) in nvals. This is the number of stored elements in scalar s, which is the size of L(s), and can only be either zero or one (see Section 3.5.1).

4.2.3.5 Scalar_setElement: Set the single element in a scalar

Set the single element of a scalar to a given value.

1545 C Syntax

```
GrB_Info GrB_Scalar_setElement(GrB_Scalar s, <a href="mailto:stype"><type</a> val);
```

1548 Parameters

1550

1557

1562

1565

s (INOUT) An existing GraphBLAS scalar for which the element is to be assigned.

val (IN) Scalar value to assign. The type must be compatible with the domain of s.

1551 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output scalar s is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

GrB_DOMAIN_MISMATCH The domains of s and val are incompatible.

First, val and output GraphBLAS scalar are tested for domain compatibility as follows: **D**(val) must be compatible with **D**(s). Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Scalar_setElement ends and the domain mismatch error listed above is returned.

We are now ready to carry out the assignment val; that is:

```
\mathsf{s}(0) = \mathsf{val}
```

1575 If s already had a stored value, it will be overwritten; otherwise, the new value is stored in s.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of s is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of scalar s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

1580 4.2.3.6 Scalar_extractElement: Extract a single element from a scalar.

Assign a non-opaque scalar with the value of the element stored in a GraphBLAS scalar.

1582 C Syntax

```
GrB_Info GrB_Scalar_extractElement(<type> *val,
const GrB_Scalar s);
```

1585 Parameters

1589

1591

1592

1593

1594

1595

1596

val (INOUT) Pointer to a non-opaque scalar of type that is compatible with the domain of scalar s. On successful return, val holds the result of the operation, and any previous value in val is overwritten.

s (IN) The GraphBLAS scalar from which an element is extracted.

1590 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val, has been computed and is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

 $_{1602}$ GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to $_{1603}$ Scalar_new or Scalar_dup.

GrB_NULL_POINTER val pointer is NULL.

GrB_DOMAIN_MISMATCH The domains of the scalar or scalar are incompatible.

GrB_NO_VALUE There is no stored value in the scalar.

1607 Description

1601

1605

1606

First, val and input GraphBLAS scalar are tested for domain compatibility as follows: **D**(val) must be compatible with **D**(s). Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Scalar_extractElement ends and the domain mismatch error listed above is returned.

Then, if no value is currently stored in the GraphBLAS scalar, the method returns GrB_NO_VALUE and val remains unchanged.

Finally the extract into the output argument, val can be performed; that is:

$$val = s(0)$$

In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value GrB_SUCCESS, the new contents of val are as defined above.

1620 4.2.4 Vector methods

1621 4.2.4.1 Vector_new: Construct new vector

1622 Creates a new vector with specified domain and size.

1623 C Syntax

```
GrB_Info GrB_Vector_new(GrB_Vector *v,
GrB_Type d,
GrB_Index nsize);
```

Parameters

1627

1639

- v (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.
- d (IN) The type corresponding to the domain of the vector being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nsize (IN) The size of the vector being created.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector v is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).

GrB_NULL_POINTER The v pointer is NULL.

Grb Invalid Value nsize is zero or outside the range of the type Grb Index.

1649 Description

Creates a new vector \mathbf{v} of domain $\mathbf{D}(\mathsf{d})$, size nsize, and empty $\mathbf{L}(\mathbf{v})$. The method returns a handle to the new vector in \mathbf{v} .

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

1654 4.2.4.2 Vector_dup: Construct a copy of a GraphBLAS vector

1655 Creates a new vector with the same domain, size, and contents as another vector.

1656 C Syntax

```
GrB_Info GrB_Vector_dup(GrB_Vector *w, const GrB_Vector u);
```

1659 Parameters

1662

w (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.

u (IN) The GraphBLAS vector to be duplicated.

1663 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, u, has not been initialized by a call to Vector_new or Vector_dup.

GrB_NULL_POINTER The w pointer is NULL.

1677 Description

Creates a new vector \mathbf{w} of domain $\mathbf{D}(u)$, size $\mathbf{size}(u)$, and contents $\mathbf{L}(u)$. The method returns a handle to the new vector in \mathbf{w} .

1680 It is not an error to call this method more than once on the same variable; however, the handle to 1681 the previously created object will be overwritten.

1682 4.2.4.3 Vector_resize: Resize a vector

1683 Changes the size of an existing vector.

1684 C Syntax

```
GrB_Info GrB_Vector_resize(GrB_Vector w,
GrB_Index nsize);
```

1687 Parameters

1689

1695

1702

w (INOUT) An existing Vector object that is being resized.

nsize (IN) The new size of the vector. It can be smaller or larger than the current size.

1690 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

Grb Out Of Memory Not enough memory available for operation.

GrB_NULL_POINTER The w pointer is NULL.

GrB_INVALID_VALUE nsize is zero or outside the range of the type GrB_Index.

1703 Description

Changes the size of w to nsize. The domain $\mathbf{D}(w)$ of vector w remains the same. The contents $\mathbf{L}(w)$ are modified as described below.

Let $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), N, \mathbf{L}(\mathbf{w}) \rangle$ when the method is called. When the method returns, $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathsf{nsize}, \mathbf{L}'(\mathbf{w}) \rangle$ where $\mathbf{L}'(\mathbf{w}) = \{(i, w_i) : (i, w_i) \in \mathbf{L}(\mathbf{w}) \land (i < \mathsf{nsize})\}$. That is, all elements of \mathbf{w} with index greater than or equal to the new vector size (nsize) are dropped.

1709 4.2.4.4 Vector_clear: Clear a vector

1710 Removes all the elements (tuples) from a vector.

```
C Syntax
1711
              GrB_Info GrB_Vector_clear(GrB_Vector v);
1712
    Parameters
                  v (INOUT) An existing GraphBLAS vector to clear.
1714
    Return Values
1715
                    GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
1716
                                    blocking mode, this indicates that the API checks for the input
1717
                                    arguments passed successfully. Either way, output vector v is ready
1718
                                    to be used in the next method of the sequence.
1719
                       GrB PANIC Unknown internal error.
1720
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
1721
                                    GraphBLAS objects (input or output) is in an invalid state caused
1722
                                    by a previous execution error. Call GrB_error() to access any error
1723
                                    messages generated by the implementation.
1724
        GrB_OUT_OF_MEMORY Not enough memory available for operation.
1725
   GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to
1726
                                    Vector_new or Vector_dup.
1727
    Description
1728
    Removes all elements (tuples) from an existing vector. After the call to GrB_Vector_clear(v),
1729
    \mathbf{L}(\mathbf{v}) = \emptyset. The size of the vector does not change.
1730
    4.2.4.5
              Vector_size: Size of a vector
    Retrieve the size of a vector.
    C Syntax
1733
```

const GrB_Vector v);

*nsize,

GrB_Info GrB_Vector_size(GrB_Index

1734

1735

736 Parameters

1738

nsize (OUT) On successful return, is set to the size of the vector.

v (IN) An existing GraphBLAS vector being queried.

1739 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nsize has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.

GrB_NULL_POINTER nsize pointer is NULL.

1750 Description

1751 Return size(v) in nsize.

1752 4.2.4.6 Vector_nvals: Number of stored elements in a vector

1753 Retrieve the number of stored elements (tuples) in a vector.

1754 C Syntax

```
GrB_Info GrB_Vector_nvals(GrB_Index *nvals, const GrB_Vector v);
```

Parameters

1757

1760

nvals (OUT) On successful return, this is set to the number of stored elements (tuples) in the vector.

v (IN) An existing GraphBLAS vector being queried.

1761 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

Grb Panic Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.

GrB_NULL_POINTER The nvals pointer is NULL.

1773 Description

Return nvals(v) in nvals. This is the number of stored elements in vector v, which is the size of L(v) (see Section 3.5.2).

1776 4.2.4.7 Vector_build: Store elements from tuples into a vector

1777 C Syntax

```
        1778
        GrB_Info GrB_Vector_build(GrB_Vector
        w,

        1779
        const GrB_Index
        *indices,

        1780
        const <type>
        *values,

        1781
        GrB_Index
        n,

        1782
        const GrB_BinaryOp
        dup);
```

1783 Parameters

1788

w (INOUT) An existing Vector object to store the result.

indices (IN) Pointer to an array of indices.

values (IN) Pointer to an array of scalars of a type that is compatible with the domain of vector w.

n (IN) The number of entries contained in each array (the same for indices and values).

dup (IN) An associative and commutative binary operator to apply when duplicate values for the same location are present in the input arrays. All three domains of dup must be the same; hence $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$. If dup is GrB_NULL, then duplicate locations will result in an error.

Return Values

1789

1790

1791

1792

1798

1803

1807

1814

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT Either w has not been initialized by a call to by GrB_Vector_new or by GrB_Vector_dup, or dup has not been initialized by a call to by GrB_BinaryOp_new.

GrB NULL POINTER indices or values pointer is NULL.

1808 GrB INDEX OUT OF BOUNDS A value in indices is outside the allowed range for w.

GrB_DOMAIN_MISMATCH Either the domains of the GraphBLAS binary operator dup are not all the same, or the domains of values and w are incompatible with each other or D_{dup} .

GrB_OUTPUT_NOT_EMPTY Output vector w already contains valid tuples (elements). In other words, GrB_Vector_nvals(C) returns a positive value.

GrB_INVALID_VALUE indices contains a duplicate location and dup is GrB_NULL.

1815 Description

If dup is not GrB_NULL, an internal vector $\widetilde{\mathbf{w}} = \langle D_{dup}, \mathbf{size}(\mathbf{w}), \emptyset \rangle$ is created, which only differs from w in its domain; otherwise, $\widetilde{\mathbf{w}} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \emptyset \rangle$.

Each tuple {indices[k], values[k]}, where $0 \le k < n$, is a contribution to the output in the form of

$$\widetilde{\mathbf{w}}(\mathsf{indices}[\mathsf{k}]) = \begin{cases} (D_{\mathit{dup}})\,\mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB_NULL} \\ (\mathbf{D}(\mathsf{w}))\,\mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB_NULL, dup is used to reduce the values before assignment into $\widetilde{\mathbf{w}}$ as follows:

$$\widetilde{\mathbf{w}}_i = \bigoplus_{k: \, \mathsf{indices[k]} = i} (D_{dup}) \, \mathsf{values[k]},$$

where \oplus is the dup binary operator. Finally, the resulting $\widetilde{\mathbf{w}}$ is copied into \mathbf{w} via typecasting its values to $\mathbf{D}(\mathbf{w})$ if necessary. If \oplus is not associative or not commutative, the result is undefined.

The nonopaque input arrays, indices and values, must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB_Vector_nvals(w) should evaluate to zero prior to calling this function.

After GrB_Vector_build returns, it is safe for a programmer to modify or delete the arrays indices or values.

1830 4.2.4.8 Vector_setElement: Set a single element in a vector

1831 Set one element of a vector to a given value.

1832 C Syntax

```
// scalar value
1833
             GrB_Info GrB_Vector_setElement(GrB_Vector
1834
                                                                      W,
                                                  <type>
                                                                      val,
1835
                                                  GrB_Index
                                                                      index);
1836
1837
              // GraphBLAS scalar
1838
             GrB_Info GrB_Vector_setElement(GrB_Vector
1839
                                                  const GrB_Scalar
1840
                                                  GrB_Index
                                                                      index);
1841
```

Parameters

1842

1844

1845

1847

1848

1849

w (INOUT) An existing GraphBLAS vector for which an element is to be assigned.

val or s (IN) Scalar assign. Its domain (type) must be compatible with the domain of w.

index (IN) The location of the element to be assigned.

1846 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed suc-

cessfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

1858 GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, w, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

GrB_DOMAIN_MISMATCH The domains of the vector and the scalar are incompatible.

Description

1852

1861

First, the scalar and output vector are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{w})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Vector_setElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 < \mathsf{index} < \mathsf{size}(\mathsf{w})$$

If this condition is violated, execution of GrB_Vector_setElement ends and the invalid index error listed above is returned.

1873 We are now ready to carry out the assignment; that is:

$$w(\mathsf{index}) = \begin{cases} \mathbf{L}(\mathsf{s}), & \mathrm{GraphBLAS\ scalar.} \\ \mathsf{val}, & \mathrm{otherwise.} \end{cases}$$

In the case of a transparent scalar or if $\mathbf{L}(s)$ is not empty, then a value will be stored at the specified location in \mathbf{w} , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar, if $\mathbf{L}(s)$ is empty, then any value stored at the specified location in \mathbf{w} will be removed.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

1883 4.2.4.9 Vector_removeElement: Remove an element from a vector

1884 Remove (annihilate) one stored element from a vector.

1885 C Syntax

```
GrB_Info GrB_Vector_removeElement(GrB_Vector w,

GrB_Index index);
```

1888 Parameters

1890

1897

1902

w (INOUT) An existing GraphBLAS vector from which an element is to be removed.

index (IN) The location of the element to be removed.

1891 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, w, has not been initialized by a call to Vector_new or Vector_dup.

GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

1906 Description

First, the index parameter is checked for a valid value where the following condition must hold:

$$0 < \mathsf{index} < \mathsf{size}(\mathsf{w})$$

If this condition is violated, execution of GrB_Vector_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by index. If a value does not exist at the specified location in w, no error is reported and the operation has no effect on the state of w. In either case, the following will be true on return from the method: index \notin ind(w).

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

1919 4.2.4.10 Vector_extractElement: Extract a single element from a vector.

1920 Extract one element of a vector into a scalar.

1921 C Syntax

```
// scalar value
1922
             GrB_Info GrB_Vector_extractElement(<type>
                                                                          *val,
1923
                                                      const GrB_Vector
                                                                           u,
1924
                                                      GrB_Index
                                                                           index);
1925
1926
             // GraphBLAS scalar
1927
             GrB_Info GrB_Vector_extractElement(GrB_Scalar
                                                                           s,
1928
                                                      const GrB Vector
1929
                                                                           u,
                                                      GrB_Index
                                                                           index);
1930
```

1931 Parameters

1935

1936

1938

1939

1940

1941

1942

val or s (INOUT) An existing scalar of whose domain is compatible with the domain of vector
u. On successful return, this scalar holds the result of the extract. Any previous
value stored in val or s is overwritten.

u (IN) The GraphBLAS vector from which an element is extracted.

index (IN) The location in u to extract.

1937 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val or s, has been computed and is ready to be used in the next method of the sequence.

GrB_NO_VALUE When using the transparent scalar, val, this is returned when there is no stored value at specified location.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

1951 GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, u, or scalar, s, has not been initialized by a call to a corresponding constructor.

1953 GrB_NULL_POINTER val pointer is NULL.

GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

GrB_DOMAIN_MISMATCH The domains of the vector and scalar are incompatible.

1956 Description

1945

1950

1955

1971

1972

First, the scalar and input vector are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{u})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of $\mathsf{GrB_Vector_extractElement}$ ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 < \mathsf{index} < \mathsf{size}(\mathsf{u})$$

1965 If this condition is violated, execution of GrB_Vector_extractElement ends and the invalid index error listed above is returned.

1967 We are now ready to carry out the extract into the output scalar; that is:

$$\left. \begin{array}{c} \mathbf{L}(\mathsf{s}) \\ \mathsf{val} \end{array} \right\} = \mathsf{u}(\mathsf{index})$$

If $index \in ind(u)$, then the corresponding value from u is copied into s or val with casting as necessary. If $index \notin ind(u)$, then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB_SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB_NO_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB_BLOCKING mode GrB_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB_SUCCESS or GrB_NO_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB_BLOCKING mode. In GrB_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

1980 4.2.4.11 Vector_extractTuples: Extract tuples from a vector

1981 Extract the contents of a GraphBLAS vector into non-opaque data structures.

1982 C Syntax

1983	<pre>GrB_Info GrB_Vector_extractTuples</pre>	s(GrB_Index	*indices,
1984		<type></type>	*values,
1985		<pre>GrB_Index</pre>	*n,
1986		<pre>const GrB_Vector</pre>	v);
1987			
1988	indices (OUT) Pointer to an array of ind	dices that is large enough	to hold all of the stored
1989	values' indices.		
	(0117) 7		1 . 1 11 11 6
1990	values (OUT) Pointer to an array of sc	v	ge enough to hold all of
1991	the stored values whose type is	compatible with $\mathbf{D}(\mathbf{v})$.	
1992	n (INOUT) Pointer to a value in	dicating (on input) the r	number of elements the
1993	values and indices arrays can he	_ ` _ ,	
1994	values written to the arrays.	ora. Opon retain, it win	contain the named of
233.	. arabs restain to the drings.		
1995	v (IN) An existing GraphBLAS ve	ector.	

996 Return Values

1997	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed suc-
1998		cessfully. This indicates that the compatibility tests on the input
1999		argument passed successfully, and the output arrays, indices and
2000		values, have been computed.
2001	GrB_PANIC	Unknown internal error.
2002	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
2003		GraphBLAS objects (input or output) is in an invalid state caused
2004		by a previous execution error. Call GrB_error() to access any error
2005		messages generated by the implementation.

- GrB_OUT_OF_MEMORY Not enough memory available for operation. 2006
- GrB INSUFFICIENT SPACE Not enough space in indices and values (as indicated by the n pa-2007 rameter) to hold all of the tuples that will be extacted. 2008
- GrB UNINITIALIZED OBJECT The GraphBLAS vector, v, has not been initialized by a call to 2009 Vector new or Vector dup. 2010
- GrB_NULL_POINTER indices, values, or n pointer is NULL. 2011
- GrB_DOMAIN_MISMATCH The domains of the v vector or values array are incompatible with 2012 one another. 2013

2021

- This method will extract all the tuples from the GraphBLAS vector v. The values associated 2015 with those tuples are placed in the values array and the indices are placed in the indices array. Both indices and values must be pre-allocated by the user to have enough space to hold at least 2017 GrB Vector nvals(v) elements before calling this function. 2018
- Upon return of this function, n will be set to the number of values (and indices) copied. Also, the 2019 entries of indices are unique, but not necessarily sorted. Each tuple (i, v_i) in v is unzipped and 2020 copied into a distinct kth location in output vectors:

$$\{\text{indices}[k], \text{values}[k]\} \leftarrow (i, v_i),$$

- where $0 \le k \le GrB$ Vector nvals(v). No gaps in output vectors are allowed; that is, if indices[k] 2022 and values[k] exist upon return, so does indices[j] and values[j] for all j such that $0 \le j < k$. 2023
- Note that if the value in n on input is less than the number of values contained in the vector v, 2024 then a Grb_INSUFFICIENT_SPACE error is returned because it is undefined which subset of values 2025 would be extracted otherwise. 2026
- In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value 2027 GrB_SUCCESS, the new contents of the arrays indices and values are as defined above. 2028

4.2.5Matrix methods 2029

4.2.5.1Matrix new: Construct new matrix 2030

Creates a new matrix with specified domain and dimensions. 2031

C Syntax 2032

2035 2036		<pre>GrB_Index nrows, GrB_Index ncols);</pre>	
2037	Parameters		
2038 2039	A (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.		
2040 2041 2042	d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.		
2043	nrows (IN) The number of rows of the matrix being created.		
2044	ncols (IN) The number of columns of the matrix being created.		
2045 Return Values			
2046 2047 2048 2049	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.	
2050	GrB_PANIC	Unknown internal error.	
2051 2052 2053 2054	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.	
2055	GrB_OUT_OF_MEMORY	Not enough memory available for operation.	
2056	GrB_UNINITIALIZED_OBJECT	The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).	

2058

2059

Creates a new matrix $\bf A$ of domain $\bf D(d)$, size nrows \times ncols, and empty $\bf L(A)$. The method returns a handle to the new matrix in $\bf A$.

GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.

GrB_NULL_POINTER The A pointer is NULL.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

2065 4.2.5.2 Matrix_dup: Construct a copy of a GraphBLAS matrix

2006 Creates a new matrix with the same domain, dimensions, and contents as another matrix.

2067 C Syntax

```
GrB_Info GrB_Matrix_dup(GrB_Matrix *C,
const GrB Matrix A);
```

2070 Parameters

2073

2075

2076

2077

2078

2079

2087

C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.

A (IN) The GraphBLAS matrix to be duplicated.

2074 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

2085 GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER The C pointer is NULL.

2088 Description

Creates a new matrix C of domain D(A), size $\mathbf{nrows}(A) \times \mathbf{ncols}(A)$, and contents L(A). It returns a handle to it in C.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

2093 4.2.5.3 Matrix_diag: Construct a diagonal GraphBLAS matrix

Creates a new matrix with the same domain and contents as a GrB_Vector, and square dimensions appropriate for placing the contents of the vector along the specified diagonal of the matrix.

2096 C Syntax

```
GrB_Info GrB_Matrix_diag(GrB_Matrix *C,

const GrB_Vector v,

int64_t k);
```

2100 Parameters

- C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix. The matrix is square with each dimension equal to $\operatorname{size}(\mathsf{v}) + |k|$.
- v (IN) The GraphBLAS vector whose contents will be copied to the diagonal of the matrix.
- k (IN) The diagonal to which the vector is assigned. k = 0 represents the main diagonal, k > 0 is above the main diagonal, and k < 0 is below.

2107 Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
 GraphBLAS objects (input or output) is in an invalid state caused
 by a previous execution error. Call GrB_error() to access any error
 messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for the operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.
- 2120 GrB_NULL_POINTER The C pointer is NULL.

2121

Creates a new matrix C of domain $\mathbf{D}(\mathbf{v})$, size $(\mathbf{size}(\mathbf{v}) + |k|) \times (\mathbf{size}(\mathbf{v}) + |k|)$, and contents

```
L(C) = \{(i, i + k, v_i) : (i, v_i) \in \mathbf{L}(\mathbf{v})\} if k \ge 0 or 
L(C) = \{(i - k, i, v_i) : (i, v_i) \in \mathbf{L}(\mathbf{v})\} if k < 0.
```

It returns a handle to it in C. It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

2127 4.2.5.4 Matrix_resize: Resize a matrix

2128 Changes the dimensions of an existing matrix.

2129 C Syntax

```
GrB_Info GrB_Matrix_resize(GrB_Matrix C,

GrB_Index nrows,

GrB_Index ncols);
```

2133 Parameters

2134

2135

2136

2137

2138

2144

2149

C (INOUT) An existing Matrix object that is being resized.

nrows (IN) The new number of rows of the matrix. It can be smaller or larger than the current number of rows.

ncols (IN) The new number of columns of the matrix. It can be smaller or larger than the current number of columns.

2139 Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

```
GrB_NULL_POINTER The C pointer is NULL.
2150
             GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.
2151
    Description
2152
     Changes the number of rows and columns of C to nrows and ncols, respectively. The domain \mathbf{D}(\mathsf{C})
2153
     of matrix C remains the same. The contents L(C) are modified as described below.
2154
     Let C = \langle \mathbf{D}(C), M, N, \mathbf{L}(C) \rangle when the method is called. When the method returns C is modified
2155
     to C = \langle D(C), \text{nrows}, \text{ncols}, L'(C) \rangle where L'(C) = \{(i, j, C_{ij}) : (i, j, C_{ij}) \in L(C) \land (i < \text{nrows}) \land (j < i) \}
     ncols). That is, all elements of C with row index greater than or equal to nrows or column index
2157
     greater than or equal to ncols are dropped.
2158
     4.2.5.5
               Matrix_clear: Clear a matrix
2159
     Removes all elements (tuples) from a matrix.
2160
     C Syntax
2161
               GrB_Info GrB_Matrix_clear(GrB_Matrix A);
2162
     Parameters
2163
                   A (IN) An exising GraphBLAS matrix to clear.
2164
     Return Values
2165
                     GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
2166
                                      blocking mode, this indicates that the API checks for the input ar-
2167
                                      guments passed successfully. Either way, output matrix A is ready
2168
                                      to be used in the next method of the sequence.
2169
                         GrB PANIC Unknown internal error.
2170
           GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2171
                                      GraphBLAS objects (input or output) is in an invalid state caused
2172
                                      by a previous execution error. Call GrB_error() to access any error
2173
                                      messages generated by the implementation.
2174
```

2176 GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

Grb Out of Memory Not enough memory available for operation.

2175

Removes all elements (tuples) from an existing matrix. After the call to $GrB_Matrix_clear(A)$, $L(A) = \emptyset$. The dimensions of the matrix do not change.

2181 4.2.5.6 Matrix_nrows: Number of rows in a matrix

2182 Retrieve the number of rows in a matrix.

2183 C Syntax

```
GrB_Info GrB_Matrix_nrows(GrB_Index *nrows, const GrB_Matrix A);
```

2186 Parameters

2188

2192

2187 nrows (OUT) On successful return, contains the number of rows in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

2189 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nrows has been set.

GrB PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

2197 GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER nrows pointer is NULL.

2200 Description

2201 Return **nrows**(A) in **nrows** (the number of rows).

2202 4.2.5.7 Matrix ncols: Number of columns in a matrix

2203 Retrieve the number of columns in a matrix.

```
C Syntax
2204
              GrB_Info GrB_Matrix_ncols(GrB_Index
                                                                *ncols,
2205
                                            const GrB_Matrix
                                                               A);
2206
    Parameters
               ncols (OUT) On successful return, contains the number of columns in the matrix.
2208
                  A (IN) An existing GraphBLAS matrix being queried.
2209
    Return Values
                   GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2211
                                   cessfully and the value of ncols has been set.
2212
                      GrB_PANIC Unknown internal error.
2213
          GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
2214
                                   GraphBLAS objects (input or output) is in an invalid state caused
2215
                                   by a previous execution error. Call GrB_error() to access any error
2216
                                   messages generated by the implementation.
2217
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to
2218
                                   any matrix constructor.
2219
            GrB_NULL_POINTER ncols pointer is NULL.
2220
    Description
2221
    Return ncols(A) in ncols (the number of columns).
2222
    4.2.5.8
              Matrix_nvals: Number of stored elements in a matrix
2223
    Retrieve the number of stored elements (tuples) in a matrix.
    C Syntax
2225
              GrB_Info GrB_Matrix_nvals(GrB_Index
                                                                *nvals,
2226
                                            const GrB_Matrix A);
2227
```

Parameters

2228

2231

nvals (OUT) On successful return, contains the number of stored elements (tuples) in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

2232 Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

2235 GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER The nvals pointer is NULL.

2244 Description

Return nvals(A) in nvals. This is the number of tuples stored in matrix A, which is the size of L(A) (see Section 3.5.3).

2247 4.2.5.9 Matrix_build: Store elements from tuples into a matrix

2248 C Syntax

Parameters Parameters

2250

C (INOUT) An existing Matrix object to store the result.

row_indices (IN) Pointer to an array of row indices. 2251 col_indices (IN) Pointer to an array of column indices. 2252 values (IN) Pointer to an array of scalars of a type that is compatible with the domain of 2253 matrix, C. 2254 n (IN) The number of entries contained in each array (the same for row indices, 2255 col indices, and values). 2256 dup (IN) An associative and commutative binary operator to apply when duplicate 2257 values for the same location are present in the input arrays. All three domains of 2258 dup must be the same; hence $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$. If dup is GrB_NULL, 2259 then duplicate locations will result in an error. 2260 Return Values GrB_SUCCESS In blocking mode, the operation completed successfully. In non-2262 blocking mode, this indicates that the API checks for the input 2263 arguments passed successfully. Either way, output matrix C is 2264 ready to be used in the next method of the sequence. 2265 GrB_PANIC Unknown internal error. 2266 GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the 2267 opaque GraphBLAS objects (input or output) is in an invalid 2268 state caused by a previous execution error. Call GrB error() to 2269 access any error messages generated by the implementation. 2270 GrB_OUT_OF_MEMORY Not enough memory available for operation. 2271 GrB UNINITIALIZED OBJECT Either C has not been initialized by a call to any matrix construc-2272 tor, or dup has not been initialized by a call to by GrB BinaryOp new. 2273 GrB_NULL_POINTER row_indices, col_indices or values pointer is NULL. 2274 GrB_INDEX_OUT_OF_BOUNDS A value in row_indices or col_indices is outside the allowed range 2275 for C. 2276 Grb_DOMAIN_MISMATCH Either the domains of the GraphBLAS binary operator dup are 2277 not all the same, or the domains of values and C are incompatible 2278 with each other or D_{dup} . 2279 Grb Output NOT EMPTY Output matrix C already contains valid tuples (elements). In 2280 other words, GrB_Matrix_nvals(C) returns a positive value.

2282

GrB_INVALID_VALUE indices contains a duplicate location and dup is GrB_NULL.

If dup is not GrB_NULL, an internal matrix $\widetilde{\mathbf{C}} = \langle D_{dup}, \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$ is created, which only differs from C in its domain; otherwise, $\widetilde{\mathbf{C}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$.

Each tuple {row_indices[k], col_indices[k], values[k]}, where $0 \le k < n$, is a contribution to the output in the form of

$$\widetilde{\mathbf{C}}(\mathsf{row_indices}[\mathsf{k}],\mathsf{col_indices}[\mathsf{k}]) = \begin{cases} (D_{dup})\,\mathsf{values}[\mathsf{k}] & \text{if } \mathsf{dup} \neq \mathsf{GrB_NULL} \\ (\mathbf{D}(\mathsf{C}))\,\mathsf{values}[\mathsf{k}] & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB_NULL, dup is used to reduce the values before assignment into $\tilde{\mathbf{C}}$ as follows:

$$\widetilde{\mathbf{C}}_{ij} = \bigoplus_{k:\, \mathsf{row_indices}[\mathtt{k}] = i \, \land \, \mathsf{col_indices}[\mathtt{k}] = j} (D_{dup}) \, \mathsf{values}[\mathtt{k}],$$

where \oplus is the dup binary operator. Finally, the resulting $\widetilde{\mathbf{C}}$ is copied into C via typecasting its values to $\mathbf{D}(C)$ if necessary. If \oplus is not associative or not commutative, the result is undefined.

The nonopaque input arrays row_indices, col_indices, and values must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB_Matrix_nvals(C) should evaluate to zero prior to calling this function.

After GrB_Matrix_build returns, it is safe for a programmer to modify or delete the arrays row_indices, col_indices, or values.

4.2.5.10 Matrix_setElement: Set a single element in matrix

2300 Set one element of a matrix to a given value.

2301 C Syntax

```
// scalar value
2302
             GrB Info GrB Matrix setElement(GrB Matrix
                                                                        C,
2303
                                                  <type>
                                                                        val,
2304
                                                  GrB_Index
                                                                        row_index,
2305
                                                  GrB Index
                                                                        col_index);
2306
2307
              // GraphBLAS scalar
2308
             GrB_Info GrB_Matrix_setElement(GrB_Matrix
                                                                        С,
2309
                                                  const GrB_Scalar
2310
                                                  GrB_Index
                                                                        row_index,
2311
                                                  GrB Index
                                                                        col index);
2312
```

Parameters 2313

```
C (INOUT) An existing GraphBLAS matrix for which an element is to be assigned.
2314
            val or s (IN) Scalar to assign. Its domain (type) must be compatible with the domain of
2315
2316
          row_index (IN) Row index of element to be assigned
2317
          col_index (IN) Column index of element to be assigned
2318
```

Return Values

2319

2320	GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
2321	blocking mode, this indicates that the compatibility tests on in-
2322	dex/dimensions and domains for the input arguments passed suc-
2323	cessfully. Either way, the output matrix C is ready to be used in
2324	the next method of the sequence.
2325	GrB PANIC Unknown internal error.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque 2326 GraphBLAS objects (input or output) is in an invalid state caused 2327 by a previous execution error. Call GrB_error() to access any error 2328 messages generated by the implementation. 2329

GrB OUT OF MEMORY Not enough memory available for operation. 2330

GrB UNINITIALIZED OBJECT The GraphBLAS matrix, A, or GraphBLAS scalar, s, has not been 2331 initialized by a call to a respective constructor. 2332

GrB_INVALID_INDEX row_index or col_index is outside the allowable range (i.e., not less 2333 than $\mathbf{nrows}(\mathsf{C})$ or $\mathbf{ncols}(\mathsf{C})$, respectively). 2334

GrB_DOMAIN_MISMATCH The domains of the matrix and the scalar are incompatible.

Description 2336

2335

2344

First, the scalar and output matrix are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or 2337 $\mathbf{D}(s)$ must be compatible with $\mathbf{D}(\mathsf{C})$. Two domains are compatible with each other if values from 2338 one domain can be cast to values in the other domain as per the rules of the C language. In 2339 particular, domains from Table 3.2 are all compatible with each other. A domain from a user-2340 defined type is only compatible with itself. If any compatibility rule above is violated, execution of 2341 GrB_Matrix_setElement ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold: 2343

$$0 \le \text{row_index} < \mathbf{nrows}(C),$$

 $0 \le \text{col_index} < \mathbf{ncols}(C)$

If either of these conditions is violated, execution of GrB_Matrix_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$C(row_index, col_index) = \begin{cases} \mathbf{L}(s), & \operatorname{GraphBLAS\ scalar.} \\ val, & \operatorname{otherwise.} \end{cases}$$

In the case of a transparent scalar or if $\mathbf{L}(s)$ is not empty, then a value will be stored at the specified location in C , overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar and if $\mathbf{L}(s)$ is empty, then any value stored at the specified location in C will be removed.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

2357 4.2.5.11 Matrix_removeElement: Remove an element from a matrix

2358 Remove (annihilate) one stored element from a matrix.

2359 C Syntax

```
GrB_Info GrB_Matrix_removeElement(GrB_Matrix C,
GrB_Index row_index,
GrB_Index col_index);
```

63 Parameters

2365

2367

2368

2369

2370

2371

2372

2373

²³⁶⁴ C (INOUT) An existing GraphBLAS matrix from which an element is to be removed.

row_index (IN) Row index of element to be removed

col index (IN) Column index of element to be removed

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

```
GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.
```

GrB_OUT_OF_MEMORY Not enough memory available for operation.

2379 GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, C, has not been initialized by a call to any matrix constructor.

GrB_INVALID_INDEX row_index or col_index is outside the allowable range (i.e., not less than nrows(C) or ncols(C), respectively).

2383 Description

2378

First, both index parameters are checked for valid values where following conditions must hold:

```
0 \le \text{row\_index} < \mathbf{nrows}(\mathsf{C}),0 \le \text{col index} < \mathbf{ncols}(\mathsf{C})
```

If either of these conditions is violated, execution of GrB_Matrix_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by (row_index, col_index). If a value does not exist at the specified location in C, no error is reported and the operation has no effect on the state of C. In either case, the following will be true on return from this method: (row_index, col_index) \notin ind(C)

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

2396 4.2.5.12 Matrix_extractElement: Extract a single element from a matrix

2397 Extract one element of a matrix into a scalar.

2398 C Syntax

```
// scalar value
2399
             GrB_Info GrB_Matrix_extractElement(<type>
                                                                          *val,
2400
                                                      const GrB_Matrix
2401
                                                      GrB_Index
                                                                           row_index,
2402
                                                      GrB_Index
                                                                           col_index);
2403
2404
              // GraphBLAS scalar
2405
```

```
GrB_Info GrB_Matrix_extractElement(GrB_Scalar
                                                                              s,
2406
                                                        const GrB_Matrix
                                                                              Α,
2407
                                                        GrB_Index
                                                                              row_index,
2408
                                                        GrB_Index
                                                                              col_index);
2409
2410
    Parameters
2411
           val or s (INOUT) An existing scalar whose domain is compatible with the domain of matrix
2412
                   A. On successful return, this scalar holds the result of the extract. Any previous
2413
                   value stored in val or s is overwritten.
2414
                 A (IN) The GraphBLAS matrix from which an element is extracted.
2415
        row index (IN) The row index of location in A to extract.
2416
         col_index (IN) The column index of location in A to extract.
2417
    Return Values
2418
                    GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-
2419
                                    cessfully. This indicates that the compatibility tests on dimensions
2420
                                    and domains for the input arguments passed successfully, and the
2421
                                    output scalar, val or s, has been computed and is ready to be used
2422
                                    in the next method of the sequence.
2423
                  GrB_NO_VALUE When using the transparent scalar, val, this is returned when there
2424
                                    is no stored value at specified location.
2425
                       GrB_PANIC Unknown internal error.
2426
           Grb INVALID OBJECT This is returned in any execution mode whenever one of the opaque
2427
                                    GraphBLAS objects (input or output) is in an invalid state caused
2428
                                    by a previous execution error. Call GrB_error() to access any error
2429
                                    messages generated by the implementation.
2430
         GrB OUT OF MEMORY Not enough memory available for operation.
2431
   GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, or scalar, s, has not been initialized by
2432
                                    a call to a corresponding constructor.
2433
            GrB_NULL_POINTER val pointer is NULL.
2434
            GrB INVALID INDEX row index or col index is outside the allowable range (i.e. less than
2435
                                    zero or greater than or equal to \mathbf{nrows}(A) or \mathbf{ncols}(A), respec-
2436
                                    tively).
2437
```

GrB_DOMAIN_MISMATCH The domains of the matrix and scalar are incompatible.

2438

First, the scalar and input matrix are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{A})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of $\mathsf{GrB}_\mathsf{Matrix_extractElement}$ ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

If either condition is violated, execution of GrB_Matrix_extractElement ends and the invalid index error listed above is returned.

²⁴⁵⁰ We are now ready to carry out the extract into the output scalar; that is,

$$\frac{\mathbf{L}(s)}{\text{val}} \left. \right\} = \mathsf{A}(\mathsf{row_index}, \mathsf{col_index})$$

If $(row_index, col_index) \in ind(A)$, then the corresponding value from A is copied into s or val with casting as necessary. If $(row_index, col_index) \notin ind(A)$, then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB_NO_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB_BLOCKING mode GrB_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB_SUCCESS or GrB_NO_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB_BLOCKING mode.

In GrB_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

2464 4.2.5.13 Matrix extractTuples: Extract tuples from a matrix

Extract the contents of a GraphBLAS matrix into non-opaque data structures.

2466 C Syntax

2455

2456

2469 2470 2471		<pre><type> *values, GrB_Index *n, const GrB_Matrix A);</type></pre>	
2472	Parameters		
2473 2474	row_indices (OUT) Pointer to an array of row indices that is large enough to hold all of the row indices.		
2475 2476	$\operatorname{col_indices}$ (OUT) Pointer to an array of column indices that is large enough to hold all of the column indices.		
2477 2478	values (OUT) Pointer to an array of scalars of a type that is large enough to hold all of the stored values whose type is compatible with $\mathbf{D}(\mathbf{A})$.		
2479 2480 2481	n (INOUT) Pointer to a value indicating (in input) the number of elements the values, row_indices, and col_indices arrays can hold. Upon return, it will contain the number of values written to the arrays.		
2482	A (IN) An existing	ng GraphBLAS matrix.	
2483	Return Values		
2484 2485 2486 2487	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indices and values, have been computed.	
2488	GrB_PANIC	Unknown internal error.	
2489 2490 2491 2492	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.	
2493	GrB_OUT_OF_MEMORY	Not enough memory available for operation.	
2494 2495 2496	GrB_INSUFFICIENT_SPACE	Not enough space in $row_indices$, $col_indices$, and values (as indicated by the n parameter) to hold all of the tuples that will be extacted.	
2497 2498	GrB_UNINITIALIZED_OBJECT	The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.	
2499	GrB_NULL_POINTER	row_indices, col_indices, values or n pointer is NULL.	
2500 2501	GrB_DOMAIN_MISMATCH	The domains of the ${\sf A}$ matrix and ${\sf values}$ array are incompatible with one another.	

This method will extract all the tuples from the GraphBLAS matrix A. The values associated with those tuples are placed in the values array, the column indices are placed in the col_indices array, and the row indices are placed in the row_indices array. These output arrays are pre-allocated by the user before calling this function such that each output array has enough space to hold at least GrB_Matrix_nvals(A) elements.

Upon return of this function, a pair of $\{\text{row_indices}[k], \text{col_indices}[k]\}$ are unique for every valid k, but they are not required to be sorted in any particular order. Each tuple (i, j, A_{ij}) in A is unzipped and copied into a distinct kth location in output vectors:

 $\{\text{row_indices}[k], \text{col_indices}[k], \text{values}[k]\} \leftarrow (i, j, A_{ij}),$

where $0 \le k < GrB_Matrix_nvals(v)$. No gaps in output vectors are allowed; that is, if row_indices[k], col_indices[k] and values[k] exist upon return, so does row_indices[j], col_indices[j] and values[j] for all j such that $0 \le j < k$.

Note that if the value in n on input is less than the number of values contained in the matrix A, then a GrB_INSUFFICIENT_SPACE error is returned since it is undefined which subset of values would be extracted.

In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value GrB_SUCCESS, the new contents of the arrays row_indices, col_indices and values are as defined above.

4.2.5.14 Matrix_exportHint: Provide a hint as to which storage format might be most efficient for exporting a matrix

2522 C Syntax

2523 Parameters

2524

2525

2526

2529

hint (OUT) Pointer to a value of type GrB Format.

A (IN) A GraphBLAS matrix object.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of hint has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

2534 GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER hint is NULL.

GrB_NO_VALUE If the implementation does not have a preferred format, it may return the value GrB_NO_VALUE.

2540 Description

2535

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2539

Given a GraphBLAS matrix A, provide a hint as to which format might be most efficient for exporting the matrix A. GraphBLAS implementations might return the current storage format of the matrix, or the format to which it could most efficiently be exported. However, implementations are free to return any value for format defined in Section 3.5.3.1. Note that an implementation is free to refuse to provide a format hint, returning GrB_NO_VALUE.

2546 4.2.5.15 Matrix_exportSize: Return the array sizes necessary to export a GraphBLAS matrix object

2548 C Syntax

```
GrB_Info GrB_Matrix_exportSize(GrB_Index *n_indptr,
GrB_Index *n_indices,
GrB_Index *n_values,
GrB_Format format,
GrB_Matrix A);
```

2549 Parameters

2555

2550 n_indptr (OUT) Pointer to a value of type GrB_Index.

n_indices (OUT) Pointer to a value of type GrB_Index.

n_values (OUT) Pointer to a value of type GrB_Index.

format (IN) a value indicating the format in which the matrix will be exported, as defined in Section 3.5.3.1.

A (IN) A GraphBLAS matrix object.

$_{556}$ Return Values

2562

2567

GrB_SUCCESS In blocking mode or non-blocking mode, the operation completed successfully. This indicates that the API checks for the input arguments passed successfully, and the number of elements necessary for the export buffers have been written to n_indptr, n_indices, and n_values, respectively.

GrB PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS Matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER n_indptr, n_indices, or n_values is NULL.

2571 Description

Given a matrix **A**, returns the required capacities of arrays values, indptr, and indices necessary to export the matrix in the format specified by format. The output values n_values, n_indptr, and indices will contain the corresponding sizes of the arrays (in number of elements) that must be allocated to hold the exported matrix. The argument format can be chosen arbitrarily by the user as one of the values defined in Section 3.5.3.1.

2577 4.2.5.16 Matrix_export: Export a GraphBLAS matrix to a pre-defined format

2578 C Syntax

```
GrB_Info GrB_Matrix_export(GrB_Index
                                                    *indptr,
                             GrB_Index
                                                    *indices,
                             <type>
                                                    *values,
                             GrB_Index
                                                    *n_indptr,
                             GrB_Index
                                                    *n_indices,
                             GrB Index
                                                    *n_values,
                             GrB_Format
                                                     format,
                             GrB_Matrix
                                                     A);
```

2579 Parameters

indptr (INOUT) Pointer to an array that will hold row or column offsets, or row in-2580 dices, depending on the value of format. It must be large enough to hold at 2581 least n indptr elements of type GrB Index, where n indices was returned from 2582 GrB_Matrix_exportSize() method. 2583 indices (INOUT) Pointer to an array that will hold row or column indices of the elements 2584 in values, depending on the value of format. It must be large enough to hold at 2585 least n_indices elements of type GrB_Index, where n_indices was returned from 2586 GrB Matrix exportSize() method. 2587 values (INOUT) Pointer to an array that will hold stored values. The type of ele-2588 ment must match the type of the values stored in A. It must be large enough 2589 to hold at least n_values elements of that type, where n_values was returned from 2590 GrB_Matrix_exportSize. 2591 n_indptr (INOUT) Pointer to a value indicating (on input) the number of elements the indptr 2592 array can hold. Upon return, it will contain the number of elements written to the 2593 array. 2594 n_indices (INOUT) Pointer to a value indicating (on input) the number of elements the indices 2595 array can hold. Upon return, it will contain the number of elements written to the 2596 array. 2597 n_values (INOUT) Pointer to a value indicating (on input) the number of elements the values 2598 array can hold. Upon return, it will contain the number of elements written to the 2599 array. 2600 format (IN) a value indicating the format in which the matrix will be exported, as defined 2601 in Section 3.5.3.1. 2602 A (IN) A GraphBLAS matrix object. 2603

Return Values

2605 2606 2607 2608	GrB_SUCCESS	In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indptr, indices and values, have been computed.
2609	GrB_PANIC	Unknown internal error.
2610	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
2611		opaque GraphBLAS objects (input or output) is in an invalid
2612		state caused by a previous execution error. Call GrB_error() to
2613		access any error messages generated by the implementation.
2614	GrB_OUT_OF_MEMORY	Not enough memory available for operation.

GrB_INSUFFICIENT_SPACE Not enough space in indptr, indices, and/or values (as indicated 2615 by the corresponding n_* parameter) to hold all of the corre-2616 sponding elements that will be extacted. 2617 GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to 2618 any matrix constructor. 2619 GrB_NULL_POINTER indptr, indices, values n_indptr, n_indices, n_values pointer is 2620 NULL. 2621 GrB_DOMAIN_MISMATCH The domain of A does not match with the type of values. 2622

Description

2623

Given a matrix **A**, this method exports the contents of the matrix into one of the pre-defined GrB_Format formats from Section 3.5.3.1. The user-allocated arrays pointed to by indptr, indices, and values must be at least large enough to hold the corresponding number of elements returned by calling GrB_Matrix_exportSize. The value of format can be chosen arbitrarily, but a call to GrB_Matrix_exportHint may suggest a format that results in the most efficient export. Details of the contents of indptr, indices, and values corresponding to each supported format is given in Appendix B.

2631 4.2.5.17 Matrix_import: Import a matrix into a GraphBLAS object

2632 C Syntax

```
GrB_Info GrB_Matrix_import(GrB_Matrix
                                                    *A,
                                                     d,
                             GrB_Type
                             GrB_Index
                                                     nrows,
                             GrB_Index
                                                     ncols
                             const GrB_Index
                                                    *indptr,
                             const GrB Index
                                                    *indices,
                             const <type>
                                                    *values,
                             GrB Index
                                                     n indptr,
                             GrB_Index
                                                     n indices,
                             GrB_Index
                                                     n_values,
                             GrB_Format
                                                     format);
```

Parameters

2633

2636

2637

2638

- A (INOUT) On a successful return, contains a handle to the newly created Graph-BLAS matrix.
 - d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

nrows (IN) Integer value holding the number of rows in the matrix. 2639 ncols (IN) Integer value holding the number of columns in the matrix. 2640 indptr (IN) Pointer to an array of row or column offsets, or row indices, depending on the 2641 value of format. 2642 indices (IN) Pointer to an array row or column indices of the elements in values, depending 2643 on the value of format. 2644 values (IN) Pointer to an array of values. Type must match the type of d. 2645 n indptr (IN) Integer value holding the number of elements in the array pointed to by indptr. 2646 n_indices (IN) Integer value holding the number of elements in the array pointed to by indices. 2647 n values (IN) Integer value holding the number of elements in the array pointed to by values. 2648 format (IN) a value indicating the format of the matrix being imported, as defined in 2649 Section 3.5.3.1. 2650 Return Values 2651 GrB SUCCESS In blocking mode, the operation completed successfully. In non-2652 blocking mode, this indicates that the API checks for the input 2653 arguments passed successfully and the input arrays have been 2654 consumed. Either way, output matrix A is ready to be used in 2655 the next method of the sequence. 2656 GrB PANIC Unknown internal error. 2657 Grb Out of Memory Not enough memory available for operation. 2658 GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new 2659 (needed for user-defined types). 2660 GrB_NULL_POINTER A, indptr, indices or values pointer is NULL. 2661 GrB_INDEX_OUT_OF_BOUNDS A value in indptr or indices is outside the allowed range for indices 2662 in A and or the size of values, n_values, depending on the value 2663 of format. 2664 GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index. 2665 GrB_DOMAIN_MISMATCH The domain given in parameter d does not match the element 2666 type of values.

2667

2668 Description

Creates a new matrix **A** of domain **D**(d) and dimension nrows × ncols. The new GraphBLAS matrix will be filled with the contents of the matrix pointed to by indptr, and indices, and values. The method returns a handle to the new matrix in **A**. The structure of the data being imported is defined by format, which must be equal to one of the values defined in Section 3.5.3.1. Details of the contents of indptr, indices and values for each supported format is given in Appendix B.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

2676 4.2.5.18 Matrix_serializeSize: Compute the serialize buffer size

²⁶⁷⁷ Compute the buffer size (in bytes) necessary to serialize a GrB_Matrix using GrB_Matrix_serialize.

2678 C Syntax

2679 Parameters

2682

2687

2688

2689

size (OUT) Pointer to GrB_Index value where size in bytes of serialized object will be written.

A (IN) A GraphBLAS matrix object.

2683 Return Values

GrB_SUCCESS The operation completed successfully and the value pointed to by *size has been computed and is ready to use.

Grb Panic Unknown internal error.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_NULL_POINTER size is NULL.

Description

Returns the size in bytes of the data buffer necessary to serialize the GraphBLAS matrix object A.
Users may then allocate a buffer of size bytes to pass as a parameter to GrB_Matrix_serialize.

2692 4.2.5.19 Matrix_serialize: Serialize a GraphBLAS matrix.

²⁶⁹³ Serialize a GraphBLAS Matrix object into an opaque stream of bytes.

2694 C Syntax

2695 Parameters

serialized_data (INOUT) Pointer to the preallocated buffer where the serialized matrix will be written.

serialized_size (INOUT) On input, the size in bytes of the buffer pointed to by serialized_data.

On output, the number of bytes written to serialized_data.

A (IN) A GraphBLAS matrix object.

2701 Return Values

2700

2707

2713

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output buffer serialized_data and serialized_size, have been computed and are ready to use.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

Grb_OUT_OF_MEMORY Not enough memory available for operation.

GrB_NULL_POINTER serialized_data or serialize_size is NULL.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_INSUFFICIENT_SPACE The size of the buffer serialized_data (provided as an input serialized_size) was not large enough.

2718 Description

Serializes a GraphBLAS matrix object to an opaque buffer. To guarantee successful execution, the size of the buffer pointed to by serialized_data, provided as an input by serialized_size, must be of at least the number of bytes returned from GrB_Matrix_serializeSize. The actual size of the serialized matrix written to serialized_data is provided upon completion as an output written to serialized_size.

The contents of the serialized buffer are implementation defined. Thus, a serialized matrix created with one library implementation is not necessarily valid for descrialization with another implementation.

2727 4.2.5.20 Matrix_deserialize: Deserialize a GraphBLAS matrix.

2728 Construct a new GraphBLAS matrix from a serialized object.

2729 C Syntax

2730 Parameters

A (INOUT) On a successful return, contains a handle to the newly created Graph-BLAS matrix.

d (IN) the type of the matrix that was serialized in serialized_data.

If d is GrB_NULL, the implementation must attempt to describing the matrix without a provided type object.

serialized_data (IN) a pointer to a serialized GraphBLAS matrix created with GrB_Matrix_serialize.

serialized_size (IN) the size of the buffer pointed to by serialized_data in bytes.

2738 Return Values

2743

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned if serialized_data is invalid or corrupted.

GrB_OUT_OF_MEMORY Not enough memory available for operation. 2745 GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new 2746 (needed for user-defined types). 2747 GrB NULL POINTER serialized data or A is NULL. 2748 GrB DOMAIN MISMATCH The type given in d does not match the type of the matrix 2749 serialized in serialized_data, or GrB_NULL was passed in and 2750 the implementation is unable to construct the matrix without 2751 the explicitly provided GrB_Type.

Description

2752

Creates a new matrix A using the serialized matrix object pointed to by serialized_data. The object 2754 pointed to by serialized_data must have been created using the method GrB_Matrix_serialize. The 2755 domain of the matrix is given as an input in d, which must match the domain of the matrix serialized 2756 in serialized data. Note that for user-defined types, only the size of the type will be checked. 2757

Since the format of a serialized matrix is implementation-defined, it is not guaranteed that a matrix 2758 serialized in one library implementation can be deserialized by another. 2759

It is not an error to call this method more than once on the same output matrix; however, the 2760 handle to the previously created object will be overwritten. 2761

4.2.6 Descriptor methods

The methods in this section create and set values in descriptors. A descriptor is an opaque Graph-2763 BLAS object the values of which are used to modify the behavior of GraphBLAS operations. 2764

Descriptor_new: Create new descriptor 4.2.6.12765

Creates a new (empty or default) descriptor. 2766

C Syntax 2767

GrB_Info GrB_Descriptor_new(GrB_Descriptor *desc); 2768

Parameters 2769

desc (INOUT) On successful return, contains a handle to the newly created GraphBLAS 2770 descriptor. 2771

Return Value

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2773 GrB SUCCESS The method completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_NULL_POINTER desc pointer is NULL.

2777 Description

Creates a new descriptor object and returns a handle to it in desc. A newly created descriptor can be populated by calls to Descriptor_set.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

2782 4.2.6.2 Descriptor_set: Set content of descriptor

2783 Sets the content for a field for an existing descriptor.

2784 C Syntax

```
GrB_Info GrB_Descriptor_set(GrB_Descriptor desc,

GrB_Desc_Field field,

GrB_Desc_Value val);
```

788 Parameters

desc (IN) An existing GraphBLAS descriptor to be modified.

field (IN) The field being set.

val (IN) New value for the field being set.

2792 Return Values

2793 GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

2796 GrB_UNINITIALIZED_OBJECT the desc parameter has not been initialized by a call to new.

GrB_INVALID_VALUE invalid value set on the field, or invalid field.

2798 Description

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For a given descriptor, the GrB_Descriptor_set method can be called for each field in the descriptor to set the value associated with that field. Valid values for the field parameter include the following:

Grb OUTP refers to the output parameter (result) of the operation.

GrB_MASK refers to the mask parameter of the operation.

GrB INPO refers to the first input parameters of the operation (matrices and vectors).

GrB INP1 refers to the second input parameters of the operation (matrices and vectors).

2805 Valid values for the val parameter are:

GrB_STRUCTURE Use only the structure of the stored values of the corresponding mask (GrB_MASK) parameter.

GrB_COMP Use the complement of the corresponding mask (GrB_MASK) parameter. When combined with GrB_STRUCTURE, the complement of the structure of the mask is used without evaluating the values stored.

GrB_TRAN Use the transpose of the corresponding matrix parameter (valid for input matrix parameters only).

GrB_REPLACE When assigning the masked values to the output matrix or vector, clear the matrix first (or clear the non-masked entries). The default behavior is to leave non-masked locations unchanged. Valid for the GrB_OUTP parameter only.

Descriptor values can only be set, and once set, cannot be cleared. As, in the case of GrB_MASK, multiple values can be set and all will apply (for example, both GrB_COMP and GrB_STRUCTURE).

A value for a given field may be set multiple times but will have no additional effect. Fields that have no values set result in their default behavior, as defined in Section 3.6.

4.2.7 free: Destroy an object and release its resources

Destroys a previously created GraphBLAS object and releases any resources associated with the object.

2824 C Syntax

6 Parameters

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obj (INOUT) An existing GraphBLAS object to be destroyed. The object must have been created by an explicit call to a GraphBLAS constructor. It can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful completion of GrB_free, obj behaves as an uninitialized object.

2 Return Values

GrB_SUCCESS operation completed successfully

GrB_PANIC unknown internal error. If this return value is encountered when in nonblocking mode, the error responsible for the panic condition could be from any method involved in the computation of the input object. The GrB_error() method should be called for additional information.

2839 Description

GraphBLAS objects consume memory and other resources managed by the GraphBLAS runtime system. A call to GrB_free frees those resources so they are available for use by other GraphBLAS objects.

The parameter passed into GrB_free is a handle referencing a GraphBLAS opaque object of a data type from table 2.1. The object must have been created by an explicit call to a GraphBLAS constructor. The behavior of a program that calls GrB_free on a pre-defined object is implementation defined.

After the GrB_free method returns, the object referenced by the input handle is destroyed and the handle has the value GrB_INVALID_HANDLE. The handle can be used in subsequent GraphBLAS methods but only after the handle has been reinitialized with a call the the appropriate _new or _dup method.

Note that unlike other GraphBLAS methods, calling GrB_free with an object with an invalid handle is legal. The system may attempt to free resources that might be associated with that object, if possible, and return normally.

When using GrB_free it is possible to create a dangling reference to an object. This would occur when a handle is assigned to a second variable of the same opaque type. This creates two handles that reference the same object. If GrB_free is called with one of the variables, the object is destroyed and the handle associated with the other variable no longer references a valid object. This is not an error condition that the implementation of the GraphBLAS API can be expected to catch, hence programmers must take care to prevent this situation from occurring.

2860 4.2.8 wait: Return once an object is either complete or materialized

Wait until method calls in a sequence put an object into a state of completion or materialization.

2862 C Syntax

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GrB_Info GrB_wait(GrB_Object obj, GrB_WaitMode mode);

2864 Parameters

obj (INOUT) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful return of GrB_wait, the obj can be safely read from another thread (completion) or all computing to produce obj by all GraphBLAS operations in its sequence have finished (materialization).

mode (IN) Set's the mode for GrB_wait for whether it is waiting for obj to be in the state of *completion* or *materialization*. Acceptable values are GrB_COMPLETE or GrB_MATERIALIZE.

74 Return values

GrB_SUCCESS operation completed successfully.

GrB_INDEX_OUT_OF_BOUNDS an index out-of-bounds execution error happened during completion of pending operations.

GrB_OUT_OF_MEMORY and out-of-memory execution error happened during completion of pending operations.

GrB_UNINITIALIZED_OBJECT object has not been initialized by a call to the respective *_new, or other constructor, method.

GrB_PANIC unknown internal error.

GrB_INVALID_VALUE method called with a GrB_WaitMode other than GrB_COMPLETE GrB_MATERIALIZE.

Description

On successful return from GrB_wait(), the input object, obj is in one of two states depending on the mode of GrB_wait:

- complete: obj can be used in a happens-before relation, so in a properly synchronized program it can be safely used as an IN or INOUT parameter in a GraphBLAS method call from another thread. This result occurs when the mode parameter is set to GrB_COMPLETE.
- materialized: obj is complete, but in addition, no further computing will be carried out on behalf of obj and error information is available. This result occurs when the mode parameter is set to GrB MATERIALIZE.
- Since in blocking mode OUT or INOUT parameters to any method call are materialized upon return,

 GrB_wait(obj,mode) has no effect when called in blocking mode.
- In non-blocking mode, the status of any pending method calls, other than those associated with producing the *complete* or *materialized* state of obj, are not impacted by the call to GrB_wait(obj,mode).
- Methods in the sequence for obj, however, most likely would be impacted by a call to GrB_wait(obj,mode);
- especially in the case of the *materialized* mode for which any computing on behalf of obj must be
- 2900 finished prior to the return from GrB_wait(obj,mode).

2901 4.2.9 error: Retrieve an error string

Retrieve an error-message about any errors encountered during the processing associated with an object.

2904 C Syntax

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```
GrB_Info GrB_error(const char **error, const GrB_Object obj);
```

2907 Parameters

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- error (OUT) A pointer to a null-terminated string. The contents of the string are implementation defined.
 - obj (IN) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, or type.

2914 Return value

GrB_SUCCESS operation completed successfully.

GrB_UNINITIALIZED_OBJECT object has not been initialized by a call to the respective *_new, or other constructor, method.

GrB_PANIC unknown internal error.

2919 Description

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This method retrieves a message related to any errors that were encountered during the last Graph-2920 BLAS method that had the opaque GraphBLAS object, obj, as an OUT or INOUT parameter. 2921 The function returns a pointer to a null-terminated string and the contents of that string are 2922 implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error 2923 string. The string that is returned is owned by obj and will be valid until the next time obj is 2924 used as an OUT or INOUT parameter or the object is freed by a call to GrB_free(obj). This is a 2925 thread-safe function. It can be safely called by multiple threads for the same object in a race-free 2926 program. 2927

4.3 GraphBLAS operations

The GraphBLAS operations are defined in the GraphBLAS math specification and summarized in Table 4.1. In addition to methods that implement these fundamental GraphBLAS operations, we support a number of variants that have been found to be especially useful in algorithm development.

A flowchart of the overall behavior of a GraphBLAS operation is shown in Figure 4.1.

2933 Domains and Casting

A GraphBLAS operation is only valid when the domains of the GraphBLAS objects are mathemat-2934 ically consistent. The C programming language defines implicit casts between built-in data types. 2935 For example, floats, doubles, and ints can be freely mixed according to the rules defined for implicit 2936 casts. It is the responsibility of the user to assure that these casts are appropriate for the algorithm 2937 in question. For example, a cast to int implies truncation of a floating point type. Depending on 2938 the operation, this truncation error could lead to erroneous results. Furthermore, casting a wider 2930 type onto a narrower type can lead to overflow errors. The GraphBLAS operations do not attempt 2940 to protect a user from these sorts of errors. 2941

When user-define types are involved, however, GraphBLAS requires strict equivalence between types and no casting is supported. If GraphBLAS detects these mismatches, it will return a domain mismatch error.

2945 Dimensions and Transposes

GraphBLAS operations also make assumptions about the numbers of dimensions and the sizes of vectors and matrices in an operation. An operation will test these sizes and report an error if they are not *shape compatible*. For example, when multiplying two matrices, $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, the number of rows of \mathbf{C} must equal the number of rows of \mathbf{A} , the number of columns of \mathbf{A} must match the number of rows of \mathbf{B} , and the number of columns of \mathbf{C} must match the number of columns of \mathbf{B} . This is the behavior expected given the mathematical definition of the operations.

For most of the GraphBLAS operations involving matrices, an optional descriptor can modify the matrix associated with an input GraphBLAS matrix object. For example, if an input matrix is an

Table 4.1: A mathematical notation for the fundamental GraphBLAS operations supported in this specification. Input matrices \mathbf{A} and \mathbf{B} may be optionally transposed (not shown). Use of an optional accumulate with existing values in the output object is indicated with \odot . Use of optional write masks and replace flags are indicated as $\mathbf{C}\langle\mathbf{M},r\rangle$ when applied to the output matrix, \mathbf{C} . The mask controls which values resulting from the operation on the right-hand side are written into the output object (complement and structure flags are not shown). The "replace" option, indicated by specifying the r flag, means that all values in the output object are removed prior to assignment. If "replace" is not specified, only the values/locations computed on the right-hand side and allowed by the mask will be written to the output ("merge" mode).

Operation Name]	Math	nematical N	Votation
mxm	$\mathbf{C}\langle \mathbf{M}, r \rangle$	=	C ⊙	$\mathbf{A} \oplus . \otimes \mathbf{B}$
mxv	$\mathbf{w}\langle\mathbf{m},r angle$	=	\mathbf{w} \odot	
vxm	$\mathbf{w}^T \langle \mathbf{m}^T, r \rangle$	=	\mathbf{w}^T \odot	$\mathbf{u}^T \oplus . \otimes \mathbf{A}$
eWiseMult	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$\mathbf{A} \otimes \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$\mathbf{u} \otimes \mathbf{v}$
eWiseAdd	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$\mathbf{A} \oplus \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$\mathbf{u} \oplus \mathbf{v}$
extract	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$\mathbf{A}(m{i},m{j})$
	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$\mathbf{u}(m{i})$
assign	$\mathbf{C}\langle\mathbf{M},r\rangle(\pmb{i},\pmb{j})$	=	$\mathbf{C}(m{i},m{j})$ \odot) A
	$\mathbf{w}\langle\mathbf{m},r\rangle(i)$	=	$\mathbf{w}(i)$ \odot	\mathbf{u}
reduce (row)	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$[\oplus_j \mathbf{A}(:,j)]$
reduce (scalar)	s	=	s \odot	$[\oplus_{i,j} \mathbf{A}(i,j)]$
	s	=	s \odot	$[\oplus_i \mathbf{u}(i)]$
apply	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$f_u({f A})$
	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$f_u(\mathbf{u})$
apply(indexop)	$\mathbf{C}\langle\mathbf{M},r\rangle$	=	C ⊙	$f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s)$
	$\mathbf{w}\langle\mathbf{m},r angle$	=	\mathbf{w} \odot	$f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s)$
select	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$\mathbf{A}\langle f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s) \rangle$
	$\mathbf{w}\langle\mathbf{m},r\rangle$	=	\mathbf{w} \odot	$\mathbf{u}\langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s) \rangle$
transpose	$\mathbf{C}\langle\mathbf{M},r angle$	=	C •	\mathbf{A}^T
kronecker	$\mathbf{C}\langle\mathbf{M},r angle$	=	\mathbf{C} \odot	$\mathbf{A} \otimes \mathbf{B}$

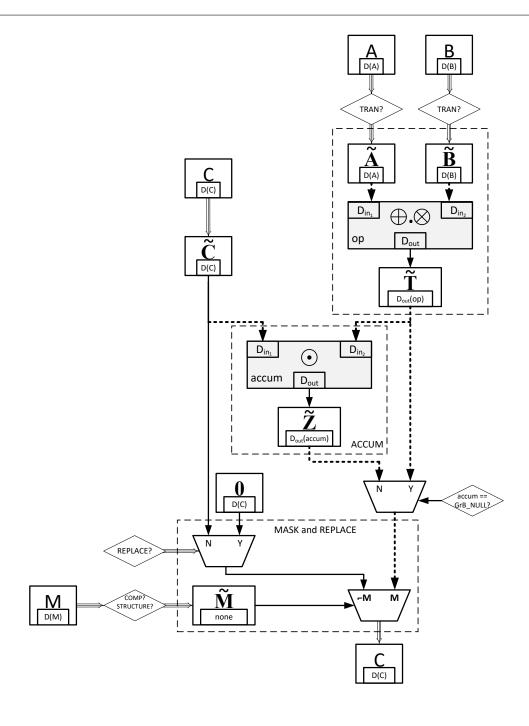


Figure 4.1: Flowchart for the GraphBLAS operations. Although shown specifically for the mxm operation, many elements are common to all operations: such as the "ACCUM" and "MASK and REPLACE" blocks. The triple arrows (\Rrightarrow) denote where "as if copy" takes place (including both collections and descriptor settings). The bold, dotted arrows indicate where casting may occur between different domains.

argument to a GraphBLAS operation and the associated descriptor indicates the transpose option, then the operation occurs as if on the transposed matrix. In this case, the relationships between the sizes in each dimension shift in the mathematically expected way.

²⁹⁵⁷ Masks: Structure-only, Complement, and Replace

When a GraphBLAS operation supports the use of an optional mask, that mask is specified through 2958 a GraphBLAS vector (for one-dimensional masks) or a GraphBLAS matrix (for two-dimensional 2959 masks). When a mask is used and the GTB_STRUCTURE descriptor value is not set, it is applied 2960 to the result from the operation wherever the stored values in the mask evaluate to true. If the 2961 GrB_STRUCTURE descriptor is set, the mask is applied to the result from the operation wherever the 2962 mask as a stored value (regardless of that value). Wherever the mask is applied, the result from 2963 the operation is either assigned to the provided output matrix/vector or, if a binary accumulation 2964 operation is provided, the result is accumulated into the corresponding elements of the provided 2965 output matrix/vector. 2966

Given a GraphBLAS vector $\mathbf{v} = \langle D, N, \{(i, v_i)\} \rangle$, a one-dimensional mask is derived for use in the operation as follows:

$$\mathbf{m} = \begin{cases} \langle N, \{ \mathbf{ind}(\mathbf{v}) \} \rangle, & \text{if GrB_STRUCTURE is specified,} \\ \langle N, \{ i : (\mathsf{bool}) v_i = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool) v_i denotes casting the value v_i to a Boolean value (true or false). Likewise, given a GraphBLAS matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\} \rangle$, a two-dimensional mask is derived for use in the operation as follows:

$$\mathbf{M} = \begin{cases} \langle M, N, \{ \mathbf{ind}(\mathbf{A}) \} \rangle, & \text{if GrB_STRUCTURE is specified,} \\ \langle M, N, \{ (i,j) : (\mathsf{bool}) A_{ij} = \mathsf{true} \} \rangle, & \text{otherwise} \end{cases}$$

where (bool) A_{ij} denotes casting the value A_{ij} to a Boolean value. (true or false)

In both the one- and two-dimensional cases, the mask may also have a subsequent complement operation applied (Section 3.5.4) as specified in the descriptor, before a final mask is generated for use in the operation.

When the descriptor of an operation with a mask has specified that the GrB_REPLACE value is to be applied to the output (GrB_OUTP), then anywhere the mask is not true, the corresponding location in the output is cleared.

Invalid and uninitialized objects

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Upon entering a GraphBLAS operation, the first step is a check that all objects are valid and initialized. (Optional parameters can be set to GrB_NULL, which always counts as a valid object.) An invalid object is one that could not be computed due to a previous execution error. An unitialized object is one that has not yet been created by a corresponding new or dup method. Appropriate error codes are returned if an object is not initialized (GrB_UNINITIALIZED_OBJECT) or invalid (GrB_INVALID_OBJECT).

To support the detection of as many cases of uninitialized objects as possible, it is strongly recommended to initialize all GraphBLAS objects to the predefined value GrB_INVALID_HANDLE at the point of their declaration, as shown in the following examples:

```
GrB_Type type = GrB_INVALID_HANDLE;

GrB_Semiring semiring = GrB_INVALID_HANDLE;

GrB_Matrix matrix = GrB_INVALID_HANDLE;
```

2994 Compliance

We follow a *prescriptive* approach to the definition of the semantics of GraphBLAS operations.

That is, for each operation we give a recipe for producing its outcome. Any implementation that produces the same outcome, and follows the GraphBLAS execution model (Section 2.5) and error model (Section 2.6) is a conforming implementation.

2999 4.3.1 mxm: Matrix-matrix multiply

3000 Multiplies a matrix with another matrix on a semiring. The result is a matrix.

3001 C Syntax

```
GrB_Info GrB_mxm(GrB_Matrix
                                                             С,
3002
                                  const GrB_Matrix
                                                             Mask,
3003
                                  const GrB_BinaryOp
                                                             accum,
3004
                                  const GrB_Semiring
                                                             op,
3005
                                  const GrB_Matrix
                                                             Α,
3006
                                  const GrB Matrix
                                                             В,
3007
                                  const GrB_Descriptor
                                                             desc);
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```

3009 Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the matrix product. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The semiring used in the matrix-matrix multiply.
 - A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
 - B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
 - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
3030				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
	В	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

$_{\scriptscriptstyle{031}}$ Return Values

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3032	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3033		blocking mode, this indicates that the compatibility tests on di-
3034		mensions and domains for the input arguments passed successfully.
3035		Either way, output matrix C is ready to be used in the next method
3036		of the sequence.
3037	GrB_PANIC	Unknown internal error.
3038	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3039		GraphBLAS objects (input or output) is in an invalid state caused
3040		by a previous execution error. Call GrB_error() to access any error
3041		messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

₀₄₅ GrB_DIMENSION_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3050 Description

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GrB_mxm computes the matrix product $C = A \oplus . \otimes B$ or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot (A \oplus . \otimes B)$ (where matrices A and B can be optionally transposed). Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3056 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 3058 Up to four argument matrices are used in the GrB_mxm operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3060 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
- 3061 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4. $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
- 3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
- 4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

- If any compatibility rule above is violated, execution of GrB_mxm ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed $(\leftarrow \text{denotes copy})$:
- 3080 1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 3081 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
- (b) If Mask \neq GrB NULL,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
 - (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix $\tilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP1}].\mathsf{GrB_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}).$
- 3095 2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$
- 3097 4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{B}}).$
- $5. \ \mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathbf{nrows}(\widetilde{\mathbf{B}}).$
- If any compatibility rule above is violated, execution of GrB_mxm ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix multiplication and any additional associated operations.
- We describe this in terms of two intermediate matrices:
 - $\widetilde{\mathbf{T}}$: The matrix holding the product of matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
 - ullet $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(: ,j)) \neq \emptyset \} \rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(:,j))} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{B}}(k,j)),$$

where \oplus and \otimes are the additive and multiplicative operators of semiring op, respectively.

The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{-}\operatorname{\mathsf{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

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• If \mathbf{z} is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C , and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

3138 4.3.2 vxm: Vector-matrix multiply

Multiplies a (row) vector with a matrix on an semiring. The result is a vector.

3140 C Syntax

```
GrB_Info GrB_vxm(GrB_Vector
                                                             W,
3141
                                 const GrB_Vector
                                                             mask,
3142
                                 const GrB_BinaryOp
                                                             accum,
3143
                                 const GrB_Semiring
3144
                                                             op,
                                 const GrB_Vector
3145
                                                             u,
                                 const GrB Matrix
                                                             Α,
3146
                                  const GrB_Descriptor
                                                             desc);
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```

3148 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the vector-matrix product. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) Semiring used in the vector-matrix multiply.
 - u (IN) The GraphBLAS vector holding the values for the left-hand vector in the multiplication.
 - A (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.
 - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
Α	GrB_INP1	GrB_TRAN	Use transpose of A for the operation.

Return Values

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3171 3172 3173 3174 3175	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
3176	GrB_PANIC	Unknown internal error.
3177 3178 3179 3180	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
3181	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
3182 3183	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).
3184	GrB_DIMENSION_MISMATCH	Mask, vector, and/or matrix dimensions are incompatible.
3185 3186 3187 3188	GrB_DOMAIN_MISMATCH	The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3189 **Description**

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GrB_vxm computes the vector-matrix product $\mathbf{w}^T = \mathbf{u}^T \oplus . \otimes \mathsf{A}$, or, if an optional binary accumulation operator (\odot) is provided, $\mathbf{w}^T = \mathbf{w}^T \odot \left(\mathbf{u}^T \oplus . \otimes \mathsf{A} \right)$ (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

Setup The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

³¹⁹⁷ Up to four argument vectors or matrices are used in the GrB_vxm operation:

```
3198 1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

3199 2.
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$$

3200 3.
$$\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$$

3201

4.
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 3206 2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
- 3207 3. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
- 4. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(w)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB_vxm ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed (\leftarrow denotes copy):

- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3226 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

- 4. Matrix $\tilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP1}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- The internal matrices and masks are checked for shape compatibility. The following conditions must hold:
- 3230 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}).$
- 3231 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$
- 3232 3. $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

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- If any compatibility rule above is violated, execution of GrB_vxm ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the vector-matrix multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$: The vector holding the product of vector $\tilde{\mathbf{u}}^T$ and matrix $\tilde{\mathbf{A}}$.
 - $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{ncols}(\tilde{\mathbf{A}}), \{(j, t_j) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{A}}(:, j)) \neq \emptyset \} \rangle$ is created.

 The value of each of its elements is computed by

$$t_j = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{A}}(:,j))} (\widetilde{\mathbf{u}}(k) \otimes \widetilde{\mathbf{A}}(k,j)),$$

- where \oplus and \otimes are the additive and multiplicative operators of semiring op, respectively.
- The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:
- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_{i} = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_{i} = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_{i} = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$L(w) = \{(i, z_i) : i \in (ind(\widetilde{z}) \cap ind(\widetilde{m}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

3272 4.3.3 mxv: Matrix-vector multiply

Multiplies a matrix by a vector on a semiring. The result is a vector.

3274 C Syntax

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```
GrB_Info GrB_mxv(GrB_Vector
                                                             W,
3275
                                  const GrB_Vector
                                                             mask,
3276
                                  const GrB_BinaryOp
                                                             accum,
3277
                                  const GrB Semiring
3278
                                                             op,
                                  const GrB_Matrix
                                                             Α,
3279
                                  const GrB Vector
3280
                                                             u,
                                  const GrB Descriptor
                                                             desc);
3281
```

Parameters

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w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the matrix-vector product. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain

3289 3290 3291	in T	of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.			
3292 3293 3294	entr	n (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.			
3295	op (IN)	Semiring used	in the vector-matrix	multiply.	
3296 3297	` '	(IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.			
3298 3299	` /	(IN) The GraphBLAS vector holding the values for the right-hand vector in the multiplication.			
3300 3301 3302	` /	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:			
	Param	Field	Value	Description	
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.	
3303	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.	

GrB_COMP

GrB_TRAN

Use the complement of mask.

Use transpose of A for the operation.

Neturn Values

mask

Α

GrB_MASK

GrB_INP0

3305	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3306		blocking mode, this indicates that the compatibility tests on di-
3307		mensions and domains for the input arguments passed successfully.
3308		Either way, output vector w is ready to be used in the next method
3309		of the sequence.
3310	GrB_PANIC	Unknown internal error.
3311	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3312		GraphBLAS objects (input or output) is in an invalid state caused
3313		by a previous execution error. Call GrB_error() to access any error
3314		messages generated by the implementation.
3315	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
3316 3317	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).

3318 GrB_DIMENSION_MISMATCH Mask, vector, and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3323 Description

GrB_mxv computes the matrix-vector product $w = A \oplus . \otimes u$, or, if an optional binary accumulation operator (\odot) is provided, $w = w \odot (A \oplus . \otimes u)$ (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

- Setup The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.
- 3329 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 3331 Up to four argument vectors or matrices are used in the GrB_mxv operation:
- 3332 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3333 2. $\operatorname{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3335 4. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
- 3. $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
- 3342 4. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(w)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB_mxv ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed (\leftarrow denotes copy):

- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If mask \neq GrB NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3360 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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- The internal matrices and masks are checked for shape compatibility. The following conditions must hold:
- 3364 1. $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}}).$
- 3365 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- 3. $\operatorname{size}(\widetilde{\mathbf{u}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB_mxv ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the matrix-vector multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$: The vector holding the product of matrix $\tilde{\mathbf{A}}$ and vector $\tilde{\mathbf{u}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(i,t_i) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}}) \neq \emptyset \} \rangle$ is created.

 The value of each of its elements is computed by

$$t_i = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}})} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{u}}(k)),$$

where \oplus and \otimes are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

• If $\operatorname{accum} = \operatorname{GrB} \ \operatorname{NULL}$, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.4 eWiseMult: Element-wise multiplication

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

3412 **4.3.4.1** eWiseMult: Vector variant

Perform element-wise (general) multiplication on the intersection of elements of two vectors, producing a third vector as result.

3415 C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Vector
                                                                    W,
3416
                                         const GrB_Vector
                                                                   mask,
3417
                                         const GrB_BinaryOp
                                                                    accum,
3418
                                         const GrB_Semiring
                                                                    op,
3419
                                         const GrB_Vector
3420
                                                                    u,
                                         const GrB_Vector
                                                                    v,
3421
                                         const GrB_Descriptor
                                                                    desc);
3422
3423
              GrB_Info GrB_eWiseMult(GrB_Vector
3424
                                                                    W,
                                         const GrB_Vector
                                                                   mask,
3425
                                         const GrB_BinaryOp
                                                                    accum,
3426
                                         const GrB_Monoid
                                                                    op,
3427
                                         const GrB Vector
3428
                                                                    u,
                                         const GrB Vector
                                                                    v,
3429
                                         const GrB_Descriptor
                                                                    desc);
3430
3431
              GrB_Info GrB_eWiseMult(GrB_Vector
3432
                                                                    W,
                                         const GrB_Vector
                                                                   mask,
3433
                                         const GrB_BinaryOp
3434
                                                                    accum,
                                         const GrB_BinaryOp
3435
                                                                    op,
                                         const GrB_Vector
                                                                    u,
3436
                                         const GrB_Vector
                                                                    v,
3437
                                         const GrB_Descriptor
                                                                    desc);
3438
```

3439 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
 - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
 - accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB_NULL should be 3450 specified. 3451

> op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), (\mathsf{op}) \rangle$; the identity element is ig-

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes (\mathsf{op}) \rangle$; the additive monoid is ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB MASK	GrB COMP	Use the complement of mask.

Return Values 3468

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GrB_SUCCESS In blocking mode, the operation completed successfully. In non-3469 blocking mode, this indicates that the compatibility tests on di-3470 mensions and domains for the input arguments passed successfully. 3471 Either way, output vector w is ready to be used in the next method 3472 of the sequence. 3473 GrB_PANIC Unknown internal error. 3474 GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque 3475 GraphBLAS objects (input or output) is in an invalid state caused 3476 by a previous execution error. Call GrB_error() to access any error

messages generated by the implementation. GrB_OUT_OF_MEMORY Not enough memory available for the operation. GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3482 GrB_DIMENSION_MISMATCH Mask or vector dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_eWiseMult computes the element-wise "product" of two GraphBLAS vectors: $w = u \otimes v$, or, if an optional binary accumulation operator (\odot) is provided, $w = w \odot (u \otimes v)$. Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3493 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB_eWiseMult operation:

- 3496 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3497 2. mask = $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 3. $u = \langle \mathbf{D}(u), \mathbf{size}(u), \mathbf{L}(u) = \{(i, u_i)\} \rangle$
- 3499 4. $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3504 2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
- 3. $\mathbf{D}(\mathbf{v})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
- 4. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If $mask \neq GrB_NULL$,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- 3522 ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- $(c) \ \text{If desc[GrB_MASK].GrB_COMP is set, then } \widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}.$
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 4. Vector $\tilde{\mathbf{v}} \leftarrow \mathbf{v}$.

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- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 3528 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.
- We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$: The vector holding the element-wise "product" of $\tilde{\mathbf{u}}$ and vector $\tilde{\mathbf{v}}$.
 - $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$ is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \otimes \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

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• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
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$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$
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 $z_i = \widetilde{\mathbf{t}}(i), \ \mathrm{if} \ i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.4.2 eWiseMult: Matrix variant

Perform element-wise (general) multiplication on the intersection of elements of two matrices, producing a third matrix as result.

3570 C Syntax

```
3571
             GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    С,
                                         const GrB_Matrix
                                                                   Mask,
3572
                                         const GrB_BinaryOp
                                                                    accum,
3573
                                         const GrB_Semiring
                                                                    op,
3574
                                         const GrB_Matrix
                                                                    Α,
3575
                                         const GrB_Matrix
                                                                    В,
3576
                                         const GrB Descriptor
                                                                    desc);
3577
3578
             GrB Info GrB eWiseMult(GrB Matrix
                                                                    C,
3579
                                         const GrB Matrix
                                                                   Mask,
3580
                                         const GrB BinaryOp
                                                                    accum,
3581
                                         const GrB_Monoid
                                                                    op,
3582
                                         const GrB_Matrix
                                                                   Α,
3583
                                         const GrB_Matrix
                                                                   В,
3584
                                         const GrB_Descriptor
                                                                    desc);
3585
3586
             GrB_Info GrB_eWiseMult(GrB_Matrix
                                                                    C,
3587
                                         const GrB_Matrix
                                                                   Mask,
3588
                                         const GrB_BinaryOp
                                                                    accum,
3589
                                         const GrB_BinaryOp
                                                                    op,
3590
                                         const GrB_Matrix
                                                                    Α,
3591
                                         const GrB Matrix
3592
                                                                   В,
                                         const GrB_Descriptor
                                                                    desc);
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```

3594 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

3610		BinaryOp: F_t	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$(p), \mathbf{D}_{in_2}(op), \bigcirc(op) \rangle.$
3611		Monoid: F_t	$_{o} = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{D}$	$O(op), \bigcirc(op)$; the identity element is ig-
3612		no	ored.	
3613		Semiring: F_t	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$p), \mathbf{D}_{in_2}(op), \bigotimes(op)$; the additive monoid
3614		is	ignored.	
3615	` ,	-	LAS matrix holding t	the values for the left-hand matrix in the
3616	oper	ation.		
3617	B (IN)	The GraphBL	AS matrix holding th	he values for the right-hand matrix in the
3618	oper	ation.		
3619	desc (IN)	An optional of	peration descriptor. If	f a default descriptor is desired, GrB_NULL
3620	, ,			alue pairs are listed as follows:
3621				
	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
3622				structure (pattern of stored values) of the
3022				input Mask matrix. The stored values are not examined.

GrB_COMP

GrB_TRAN

GrB_TRAN

Use the complement of $\mathsf{Mask}.$

Use transpose of A for the operation.

Use transpose of B for the operation.

3623 Return Values

Mask

Α

В

GrB_MASK

GrB_INP0

GrB_INP1

3624	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3625		blocking mode, this indicates that the compatibility tests on di-
3626		mensions and domains for the input arguments passed successfully.
3627		Either way, output matrix C is ready to be used in the next method
3628		of the sequence.
3629	GrB_PANIC	Unknown internal error.
3630	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3631		GraphBLAS objects (input or output) is in an invalid state caused
3632		by a previous execution error. Call GrB_error() to access any error
3633		messages generated by the implementation.
3634	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
	B_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
3636		a can to new (or matrix_dup for matrix parameters).
3637 Gr	B_DIMENSION_MISMATCH	Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3642 Description

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This variant of $GrB_eWiseMult$ computes the element-wise "product" of two GraphBLAS matrices: $C = A \otimes B$, or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot (A \otimes B)$. Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 3648 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to four argument matrices are used in the GrB_eWiseMult operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3653 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
- 3660 3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
- 4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any

- compatibility rule above is violated, execution of GrB_eWiseMult ends and the domain mismatch error listed above is returned.
- From the argument matrices, the internal matrices and mask used in the computation are formed $(\leftarrow \text{denotes copy})$:
- 1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathbf{C}$.

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- \mathbf{M} , is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
 - (b) If Mask \neq GrB_NULL,
 - i. If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_STRUCTURE}$ is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - $$\begin{split} \text{ii. Otherwise, } \mathbf{M} &= \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ &\{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle. \end{split}$$
 - (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix $\tilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP1}].\mathsf{GrB_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 3686 1. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}) = \operatorname{nrows}(\widetilde{\mathbf{C}}).$
- 2. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "product" and any additional associated operations.
 We describe this in terms of two intermediate matrices:
- We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$: The matrix holding the element-wise product of $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \otimes \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\hspace{-.07em}\underline{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

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• If accum is a binary operator, then $\tilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C , and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.5 eWiseAdd: Element-wise addition

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

3732 **4.3.5.1** eWiseAdd: Vector variant

Perform element-wise (general) addition on the elements of two vectors, producing a third vector as result.

3735 C Syntax

```
GrB_Info GrB_eWiseAdd(GrB_Vector
3736
                                                                  W,
                                       const GrB_Vector
                                                                  mask,
3737
                                        const GrB_BinaryOp
                                                                  accum,
3738
                                        const GrB_Semiring
                                                                  op,
3739
                                        const GrB_Vector
3740
                                                                  u,
                                        const GrB_Vector
                                                                  v,
3741
                                        const GrB_Descriptor
                                                                  desc);
3742
3743
              GrB_Info GrB_eWiseAdd(GrB_Vector
3744
                                                                  w,
                                       const GrB_Vector
                                                                  mask,
3745
                                       const GrB BinaryOp
                                                                  accum,
3746
                                       const GrB_Monoid
3747
                                                                  op,
                                        const GrB Vector
3748
                                                                  u,
                                       const GrB Vector
                                                                  v,
3749
                                        const GrB_Descriptor
                                                                  desc);
3750
3751
              GrB_Info GrB_eWiseAdd(GrB_Vector
3752
                                                                  W,
                                       const GrB_Vector
                                                                  mask,
3753
                                       const GrB_BinaryOp
3754
                                                                  accum,
                                        const GrB_BinaryOp
3755
                                                                  op,
                                        const GrB_Vector
                                                                  u,
3756
                                        const GrB_Vector
3757
                                                                  v,
                                       const GrB_Descriptor
                                                                  desc);
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```

3759 Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.
 - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
 - accum (IN) An optional binary operator used for accumulating entries into existing w

entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), (\mathsf{op}) \rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigoplus(\mathsf{op}) \rangle$; the multiplicative binary op and additive identity are ignored.

- u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.
- v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
			removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB MASK	GrB COMP	Use the complement of mask.

3788 Return Values

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GrB_SUCCESS In blocking mode, the operation completed successfully. In non-3789 blocking mode, this indicates that the compatibility tests on di-3790 mensions and domains for the input arguments passed successfully. 3791 Either way, output vector w is ready to be used in the next method 3792 of the sequence. 3793 GrB_PANIC Unknown internal error. 3794 GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque 3795 GraphBLAS objects (input or output) is in an invalid state caused 3796 by a previous execution error. Call GrB_error() to access any error 3797 messages generated by the implementation. 3798

Grb_OUT_OF_MEMORY Not enough memory available for the operation.

3800 GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

3802 GrB_DIMENSION_MISMATCH Mask or vector dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3807 Description

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This variant of GrB_eWiseAdd computes the element-wise "sum" of two GraphBLAS vectors: $w = u \oplus v$, or, if an optional binary accumulation operator (\odot) is provided, $w = w \odot (u \oplus v)$. Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

3813 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB_eWiseAdd operation:

- 3816 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 3817 2. mask = $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 3818 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- 3819 4. $\mathbf{v} = \langle \mathbf{D}(\mathbf{v}), \mathbf{size}(\mathbf{v}), \mathbf{L}(\mathbf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 3824 2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
- 3825 3. $\mathbf{D}(\mathbf{v})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
- 3826 4. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 5. $\mathbf{D}(\mathbf{u})$ and $\mathbf{D}(\mathbf{v})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 6. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

- 3838 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
 - (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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- 4. Vector $\tilde{\mathbf{v}} \leftarrow \mathbf{v}$.
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{v}}).$
- If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.
 We describe this in terms of two intermediate vectors:
 - $\widetilde{\mathbf{t}}$: The vector holding the element-wise "sum" of $\widetilde{\mathbf{u}}$ and vector $\widetilde{\mathbf{v}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\tilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\tilde{\mathbf{u}}) \cup \mathbf{ind}(\tilde{\mathbf{v}}) \neq \emptyset \} \rangle$ is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \oplus \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$
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$$t_i = \widetilde{\mathbf{u}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

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$$t_i = \widetilde{\mathbf{v}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{v}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

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• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.5.2 eWiseAdd: Matrix variant

Perform element-wise (general) addition on the elements of two matrices, producing a third matrix as result.

3896 C Syntax

```
GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                  С,
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                                        const GrB_Matrix
                                                                  Mask,
3898
                                        const GrB_BinaryOp
                                                                  accum,
3899
                                        const GrB_Semiring
                                                                  op,
3900
                                        const GrB_Matrix
                                                                  Α,
3901
                                        const GrB_Matrix
                                                                  Β,
3902
                                        const GrB Descriptor
                                                                  desc);
3903
3904
              GrB Info GrB eWiseAdd(GrB Matrix
                                                                  С,
3905
                                        const GrB Matrix
                                                                  Mask,
3906
                                        const GrB BinaryOp
                                                                  accum,
3907
                                       const GrB_Monoid
                                                                  op,
3908
                                       const GrB_Matrix
                                                                  Α,
3909
                                       const GrB_Matrix
                                                                  В,
3910
                                       const GrB_Descriptor
                                                                  desc);
3911
3912
              GrB_Info GrB_eWiseAdd(GrB_Matrix
                                                                  С,
3913
                                       const GrB_Matrix
                                                                  Mask,
3914
                                       const GrB_BinaryOp
                                                                  accum,
3915
                                        const GrB_BinaryOp
                                                                  op,
3916
                                        const GrB_Matrix
                                                                  Α,
3917
                                       const GrB Matrix
3918
                                                                  В,
                                       const GrB_Descriptor
                                                                  desc);
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```

3920 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$, used:

3936		BinaryOp: F_l	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$(p), \mathbf{D}_{in_2}(op), \bigcirc(op) \rangle.$			
3937		Monoid: F_t	$\mathbf{D}_{o} = \langle \mathbf{D}(op), \mathbf{D}(op), \mathbf{D}(op) \rangle$	$O(op), \bigcirc(op)$; the identity element is ig-			
3938		no	ored.				
3939		Semiring: F_{ℓ}	$\mathbf{p} = \langle \mathbf{D}_{out}(op), \mathbf{D}_{in_1}(op) \rangle$	$(p), \mathbf{D}_{in_2}(op), \bigoplus(op)$; the multiplicative bi-			
3940		na	ary op and additive id	lentity are ignored.			
3941	A (IN)	The GraphBI	AS matrix holding t	the values for the left-hand matrix in the			
3942	` /	ation.					
	D (INI)	mi C i Di	A.C 1 11:				
3943	` /	B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the					
3944	oper	operation.					
3945	desc (IN)	An optional of	peration descriptor. If	f a default descriptor is desired, GrB_NULL			
3946	shou	should be specified. Non-default field/value pairs are listed as follows:					
3947		-	,	•			
	Param	Field	Value	Description			
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements			
				removed) before the result is stored in it.			
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the			
		_	_	structure (pattern of stored values) of the			
3948				input Mack matrix. The stored values are			

GrB_COMP

GrB_TRAN

GrB_TRAN

input Mask matrix. The stored values are

Use transpose of A for the operation.

Use transpose of B for the operation.

Use the complement of Mask.

not examined.

Return Values

Mask

Α

В

GrB_MASK

GrB_INP0

GrB_INP1

3950	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
3951		blocking mode, this indicates that the compatibility tests on di-
3952		mensions and domains for the input arguments passed successfully.
3953		Either way, output matrix C is ready to be used in the next method
3954		of the sequence.
3955	GrB_PANIC	Unknown internal error.
3956	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque
3957		GraphBLAS objects (input or output) is in an invalid state caused
3958		by a previous execution error. Call GrB_error() to access any error
3959		messages generated by the implementation.
3960	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
3961	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by
3962		a call to new (or Matrix_dup for matrix parameters).
3963	GrB_DIMENSION_MISMATCH	Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

3968 Description

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This variant of $GrB_eWiseAdd$ computes the element-wise "sum" of two GraphBLAS matrices: $C = A \oplus B$, or, if an optional binary accumulation operator O0 is provided, $C = C \odot (A \oplus B)$. Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 3976 Up to four argument matrices are used in the GrB_eWiseAdd operation:
- 3977 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 3978 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
- 3979 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 3980 4. $B = \langle \mathbf{D}(B), \mathbf{nrows}(B), \mathbf{ncols}(B), \mathbf{L}(B) = \{(i, j, B_{ij})\} \rangle$
- The argument matrices, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 3985 2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
- 3986 3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
- 3987 4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 3988 5. $\mathbf{D}(\mathsf{A})$ and $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 6. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed \leftarrow denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 4000 2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
- 4001 (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{ncols}(\mathsf{C})\} \rangle$.
- 4003 (b) If $Mask \neq GrB_NULL$,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- 4008 (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. Matrix $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP1}].\mathsf{GrB_TRAN} ? \mathsf{B}^T : \mathsf{B}.$
- The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{M}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}).$
- 2. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) = \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the element-wise "sum" and any additional associated operations.
- 4020 We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$: The matrix holding the element-wise sum of $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
 - $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i, j, T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cup \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset \} \rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \oplus \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

$$T_{ij} = \widetilde{\mathbf{A}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

$$T_{ij} = \widetilde{\mathbf{B}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

$$T_{ij} = \widetilde{\mathbf{B}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{B}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}}_{-}\operatorname{\mathsf{NULL}}, \, \operatorname{then} \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

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• If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
4040
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\dot{\mathbf{Z}}$ are written into the final result matrix C , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6 extract: Selecting sub-graphs

4059 Extract a subset of a matrix or vector.

4060 4.3.6.1 extract: Standard vector variant

Extract a sub-vector from a larger vector as specified by a set of indices. The result is a vector whose size is equal to the number of indices.

4063 C Syntax

```
GrB_Info GrB_extract(GrB_Vector
4064
                                                                  W,
                                      const GrB_Vector
                                                                  mask,
4065
                                      const GrB_BinaryOp
                                                                  accum,
4066
                                      const GrB_Vector
                                                                  u,
4067
                                      const GrB_Index
                                                                 *indices,
4068
                                      GrB_Index
                                                                  nindices,
4069
                                      const GrB_Descriptor
                                                                  desc);
4070
```

Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - u (IN) The GraphBLAS vector from which the subset is extracted.
- indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations of elements from u that are extracted. If all elements of u are to be extracted in order from 0 to nindices 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.
- nindices (IN) The number of values in indices array. Must be equal to size(w).

desc	(IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL
	should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4094				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

95 Return Values

4091 4092 4093

4096 4097 4098 4099 4100	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
4101	GrB_PANIC	Unknown internal error.
4102 4103 4104 4105	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4106	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4107 4108	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
4109 4110	GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to $\mathbf{size}(u).$ In non-blocking mode, this error can be deferred.
4111	GrB_DIMENSION_MISMATCH	$mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nindices \neq \mathbf{size}(w).$
4112 4113 4114 4115	GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4116	GrB_NULL_POINTER	Argument row_indices is a NULL pointer.

Description

This variant of $GrB_extract$ computes the result of extracting a subset of locations from a Graph-BLAS vector in a specific order: w = u(indices); or, if an optional binary accumulation operator

 (\odot) is provided, $w = w \odot u$ (indices). More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \mathsf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices}, \ \ \mathsf{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{u}(\mathsf{indices}[i]), \ \forall \ i: \ 0 \leq i < \mathsf{nindices} \end{aligned}$$

Logically, this operation occurs in three steps:

- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4125 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4127 Up to three argument vectors are used in this GrB_extract operation:
- 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 1129 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4135 2. $\mathbf{D}(w)$ must be compatible with $\mathbf{D}(u)$.
- 3. If accum is not GrB_NULL , then D(w) must be compatible with $D_{in_1}(accum)$ and $D_{out}(accum)$ of the accumulation operator and D(u) must be compatible with $D_{in_2}(accum)$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask, and index array used in the computation are formed (\leftarrow denotes copy):
- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.

- (b) If mask \neq GrB_NULL,
- i. If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_STRUCTURE} \ \mathrm{is} \ \mathrm{set}, \ \mathrm{then} \ \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle,$
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 4. The internal index array, \widetilde{I} , is computed from argument indices as follows:
- (a) If indices = GrB_ALL, then $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nindices}$.
- (b) Otherwise, $\tilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$
- The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 2. nindices = $\mathbf{size}(\widetilde{\mathbf{w}})$.

- If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$: The vector holding the extraction from $\widetilde{\mathbf{u}}$ in their destination locations relative to $\widetilde{\mathbf{w}}$.
 - $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, \widetilde{\mathbf{u}}(\widetilde{\boldsymbol{I}}[i])) \ \forall \ i, 0 \leq i < \mathsf{nindices} : \widetilde{\boldsymbol{I}}[i] \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

- 4171 At this point, if any value in \widetilde{I} is not in the valid range of indices for vector $\widetilde{\mathbf{u}}$, the execution of
- GrB_extract ends and the index-out-of-bounds error listed above is generated. In GrB_NONBLOCKING
- mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the
- result vector, w, is invalid from this point forward in the sequence.
- The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:
- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

```
z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),
z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),
```

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6.2 extract: Standard matrix variant

Extract a sub-matrix from a larger matrix as specified by a set of row indices and a set of column indices. The result is a matrix whose size is equal to size of the sets of indices.

4205 C Syntax

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```
C,
             GrB_Info GrB_extract(GrB_Matrix
4206
                                      const GrB_Matrix
                                                               Mask,
4207
                                      const GrB_BinaryOp
                                                               accum,
4208
                                      const GrB_Matrix
                                                               Α,
4209
                                      const GrB_Index
                                                              *row_indices,
4210
                                      GrB_Index
                                                               nrows,
4211
                                      const GrB_Index
                                                              *col_indices,
4212
                                      GrB_Index
                                                               ncols,
4213
                                      const GrB Descriptor
                                                               desc);
4214
```

4215 Parameters

4216 4217 4218	C	that	may be accum	~ .	ix. On input, the matrix provides values to f the extract operation. On output, the n.
4219 4220 4221 4222 4223 4224	Mask	stored matri of the in Ta	d into the out ix C. If the Grie Mask matrix able 3.2. If the	put matrix C. The mB_STRUCTURE desc must be of type boo	crols which results from this operation are mask dimensions must match those of the riptor is <i>not</i> set for the mask, the domain I or any of the predefined "built-in" types ared (i.e., a mask that is all true with the specified.
4225 4226 4227	accum	` ,	es. If assignme		for accumulating entries into existing C mulation is desired, GrB_NULL should be
4228	А	(IN)	The GraphBL	AS matrix from which	h the subset is extracted.
4229 4230 4231 4232 4233	row_indices	from in ordinate value	which element der, GrB_ALL , this array m	s are extracted. If elesshould be specified. nay be manipulated by	of indices corresponding to the rows of A ements in all rows of A are to be extracted Regardless of execution mode and return by the caller after this operation returns tions for this operation.
4234	nrows	(IN)	The number of	f values in the row_ir	ndices array. Must be equal to $\mathbf{nrows}(C)$.
4235 4236 4237 4238 4239	col_indices	of A be ex mode	from which electracted in order and return v	ements are extracted er, then GrB_ALL sho value, this array may	of indices corresponding to the columns d. If elements in all columns of A are to ould be specified. Regardless of execution be manipulated by the caller after this deferred computations for this operation.
4240	ncols	(IN)	The number of	f values in the col_ine	dices array. Must be equal to $\mathbf{ncols}(C)$.
4241 4242 4243	desc	` ′		-	a default descriptor is desired, GrB_NULL alue pairs are listed as follows:
	Pa	ram	Field GrB_OUTP	Value GrB_REPLACE	Description Output matrix C is cleared (all elements removed) before the result is stored in it.
4244	Ma	ask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
	Ma A	ask	GrB_MASK GrB_INP0	GrB_COMP GrB_TRAN	Use the complement of Mask. Use transpose of A for the operation.

Return Values

4246 4247 4248 4249 4250	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
4251	GrB_PANIC	Unknown internal error.
4252 4253 4254 4255	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4256	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
4257 4258	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
4259 4260 4261	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$, or a value in $\mathbf{col_indices}$ is greater than or equal to $\mathbf{ncols}(A)$. In non-blocking mode, this error can be deferred.
4262 4263	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows \neq $\mathbf{nrows}(C)$, or $\mathbf{ncols} \neq \mathbf{ncols}(C)$.
4264 4265 4266 4267	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4268 4269	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

4270 Description

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This variant of GrB_extract computes the result of extracting a subset of locations from specified rows and columns of a GraphBLAS matrix in a specific order: $C = A(row_indices, col_indices)$; or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot A(row_indices, col_indices)$.

More explicitly (not accounting for an optional transpose of A):

```
\mathsf{C}(i,j) = \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols}, \ \mathsf{or} \\ \mathsf{C}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \ \forall \ i,j \ : \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \\ \mathsf{ncols}(i,j) = \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{ncols}(i), \mathsf{ncols}(i), \mathsf{
```

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

- Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4281 Up to three argument matrices are used in the GrB_extract operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.
- From the arguments, the internal matrices, mask, and index arrays used in the computation are formed (\leftarrow denotes copy):
- 1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$.
- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
- (b) If Mask \neq GrB_NULL,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
- 4307 ii. Otherwise, $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \\ \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}.$

- 4. The internal row index array, \tilde{I} , is computed from argument row_indices as follows:
- 4312 (a) If row_indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \forall i: 0 \leq i < \text{nrows}.$
- (b) Otherwise, $\tilde{I}[i] = \text{row_indices}[i], \forall i : 0 \le i < \text{nrows}.$
- 5. The internal column index array, \tilde{J} , is computed from argument col_indices as follows:
- 4315 (a) If col_indices = GrB_ALL, then $\widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \mathsf{col_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 4320 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathsf{nrows}.$
- 4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathsf{ncols}.$
- If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$: The matrix holding the extraction from $\widetilde{\mathbf{A}}$.
- \bullet $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i, j, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j])) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j]) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

- At this point, if any value in the \widetilde{I} array is not in the range $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$ or any value in the \widetilde{J} array is not in the range $[0, \mathbf{ncols}(\widetilde{\mathbf{A}}))$, the execution of $\mathsf{GrB_extract}$ ends and the index out-of-bounds error listed above is generated. In $\mathsf{GrB_NONBLOCKING}$ mode, the error can be deferred until a sequence-terminating $\mathsf{GrB_wait}()$ is called. Regardless, the result matrix C is invalid from this point forward in the sequence.
- The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:
- If $\mathsf{accum} = \mathsf{GrB} _\mathsf{NULL}, \, \mathsf{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

• If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

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$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C , and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6.3 extract: Column (and row) variant

Extract from one column of a matrix into a vector. Note that with the transpose descriptor for the source matrix, elements of an arbitrary row of the matrix can be extracted with this function as well.

4369 C Syntax

4370	<pre>GrB_Info GrB_extract(GrB_Vector</pre>	W,
4371	const GrB_Vector	mask,
4372	const GrB_BinaryOp	accum,
4373	const GrB_Matrix	Α,
4374	const GrB_Index	*row_indices,
4375	${ t GrB_Index}$	nrows,
4376	${ t GrB_Index}$	col_index,
4377	const GrB_Descriptor	desc);

4378 Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - A (IN) The GraphBLAS matrix from which the column subset is extracted.
- row_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations
 within the specified column of A from which elements are extracted. If elements in
 all rows of A are to be extracted in order, GrB_ALL should be specified. Regardless
 of execution mode and return value, this array may be manipulated by the caller
 after this operation returns without affecting any deferred computations for this
 operation.
 - nrows (IN) The number of indices in the row indices array. Must be equal to size(w).
- col_index (IN) The index of the column of A from which to extract values. It must be in the range $[0, \mathbf{ncols}(A))$.
 - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
-	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4404				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

4406 4407 4408 4409 4410	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
4411	GrB_PANIC	Unknown internal error.
4412 4413 4414 4415	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4416	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4417 4418	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
4419	GrB_INVALID_INDEX	${\color{blue} \textbf{col_index} \ is \ outside \ the \ allowable \ range \ (i.e., \ greater \ than \ \textbf{ncols}(A)).}$
4420 4421	GrB_INDEX_OUT_OF_BOUNDS	A value in $row_indices$ is greater than or equal to $nrows(A)$. In non-blocking mode, this error can be deferred.
4422	GrB_DIMENSION_MISMATCH	$mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nrows \neq \mathbf{size}(w).$
4423 4424 4425 4426	GrB_DOMAIN_MISMATCH	The domains of the vector or matrix are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4427	GrB_NULL_POINTER	Argument row_indices is a NULL pointer.

Description

This variant of GrB_extract computes the result of extracting a subset of locations (in a specific order) from a specified column of a GraphBLAS matrix: $w = A(:, col_index)(row_indices)$; or, if 4430

an optional binary accumulation operator (\odot) is provided, $w = w \odot A(:,col_index)(row_indices)$.

More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \mathsf{A}(\mathsf{row_indices}[i], \mathsf{col_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows}, \; \; \mathsf{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{A}(\mathsf{row_indices}[i], \mathsf{col_index}) \; \forall \; i: \; 0 \leq i < \mathsf{nrows} \end{aligned}$$

Logically, this operation occurs in three steps:

- Setup The internal matrices, vectors, and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4437 **Compute** The indicated computations are carried out.
- Output The result is written into the output vector, possibly under control of a mask.
- 4439 Up to three argument vectors and matrices are used in this GrB_extract operation:
- 4440 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$

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- 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument vectors, matrix and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 4447 2. $\mathbf{D}(w)$ must be compatible with $\mathbf{D}(A)$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.

- From the arguments, the internal vector, matrix, mask, and index array used in the computation are formed (\leftarrow denotes copy):
- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.

- (b) If $mask \neq GrB_NULL$,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$
- 4. The internal row index array, \tilde{I} , is computed from argument row_indices as follows:
- (a) If indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i: 0 \leq i < \text{nrows}.$
- (b) Otherwise, $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$
- The internal vector, mask, and index array are checked for dimension compatibility. The following conditions must hold:
- 4471 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4472 2. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathsf{nrows}.$
- If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.
- The col_index parameter is checked for a valid value. The following condition must hold:
- $1. 0 \leq \text{col_index} < \mathbf{ncols}(A)$
- If the rule above is violated, execution of GrB_extract ends and the invalid index error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$: The vector holding the extraction from a column of $\widetilde{\mathbf{A}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{A}), \mathsf{nrows}, \{(i, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \mathsf{col_index})) \ \forall \ i, 0 \leq i < \mathsf{nrows} : (\widetilde{\boldsymbol{I}}[i], \mathsf{col_index}) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle.$$

- At this point, if any value in \widetilde{I} is not in the range $[0, \mathbf{nrows}(\widetilde{\mathbf{A}}))$, the execution of GrB_extract ends and the index-out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result
- vector, w, is invalid from this point forward in the sequence.
- The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

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• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7 assign: Modifying sub-graphs

4519 Assign the contents of a subset of a matrix or vector.

4.3.7.1 assign: Standard vector variant

Assign values from one GraphBLAS vector to a subset of a vector as specified by a set of indices.

The size of the input vector is the same size as the index array provided.

4523 C Syntax

```
GrB_Info GrB_assign(GrB_Vector
                                                               W,
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                                     const GrB Vector
                                                              mask,
4525
                                     const GrB_BinaryOp
                                                               accum,
4526
                                     const GrB Vector
                                                               u,
4527
                                     const GrB_Index
                                                             *indices.
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                                     GrB_Index
                                                              nindices,
4529
                                     const GrB_Descriptor
                                                              desc);
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```

Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
 - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
 - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - u (IN) The GraphBLAS vector whose contents are assigned to a subset of w.
 - indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0 to nindices 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
 - nindices (IN) The number of values in indices array. Must be equal to size(u).
 - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4556				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

4558 4559 4560 4561 4562	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
4563	GrB_PANIC	Unknown internal error.
4564 4565 4566 4567	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
4568	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
4569 4570	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
4571 4572	GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to ${\bf size}(w).$ In non-blocking mode, this can be reported as an execution error.
4573	GrB_DIMENSION_MISMATCH	$mask \ \mathrm{and} \ w \ \mathrm{dimensions} \ \mathrm{are} \ \mathrm{incompatible}, \ \mathrm{or} \ nindices \neq \mathbf{size}(u).$
4574 4575 4576 4577	GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4578	GrB_NULL_POINTER	Argument indices is a NULL pointer.

Description

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This variant of GrB_assign computes the result of assigning elements from a source GraphBLAS 4580 vector to a destination GraphBLAS vector in a specific order: w(indices) = u; or, if an optional 4581 binary accumulation operator (\odot) is provided, $w(indices) = w(indices) \odot u$. More explicitly: 4582

$$\begin{aligned} & \text{w(indices}[i]) = & \text{u}(i), \ \forall \ i \ : \ 0 \leq i < \text{nindices}, \ \text{ or } \\ & \text{w(indices}[i]) = \text{w(indices}[i]) \odot \text{u}(i), \ \forall \ i \ : \ 0 \leq i < \text{nindices}. \end{aligned}$$

- Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4587 **Compute** The indicated computations are carried out.
- 4588 Output The result is written into the output vector, possibly under control of a mask.
- 4589 Up to three argument vectors are used in the GrB_assign operation:
- 4590 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 4591 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 4597 2. $\mathbf{D}(w)$ must be compatible with $\mathbf{D}(u)$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal vectors, mask and index array used in the computation are formed (← denotes copy):
- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- 4611 (b) If $mask \neq GrB_NULL$,
- i. If desc[GrB MASK].GrB STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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- 4. The internal index array, \widetilde{I} , is computed from argument indices as follows:
 - (a) If indices = GrB_ALL, then $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nindices}$.
 - (b) Otherwise, $\widetilde{I}[i] = \text{indices}[i], \ \forall \ i : 0 \le i < \text{nindices}.$

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 2. nindices = $\mathbf{size}(\widetilde{\mathbf{u}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$: The vector holding the elements from $\tilde{\mathbf{u}}$ in their destination locations relative to $\tilde{\mathbf{w}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\mathbf{u}}(i)) \forall i, 0 \leq i < \mathsf{nindices} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

At this point, if any value of $\tilde{I}[i]$ is outside the valid range of indices for vector $\tilde{\mathbf{w}}$, computation ends and the method returns the index-out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows:

• If $accum = GrB \ \ NULL$, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector $\tilde{\mathbf{z}}$ as follows: We start with the structure of $\tilde{\mathbf{w}}$ ($\mathbf{ind}(\tilde{\mathbf{w}})$) and remove from it all the indices of $\tilde{\mathbf{w}}$ that are in the set of indices being assigned ($\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{w}})$). Finally, we add the structure of $\tilde{\mathbf{t}}$ ($\mathbf{ind}(\tilde{\mathbf{t}})$).

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.2 assign: Standard matrix variant

Assign values from one GraphBLAS matrix to a subset of a matrix as specified by a set of indices.

The dimensions of the input matrix are the same size as the row and column index arrays provided.

4677 C Syntax

```
GrB_Info GrB_assign(GrB_Matrix C,

4679 const GrB_Matrix Mask,

4680 const GrB_BinaryOp accum,

4681 const GrB_Matrix A,
```

4682	const GrB_Index	*row_indices,
4683	<pre>GrB_Index</pre>	nrows,
4684	const GrB_Index	$*col_indices,$
4685	GrB_Index	ncols,
4686	${\tt const~GrB_Descriptor}$	desc);

4687 Parameters

- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - A (IN) The GraphBLAS matrix whose contents are assigned to a subset of C.
- row_indices (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C
 that are assigned. If all rows of C are to be assigned in order from 0 to nrows 1,
 then GrB_ALL can be specified. Regardless of execution mode and return value,
 this array may be manipulated by the caller after this operation returns without
 affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location
 which leads to undefined results.
 - nrows (IN) The number of values in the row_indices array. Must be equal to **nrows**(A) if A is not transposed, or equal to **ncols**(A) if A is transposed.
 - col_indices (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C that are assigned. If all columns of C are to be assigned in order from 0 to ncols 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.
 - ncols (IN) The number of values in col_indices array. Must be equal to $\mathbf{ncols}(A)$ if A is not transposed, or equal to $\mathbf{nrows}(A)$ if A is transposed.

9	desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL
)	should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
-	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
4722				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

4723 Return Values

4724	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
4725		blocking mode, this indicates that the compatibility tests on
4726		dimensions and domains for the input arguments passed suc-
4727		cessfully. Either way, output matrix C is ready to be used in the
4728		next method of the sequence.
4729	GrB_PANIC	Unknown internal error.
4730	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
4731		opaque GraphBLAS objects (input or output) is in an invalid
4732		state caused by a previous execution error. Call GrB_error() to
4733		access any error messages generated by the implementation.
4734	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
4735	GrB UNINITIALIZED OBJECT	One or more of the GraphBLAS objects has not been initialized
4736		by a call to new (or Matrix_dup for matrix parameters).
4737	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(C)$, or
4738		a value in col_indices is greater than or equal to ncols(C). In
4739		non-blocking mode, this can be reported as an execution error.
4740	GrB DIMENSION MISMATCH	Mask and C dimensions are incompatible, nrows \neq nrows(A),
4741	615_511112116141_1111611#	or $ncols \neq ncols(A)$.
4742	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with each
4743		other or the corresponding domains of the accumulation oper-
4744		ator, or the mask's domain is not compatible with bool (in the
4745		case where desc[GrB_MASK].GrB_STRUCTURE is not set).
4746	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices
4747		is a NULL pointer, or both.

4748 Description

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This variant of GrB_assign computes the result of assigning the contents of A to a subset of rows and columns in C in a specified order: $C(row_indices, col_indices) = A$; or, if an optional binary accumulation operator (\odot) is provided, $C(row_indices, col_indices) = C(row_indices, col_indices) \odot$ A. More explicitly (not accounting for an optional transpose of A):

C(row_indices[
$$i$$
], col_indices[j]) = A(i , j), \forall i , j : $0 \le i < \text{nrows}$, $0 \le j < \text{ncols}$, or C(row_indices[i], col_indices[j]) = C(row_indices[i], col_indices[j]) \odot A(i , j), \forall (i , j) : $0 \le i < \text{nrows}$, $0 \le j < \text{ncols}$

Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 4757 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 4759 Up to three argument matrices are used in the GrB assign operation:
- 4760 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
 - 2. $Mask = \langle \mathbf{D}(Mask), \mathbf{nrows}(Mask), \mathbf{ncols}(Mask), \mathbf{L}(Mask) = \{(i, j, M_{ij})\} \rangle$ (optional)
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:
 - 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(Mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.
- From the arguments, the internal matrices, mask, and index arrays used in the computation are formed (\leftarrow denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 2. Two-dimensional mask M is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
- (b) If Mask \neq GrB_NULL,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
 - (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}$
- 4. The internal row index array, \tilde{I} , is computed from argument row_indices as follows:
- (a) If row_indices = GrB_ALL, then $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$.
- (b) Otherwise, $\widetilde{I}[i] = \text{row_indices}[i], \forall i : 0 \leq i < \text{nrows}.$
- 5. The internal column index array, \tilde{J} , is computed from argument col_indices as follows:
- 4793 (a) If col_indices = GrB_ALL, then $\widetilde{m{J}}[j] = j, \forall j: 0 \leq j < ext{ncols}.$
- (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \text{col_indices}[j], \ \forall \ j: 0 \leq j < \text{ncols.}$
- The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:
- 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 4798 2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}).$
- 3. $\mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathsf{nrows}.$
- 4800 4. $\mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathsf{ncols}.$
- If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{T}}$: The matrix holding the contents from $\widetilde{\mathbf{A}}$ in their destination locations relative to $\widetilde{\mathbf{C}}$.
 - ullet $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \widetilde{\mathbf{A}}(i,j)) \ \forall \ (i,j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle. \end{split}$$

At this point, if any value in the \widetilde{I} array is not in the range $[0, \mathbf{nrows}(\widetilde{\mathbf{C}}))$ or any value in the \widetilde{J} array is not in the range $[0, \mathbf{ncols}(\widetilde{\mathbf{C}}))$, the execution of $\mathsf{GrB_assign}$ ends and the index out-of-bounds error listed above is generated. In $\mathsf{GrB_NONBLOCKING}$ mode, the error can be deferred until a sequence-terminating $\mathsf{GrB_wait}()$ is called. Regardless, the result matrix C is invalid from this point forward in the sequence.

The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows:

• If $accum = GrB_NULL$, then $\widetilde{\mathbf{Z}}$ is defined as

$$egin{array}{ll} \widetilde{\mathbf{Z}} &= \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i,j,Z_{ij}) orall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{m{I}}[k], \widetilde{m{J}}[l]), orall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \}
angle. \end{array}$$

The above expression defines the structure of matrix $\widetilde{\mathbf{Z}}$ as follows: We start with the structure of $\widetilde{\mathbf{C}}$ ($\mathbf{ind}(\widetilde{\mathbf{C}})$) and remove from it all the indices of $\widetilde{\mathbf{C}}$ that are in the set of indices being assigned ($\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$). Finally, we add the structure of $\widetilde{\mathbf{T}}$ ($\mathbf{ind}(\widetilde{\mathbf{T}})$).

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C , and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4854 4.3.7.3 assign: Column variant

Assign the contents a vector to a subset of elements in one column of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a row of a matrix.

4858 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
4859
                                     const GrB Vector
                                                               mask,
4860
                                     const GrB BinaryOp
                                                               accum,
4861
                                     const GrB Vector
                                                               u,
4862
                                     const GrB_Index
                                                              *row_indices,
4863
                                     GrB Index
                                                               nrows,
4864
                                     GrB Index
                                                               col_index,
4865
                                     const GrB Descriptor
                                                               desc);
4866
```

Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified column of the output matrix C. The mask dimensions must match those of a single column of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type

4875 4876 4877	bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a column of C), GrB_NULL should be specified.
4878 4879	accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be
4880	specified.

u (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a column of C.

row_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified column of C that are to be assigned. If all elements of the column in C are to be assigned in order from index 0 to nrows — 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.

nrows (IN) The number of values in row_indices array. Must be equal to size(u).

col_index (IN) The index of the column in C to assign. Must be in the range [0, ncols(C)).

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
С	GrB_OUTP	GrB_REPLACE	Output column in C is cleared (all ele-
			ments removed) before result is stored in
			it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input mask vector. The stored values are
			not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

```
GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the
4904
                                     opaque GraphBLAS objects (input or output) is in an invalid
4905
                                     state caused by a previous execution error. Call GrB_error() to
4906
                                     access any error messages generated by the implementation.
4907
           GrB_OUT_OF_MEMORY Not enough memory available for operation.
4908
      Grb_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized
4909
                                     by a call to new (or dup for vector or matrix parameters).
               GrB INVALID INDEX col index is outside the allowable range (i.e., greater than ncols(C)).
4911
    GrB_INDEX_OUT_OF_BOUNDS A value in row_indices is greater than or equal to nrows(C). In
4912
                                     non-blocking mode, this can be reported as an execution error.
4913
      GrB_DIMENSION_MISMATCH mask size and number of rows in C are not the same, or nrows \neq
4914
                                     size(u).
4915
         Grb DOMAIN MISMATCH The domains of the matrix and vector are incompatible with
4916
                                     each other or the corresponding domains of the accumulation
4917
                                     operator, or the mask's domain is not compatible with bool (in
4918
                                     the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
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```

4921 Description

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```
This variant of GrB_assign computes the result of assigning a subset of locations in a column of a GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector:

C(:, col_index) = u; or, if an optional binary accumulation operator (\odot) is provided, C(:, col_index) = C(:, col_index) \odot u. Taking order of row_indices into account, it is more explicitly written as:
```

C(row_indices[i], col_index) = u(i),
$$\forall i : 0 \le i < \text{nrows}$$
, or C(row_indices[i], col_index) = C(row_indices[i], col_index) \odot u(i), $\forall i : 0 \le i < \text{nrows}$.

GrB_NULL_POINTER Argument row_indices is a NULL pointer.

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

4930 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

4932 Up to three argument vectors and matrices are used in this GrB_assign operation:

```
4933 1. C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle
```

4934 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 4940 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(C)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

The col_index parameter is checked for a valid value. The following condition must hold:

```
1. 0 \leq \text{col\_index} < \mathbf{ncols}(C)
```

If the rule above is violated, execution of GrB_assign ends and the invalid index error listed above is returned.

- From the arguments, the internal vectors, mask, and index array used in the computation are formed (\leftarrow denotes copy):
- 1. The vector, $\tilde{\mathbf{c}}$, is extracted from a column of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \{(i, C_{ij}) \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}), j = \mathsf{col_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
 - (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{nrows}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
 - i. If $desc[GrB_MASK].GrB_STRUCTURE$ is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool}) \mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 4963 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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- 4. The internal row index array, \tilde{I} , is computed from argument row_indices as follows:
 - (a) If row_indices = GrB_ALL, then $\tilde{I}[i] = i, \ \forall \ i : 0 \le i < \text{nrows}$.

(b) Otherwise, $\tilde{I}[i] = \text{row_indices}[i], \ \forall \ i : 0 \le i < \text{nrows}.$

The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:

- 4969 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 4970 2. nrows = $\mathbf{size}(\widetilde{\mathbf{u}})$.

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If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$: The vector holding the elements from $\tilde{\mathbf{u}}$ in their destination locations relative to $\tilde{\mathbf{c}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(\widetilde{I}[i], \widetilde{\mathbf{u}}(i)) \ \forall \ i, \ 0 \le i < \mathsf{nrows} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle.$$

At this point, if any value of $\tilde{I}[i]$ is outside the valid range of indices for vector $\tilde{\mathbf{c}}$, computation ends and the method returns the index out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows:

• If $accum = GrB_NULL$, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector $\tilde{\mathbf{z}}$ as follows: We start with the structure of $\tilde{\mathbf{c}}$ ($\mathbf{ind}(\tilde{\mathbf{c}})$) and remove from it all the indices of $\tilde{\mathbf{c}}$ that are in the set of indices being assigned ($\{\tilde{I}[k], \forall k\} \cap \mathbf{ind}(\tilde{\mathbf{c}})$). Finally, we add the structure of $\tilde{\mathbf{t}}$ ($\mathbf{ind}(\tilde{\mathbf{t}})$).

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

where the difference operator refers to set difference.

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$
5002
$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$
5004
$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where $\odot = \bigcirc(\text{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up the $\tilde{\mathbf{z}}$ vector are written into the column of the final result matrix, $C(:, col_index)$. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C(:,col_index) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : j \neq \mathsf{col_index}\} \cup \{(i, \mathsf{col_index}, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the column of the final result matrix, $C(:,col_index)$, and elements of this column that fall outside the set indicated by the mask are unchanged:

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$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}): j \neq \mathsf{col_index}\} \cup \\ \{(i,\mathsf{col_index},\widetilde{\mathbf{c}}(i)): i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ \{(i,\mathsf{col_index},z_i): i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.4 assign: Row variant

Assign the contents a vector to a subset of elements in one row of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a column of a matrix.

5027 C Syntax

```
5028
             GrB_Info GrB_assign(GrB_Matrix
                                                              С,
                                     const GrB_Vector
                                                              mask,
5029
                                     const GrB_BinaryOp
                                                              accum.
5030
                                     const GrB_Vector
                                                              u,
5031
                                     GrB_Index
                                                              row_index,
5032
                                     const GrB_Index
                                                             *col_indices,
5033
                                                              ncols,
                                     GrB Index
5034
                                     const GrB_Descriptor
                                                              desc);
5035
```

5036 Parameters

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- C (INOUT) An existing GraphBLAS Matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified row of the output matrix C. The mask dimensions must match those of a single row of the matrix C. If the GrB_STRUCTURE descriptor is not set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a row of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - u (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a row of C.
- row_index (IN) The index of the row in C to assign. Must be in the range [0, nrows(C)).
- col_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified row of C that are to be assigned. If all elements of the row in C are to be assigned in order from index 0 to ncols 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
 - ncols (IN) The number of values in col_indices array. Must be equal to size(u).
 - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output row in C is cleared (all elements
				removed) before result is stored in it.
5054	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
5064				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

5066 5067 5068 5069 5070	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
5071	GrB_PANIC	Unknown internal error.
5072 5073 5074 5075	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5076	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
5077 5078	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
5079	GrB_INVALID_INDEX	${\sf row_index} \ {\rm is} \ {\rm outside} \ {\rm the} \ {\rm allowable} \ {\rm range} \ ({\rm i.e., greater} \ {\rm than} \ {\bf nrows}(C)).$
5080 5081	GrB_INDEX_OUT_OF_BOUNDS	A value in $col_indices$ is greater than or equal to $ncols(C)$. In non-blocking mode, this can be reported as an execution error.
5082 5083	GrB_DIMENSION_MISMATCH	mask size and number of columns in C are not the same, or $n\text{cols} \neq \mathbf{size}(u).$
5084 5085 5086 5087	GrB_DOMAIN_MISMATCH	The domains of the matrix and vector are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5088	GrB_NULL_POINTER	Argument col_indices is a NULL pointer.

Description 5089

This variant of GrB_assign computes the result of assigning a subset of locations in a row of a 5090 GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector: 5091

 $C(row_index,:) = u;$ or, if an optional binary accumulation operator (\odot) is provided, $C(row_index,:) = C(row_index,:) \odot u$. Taking order of col_indices into account it is more explicitly written as:

5095 Logically, this operation occurs in three steps:

- Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 5098 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- 5100 Up to three argument vectors and matrices are used in this GrB_assign operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 5102 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5108 2. $\mathbf{D}(C)$ must be compatible with $\mathbf{D}(u)$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(C)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.
- 5117 The row_index parameter is checked for a valid value. The following condition must hold:
- 1. $0 \le \text{row_index} < \mathbf{nrows}(C)$
- If the rule above is violated, execution of GrB_assign ends and the invalid index error listed above is returned.
- From the arguments, the internal vectors, mask, and index array used in the computation are formed (\leftarrow denotes copy):

1. The vector, $\tilde{\mathbf{c}}$, is extracted from a row of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(j, C_{ij}) \ \forall \ j : 0 \le j < \mathbf{ncols}(\mathsf{C}), i = \mathsf{row_index}, (i, j) \in \mathbf{ind}(\mathsf{C}) \} \rangle$$

- 5125 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- 5126 (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{ncols}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- $\text{ (c) If desc[GrB_MASK].GrB_COMP is set, then } \widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}.$
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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- 5132 4. The internal column index array, \widetilde{J} , is computed from argument col_indices as follows:
- (a) If col_indices = GrB_ALL, then $\widetilde{J}[j] = j, \ \forall \ j : 0 \le j < \text{ncols.}$
- 5134 (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \text{col_indices}[j], \ \forall \ j: 0 \leq j < \text{ncols.}$
- The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{c}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 5138 2. $\operatorname{ncols} = \operatorname{size}(\widetilde{\mathbf{u}}).$
- 5139 If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mis-5140 match error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$: The vector holding the elements from $\tilde{\mathbf{u}}$ in their destination locations relative to $\tilde{\mathbf{c}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

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$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{ (\widetilde{J}[j], \widetilde{\mathbf{u}}(j)) \ \forall \ j, \ 0 \le j < \mathsf{ncols} : j \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle.$$

- At this point, if any value of $\widetilde{J}[j]$ is outside the valid range of indices for vector $\widetilde{\mathbf{c}}$, computation ends and the method returns the index out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.
- The intermediate vector $\tilde{\mathbf{z}}$ is created as follows:

• If $accum = GrB_NULL$, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{c}}$ ($\mathbf{ind}(\widetilde{\mathbf{c}})$) and remove from it all the indices of $\widetilde{\mathbf{c}}$ that are in the set of indices being assigned ($\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$

where the difference operator refers to set difference.

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(j, z_i) \ \forall \ j \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_j = \widetilde{\mathbf{c}}(j) \odot \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$

$$z_j = \widetilde{\mathbf{c}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_j = \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up the $\tilde{\mathbf{z}}$ vector are written into the column of the final result matrix, $C(row_index,:)$. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C(row_index,:) on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : i \neq \mathsf{row_index}\} \cup \{(\mathsf{row_index},j,z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the column of the final result matrix, $C(row_index,:)$, and elements of this column that fall outside the set indicated by the mask are unchanged:

$$\begin{split} \mathbf{L}(\mathsf{C}) &= & \{(i,j,C_{ij}): i \neq \mathsf{row_index}\} \cup \\ & \{(\mathsf{row_index},j,\widetilde{\mathbf{c}}(j)): j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \\ & \{(\mathsf{row_index},j,z_j): j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}. \end{split}$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

5191 4.3.7.5 assign: Constant vector variant[Scott: NEW CONTENT]

Assign the same value to a specified subset of vector elements. With the use of GrB_ALL, the entire destination vector can be filled with the constant.

5194 C Syntax

```
GrB_Info GrB_assign(GrB_Vector
                                                               w,
5195
                                     const GrB_Vector
5196
                                                               mask,
                                     const GrB BinaryOp
                                                               accum,
5197
                                     <type>
                                                               val,
5198
                                     const GrB_Index
                                                              *indices.
5199
                                     GrB_Index
                                                               nindices,
5200
                                     const GrB_Descriptor
                                                               desc);
5201
              GrB_Info GrB_assign(GrB_Vector
                                                               W,
5202
                                     const GrB_Vector
                                                               mask,
5203
                                     const GrB_BinaryOp
                                                               accum,
5204
                                     const GrB_Scalar
                                                               s,
5205
                                                              *indices,
                                     const GrB_Index
5206
                                     GrB Index
                                                               nindices,
5207
                                     const GrB_Descriptor
                                                               desc);
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```

Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.
 - mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
 - accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - val (IN) Scalar value to assign to (a subset of) w.
- s (IN) Scalar value to assign to (a subset of) w.
 - indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0

to nindices – 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. In this variant, the specific order of the values in the array has no effect on the result. Unlike other variants, if there are duplicated values in this array the result is still defined.

nindices (IN) The number of values in indices array. Must be in the range: [0, size(w)]. If nindices is zero, the operation becomes a NO-OP.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
5237				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

5239	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
5240		blocking mode, this indicates that the compatibility tests on
5241		dimensions and domains for the input arguments passed suc-
5242		cessfully. Either way, output vector w is ready to be used in the
5243		next method of the sequence.
5244	GrB_PANIC	Unknown internal error.
5245	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
5246		opaque GraphBLAS objects (input or output) is in an invalid
5247		state caused by a previous execution error. Call GrB_error() to
5248		access any error messages generated by the implementation.
5249	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
5250	GrB UNINITIALIZED OBJECT	One or more of the GraphBLAS objects has not been initialized
5251		by a call to new (or dup for vector parameters).
		(
5252	GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to $\mathbf{size}(w)$. In non-
5253		blocking mode, this can be reported as an execution error.
5254 5255	GrB_DIMENSION_MISMATCH	mask and w dimensions are incompatible, or nindices is not less than $\mathbf{size}(w)$.

GrB_DOMAIN_MISMATCH The domains of the vector and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB NULL POINTER Argument indices is a NULL pointer.

Description

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This variant of GrB_assign computes the result of assigning a constant scalar value – either val or s - to locations in a destination GraphBLAS vector. Either w(indices) = val or w(indices) = s is performed. If an optional binary accumulation operator (\odot) is provided, then either $w(indices) = w(indices) \odot val$ or $w(indices) = w(indices) \odot s$ is performed. More explicitly, if a non-opaque value val is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}, \ \text{ or } \\ \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{val}, \ \forall \ i : 0 \le i < \mathsf{nindices}.$$

5268 Correspondingly, if a GrB_Scalar s is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{s}, \ \forall \ i : 0 \le i < \mathsf{nindices}, \ \text{ or } \\ \mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{s}, \ \forall \ i : 0 \le i < \mathsf{nindices}.$$

5270 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5273 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

5275 Up to two argument vectors are used in the GrB_assign operation:

```
1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

5277 2.
$$\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$$

The argument scalar, vectors, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(w)$ must be compatible with either $\mathbf{D}(val)$ or $\mathbf{D}(s)$, depending on the signature of the method.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(w)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator.

4. If accum is not GrB_NULL, then either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal vectors, mask and index array used in the computation are formed (← denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.

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- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If mask \neq GrB NULL,
 - i. If desc[GrB MASK].GrB STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Scalar $\tilde{s} \leftarrow s$ (GrB Scalar version only).
- 5303 4. The internal index array, \tilde{I} , is computed from argument indices as follows:
- (a) If indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i: 0 \leq i < \text{nindices}$.
- 5305 (b) Otherwise, $\tilde{I}[i] = \mathsf{indices}[i], \ \forall \ i: 0 \leq i < \mathsf{nindices}.$

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- $2. 0 < \text{nindices} < \text{size}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

• $\tilde{\mathbf{t}}$: The vector holding the copies of the scalar, either val or \tilde{s} , in their destination locations relative to $\tilde{\mathbf{w}}$.

• $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows. If a non-opaque scalar val is provided:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{val}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{I}[i], \mathsf{val}) \ \forall \ i, \ 0 \leq i < \mathsf{nindices}\} \rangle.$$

Correspondingly, if a non-empty GrB_Scalar \tilde{s} is provided (i.e., $\mathbf{size}(\tilde{s}) = 1$):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{I}[i], \mathbf{val}(\widetilde{s})) \ \forall \ i, \ 0 \le i < \mathsf{nindices}\} \rangle.$$

Finally, if an empty GrB_Scalar \tilde{s} is provided (i.e., $\mathbf{size}(\tilde{s}) = 0$):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \emptyset \rangle.$$

If \tilde{I} is empty, this operation results in an empty vector, $\tilde{\mathbf{t}}$. Otherwise, if any value in the \tilde{I} array is not in the range $[0, \mathbf{size}(\tilde{\mathbf{w}}))$, the execution of GrB_assign ends and the index out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, \mathbf{w} , is invalid from this point forward in the sequence.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows:

• If $accum = GrB \ \ NULL$, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{w}}$ ($\mathbf{ind}(\widetilde{\mathbf{w}})$) and remove from it all the indices of $\widetilde{\mathbf{w}}$ that are in the set of indices being assigned ($\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{I}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$

where the difference operator refers to set difference. We note that in this case of assigning a constant, $\{\tilde{I}[k], \forall k\}$ and $\mathbf{ind}(\tilde{\mathbf{t}})$ are identical.

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc(\text{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

5368 4.3.7.6 assign: Constant matrix variant[Scott: NEW CONTENT]

Assign the same value to a specified subset of matrix elements. With the use of GrB_ALL, the entire destination matrix can be filled with the constant.

5371 C Syntax

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```
GrB_Info GrB_assign(GrB_Matrix
                                                               С,
5372
                                     const GrB_Matrix
                                                               Mask,
5373
                                                               accum,
                                     const GrB BinaryOp
5374
                                     <type>
                                                               val,
5375
                                     const GrB_Index
                                                              *row_indices,
5376
                                     GrB_Index
                                                               nrows,
5377
                                     const GrB_Index
                                                              *col_indices,
5378
                                     GrB_Index
                                                               ncols,
5379
                                     const GrB_Descriptor
                                                               desc);
5380
                                                               С,
             GrB_Info GrB_assign(GrB_Matrix
5381
                                     const GrB_Matrix
                                                               Mask,
5382
                                     const GrB_BinaryOp
                                                               accum,
5383
                                     const GrB_Scalar
5384
                                                               s,
                                     const GrB_Index
                                                              *row_indices,
5385
                                     GrB_Index
                                                               nrows,
5386
```

5387 5388 5389		<pre>const GrB_Index *col_indices, GrB_Index ncols, const GrB_Descriptor desc);</pre>
5390	Parameters	
5391	С	(INOUT) An existing GraphBLAS matrix. On input, the matrix provides values
5392		that may be accumulated with the result of the assign operation. On output, the
5393		matrix holds the results of the operation.
5394	Mask	(IN) An optional "write" mask that controls which results from this operation are
5395		stored into the output matrix C. The mask dimensions must match those of the
5396		matrix C. If the GrB_STRUCTURE descriptor is <i>not</i> set for the mask, the domain
5397		of the Mask matrix must be of type bool or any of the predefined "built-in" types
5398		in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the
5399		dimensions of C), GrB_NULL should be specified.
5400	accum	(IN) An optional binary operator used for accumulating entries into existing C
5401		entries. If assignment rather than accumulation is desired, GrB_NULL should be
5402		specified.
5403	val	(IN) Scalar value to assign to (a subset of) $C.$
5404	S	(IN) Scalar value to assign to (a subset of) $C.$
5405	row_indices	(IN) Pointer to the ordered set (array) of indices corresponding to the rows of C
5406		that are assigned. If all rows of ${\sf C}$ are to be assigned in order from 0 to ${\sf nrows}-1,$
5407		then GrB_ALL can be specified. Regardless of execution mode and return value,
5408		this array may be manipulated by the caller after this operation returns without
5409		affecting any deferred computations for this operation. Unlike other variants, if
5410		there are duplicated values in this array the result is still defined.
5411	nrows	(IN) The number of values in $row_indices$ array. Must be in the range: $[0, \mathbf{nrows}(C)]$.
5412		If nrows is zero, the operation becomes a NO-OP.
5413	col indices	(IN) Pointer to the ordered set (array) of indices corresponding to the columns of C
5414		that are assigned. If all columns of C are to be assigned in order from 0 to ncols – 1,
5415		then GrB_ALL should be specified. Regardless of execution mode and return value,
5416		this array may be manipulated by the caller after this operation returns without
5417		affecting any deferred computations for this operation. Unlike other variants, if
5418		there are duplicated values in this array the result is still defined.
5419	ncols	(IN) The number of values in col_indices array. Must be in the range: $[0, \mathbf{ncols}(C)]$.
5420		If ncols is zero, the operation becomes a NO-OP.
5421	desc	(IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL

 should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
5424				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.

5425 Return Values

5426 5427 5428 5429 5430	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
5431	GrB_PANIC	Unknown internal error.
5432543354345435	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5436	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
5437 5438	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
5439 5440 5441	GrB_INDEX_OUT_OF_BOUNDS	A value in $row_indices$ is greater than or equal to $nrows(C)$, or a value in $col_indices$ is greater than or equal to $ncols(C)$. In non-blocking mode, this can be reported as an execution error.
5442 5443	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows is not less than $\mathbf{nrows}(C)$, or ncols is not less than $\mathbf{ncols}(C)$.
5444 5445 5446 5447	GrB_DOMAIN_MISMATCH	The domains of the matrix and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5448 5449	GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

Description

This variant of GrB_assign computes the result of assigning a constant scalar value – either val or s - to locations in a destination GraphBLAS matrix: Either $C(row_indices, col_indices) = val$

or $C(row_indices, col_indices) = s$ is performed. If an optional binary accumulation operator (\odot) is provided, then either $C(row_indices, col_indices) = C(row_indices, col_indices)$ or $C(row_indices, col_indices) = C(row_indices, col_indices)$ or $C(row_indices, col_indices) = C(row_indices, col_indices)$ or $C(row_indices, col_indices)$ or $C(row_indices$

5458 Correspondingly, if a GrB Scalar s is provided:

5460 Logically, this operation occurs in three steps:

- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 5463 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.

5465 Up to two argument matrices are used in the GrB_assign operation:

- 5466 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 5467 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$

The argument scalar, matrices, and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 5472 2. $\mathbf{D}(C)$ must be compatible with either $\mathbf{D}(val)$ or $\mathbf{D}(val)$, depending on the signature of the method.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator.
- 4. If accum is not GrB_NULL, then either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, index arrays, and mask used in the computation are formed (\leftarrow denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 5486 2. Two-dimensional mask $\widetilde{\mathbf{M}}$ is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
 - (b) If Mask \neq GrB_NULL,
 - i. If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_STRUCTURE}$ is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
 - (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
- 3. Scalar $\tilde{s} \leftarrow s$ (GrB_Scalar version only).
- 4. The internal row index array, \tilde{I} , is computed from argument row_indices as follows:
- (a) If row_indices = GrB_ALL, then $\widetilde{I}[i] = i, \forall i : 0 \leq i < \text{nrows}$.
- $\text{ (b) Otherwise, } \widetilde{\boldsymbol{I}}[i] = \mathsf{row_indices}[i], \forall i: 0 \leq i < \mathsf{nrows}.$
- 5. The internal column index array, \widetilde{J} , is computed from argument col_indices as follows:
- 5500 (a) If col_indices = GrB_ALL, then $\widetilde{\boldsymbol{J}}[j] = j, \forall j: 0 \leq j < \text{ncols.}$
- 5501 (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \mathsf{col_indices}[j], \forall j: 0 \leq j < \mathsf{ncols}.$
- The internal matrix and mask are checked for dimension compatibility. The following conditions must hold:
- 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 5505 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- $0 \le \text{nrows}(\widetilde{\mathbf{C}}).$
- 4. $0 \le \operatorname{ncols} \le \operatorname{ncols}(\widetilde{\mathbf{C}}).$
- If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the copies of the scalar, either val or \widetilde{s} , in their destination locations relative to $\widetilde{\mathbf{C}}$.
- $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\tilde{\mathbf{T}}$, is created as follows. If a non-opaque scalar val is provided:

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\mathsf{val}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathsf{val}) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Correspondingly, if a non-empty GrB_Scalar \tilde{s} is provided (i.e., $\mathbf{size}(\tilde{s}) = 1$):

$$\begin{split} \widetilde{\mathbf{T}} &= \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathbf{val}(\widetilde{s})) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} \} \rangle. \end{split}$$

Finally, if an empty GrB_Scalar \tilde{s} is provided (i.e., $\mathbf{size}(\tilde{s}) = 0$):

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\widetilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \emptyset \rangle.$$

If either \tilde{I} or \tilde{J} is empty, this operation results in an empty matrix, $\tilde{\mathbf{T}}$. Otherwise, if any value in the \tilde{I} array is not in the range $[0, \mathbf{nrows}(\tilde{\mathbf{C}}))$ or any value in the \tilde{J} array is not in the range $[0, \mathbf{ncols}(\tilde{\mathbf{C}}))$, the execution of $\mathsf{GrB_assign}$ ends and the index out-of-bounds error listed above is generated. In $\mathsf{GrB_NONBLOCKING}$ mode, the error can be deferred until a sequence-terminating $\mathsf{GrB_wait}()$ is called. Regardless, the result matrix C is invalid from this point forward in the sequence.

The intermediate matrix $\hat{\mathbf{Z}}$ is created as follows:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}$, then $\widetilde{\mathbf{Z}}$ is defined as

$$\begin{split} \widetilde{\mathbf{Z}} &= \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ &\{ (i, j, Z_{ij}) \forall (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle. \end{split}$$

The above expression defines the structure of matrix $\widetilde{\mathbf{Z}}$ as follows: We start with the structure of $\widetilde{\mathbf{C}}$ ($\mathbf{ind}(\widetilde{\mathbf{C}})$) and remove from it all the indices of $\widetilde{\mathbf{C}}$ that are in the set of indices being assigned ($\{(\widetilde{I}[k], \widetilde{J}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$). Finally, we add the structure of $\widetilde{\mathbf{T}}$ ($\mathbf{ind}(\widetilde{\mathbf{T}})$).

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k, l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

where the difference operator refers to set difference. We note that, in this particular case of assigning a constant to a matrix, the sets $\{(\tilde{I}[k], \tilde{J}[l]), \forall k, l\}$ and $\mathbf{ind}(\tilde{\mathbf{T}})$ are identical.

• If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

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$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\dot{\mathbf{Z}}$ are written into the final result matrix C , using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8 apply: Apply a function to the elements of an object

Computes the transformation of the values of the elements of a vector or a matrix using a unary function, or a binary function where one argument is bound to a scalar.

4.3.8.1 apply: Vector variant

5572 Computes the transformation of the values of the elements of a vector using a unary function.

5573 C Syntax

5574	<pre>GrB_Info GrB_apply(GrB_Vector</pre>	W,
5575	const GrB_Vector	mask,
5576	const GrB_BinaryOp	accum,
5577	const GrB_UnaryOp	op,
5578	const GrB_Vector	u,
5579	const GrB Descriptor	desc);

5580 Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) A unary operator applied to each element of input vector u.
 - u (IN) The GraphBLAS vector to which the unary function is applied.
 - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
598	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque
GraphBLAS objects (input or output) is in an invalid state caused
by a previous execution error. Call GrB_error() to access any error
messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

5613 GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or unary function, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

5618 Description

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This variant of GrB_apply computes the result of applying a unary function to the elements of a GraphBLAS vector: $\mathbf{w} = f(\mathbf{u})$; or, if an optional binary accumulation operator (\odot) is provided, $\mathbf{w} = \mathbf{w} \odot f(\mathbf{u})$.

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5625 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

5627 Up to three argument vectors are used in this GrB_apply operation:

- 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5629 2. mask = $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5635 2. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\mathbf{op})$ of the unary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 5639 4. $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in}(\mathsf{op})$.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathsf{w}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 5658 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\tilde{\mathbf{t}}$: The vector holding the result from applying the unary operator to the input vector $\tilde{\mathbf{u}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

where f = f(op).

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
- If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ orall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \}
angle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

5697 4.3.8.2 apply: Matrix variant

5698 Computes the transformation of the values of the elements of a matrix using a unary function.

5699 C Syntax

```
GrB_Info GrB_apply(GrB_Matrix
                                                             С,
5700
                                    const GrB_Matrix
                                                             Mask,
5701
                                    const GrB_BinaryOp
                                                             accum,
5702
                                    const GrB_UnaryOp
5703
                                                             op,
                                    const GrB_Matrix
                                                             Α,
5704
                                    const GrB_Descriptor
                                                             desc);
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```

5706 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) A unary operator applied to each element of input matrix A.
 - A (IN) The GraphBLAS matrix to which the unary function is applied.
 - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
5724				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

25 Return Values

5726	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-
5727		blocking mode, this indicates that the compatibility tests on
5728		dimensions and domains for the input arguments passed suc-
5729		cessfully. Either way, output matrix C is ready to be used in the
5730		next method of the sequence.
5731	GrB_PANIC	Unknown internal error.
5732	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the
5733		opaque GraphBLAS objects (input or output) is in an invalid
5734		state caused by a previous execution error. Call GrB_error() to
5735		access any error messages generated by the implementation.
5736	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
5737	GrB UNINITIALIZED OBJECT	One or more of the GraphBLAS objects has not been initialized
5738		by a call to new (or Matrix_dup for matrix parameters).
5739	GrB DIMENSION MISMATCH	Mask and C dimensions are incompatible, nrows \neq nrows (C), or
5740		$ncols \neq ncols(C)$.
5741	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the
5742		corresponding domains of the accumulation operator or unary
		· ·

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function, or the mask's domain is not compatible with bool (in

the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

5745 Description

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This variant of GrB_apply computes the result of applying a unary function to the elements of a GraphBLAS matrix: C = f(A); or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot f(A)$.

5749 Logically, this operation occurs in three steps:

5750 **Setup** The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5752 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_apply operation:

```
5755 1. C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle
```

2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)

```
5757 3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument matrices, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 5762 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 5766 4. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in}(\mathsf{op})$ of the unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed (\leftarrow denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 2. Two-dimensional mask, M, is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \le i < \mathbf{nrows}(\mathsf{C}), 0 \le j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
 - (b) If Mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

4.
$$\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{A}}).$$

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the unary operator to the input matrix $\widetilde{\mathbf{A}}$.
- \bullet $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\tilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

where f = f(op).

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The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a standard matrix accumulate:

- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \ \operatorname{\mathsf{NULL}}, \operatorname{then} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$
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$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
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$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C , and elements of C that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.
```

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

5829 4.3.8.3 apply: Vector-BinaryOp variants[Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a vector using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the vector are passed as the second argument. In the *bind-second* variant, the elements of the vector are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

5836 C Syntax

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```
// bind-first + scalar value
5837
             GrB_Info GrB_apply(GrB_Vector
5838
                                                              W,
                                    const GrB Vector
                                                              mask,
5839
                                    const GrB_BinaryOp
                                                              accum,
5840
                                    const GrB_BinaryOp
5841
                                                              op,
                                    <type>
                                                              val,
5842
                                    const GrB_Vector
5843
                                                              u,
                                    const GrB_Descriptor
                                                              desc);
5844
             // bind-first + GraphBLAS scalar
5845
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5846
                                    const GrB Vector
                                                              mask,
5847
                                    const GrB_BinaryOp
                                                              accum,
5848
                                    const GrB_BinaryOp
5849
                                                              op,
                                    const GrB_Scalar
                                                              s,
5850
                                    const GrB_Vector
5851
                                                              u,
                                    const GrB_Descriptor
5852
                                                              desc);
             // bind-second + scalar value
5853
             GrB_Info GrB_apply(GrB_Vector
                                                              w,
5854
                                    const GrB_Vector
                                                              mask,
5855
```

```
const GrB_BinaryOp
                                                               accum,
5856
                                    const GrB_BinaryOp
5857
                                                               op,
                                    const GrB_Vector
5858
                                                               u,
                                    <type>
                                                               val,
5859
                                    const GrB Descriptor
                                                               desc);
5860
             // bind-second + GraphBLAS scalar
5861
             GrB_Info GrB_apply(GrB_Vector
5862
                                                               W,
                                    const GrB Vector
                                                               mask,
5863
                                    const GrB_BinaryOp
                                                               accum,
5864
                                    const GrB_BinaryOp
5865
                                                               op,
                                    const GrB_Vector
                                                               u,
5866
                                    const GrB_Scalar
5867
                                                               s,
                                    const GrB_Descriptor
                                                               desc);
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```

Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) A binary operator applied to each element of input vector, u, and the scalar value, val.
 - u (IN) The GraphBLAS vector whose elements are passed to the binary operator as the right-hand (second) argument in the *bind-first* variant, or the left-hand (first) argument in the *bind-second* variant.
 - val (IN) Scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant.
 - s (IN) A GraphBLAS scalar that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant. It must not be empty.

desc (IN)	An optional operation descri	ptor. If a default descrip	ptor is desired, GrB_NULL
shoul	d be specified. Non-default	field/value pairs are lis	sted as follows:

5895				
	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
5896	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	$GrB _MASK$	GrB_COMP	Use the complement of mask.

Return Values

5898 5899 5900 5901 5902	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
5903	GrB_PANIC	Unknown internal error.
5904 5905 5906 5907	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
5908	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
5909 5910	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
5911	GrB_DIMENSION_MISMATCH	mask,w and/or u dimensions are incompatible.
5912 5913 5914 5915	GrB_DOMAIN_MISMATCH	The domains of the various vectors and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
5916 5917	GrB_EMPTY_OBJECT	The $GrB_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s) = 0)$ and therefore a value cannot be passed to the binary operator.

Description

This variant of GrB_apply computes the result of applying a binary operator to the elements of a GraphBLAS vector each composed with a scalar constant, either val or s:

bind-first: w = f(val, u) or w = f(s, u)

bind-second: w = f(u, val) or w = f(u, s),

or if an optional binary accumulation operator (\odot) is provided:

bind-first: $w = w \odot f(val, u)$ or $w = w \odot f(s, u)$

bind-second: $w = w \odot f(u, val)$ or $w = w \odot f(u, s)$.

5926 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

5929 Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

⁵⁹³¹ Up to three argument vectors are used in this GrB_apply operation:

- 5932 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 5933 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 5939 2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
- 5. If bind-first:

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- (a) $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.
- (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\text{val})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
- (c) If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

6. If bind-second:

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- (a) $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
- (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\text{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.
 - (c) If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- 5965 (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- 5966 (b) If mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
 - (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 5970 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 5971 4. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 5974 1. $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}})$
- 5975 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB Scalar \tilde{s} is provided (**nvals**(\tilde{s}) = 0), the method returns with code GrB EMPTY OBJECT.

If a non-empty GrB_Scalar, \tilde{s} , is provided (i.e., $\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val with the same domain as \tilde{s} and set $\mathbf{val} = \mathbf{val}(\tilde{s})$.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the binary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\tilde{\mathbf{t}}$, is created as one of the following:

```
bind-first: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\mathsf{val}, \widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
bind-second: \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i), \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\} \rangle,
```

5990 where f = f(op).

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The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \, \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$
 - If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

6018 4.3.8.4 apply: Matrix-BinaryOp variants[Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a matrix using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the matrix are passed as the second argument. In the *bind-second* variant, the elements of the matrix are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

6025 C Syntax

```
// bind-first + scalar value
6026
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
6027
                                    const GrB_Matrix
                                                             Mask,
6028
                                    const GrB_BinaryOp
                                                             accum,
6029
                                    const GrB_BinaryOp
6030
                                                             op,
                                    <type>
                                                             val,
6031
                                    const GrB_Matrix
                                                             Α,
6032
                                    const GrB_Descriptor
                                                             desc);
6033
6034
             // bind-first + GraphBLAS scalar
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
6035
                                    const GrB_Matrix
                                                             Mask,
6036
                                    const GrB_BinaryOp
                                                             accum,
6037
                                    const GrB_BinaryOp
6038
                                                             op,
                                    const GrB_Scalar
                                                             s,
6039
                                    const GrB Matrix
                                                             Α,
6040
                                    const GrB_Descriptor
                                                             desc);
6041
             // bind-second + scalar value
6042
             GrB_Info GrB_apply(GrB_Matrix
                                                             C,
6043
                                    const GrB_Matrix
                                                             Mask,
6044
                                    const GrB BinaryOp
6045
                                                             accum,
                                    const GrB_BinaryOp
                                                             op,
6046
                                    const GrB_Matrix
                                                             Α,
6047
                                    <type>
                                                             val.
6048
                                    const GrB_Descriptor
                                                             desc);
6049
             // bind-second + GraphBLAS scalar
6050
             GrB_Info GrB_apply(GrB_Matrix
                                                             С,
6051
                                    const GrB_Matrix
                                                             Mask,
6052
                                    const GrB_BinaryOp
                                                             accum,
6053
                                    const GrB_BinaryOp
                                                             op,
6054
                                    const GrB_Matrix
                                                             Α,
6055
```

6056	const	GrB_Scalar	s,
6057	const	<pre>GrB_Descriptor</pre>	desc)

6058 Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values 6059 that may be accumulated with the result of the apply operation. On output, the 6060 matrix holds the results of the operation. 6061 Mask (IN) An optional "write" mask that controls which results from this operation are 6062 stored into the output matrix C. The mask dimensions must match those of the 6063 matrix C. If the GrB_STRUCTURE descriptor is not set for the mask, the domain 6064 of the Mask matrix must be of type bool or any of the predefined "built-in" types 6065 in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the 6066 dimensions of C), GrB_NULL should be specified. 6067 accum (IN) An optional binary operator used for accumulating entries into existing C 6068 entries. If assignment rather than accumulation is desired, GrB_NULL should be 6069 specified. 6070 op (IN) A binary operator applied to each element of input matrix, A, with the element 6071 of the input matrix used as the left-hand argument, and the scalar value, val, used 6072 as the right-hand argument. 6073 A (IN) The GraphBLAS matrix whose elements are passed to the binary operator as 6074 the right-hand (second) argument in the bind-first variant, or the left-hand (first) 6075 argument in the bind-second variant. 6076 val (IN) Scalar value that is passed to the binary operator as the left-hand (first) 6077 argument in the bind-first variant, or the right-hand (second) argument in the 6078 bind-second variant. 6079 s (IN) GraphBLAS scalar value that is passed to the binary operator as the left-hand 6080 (first) argument in the bind-first variant, or the right-hand (second) argument in 6081 the bind-second variant. It must not be empty. 6082
 - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
6086				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation
				(bind-second variant only).
	Α	GrB_INP1	GrB_TRAN	Use transpose of A for the operation
				(bind-first variant only).

6087 Return Values

6088 6089 6090 6091 6092	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
6093	GrB_PANIC	Unknown internal error.
6094 6095 6096 6097	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
6098	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
6099 6100	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
6101 6102 6103	GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to $\mathbf{nrows}(A)$, or a value in $\mathbf{col_indices}$ is greater than or equal to $\mathbf{ncols}(A)$. In non-blocking mode, this can be reported as an execution error.
6104 6105	GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows \neq $\mathbf{nrows}(C)$, or $\mathbf{ncols} \neq \mathbf{ncols}(C)$.
6106 6107 6108 6109 6110	GrB_DOMAIN_MISMATCH	The domains of the various matrices and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
6111 6112	GrB_EMPTY_OBJECT	The $GrB_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the binary operator.

6113 Description

This variant of GrB_apply computes the result of applying a binary operator to the elements of a GraphBLAS matrix each composed with a scalar constant, val or s:

bind-first:
$$C = f(val, A)$$
 or $C = f(s, A)$

bind-second:
$$C = f(A, val)$$
 or $C = f(A, s)$,

or if an optional binary accumulation operator (①) is provided:

bind-first:
$$C = C \odot f(val, A)$$
 or $C = C \odot f(s, A)$

bind-second:
$$C = C \odot f(A, val)$$
 or $C = C \odot f(A, s)$.

6121 Logically, this operation occurs in three steps:

- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6124 **Compute** The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- Up to three argument matrices are used in the GrB_apply operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
 - 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument scalar, matrices, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6134 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
- 5. If bind-first:

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(a) $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

- (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\text{val})$ must be compatible with $\mathbf{D}_{in_1}(\text{op})$ of the binary operator.
- (c) If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

6. If bind-second:

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- (a) $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
- (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\text{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.
- (c) If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed (← denotes copy):

- 1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
- 2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
 - (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
 - (b) If Mask \neq GrB NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
 - (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows:

```
bind-first: \widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP1}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}
bind-second: \widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB\_INP0}].\mathsf{GrB\_TRAN} ? \mathsf{A}^T : \mathsf{A}
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4. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}).$

- 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$
- 4. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB_Scalar \tilde{s} is provided ($\mathbf{nvals}(\tilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT.
- If a non-empty GrB_Scalar , \tilde{s} , is provided (i.e., $\mathbf{nvals}(\tilde{s})=1$), we then create an internal variable
- val with the same domain as \tilde{s} and set $val = val(\tilde{s})$.
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:
- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the binary operator to the input matrix $\widetilde{\mathbf{A}}$.
- $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.
- The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as one of the following:

bind-first:
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\mathsf{val}, \widetilde{\mathbf{A}}(i, j))) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

bind-second:
$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i, j), \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle$$

6192 where f = f(op).

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- The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:
- If $\mathsf{accum} = \mathsf{GrB} _\mathsf{NULL}, \, \mathrm{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij}=\widetilde{\mathbf{T}}(i,j), ext{ if } (i,j)\in (\mathbf{ind}(\widetilde{\mathbf{T}})-(\mathbf{ind}(\widetilde{\mathbf{T}})\cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C , 6205 using what is called a standard matrix mask and replace. This is carried out under control of the 6206 mask which acts as a "write mask". 6207

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB NONBLOCKING mode, the method 6216 exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a 6218 sequence. 6219

apply: Vector index unary operator variant[Scott: NEW CONTENT] 4.3.8.56220

Computes the transformation of the values of the stored elements of a vector using an index unary 6221 operator that is a function of the stored value, its location indices, and an user provided scalar 6222 value. The scalar can be passed either as a non-opaque variable or as a GrB Scalar object. 6223

C Syntax 6224

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```
GrB_Info GrB_apply(GrB_Vector
                                                                W,
6225
                                    const GrB_Vector
6226
                                                                mask,
                                    const GrB_BinaryOp
                                                                accum,
6227
                                    const GrB_IndexUnaryOp
6228
                                                                op,
                                    const GrB_Vector
                                                                u,
6229
                                    <type>
                                                                val,
6230
                                    const GrB_Descriptor
                                                                desc);
6231
             GrB_Info GrB_apply(GrB_Vector
6232
                                                                W,
                                    const GrB_Vector
                                                                mask,
6233
                                    const GrB_BinaryOp
6234
                                                                accum,
                                    const GrB_IndexUnaryOp
                                                                op,
6235
                                    const GrB_Vector
                                                                u,
6236
                                    const GrB_Scalar
6237
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
6238
```

6239 Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values 6240 that may be accumulated with the result of the apply operation. On output, this 6241 vector holds the results of the operation. 6242 mask (IN) An optional "write" mask that controls which results from this operation are 6243 stored into the output vector w. The mask dimensions must match those of the 6244 vector w. If the GrB STRUCTURE descriptor is not set for the mask, the domain 6245 of the mask vector must be of type bool or any of the predefined "built-in" types 6246 in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the 6247 dimensions of w), GrB_NULL should be specified. 6248 accum (IN) An optional binary operator used for accumulating entries into existing w 6249 entries. If assignment rather than accumulation is desired, GrB_NULL should be 6250 specified. 6251 op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB_Index}), D_{in_2}, f_i \rangle$, applied 6252 to each element stored in the input vector, u. It is a function of the stored element's 6253 value, its location index, and a user supplied scalar value (either s or val). 6254 u (IN) The GraphBLAS vector whose elements are passed to the index unary oper-6255 ator. 6256 val (IN) An additional scalar value that is passed to the index unary operator. 6257 s (IN) An additional GraphBLAS scalar that is passed to the index unary operator. 6258 It must not be empty. 6259 desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL 6260 should be specified. Non-default field/value pairs are listed as follows: 6261 6262 Param Value Field Description GrB OUTP **GrB_REPLACE** Output vector w is cleared (all elements W removed) before the result is stored in it. mask GrB_MASK GrB_STRUCTURE The write mask is constructed from the 6263 structure (pattern of stored values) of the input mask vector. The stored values are not examined.

6264 Return Values

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mask

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

Use the complement of mask.

GrB_MASK GrB_COMP

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to

access any error messages generated by the implementation.

6275 GrB_OUT_OF_MEMORY Not enough memory available for operation.

6276 GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

6278 GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary

operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty $(\mathbf{nvals}(s) = 0)$ and therefore a value cannot be passed to the index unary operator.

6285 Description

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This variant of GrB_apply computes the result of applying an index unary operator to the elements of a GraphBLAS vector each composed with the element's index and a scalar constant, val or s:

$$w = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val}) \text{ or } \mathsf{w} = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}),$$

or if an optional binary accumulation operator (①) is provided:

w = w
$$\odot f_i(u, \mathbf{ind}(u), 0, \mathsf{val})$$
 or w = w $\odot f_i(u, \mathbf{ind}(u), 0, \mathsf{s})$.

6291 Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6294 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

6296 Up to three argument vectors are used in this GrB_apply operation:

- 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 6298 2. mask = $\langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)

```
3. \mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle
```

The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
- 5. If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.
- 6. If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.

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- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \forall i : 0 \le i < \mathbf{size}(\mathbf{w})\} \rangle$.
- (b) If mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool}) \mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 4. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 1. $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{size}(\widetilde{\mathbf{m}})$
- 6332 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$
- If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- If an empty GrB_Scalar \tilde{s} is provided (nvals(\tilde{s}) = 0), the method returns with code GrB_EMPTY_OBJECT.
- If a non-empty GrB_Scalar, \tilde{s} , is provided ($\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val
- with the same domain as \tilde{s} and set $val = val(\tilde{s})$.
- We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the index unary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f_i(\widetilde{\mathbf{u}}(i), [i], 0, \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \} \rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

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- The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:
- If accum = GrB NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

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$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

```
\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.
```

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

6375 4.3.8.6 apply: Matrix index unary operator variant[Scott: NEW CONTENT]

Computes the transformation of the values of the stored elements of a matrix using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

6379 C Syntax

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```
GrB_Info GrB_apply(GrB_Matrix
                                                                С,
6380
                                    const GrB_Matrix
                                                                Mask,
6381
                                    const GrB_BinaryOp
                                                                accum,
6382
                                    const GrB_IndexUnaryOp
                                                                op,
6383
                                    const GrB_Matrix
                                                                Α,
6384
                                                                val,
                                    <type>
6385
                                    const GrB_Descriptor
                                                                desc);
6386
             GrB_Info GrB_apply(GrB_Matrix
                                                                C,
6387
                                    const GrB_Matrix
                                                                Mask,
6388
                                    const GrB_BinaryOp
                                                                accum,
6389
                                    const GrB_IndexUnaryOp
                                                                op,
6390
                                    const GrB_Matrix
                                                                Α,
6391
                                    const GrB_Scalar
6392
                                                                s,
                                    const GrB_Descriptor
                                                                desc);
6393
```

6394 Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
 - accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
 - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
 - val (IN) An additional scalar value that is passed to the index unary operator.
 - s (IN) An additional GraphBLAS scalar that is passed to the index unary operator.
 - desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
			removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
			structure (pattern of stored values) of the
			input Mask matrix. The stored values are
			not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

6419	GrB_SUCCESS In blocking mode, the operation completed successfully. In non-
6420	blocking mode, this indicates that the compatibility tests on di-
6421	mensions and domains for the input arguments passed successfully.
6422	Either way, output matrix C is ready to be used in the next method
6423	of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

6430 GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

Grb_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty ($\mathbf{nvals}(s) = 0$) and therefore a value cannot be passed to the index unary operator.

6439 Description

This variant of GrB_apply computes the result of applying a index unary operator to the elements of a GraphBLAS matrix each composed with the elements row and column indices, and a scalar constant, val or s:

$$C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{sol}(\mathbf{ind}(A)), \mathsf{sol}(A))$$

or if an optional binary accumulation operator (\odot) is provided:

$$C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val}) \text{ or } C = C \odot f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{sol}).$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

6448 Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6451 **Compute** The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

6453 Up to three argument matrices are used in the GrB_apply operation:

- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 6455 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$

```
3. A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle
```

The argument scalar, matrices, index unary operator and the accumulation operator (if provided)
are tested for domain compatibility as follows:

- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6461 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
- 5. If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.
- 6. If the GrB_Scalar s is provided, then $\mathbf{D}(s)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed (\leftarrow denotes copy):

1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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- 2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
- (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$.
 - (b) If Mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows:

$$\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB}_\mathsf{INP0}].\mathsf{GrB}_\mathsf{TRAN} \; ? \; \mathsf{A}^T : \mathsf{A}$$

4. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

- 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$
- 3. $\operatorname{\mathbf{nrows}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- 4. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar \tilde{s} is provided (**nvals**(\tilde{s}) = 0), the method returns with code GrB_EMPTY_OBJECT.

If a non-empty GrB_Scalar, \tilde{s} , is provided (i.e., $\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable

val with the same domain as \tilde{s} and set val = val(\tilde{s}).

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the index unary operator to the input matrix $\widetilde{\mathbf{A}}$.
 - \bullet $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\tilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val})) \ \forall \ (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \} \rangle,$$

6510 where $f_i = \mathbf{f}(op)$.

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The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\mathsf{accum} = \mathsf{GrB} _\mathsf{NULL}, \, \mathrm{then} \,\, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) orall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \}
angle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\hat{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

6538 **4.3.9** select:

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Apply a select operator to the stored elements of an object to determine whether or not to keep them.

$_{6541}$ 4.3.9.1 select: Vector variant[Scott: NEW CONTENT]

6542 Apply a select operator (an index unary operator) to the elements of a vector.

6543 C Syntax

```
// scalar value variant
6544
             GrB Info GrB select(GrB Vector
                                                                 W,
6545
                                     const GrB Vector
                                                                 mask,
6546
                                     const GrB_BinaryOp
                                                                 accum.
6547
                                     const GrB_IndexUnaryOp
                                                                 op,
6548
                                     const GrB_Vector
                                                                 u,
6549
                                     <type>
                                                                 val,
6550
                                     const GrB_Descriptor
6551
                                                                 desc);
6552
              // GraphBLAS scalar variant
6553
             GrB_Info GrB_select(GrB_Vector
6554
                                                                 W,
                                     const GrB_Vector
                                                                 mask.
6555
```

6556	const	<pre>GrB_BinaryOp</pre>	accum,
6557	const	<pre>GrB_IndexUnaryOp</pre>	op,
6558	const	<pre>GrB_Vector</pre>	u,
6559	const	<pre>GrB_Scalar</pre>	s,
6560	const	<pre>GrB_Descriptor</pre>	<pre>desc);</pre>

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the select operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input vector, \mathbf{u} . It is a function of the stored element's value, its location index, and a user supplied scalar value (either \mathbf{s} or val).
 - u (IN) The GraphBLAS vector whose elements are passed to the index unary operator.
 - val (IN) An additional scalar value that is passed to the index unary operator.
 - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
6586	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.

6587 Return Values

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GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

6599 GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.

6601 GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty (nvals(s) = 0) and therefore a value cannot be passed to the index unary operator.

6608 Description

This variant of GrB_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS vector. The operator takes, as input, the value of each stored element, along with the element's index and a scalar constant – either val or s. The corresponding element of the input vector is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input vector as follows:

$$\mathbf{w} = \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle,$$

$$\mathbf{w} = \mathbf{w} \odot \mathbf{u} \langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), 0, \mathsf{val}) \rangle.$$

6616 Correspondingly, if a GrB_Scalar, s, is provided:

6617
$$\mathsf{w} = \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle,$$

$$\mathsf{w} = \mathsf{w} \odot \mathsf{u} \langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}) \rangle.$$

- 6619 Logically, this operation occurs in three steps:
- Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6622 Compute The indicated computations are carried out.
- 6623 Output The result is written into the output vector, possibly under control of a mask.
- 6624 Up to three argument vectors are used in this GrB_select operation:
- 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 6626 2. $\operatorname{mask} = \langle \mathbf{D}(\operatorname{mask}), \operatorname{\mathbf{size}}(\operatorname{\mathsf{mask}}), \mathbf{L}(\operatorname{\mathsf{mask}}) = \{(i, m_i)\} \rangle \text{ (optional)}$
- 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(mask) must be from one of the pre-defined types of Table 3.2.
- 6632 2. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}(\mathbf{u})$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(w)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
- 5. $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathbf{op})$ of the index unary operator.
- 6639 6. $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.
- Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_select ends and the domain mismatch error listed above is returned.
- From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):
- 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.

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2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:

```
(a) If mask = GrB_NULL, then \widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle.
```

- (b) If mask \neq GrB_NULL,
- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 4. Scalar $\widetilde{s} \leftarrow s$ (GrB Scalar version only).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6660 2. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{u}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}).$

If any compatibility rule above is violated, execution of GrB_select ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty $GrB_Scalar\ \widetilde{s}$ is provided (i.e., $nvals(\widetilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT .

If a non-empty GrB_Scalar , \widetilde{s} , is provided (i.e., $nvals(\widetilde{s}) = 1$), we then create an internal variable val with the same domain as \widetilde{s} and set $val = val(\widetilde{s})$.

We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the index unary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathsf{u}}), \{(i, \widetilde{\mathsf{u}}(i), : i \in \mathbf{ind}(\widetilde{\mathsf{u}}) \land (\mathsf{bool}) f_i(\widetilde{\mathsf{u}}(i), i, 0, \mathsf{val}) = \mathsf{true} \} \rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}$, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$
 $z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$
 $z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\tilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, \mathbf{w} , and elements of \mathbf{w} that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.9.2 select: Matrix variant[Scott: NEW CONTENT]

Apply a select operator (an index unary operator) to the elements of a matrix.

6705 C Syntax

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```
// scalar value variant
6706
             GrB_Info GrB_select(GrB_Matrix
                                                                 С,
6707
                                     const GrB_Matrix
                                                                 Mask,
6708
                                     const GrB_BinaryOp
                                                                 accum,
6709
                                     const GrB_IndexUnaryOp
6710
                                                                 op,
                                     const GrB_Matrix
                                                                 Α,
6711
                                     <type>
                                                                 val,
6712
                                     const GrB_Descriptor
                                                                 desc);
6713
```

```
// GraphBLAS scalar variant
6715
             GrB_Info GrB_select(GrB_Matrix
                                                                С,
6716
                                    const GrB_Matrix
                                                                Mask,
6717
                                     const GrB_BinaryOp
                                                                accum,
6718
                                     const GrB IndexUnaryOp
6719
                                                                op,
                                     const GrB Matrix
                                                                Α,
6720
                                     const GrB Scalar
                                                                s,
6721
                                     const GrB_Descriptor
                                                                desc);
6722
```

Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the select operation. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).
 - A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.
 - val (IN) An additional scalar value that is passed to the index unary operator.
 - s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.
 - desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
6747				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

6749 6750 6751 6752 6753	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output mattrix C is ready to be used in the next method of the sequence.
6754	GrB_PANIC	Unknown internal error.
6755 6756 6757 6758	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
6759	GrB_OUT_OF_MEMORY	Not enough memory available for operation.
6760 6761	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.
6762	GrB_DIMENSION_MISMATCH	Mask,C and/or A dimensions are incompatible.
6763 6764 6765 6766	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
6767 6768	GrB_EMPTY_OBJECT	The $GrB_Scalar\ s$ used in the call is empty $(\mathbf{nvals}(s)=0)$ and therefore a value cannot be passed to the index unary operator.

Description

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This variant of GrB_select computes the result of applying a index unary operator to select the 6770 elements of the input GraphBLAS matrix. The operator takes, as input, the value of each stored element, along with the element's row and column indices and a scalar constant - from either val 6772 or s. The corresponding element of the input matrix is selected (kept) if the function evaluates 6773 to true when cast to bool. This acts like a functional mask on the input matrix as follows when 6774 specifying a transparent scalar value: 6775

6776
$$\mathsf{C} = \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle, \text{ or}$$
6777
$$\mathsf{C} = \mathsf{C} \odot \mathsf{A}\langle f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val})\rangle.$$

6778 Correspondingly, if a GrB_Scalar, s, is provided:

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$$C = A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle, \text{ or}$$

$$C = C \odot A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s) \rangle.$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

- 6783 Logically, this operation occurs in three steps:
- Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 6786 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.
- ⁶⁷⁸⁸ Up to three argument matrices are used in the GrB_select operation:
- 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 6790 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:
- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 6796 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$.

- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 4. $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
 - 5. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
- 6803 6. $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all 6806 compatible with each other. A domain from a user-defined type is only compatible with itself. If 6807 any compatibility rule above is violated, execution of GrB_select ends and the domain mismatch 6808 error listed above is returned. 6809

From the argument matrices, the internal matrices, mask, and index arrays used in the computation 6810 are formed (\leftarrow denotes copy):

1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$. 6812

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2. Two-dimensional mask, M, is computed from argument Mask as follows: 6813

```
(a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq i < \mathbf{nr
6814
                                                                                                                                                                                                                                                                                                                                                                                                                                                              j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
6815
```

- (b) If Mask \neq GrB NULL,
 - i. If desc[GrB MASK].GrB STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : \}$ $(i, j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$ $\{(i,j):(i,j)\in\mathbf{ind}(\mathsf{Mask})\land(\mathsf{bool})\mathsf{Mask}(i,j)=\mathsf{true}\}$.
- (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
- 3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows: $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$ 6822
- 4. Scalar $\tilde{s} \leftarrow s$ (GrB_Scalar version only). 6823

The internal matrices and mask are checked for dimension compatibility. The following conditions 6824 must hold: 6825

- 1. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{M}}).$ 6826
- 2. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{M}}).$ 6827
- 3. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}})$. 6828
- 4. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$ 6829

If any compatibility rule above is violated, execution of GrB_select ends and the dimension mismatch 6830 error listed above is returned. 6831

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with 6832 GrB SUCCESS return code and defer any computation and/or execution error codes. 6833

If an empty GrB Scalar \tilde{s} is provided (i.e., $nvals(\tilde{s}) = 0$), the method returns with code GrB EMPTY OBJECT. 6834

If a non-empty GrB_Scalar , \tilde{s} , is provided (i.e., $nvals(\tilde{s}) = 1$), we then create an internal variable 6835

val with the same domain as \tilde{s} and set $val = val(\tilde{s})$. 6836

We are now ready to carry out the select and any additional associated operations. We describe 6837 this in terms of two intermediate matrices: 6838

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the index unary operator to the input matrix $\widetilde{\mathbf{A}}$.
 - $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\tilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \\ \{(i, j, \widetilde{\mathbf{A}}(i, j) : i, j \in \mathbf{ind}(\widetilde{\mathbf{A}}) \land (\mathsf{bool}) f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val}) = \mathsf{true} \} \rangle,$$

where $f_i = f(op)$.

The intermediate matrix $\hat{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

6872 4.3.10 reduce: Perform a reduction across the elements of an object

6873 Computes the reduction of the values of the elements of a vector or matrix.

6874 4.3.10.1 reduce: Standard matrix to vector variant

This performs a reduction across rows of a matrix to produce a vector. If reduction down columns is desired, the input matrix should be transposed using the descriptor.

6877 C Syntax

```
GrB_Info GrB_reduce(GrB_Vector
                                                                W,
6878
                                     const GrB_Vector
6879
                                                                mask,
                                     const GrB_BinaryOp
                                                                accum
6880
                                     const GrB Monoid
                                                                op,
6881
                                     const GrB_Matrix
                                                                Α,
6882
                                     const GrB Descriptor
                                                                desc);
6883
6884
             GrB_Info GrB_reduce(GrB_Vector
6885
                                                                w,
                                     const GrB_Vector
                                                                mask,
6886
                                     const GrB_BinaryOp
                                                                accum,
6887
                                     const GrB BinaryOp
6888
                                                                op,
                                     const GrB_Matrix
6889
                                                                Α,
                                     const GrB Descriptor
                                                                desc);
6890
```

Parameters

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- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the reduction operation. On output, this vector holds the results of the operation.
- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The monoid or binary operator used in the element-wise reduction operation. Depending on which type is passed, the following defines the binary operator with one domain, $F_b = \langle D, D, D, \oplus \rangle$, that is used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$, the identity element of the monoid is ignored.

If op is a $GrB_BinaryOp$, then all its domains must be the same. Furthermore, in both cases $\bigcirc(op)$ must be commutative and associative. Otherwise, the outcome of the operation is undefined.

A (IN) The GraphBLAS matrix on which reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	W	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements
				removed) before the result is stored in it.
	mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
6917				structure (pattern of stored values) of the
				input mask vector. The stored values are
				not examined.
	mask	GrB_MASK	GrB_COMP	Use the complement of mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

6919 6920 6921 6922 6923	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
6924	GrB_PANIC	Unknown internal error.
6925 6926 6927 6928	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
6929	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
6930 6931	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
6932	GrB_DIMENSION_MISMATCH	mask,w and/or u dimensions are incompatible.
6933 6934 6935	GrB_DOMAIN_MISMATCH	Either the domains of the various vectors and matrices are incompatible with the corresponding domains of the accumulation operator or reduce function, or the domains of the GraphBLAS binary

operator op are not all the same, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE
is not set).

6939 Description

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This variant of GrB_reduce computes the result of performing a reduction across each of the rows of an input matrix: $w(i) = \bigoplus A(i,:) \forall i$; or, if an optional binary accumulation operator is provided, $w(i) = w(i) \odot (\bigoplus A(i,:)) \forall i$, where $\bigoplus = \bigodot (F_b)$ and $\odot = \bigodot (accum)$.

6943 Logically, this operation occurs in three steps:

Setup The internal vector, matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

6946 **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

6948 Up to two vector and one matrix argument are used in this GrB_reduce operation:

```
6949 1. \mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle
```

6950 2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)

6951 3.
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument vector, matrix, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then **D**(mask) must be from one of the pre-defined types of Table 3.2.
- 2. $\mathbf{D}(w)$ must be compatible with the domain of the reduction binary operator, $\mathbf{D}(F_b)$.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(F_b)$, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
 - 4. $\mathbf{D}(A)$ must be compatible with the domain of the binary reduction operator, $\mathbf{D}(F_b)$.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed (\leftarrow denotes copy):

6968 1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathbf{w}$.

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- 2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
- 6970 (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathbf{w}), \{i, \ \forall \ i : 0 \le i < \mathbf{size}(\mathbf{w}) \} \rangle$.
- (b) If mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
- 6973 ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \land (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true} \} \rangle$.
- 6974 (c) If desc[GrB MASK].GrB COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP0}].\mathsf{GrB_TRAN} ? \mathsf{A}^T : \mathsf{A}.$
- The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:
- 1. $\operatorname{\mathbf{size}}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{size}}(\widetilde{\mathbf{m}})$
- 6979 2. $\operatorname{size}(\widetilde{\mathbf{w}}) = \operatorname{\mathbf{nrows}}(\widetilde{\mathbf{A}}).$
- If any compatibility rule above is violated, execution of GrB_reduce ends and the dimension mismatch error listed above is returned.
- From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.
- We carry out the reduce and any additional associated operations. We describe this in terms of two intermediate vectors:
- $\tilde{\mathbf{t}}$: The vector holding the result from reducing along the rows of input matrix $\tilde{\mathbf{A}}$.
- $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.
- The intermediate vector, $\tilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i,t_i) : \mathbf{ind}(A(i,:)) \neq \emptyset \} \rangle.$$

6990 The value of each of its elements is computed by

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$$t_i = \bigoplus_{j \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:))} \widetilde{\mathbf{A}}(i,j),$$

where $\bigoplus = \bigcirc(F_b)$.

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The intermediate vector $\tilde{\mathbf{z}}$ is created as follows, using what is called a standard vector accumulate:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \operatorname{\mathsf{NULL}}, \, \operatorname{\mathsf{then}} \ \widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}.$

• If accum is a binary operator, then $\tilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and \mathbf{t} .

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\tilde{\mathbf{z}}$ are written into the final result vector w, 7005 using what is called a standard vector mask and replace. This is carried out under control of the 7006 mask which acts as a "write mask". 7007

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

• If $\operatorname{desc}[\operatorname{GrB}_{\operatorname{OUTP}}].\operatorname{GrB}_{\operatorname{REPLACE}}$ is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a 7018 sequence.

reduce: Vector-scalar variant[Scott: NEW CONTENT] 4.3.10.2 7020

Reduce all stored values into a single scalar. 7021

C Syntax 7022

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```
// scalar value + monoid (only)
7023
             GrB_Info GrB_reduce(<type>
                                                             *val,
7024
                                    const GrB_BinaryOp
                                                              accum,
7025
                                     const GrB_Monoid
                                                              op,
7026
                                     const GrB_Vector
7027
                                                              u,
```

```
const GrB_Descriptor
                                                              desc);
7028
7029
              // GraphBLAS Scalar + monoid
7030
             GrB_Info GrB_reduce(GrB_Scalar
7031
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7032
                                     const GrB_Monoid
                                                              op,
7033
                                     const GrB_Vector
                                                              u,
7034
                                     const GrB_Descriptor
                                                              desc);
7035
7036
              // GraphBLAS Scalar + binary operator
7037
             GrB_Info GrB_reduce(GrB_Scalar
7038
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7039
                                     const GrB_BinaryOp
                                                              op,
7040
                                     const GrB_Vector
                                                              u,
7041
                                     const GrB_Descriptor
                                                              desc);
7042
```

Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.
 - accum (IN) An optional binary operator used for accumulating entries into an existing scalar (s or val) value. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The monoid $(M = \langle D, \oplus, 0 \rangle)$ or binary operator $(F_b = \langle D, D, D, \oplus \rangle)$ used in the reduction operation. The \oplus operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
 - u (IN) The GraphBLAS vector on which reduction will be performed.
- desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param Field Value Description

Note: This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

by a previous execution error. Call GrB_error() to access any error

messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB_NULL_POINTER val pointer is NULL.

GrB_DOMAIN_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

7077 Description

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This variant of GrB_reduce computes the result of performing a reduction across all of the stored elements of an input vector storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} \; = \left\{ \begin{aligned} &\bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i), & \text{ or } \\ &\mathsf{val} \; \odot \; \left[\bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i) \right], & \text{if the the optional accumulator is specified.} \end{aligned} \right.$$

where $\bigoplus = \bigcirc(\mathsf{op})$ and $\odot = \bigcirc(\mathsf{accum})$.

TOB3 Logically, this operation occurs in three steps:

7084 **Setup** The internal vector used in the computation is formed and its domain is tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One vector argument is used in this GrB_reduce operation:

7089 1.
$$\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$$

The output scalar, argument vector, reduction operator and accumulation operator (if provided)
are tested for domain compatibility as follows:

1. If accum is GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

- 2. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator, and $\mathbf{D}(\mathsf{op})$ from M (or $\mathbf{D}_{out}(\mathsf{op})$ from F_b) must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
- 3. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, u, is checked. If there are no stored values in u, then one of the following occurs depending on the output variant:

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \text{GrB_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$

$$\text{val} = \begin{cases} \mathbf{0}(\text{op}), & \text{(cleared) if accum} = \text{GrB_NULL}, \\ \\ \text{val} \odot \mathbf{0}(\text{op}), & \text{otherwise}, \end{cases}$$

where $\mathbf{0}(\mathsf{op})$ is the identity of the monoid. The operation returns immediately with GrB SUCCESS.

For all other cases, the internal vector and scalar used in the computation is formed (\leftarrow denotes copy):

7111 1. Vector $\widetilde{\mathbf{u}} \leftarrow \mathbf{u}$.

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7112 2. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

We are now ready to carry out the reduction and any additional associated operations. An intermediate scalar result t is computed as follows:

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$$t = \bigoplus_{i \in \mathbf{ind}(\widetilde{\mathbf{u}})} \widetilde{\mathbf{u}}(i),$$

where $\oplus = \bigcirc(\mathsf{op})$.

7117 The final reduction value is computed as follows:

In both GrB_BLOCKING and GrB_NONBLOCKING modes, the method exits with return value GrB_SUCCESS and the new contents of the output scalar is as defined above.

7123 4.3.10.3 reduce: Matrix-scalar variant[Scott: NEW CONTENT]

7124 Reduce all stored values into a single scalar.

7125 C Syntax

```
// scalar value + monoid (only)
7126
             GrB_Info GrB_reduce(<type>
                                                             *val,
7127
                                    const GrB_BinaryOp
                                                              accum,
7128
                                    const GrB_Monoid
7129
                                                              op,
                                     const GrB_Matrix
                                                              Α,
7130
                                     const GrB_Descriptor
                                                              desc);
7131
7132
             // GraphBLAS Scalar + monoid
7133
             GrB_Info GrB_reduce(GrB_Scalar
7134
                                                              s,
                                     const GrB_BinaryOp
                                                              accum,
7135
                                    const GrB_Monoid
                                                              op,
7136
                                     const GrB_Matrix
                                                              Α,
7137
                                     const GrB Descriptor
                                                              desc);
7138
7139
             // GraphBLAS Scalar + binary operator
7140
             GrB_Info GrB_reduce(GrB_Scalar
7141
                                                              s,
                                    const GrB_BinaryOp
                                                              accum,
7142
                                    const GrB_BinaryOp
7143
                                                              op,
                                     const GrB_Matrix
                                                              Α,
7144
                                     const GrB_Descriptor
                                                              desc);
7145
```

Parameters

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- val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides
 a value that may be accumulated (optionally) with the result of the reduction
 operation. On output, this scalar holds the results of the operation.
 - accum (IN) An optional binary operator used for accumulating entries into existing (s or val) value. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The monoid $(M = \langle D, \oplus, 0 \rangle)$ or binary operator $(F_b = \langle D, D, D, \oplus \rangle)$ used in the reduction operation. The \oplus operator must be commutative and associative; otherwise, the outcome of the operation is undefined.
 - A (IN) The GraphBLAS matrix on which the reduction will be performed.

desc (IN) An optional operation descriptor. If a default descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

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Param Field Value Description

Note: This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

Return Values 7164

GrB_SUCCESS In blocking or non-blocking mode, the operation completed suc-7165 cessfully, and the output scalar (s or val) is ready to be used in the 7166 next method of the sequence. 7167

Grb Panic Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused 7170 by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

Grb_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by 7174 a call to a respective constructor. 7175

GrB_NULL_POINTER val pointer is NULL. 7176

GrB_DOMAIN_MISMATCH The domains of input and output arguments are incompatible with 7177 the corresponding domains of the accumulation operator, or reduce 7178 operator. 7179

Description 7180

This variant of GrB_reduce computes the result of performing a reduction across all of the stored 7181 elements of an input matrix storing the result into either s or val. This corresponds to (shown here 7182 for the scalar value case only): 7183

val =
$$\begin{cases} \bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j), & \text{or} \\ \\ \mathsf{val} \ \odot \ \left[\bigoplus_{(i,j) \in \mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j) \right], & \text{if the the optional accumulator is specified.} \end{cases}$$

where $\bigoplus = \bigcirc(\mathsf{op})$ and $\odot = \bigcirc(\mathsf{accum})$. 7185

Logically, this operation occurs in three steps: 7186

Setup The internal matrix used in the computation is formed and its domain is tested for compatibility.

7189 Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One matrix argument is used in this GrB_reduce operation:

1.
$$A = \langle \mathbf{D}(A), \mathbf{size}(A), \mathbf{L}(A) = \{(i, j, A_{i,j})\} \rangle$$

The output scalar, argument matrix, reduction operator and accumulation operator (if provided)
are tested for domain compatibility as follows:

- 1. If accum is GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).
 - 2. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator, and $\mathbf{D}(\mathsf{op})$ from M (or $\mathbf{D}_{out}(\mathsf{op})$ from F_b) must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
 - 3. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, A, is checked. If there are no stored values in A, then one of the following occurs depending on the output variant:

$$\mathbf{L}(s) = \begin{cases} \{\}, & \text{(cleared) if accum} = \mathsf{GrB_NULL}, \\ \\ \mathbf{L}(s), & \text{(unchanged) otherwise}, \end{cases}$$

7209 Or

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where $\mathbf{0}(\mathsf{op})$ is the identity of the monoid. The operation returns immediately with $\mathsf{GrB_SUCCESS}$.

For all other cases, the internal matrix and scalar used in the computation is formed (\leftarrow denotes copy):

- 1. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{A}$.
- 7215 2. Scalar $\tilde{s} \leftarrow s$ (GraphBLAS scalar case).

We are now ready to carry out the reduce and any additional associated operations. An intermediate scalar result t is computed as follows:

$$t = \bigoplus_{(i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})} \widetilde{\mathbf{A}}(i,j),$$

where $\oplus = \bigcirc(\mathsf{op})$.

7220 The final reduction value is computed as follows:

The second when
$$\mathbf{L}(\mathbf{s}) \leftarrow \begin{cases} \{t\}, & \text{when accum} = \mathsf{GrB_NULL} \text{ or } \tilde{s} \text{ is empty, or} \\ \{\mathbf{val}(\tilde{s}) \odot t\}, & \text{otherwise;} \end{cases}$$

The second when $\mathbf{ccum} = \mathsf{GrB_NULL}$, or $\mathbf{val} \leftarrow \begin{cases} t, & \text{when accum} = \mathsf{GrB_NULL}, \text{ or} \\ \mathsf{val} \odot t, & \text{otherwise;} \end{cases}$

In both GrB_BLOCKING and GrB_NONBLOCKING modes, the method exits with return value GrB_SUCCESS and the new contents of the output scalar is as defined above.

7226 4.3.11 transpose: Transpose rows and columns of a matrix

7227 This version computes a new matrix that is the transpose of the source matrix.

7228 C Syntax

```
GrB_Info GrB_transpose(GrB_Matrix C,
const GrB_Matrix Mask,
const GrB_BinaryOp accum,
const GrB_Matrix A,
const GrB_Descriptor desc);
```

Parameters

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C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the transpose operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

7244	accum (IN) An optional binary operator used for accumulating entries into existing C
7245	entries. If assignment rather than accumulation is desired, GrB_NULL should be
7246	specified.

A (IN) The GraphBLAS matrix on which transposition will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
7251				structure (pattern of stored values) of the
				input Mask matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

7253 7254 7255 7256 7257		In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
7258	GrB_PANIC	Unknown internal error.
7259 7260 7261 7262		This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
7263	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
7264 7265		One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
7266	GrB_DIMENSION_MISMATCH	mask,Cand/orAdimensionsareincompatible.
7267 7268 7269 7270		The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCT is not set).

7271 Description

GrB_transpose computes the result of performing a transpose of the input matrix: $C = A^T$; or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot A^T$. We note that the input matrix A can itself be optionally transposed before the operation, which would cause either an assignment from A to C or an accumulation of A into C.

Logically, this operation occurs in three steps:

Setup The internal matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

7279 Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

7281 Up to three matrix arguments are used in this GrB_transpose operation:

```
1. C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle
```

7283 2.
$$\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle \text{ (optional)}$$

3.
$$A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$$

The argument matrices and accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
 - 2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$ of the input matrix.
- 3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ of the input matrix must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_transpose ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed \leftarrow denotes copy):

1. Matrix $\tilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:

```
7302 (a) If Mask = GrB_NULL, then \widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle.
```

(b) If Mask \neq GrB_NULL,

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- i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j): (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
 - (c) If $\mathsf{desc}[\mathsf{GrB_MASK}].\mathsf{GrB_COMP}$ is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}.$
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 7313 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 7314 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}).$
- 4. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}).$

If any compatibility rule above is violated, execution of GrB_transpose ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix transposition and any additional associated operations.
We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the transpose of $\widetilde{\mathbf{A}}$.
- $\tilde{\mathbf{z}}$: The matrix holding the result after application of the (optional) accumulation operator.

7325 The intermediate matrix

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(j, i, A_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \}$$

7327 is created.

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The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\mathsf{accum} = \mathsf{GrB} _\mathsf{NULL}, \, \mathrm{then} \, \, \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\tilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in \mathbf{C} and \mathbf{T} .

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$
where $\odot = \mathfrak{O}(\mathbf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C , 7340 using what is called a standard matrix mask and replace. This is carried out under control of the 7341 mask which acts as a "write mask". 7342

• If desc[GrB OUTP].GrB REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In Grb BLOCKING mode, the method exits with return value Grb SUCCESS and the new content 7350 of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method 7351 exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but 7352 may not be fully computed. However, it can be used in the next GraphBLAS method call in a 7353 sequence. 7354

kronecker: Kronecker product of two matrices 4.3.12

Computes the Kronecker product of two matrices. The result is a matrix. 7356

C Syntax 7357

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```
GrB_Info GrB_kronecker(GrB_Matrix
                                                                    С,
7358
                                         const GrB_Matrix
                                                                    Mask,
7359
                                         const GrB_BinaryOp
                                                                    accum,
7360
                                         const GrB_Semiring
7361
                                                                    op,
                                         const GrB_Matrix
                                                                    Α,
7362
                                         const GrB Matrix
                                                                    В,
7363
                                         const GrB_Descriptor
                                                                    desc);
7364
7365
```

```
С,
             GrB_Info GrB_kronecker(GrB_Matrix
7366
                                         const GrB_Matrix
                                                                   Mask,
7367
                                         const GrB_BinaryOp
7368
                                                                    accum,
                                         const GrB_Monoid
                                                                    op,
7369
                                         const GrB Matrix
                                                                    Α,
7370
                                         const GrB Matrix
                                                                   В,
7371
                                         const GrB Descriptor
                                                                    desc);
7372
7373
             GrB_Info GrB_kronecker(GrB_Matrix
                                                                    C,
7374
                                         const GrB_Matrix
                                                                   Mask,
7375
                                         const GrB_BinaryOp
7376
                                                                    accum,
                                         const GrB_BinaryOp
                                                                    op,
7377
                                         const GrB_Matrix
                                                                    Α,
7378
                                         const GrB_Matrix
                                                                    Β,
7379
                                         const GrB_Descriptor
                                                                    desc);
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```

Parameters

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- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the Kronecker product. On output, the matrix holds the results of the operation.
- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.
- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
 - op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

```
BinaryOp: F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle.
```

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigcirc(\mathsf{op}) \rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes (\mathsf{op}) \rangle$; the additive monoid is ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the product.

7404	B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the
7405	product.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

	Param	Field	Value	Description
	С	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements
				removed) before the result is stored in it.
	Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the
7400				structure (pattern of stored values) of the
7409				input $Mask$ matrix. The stored values are
				not examined.
	Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
	Α	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
	В	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

o Return Values

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7411 7412 7413 7414 7415	GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
7416	GrB_PANIC	Unknown internal error.
7417 7418 7419 7420	GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
7421	GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
7422 7423	GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to <code>new</code> (or <code>Matrix_dup</code> for matrix parameters).
7424	GrB_DIMENSION_MISMATCH	Mask and/or matrix dimensions are incompatible.
7425 7426 7427 7428	GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description 129

GrB_kronecker computes the Kronecker product $C = A \otimes B$ or, if an optional binary accumulation operator (\odot) is provided, $C = C \odot (A \otimes B)$ (where matrices A and B can be optionally transposed).

7432 The Kronecker product is defined as follows:

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$$\mathsf{C} = \mathsf{A} \ \otimes \ \mathsf{B} = \left[\begin{array}{cccccc} A_{0,0} \otimes \mathsf{B} & A_{0,1} \otimes \mathsf{B} & \dots & A_{0,n_A-1} \otimes \mathsf{B} \\ A_{1,0} \otimes \mathsf{B} & A_{1,1} \otimes \mathsf{B} & \dots & A_{1,n_A-1} \otimes \mathsf{B} \\ \vdots & \vdots & \ddots & & \vdots \\ A_{m_A-1,0} \otimes \mathsf{B} & A_{m_A-1,1} \otimes \mathsf{B} & \dots & A_{m_A-1,n_A-1} \otimes \mathsf{B} \end{array} \right]$$

where $A: \mathbb{S}^{m_A \times n_A}$, $B: \mathbb{S}^{m_B \times n_B}$, and $C: \mathbb{S}^{m_A m_B \times n_A n_B}$. More explicitly, the elements of the Kronecker product are defined as

$$C(i_A m_B + i_B, j_A n_B + j_B) = A_{i_A, j_A} \otimes B_{i_B, j_B},$$

- where \otimes is the multiplicative operator specified by the op parameter.
- Logically, this operation occurs in three steps:
- The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.
- 7442 Compute The indicated computations are carried out.
- Output The result is written into the output matrix, possibly under control of a mask.

7444 Up to four argument matrices are used in the GrB_kronecker operation:

- 7445 1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\} \rangle$
- 7446 2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
- 3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\} \rangle$
- 4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$

The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

- 1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then D(Mask) must be from one of the pre-defined types of Table 3.2.
- 7453 2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
- 7454 3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
- 4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
- 5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_kronecker ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed (
denotes copy):

7466 1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

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7467 2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:

7468 (a) If Mask = GrB_NULL, then
$$\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j: 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C}) \} \rangle$$
.

- (b) If Mask \neq GrB_NULL,
 - i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
 - ii. Otherwise, $\mathbf{M} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \land (\mathsf{bool}) \mathsf{Mask}(i,j) = \mathsf{true} \} \rangle.$
- (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
- 3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc}[\mathsf{GrB} \ \mathsf{INP0}].\mathsf{GrB} \ \mathsf{TRAN} \ ? \ \mathsf{A}^T : \mathsf{A}.$
- 4. Matrix $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc}[\mathsf{GrB_INP1}].\mathsf{GrB_TRAN} ? \mathsf{B}^T : \mathsf{B}.$

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

- 7480 1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}).$
- 2. $\operatorname{\mathbf{ncols}}(\widetilde{\mathbf{C}}) = \operatorname{\mathbf{ncols}}(\widetilde{\mathbf{M}}).$
- 3. $\operatorname{nrows}(\widetilde{\mathbf{C}}) = \operatorname{nrows}(\widetilde{\mathbf{A}}) \cdot \operatorname{nrows}(\widetilde{\mathbf{B}}).$
- 4. $\operatorname{ncols}(\widetilde{\mathbf{C}}) = \operatorname{ncols}(\widetilde{\mathbf{A}}) \cdot \operatorname{ncols}(\widetilde{\mathbf{B}}).$

If any compatibility rule above is violated, execution of GrB_kronecker ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the Kronecker product and any additional associated operations.
We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the Kronecker product of matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\tilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}) \times \mathbf{nrows}(\widetilde{\mathbf{B}}), \mathbf{ncols}(\widetilde{\mathbf{A}}) \times \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij}) \text{ where } i = i_A \cdot m_B + i_B, \ j = j_A \cdot n_B + j_B, \ \forall \ (i_A, j_A) = \mathbf{ind}(\widetilde{\mathbf{A}}), \ (i_B, j_B) = \mathbf{ind}(\widetilde{\mathbf{B}}) \rangle$ is created. The value of each of its elements is computed by

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$$T_{i_A \cdot m_B + i_B, \ j_A \cdot n_B + j_B} = \widetilde{\mathbf{A}}(i_A, j_A) \otimes \widetilde{\mathbf{B}}(i_B, j_B)),$$

where \otimes is the multiplicative operator specified by the **op** parameter.

The intermediate matrix $\tilde{\mathbf{Z}}$ is created as follows, using what is called a standard matrix accumulate:

• If $\operatorname{\mathsf{accum}} = \operatorname{\mathsf{GrB}} \ \ \operatorname{\mathsf{NULL}}, \ \operatorname{\mathsf{then}} \ \widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}.$

• If \mathbf{z} is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}}) \} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigcirc$ (accum), and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\tilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

• If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

• If $desc[GrB_OUTP].GrB_REPLACE$ is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence. s

Chapter 5

Nonpolymorphic interface[Scott:

NEW CONTENT

Each polymorphic GraphBLAS method (those with multiple parameter signatures under the same name) has a corresponding set of long-name forms that are specific to each parameter signature.
That is show in Tables 5.1 through 5.11.

Table 5.1: Long-name, nonpolymorphic form of GraphBLAS methods.

Polymorphic signature	Nonpolymorphic signature
GrB_Monoid_new(GrB_Monoid*,,bool)	GrB_Monoid_new_BOOL(GrB_Monoid*,GrB_BinaryOp,bool)
$GrB_Monoid_new(GrB_Monoid*,,int8_t)$	$GrB_Monoid_new_INT8(GrB_Monoid*,GrB_BinaryOp,int8_t)$
$GrB_Monoid_new(GrB_Monoid*,,uint8_t)$	GrB_Monoid_new_UINT8(GrB_Monoid*,GrB_BinaryOp,uint8_t)
$GrB_Monoid_new(GrB_Monoid*,,int16_t)$	$GrB_Monoid_new_INT16(GrB_Monoid*,GrB_BinaryOp,int16_t)$
$GrB_Monoid_new(GrB_Monoid*,,uint16_t)$	GrB_Monoid_new_UINT16(GrB_Monoid*,GrB_BinaryOp,uint16_t)
$GrB_Monoid_new(GrB_Monoid*,,int32_t)$	$\label{lem:grb_Monoid_new_INT32} GrB_Monoid^*, GrB_BinaryOp, int 32_t)$
$GrB_Monoid_new(GrB_Monoid*,,uint32_t)$	GrB_Monoid_new_UINT32(GrB_Monoid*,GrB_BinaryOp,uint32_t)
$GrB_Monoid_new(GrB_Monoid*,,int64_t)$	$GrB_Monoid_new_INT64(GrB_Monoid*,GrB_BinaryOp,int64_t)$
$GrB_Monoid_new(GrB_Monoid*,,uint64_t)$	GrB_Monoid_new_UINT64(GrB_Monoid*,GrB_BinaryOp,uint64_t)
$GrB_Monoid_new(GrB_Monoid*,,float)$	${\sf GrB_Monoid_new_FP32}({\sf GrB_Monoid*,GrB_BinaryOp,float})$
$GrB_Monoid_new(GrB_Monoid*,,double)$	${\sf GrB_Monoid_new_FP64(GrB_Monoid*,GrB_BinaryOp,double)}$
$GrB_Monoid_new(GrB_Monoid*,,other)$	${\sf GrB_Monoid_new_UDT(GrB_Monoid*,GrB_BinaryOp,void*)}$

Table 5.2: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Nonpolymorphic signature

Polymorphic signature	Nonpolymorphic signature
GrB_Scalar_setElement(, bool,)	GrB_Scalar_setElement_BOOL(, bool,)
$GrB_Scalar_setElement(, int8_t,)$	$GrB_Scalar_setElement_INT8(, int8_t,)$
$GrB_Scalar_setElement(, uint8_t,)$	$GrB_Scalar_setElement_UINT8(, uint8_t,)$
$GrB_Scalar_setElement(, int16_t,)$	$GrB_Scalar_setElement_INT16(, int16_t,)$
$GrB_Scalar_setElement(, uint16_t,)$	GrB_Scalar_setElement_UINT16(, uint16_t,)
$GrB_Scalar_setElement(, int32_t,)$	GrB_Scalar_setElement_INT32(, int32_t,)
$GrB_Scalar_setElement(, uint32_t,)$	GrB_Scalar_setElement_UINT32(, uint32_t,)
$GrB_Scalar_setElement(, int64_t,)$	$GrB_Scalar_setElement_INT64(, int64_t,)$
$GrB_Scalar_setElement(, uint64_t,)$	GrB_Scalar_setElement_UINT64(, uint64_t,)
$GrB_Scalar_setElement(, float,)$	GrB_Scalar_setElement_FP32(, float,)
$GrB_Scalar_setElement(, double,)$	GrB_Scalar_setElement_FP64(, double,)
$GrB_Scalar_setElement(,other,)$	GrB_Scalar_setElement_UDT(,const void*,)
GrB_Scalar_extractElement(bool*,)	GrB_Scalar_extractElement_BOOL(bool*,)
$GrB_Scalar_extractElement(int8_t*,)$	GrB_Scalar_extractElement_INT8(int8_t*,)
$GrB_Scalar_extractElement(uint8_t*,)$	GrB_Scalar_extractElement_UINT8(uint8_t*,)
$GrB_Scalar_extractElement(int16_t^*,)$	GrB_Scalar_extractElement_INT16(int16_t*,)
$GrB_Scalar_extractElement(uint16_t^*,)$	GrB_Scalar_extractElement_UINT16(uint16_t*,)
$GrB_Scalar_extractElement(int32_t^*,)$	GrB_Scalar_extractElement_INT32(int32_t*,)
$GrB_Scalar_extractElement(uint32_t^*,)$	GrB_Scalar_extractElement_UINT32(uint32_t*,)
$GrB_Scalar_extractElement(int64_t^*,)$	GrB_Scalar_extractElement_INT64(int64_t*,)
$GrB_Scalar_extractElement(uint64_t^*,)$	GrB_Scalar_extractElement_UINT64(uint64_t*,)
$GrB_Scalar_extractElement(float*,)$	GrB_Scalar_extractElement_FP32(float*,)
$GrB_Scalar_extractElement(double*,)$	GrB_Scalar_extractElement_FP64(double*,)
GrB_Scalar_extractElement(other*,)	GrB_Scalar_extractElement_UDT(void*,)

Table 5.3: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                 Nonpolymorphic signature
GrB_Vector_build(...,const bool*,...)
                                                 GrB\_Vector\_build\_BOOL(...,const bool*,...)
                                                 \label{eq:GrB_Vector_build_INT8(...,const int8\_t*,...)} $$\operatorname{GrB\_Vector\_build\_UINT8(...,const uint8\_t*,...)}$$
GrB_Vector_build(...,const int8_t*,...)
GrB_Vector_build(...,const uint8_t*,...)
GrB_Vector_build(...,const int16_t*,...)
                                                 GrB_Vector_build_INT16(...,const int16_t*,...)
GrB_Vector_build(...,const uint16_t*,...)
                                                 GrB_Vector_build_UINT16(...,const uint16_t*,...)
\mathsf{GrB\_Vector\_build}(\dots, \mathsf{const\ int} 32\_t^*, \dots)
                                                 \mathsf{GrB}\_\mathsf{Vector}\_\mathsf{build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32}\_\mathsf{t*},\dots)
GrB_Vector_build(...,const uint32_t*,...)
                                                  GrB_Vector_build_UINT32(...,const_uint32_t*,...)
                                                 GrB_Vector_build_INT64(...,const int64_t*,...)
GrB_Vector_build(...,const int64_t*,...)
GrB_Vector_build(...,const uint64_t*,...)
                                                 GrB_Vector_build_UINT64(...,const uint64_t*,...)
GrB_Vector_build(...,const float*,...)
                                                 GrB_Vector_build_FP32(...,const float*,...)
GrB_Vector_build(...,const double*,...)
                                                 GrB_Vector_build_FP64(...,const double*,...)
GrB_Vector_build(...,const other*,...)
                                                  GrB_Vector_build_UDT(...,const void*,...)
                                                 GrB_Vector_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Vector\_setElement(...,GrB\_Scalar,...)
                                                 GrB Vector_setElement_BOOL(..., bool,...)
GrB Vector_setElement(...,bool,...)
GrB\_Vector\_setElement(...,int8\_t,...)
                                                 GrB_Vector_setElement_INT8(..., int8_t,...)
GrB\_Vector\_setElement(...,uint8\_t,...)
                                                 GrB\_Vector\_setElement\_UINT8(..., uint8\_t,...)
GrB_Vector_setElement(...,int16_t,...)
                                                  GrB\_Vector\_setElement\_INT16(..., int16\_t,...)
                                                 GrB_Vector_setElement_UINT16(..., uint16_t,...)
GrB\_Vector\_setElement(...,uint16\_t,...)
GrB_Vector_setElement(...,int32_t,...)
                                                 GrB_Vector_setElement_INT32(..., int32_t,...)
GrB_Vector_setElement(...,uint32_t,...)
                                                 GrB_Vector_setElement_UINT32(..., uint32_t,...)
GrB_Vector_setElement(...,int64_t,...)
                                                 GrB\_Vector\_setElement\_INT64(..., int64\_t,...)
                                                 GrB_Vector_setElement_UINT64(..., uint64_t,...)
GrB_Vector_setElement(...,uint64_t,...)
                                                 GrB_Vector_setElement_FP32(..., float,...)
GrB\_Vector\_setElement(...,float,...)
GrB\_Vector\_setElement(...,double,...)
                                                 GrB_Vector_setElement_FP64(..., double,...)
GrB_Vector_setElement(...,other,...)
                                                 GrB_Vector_setElement_UDT(...,const void*,...)
                                                 GrB\_Vector\_extractElement\_Scalar(GrB\_Scalar,...)
GrB\_Vector\_extractElement(GrB\_Scalar,...)
GrB_Vector_extractElement(bool*,...)
                                                  GrB_Vector_extractElement_BOOL(bool*,...)
                                                 GrB_Vector_extractElement_INT8(int8_t*,...)
GrB_Vector_extractElement(int8_t*,...)
GrB_Vector_extractElement(uint8_t*,...)
                                                 GrB_Vector_extractElement_UINT8(uint8_t*,...)
                                                 GrB\_Vector\_extractElement\_INT16(int16\_t^*,...)
GrB_Vector_extractElement(int16_t*,...)
GrB_Vector_extractElement(uint16_t*,...)
                                                 \label{lement_UINT16} GrB\_Vector\_extractElement\_UINT16(uint16\_t^*,\dots)
GrB_Vector_extractElement(int32_t*,...)
                                                 GrB_Vector_extractElement_INT32(int32_t*,...)
GrB_Vector_extractElement(uint32_t*,...)
                                                 GrB_Vector_extractElement_UINT32(uint32_t*,...)
GrB_Vector_extractElement(int64_t*,...)
                                                 GrB_Vector_extractElement_INT64(int64_t*,...)
GrB_Vector_extractElement(uint64_t*,...)
                                                 GrB_Vector_extractElement_UINT64(uint64_t*,...)
GrB_Vector_extractElement(float*,...)
                                                 GrB_Vector_extractElement_FP32(float*,...)
GrB_Vector_extractElement(double*,...)
                                                 GrB_Vector_extractElement_FP64(double*,...)
GrB_Vector_extractElement(other*,...)
                                                 GrB_Vector_extractElement_UDT(void*,...)
GrB\_Vector\_extractTuples(...,bool*,...)
                                                 GrB_Vector_extractTuples_BOOL(..., bool*,...)
GrB\_Vector\_extractTuples(...,int8\_t*,...)
                                                 GrB\_Vector\_extractTuples\_INT8(..., int8\_t*,...)
GrB\_Vector\_extractTuples(...,uint8\_t*,...)
                                                 \label{linear_struct_toples} GrB\_Vector\_extractTuples\_UINT8(..., uint8\_t^*,...)
GrB\_Vector\_extractTuples(...,int16\_t*,...)
                                                 GrB_Vector_extractTuples_INT16(..., int16_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT16(..., uint16\_t*,...)
GrB\_Vector\_extractTuples(...,uint16\_t^*,...)
GrB\_Vector\_extractTuples(...,int32\_t^*,...)
                                                 GrB_Vector_extractTuples_INT32(..., int32_t*,...)
GrB_Vector_extractTuples(...,uint32_t*,...)
                                                 GrB\_Vector\_extractTuples\_UINT32(..., uint32\_t*,...)
GrB\_Vector\_extractTuples(...,int64\_t*,...)
                                                 \label{linear_continuity} GrB\_Vector\_extractTuples\_INT64(\dots,\ int64\_t^*,\dots)
GrB_Vector_extractTuples(...,uint64_t*,...)
                                                  GrB\_Vector\_extractTuples\_UINT64(..., uint64\_t*,...)
                                                 GrB\_Vector\_extractTuples\_FP32(..., float*,...)
GrB\_Vector\_extractTuples(...,float*,...)
                                                 GrB_Vector_extractTuples_FP64(..., double*,...)
GrB\_Vector\_extractTuples(...,double*,...)
GrB_Vector_extractTuples(...,other*,...)
                                                 GrB_Vector_extractTuples_UDT(..., void*,...)
```

Table 5.4: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Nonpolymorphic signature
Polymorphic signature
GrB_Matrix_build(...,const bool*,...)
                                                 GrB_Matrix_build_BOOL(...,const bool*,...)
                                                 GrB_Matrix_build_INT8(...,const int8_t*,...)
GrB_Matrix_build(...,const int8_t*,...)
GrB_Matrix_build(...,const uint8_t*,...)
                                                 GrB_Matrix_build_UINT8(...,const uint8_t*,...)
GrB_Matrix_build(...,const int16_t*,...)
                                                 GrB_Matrix_build_INT16(...,const int16_t*,...)
GrB_Matrix_build(...,const uint16_t*,...)
                                                 GrB_Matrix_build_UINT16(...,const uint16_t*,...)
                                                 \mathsf{GrB}\_\mathsf{Matrix\_build}\_\mathsf{INT32}(\dots,\mathsf{const\ int32\_t*},\dots)
GrB_Matrix_build(...,const int32_t*,...)
GrB_Matrix_build(...,const uint32_t*,...)
                                                 GrB_Matrix_build_UINT32(...,const_uint32_t*,...)
                                                 \label{local_gradient} $\sf GrB\_Matrix\_build\_INT64(\dots,const\ int64\_t^*,\dots)$}
GrB_Matrix_build(...,const int64_t*,...)
GrB_Matrix_build(...,const uint64_t*,...)
                                                 GrB_Matrix_build_UINT64(...,const uint64_t*,...)
GrB_Matrix_build(...,const float*,...)
                                                 GrB_Matrix_build_FP32(...,const float*,...)
GrB_Matrix_build(...,const double*,...)
                                                 GrB_Matrix_build_FP64(...,const double*,...)
GrB_Matrix_build(...,const other*,...)
                                                 GrB_Matrix_build_UDT(...,const void*,...)
                                                 GrB_Matrix_setElement_Scalar(...,const GrB_Scalar,...)
GrB\_Matrix\_setElement(...,GrB\_Scalar,...)
GrB Matrix_setElement(...,bool,...)
                                                 GrB Matrix_setElement_BOOL(..., bool,...)
GrB\_Matrix\_setElement(...,int8\_t,...)
                                                 GrB_Matrix_setElement_INT8(..., int8_t,...)
GrB_Matrix_setElement(...,uint8_t,...)
                                                 \label{lement_UINT8} GrB\_Matrix\_setElement\_UINT8(..., uint8\_t,...)
GrB_Matrix_setElement(...,int16_t,...)
                                                 GrB\_Matrix\_setElement\_INT16(..., int16\_t,...)
GrB_Matrix_setElement(...,uint16_t,...)
                                                 GrB_Matrix_setElement_UINT16(..., uint16_t,...)
                                                 GrB_Matrix_setElement_INT32(..., int32_t,...)
GrB_Matrix_setElement(...,int32_t,...)
GrB_Matrix_setElement(...,uint32_t,...)
                                                 GrB_Matrix_setElement_UINT32(..., uint32_t,...)
                                                 GrB\_Matrix\_setElement\_INT64(..., int64\_t,...)
GrB_Matrix_setElement(...,int64_t,...)
GrB_Matrix_setElement(...,uint64_t,...)
                                                 GrB\_Matrix\_setElement\_UINT64(..., uint64\_t,...)
                                                 GrB_Matrix_setElement_FP32(..., float,...)
GrB\_Matrix\_setElement(...,float,...)
GrB_Matrix_setElement(...,double,...)
                                                 GrB_Matrix_setElement_FP64(..., double,...)
                                                 {\sf GrB\_Matrix\_setElement\_UDT}(\dots, {\sf const\ void*}, \dots)
GrB\_Matrix\_setElement(...,other,...)
GrB_Matrix_extractElement(GrB_Scalar,...)
                                                 GrB_Matrix_extractElement_Scalar(GrB_Scalar,...)
GrB_Matrix_extractElement(bool*,...)
                                                 GrB_Matrix_extractElement_BOOL(bool*,...)
                                                 GrB_Matrix_extractElement_INT8(int8_t*,...)
GrB_Matrix_extractElement(int8_t*,...)
GrB\_Matrix\_extractElement(uint8\_t^*,...)
                                                 GrB_Matrix_extractElement_UINT8(uint8_t*,...)
GrB_Matrix_extractElement(int16_t*,...)
                                                 GrB_Matrix_extractElement_INT16(int16_t*,...)
GrB_Matrix_extractElement(uint16_t*,...)
                                                 GrB_Matrix_extractElement_UINT16(uint16_t*,...)
GrB_Matrix_extractElement(int32_t*,...)
                                                 GrB_Matrix_extractElement_INT32(int32_t*,...)
GrB_Matrix_extractElement(uint32_t*,...)
                                                 GrB_Matrix_extractElement_UINT32(uint32_t*,...)
GrB\_Matrix\_extractElement(int64\_t^*,...)
                                                 GrB_Matrix_extractElement_INT64(int64_t*,...)
GrB_Matrix_extractElement(uint64_t*,...)
                                                 GrB_Matrix_extractElement_UINT64(uint64_t*,...)
GrB_Matrix_extractElement(float*,...)
                                                 GrB_Matrix_extractElement_FP32(float*,...)
GrB_Matrix_extractElement(double*,...)
                                                 GrB_Matrix_extractElement_FP64(double*,...)
                                                 GrB_Matrix_extractElement_UDT(void*,...)
GrB_Matrix_extractElement(other,...)
GrB_Matrix_extractTuples(..., bool*,...)
                                                 GrB\_Matrix\_extractTuples\_BOOL(..., bool*,...)
GrB_Matrix_extractTuples(..., int8_t*,...)
                                                 GrB\_Matrix\_extractTuples\_INT8(..., int8\_t*,...)
                                                 GrB\_Matrix\_extractTuples\_UINT8(..., uint8\_t*,...)
GrB_Matrix_extractTuples(..., uint8_t*,...)
GrB_Matrix_extractTuples(..., int16_t*,...)
                                                 GrB_Matrix_extractTuples_INT16(..., int16_t*,...)
                                                 \label{linear_gradient} GrB\_Matrix\_extractTuples\_UINT16(\dots, uint16\_t^*,\dots)
GrB_Matrix_extractTuples(..., uint16_t*,...)
GrB_Matrix_extractTuples(..., int32_t*,...)
                                                 GrB_Matrix_extractTuples_INT32(..., int32_t*,...)
GrB_Matrix_extractTuples(..., uint32_t*,...)
                                                 GrB_Matrix_extractTuples_UINT32(..., uint32_t*,...)
GrB_Matrix_extractTuples(..., int64_t*,...)
                                                 GrB_Matrix_extractTuples_INT64(..., int64_t*,...)
GrB_Matrix_extractTuples(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_UINT64(..., uint64_t*,...)
                                                 GrB_Matrix_extractTuples_FP32(..., float*,...)
GrB\_Matrix\_extractTuples(..., float*,...)
                                                 GrB_Matrix_extractTuples_FP64(..., double*,...)
GrB_Matrix_extractTuples(..., double*,...)
GrB_Matrix_extractTuples(...,other*,...)
                                                 GrB_Matrix_extractTuples_UDT(..., void*,...)
```

Table 5.5: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                    Nonpolymorphic signature
GrB_Matrix_import(...,const bool*,...)
                                                    GrB_Matrix_import_BOOL(...,const bool*,...)
GrB_Matrix_import(...,const int8_t*,...)
                                                    GrB_Matrix_import_INT8(...,const int8_t*,...)
GrB_Matrix_import(...,const uint8_t*,...)
                                                    GrB_Matrix_import_UINT8(...,const uint8_t*,...)
GrB_Matrix_import(...,const int16_t*,...)
                                                    GrB_Matrix_import_INT16(...,const int16_t*,...)
                                                    GrB_Matrix_import_UINT16(...,const uint16_t*,...)
GrB_Matrix_import(...,const uint16_t*,...)
                                                    GrB_Matrix_import_INT32(...,const int32_t*,...)
GrB_Matrix_import(...,const int32_t*,...)
GrB_Matrix_import(...,const uint32_t*,...)
                                                    GrB_Matrix_import_UINT32(...,const uint32_t*,...)
GrB_Matrix_import(...,const int64_t*,...)
                                                    \mathsf{GrB}\_\mathsf{Matrix}\_\mathsf{import}\_\mathsf{INT64}(\dots,\mathsf{const}\;\mathsf{int64}\_\mathsf{t*},\dots)
GrB_Matrix_import(...,const uint64_t*,...)
                                                    GrB_Matrix_import_UINT64(...,const uint64_t*,...)
                                                    GrB_Matrix_import_FP32(...,const float*,...)
GrB_Matrix_import(...,const float*,...)
GrB_Matrix_import(...,const double*,...)
                                                    GrB_Matrix_import_FP64(...,const double*,...)
                                                    GrB_Matrix_import_UDT(...,const void*,...)
GrB_Matrix_import(...,const other,...)
\overline{\mathsf{GrB}}_Matrix_export(...,bool*,...)
                                                    GrB\_Matrix\_export\_BOOL(...,bool*,...)
GrB_Matrix_export(...,int8_t*,...)
                                                    GrB_Matrix_export_INT8(...,int8_t*,...)
                                                    GrB_Matrix_export_UINT8(...,uint8_t*,...)
GrB_Matrix_export(...,uint8_t*,...)
GrB_Matrix_export(...,int16_t*,...)
                                                    GrB_Matrix_export_INT16(...,int16_t*,...)
GrB\_Matrix\_export(...,uint16\_t^*,...)
                                                    GrB\_Matrix\_export\_UINT16(...,uint16\_t*,...)
                                                    GrB\_Matrix\_export\_INT32(...,int32\_t*,...)
GrB_Matrix_export(...,int32_t*,...)
GrB_Matrix_export(...,uint32_t*,...)
                                                    GrB_Matrix_export_UINT32(...,uint32_t*,...)
                                                    GrB_Matrix_export_INT64(...,int64_t*,...)
GrB_Matrix_export(...,int64_t*,...)
GrB_Matrix_export(...,uint64_t*,...)
                                                    GrB_Matrix_export_UINT64(...,uint64_t*,...)
GrB_Matrix_export(...,float*,...)
                                                    GrB_Matrix_export_FP32(...,float*,...)
GrB\_Matrix\_export(...,double*,...)
                                                    GrB_Matrix_export_FP64(...,double*,...)
GrB_Matrix_export(...,other,...)
                                                    GrB_Matrix_export_UDT(...,void*,...)
GrB_free(GrB_Type*
                                                    GrB_Type_free(GrB_Type*)
GrB_free(GrB_UnaryOp*)
                                                    GrB_UnaryOp_free(GrB_UnaryOp*)
                                                    {\sf GrB\_IndexUnaryOp\_free}({\sf GrB\_IndexUnaryOp*})
GrB_free(GrB_IndexUnaryOp*)
GrB_free(GrB_BinaryOp*)
                                                    GrB_BinaryOp_free(GrB_BinaryOp*)
GrB_free(GrB_Monoid*)
                                                    GrB_Monoid_free(GrB_Monoid*)
                                                    GrB_Semiring_free(GrB_Semiring*)
GrB_free(GrB_Semiring*)
GrB_free(GrB_Scalar*)
                                                    GrB_Scalar_free(GrB_Scalar*)
                                                    GrB_Vector_free(GrB_Vector*)
GrB_free(GrB_Vector*)
GrB_free(GrB_Matrix*)
                                                    GrB_Matrix_free(GrB_Matrix*)
GrB_free(GrB_Descriptor*)
                                                    GrB_Descriptor_free(GrB_Descriptor*)
GrB_wait(GrB_Type, GrB_WaitMode)
                                                    GrB_Type_wait(GrB_Type, GrB_WaitMode)
GrB_wait(GrB_UnaryOp, GrB_WaitMode)
                                                    GrB_UnaryOp_wait(GrB_UnaryOp, GrB_WaitMode)
GrB_wait(GrB_IndexUnaryOp, GrB_WaitMode)
                                                    GrB_IndexUnaryOp_wait(GrB_IndexUnaryOp, GrB_WaitMode)
GrB_wait(GrB_BinaryOp, GrB_WaitMode)
                                                    GrB_BinaryOp_wait(GrB_BinaryOp, GrB_WaitMode)
GrB_wait(GrB_Monoid, GrB_WaitMode)
                                                    GrB_Monoid_wait(GrB_Monoid, GrB_WaitMode)
GrB_wait(GrB_Semiring, GrB_WaitMode)
                                                    GrB_Semiring_wait(GrB_Semiring, GrB_WaitMode)
GrB_wait(GrB_Scalar, GrB_WaitMode)
                                                    GrB_Scalar_wait(GrB_Scalar, GrB_WaitMode)
GrB_wait(GrB_Vector, GrB_WaitMode)
                                                    {\sf GrB\_Vector\_wait}({\sf GrB\_Vector},\ {\sf GrB\_WaitMode})
GrB_wait(GrB_Matrix, GrB_WaitMode)
                                                    GrB_Matrix_wait(GrB_Matrix, GrB_WaitMode)
GrB_wait(GrB_Descriptor, GrB_WaitMode)
                                                    GrB_Descriptor_wait(GrB_Descriptor, GrB_WaitMode)
GrB_error(const char**, const GrB_Type)
                                                    GrB_Type_error(const char**, const GrB_Type)
GrB_error(const char**, const GrB_UnaryOp)
                                                    GrB_UnaryOp_error(const char**, const GrB_UnaryOp)
GrB_error(const char**, const GrB_IndexUnaryOp)
                                                    GrB_IndexUnaryOp_error(const char**, const GrB_IndexUnaryOp)
GrB_error(const char**, const GrB_BinaryOp)
GrB_error(const char**, const GrB_Monoid)
                                                    GrB_BinaryOp_error(const char**, const GrB_BinaryOp)
                                                    GrB_Monoid_error(const char**, const GrB_Monoid)
                                                    GrB_Semiring_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Scalar)
                                                    GrB_Scalar_error(const char**, const GrB_Scalar)
                                                    GrB_Vector_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Matrix)
GrB_error(const char**, const GrB_Descriptor)
                                                    GrB_Matrix_error(const char**, const GrB_Matrix)
                                                    GrB_Descriptor_error(const char**, const GrB_Descriptor)
```

Table 5.6: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

```
Polymorphic signature
                                                                    Nonpolymorphic signature
GrB_eWiseMult(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseMult_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB\_Vector\_eWiseMult\_Monoid(GrB\_Vector,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_BinaryOp,...)
                                                                    GrB\_Vector\_eWiseMult\_BinaryOp(GrB\_Vector, ..., GrB\_BinaryOp, ...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB_Matrix_eWiseMult_Semiring(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseMult\_Monoid(GrB\_Matrix,...,GrB\_Monoid,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Monoid,...)
\mathsf{GrB\_eWiseMult}(\mathsf{GrB\_Matrix}, \ldots, \mathsf{GrB\_BinaryOp}, \ldots)
                                                                    GrB_Matrix_eWiseMult_BinaryOp(GrB_Matrix,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_Semiring,...)
                                                                    GrB_Vector_eWiseAdd_Semiring(GrB_Vector,...,GrB_Semiring,...)
                                                                     GrB\_Vector\_eWiseAdd\_Monoid(GrB\_Vector, \dots, GrB\_Monoid, \dots) 
GrB_eWiseAdd(GrB_Vector,...,GrB_Monoid,...)
                                                                    \label{lem:grb_vector_eWiseAdd_BinaryOp} GrB\_Vector, \dots, GrB\_BinaryOp, \dots)
GrB_eWiseAdd(GrB_Vector,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Semiring,...)
                                                                    GrB\_Matrix\_eWiseAdd\_Semiring(GrB\_Matrix,...,GrB\_Semiring,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Monoid,...)
                                                                    GrB Matrix eWiseAdd Monoid(GrB Matrix,...,GrB Monoid,...)
GrB\_eWiseAdd(GrB\_Matrix,...,GrB\_BinaryOp,...)
                                                                    \label{lem:grb_matrix_eWiseAdd_BinaryOp} GrB\_Matrix, \dots, GrB\_BinaryOp, \dots
GrB_extract(GrB_Vector,...,GrB_Vector,...
                                                                    GrB\_Vector\_extract(GrB\_Vector,...,GrB\_Vector,...)
GrB\_extract(GrB\_Matrix,...,GrB\_Matrix,...)
                                                                    GrB_Matrix_extract(GrB_Matrix,...,GrB_Matrix,...)
GrB_extract(GrB_Vector,...,GrB_Matrix,...)
                                                                    GrB\_Col\_extract(GrB\_Vector,...,GrB\_Matrix,...)
GrB_assign(GrB_Vector,...,GrB_Vector,...)
                                                                    GrB\_Vector\_assign(GrB\_Vector,...,GrB\_Vector,...)
GrB_assign(GrB_Matrix,...,GrB_Matrix,...)
                                                                    GrB_Matrix_assign(GrB_Matrix,...,GrB_Matrix,...)
\label{lem:grb_assign} $$\operatorname{\mathsf{GrB\_Matrix}},\ldots,\operatorname{\mathsf{GrB\_Vector}},\operatorname{\mathsf{const}} \ \operatorname{\mathsf{GrB\_Index}}^*,\ldots)$$
                                                                    {\sf GrB\_Col\_assign}({\sf GrB\_Matrix}, \ldots, {\sf GrB\_Vector}, {\sf const}\ {\sf GrB\_Index^*}, \ldots)
                                                                     \begin{array}{lll} & GrB\_Row\_assign(GrB\_Matrix, \ldots, GrB\_Vector, GrB\_Index, \ldots) \\ & GrB\_Vector\_assign\_Scalar(GrB\_Vector, \ldots, const \ GrB\_Scalar, \ldots) \end{array} 
GrB\_assign(GrB\_Matrix,...,GrB\_Vector,GrB\_Index,...)
GrB_assign(GrB_Vector,...,GrB_Scalar,...)
GrB_assign(GrB_Vector,...,bool,...)
                                                                    GrB_Vector_assign_BOOL(GrB_Vector,..., bool,...)
GrB_assign(GrB_Vector,...,int8_t,...)
                                                                    GrB_Vector_assign_INT8(GrB_Vector,..., int8_t,...)
GrB_assign(GrB_Vector,...,uint8_t,...)
                                                                    GrB_Vector_assign_UINT8(GrB_Vector,..., uint8_t,...)
GrB_assign(GrB_Vector,...,int16_t,...)
                                                                    GrB_Vector_assign_INT16(GrB_Vector,..., int16_t,...)
GrB_assign(GrB_Vector,...,uint16_t,...)
                                                                    GrB_Vector_assign_UINT16(GrB_Vector,..., uint16_t,...)
GrB_assign(GrB_Vector,...,int32_t,...)
                                                                    GrB_Vector_assign_INT32(GrB_Vector,..., int32_t,...)
GrB_assign(GrB_Vector,...,uint32_t,...)
                                                                    GrB_Vector_assign_UINT32(GrB_Vector,..., uint32_t,...)
GrB_assign(GrB_Vector,...,int64_t,...)
                                                                    GrB\_Vector\_assign\_INT64(GrB\_Vector,..., int64\_t,...)
GrB_assign(GrB_Vector,...,uint64_t,...)
                                                                    GrB_Vector_assign_UINT64(GrB_Vector,..., uint64_t,...)
GrB\_assign(GrB\_Vector,...,float,...)
                                                                    GrB_Vector_assign_FP32(GrB_Vector,..., float,...)
                                                                    GrB_Vector_assign_FP64(GrB_Vector,..., double,...)
GrB_assign(GrB_Vector,...,double,...)
GrB_assign(GrB_Vector,...,other,...)
                                                                    GrB_Vector_assign_UDT(GrB_Vector,...,const void*,...)
GrB_assign(GrB_Matrix,...,GrB_Scalar,...)
                                                                    GrB_Matrix_assign_Scalar(GrB_Matrix,...,const GrB_Scalar,...)
GrB_assign(GrB_Matrix,...,bool,...)
                                                                    GrB_Matrix_assign_BOOL(GrB_Matrix,..., bool,...)
                                                                    GrB\_Matrix\_assign\_INT8(GrB\_Matrix,..., int8\_t,...)
GrB_assign(GrB_Matrix,...,int8_t,...)
GrB_assign(GrB_Matrix,...,uint8_t,...)
                                                                    GrB_Matrix_assign_UINT8(GrB_Matrix,..., uint8_t,...)
GrB_assign(GrB_Matrix,...,int16_t,...)
                                                                    GrB_Matrix_assign_INT16(GrB_Matrix,..., int16_t,...)
GrB\_assign(GrB\_Matrix,...,uint16\_t,...)
                                                                    GrB\_Matrix\_assign\_UINT16(GrB\_Matrix,..., uint16\_t,...)
                                                                    GrB_Matrix_assign_INT32(GrB_Matrix,..., int32_t,...)
GrB_assign(GrB_Matrix,...,int32_t,...)
GrB_assign(GrB_Matrix,...,uint32_t,...)
                                                                    GrB_Matrix_assign_UINT32(GrB_Matrix,..., uint32_t,...)
GrB_assign(GrB_Matrix,...,int64_t,...)
                                                                    GrB_Matrix_assign_INT64(GrB_Matrix,..., int64_t,...)
GrB_assign(GrB_Matrix,...,uint64_t,...)
                                                                    GrB_Matrix_assign_UINT64(GrB_Matrix,..., uint64_t,...)
GrB_assign(GrB_Matrix,...,float,...)
                                                                    {\sf GrB\_Matrix\_assign\_FP32}({\sf GrB\_Matrix}, \ldots, \ {\sf float}, \ldots)
                                                                    GrB_Matrix_assign_FP64(GrB_Matrix,..., double,...)
GrB_assign(GrB_Matrix,...,double,...)
GrB_assign(GrB_Matrix,...,other,...)
                                                                    GrB_Matrix_assign_UDT(GrB_Matrix,...,const void*,...)
```

Table 5.7: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

	Polymorphic signature	Nonpolymorphic signature
-	GrB_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)	GrB_Vector_apply(GrB_Vector,,GrB_UnaryOp,GrB_Vector,)
	$GrB_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)$	GrB_Matrix_apply(GrB_Matrix,,GrB_UnaryOp,GrB_Matrix,)
-	GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Scalar,GrB_Vector,)	$GrB_Vector_apply_BinaryOp1st_Scalar(GrB_Vector,,GrB_BinaryOp,GrB_Scalar,GrB_Vector,)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,bool,GrB_Vector,)$	$GrB_Vector_apply_BinaryOp1st_BOOL(GrB_Vector, , GrB_BinaryOp,bool,GrB_Vector,)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,int8_t,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT8(GrB_Vector,,GrB_BinaryOp,int8_t,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,uint8_t,GrB_Vector,)$	$\label{linear_grb_def} Grb_Vector, \dots, Grb_BinaryOp, uint8_t, Grb_Vector, \dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,int16_t,GrB_Vector,)$	$\label{linear_grb_def} Grb_Vector, \dots, Grb_BinaryOp, int16_t, Grb_Vector, \dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,uint16_t,GrB_Vector,)$	$GrB_Vector_apply_BinaryOp1st_UINT16(GrB_Vector,,GrB_BinaryOp,uint16_t,GrB_Vector,)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,int32_t,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT32(GrB_Vector,,GrB_BinaryOp,int32_t,GrB_Vector,)
	$\label{lem:grb_apply} $$\operatorname{GrB_Vector},\ldots,\operatorname{GrB_BinaryOp,uint32_t,GrB_Vector},\ldots)$$	GrB_Vector_apply_BinaryOp1st_UINT32(GrB_Vector,,GrB_BinaryOp,uint32_t,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,int64_t,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_INT64(GrB_Vector,,GrB_BinaryOp,int64_t,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,uint64_t,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_UINT64(GrB_Vector,,GrB_BinaryOp,uint64_t,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,float,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_FP32(GrB_Vector,,GrB_BinaryOp,float,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,double,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_FP64(GrB_Vector,,GrB_BinaryOp,double,GrB_Vector,)
_	$GrB_apply(GrB_Vector,,GrB_BinaryOp,other,GrB_Vector,)$	GrB_Vector_apply_BinaryOp1st_UDT(GrB_Vector,,GrB_BinaryOp,const void*,GrB_Vector,)
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,GrB_Scalar,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_Scalar(GrB_Vector,\dots,GrB_BinaryOp,GrB_Vector,GrB_Scalar,\dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,bool,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_BOOL(GrB_Vector,,GrB_BinaryOp,GrB_Vector,bool,)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int8_t,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_INT8(GrB_Vector,\dots,GrB_BinaryOp,GrB_Vector,int8_t,\dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint8_t,)$	$\label{linear_gradient} GrB_Vector, \dots, GrB_BinaryOp, GrB_Vector, uint8_t, \dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int16_t,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_INT16 (GrB_Vector,,GrB_BinaryOp,GrB_Vector,int16_t,)$
\mathbf{c}	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint16_t,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_UINT16(GrB_Vector,\dots,GrB_BinaryOp,GrB_Vector,uint16_t,\dots)$
7	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int32_t,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_INT32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int32_t,)$
•	$\label{lem:grb_apply} $$\operatorname{GrB_Vector}_{,\dots,\operatorname{GrB_BinaryOp}_{,\operatorname{GrB_Vector}_{,\operatorname{uint}32_t},\dots)}$$$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_UINT32(GrB_Vector,\dots,GrB_BinaryOp,GrB_Vector,uint32_t,\dots)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int64_t,)$	$\label{linear_gradient} GrB_Vector_apply_BinaryOp2nd_INT64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,int64_t,)$
	$GrB_apply(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint64_t,)$	GrB_Vector_apply_BinaryOp2nd_UINT64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,uint64_t,)
	$\label{lem:grb_apply} $$\operatorname{GrB_Vector}_{,\dots,\operatorname{GrB_BinaryOp}_{,\operatorname{GrB_Vector}_{,\operatorname{float}_{,\dots}}})$$$	GrB_Vector_apply_BinaryOp2nd_FP32(GrB_Vector,,GrB_BinaryOp,GrB_Vector,float,)
	$GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, double,)$	GrB_Vector_apply_BinaryOp2nd_FP64(GrB_Vector,,GrB_BinaryOp,GrB_Vector,double,)
_	$GrB_apply(GrB_Vector,, GrB_BinaryOp, GrB_Vector, other,)$	GrB_Vector_apply_BinaryOp2nd_UDT(GrB_Vector,,GrB_BinaryOp,GrB_Vector,const void*,)

 ${\it Table 5.8: Long-name, nonpolymorphic form of GraphBLAS methods (continued)}.$

	Polymorphic signature	Nonpolymorphic signature
_	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_Scalar(GrB_Matrix,,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,bool,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_BOOL(GrB_Matrix,,GrB_BinaryOp,bool,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,int8_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT8(GrB_Matrix,,GrB_BinaryOp,int8_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,uint8_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT8(GrB_Matrix,,GrB_BinaryOp,uint8_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,int16_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT16(GrB_Matrix,,GrB_BinaryOp,int16_t,GrB_Matrix,)
	GrB_apply(GrB_Matrix,,GrB_BinaryOp,uint16_t,GrB_Matrix,)	GrB_Matrix_apply_BinaryOp1st_UINT16(GrB_Matrix,,GrB_BinaryOp,uint16_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,int32_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT32(GrB_Matrix,,GrB_BinaryOp,int32_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,uint32_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT32(GrB_Matrix,,GrB_BinaryOp,uint32_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,int64_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_INT64(GrB_Matrix,,GrB_BinaryOp,int64_t,GrB_Matrix,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,uint64_t,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UINT64(GrB_Matrix,,GrB_BinaryOp,uint64_t,GrB_Matrix,)
2	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,float,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_FP32(GrB_Matrix,,GrB_BinaryOp,float,GrB_Matrix,)
78	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,double,GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_FP64(GrB_Matrix,,GrB_BinaryOp,double,GrB_Matrix,)
•	$GrB_apply(GrB_Matrix,,GrB_BinaryOp, other, GrB_Matrix,)$	GrB_Matrix_apply_BinaryOp1st_UDT(GrB_Matrix,,GrB_BinaryOp,const void*,GrB_Matrix,)
	$\label{lem:grb_apply} $$\operatorname{GrB_Matrix}_{,\dots,\operatorname{GrB_BinaryOp}},\operatorname{GrB_Matrix}_{,\operatorname{GrB_Scalar}_{,\dots}}$$$	$\label{linear_grb_matrix} GrB_Matrix_apply_BinaryOp2nd_Scalar(GrB_Matrix,\ldots,GrB_BinaryOp,GrB_Matrix,GrB_Scalar,\ldots)$
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,bool,)$	GrB_Matrix_apply_BinaryOp2nd_BOOL(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,bool,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int8_t,)$	GrB_Matrix_apply_BinaryOp2nd_INT8(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int8_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint8_t,)$	GrB_Matrix_apply_BinaryOp2nd_UINT8(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint8_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int16_t,)$	GrB_Matrix_apply_BinaryOp2nd_INT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int16_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint16_t,)$	GrB_Matrix_apply_BinaryOp2nd_UINT16(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint16_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int32_t,)$	GrB_Matrix_apply_BinaryOp2nd_INT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int32_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint32_t,)$	GrB_Matrix_apply_BinaryOp2nd_UINT32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint32_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int64_t,)$	GrB_Matrix_apply_BinaryOp2nd_INT64(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,int64_t,)
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,uint64_t,)$	$\label{linear_gradient} GrB_Matrix_apply_BinaryOp2nd_UINT64(GrB_Matrix, \dots, GrB_BinaryOp, GrB_Matrix, uint64_t, \dots)$
	$GrB_apply(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,float,)$	GrB_Matrix_apply_BinaryOp2nd_FP32(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,float,)
	$GrB_apply(GrB_Matrix, \dots, GrB_BinaryOp, GrB_Matrix, double, \dots)$	$\label{linear_gradient} GrB_Matrix_apply_BinaryOp2nd_FP64 (GrB_Matrix, \dots, GrB_BinaryOp, GrB_Matrix, double, \dots)$
_	$GrB_apply(GrB_Matrix, \dots, GrB_BinaryOp, GrB_Matrix, other, \dots)$	GrB_Matrix_apply_BinaryOp2nd_UDT(GrB_Matrix,,GrB_BinaryOp,GrB_Matrix,const void*,)

Table 5.9: Long-name, nonpolymorphic form of GraphBLAS methods (continued). [Scott: NEW CONTENT]

Polymorphic signature	Nonpolymorphic signature
$\label{lem:grb_apply} $$\operatorname{GrB_Vector}_{,\dots,GrB_IndexUnaryOp}_{GrB_Vector}_{GrB_Scalar}_{,\dots)}$$	GrB_Vector_apply_IndexOp_Scalar(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,)
$GrB_apply(GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, bool, \dots)$	GrB_Vector_apply_IndexOp_BOOL(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,bool,)
$GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)$	GrB_Vector_apply_IndexOp_INT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)
$GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint8_t,)$	GrB_Vector_apply_IndexOp_UINT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint8_t,)
$GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int16_t,)$	GrB_Vector_apply_IndexOp_INT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int16_t,)
$\label{lem:grb_apply} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, uint16_t, \dots)$	GrB_Vector_apply_IndexOp_UINT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint16_t,)
$GrB_apply(GrB_Vector,, GrB_IndexUnaryOp, GrB_Vector, int 32_t,)$	GrB_Vector_apply_IndexOp_INT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int32_t,)
$GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)$	GrB_Vector_apply_IndexOp_UINT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint32_t,)
$GrB_apply(GrB_Vector,\ldots,GrB_IndexUnaryOp,GrB_Vector,int64_t,\ldots)$	GrB_Vector_apply_IndexOp_INT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int64_t,)
$GrB_apply(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint64_t,)$	GrB_Vector_apply_IndexOp_UINT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint64_t,)
$GrB_apply(GrB_Vector,\ldots,GrB_IndexUnaryOp,GrB_Vector,float,\ldots)$	GrB_Vector_apply_IndexOp_FP32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,float,)
$GrB_apply(GrB_Vector,\ldots,GrB_IndexUnaryOp,GrB_Vector,double,\ldots)$	GrB_Vector_apply_IndexOp_FP64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,double,)
$\label{eq:GrB_apply} $$ GrB_apply(GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, other, \dots)$$	GrB_Vector_apply_IndexOp_UDT(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,const void*,)
$\label{lem:grb_apply} $$\operatorname{GrB_Apply}(GrB_Matrix,\ldots,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,\ldots)$$$	GrB_Matrix_apply_IndexOp_Scalar(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,)
$GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, bool, \dots)$	GrB_Matrix_apply_IndexOp_BOOL(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,bool,)
$GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, int8_t, \dots)$	GrB_Matrix_apply_IndexOp_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)
$GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, uint8_t, \dots)$	$\begin{tabular}{ll} GrB_Matrix_apply_IndexOp_UINT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,) \end{tabular}$
$GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, int16_t, \dots)$	GrB_Matrix_apply_IndexOp_INT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int16_t,)
$GrB_apply(GrB_Matrix,\ldots,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,\ldots)$	GrB_Matrix_apply_IndexOp_UINT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)
$\label{lem:grb_apply} GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, int 32_t, \dots)$	GrB_Matrix_apply_IndexOp_INT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int32_t,)
$_{f f \bigcirc}$ GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)	GrB_Matrix_apply_IndexOp_UINT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)
\ccite{G} GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)	GrB_Matrix_apply_IndexOp_INT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int64_t,)
$\label{lem:grb_apply} GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, uint64_t, \dots)$	GrB_Matrix_apply_IndexOp_UINT64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,)
$GrB_apply(GrB_Matrix,\ldots,GrB_IndexUnaryOp,GrB_Matrix,float,\ldots)$	GrB_Matrix_apply_IndexOp_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)
$GrB_apply(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, double, \dots)$	GrB_Matrix_apply_IndexOp_FP64(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,double,)
$GrB_apply(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,other,)$	GrB_Matrix_apply_IndexOp_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)

Table 5.10: Long-name, nonpolymorphic form of GraphBLAS methods (continued).[Scott: NEW CONTENT]

	Polymorphic signature	Nonpolymorphic signature
	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,)$	
	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,bool,)$	$ GrB_Vector_select_BOOL(GrB_Vector, \ldots, GrB_IndexUnaryOp, GrB_Vector, bool, \ldots) $
	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)$	GrB_Vector_select_INT8(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int8_t,)
	$\label{lem:conditional} GrB_select(GrB_Vector,\ldots,GrB_IndexUnaryOp,GrB_Vector,uint8_t,\ldots)$	$\label{lem:grb_vector_select_UINT8} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, uint8_t, \dots)$
	$\label{lem:grb_select} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, int16_t, \dots)$	$\label{linear_gradient} GrB_Vector_select_INT16(GrB_Vector,\dots,GrB_IndexUnaryOp,GrB_Vector,int16_t,\dots)$
	$\label{lem:grb_select} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, uint16_t, \dots)$	GrB_Vector_select_UINT16(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint16_t,)
	$\label{lem:grb_select} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, int 32_t, \dots)$	GrB_Vector_select_INT32(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int32_t,)
	$\label{lem:grb_select} GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, uint 32_t, \dots)$	$\label{lem:conditional_gradient} GrB_Vector_select_UINT32(GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, uint32_t, \dots)$
	$GrB_select(GrB_Vector, \dots, GrB_IndexUnaryOp, GrB_Vector, int64_t, \dots)$	GrB_Vector_select_INT64(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,int64_t,)
	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,uint64_t,)$	$\label{lem:conditional_gradient} GrB_Vector_select_UINT64(GrB_Vector,\ldots,GrB_IndexUnaryOp,GrB_Vector,uint64_t,\ldots)$
2	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,float,)$	$\label{lem:grb_vector_select_FP32} GrB_Vector, GrB_IndexUnaryOp, GrB_Vector, float, \dots)$
80	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,double,)$	$\label{lem:grb_vector_select_FP64} GrB_Vector, GrB_IndexUnaryOp, GrB_Vector, double, \dots)$
	$GrB_select(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,other,)$	GrB_Vector_select_UDT(GrB_Vector,,GrB_IndexUnaryOp,GrB_Vector,const void*,)
	$\label{lem:grb_select} $$\operatorname{GrB_Matrix},\ldots,\operatorname{GrB_IndexUnaryOp},\operatorname{GrB_Matrix},\operatorname{GrB_Scalar},\ldots)$$$	$\label{lem:grb_matrix_select_Scalar} GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, GrB_Scalar, \dots)$
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,bool,)$	$\label{lem:grb_matrix} Grb_Matrix_select_BOOL(Grb_Matrix,\dots,Grb_IndexUnaryOp,Grb_Matrix,bool,\dots)$
	GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)	GrB_Matrix_select_INT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int8_t,)
	GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,)	GrB_Matrix_select_UINT8(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,)
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int16_t,)$	$\label{linear_gradient} GrB_Matrix_select_INT16 \big(GrB_Matrix,\dots,GrB_IndexUnaryOp,GrB_Matrix,int16_t,\dots\big)$
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)$	GrB_Matrix_select_UINT16(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,)
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,int32_t,)$	$\label{linear_gradient} GrB_Matrix_select_INT32 (GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, int32_t, \dots)$
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)$	GrB_Matrix_select_UINT32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,)
	$\label{lem:grb_select} GrB_Select \big(GrB_Matrix, \ldots, GrB_IndexUnaryOp, GrB_Matrix, int 64_t, \ldots \big)$	$\label{lem:grb_matrix} GrB_Matrix_select_INT64(GrB_Matrix,\dots,GrB_IndexUnaryOp,GrB_Matrix,int64_t,\dots)$
	$\label{lem:grb_select} GrB_Matrix, \ldots, GrB_IndexUnaryOp, GrB_Matrix, uint 64_t, \ldots)$	$\label{lem:grb_matrix} Grb_Matrix_select_UINT64 (Grb_Matrix,\dots,Grb_IndexUnaryOp,Grb_Matrix,uint64_t,\dots)$
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)$	GrB_Matrix_select_FP32(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,float,)
	$GrB_select(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,double,)$	$\label{lem:grb_matrix} GrB_Matrix_select_FP64 \big(GrB_Matrix,\dots,GrB_IndexUnaryOp,GrB_Matrix,double,\dots\big)$
	$GrB_select(GrB_Matrix, \dots, GrB_IndexUnaryOp, GrB_Matrix, other, \dots)$	GrB_Matrix_select_UDT(GrB_Matrix,,GrB_IndexUnaryOp,GrB_Matrix,const void*,)

 ${\it Table 5.11: Long-name, nonpolymorphic form of GraphBLAS methods (continued).}$

Polymorphic signature	Nonpolymorphic signature
GrB_reduce(GrB_Vector,,GrB_Monoid,)	GrB_Matrix_reduce_Monoid(GrB_Vector,,GrB_Monoid,)
$GrB_reduce(GrB_Vector,,GrB_BinaryOp,)$	GrB_Matrix_reduce_BinaryOp(GrB_Vector,,GrB_BinaryOp,)
GrB_reduce(GrB_Scalar,,GrB_Monoid,GrB_Vector,)	GrB_Vector_reduce_Monoid_Scalar(GrB_Scalar,,GrB_Vector,)
GrB_reduce(GrB_Scalar,,GrB_BinaryOp,GrB_Vector,)	GrB_Vector_reduce_BinaryOp_Scalar(GrB_Scalar,,GrB_Vector,)
GrB_reduce(bool*,,GrB_Vector,)	GrB_Vector_reduce_BOOL(bool*,,GrB_Vector,)
$GrB_reduce(int8_t^*,,GrB_Vector,)$	GrB_Vector_reduce_INT8(int8_t*,,GrB_Vector,)
GrB_reduce(uint8_t*,,GrB_Vector,)	GrB_Vector_reduce_UINT8(uint8_t*,,GrB_Vector,)
$GrB_reduce(int16_t^*,,GrB_Vector,)$	$GrB_Vector_reduce_INT16(int16_t^*,,GrB_Vector,)$
$GrB_reduce(uint16_t^*,,GrB_Vector,)$	GrB_Vector_reduce_UINT16(uint16_t*,,GrB_Vector,)
$GrB_reduce(int32_t^*,,GrB_Vector,)$	GrB_Vector_reduce_INT32(int32_t*,,GrB_Vector,)
$GrB_reduce(uint32_t^*,,GrB_Vector,)$	GrB_Vector_reduce_UINT32(uint32_t*,,GrB_Vector,)
$GrB_reduce(int64_t^*,,GrB_Vector,)$	GrB_Vector_reduce_INT64(int64_t*,,GrB_Vector,)
$GrB_reduce(uint64_t^*,,GrB_Vector,)$	GrB_Vector_reduce_UINT64(uint64_t*,,GrB_Vector,)
$GrB_reduce(float*,,GrB_Vector,)$	GrB_Vector_reduce_FP32(float*,,GrB_Vector,)
$GrB_reduce(double*,,GrB_Vector,)$	$GrB_Vector_reduce_FP64(double*,,GrB_Vector,)$
$GrB_reduce(other,,GrB_Vector,)$	$GrB_Vector_reduce_UDT(void*,,GrB_Vector,)$
$GrB_reduce(GrB_Scalar, \dots, GrB_Monoid, GrB_Matrix, \dots)$	$\label{lem:grb_matrix} GrB_Matrix_reduce_Monoid_Scalar(GrB_Scalar, \dots, GrB_Monoid, GrB_Matrix, \dots)$
$GrB_reduce(GrB_Scalar,,GrB_BinaryOp,GrB_Matrix,)$	$\label{lem:grb_matrix} GrB_Matrix_reduce_BinaryOp_Scalar(GrB_Scalar, \dots, GrB_BinaryOp, GrB_Matrix, \dots)$
$GrB_reduce(bool*,\ldots,GrB_Matrix,\ldots)$	$GrB_Matrix_reduce_BOOL(bool*,\ldots,GrB_Matrix,\ldots)$
$GrB_reduce(int8_t^*, \dots, GrB_Matrix, \dots)$	$GrB_Matrix_reduce_INT8(int8_t^*,,GrB_Matrix,)$
$GrB_reduce(uint8_t*,\ldots,GrB_Matrix,\ldots)$	GrB_Matrix_reduce_UINT8(uint8_t*,,GrB_Matrix,)
$GrB_reduce(int16_t^*,\ldots,GrB_Matrix,\ldots)$	GrB_Matrix_reduce_INT16(int16_t*,,GrB_Matrix,)
$GrB_reduce(uint16_t^*,\ldots,GrB_Matrix,\ldots)$	GrB_Matrix_reduce_UINT16(uint16_t*,,GrB_Matrix,)
$GrB_reduce(int32_t^*,\ldots,GrB_Matrix,\ldots)$	GrB_Matrix_reduce_INT32(int32_t*,,GrB_Matrix,)
$GrB_reduce(uint32_t^*, \dots, GrB_Matrix, \dots)$	GrB_Matrix_reduce_UINT32(uint32_t*,,GrB_Matrix,)
$GrB_reduce(int64_t^*, \dots, GrB_Matrix, \dots)$	$GrB_Matrix_reduce_INT64(int64_t^*, \dots, \mathsf{GrB_Matrix, \dots)}$
$GrB_reduce(uint64_t*,\ldots,GrB_Matrix,\ldots)$	$GrB_Matrix_reduce_UINT64(uint64_t^*,,GrB_Matrix,)$
GrB_reduce(float*,,GrB_Matrix,)	GrB_Matrix_reduce_FP32(float*,,GrB_Matrix,)
$GrB_reduce(double*,\ldots,GrB_Matrix,\ldots)$	GrB_Matrix_reduce_FP64(double*,,GrB_Matrix,)
GrB_reduce(other,,GrB_Matrix,)	GrB_Matrix_reduce_UDT(void*,,GrB_Matrix,)
$GrB_kronecker(GrB_Matrix,,GrB_Semiring,)$	$\label{lem:grb_matrix} GrB_Matrix_kronecker_Semiring(GrB_Matrix,\dots,GrB_Semiring,\dots)$
$GrB_kronecker(GrB_Matrix,,GrB_Monoid,)$	GrB_Matrix_kronecker_Monoid(GrB_Matrix,,GrB_Monoid,)
$GrB_kronecker(GrB_Matrix,,GrB_BinaryOp,)$	GrB_Matrix_kronecker_BinaryOp(GrB_Matrix,,GrB_BinaryOp,)

Table 5.12: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Table 5.12: Long-name, nonpol	ymorphic form of GraphBLAS methods (continued).
Polymorphic signature	Nonpolymorphic signature
GrB_get(GrB_Scalar,GrB_Scalar,GrB_Field)	GrB_Scalar_get_Scalar(GrB_Scalar,GrB_Scalar,GrB_Field)
$GrB_get(GrB_Scalar, char*, GrB_Field)$	GrB_Scalar_get_String(GrB_Scalar,char*,GrB_Field)
GrB_get(GrB_Scalar,int32_t*,GrB_Field)	GrB_Scalar_get_INT32(GrB_Scalar,int32_t*,GrB_Field)
GrB_get(GrB_Scalar,size_t*,GrB_Field)	GrB_Scalar_get_SIZE(GrB_Scalar,size_t*,GrB_Field)
GrB_get(GrB_Scalar,void*,GrB_Field)	GrB_Scalar_get_VOID(GrB_Scalar,void*,GrB_Field)
GrB get(GrB Vector, GrB Scalar, GrB Field)	GrB_Vector_get_Scalar(GrB_Vector,GrB_Scalar,GrB_Field)
GrB_get(GrB_Vector,char*,GrB_Field)	GrB_Vector_get_String(GrB_Vector,char*,GrB_Field)
GrB_get(GrB_Vector,int32_t*,GrB_Field)	GrB_Vector_get_INT32(GrB_Vector,int32_t*,GrB_Field)
GrB_get(GrB_Vector,size_t*,GrB_Field)	GrB_Vector_get_SIZE(GrB_Vector,size_t*,GrB_Field)
GrB_get(GrB_Vector,void*,GrB_Field)	GrB_Vector_get_VOID(GrB_Vector,void*,GrB_Field)
GrB_get(GrB_Matrix,GrB_Scalar,GrB_Field)	GrB Matrix get Scalar(GrB Matrix, GrB Scalar, GrB Field)
GrB_get(GrB_Matrix,char*,GrB_Field)	GrB_Matrix_get_String(GrB_Matrix,char*,GrB_Field)
GrB_get(GrB_Matrix,int32_t*,GrB_Field)	GrB_Matrix_get_INT32(GrB_Matrix,int32_t*,GrB_Field)
GrB_get(GrB_Matrix,size_t*,GrB_Field)	GrB_Matrix_get_SIZE(GrB_Matrix,size_t*,GrB_Field)
GrB_get(GrB_Matrix,void*,GrB_Field)	GrB_Matrix_get_VOID(GrB_Matrix,void*,GrB_Field)
GrB_get(GrB_UnaryOp,GrB_Scalar,GrB_Field)	GrB_UnaryOp_get_Scalar(GrB_UnaryOp,GrB_Scalar,GrB_Field)
GrB_get(GrB_UnaryOp,GrB_Scalar,GrB_Field)	GrB_UnaryOp_get_String(GrB_UnaryOp,char*,GrB_Field)
GrB_get(GrB_UnaryOp,int32_t*,GrB_Field)	GrB_UnaryOp_get_INT32(GrB_UnaryOp,int32_t*,GrB_Field)
GrB_get(GrB_UnaryOp,size_t*,GrB_Field)	GrB_UnaryOp_get_SIZE(GrB_UnaryOp,size_t*,GrB_Field)
GrB_get(GrB_UnaryOp,void*,GrB_Field)	GrB_UnaryOp_get_VOID(GrB_UnaryOp,void*,GrB_Field)
GrB_get(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)	GrB_IndexUnaryOp_get_Scalar(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)
GrB_get(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)	
GrB_get(GrB_IndexUnaryOp,char*,GrB_Field) GrB_get(GrB_IndexUnaryOp,int32_t*,GrB_Field)	GrB_IndexUnaryOp_get_String(GrB_IndexUnaryOp,char*,GrB_Field)
	GrB_IndexUnaryOp_get_INT32(GrB_IndexUnaryOp,int32_t*,GrB_Field) GrB_IndexUnaryOp_get_SIZE(GrB_IndexUnaryOp,size_t*,GrB_Field)
GrB_get(GrB_IndexUnaryOp,size_t*,GrB_Field)	
GrB_get(GrB_IndexUnaryOp,void*,GrB_Field)	GrB_IndexUnaryOp_get_VOID(GrB_IndexUnaryOp,void*,GrB_Field)
GrB_get(GrB_BinaryOp,GrB_Scalar,GrB_Field)	GrB_BinaryOp_get_Scalar(GrB_BinaryOp,GrB_Scalar,GrB_Field)
GrB_get(GrB_BinaryOp,char*,GrB_Field)	GrB_BinaryOp_get_String(GrB_BinaryOp,char*,GrB_Field)
GrB_get(GrB_BinaryOp,int32_t*,GrB_Field)	GrB_BinaryOp_get_INT32(GrB_BinaryOp,int32_t*,GrB_Field)
GrB_get(GrB_BinaryOp,size_t*,GrB_Field)	GrB_BinaryOp_get_SIZE(GrB_BinaryOp,size_t*,GrB_Field)
GrB_get(GrB_BinaryOp,void*,GrB_Field)	GrB_BinaryOp_get_VOID(GrB_BinaryOp,void*,GrB_Field)
GrB_get(GrB_Monoid,GrB_Scalar,GrB_Field)	GrB_Monoid_get_Scalar(GrB_Monoid,GrB_Scalar,GrB_Field)
GrB_get(GrB_Monoid,char*,GrB_Field)	GrB_Monoid_get_String(GrB_Monoid,char*,GrB_Field)
GrB_get(GrB_Monoid,int32_t*,GrB_Field)	GrB_Monoid_get_INT32(GrB_Monoid,int32_t*,GrB_Field)
GrB_get(GrB_Monoid,size_t*,GrB_Field)	GrB_Monoid_get_SIZE(GrB_Monoid,size_t*,GrB_Field)
GrB_get(GrB_Monoid,void*,GrB_Field)	GrB_Monoid_get_VOID(GrB_Monoid,void*,GrB_Field)
GrB_get(GrB_Semiring,GrB_Scalar,GrB_Field)	GrB_Semiring_get_Scalar(GrB_Semiring,GrB_Scalar,GrB_Field)
GrB_get(GrB_Semiring,char*,GrB_Field)	GrB_Semiring_get_String(GrB_Semiring,char*,GrB_Field)
$GrB_get(GrB_Semiring,int32_t*,GrB_Field)$	GrB_Semiring_get_INT32(GrB_Semiring,int32_t*,GrB_Field)
$GrB_get(GrB_Semiring,size_t^*,GrB_Field)$	GrB_Semiring_get_SIZE(GrB_Semiring,size_t*,GrB_Field)
GrB_get(GrB_Semiring,void*,GrB_Field)	GrB_Semiring_get_VOID(GrB_Semiring,void*,GrB_Field)
$GrB_get(GrB_Descriptor,GrB_Scalar,GrB_Field)$	GrB_Descriptor_get_Scalar(GrB_Descriptor,GrB_Scalar,GrB_Field)
$GrB_get(GrB_Descriptor, char*, GrB_Field)$	GrB_Descriptor_get_String(GrB_Descriptor,char*,GrB_Field)
$GrB_get(GrB_Descriptor,int32_t*,GrB_Field)$	GrB_Descriptor_get_INT32(GrB_Descriptor,int32_t*,GrB_Field)
$GrB_get(GrB_Descriptor, size_t^*, GrB_Field)$	GrB_Descriptor_get_SIZE(GrB_Descriptor,size_t*,GrB_Field)
$GrB_get(GrB_Descriptor, void*, GrB_Field)$	GrB_Descriptor_get_VOID(GrB_Descriptor,void*,GrB_Field)
GrB_get(GrB_Type,GrB_Scalar,GrB_Field)	GrB_Type_get_Scalar(GrB_Type,GrB_Scalar,GrB_Field)
$GrB_get(GrB_Type, char*, GrB_Field)$	GrB_Type_get_String(GrB_Type,char*,GrB_Field)
$GrB_get(GrB_Type,int32_t*,GrB_Field)$	GrB_Type_get_INT32(GrB_Type,int32_t*,GrB_Field)
$GrB_get(GrB_Type,size_t^*,GrB_Field)$	GrB_Type_get_SIZE(GrB_Type,size_t*,GrB_Field)
GrB_get(GrB_Type,void*,GrB_Field)	GrB_Type_get_VOID(GrB_Type,void*,GrB_Field)
GrB_get(GrB_Global,GrB_Scalar,GrB_Field)	GrB_Global_get_Scalar(GrB_Global,GrB_Scalar,GrB_Field)
GrB_get(GrB_Global,char*,GrB_Field)	GrB_Global_get_String(GrB_Global,char*,GrB_Field)
GrB_get(GrB_Global,int32_t*,GrB_Field)	GrB_Global_get_INT32(GrB_Global,int32_t*,GrB_Field)
GrB_get(GrB_Global,size_t*,GrB_Field)	GrB_Global_get_SIZE(GrB_Global,size_t*,GrB_Field)
GrB_get(GrB_Global,void*,GrB_Field)	GrB_Global_get_VOID(GrB_Global,void*,GrB_Field)

Table 5.13: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

9 , 1	ymorphic form of Graphblas methods (continued).
Polymorphic signature	Nonpolymorphic signature
GrB_set(GrB_Scalar,GrB_Scalar,GrB_Field)	GrB_Scalar_set_Scalar(GrB_Scalar,GrB_Scalar,GrB_Field)
$GrB_set(GrB_Scalar,char*,GrB_Field)$	GrB_Scalar_set_String(GrB_Scalar,char*,GrB_Field)
GrB_set(GrB_Scalar,int32_t,GrB_Field)	GrB_Scalar_set_INT32(GrB_Scalar,int32_t,GrB_Field)
GrB_set(GrB_Scalar,void*,GrB_Field,size_t)	GrB_Scalar_set_VOID(GrB_Scalar,void*,GrB_Field,size_t)
GrB_set(GrB_Vector,GrB_Scalar,GrB_Field)	GrB_Vector_set_Scalar(GrB_Vector,GrB_Scalar,GrB_Field)
GrB_set(GrB_Vector,char*,GrB_Field)	GrB_Vector_set_String(GrB_Vector,char*,GrB_Field)
$GrB_set(GrB_Vector,int32_t,GrB_Field)$	GrB_Vector_set_INT32(GrB_Vector,int32_t,GrB_Field)
$GrB_set(GrB_Vector,void*,GrB_Field,size_t)$	GrB_Vector_set_VOID(GrB_Vector,void*,GrB_Field,size_t)
$GrB_set(GrB_Matrix,GrB_Scalar,GrB_Field)$	GrB_Matrix_set_Scalar(GrB_Matrix,GrB_Scalar,GrB_Field)
$GrB_set(GrB_Matrix,char*,GrB_Field)$	GrB_Matrix_set_String(GrB_Matrix,char*,GrB_Field)
$GrB_set(GrB_Matrix,int32_t,GrB_Field)$	GrB_Matrix_set_INT32(GrB_Matrix,int32_t,GrB_Field)
$GrB_set(GrB_Matrix,void*,GrB_Field,size_t)$	GrB_Matrix_set_VOID(GrB_Matrix,void*,GrB_Field,size_t)
GrB_set(GrB_UnaryOp,GrB_Scalar,GrB_Field)	GrB_UnaryOp_set_Scalar(GrB_UnaryOp,GrB_Scalar,GrB_Field)
$GrB_set(GrB_UnaryOp,char*,GrB_Field)$	GrB_UnaryOp_set_String(GrB_UnaryOp,char*,GrB_Field)
$GrB_set(GrB_UnaryOp,int32_t,GrB_Field)$	GrB_UnaryOp_set_INT32(GrB_UnaryOp,int32_t,GrB_Field)
$GrB_set(GrB_UnaryOp,void*,GrB_Field,size_t)$	$\label{lem:grb_unaryOp_set_VOID} GrB_UnaryOp,void*,GrB_Field,size_t)$
$GrB_set(GrB_IndexUnaryOp,GrB_Scalar,GrB_Field)$	$\label{lindexUnaryOp_set_Scalar} GrB_IndexUnaryOp, GrB_Scalar, GrB_Field)$
$GrB_set(GrB_IndexUnaryOp,char*,GrB_Field)$	$\label{lindexUnaryOpset_String} GrB_IndexUnaryOp, char*, GrB_Field)$
$GrB_set(GrB_IndexUnaryOp,int32_t,GrB_Field)$	GrB_IndexUnaryOp_set_INT32(GrB_IndexUnaryOp,int32_t,GrB_Field)
$GrB_set(GrB_IndexUnaryOp,void*,GrB_Field,size_t)$	$\label{local-condition} GrB_IndexUnaryOp_set_VOID(GrB_IndexUnaryOp,void*,GrB_Field,size_t)$
$GrB_set(GrB_BinaryOp,GrB_Scalar,GrB_Field)$	GrB_BinaryOp_set_Scalar(GrB_BinaryOp,GrB_Scalar,GrB_Field)
$GrB_set(GrB_BinaryOp,char*,GrB_Field)$	GrB_BinaryOp_set_String(GrB_BinaryOp,char*,GrB_Field)
$GrB_set(GrB_BinaryOp,int32_t,GrB_Field)$	GrB_BinaryOp_set_INT32(GrB_BinaryOp,int32_t,GrB_Field)
$GrB_set(GrB_BinaryOp,void*,GrB_Field,size_t)$	GrB_BinaryOp_set_VOID(GrB_BinaryOp,void*,GrB_Field,size_t)
$GrB_set(GrB_Monoid,GrB_Scalar,GrB_Field)$	GrB_Monoid_set_Scalar(GrB_Monoid,GrB_Scalar,GrB_Field)
$GrB_set(GrB_Monoid,char*,GrB_Field)$	GrB_Monoid_set_String(GrB_Monoid,char*,GrB_Field)
$GrB_set(GrB_Monoid,int32_t,GrB_Field)$	GrB_Monoid_set_INT32(GrB_Monoid,int32_t,GrB_Field)
GrB_set(GrB_Monoid,void*,GrB_Field,size_t)	$GrB_Monoid_set_VOID(GrB_Monoid,void*,GrB_Field,size_t)$
$GrB_set(GrB_Semiring,GrB_Scalar,GrB_Field)$	$\label{lem:grb_semiring_set_Scalar} GrB_Semiring, GrB_Scalar, GrB_Field)$
$GrB_set(GrB_Semiring, char*, GrB_Field)$	GrB_Semiring_set_String(GrB_Semiring,char*,GrB_Field)
$GrB_set(GrB_Semiring,int32_t,GrB_Field)$	GrB_Semiring_set_INT32(GrB_Semiring,int32_t,GrB_Field)
GrB_set(GrB_Semiring,void*,GrB_Field,size_t)	$\label{lem:condition} GrB_Semiring_set_VOID(GrB_Semiring,void^*,GrB_Field,size_t)$
$GrB_set(GrB_Descriptor,GrB_Scalar,GrB_Field)$	$\label{lem:grb_def} GrB_Descriptor_set_Scalar (GrB_Descriptor, GrB_Scalar, GrB_Field)$
$GrB_set(GrB_Descriptor, char*, GrB_Field)$	GrB_Descriptor_set_String(GrB_Descriptor,char*,GrB_Field)
GrB_set(GrB_Descriptor,int32_t,GrB_Field)	GrB_Descriptor_set_INT32(GrB_Descriptor,int32_t,GrB_Field)
GrB_set(GrB_Descriptor,void*,GrB_Field,size_t)	GrB_Descriptor_set_VOID(GrB_Descriptor,void*,GrB_Field,size_t)
$GrB_set(GrB_Type,GrB_Scalar,GrB_Field)$	GrB_Type_set_Scalar(GrB_Type,GrB_Scalar,GrB_Field)
GrB_set(GrB_Type,char*,GrB_Field)	GrB_Type_set_String(GrB_Type,char*,GrB_Field)
GrB_set(GrB_Type,int32_t,GrB_Field)	GrB_Type_set_INT32(GrB_Type,int32_t,GrB_Field)
GrB_set(GrB_Type,void*,GrB_Field,size_t)	GrB_Type_set_VOID(GrB_Type,void*,GrB_Field,size_t)
GrB_set(GrB_Global,GrB_Scalar,GrB_Field)	GrB_Global_set_Scalar(GrB_Global,GrB_Scalar,GrB_Field)
GrB_set(GrB_Global,char*,GrB_Field)	GrB_Global_set_String(GrB_Global,char*,GrB_Field)
GrB_set(GrB_Global,int32_t,GrB_Field)	GrB_Global_set_INT32(GrB_Global,int32_t,GrB_Field)
GrB_set(GrB_Global,void*,GrB_Field,size_t)	GrB_Global_set_VOID(GrB_Global,void*,GrB_Field,size_t)

Appendix A

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Revision history

7532 Changes in 2.0.1 (Released: ## Xxxxx 2022:

• (Issue GH-69) Fix error in description of contents of matrix constructed from GrB_Matrix_diag.

7534 Changes in 2.0.0 (Released: 15 November 2021:

- Reorganized Chapters 2 and 3: Chapter 2 contains prose regarding the basic concepts captured in the API; Chapter 3 presents all of the enumerations, literals, data types, and predefined objects required by the API. Made short captions for the List of Tables.
 - (Issue BB-49, BB-50) Updated and corrected language regarding multithreading and completion, and requirements regarding acquire-release memory orders. Methods that used to force complete no longer do.
- (Issue BB-74, BB-9) Assigned integer values to all return codes as well as all enumerations in the API to ensure run-time compatibility between libraries.
- (Issues BB-70, BB-67) Changed semantics and signature of GrB_wait(obj, mode). Added wait modes for 'complete' or 'materialize' and removed GrB_wait(void). This breaks backward compatibility.
 - (Issue GH-51) Removed deprecated GrB_SCMP literal from descriptor values. This breaks backward compatibility.
 - (Issues BB-8, BB-36) Added sparse GrB_Scalar object and its use in additional variants of extract/setElement methods, and reduce, apply, assign and select operations.
- (Issues BB-34, GH-33, GH-45) Added new select operation that uses an index unary operator.

 Added new variants of apply that take an index unary operator (matrix and vector variants).
 - (Issues BB-68, BB-51) Added serialize and deserialize methods for matrices to/from implementation defined formats.

- (Issues BB-25, GH-42) Added import and export methods for matrices to/from API specified formats. Three formats have been specified: CSC, CSR, COO. Dense row and column formats have been deferred.
- (Issue BB-75) Added matrix constructor to build a diagonal GrB_Matrix from a GrB_Vector.
- (Issue BB-73) Allow GrB_NULL for dup operator in matrix and vector build methods. Return error if duplicate locations encountered.
- (Issue BB-58) Added matrix and vector methods to remove (annihilate) elements.
- (Issue BB-17) Added GrB_ABS_T (absolute value) unary operator.
- (Issue GH-46) Adding GrB_ONEB_T binary operator that returns 1 cast to type T (not to be confused with the proposed unary operator).
- (Issue GH-53) Added language about what constitutes a "conformant" implementation. Added GrB_NOT_IMPLEMENTED return value (API error) for API any combinations of inputs to a method that is not supported by the implementation.
- Added GrB_EMPTY_OBJECT return value (execution error) that is used when an opaque object (currently only GrB_Scalar) is passed as an input that cannot be empty.
- (Issue BB-45) Removed language about annihilators.
- (Issue BB-69) Made names/symbols containing underscores searchable in PDF.
- Updated a number algorithms in the appendix to use new operations and methods.
- Numerous additions (some changes) to the non-polymorphic interface to track changes to the specification.
- Typographical error in version macros was corrected. They are all caps: GRB_VERSION and GRB_SUBVERSION.
- Typographical change to eWiseAdd Description to be consistent in order of set intersections.
- Typographical errors in eWiseAdd: cut-and-paste errors from eWiseMult/set intersection fixed to read eWiseAdd/set union.
- Typographical error (NEQ \rightarrow NE) in Description of Table 3.8.
- 7580 Changes in 1.3.0 (Released: 25 September 2019):
- (Issue BB-50) Changed definition of completion and added GrB_wait() that takes an opaque GraphBLAS object as an argument.
- (Issue BB-39) Added GrB_kronecker operation.
- (Issue BB-40) Added variants of the GrB_apply operation that take a binary function and a scalar.

- (Issue BB-59) Changed specification about how reductions to scalar (GrB_reduce) are to be performed (to minimize dependence on monoid identity).
- (Issue BB-24) Added methods to resize matrices and vectors (GrB_Matrix_resize and GrB_Vector_resize).
- (Issue BB-47) Added methods to remove single elements from matrices and vectors (GrB_Matrix_removeElement).
- (Issue BB-41) Added GrB_STRUCTURE descriptor flag for masks (consider only the structure of the mask and not the values).
- (Issue BB-64) Deprecated GrB_SCMP in favor of new GrB_COMP for descriptor values.
- (Issue BB-46) Added predefined descriptors covering all possible combinations of field, value pairs.
- Added unary operators: absolute value ($\mathsf{GrB_ABS_}T$) and bitwise complement of integers ($\mathsf{GrB_BNOT_}I$).
- (Issues BB-42, BB-62) Added binary operators: Added boolean exclusive-nor (GrB_LXNOR)

 and bitwise logical operators on integers (GrB_BOR_I, GrB_BAND_I, GrB_BXOR_I, GrB_BXNOR_I).
- (Issue BB-11) Added a set of predefined monoids and semirings.
- (Issue BB-57) Updated all examples in the appendix to take advantage of new capabilities and predefined objects.
- (Issue BB-43) Added parent-BFS example.
- (Issue BB-1) Fixed bug in the non-batch betweenness centrality algorithm in Appendix C.4 where source nodes were incorrectly assigned path counts.
- (Issue BB-3) Added compile-time preprocessor defines and runtime method for querying the GraphBLAS API version being used.
- (Issue BB-10) Clarified GrB_init() and GrB_finalize() errors.
- (Issue BB-16) Clarified behavior of boolean and integer division. Note that GrB_MINV for integer and boolean types was removed from this version of the spec.
- (Issue BB-19) Clarified aliasing in user-defined operators.
- (Issue BB-20) Clarified language about behavior of GrB_free() with predefined objects (implementation defined)
- (Issue BB-55) Clarified that multiplication does not have to distribute over addition in a GraphBLAS semiring.
- (Issue BB-45) Removed unnecessary language about annihilators.
- (Issue BB-61) Removed unnecessary language about implied zeros.
- (Issue BB-60) Added disclaimer against overspecification.

• Fixed miscellaneous typographical errors (such as $\otimes .\oplus$).

7620 Changes in 1.2.0:

• Removed "provisional" clause.

7622 Changes in 1.1.0:

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- Removed unnecessary const from nindices, nrows, and ncols parameters of both extract and assign operations.
 - Signature of GrB_UnaryOp_new changed: order of input parameters changed.
- Signature of GrB_BinaryOp_new changed: order of input parameters changed.
- Signature of GrB_Monoid_new changed: removal of domain argument which is now inferred from the domains of the binary operator provided.
- Signature of GrB_Vector_extractTuples and GrB_Matrix_extractTuples to add an in/out argument, n, which indicates the size of the output arrays provided (in terms of number of elements, not number of bytes). Added new execution error, GrB_INSUFFICIENT_SPACE which is returned when the capacities of the output arrays are insufficient to hold all of the tuples.
- Changed GrB_Column_assign to GrB_Col_assign for consistency in non-polymorphic interface.
- Added replace flag (z) notation to Table 4.1.
 - Updated the "Mathematical Description" of the assign operation in Table 4.1.
- Added triangle counting example.
- Added subsection headers for accumulate and mask/replace discussions in the Description sections of GraphBLAS operations when the respective text was the "standard" text (i.e., identical in a majority of the operations).
- Fixed typographical errors.

7643 Changes in 1.0.2:

- Expanded the definitions of Vector_build and Matrix_build to conceptually use intermediate matrices and avoid casting issues in certain implementations.
- Fixed the bug in the GrB_assign definition. Elements of the output object are no longer being erased outside the assigned area.
- Changes non-polymorphic interface:
- Renamed GrB_Row_extract to GrB_Col_extract.

- Renamed GrB_Vector_reduce_BinaryOp to GrB_Matrix_reduce_BinaryOp.
- $\ \, \text{Renamed GrB_Vector_reduce_Monoid to GrB_Matrix_reduce_Monoid}.$
- Fixed the bugs with respect to isolated vertices in the Maximal Independent Set example.
- Fixed numerous typographical errors.

Appendix B

Son-opaque data format definitions

GraphGr

In this section, the non-opaque matrix formats specified by GrB_Format and used in matrix import and export methods are defined.

7660 B.1.1 GrB_CSR_FORMAT

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The GrB_CSR_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse row (CSR) format. indptr is a pointer to an array of GrB_Index of size nrows+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th row of the matrix. indices is a pointer to an array of number of stored elements (each a GrB_Index), where each element contains the corresponding element's column index within a row of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each row are not required to be sorted by column index.

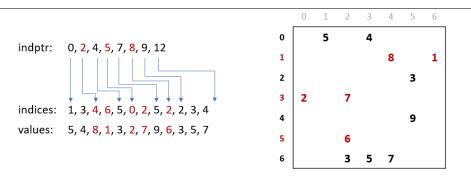


Figure B.1: Data layout for CSR format.

B.1.2 GrB_CSC_FORMAT

The GrB_CSC_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse column (CSC) format. indptr is a pointer to an array of GrB_Index of size ncols+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th column of the matrix. indices is a pointer to an array of number of stored elements (each a GrB_Index), where each element contains the corresponding element's row index within a column of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each column are not required to be sorted by row index.

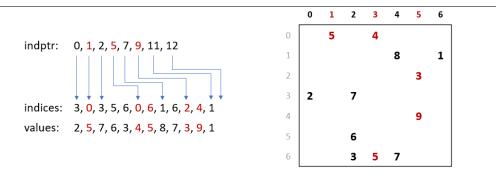


Figure B.2: Data layout for CSC format.

B.1.3 GrB_COO_FORMAT

The GrB_COO_FORMAT format indicates that a matrix will be imported or exported using the coordinate list (COO) format. indptr is a pointer to an array of GrB_Index of size number of stored elements, where each element contains the corresponding element's column index. indices will be a pointer to an array of GrB_Index of size number of stored elements, where each element contains the corresponding element's row index. values will be a pointer to an array of size number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. Elements are not required to be sorted in any order.

```
3
                                                                                    4
                                                                                         5
                                                                                             6
                                                           1
                                                                                    8
                                                                                             1
indptr:
          1, 3, 4, 6, 5, 0, 1, 5, 2, 2, 3, 4
                                                                                         3
                                                           2
indices: 0, 0, 1, 1, 2, 3, 3, 4, 5, 6, 6, 6
                                                           3
                                                                2
                                                                          7
values: 5, 4, 8, 1, 3, 2, 7, 9, 6, 3, 5, 7
                                                           4
                                                                                         9
                                                           5
                                                                          3
                                                                                   7
                                                           6
```

Figure B.3: Data layout for COO format.

 7686 Appendix C

 $_{7687}$ Examples

C.1 Example: Level breadth-first search (BFS) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reacheable from s, then v[i] = 0. (Vector v should be empty on input.)
10
11
    GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s)
13
14
      GrB_Index n;
                                                        // n = \# of rows of A
      GrB\_Matrix\_nrows(\&n,A);
15
16
                                                        // Vector < int32_t > v(n)
17
      GrB\_Vector\_new(v,GrB\_INT32,n);
18
19
      GrB_Vector q;
                                                        // vertices visited in each level
      GrB\_Vector\_new(\&q,GrB\_BOOL,n);
20
                                                        // Vector < bool > q(n)
21
      GrB_Vector_setElement(q,(bool)true,s);
                                                        // q[s] = true, false everywhere else
22
23
       * BFS traversal and label the vertices.
24
25
26
      int32 t d = 0;
                                                        // d = level in BFS traversal
27
      bool succ = false;
                                                        // succ == true when some successor found
28
      do {
29
                                                        // next level (start with 1)
30
        GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL);
                                                              // v[q] = d
31
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
                                                        // q[!v] = q ||.&& A; finds all the ||...| unvisited successors from current q
                 q, A, GrB\_DESC\_RC);
32
33
        GrB_reduce(&succ, GrB_NULL, GrB_LOR_MONOID_BOOL,
34
35
                    q, GrB_NULL);
                                                        // succ = //(q)
      } while (succ);
36
                                                        // if there is no successor in q, we are done.
37
                                                        // q vector no longer needed
38
      GrB_free(&q);
39
40
      return GrB SUCCESS;
41
```

C.2 Example: Level BFS in GraphBLAS using apply

```
#include <stdlib.h>
   #include <stdio.h>
3 #include <stdint.h>
4 #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
    * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
8
     * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
     * If i is not reachable from s, then v[i] does not have a stored element.
10
11
     * Vector v should be uninitialized on input.
12
   GrB_Info BFS(GrB_Vector *v, const GrB_Matrix A, GrB_Index s)
13
14
      GrB Index n;
15
                                                        // n = \# of rows of A
16
      GrB\_Matrix\_nrows(\&n,A);
17
18
      GrB_Vector_new(v,GrB_INT32,n);
                                                        // Vector < int32_t > v(n) = 0
19
                                                        // vertices visited in each level
20
      GrB_Vector q;
      GrB\_Vector\_new(&q,GrB\_BOOL,n);
                                                        // Vector < bool > q(n) = false
// q[s] = true, false everywhere else
21
      GrB_Vector_setElement(q,(bool)true,s);
22
23
^{24}
25
      * BFS traversal and label the vertices.
26
                                                        //\ level = depth\ in\ BFS\ traversal
27
      int32\_t level = 0;
28
      GrB_Index nvals;
29
      do {
30
        ++level;
                                                        // next level (start with 1)
        GrB_apply(*v,GrB_NULL,GrB_PLUS_INT32,
31
                   GrB\_SECOND\_INT32, q, level, GrB\_NULL); // v[q] = level
32
        GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
33
                                                        // q[!v] = q //.&&A; finds all the
34
                q, A, GrB\_DESC\_RC);
35
                                                         // unvisited successors from current q
36
        GrB_Vector_nvals(&nvals, q);
      } while (nvals);
37
                                                        // if there is no successor in q, we are done.
38
39
      GrB_free(&q);
                                                        // q vector no longer needed
40
41
      return GrB_SUCCESS;
42 }
```

C.3 Example: Parent BFS in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
   #include <stdbool.h>
   #include "GraphBLAS.h"
6
7
     * Given a binary n x n adjacency matrix A and a source vertex s, performs a BFS
8
     * traversal of the graph and sets parents[i] to the index of vertex i's parent.
     * The parent of the root vertex, s, will be set to itself (parents[s] == s). If * vertex i is not reachable from s, parents[i] will not contain a stored value.
10
11
12
    GrB\_Info\ BFS(GrB\_Vector\ *parents\ ,\ \textbf{const}\ GrB\_Matrix\ A,\ GrB\_Index\ s\ )
13
14
      GrB Index N;
15
                                                            //N = \# vertices
16
      GrB_Matrix_nrows(&N, A);
17
      GrB_Vector_new(parents, GrB_UINT64, N);
18
                                                            // parents[s] = s
      GrB_Vector_setElement(*parents, s, s);
20
21
      GrB Vector wavefront;
      GrB_Vector_new(&wavefront, GrB_UINT64, N);
22
23
      GrB_Vector_setElement(wavefront, 1UL, s);
                                                           // wavefront[s] = 1
^{24}
25
26
       * BFS traversal and label the vertices.
27
28
      GrB Index nvals;
29
      GrB_Vector_nvals(&nvals, wavefront);
30
31
      while (nvals > 0)
32
33
         // convert all stored values in wavefront to their 0-based index
        GrB_apply(wavefront, GrB_NULL, GrB_NULL, GrB_ROWINDEX_INT64,
34
35
                    wavefront , OUL, GrB_NULL);
36
        // "FIRST" because left-multiplying wavefront rows. Masking out the parent
37
         // list ensures wavefront values do not overwrite parents already stored.
38
        \label{eq:cont_state} GrB\_vxm(\,wavefront\,,\,\,*parents\,,\,\,GrB\_NULL,\,\,GrB\_MIN\_FIRST\_SEMIRING\_UINT64,
39
                  wavefront, A, GrB_DESC_RSC);
40
41
        //\ {\it Don't\ need\ to\ mask\ here\ since\ we\ did\ it\ in\ mxm.\ Merges\ new\ parents\ in}
42
         // current wavefront with existing parents: parents += wavefront
        GrB_apply(*parents, GrB_NULL, GrB_PLUS_UINT64,
44
45
                    GrB_IDENTITY_UINT64, wavefront, GrB_NULL);
46
47
        GrB_Vector_nvals(&nvals, wavefront);
48
49
50
      GrB free(&wavefront);
51
      return GrB_SUCCESS;
52
53
```

C.4 Example: Betweenness centrality (BC) in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
   #include <stdint.h>
4
   #include <stdbool.h>
   #include "GraphBLAS.h"
7
8
     * Given a boolean n x n adjacency matrix A and a source vertex s,
     st compute the BC-metric vector delta, which should be empty on input.
9
10
    GrB_Info BC(GrB_Vector *delta, GrB_Matrix A, GrB_Index s)
11
12
13
      GrB_Index n;
      GrB\_Matrix\_nrows(\&n,A);
                                                           // n = \# of vertices in graph
14
15
      GrB Vector new(delta, GrB FP32, n);
                                                           // Vector < float > delta(n)
16
17
18
      GrB_Matrix sigma;
                                                            // Matrix < int32\_t > sigma(n,n)
      GrB_Matrix_new(&sigma, GrB_INT32, n, n);
                                                           // sigma [d,k] = \# shortest paths to node k at level d
19
20
21
      GrB_Vector q;
                                                           // Vector<int32_t> q(n) of path counts
22
      GrB_Vector_new(&q, GrB_INT32, n);
                                                           // q[s] = 1
23
      GrB_Vector_setElement(q,1,s);
24
                                                            //\ \ Vector < int 32\_t > p(n) \ \ shortest \ \ path \ \ counts \ \ so \ \ far
25
      GrB_Vector p;
      GrB\_Vector\_dup(\&p, q);
26
27
      GrB\_vxm(\,q\,,p\,,GrB\_NULL,GrB\_PLUS\_TIMES\_SEMIRING\_INT32\,,
28
                                                           // get the first set of out neighbors
29
               q, A, GrB\_DESC\_RC);
30
31
       * BFS phase
32
33
      GrB\_Index d = 0;
                                                           // BFS level number
                                                           // sum == 0 when BFS phase is complete
35
      int32\_t sum = 0;
36
37
         GrB\_assign\left(sigma,GrB\_NULL,GrB\_NULL,q,d,GrB\_ALL,n,GrB\_NULL\right); \qquad // \ sigma\left[d,:\right] = q 
38
         GrB_eWiseAdd(p,GrB_NULL,GrB_NULL,GrB_PLUS_INT32,p,q,GrB_NULL); // accum path counts on this level
39
40
        GrB_vxm(q,p,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
41
                  q, A, GrB\_DESC\_RC);
                                                                                  // q = \# paths to nodes reachable
42
                                                                                        from current level
        GrB reduce(&sum, GrB NULL, GrB PLUS MONOID INT32, q, GrB NULL);
                                                                                  // sum path counts at this level
43
44
        ++d;
45
      } while (sum);
46
47
48
       * BC computation phase
49
        * (t1, t2, t3, t4) are temporary vectors
50
      GrB_Vector t1; GrB_Vector_new(&t1,GrB_FP32,n);
51
       \begin{array}{ll} GrB\_Vector & t2 \ ; & GrB\_Vector\_new(\&t2 \ ,GrB\_FP32 \ ,n \ ) \ ; \end{array} 
52
      GrB_Vector t3; GrB_Vector_new(&t3,GrB_FP32,n);
53
54
      GrB_Vector t4; GrB_Vector_new(&t4, GrB_FP32, n);
55
      for (int i=d-1; i>0; i---)
56
57
         GrB assign(t1,GrB NULL,GrB NULL,1.0f,GrB ALL,n,GrB NULL);
                                                                                      // t1 = 1 + delta
58
        GrB_eWiseAdd(t1,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,t1,*delta,GrB_NULL);
59
        GrB_extract(t2,GrB_NULL,GrB_NULL,sigma,GrB_ALL,n,i,GrB_DESC_T0);
GrB_eWiseMult(t2,GrB_NULL,GrB_NULL,GrB_DIV_FP32,t1,t2,GrB_NULL);
60
                                                                                      // t2 = sigma[i,:]
                                                                                      // t2 = (1 + delta)/sigma[i,:]
61
        GrB_mxv(t3,GrB_NULL,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_FP32,
                                                                                      // add contributions made by
62
```

```
63
64
65
66
67
68
      GrB_free(&sigma);
GrB_free(&q); GrB_free(&p);
69
70
71
      \label{eq:GrB_free} $\operatorname{GrB\_free}(\&t1)$; $\operatorname{GrB\_free}(\&t2)$; $\operatorname{GrB\_free}(\&t3)$; $\operatorname{GrB\_free}(\&t4)$;}
72
73
      return GrB_SUCCESS;
74
```

C.5 Example: Batched BC in GraphBLAS

```
#include <stdlib.h>
   #include "GraphBLAS.h" // in addition to other required C headers
2
4
    /\!/ Compute partial BC metric for a subset of source vertices, s, in graph A
   GrB Info BC update(GrB Vector *delta, GrB Matrix A, GrB Index *s, GrB Index nsver)
5
6
7
     GrB_Index n;
     GrB_Matrix_nrows(&n, A);
8
                                                            // n = \# of vertices in graph
     GrB_Vector_new(delta,GrB_FP32,n);
                                                             // Vector < float > delta(n)
9
10
     // index and value arrays needed to build numsp
11
12
     GrB_Index *i_nsver = (GrB_Index*) malloc(sizeof(GrB_Index)*nsver);
13
     int32\_t *ones = (int32\_t*) malloc(sizeof(int32\_t)*nsver);
     for(int i=0; i< nsver; ++i) {
14
15
       i_nsver[i] = i;
       ones [i] = 1;
16
17
18
     // numsp: structure holds the number of shortest paths for each node and starting vertex
19
20
      // discovered so far. Initialized to source vertices: numsp[s[i], i]=1, i=[0, nsver)
21
     GrB_Matrix numsp;
22
     GrB_Matrix_new(&numsp, GrB_INT32, n, nsver);
23
     GrB_Matrix_build(numsp,s,i_nsver,ones,nsver,GrB_PLUS_INT32);
24
     free(i_nsver); free(ones);
25
26
     // frontier: Holds the current frontier where values are path counts.
27
        Initialized to out vertices of each source node in s.
28
     GrB_Matrix frontier;
     GrB Matrix new(&frontier, GrB INT32, n, nsver);
30
     GrB_extract(frontier, numsp, GrB_NULL, A, GrB_ALL, n, s, nsver, GrB_DESC_RCT0);
31
     // sigma: stores frontier information for each level of BFS phase. The memory
32
     // for an entry in sigmas is only allocated within the do-while loop if needed.
33
      // n is an upper bound on diameter.
34
35
     GrB_Matrix *sigmas = (GrB_Matrix*) malloc(sizeof(GrB_Matrix)*n);
36
37
     int32 t d = 0;
                                                            // BFS level number
                                                            // nvals == 0 when BFS phase is complete
     GrB\_Index nvals = 0;
38
39
                           —— The BFS phase (forward sweep) —
40
41
     do {
        // sigmas [d](:,s) = d^{h} level frontier from source vertex s
42
       GrB_Matrix_new(&(sigmas[d]),GrB_BOOL,n,nsver);
43
44
       GrB\_apply(sigmas [d], GrB\_NULL, GrB\_NULL,
45
                  GrB_IDENTITY_BOOL, frontier ,GrB_NULL);
                                                            // sigmas[d](:,:) = (Boolean) frontier
46
       GrB\_eWiseAdd (numsp\,, GrB\_NULL, GrB\_NULL, GrB\_PLUS\_INT32\,,
47
48
                     numsp, frontier, GrB NULL);
                                                             // numsp += frontier (accum path counts)
       49
                                                            //\ f < !numsp > = A \ ' \ +.* \ f \ (update \ frontier)
                A, frontier, GrB_DESC_RCT0);
50
       GrB_Matrix_nvals(&nvals, frontier);
                                                            // number of nodes in frontier at this level
51
52
       d++:
53
     } while (nvals);
54
      // nspinv: the inverse of the number of shortest paths for each node and starting vertex.
55
     GrB_Matrix nspinv;
56
     GrB_Matrix_new(&nspinv,GrB_FP32,n,nsver);
57
     GrB_apply(nspinv,GrB_NULL,GrB_NULL,
58
                GrB_MINV_FP32, numsp ,GrB_NULL);
                                                            // nspinv = 1./numsp
59
60
61
      // bcu: BC updates for each vertex for each starting vertex in s
     GrB_Matrix bcu;
62
```

```
GrB_Matrix_new(&bcu,GrB_FP32,n,nsver);
63
64
      GrB assign (bcu , GrB NULL, GrB NULL,
                  1.0f, GrB_ALL, n, GrB_ALL, nsver, GrB_NULL); // filled with 1 to avoid sparsity issues
65
66
67
      GrB Matrix w;
                                                                 // temporary workspace matrix
68
      GrB_Matrix_new(&w, GrB_FP32, n, nsver);
69
70
                               — Tally phase (backward sweep) —
      for (int i=d-1; i>0; i--) {
71
        GrB\_eWiseMult (w, sigmas \cite{black} i \cite{black} i \cite{black}, GrB\_NULL,
72
73
                       74
         // add contributions by successors and mask with that BFS level's frontier
75
76
        GrB_mxm(w, sigmas[i-1], GrB_NULL, GrB_PLUS_TIMES_SEMIRING_FP32,
        \label{eq:continuous} $$ \prod_{x, w, \text{cib\_desc_R}} : // w < igmas [i-1] > = (A + .* w) $$ GrB_eWiseMult(bcu, GrB_NULL, GrB_PLUS_FP32, GrB_TIMES_FP32, w, numsp. GrB_NULL).
77
78
79
                       w, numsp, GrB_NULL);
                                                                    // bcu += w .* numsp
80
      }
81
      // row reduce bcu and subtract "nsver" from every entry to account
82
83
      // for 1 extra value per bcu row element.
      GrB_reduce(*delta,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,bcu,GrB_NULL);
84
      GrB_apply(*delta,GrB_NULL,GrB_NULL,GrB_MINUS_FP32, *delta,(float)nsver,GrB_NULL);
85
86
87
      // Release resources
88
      for (int i=0; i < d; i++) {
89
        GrB\_free(\&(sigmas[i]));
90
91
      free (sigmas);
92
93
      GrB_free(&frontier);
                                  GrB_free(&numsp);
      GrB_free(&nspinv);
                                  GrB_free(&bcu);
94
                                                          GrB_free(&w);
95
96
      return GrB_SUCCESS;
97
  }
```

C.6 Example: Maximal independent set (MIS) in GraphBLAS

```
1 #include <stdlib.h>
2 #include <stdio.h>
   #include <stdint.h>
4 #include <stdbool.h>
5 #include "GraphBLAS.h"
      Assign a random number to each element scaled by the inverse of the node's degree.
7
   // This will increase the probability that low degree nodes are selected and larger
   // sets are selected.
9
10
   void setRandom(void *out, const void *in)
11
12
      uint32\_t degree = *(uint32\_t*)in;
      *(float*)out = (0.0001f + random()/(1. + 2.*degree)); // add 1 to prevent divide by zero
13
   }
14
15
16
     * A variant of Luby's randomized algorithm [Luby 1985].
17
18
    * Given a numeric n x n adjacency matrix A of an unweighted and undirected graph (where
19
     * the value true represents an edge), compute a maximal set of independent vertices and * return it in a boolean n-vector, 'iset' where set[i] == true implies vertex\ i is a member
21
22
     * of the set (the iset vector should be uninitialized on input.)
23
24
    GrB_Info MIS(GrB_Vector *iset, const GrB_Matrix A)
25
26
      GrB Index n;
27
      GrB Matrix nrows(&n,A);
                                                        // n = \# of rows of A
28
                                                        // holds random probabilities for each node
29
      GrB Vector prob;
                                                        // holds value of max neighbor probability
30
      GrB_Vector neighbor_max;
31
      GrB_Vector new_members;
                                                        // holds set of new members to iset
                                                        // holds set of new neighbors to new iset mbrs.
      GrB_Vector new_neighbors;
32
      GrB_Vector candidates;
                                                        // candidate members to iset
33
      GrB_Vector_new(&prob, GrB_FP32, n);
35
36
      GrB_Vector_new(&neighbor_max, GrB_FP32, n);
37
      GrB_Vector_new(&new_members, GrB_BOOL, n);
38
      GrB_Vector_new(&new_neighbors,GrB_BOOL,n);
      GrB_Vector_new(&candidates, GrB_BOOL, n);
40
      GrB_Vector_new(iset ,GrB_BOOL, n);
                                                        // Initialize independent set vector, bool
41
42
      GrB_UnaryOp set_random;
      GrB\_UnaryOp\_new(\&set\_random\;, setRandom\;, GrB\_FP32\;, GrB\_UINT32\;)\;;
43
      // compute the degree of each vertex.
45
46
      GrB_Vector degrees;
      GrB\_Vector\_new(\&degrees, GrB\_FP64, n);
47
48
      GrB reduce(degrees, GrB NULL, GrB NULL, GrB PLUS FP64, A, GrB NULL);
49
50
      // Isolated vertices are not candidates: candidates[degrees !=0] = true
      GrB_assign(candidates, degrees, GrB_NULL, true, GrB_ALL, n, GrB_NULL);
51
52
      // add all singletons to iset: iset[degree == 0] = 1
53
54
      GrB_assign(*iset , degrees ,GrB_NULL, true ,GrB_ALL, n ,GrB_DESC_RC) ;
55
56
      // Iterate while there are candidates to check.
57
      GrB_Index nvals;
      GrB_Vector_nvals(&nvals, candidates);
58
59
      while (nvals > 0) {
        // compute a random probability scaled by inverse of degree
60
61
        GrB_apply(prob, candidates, GrB_NULL, set_random, degrees, GrB_DESC_R);
62
```

```
63
        // compute the max probability of all neighbors
64
        GrB mxv(neighbor max, candidates, GrB NULL, GrB MAX SECOND SEMIRING FP32, A, prob, GrB DESC R);
65
66
        //\ select\ vertex\ if\ its\ probability\ is\ larger\ than\ all\ its\ active\ neighbors\,,
        // and apply a "masked no-op" to remove stored falses
67
68
        GrB_eWiseAdd(new_members,GrB_NULL,GrB_NULL,GrB_GT_FP64,prob,neighbor_max,GrB_NULL);
69
        GrB_apply(new_members,new_members,GrB_NULL,GrB_IDENTITY_BOOL,new_members,GrB_DESC_R);
70
71
        // add new members to independent set.
        GrB_eWiseAdd(*iset,GrB_NULL,GrB_NULL,GrB_LOR,*iset,new_members,GrB_NULL);
72
73
74
        // remove new members from set of candidates c = c \mathcal{E}!new
        GrB_eWiseMult(candidates, new_members, GrB_NULL,
75
76
                       GrB_LAND, candidates, candidates, GrB_DESC_RC);
77
        GrB_Vector_nvals(&nvals, candidates);
78
79
        if (nvals == 0) { break; }
                                                       // early exit condition
80
        // Neighbors of new members can also be removed from candidates
81
        GrB_mxv(new_neighbors, candidates, GrB_NULL, GrB_LOR_LAND_SEMIRING_BOOL,
82
83
                A, new_members, GrB_NULL);
        GrB\_eWiseMult(candidates, new\_neighbors, GrB\_NULL, GrB\_LAND,
84
                       candidates, candidates, GrB_DESC_RC);
85
86
87
        GrB\_Vector\_nvals(\&nvals\;,\; candidates\;)\;;
88
89
      GrB_free(&neighbor_max);
                                                       // free all objects "new'ed"
90
91
      GrB_free(&new_members);
      GrB_free(&new_neighbors);
92
93
      GrB_free(&prob);
      GrB_free(&candidates);
94
      GrB_free(&set_random);
95
96
      GrB_free(&degrees);
97
98
      return GrB_SUCCESS;
99
```

C.7 Example: Counting triangles in GraphBLAS

```
#include <stdlib.h>
   #include <stdio.h>
 3 #include <stdint.h>
 4 #include <stdbool.h>
   #include "GraphBLAS.h"
 6
 7
     * Given an n x n boolean adjacency matrix, A, of an undirected graph, computes
 8
     st the number of triangles in the graph.
10
    uint64_t triangle_count(GrB_Matrix A)
11
12
      GrB_Index n;
13
14
      GrB_Matrix_nrows(&n, A);
                                                             // n = \# of vertices
15
      // L: NxN, lower-triangular, bool
16
      GrB_Matrix L;
17
18
      GrB_Matrix_new(&L, GrB_BOOL, n, n);
      \label{eq:conditional_grb_null} $\operatorname{GrB\_NULL}, \ \operatorname{GrB\_NULL}, \ \operatorname{GrB\_TRIL}, \ A, \ \operatorname{OUL}, \ \operatorname{GrB\_NULL});$
20
21
      GrB_Matrix C;
22
      GrB\_Matrix\_new(\&C, GrB\_UINT64, n, n);
23
24
      25
26
      uint64 t count;
      \label{eq:GrB_reduce} $$\operatorname{GrB\_NULL}, $\operatorname{GrB\_PLUS\_MONOID\_UINT64}, $\operatorname{C}, $\operatorname{GrB\_NULL})$;}
27
                                                                                        // 1-norm of C
28
29
      GrB_free(&C);
30
      GrB_free(&L);
31
32
      return count;
33 }
```