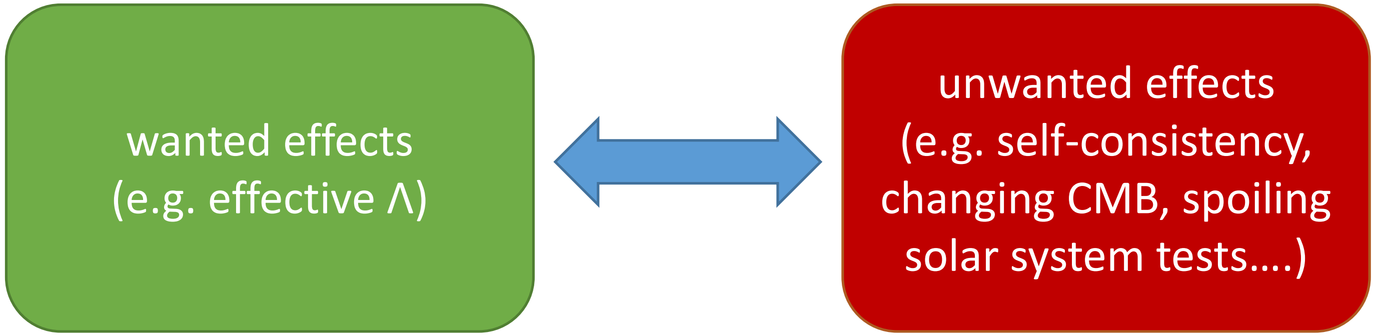
Part II: Models of dark energy and modified gravity

1. Dynamical dark energy: quintessence
2. Examples of modified gravity
3. Structure formation as a model discriminator
4. *(Observational tests)*

Theories of dark energy

- A useful approach is to introduce features as new parameters in observables. These can then be probed by experiments/observations
- This needs to be complemented by theories: provide a self-consistent description to justify a specific effect.
- Challenge: A single parameter in a consistent theoretical framework may contaminate several observables. “A bug or a feature?”



Dynamics of dark energy
Copeland et al. **hep-th/0603057**

Modified Gravity and Cosmology
Clifton et al. **arXiv:1106.2476**

Quintessence: $w_{DE}(t)$ from single scalar field

In field theory, explicit t dependence $\Leftarrow$ VEV of a (clock) field.

We now know that (at least one) scalar field exists

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_\phi$$

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - V(\phi)\right]$$

energy momentum tensor

$$T^\phi_{\mu\nu} = \left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(\partial_\alpha\phi\right)\left(\partial_\beta\phi\right) + V(\phi)\right] \qquad T^\phi_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_\phi)}{\delta g^{\mu\nu}}$$

scalar field equation

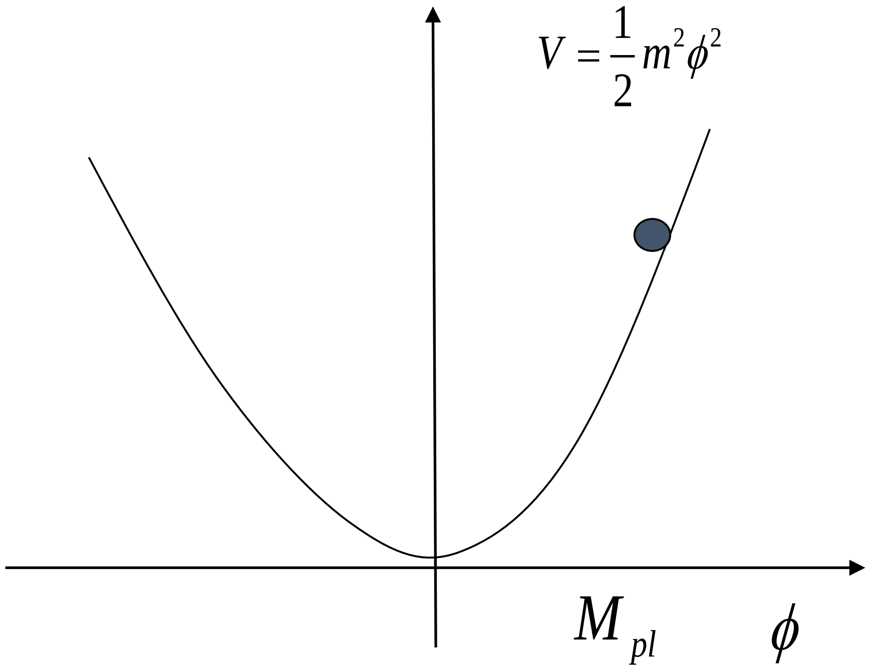
$$\frac{\delta S_\phi}{\delta\phi} = 0 \qquad \nabla^\mu\nabla_\mu\phi - V'(\phi) = 0 \Longleftrightarrow \nabla^\mu T^\phi_{\mu\nu} = 0$$

Background

- Energy density and pressure

$$\rho_\phi = -T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad \phi = \phi(t)$$

$$P_\phi = \frac{1}{3}T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$



- Scalar field equation

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + V'(\phi) = 0 \quad \Longleftrightarrow \quad \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

friction

For accelerated expansion, the scalar field needs to slowly roll down the potential

$$\dot{\phi}^2 << V(\phi) \qquad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1$$

Quintessence potentials

- Freezing model, e.g.

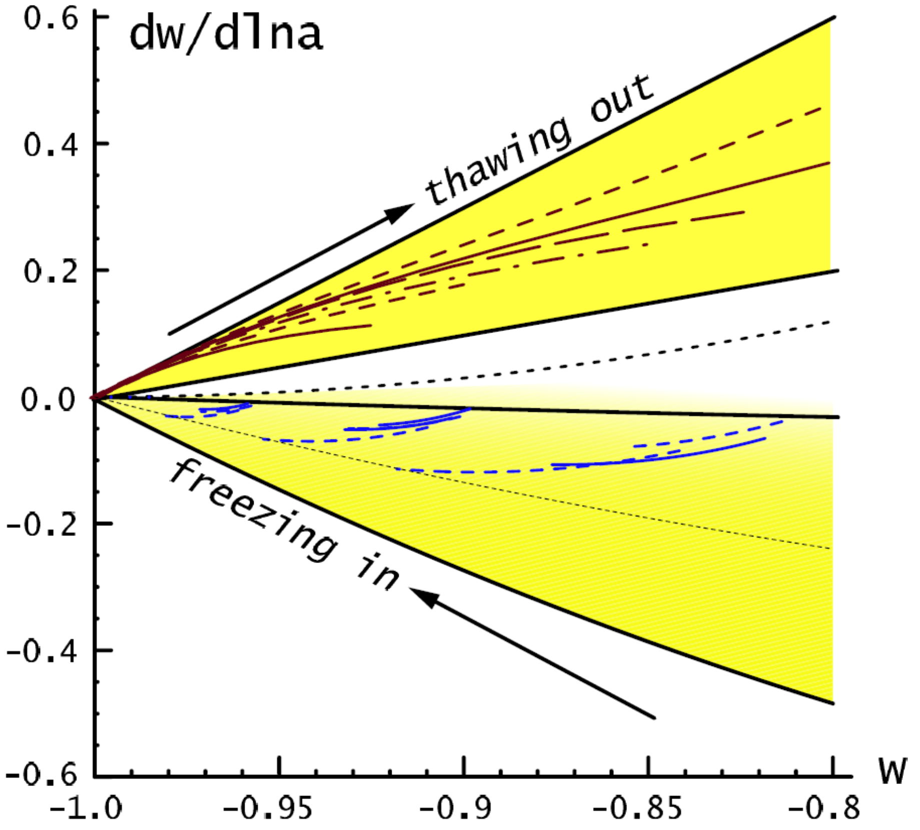
$$V(\phi) = M^{4+n}\phi^{-n}$$

The scalar field dynamics freezes at late time and approaches $w_\phi \approx -1$

- Thawing models, e.g.

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$

Scalar initially held at $w_\phi \approx -1$ by Hubble friction. Recently started to roll down towards minimum. (similar to inflation)



Linder & Caldwell astro-ph/0505494

Fundamental issues

- Consider a massive scalar field

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \ , \quad \varepsilon_\phi \equiv \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2 = \frac{2{M_{pl}}^2}{\phi^2} << 1 \quad \rightarrow \quad \phi > M_{pl} = 10^{18}\,\mathrm{GeV}$$

$$V(\phi_0) = \rho_{DE} = 10^{-48}\,\mathrm{GeV}^4 \quad \rightarrow \quad m = \left(\frac{\rho_{DE}}{\phi^2} \right)^{\frac{1}{2}} = 10^{-42}\,\mathrm{GeV} \approx H_0$$

Correction to the potential

$$V(\phi) = \frac{1}{4} \lambda \phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda < 10^{-120}$$

these small numbers are very difficult to protect against quantum corrections

Lovelock's theorem [1971]

- GR+Λ is the unique metric theory assuming:

a. EOM originates from an action
b. 3+1 dimensions
c. EOM has at most 2^{nd} order derivatives
d. no other (hidden or explicit) fields
e. diffeomorphism invariant action
f. no non-local operators

- A modified theory of GR should break at least one of these. This lecture briefly focusses only on breaking (d), as it includes breaking either of (b),(c),(e).

The fuzzy line between dynamical DE and MG

- We will modify gravity by adding new fields.
- Wait… Quintessence *is* a new field. Is it modified gravity?
- Dark energy nomenclature proposed by Clifton, Ferreira, Padilla, Skordis [arXiv:1106.2476]

New matter fields → minimal coupling to gravity, i.e. $S = \int \sqrt{-g}\, \mathcal{L}_\phi$

examples: quintessence, k-essence

Modified gravity → non-minimal coupling, e.g. $\sqrt{-g}\, f(\phi) R$

examples: $f(R)$, generic Horndeski theory, massive gravity

- Warning: not everyone follows this terminology, e.g. non-minimal quintessence

Horndeski Theory [1974]

Scalar-Tensor extension of Lovelock

Independently discovered by Deffayet et al. 2011

- **Lagrangian**

$$\mathscr{L}_H = \sum_{i=2}^{5} \mathscr{L}_i .$$

$$\mathscr{L}_2 = K(\phi, X),$$

$$\mathscr{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$\mathscr{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$\mathscr{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$X \equiv -\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Three examples:

K-essence

$K(X), G_4 = \frac{M_p^2}{2}$

→ **Dynamical DE**

Covariant Galileon

$K, G_3 \sim X, G_5 \sim X^2$

$G_4 \sim \frac{M_p^2}{2} + (..)X^2$

→ **Self-acceleration**

Fab Four

$$G_2 = 2V''_{\rm john}(\phi)X^2 - V^{(3)}_{\rm Paul}(\phi)X^3 + 6V''_{\rm george}(\phi)X + 8V^{(3)}_{\rm ringo}(\phi)X^2(3 - \ln(|X|)),$$

$$G_3 = 3V'_{\rm john}(\phi)X - \frac{5}{2}V''_{\rm paul}(\phi)X^2 + 3V'_{\rm george}(\phi) + 4V^{(3)}_{\rm ringo}(\phi)X(7 - 3\ln(|X|)),$$

$$G_4 = V_{\rm john}(\phi)X - V'_{\rm paul}(\phi)X^2 + V_{\rm george}(\phi) + 4V''_{\rm ringo}(\phi)X(2 - \ln(|X|)),$$

$$G_5 = -3V_{\rm paul}(\phi)X - 4V'_{\rm ringo}(\phi)\ln(|X|).$$

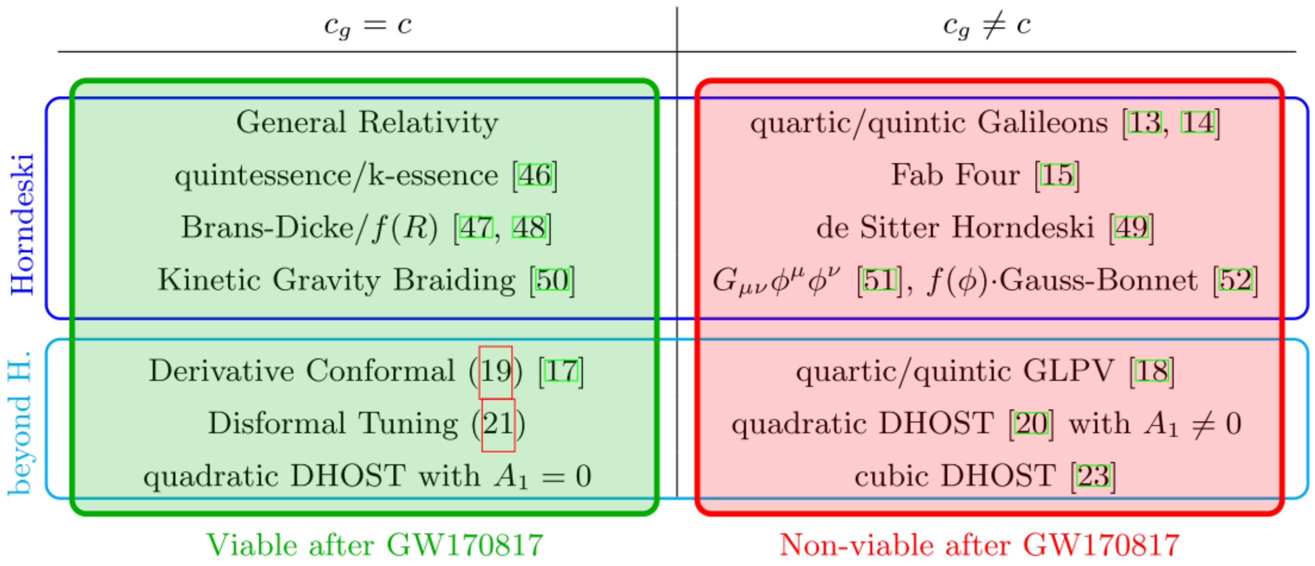
→ **Self-tuning**

Generically, Horndeski theories have (potentially dangerous) implications: $\dot{G}_N \neq 0$, speed of GW$\neq 1$ etc…

Enter GW170817/GRB170817A

- BNS merger, time delay between GW and GRB $\sim$1.74s
- Conservative limit, $d = 26\text{Mpc}$, $\Delta t \sim 10\text{s}$ gives

$$-3 \times 10^{-15} < \frac{c_g}{c} - 1 < 0.7 \times 10^{-15}$$

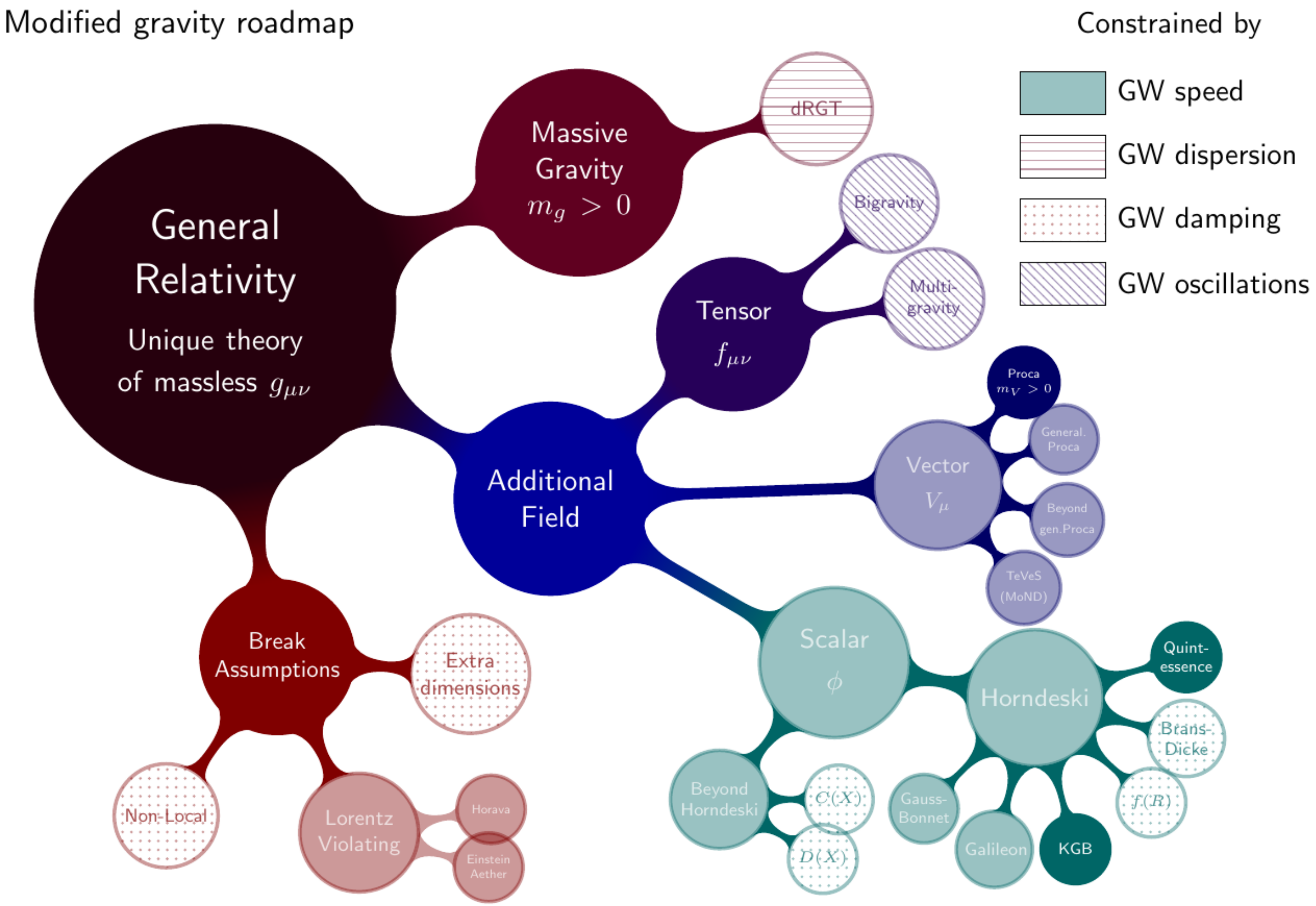


Ezquiaga, Zumalacàrregui
arXiv:1710.05901

FIG. 2: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. Only simple Horndeski theories, $G_{4,X} \approx 0$ and $G_5 \approx$ constant, and specific beyond Horndeski models, conformally related to $c_g = 1$ Horndeski or disformally tuned, remain viable.

Scalar-Tensor theory is not the only way

- Beyond Horndeski, DHOST (degeneracies)
- Other fields, e.g. massive gravity, bimetric theory;
- From higher dimensions: DBI;
- Non-local actions, e.g. non-local GR/massive gravity;
- Constrained actions, e.g. unimodular gravity;
- Non-metric, e.g. emergent scenarios, squared Yang-Mills;
- and many more…



Ezquiaga, Zumalacàrregui arXiv:1807.09241

Summary of models

- DE/MG models

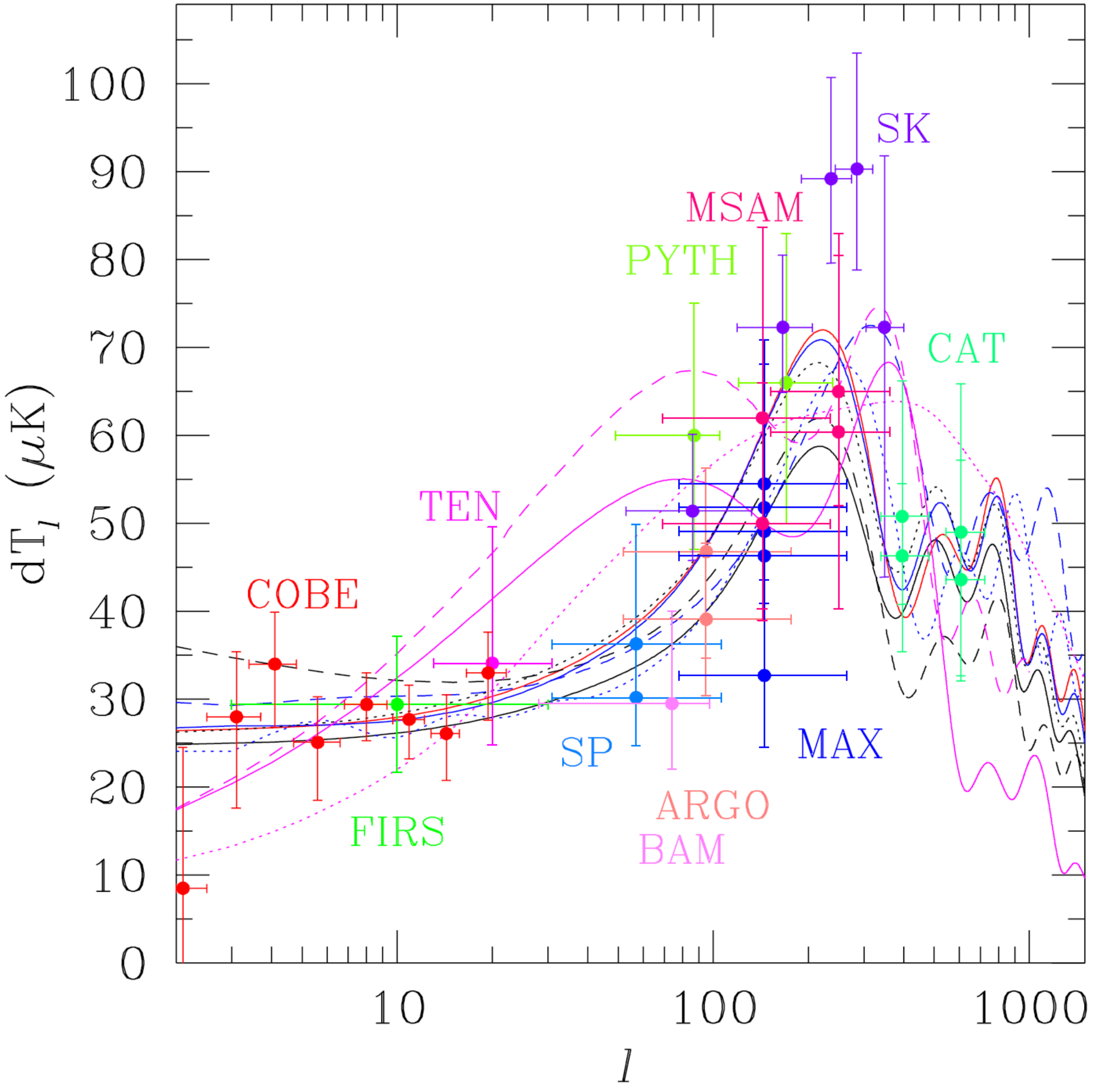
There are many models but we still do not have a compelling alternative to ΛCDM

Observations may give us a clue:

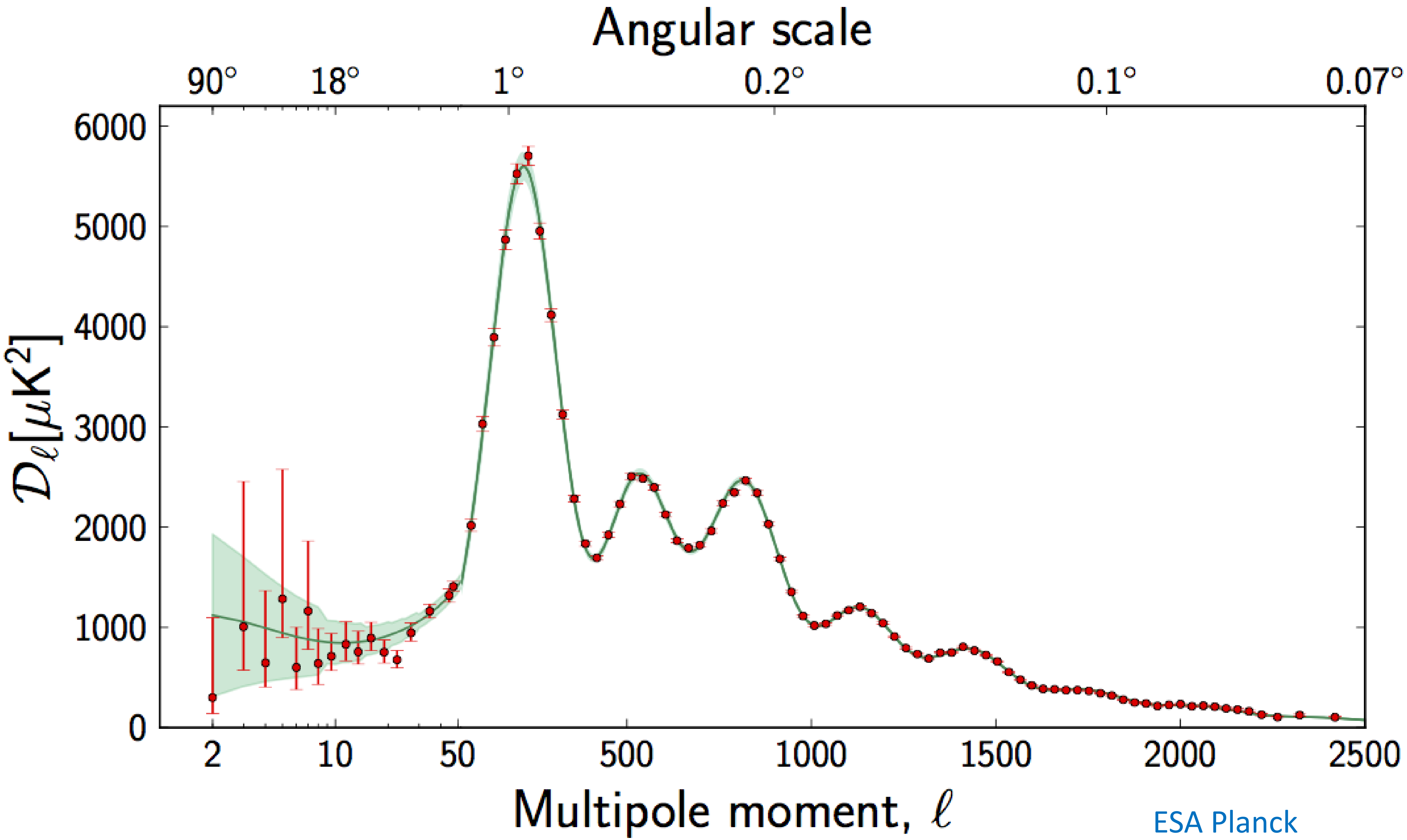
Is it the cosmological constant or a light degree of freedom?

Does it cluster or couple to matter?

cf. CMB (before WMAP)



Gawiser & Silk astro-ph/9806197



Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell[\mu K^2]$
2
10
50
500
1000
1500
2000
2500
Multipole moment, ℓ

ESA Planck

How to test DE/MG models

- Einstein equations

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{M} + \kappa^2 E_{\mu\nu}, \quad E_{\mu\nu} = T_{\mu\nu}^{DE} + G_{\mu\nu}^{MG} \qquad \nabla^{\mu}(T_{\mu\nu}^{M} + E_{\mu\nu}) = 0$$

- Background (homogeneity & Isotropy) $E_{\nu}^{\mu} = \mathrm{diag}\,(-\rho_E, P_E, P_E, P_E)$

  everything is determined by the equation of state $w_E = P_E / \rho_E$

- Small Inhomogeneity [in Newtonian gauge]

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right] \qquad H = \frac{a'}{a}$$

Linear scalar perturbations with respect to 3-space (assumed to be flat)

Cosmological perturbation theory

- Fourier transformation and decomposition
- Scalar harmonics:

$$S \propto e^{i\,k_j x^j}\,, \quad S_j = -i\,\hat{k}_j S, \quad S_{ij} = \left(\frac{\delta_{ij}}{3} - \hat{k}_i\,\hat{k}_j\right) S$$

- $\nabla^2 S = -k^2 S$ and $S^i_i = 0$
- Any scalar, vector and tensor can be written in terms of these:

$$V_i = V\,S_i, \qquad\qquad A_{ij} = A_L \delta_{ij} + A_T S_{ij}$$

Kodama & Sasaki 1985
Mukhanov, Feldman, Brandenberger 1990
Malik & Wands 2008

Matter content

- Energy momentum tensor

$$T^{\mu}_{I\nu} = \begin{pmatrix} -(\rho_I + \delta\rho_I), & (\rho_I + P_I) v_{I\,i} \\ -(\rho_I + P_I) v_I{}^i, & \left(P_I + \delta P_I\right)\delta^i_j + P_I\, \Pi_{I\,j}{}^i \end{pmatrix}$$

$$v^i = v\, S^i, \quad : \text{velocity}$$

$$\Pi^i_j = \Pi\, S^i_j : \ \text{anisotropic stress}$$

- Conservation of energy momentum tensor

for now, we assume matter and dark component obeys the conservation independently [if not assumed, *interacting DE:* $\nabla^{\mu}(E_{\mu\nu}) = Q^{DE}_{\nu}$]

$$\frac{d\delta\rho_I}{d\eta} + 3H(\delta\rho_I + \delta P_I) = -(\rho_I + P_I)(kv_I - 3\dot{\Phi})$$

$$\left(\frac{d}{d\eta} + 4H\right)\left[\frac{(\rho_I + P_I)v_I}{k}\right] = \delta P_I - \frac{2}{3} P_I\, \Pi_I + (\rho_I + P_I)\Psi$$

$$w_m = P_m / \rho_m = 0, \quad \delta P_m = \Pi_m = 0,$$

$$w_E = P_E / \rho_E, \quad \delta P_E, \quad \Pi_E,$$

Equations for linear perturbations

- Einstein equations

$$k^2\Phi = -4\pi G a^2(\rho_m\Delta_m + \rho_E\Delta_E), \quad \rho_I\Delta_I = \delta\rho_I + 3(\rho_I + P_I)\frac{\mathcal{H}}{k}v_I$$

$$k^2(\Psi - \Phi) = -8\pi G a^2 P_E\,\Pi_E$$

- Conservation of energy momentum tensor for matter $\left(k/\mathcal{H}\right)^2 >> 1$

$$\Delta_m' = -\mathcal{H}\,\theta_m, \qquad \theta_m = \left(k/\mathcal{H}\right)v_m: \text{ velocity divergence}$$

$$\theta_m' + \mathcal{H}\left(1 + \frac{\mathcal{H}'}{\mathcal{H}^2}\right)\theta_m = \frac{k^2}{\mathcal{H}}\Psi \quad \Longrightarrow \quad \Delta_m'' + \mathcal{H}\,\Delta_m' = -k^2\Psi$$

Evolution of matter is determined by the Newtonian potential

Dark component affects the evolution through the Newtonian potential

Growth of structure – LCDM

1) LCDM $E_{\mu\nu} = -\Lambda g_{\mu\nu}$

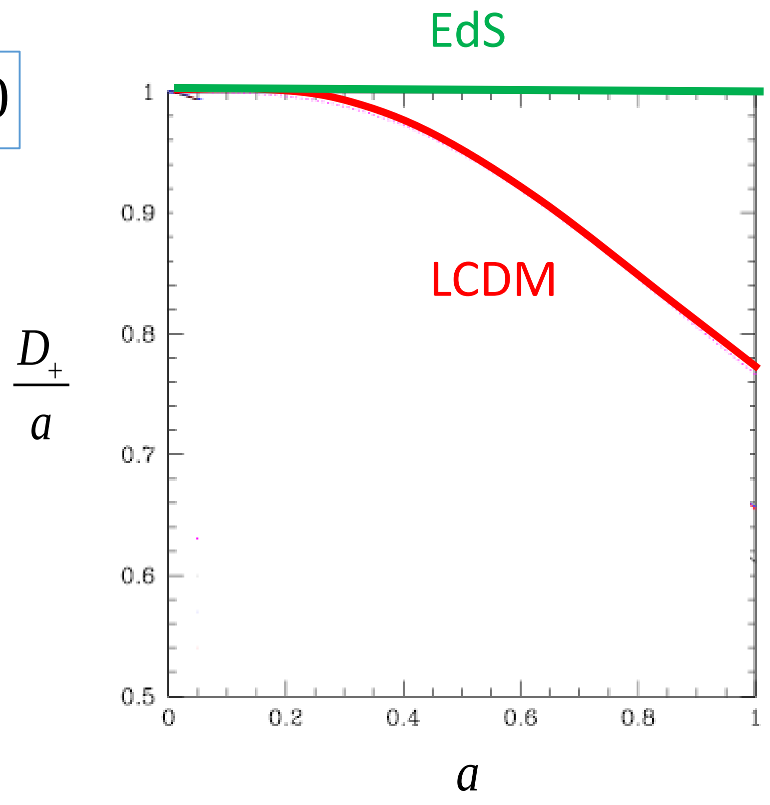
$$k^2\Phi = -4\pi G a^2 \rho_m \Delta_m$$
$$\Phi - \Psi = 0$$

$$\Rightarrow \boxed{\Delta_m'' + H\,\Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0}$$

We define the growth function D_+ as the growing mode solution for Δ_m

in MD era, $a \propto \eta^2, H^2 = 8\pi G a^2 \rho_m / 3 \Rightarrow D_+ \propto a$

at late times, due to the cosmological constant, gravity becomes weaker $H^2 = \dfrac{8\pi G a^2}{3}(\rho_m + \rho_\Lambda)$



Growth of structure – smooth DE

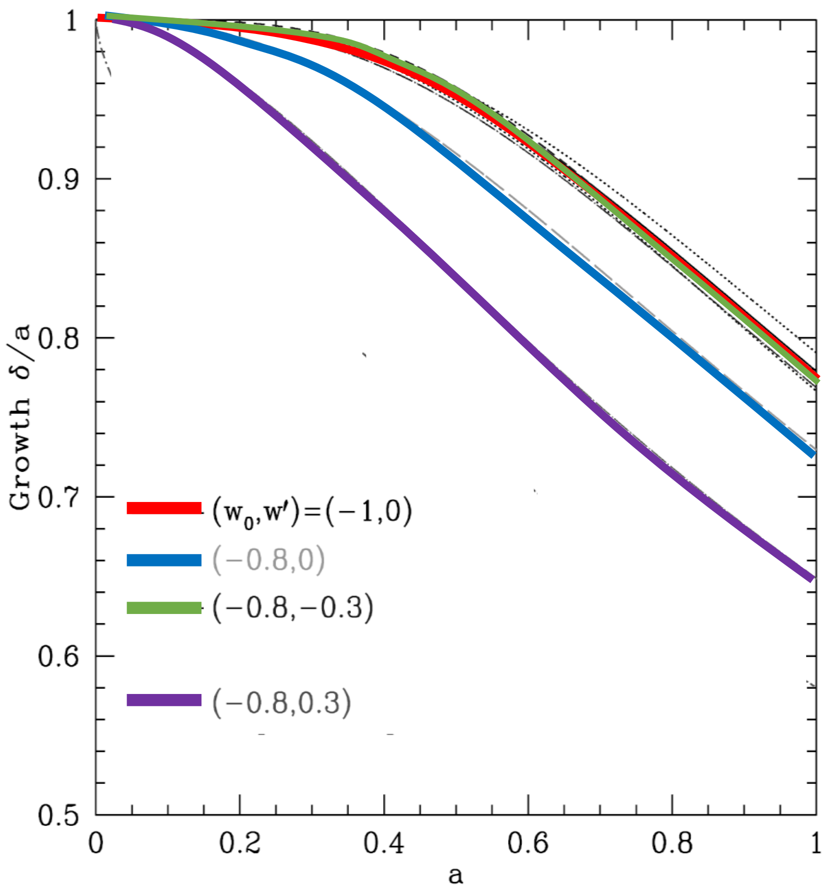
2) smooth DE $\delta\rho_E = \pi_E = 0$

using $N = \ln a$ $\Delta_m'' + \mathcal{H}\,\Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$

$$\ddot{D}_+ + \frac{1}{2}(1 - 3\Omega_{DE} w_{DE})\dot{D}_+ - \frac{3}{2}\Omega_m D_+ = 0$$

$$\dot{\Omega}_m = 3w_{DE}(1-\Omega_m)\Omega_m, \quad \Omega_{DE} = 1 - \Omega_m$$

$$\frac{D_+}{a}$$



Linder astro-ph/0305286

For a fixed present-day Ω_{DE} ,if $w_{DE} > -1$,DE density is larger in the past suppressing the growth compared with LCDM

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$

Growth rate

- Growth rate

$$f = \frac{d\ln\Delta_m}{d\ln a} = \frac{a}{D_+}\frac{dD_+}{da}$$

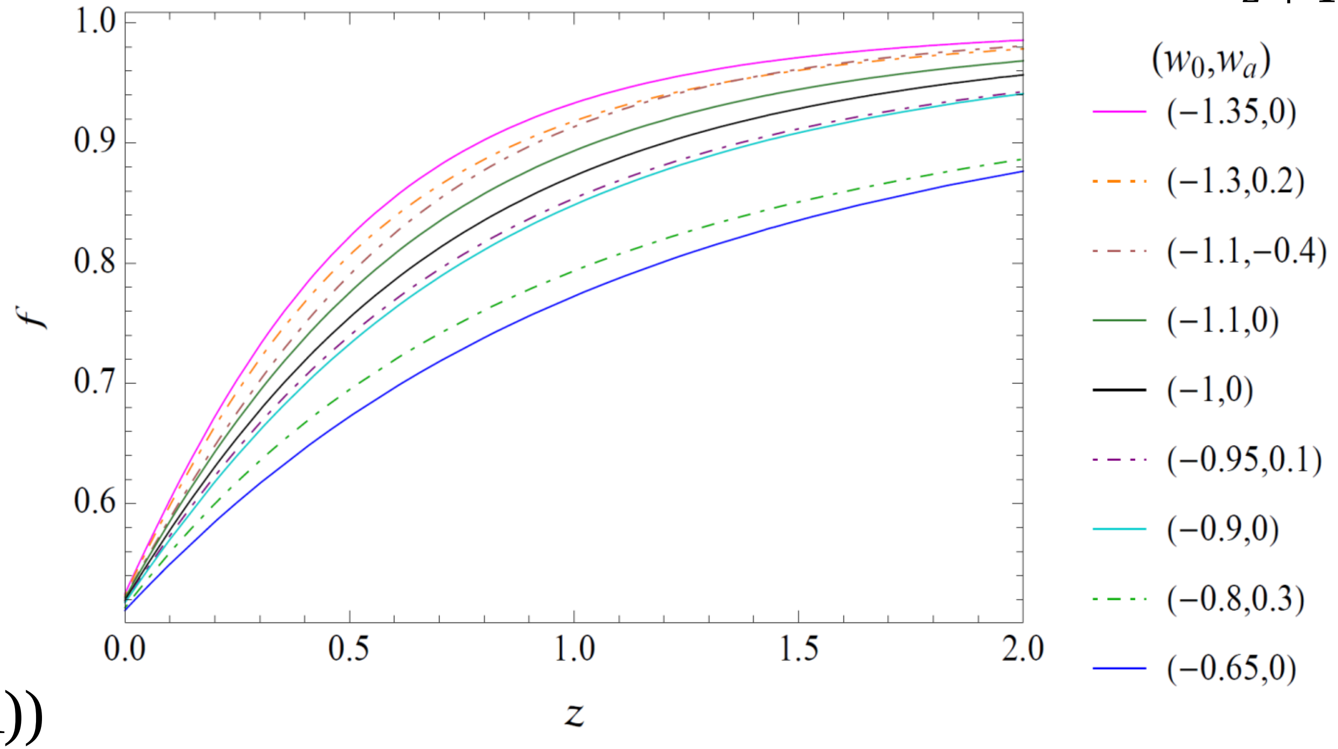
$$\dot{f} + f^2 + \left[\frac{1}{2} - \frac{3}{2}w_{DE}(1-\Omega_m)\right] f = \frac{3}{2}\Omega_m$$

$$f = \Omega_m^\gamma, \quad \gamma = 0.545 + 0.05(1 + w_{DE}(z=1))$$

γ is insensitive to the equation of state w_{DE}

(but the growth rate depends on w_{DE} through Ω_m)

$$w = w_0 + w_a \frac{z}{z+1}$$



Dossett & Ishak 1311.0726

Growth of structure – clustering DE

3) Clustering DE $\delta\rho_E \neq 0 \quad (\pi_E = 0)$

Let's consider a toy model for dark component with non-zero sound speed

$$\delta P_E = c_{sE}^2 \delta\rho_E$$

assuming that the dark component dominates the universe

$$\Delta_E'' + H\,\Delta_E' + \left(c_{sE}^2 k^2 - 4\pi G a^2 \rho_E\right)\Delta_E = 0$$

For $k > k_J = \sqrt{\dfrac{4\pi G a^2 \rho_E}{c_{sE}}}$ pressure wins over gravity and Δ_E does not grow

clustering DE requires small sound speed $\quad \left(k_J / a\right)^{-1}$: Jean's length

Quintessence

- Sound speed

$$c_{s\phi}^2 \equiv \left.\frac{\delta P_\phi}{\delta \rho_\phi}\right|_{v_\phi=0} = 1, \qquad v_\phi \propto \delta\phi$$

propagation speed

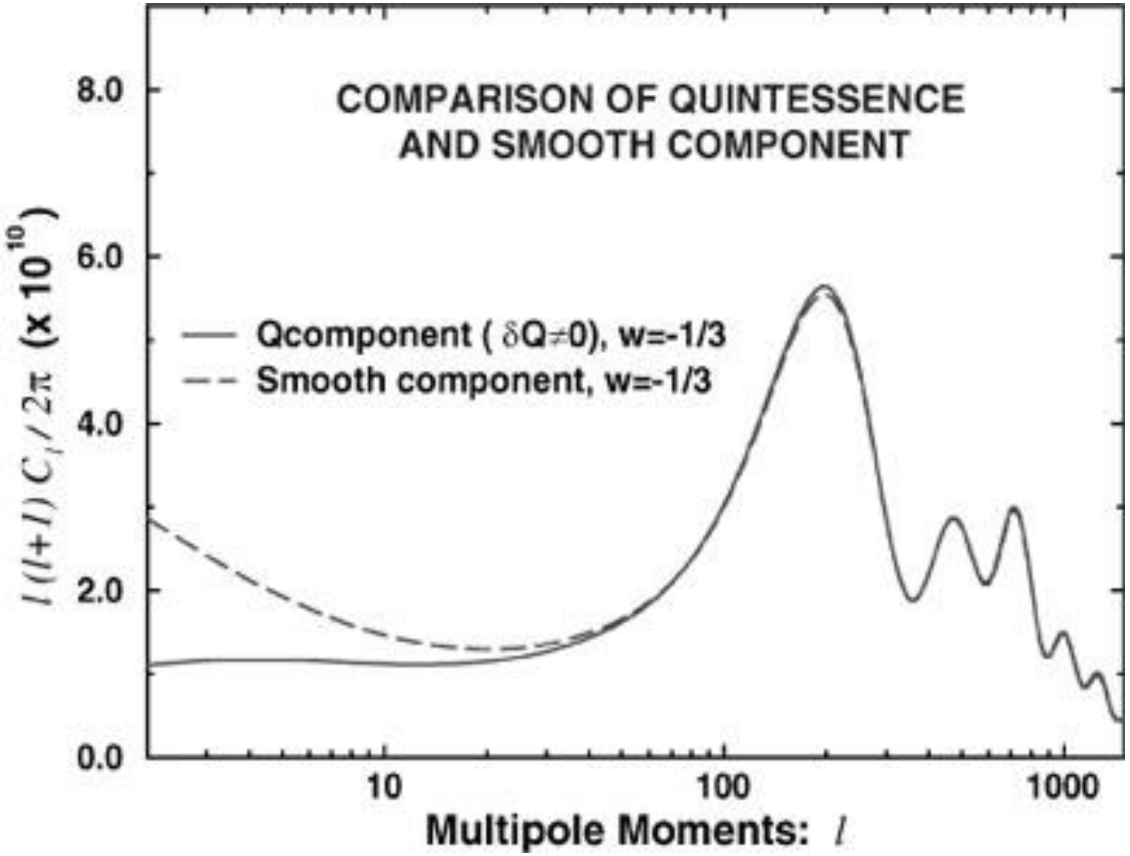
scalar field equation of motion $\quad S_\phi = \int d^4x \left[-\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi\right)\left(\partial_\nu \phi\right) - V(\phi) \right]$

$$\delta\ddot{\phi} + 3H\delta\phi + c_{p\phi}^2 \frac{k^2}{a^2} \delta\phi + \ldots = 0$$

for standard kinetic term, $c_{p\phi}^2 = c_{s\phi}^2 = 1$ thus the scalar field does not cluster below the horizon scale thus can be approximated as smooth DE $\delta\rho_E = \pi_E = 0$

Quintessence

- Note that this does not mean we can ignore the perturbations of scalar field entirely



Caldwell: An introduction to quintessence

Growth of structure – clustering DE/MG

- Poisson equation

$$k^2\Phi = -4\pi a^2 G\left(1+\frac{\rho_E\Delta_E}{\rho_m\Delta_m}\right)\rho_m\Delta_m$$

clustering DE acts like modifications of gravity for dark matter

$$k^2\Phi = -4\pi a^2 G_{\text{eff}}(k,\eta)\rho_m\Delta_m$$

- Anisotropic stress

$$k^2(\Psi-\Phi) = -8\pi G a^2 P_E\, \Pi_E$$

$\Pi_E = 0$ for quintessence but it appears typically in modified gravity models

Zoology of DE/MG models

LCDM $\Lambda\ (\Omega_E)$

Clustering DE $(\Omega_E, w_E, \delta\rho_E)$
K-essence

Modified gravity $(\Omega_E, w_E, \delta\rho_E, \pi_E)$
(screening mechanisms)

Smooth DE (Ω_E, w_E)
Quintessence

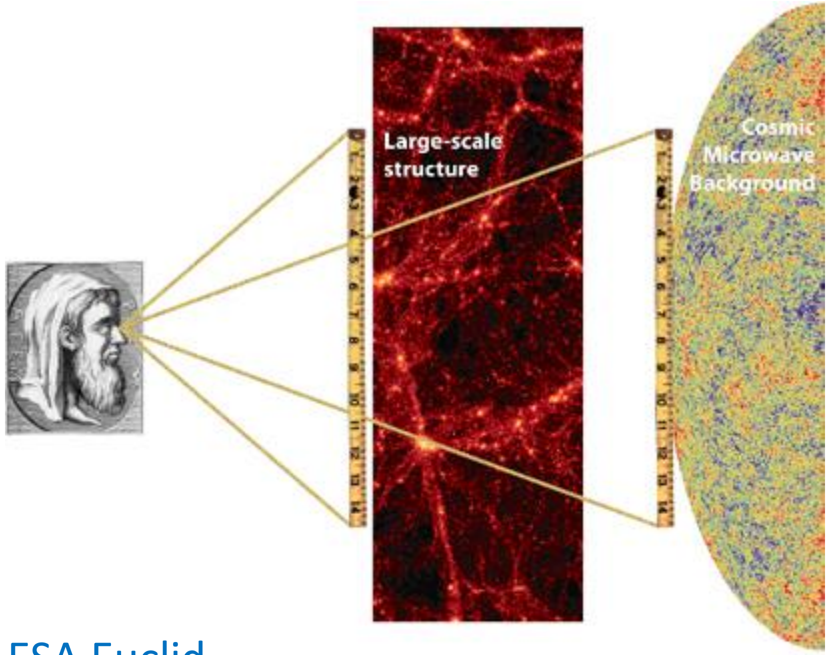
Interacting DE $(\Omega_E, w_E, \delta\rho_E, Q_E^\mu)$
(violation of equivalence principle)

Observations

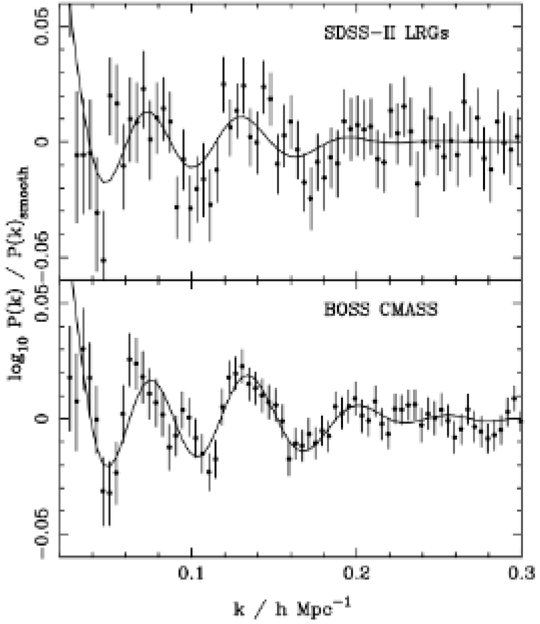
- Background $H(z)$

  Supernovae: luminosity distance

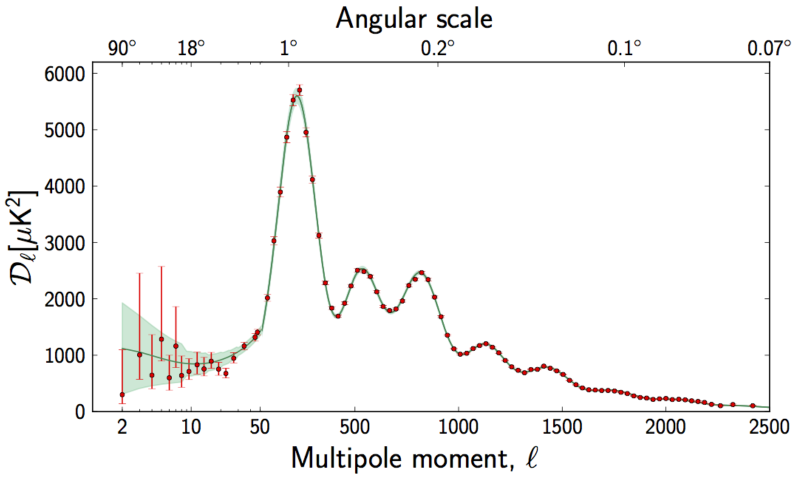
  CMB/Baryon Acoustic Oscillation (BAO): angular diameter distance



ESA Euclid



BOSS



ESA Planck

Observations

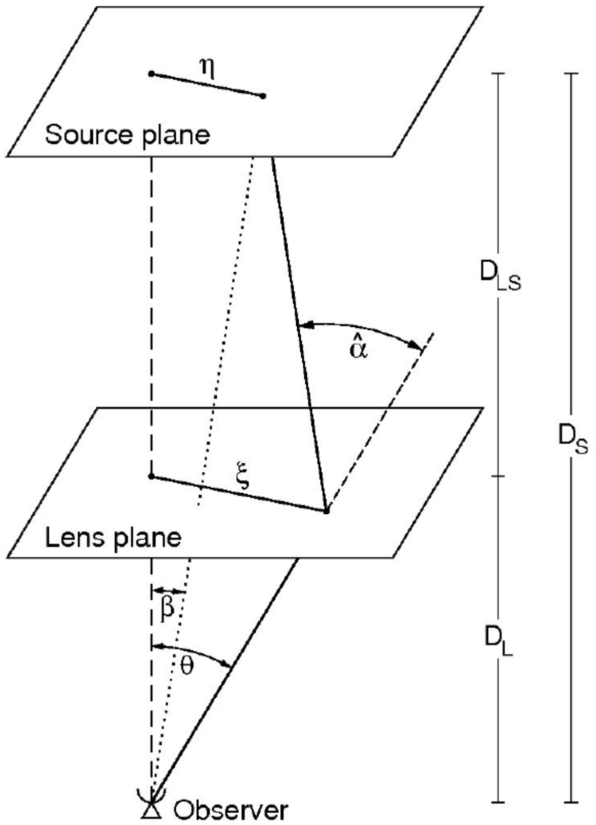
- Weak lensing

$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

Convergence (photons follow geodesic)

$$\kappa(\vec{n}) = \int d\chi \boxed{\frac{(\chi_s - \chi)\chi}{\chi_s}} \nabla_\perp{}^2 \phi_W(\eta_0 - \chi, \chi\vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

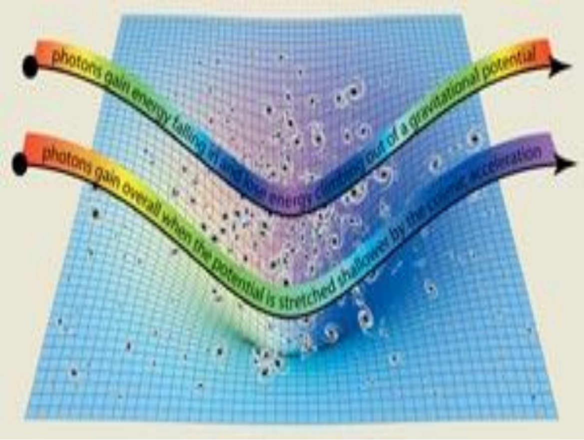
geometry



Galaxy shape is determined by shear which can be computed from convergence

Bartelmann & Schneider astro-ph/9912508

Observations



http://cmbcorrelations.pbworks.com

- CMB

  Integrated Sachs-Wolfe (ISW) effect

  The time variation of lensing potential causes

  a shift of photon temperature

$$\Theta_\ell(k) = \int_0^\eta d\eta \, \frac{\partial \phi_W(k,\eta)}{\partial \eta} \, j_\ell[k(\eta_0 - \eta)] \qquad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

  lensing

  CMB is also lensed

$$\Theta_{lensed}(\vec{n}) = \Theta(\vec{n} + \vec{d})$$

$$\vec{d} = \vec{\nabla}\psi, \quad \psi(\vec{n}) = -2\int d\chi \, \frac{(\chi_{LSS} - \chi)\chi}{\chi_{LSS}} \phi_W(\eta_0 - \chi, \chi\vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

Observations

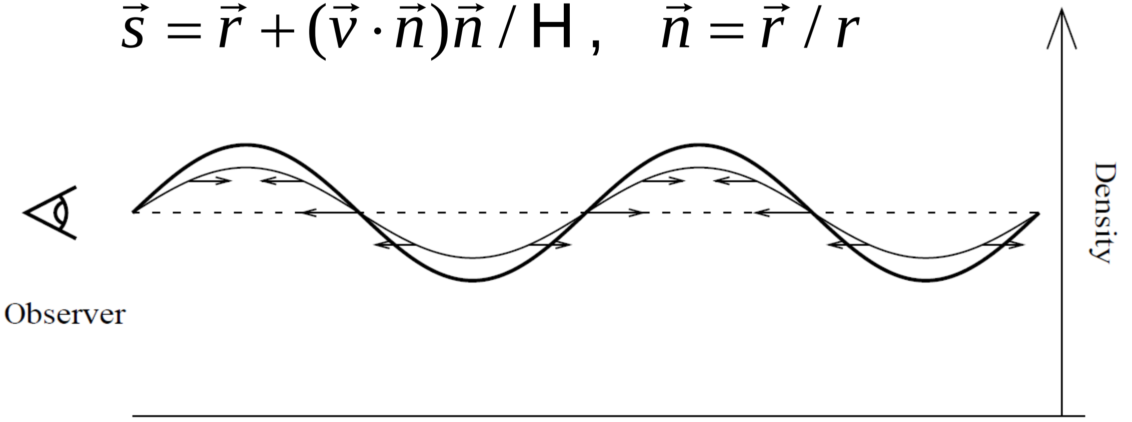
- Redshift distortions

galaxies have peculiar velocities

clustering of galaxies in redshift space

is enhanced along the line of sight

$$\vec{s} = \vec{r} + (\vec{v}\cdot\vec{n})\vec{n}/\mathrm{H}, \quad \vec{n} = \vec{r}/r$$



Hamilton astro-ph/9708102

$$\delta^s(k,\mu) = \Delta_m(k) - \mu^2\theta(k), \quad \mu^2 = \frac{\left(\vec{k}\cdot\vec{n}\right)^2}{k^2}$$

If the continuity equation holds, the velocity dispersion is related to the growth rate

$$\delta^s(k,\mu) = \Delta_m(k)\left(1-\mu^2\frac{\theta(k)}{\Delta_m(k)}\right) = \Delta_m(k)\left(1+\mu^2 f\right) \qquad \Delta_m' = -H\ \theta_m$$

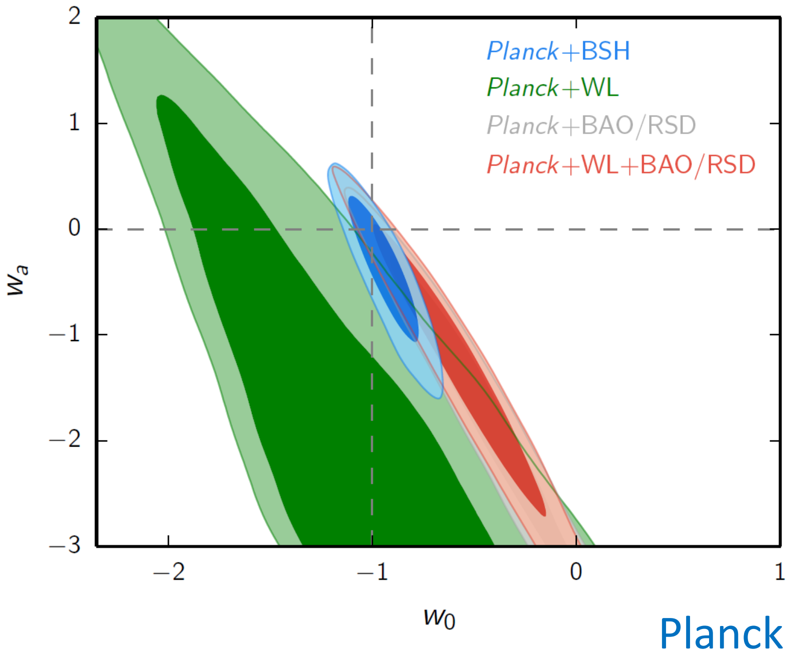
$$f = \frac{d\ln\Delta_m}{d\ln a} = \frac{a}{D_+}\frac{dD_+}{da}$$

Background expansion history

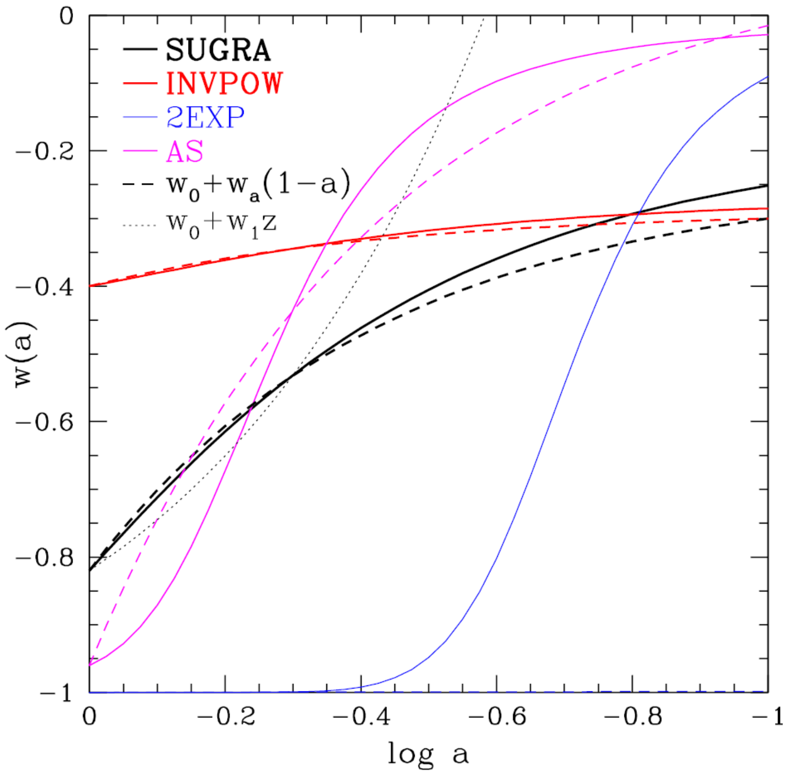
- Background expansion is determined by the equation of state

Parametrisation

$$w_{DE} = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$



Planck collaboration 1502.01590



Linder astro-ph/0311403

Model based parametrisation

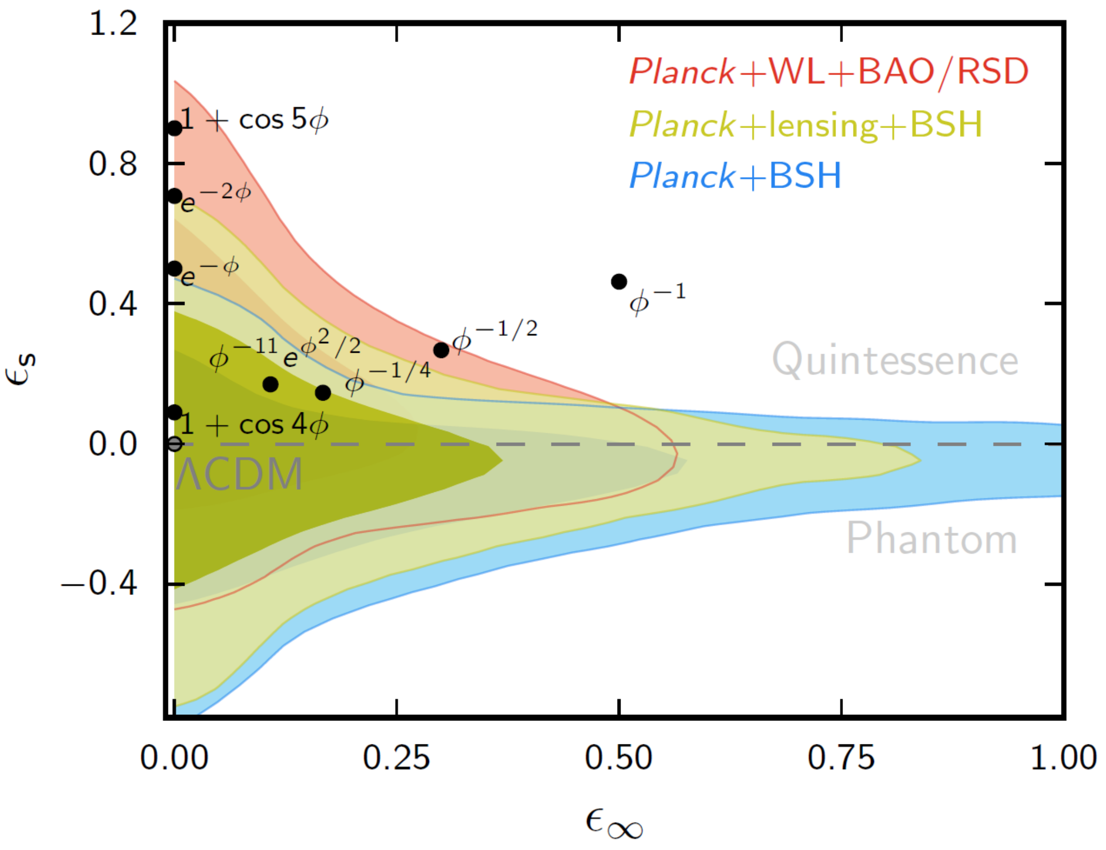
- Parametrisation

$$1 + w_\phi \approx \frac{2}{3}\varepsilon_\phi \Omega_\phi(a)$$

$$\varepsilon_\phi = \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m}$$

$$\varepsilon_s = \varepsilon_\phi(\rho_m = \rho_{DE})$$

$$\varepsilon_\infty = \varepsilon_\phi \Omega_\phi(a \to 0)$$

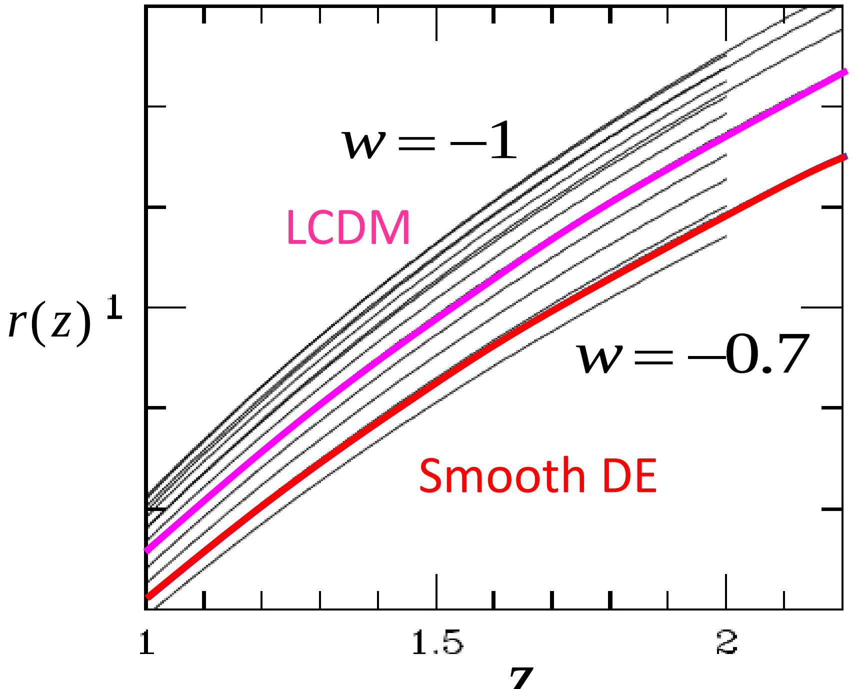


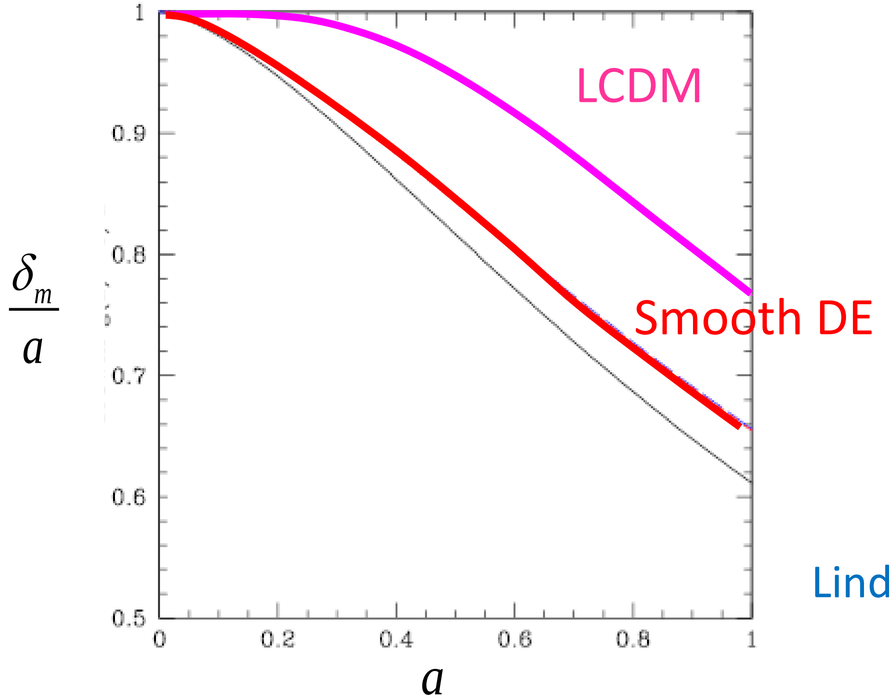
Planck 1502.01590

Expansion history v structure growth

- LCDM/Smooth DE

There is a one-to-one correspondence between background expansion history and growth of structure





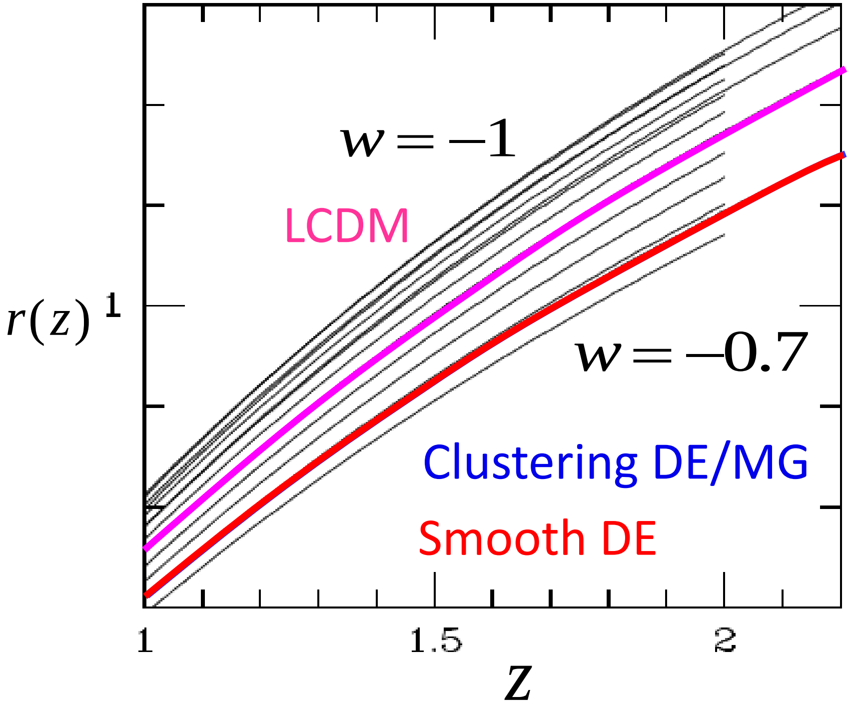
Linder astro-ph/0507263

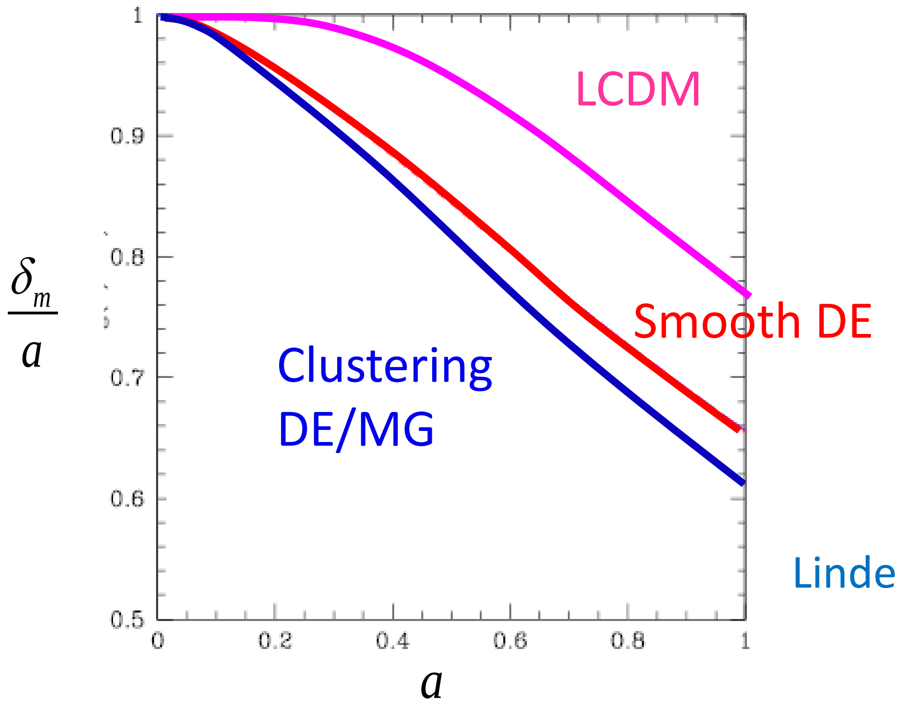
Expansion history v structure growth

- Clustering DE/MG

structure growth is controlled also by $\delta\rho_E$

Even if it has the same expansion history as smooth DE, structure growth is different



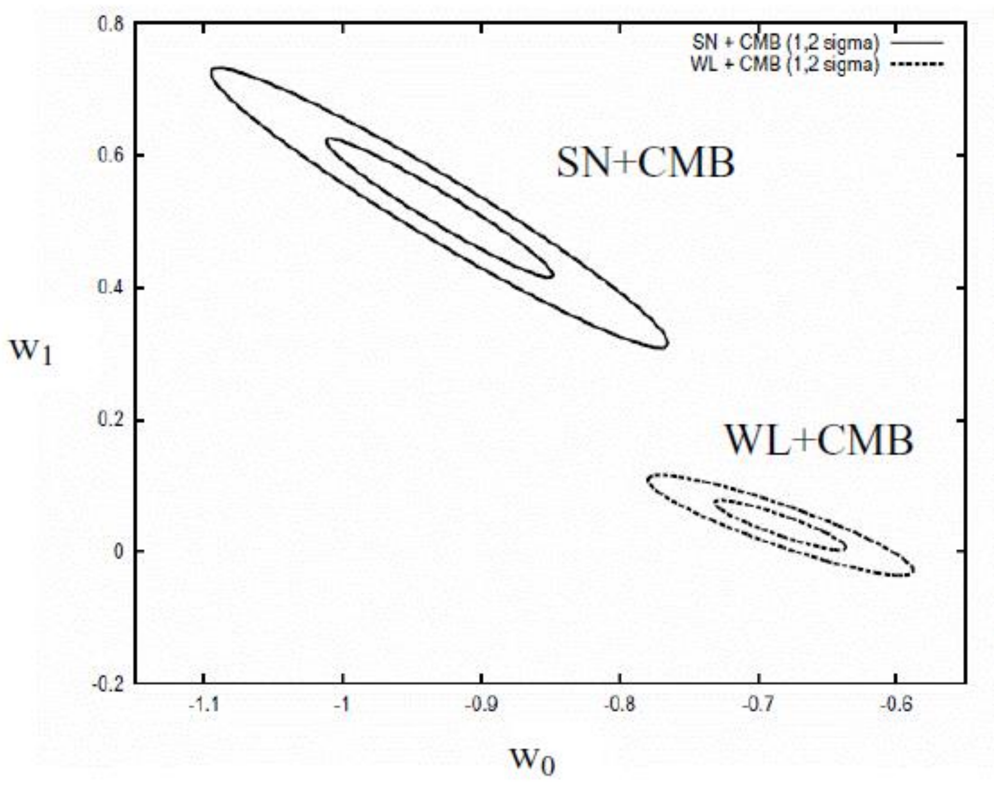


Linder astro-ph/0507263

Consistency test

Assume that the Universe is described by a clustering DE/MG model but we still try to fit the date using smooth DE

$$w(z) = w_0 + w_1 z,$$



SNe+CMB

SNe+weak lensing

Inconsistent!

Ishak et.al. astro-ph/0507184

Consistency relation

Zhang & Jain Phys. Rev. D78 (2008) 063503
Song & KK JCAP 0901(2009) 048

- In GR, gravitation equations are given by

$$H^2 = \frac{8\pi G}{3}\rho_T, \quad \rho_T = \sum_i \rho_i$$

$$\frac{k^2}{a^2}\Phi = 4\pi G a^2 \rho_T \delta_T, \quad \rho_T \delta_T = \sum_i \rho_i \delta_i$$

- Consistency relation

$$\alpha(k,t) = \frac{2k^2}{3a^2 \boxed{H^2}} \frac{\boxed{(\Phi+\Psi)} - \boxed{\Psi}}{\boxed{\delta_T}} = 1$$

background

Weak lensing

Peculiar velocity (Redshift distortions)

$$\boxed{k^2\Psi = \frac{d(a\theta_m)}{dt}}$$

Galaxy distribution

$$\boxed{\delta_g = b_T \delta_T}$$

We have just enough number of observations to check the relation

Parametrisation

- Dark component

  We need to specify $(\delta P_E, \pi_E)$

- Parametrisation of Einstein equations

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$
$$\Phi = \eta(k,a)\Psi$$

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

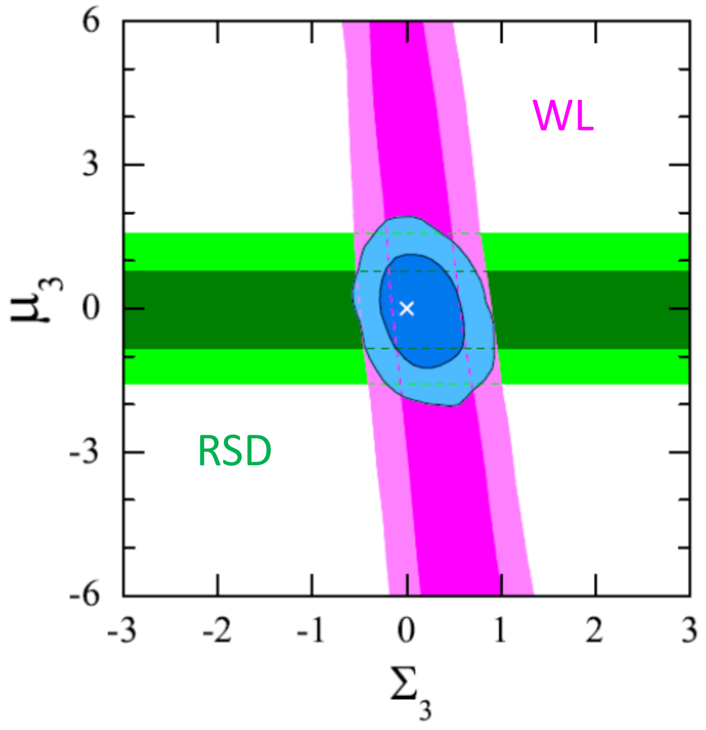
equivalently, we can also parametrise the lensing potential

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$
$$k^2\frac{(\Psi+\Phi)}{2} = -4\pi G a^2 \Sigma(k,a)\rho_m \Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

$\mu = \eta = \Sigma = 1$ for smooth DE

Weak lensing and Redshift Distortions

- Combining WL ($\phi_W = (\Phi + \Psi)/2$) and RSD (θ_m) we can break the degeneracy



Song et.al. 1011.2106

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$

$$k^2(\Psi + \Phi) = -4\pi G a^2 \Sigma(k,a)\rho_m \Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

$$\theta_m' + H\left(1 + \frac{H'}{H^2}\right)\theta_m = \frac{k^2}{H}\Psi$$

$$\Delta_m'' + H\,\Delta_m' + 4\pi G a^2 \mu(k,a)\rho_m \Delta_m = 0$$

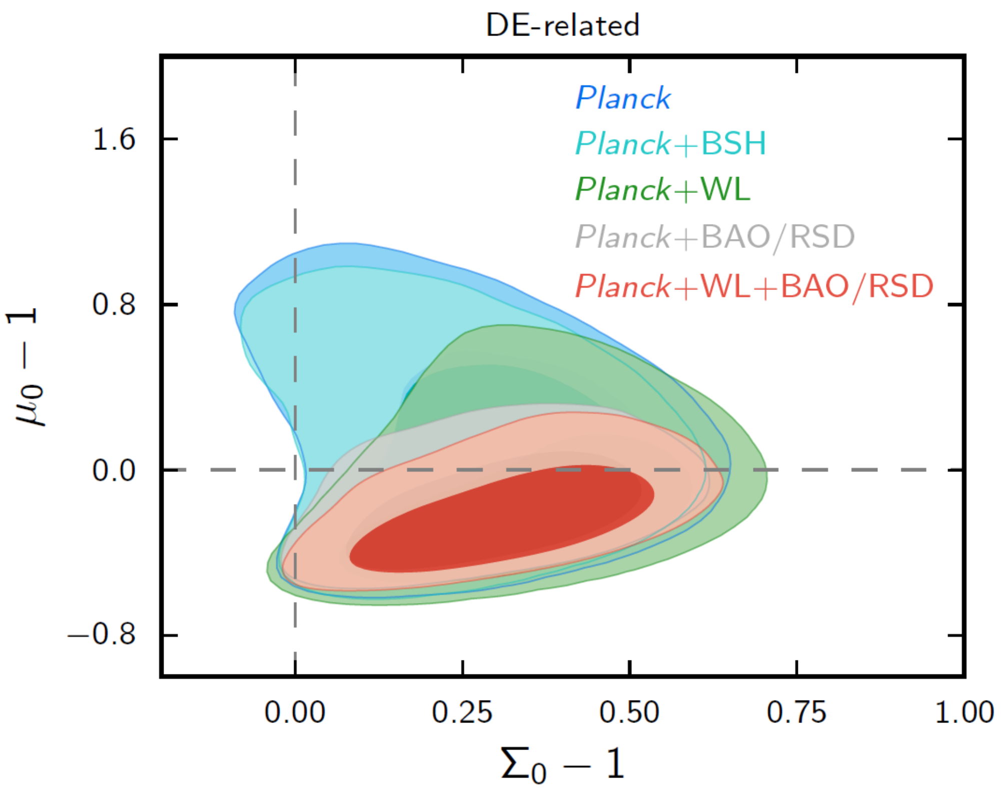
$$\mu(a) = 1 + \mu_3\, a^3, \quad \Sigma(a) = 1 + \Sigma_3\, a^3$$

Planck 2015 results

- Assuming LCDM background

$$\mu(a) = 1 + \bar{\mu}\,\Omega_{DE}(a), \quad \Sigma(a) = 1 + \bar{\Sigma}\,\Omega_{DE}(a)$$

$$\mu_0 = \mu(1), \quad \Sigma_0 = \Sigma(1)$$



Max. degeneracy	*Planck* TT+lowP	*Planck* TT+lowP +BSH	*Planck* TT+lowP +WL	*Planck* TT+lowP +BAO/RSD	*Planck* TT+lowP +WL+BAO/RSD
DE-related	$0.84^{+0.30}_{-0.40}$ (2.1σ)	$0.80^{+0.28}_{-0.39}$ (2.1σ)	$1.08^{+0.35}_{-0.42}$ (2.6σ)	$0.90^{+0.33}_{-0.37}$ (2.4σ)	1.03 ± 0.34 (3.0σ)
+ CMB lensing	$0.42^{+0.18}_{-0.34}$ (1.2σ)	$0.38^{+0.18}_{-0.28}$ (1.4σ)	$0.58^{+0.24}_{-0.37}$ (1.6σ)	$0.40^{+0.18}_{-0.28}$ (1.4σ)	$0.51^{+0.21}_{-0.30}$ (1.7σ)

tension with LCDM

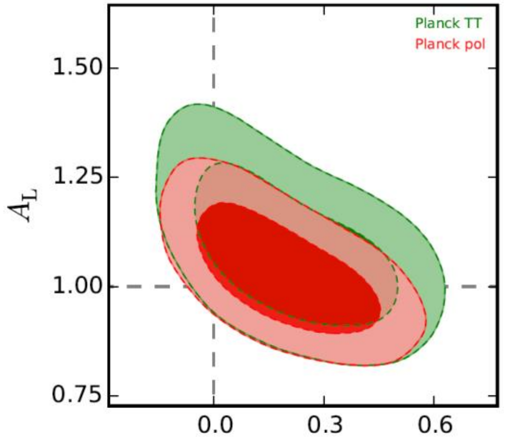
Planck 1502.01590

Tension with LCDM in Planck data

- Lensing amplitude

CMB lensing requires a larger amplitude than LCDM in the power spectrum $A_{lens} = 1.22 \pm 0.10$

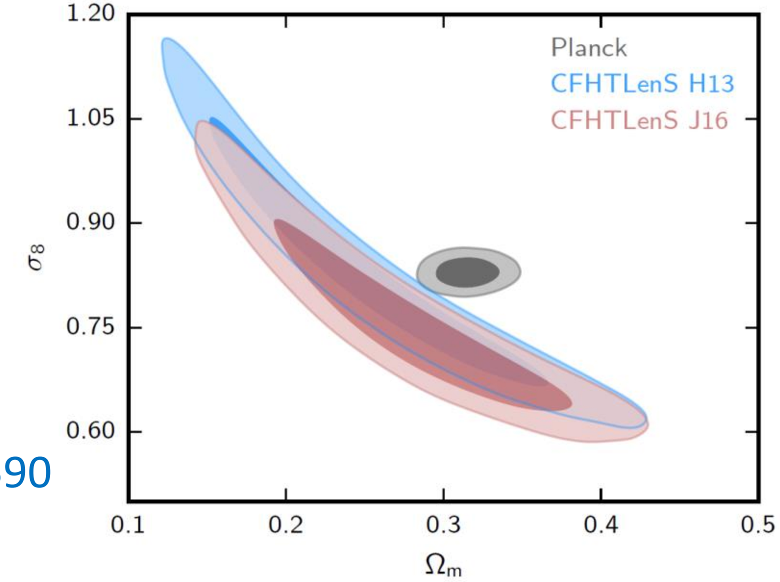
(cf. this tension does not exist for lensing measured from trispectrum)



Valentino et.al. 1509.07501

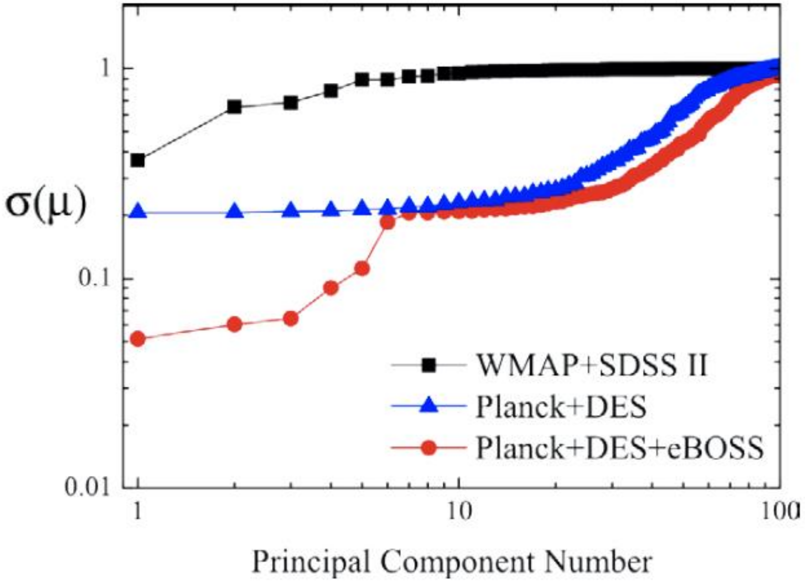
- Amplitude of fluctuations

The late time amplitude of fluctuations in LCDM predicted from primordial amplitude measured by CMB is larger than that measured by weak lensing (CFHTLS) Planck 1502.01590



Future forecasts

Next 3-5 years



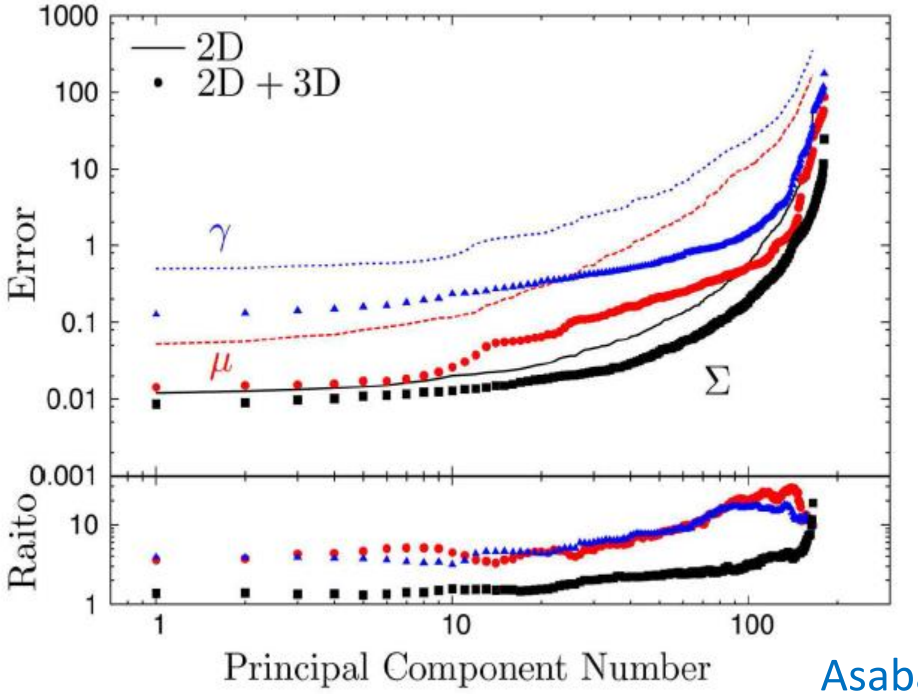
Zhao et.al. 1510.08216

DES (2012-2017) imaging

eBOSS (2014-2018) spectroscopic

Several parameters at the 5-10% level

Next 5-10 years



Asaba et.al. 1306.2546

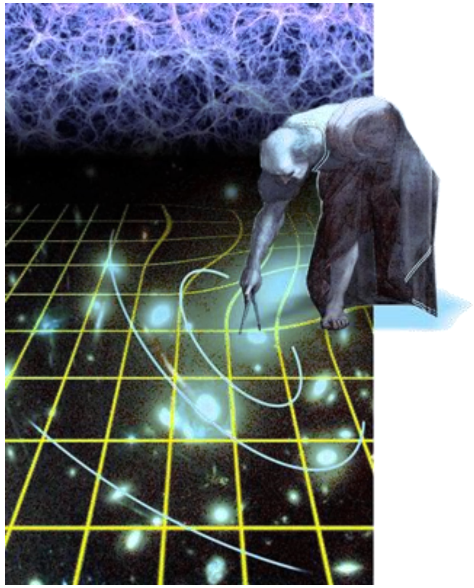
Euclid (2020-)

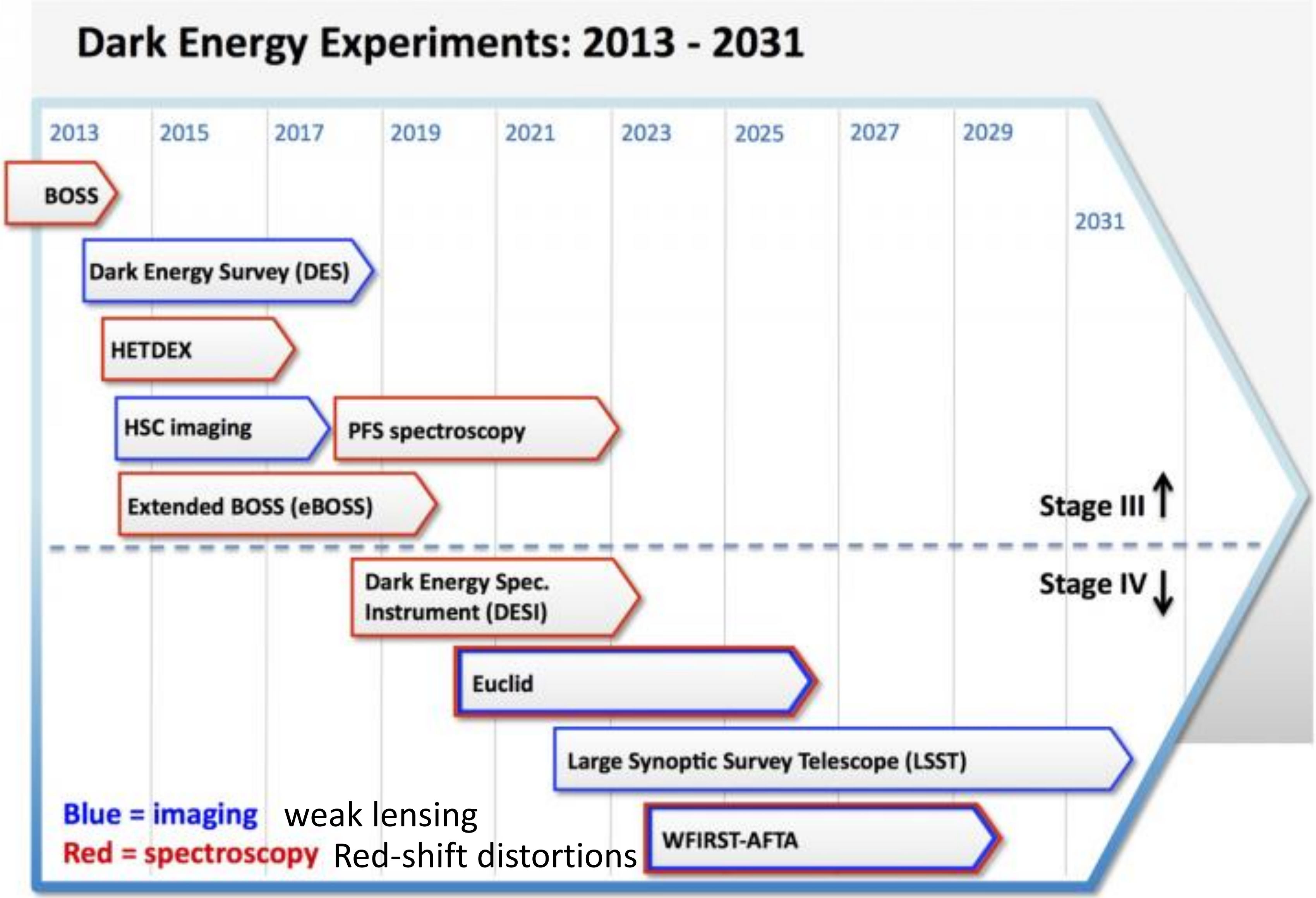
10 parameters at the 1% level

Euclid (2020-)

http://sci.esa.int/euclid/

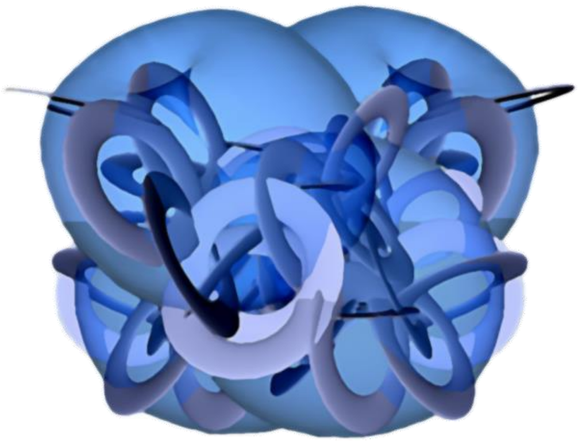
R-L0	The Euclid Mission will by itself allow us to • understand the nature of the apparent acceleration of the Universe and • test gravity on cosmological scales from the measurement of the cosmic expansion history and the growth rate of structures.
R-L0.1	To determine the nature of the apparent acceleration, Euclid will distinguish effects produced by a cosmological constant from those produced by a dynamical dark energy. This must be done by achieving a minimum *FoM*>400 from Euclid data alone.
R-L0.2	To experience effects of gravity on cosmological scales, Euclid will probe the growth of structure and will separately constrain the two relativistic potentials, Ψ and Φ. This can be done by achieving an absolute 1σ precision of 0.02 on the growth index, γ, from Euclid data alone.

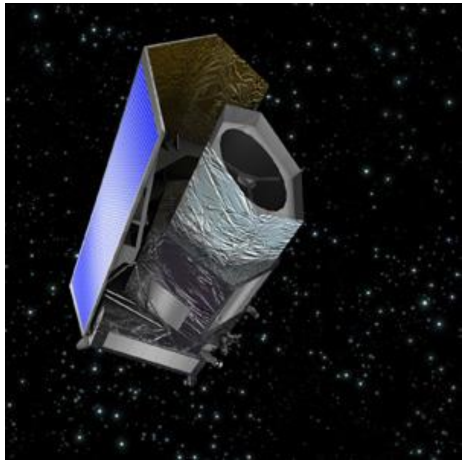




Abazajian et.al. Dark Energy and CMB [2014]

From theory to observations





initial conditions

fluid approach
Effective Field Theory

many free functions

$$\mu(k_i, z_i), \Sigma(k_i, z_i)$$

$$C_\ell^{IJ}(z)$$

galaxy count
Lensing
ISW
systematics