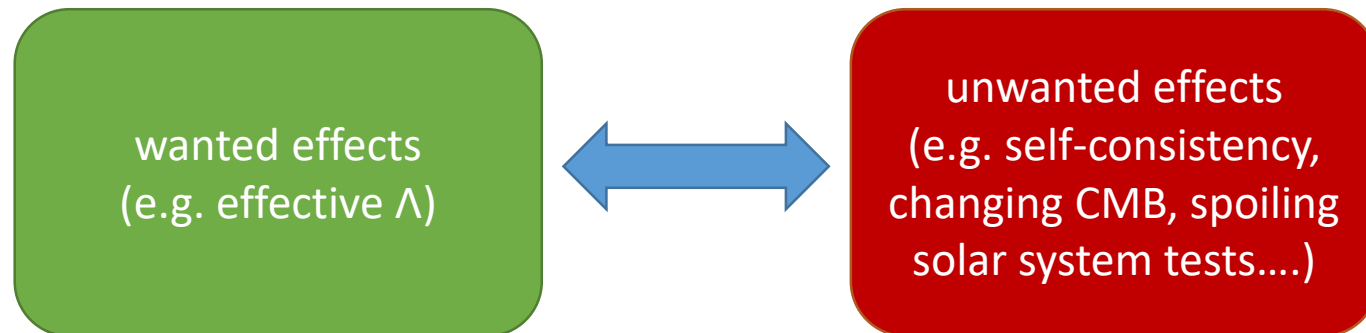


Part II: Models of dark energy and modified gravity

1. Dynamical dark energy: quintessence
2. Examples of modified gravity
3. Structure formation as a model discriminator
4. *(Observational tests)*

Theories of dark energy

- A useful approach is to introduce features as new parameters in observables. These can then be probed by experiments/observations
- This needs to be complemented by theories: provide a self-consistent description to justify a specific effect.
- Challenge: A single parameter in a consistent theoretical framework may contaminate several observables. “A bug or a feature?”



V. Scalar-field models of dark energy	20
A. Quintessence	20
B. K-essence	22
C. Tachyon field	23
D. Phantom (ghost) field	24
E. Dilatonic dark energy	25
F. Chaplygin gas	26
VI. Cosmological dynamics of scalar fields in the presence of a barotropic perfect fluid	26
A. Autonomous system of scalar-field dark energy models	27
1. Fixed or critical points	27
2. Stability around the fixed points	27
B. Quintessence	28
1. Constant λ	28
2. Dynamically changing λ	30
C. Phantom fields	30
D. Tachyon fields	30
1. Constant λ	31
2. Dynamically changing λ	31
E. Dilatonic ghost condensate	33
VII. Scaling solutions in a general Cosmological background	34
A. General Lagrangian for the existence of scaling solution	34
B. General properties of scaling solutions	35
C. Effective potential corresponding to scaling solutions	36
1. Ordinary scalar fields	36
2. Tachyon	36
3. Dilatonic ghost condensate	36
D. Autonomous system in Einstein gravity	37
VIII. The details of quintessence	37
A. Nucleosynthesis constraint	37
B. Exit from a scaling regime	38
C. Assisted quintessence	38
D. Particle physics models of Quintessence	39
1. Supergravity inspired models	39
2. Pseudo-Nambu-Goldstone models	42
E. Quintessential inflation	43
IX. Coupled dark energy	44
A. Critical points for coupled Quintessence	45
B. Stability of critical points	45
1. Ordinary field ($\epsilon = +1$)	46
2. Phantom field ($\epsilon = -1$)	47
C. General properties of fixed points	48
D. Can we have two scaling regimes ?	48
E. Varying mass neutrino scenario	50
F. Dark energy through brane-bulk energy exchange	50
X. Dark energy and varying alpha	51
A. Varying alpha from quintessence	51
B. Varying alpha from tachyon fields	52

Dynamics of dark energy
Copeland et al. hep-th/0603057

3	Alternative Theories of Gravity with Extra Fields	49
3.1	Scalar-Tensor Theories	49
3.1.1	Action, field equations, and conformal transformations	49
3.1.2	Brans-Dicke theory	52
3.1.3	General scalar-tensor theories	59
3.1.4	The chameleon mechanism	66
3.2	Einstein-Ether Theories	68
3.2.1	Modified Newtonian dynamics	68
3.2.2	Action and field equations	69
3.2.3	FLRW solutions	70
3.2.4	Cosmological perturbations	71
3.2.5	Observations and constraints	73
3.3	Bimetric Theories	75
3.3.1	Rosen's theory, and non-dynamical metrics	76
3.3.2	Drummond's theory	77
3.3.3	Massive gravity	77
3.3.4	Bigravity	79
3.3.5	Bimetric MOND	80
3.4	Tensor-Vector-Scalar Theories	81
3.4.1	Actions and field equations	82
3.4.2	Newtonian and MOND limits	84
3.4.3	Homogeneous and isotropic cosmology	86
3.4.4	Cosmological perturbation theory	91
3.4.5	Cosmological observations and constraints	93
3.5	Other Theories	96
3.5.1	The Einstein-Cartan-Sciama-Kibble Theory	96
3.5.2	Scalar-Tensor-Vector Theory	99
4	Higher Derivative and Non-Local Theories of Gravity	101
4.1	$f(R)$ Theories	101
4.1.1	Action, field equations and transformations	102
4.1.2	Weak-field limit	106
4.1.3	Exact solutions, and general behaviour	111
4.1.4	Cosmology	114
4.1.5	Stability issues	120
4.2	General combinations of Ricci and Riemann curvature.	123
4.2.1	Action and field equations	123
4.2.2	Weak-field limit	125
4.2.3	Exact solutions, and general behaviour	126
4.2.4	Physical cosmology and dark energy	128
4.2.5	Other topics	131
4.3	Horava-Lifschitz Gravity	137
4.3.1	The projectable theory	141
4.3.2	The non-projectable theory	144
4.3.3	Aspects of Horava-Lifschitz cosmology	146
4.3.4	The Θ CDM model	148
4.3.5	HMT-da Silva theory	148
4.4	Galileons	150
4.4.1	Galileon modification of gravity	151
4.4.2	Covariant galileon	157
4.4.3	DBI galileon	158
4.4.4	Galileon cosmology	160
4.4.5	Multi-galileons	161
4.5	Other Theories	164
4.5.1	Ghost condensates	164
4.5.2	Non-metric gravity	166
4.5.3	Dark energy from curvature corrections	169

5	Higher Dimensional Theories of Gravity	172
5.1	Kaluza-Klein Theories of Gravity	172
5.1.1	Kaluza-Klein compactifications	173
5.1.2	Kaluza-Klein cosmology	174
5.2	The Braneworld Paradigm	179
5.2.1	The ADD model	180
5.3	Randall-Sundrum Gravity	181
5.3.1	The RS1 model	182
5.3.2	The RS2 model	184
5.3.3	Other RS-like models	185
5.3.4	Action and equations of motion	188
5.3.5	Linear perturbations in RS1 and RS2	189
5.4	Brane Cosmology	195
5.4.1	Brane based formalism – covariant formulation	197
5.4.2	Bulk based formalism – moving branes in a static bulk	199
5.4.3	Cosmological perturbations	200
5.5	Dvali-Gabadadze-Porrati Gravity	207
5.5.1	Action, equations of motion, and vacua	207
5.5.2	Linear perturbations on the normal branch	209
5.5.3	Linear perturbations (and ghosts) on the self-accelerating branch	211
5.5.4	From strong coupling to the Vainshtein mechanism	214
5.5.5	DGP cosmology	221
5.6	Higher Co-Dimension Braneworlds	228
5.6.1	Cascading gravity	230
5.6.2	Degravitation	235
5.7	Einstein Gauss-Bonnet Gravity	236
5.7.1	Action, equations of motion, and vacua	237
5.7.2	Kaluza-Klein reduction of EGB gravity	240
5.7.3	Co-dimension one branes in EGB gravity	240
5.7.4	Co-dimension two branes in EGB gravity	244

Modified Gravity and Cosmology

Clifton et al. arXiv:1106.2476

Quintessence: $w_{DE}(t)$ from single scalar field

In field theory, explicit t dependence \leftarrow VEV of a (clock) field.

We now know that (at least one) scalar field exists

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_\phi$$

$$S_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

energy momentum tensor

$$T_{\mu\nu}^\phi = (\partial_\mu \phi)(\partial_\nu \phi) - g_{\mu\nu} \left[\frac{1}{2} g^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) + V(\phi) \right]$$

$$T_{\mu\nu}^\phi = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_\phi)}{\delta g^{\mu\nu}}$$

scalar field equation

$$\frac{\delta S_\phi}{\delta \phi} = 0 \quad \nabla^\mu \nabla_\mu \phi - V'(\phi) = 0 \quad \longleftrightarrow \quad \nabla^\mu T_{\mu\nu}^\phi = 0$$

Background

- Energy density and pressure

$$\rho_\phi = -T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \phi = \phi(t)$$

$$P_\phi = \frac{1}{3}T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

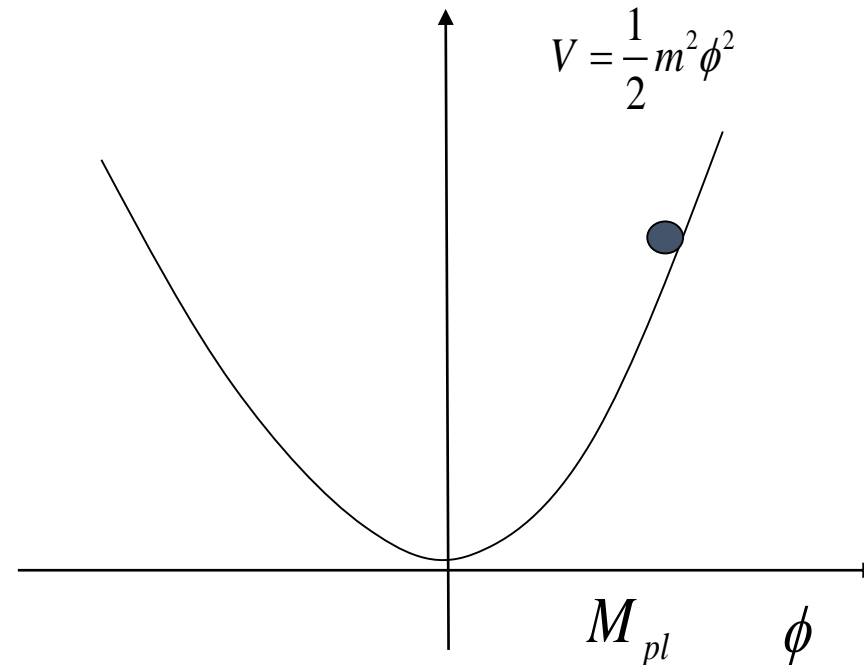
- Scalar field equation

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + V'(\phi) = 0 \quad \longleftrightarrow \quad \dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

friction

For accelerated expansion, the scalar field needs to slowly roll down the potential

$$\dot{\phi}^2 \ll V(\phi) \quad w_\phi = \frac{P_\phi}{\rho_\phi} \approx -1$$



Quintessence potentials

- Freezing model, e.g.

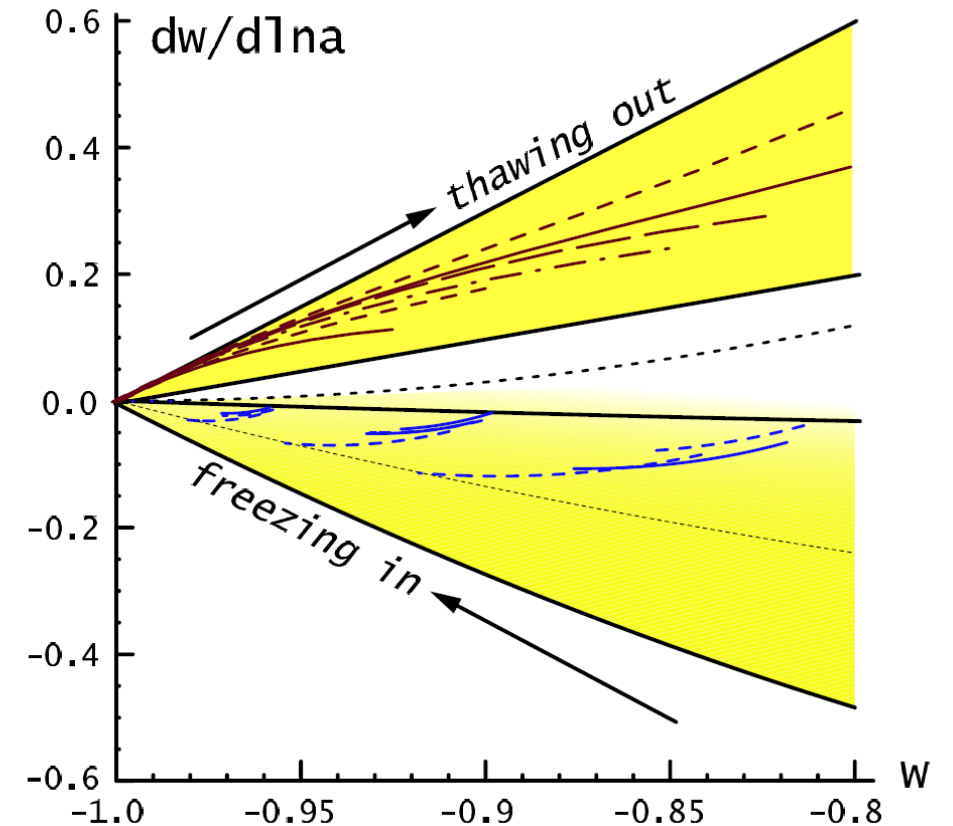
$$V(\phi) = M^{4+n} \phi^{-n}$$

The scalar field dynamics freezes at late time and approaches $w_\phi \approx -1$

- Thawing models, e.g.

$$V(\phi) = M^4 \cos^2\left(\frac{\phi}{f}\right)$$

Scalar initially held at $w_\phi \approx -1$ by Hubble friction. Recently started to roll down towards minimum. (similar to inflation)



Linder & Caldwell astro-ph/0505494

Fundamental issues

- Consider a massive scalar field

$$V(\phi) = \frac{1}{2}m^2\phi^2, \quad \varepsilon_\phi \equiv \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2 = \frac{2M_{pl}^2}{\phi^2} \ll 1 \quad \rightarrow \quad \phi > M_{pl} = 10^{18} \text{ GeV}$$

$$V(\phi_0) = \rho_{DE} = 10^{-48} \text{ GeV}^4 \quad \rightarrow \quad m = \left(\frac{\rho_{DE}}{\phi^2} \right)^{\frac{1}{2}} = 10^{-42} \text{ GeV} \approx H_0$$

Correction to the potential

$$V(\phi) = \frac{1}{4}\lambda\phi^4 < \rho_{DE} \quad \rightarrow \quad \lambda < 10^{-120}$$

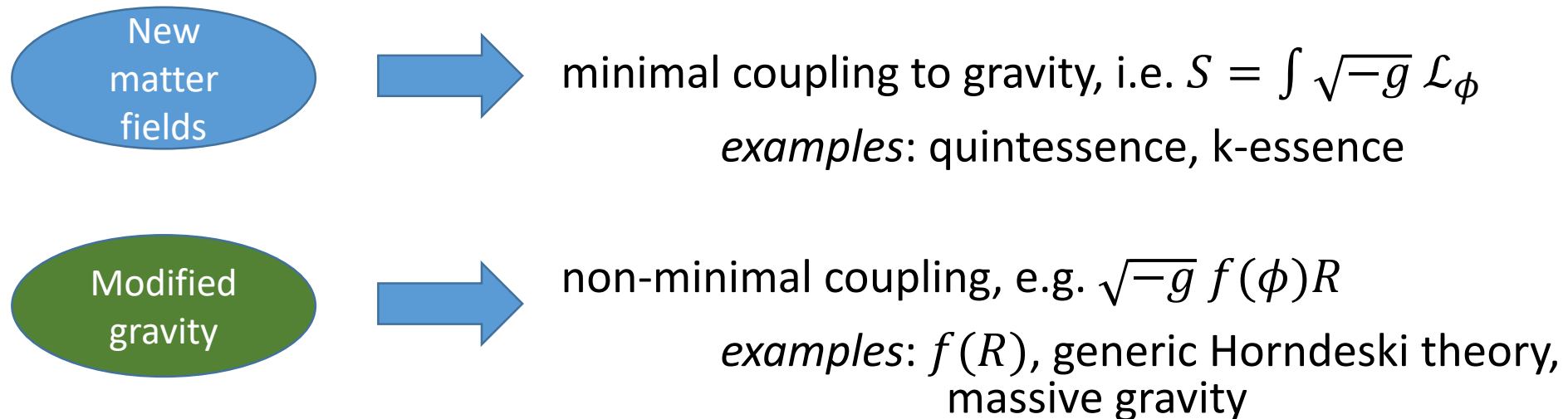
these small numbers are very difficult to protect against quantum corrections

Lovelock's theorem [1971]

- GR+ Λ is the unique metric theory assuming:
 - a. EOM originates from an action
 - b. 3+1 dimensions
 - c. EOM has at most 2nd order derivatives
 - d. no other (hidden or explicit) fields
 - e. diffeomorphism invariant action
 - f. no non-local operators
- A modified theory of GR should break at least one of these. This lecture briefly focusses only on breaking (d), as it includes breaking either of (b),(c),(e).

The fuzzy line between dynamical DE and MG

- We will modify gravity by adding new fields.
- Wait... Quintessence *is* a new field. Is it modified gravity?
- Dark energy nomenclature proposed by Clifton, Ferreira, Padilla, Skordis [arXiv:1106.2476]



- Warning: not everyone follows this terminology, e.g. non-minimal quintessence

Horndeski Theory [1974]

Scalar-Tensor extension of Lovelock

Independently discovered
by Deffayet et al. 2011

• Lagrangian $\mathcal{L}_H = \sum_{i=2}^5 \mathcal{L}_i.$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi,$$

$$X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X} \left[(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5,X} \left[(\square\phi)^3 - 3(\square\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3 \right]$$

Three examples:

K-essence
 $K(X), G_4 = \frac{M_p^2}{2}$



Dynamical DE

Covariant Galileon
 $K, G_3 \sim X, G_5 \sim X^2$
 $G_4 \sim \frac{M_p^2}{2} + (\dots)X^2$



Self-acceleration

Fab Four

$$\begin{aligned} G_2 &= 2V''_{\text{john}}(\phi)X^2 - V_{\text{Paul}}^{(3)}(\phi)X^3 + 6V''_{\text{george}}(\phi)X \\ &\quad + 8V_{\text{ringo}}^{(3)}(\phi)X^2(3 - \ln(|X|)), \\ G_3 &= 3V'_{\text{john}}(\phi)X - \frac{5}{2}V''_{\text{Paul}}(\phi)X^2 + 3V'_{\text{george}}(\phi) \\ &\quad + 4V_{\text{ringo}}^{(3)}(\phi)X(7 - 3\ln(|X|)), \\ G_4 &= V_{\text{john}}(\phi)X - V'_{\text{Paul}}(\phi)X^2 + V_{\text{george}}(\phi) \\ &\quad + 4V_{\text{ringo}}^{(3)}(\phi)X(2 - \ln(|X|)), \\ G_5 &= -3V_{\text{Paul}}(\phi)X - 4V'_{\text{ringo}}(\phi)\ln(|X|). \end{aligned}$$



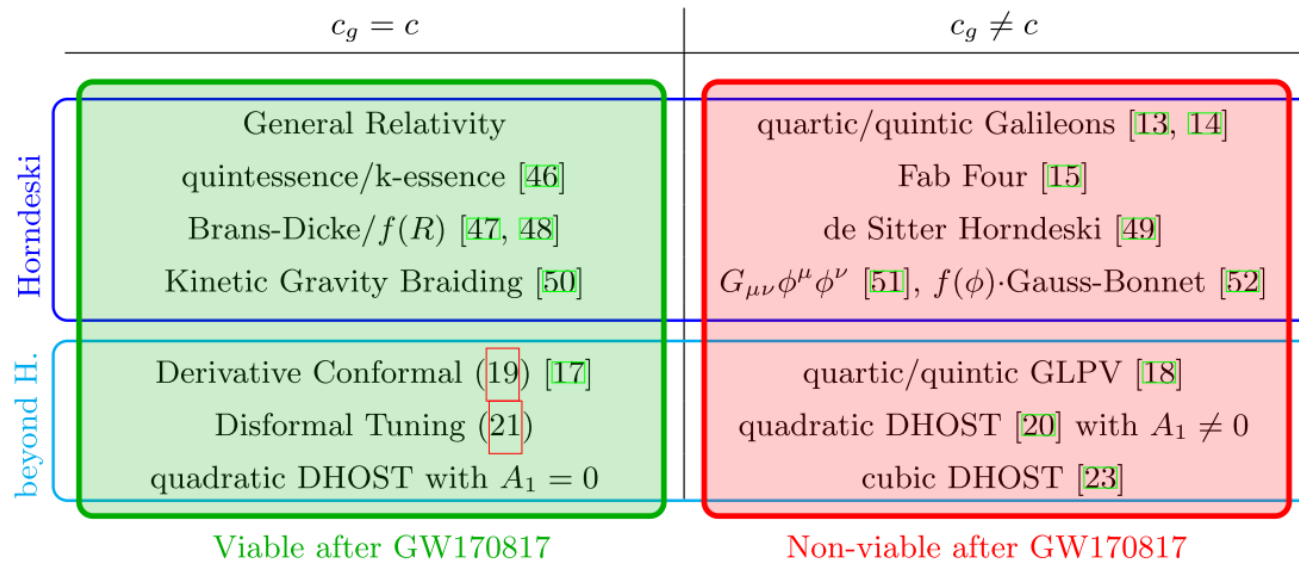
Self-tuning

Generically, Horndeski theories have (potentially dangerous) implications: $\dot{G}_N \neq 0$, speed of GW $\neq 1$ etc...

Enter GW170817/GRB170817A

- BNS merger, time delay between GW and GRB $\sim 1.74\text{s}$
- Conservative limit, $d = 26\text{Mpc}$, $\Delta t \sim 10\text{s}$ gives

$$-3 \times 10^{-15} < \frac{c_g}{c} - 1 < 0.7 \times 10^{-15}$$



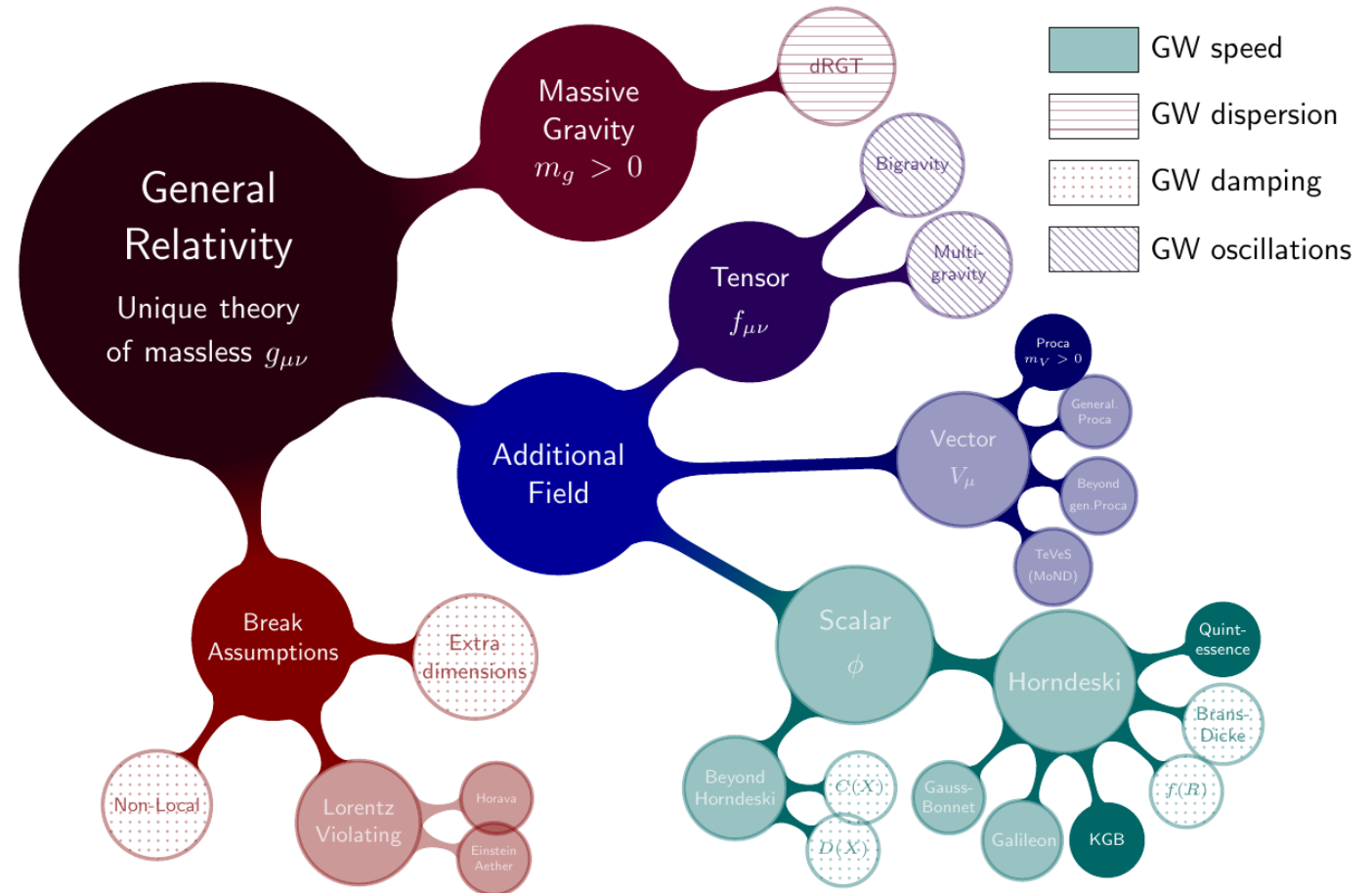
Ezquiaga, Zumalacàrregui
arXiv:1710.05901

FIG. 2: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. Only simple Horndeski theories, $G_{4,X} \approx 0$ and $G_5 \approx \text{constant}$, and specific beyond Horndeski models, conformally related to $c_g = 1$ Horndeski or disformally tuned, remain viable.

Scalar-Tensor theory is not the only way

Modified gravity roadmap

- Beyond Horndeski, DHOST (degeneracies)
- Other fields, e.g. massive gravity, bimetric theory;
- From higher dimensions: DBI;
- Non-local actions, e.g. non-local GR/massive gravity;
- Constrained actions, e.g. unimodular gravity;
- Non-metric, e.g. emergent scenarios, squared Yang-Mills;
- and many more...



Ezquiaga, Zumalacàrregui arXiv:1807.09241

Summary of models

- DE/MG models

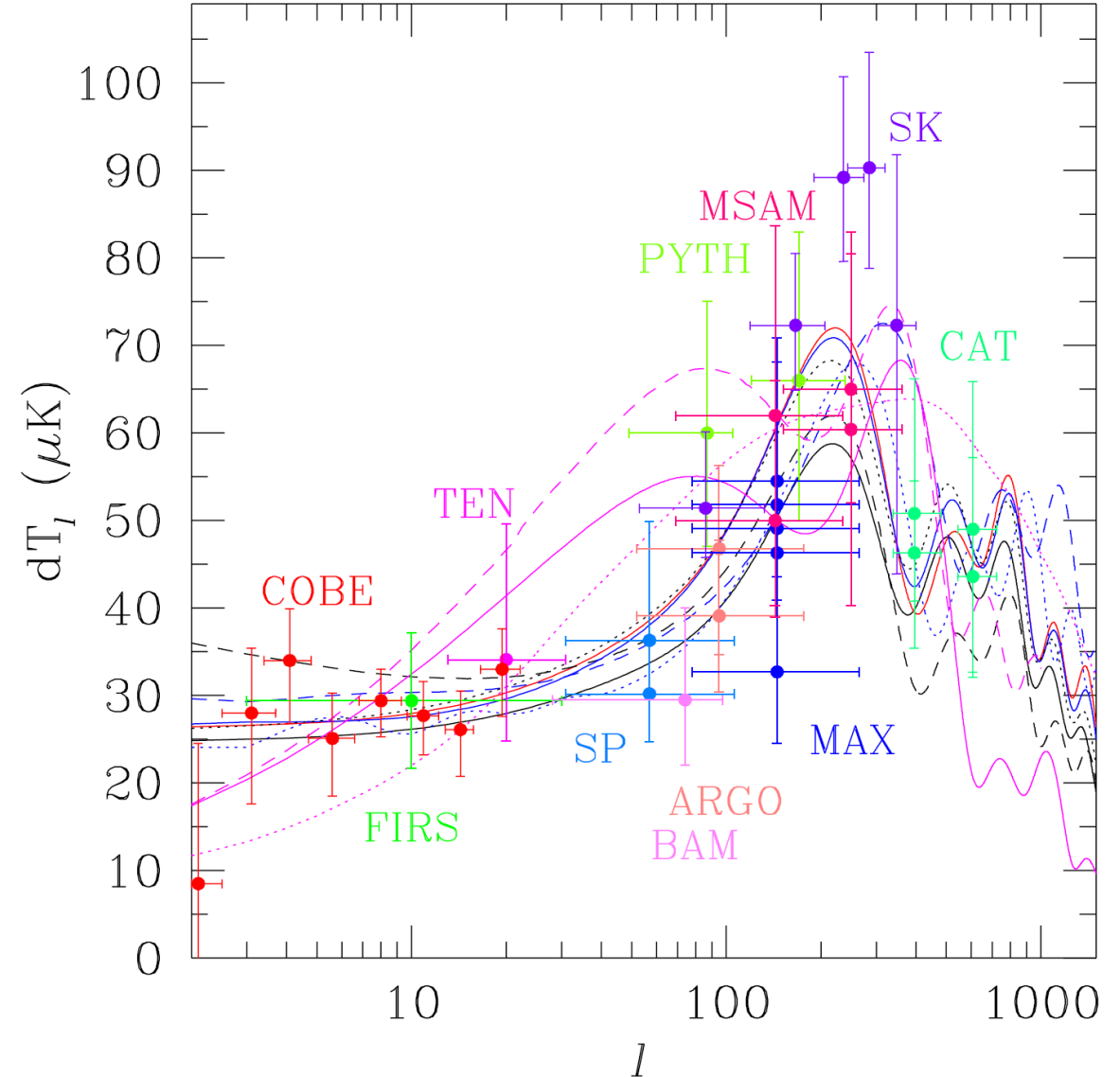
There are many models but we still do not have a compelling alternative to Λ CDM

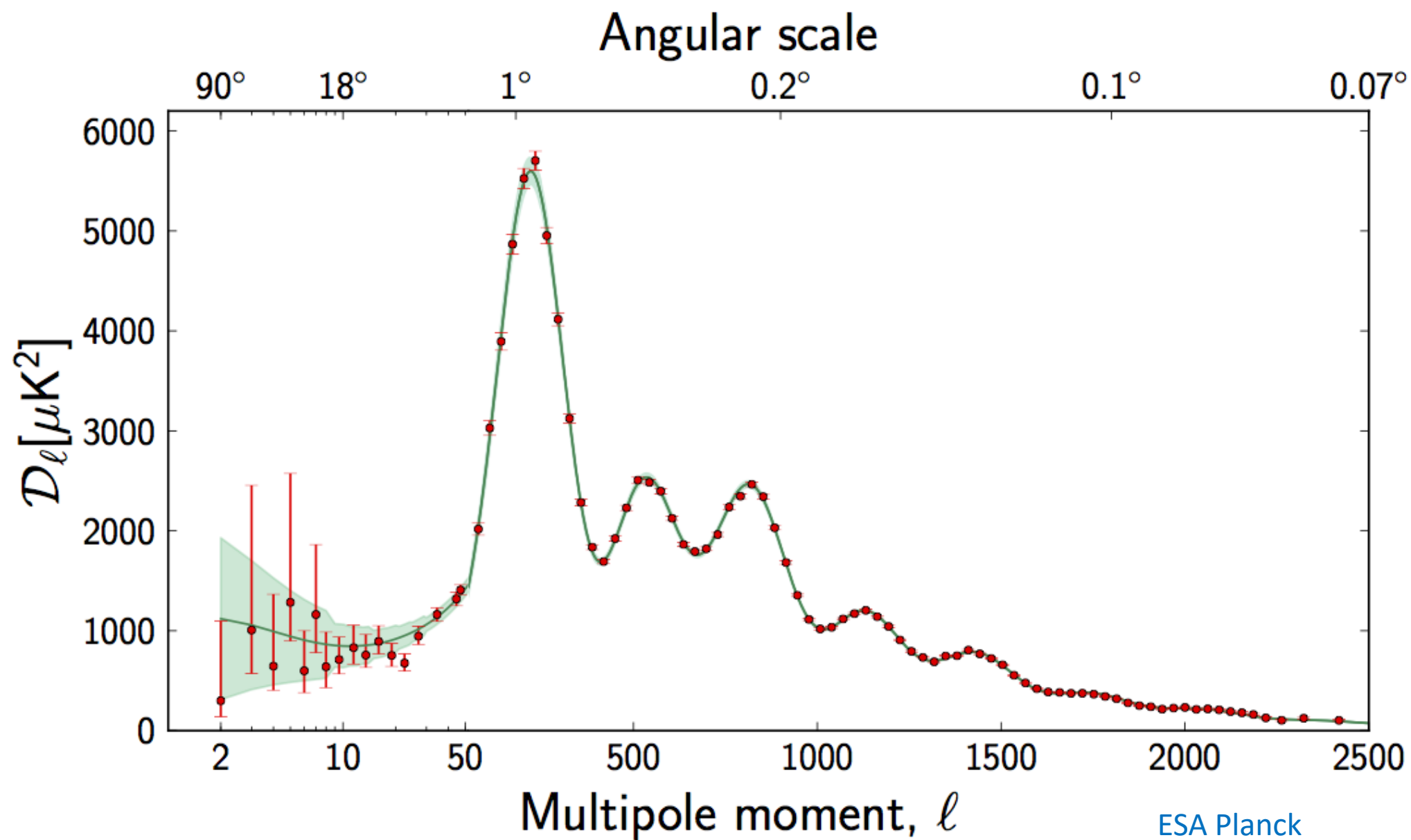
Observations may give us a clue:

Is it the cosmological constant or a light degree of freedom?

Does it cluster or couple to matter?

cf. CMB (before WMAP)





How to test DE/MG models

- Einstein equations

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^M + \kappa^2 E_{\mu\nu}, \quad E_{\mu\nu} = T_{\mu\nu}^{DE} + G_{\mu\nu}^{MG} \quad \nabla^\mu (T_{\mu\nu}^M + E_{\mu\nu}) = 0$$

- Background (homogeneity & Isotropy) $E_\nu^\mu = \text{diag}(-\rho_E, P_E, P_E, P_E)$
everything is determined by the equation of state $w_E = P_E / \rho_E$

- Small Inhomogeneity [in Newtonian gauge]

$$ds^2 = a^2(\eta) \left[-(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right] \quad H = \frac{a'}{a}$$

Linear scalar perturbations with respect to 3-space (assumed to be flat)

Cosmological perturbation theory

- Fourier transformation and decomposition
- Scalar harmonics:

$$S \propto e^{i k_j x^j}, \quad S_j = -i \hat{k}_j S, \quad S_{ij} = \left(\frac{\delta_{ij}}{3} - \hat{k}_i \hat{k}_j \right) S$$

- $\nabla^2 S = -k^2 S$ and $S_i^i = 0$
- Any scalar, vector and tensor can be written in terms of these:

$$V_i = V S_i, \quad A_{ij} = A_L \delta_{ij} + A_T S_{ij}$$

Kodama & Sasaki 1985

Mukhanov, Feldman, Brandenberger 1990

Malik & Wands 2008

Matter content

- Energy momentum tensor

$$T_{I\nu}^{\mu} = \begin{pmatrix} -(\rho_I + \delta\rho_I), & (\rho_I + P_I)v_{Ii} \\ -(\rho_I + P_I)v_I^i, & (P_I + \delta P_I)\delta_j^i + P_I \Pi_{Ij}^i \end{pmatrix} \quad \begin{array}{l} v^i = v S^i, \quad : \text{velocity} \\ \Pi_j^i = \Pi S_j^i : \text{anisotropic stress} \end{array}$$

- Conservation of energy momentum tensor

for now, we assume matter and dark component obeys the conservation independently [if not assumed, *interacting DE*: $\nabla^{\mu}(E_{\mu\nu}) = Q_{\nu}^{DE}$]

$$\begin{aligned} \frac{d\delta\rho_I}{d\eta} + 3H(\delta\rho_I + \delta P_I) &= -(\rho_I + P_I)(kv_I - 3\dot{\Phi}) & w_m &= P_m / \rho_m = 0, \quad \delta P_m = \Pi_m = 0, \\ \left(\frac{d}{d\eta} + 4H \right) \left[\frac{(\rho_I + P_I)v_I}{k} \right] &= \delta P_I - \frac{2}{3}P_I \Pi_I + (\rho_I + P_I)\Psi & w_E &= P_E / \rho_E, \quad \delta P_E, \quad \Pi_E, \end{aligned}$$

Equations for linear perturbations

- Einstein equations

$$k^2 \Phi = -4\pi G a^2 (\rho_m \Delta_m + \rho_E \Delta_E), \quad \rho_I \Delta_I = \delta\rho_I + 3(\rho_I + P_I) \frac{H}{k} v_I$$

$$k^2 (\Psi - \Phi) = -8\pi G a^2 P_E \Pi_E$$

- Conservation of energy momentum tensor for matter $(k/H)^2 \gg 1$

$$\Delta_m' = -H \theta_m, \quad \theta_m = (k/H) v_m : \text{velocity divergence}$$

$$\theta_m' + H \left(1 + \frac{H'}{H^2} \right) \theta_m = \frac{k^2}{H} \Psi \quad \Rightarrow \quad \Delta_m'' + H \Delta_m' = -k^2 \Psi$$

Evolution of matter is determined by the Newtonian potential

Dark component affects the evolution through the Newtonian potential

Growth of structure – LCDM

1) LCDM $E_{\mu\nu} = -\Lambda g_{\mu\nu}$

$$k^2 \Phi = -4\pi G a^2 \rho_m \Delta_m \quad \Rightarrow \quad \Delta_m'' + H \Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$$

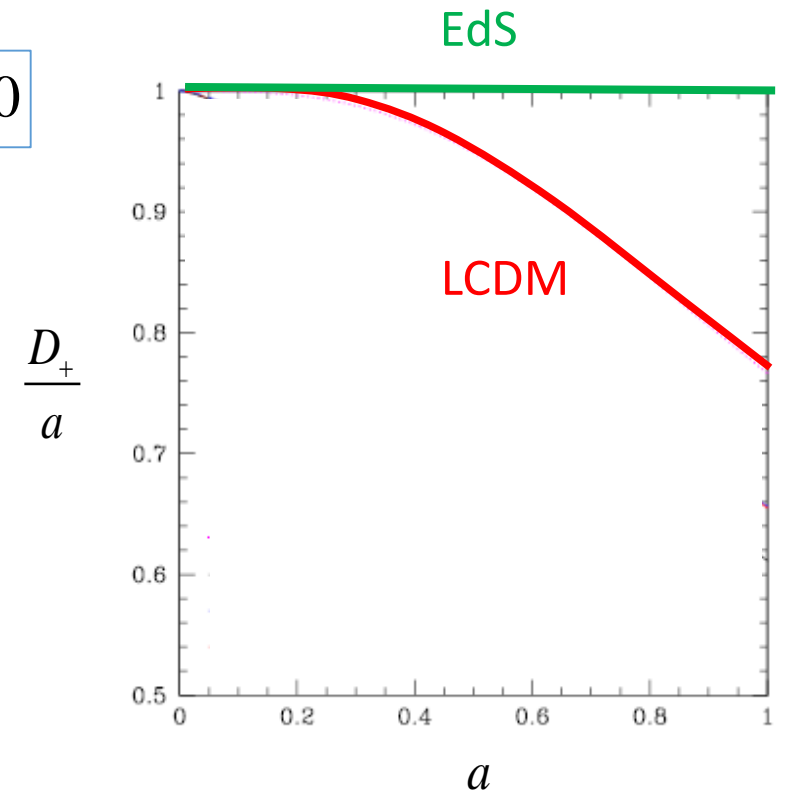
$$\Phi - \Psi = 0$$

We define the growth function D_+ as the growing mode solution for Δ_m

in MD era, $a \propto \eta^2, H^2 = 8\pi G a^2 \rho_m / 3 \Rightarrow D_+ \propto a$

at late times, due to the cosmological constant,
gravity becomes weaker

$$H^2 = \frac{8\pi G a^2}{3} (\rho_m + \rho_\Lambda)$$



Growth of structure – smooth DE

2) smooth DE $\delta\rho_E = \pi_E = 0$

using $N = \ln a$ $\Delta_m'' + H \Delta_m' - 4\pi G a^2 \rho_m \Delta_m = 0$

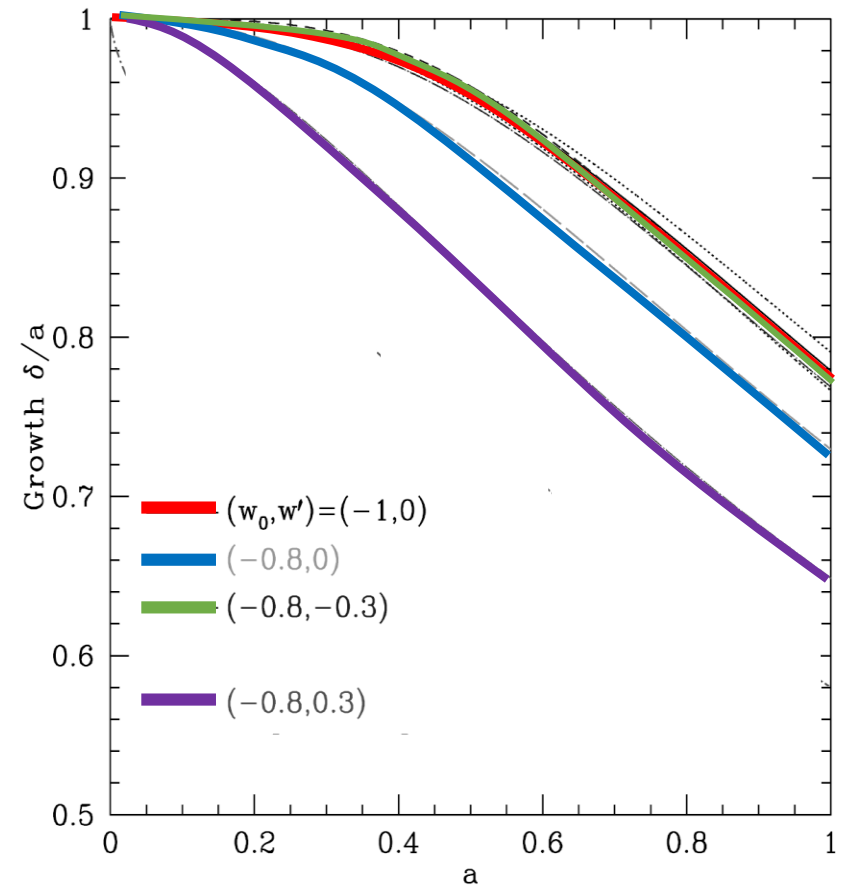
$$\ddot{D}_+ + \frac{1}{2}(1 - 3\Omega_{DE}w_{DE})\dot{D}_+ - \frac{3}{2}\Omega_m D_+ = 0$$

$$\dot{\Omega}_m = 3w_{DE}(1 - \Omega_m)\Omega_m, \quad \Omega_{DE} = 1 - \Omega_m$$

$$\frac{D_+}{a}$$

For a fixed present-day Ω_{DE} , if $w_{DE} > -1$, DE density is larger in the past suppressing the growth compared with LCDM

$$w_{DE}(z) = w_0 + w_a \frac{z}{1+z}$$



Linder astro-ph/0305286

Growth rate

- Growth rate

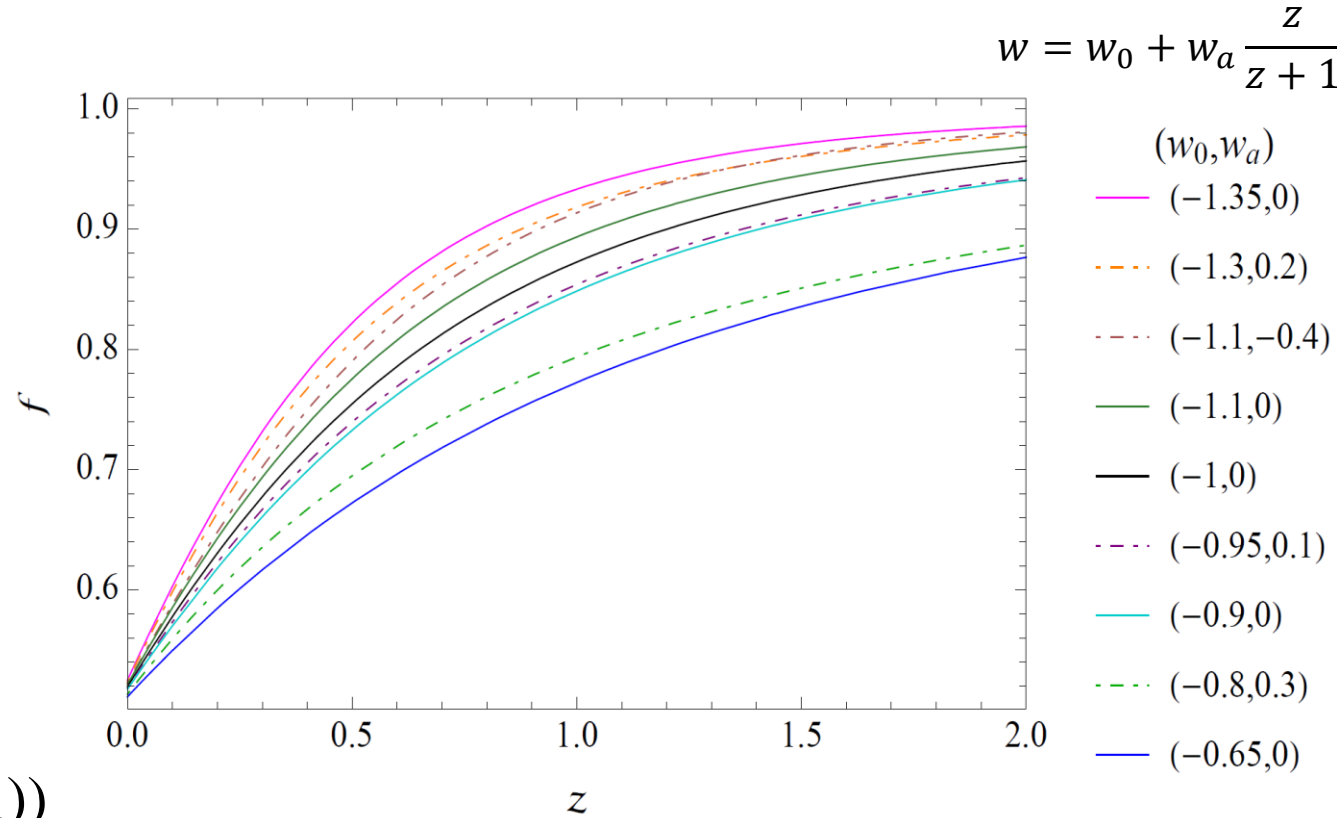
$$f = \frac{d \ln \Delta_m}{d \ln a} = \frac{a}{D_+} \frac{dD_+}{da}$$

$$\dot{f} + f^2 + \left[\frac{1}{2} - \frac{3}{2} w_{DE} (1 - \Omega_m) \right] f = \frac{3}{2} \Omega_m$$

$$f = \Omega_m^\gamma, \quad \gamma = 0.545 + 0.05(1 + w_{DE}(z=1))$$

γ is insensitive to the equation of state w_{DE}

(but the growth rate depends on w_{DE} through Ω_m)



Dossett & Ishak 1311.0726

Growth of structure – clustering DE

3) Clustering DE $\delta\rho_E \neq 0$ ($\pi_E = 0$)

Let's consider a toy model for dark component with non-zero sound speed

$$\delta P_E = c_{sE}^2 \delta\rho_E$$

assuming that the dark component dominates the universe

$$\Delta_E'' + H \Delta_E' + \left(c_{sE}^2 k^2 - 4\pi G a^2 \rho_E \right) \Delta_E = 0$$

For $k > k_J = \sqrt{\frac{4\pi G a^2 \rho_E}{c_{sE}}}$ pressure wins over gravity and Δ_E does not grow

clustering DE requires small sound speed $(k_J / a)^{-1}$: Jean's length

Quintessence

- Sound speed

$$c_{s\phi}^2 \equiv \left. \frac{\delta P_\phi}{\delta \rho_\phi} \right|_{v_\phi=0} = 1, \quad v_\phi \propto \delta\phi$$

propagation speed

scalar field equation of motion

$$S_\phi = \int d^4x \left[-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

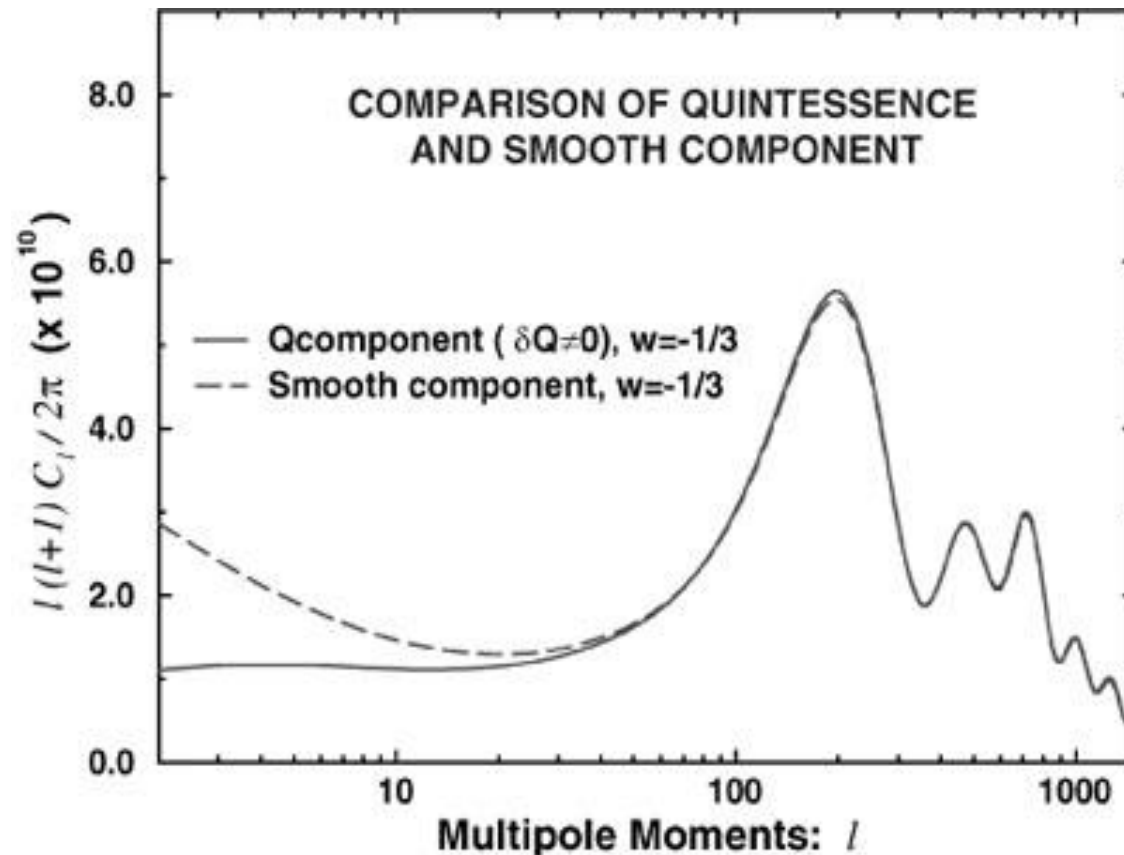
$$\delta\ddot{\phi} + 3H\delta\phi + c_{p\phi}^2 \frac{k^2}{a^2} \delta\phi + \dots = 0$$

for standard kinetic term, $c_{p\phi}^2 = c_{s\phi}^2 = 1$ thus the scalar field does not cluster

below the horizon scale thus can be approximated as smooth DE $\delta\rho_E = \pi_E = 0$

Quintessence

- Note that this does not mean we can ignore the perturbations of scalar field entirely



Caldwell: An introduction to quintessence

Growth of structure – clustering DE/MG

- Poisson equation

$$k^2\Phi = -4\pi a^2 G \left(1 + \frac{\rho_E \Delta_E}{\rho_m \Delta_m} \right) \rho_m \Delta_m$$

clustering DE acts like modifications of gravity for dark matter

$$k^2\Phi = -4\pi a^2 G_{\text{eff}}(k, \eta) \rho_m \Delta_m$$

- Anisotropic stress

$$k^2(\Psi - \Phi) = -8\pi G a^2 P_E \Pi_E$$

$\Pi_E = 0$ for quintessence but it appears typically in modified gravity models

Zoology of DE/MG models

LCDM Λ (Ω_E)

Clustering DE ($\Omega_E, w_E, \delta\rho_E$)
K-essence

Smooth DE (Ω_E, w_E)
Quintessence

Modified gravity ($\Omega_E, w_E, \delta\rho_E, \pi_E$)
(screening mechanisms)

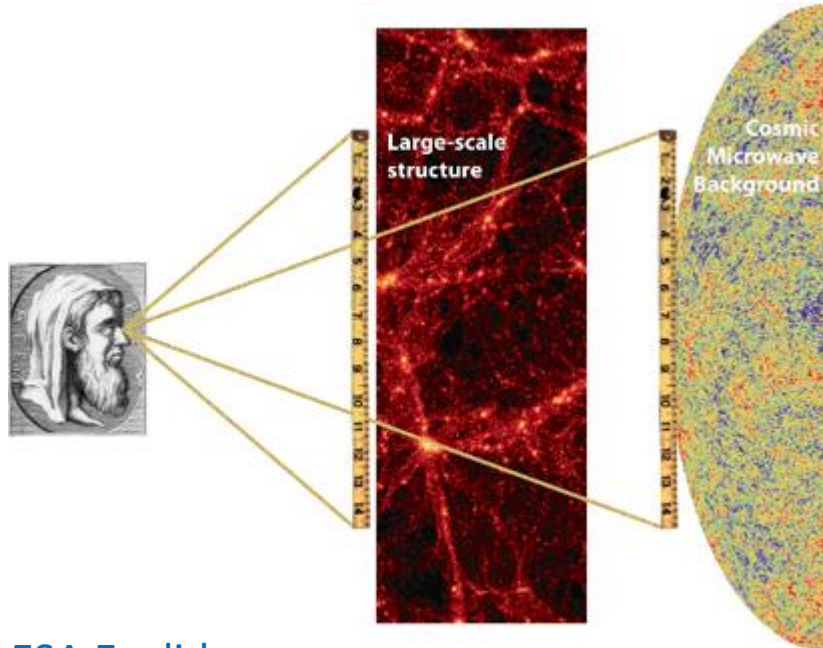
Interacting DE ($\Omega_E, w_E, \delta\rho_E, Q_E^\mu$)
(violation of equivalence principle)

Observations

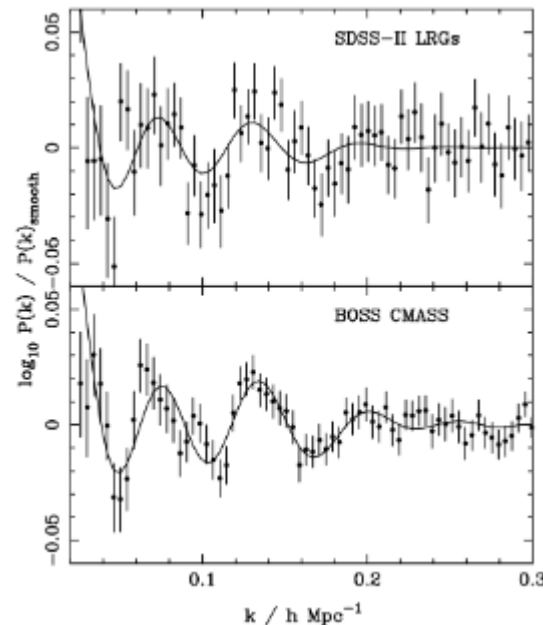
- Background $H(z)$

Supernovae: luminosity distance

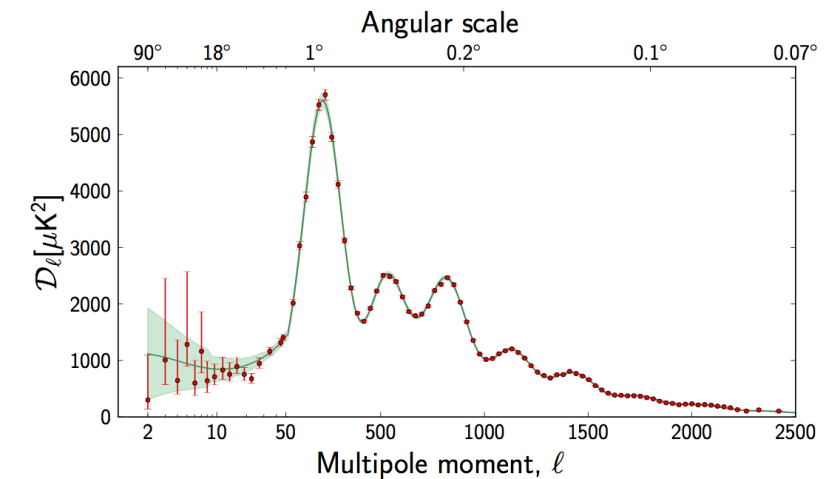
CMB/Baryon Acoustic Oscillation (BAO): angular diameter distance



ESA Euclid



BOSS



ESA Planck

Observations

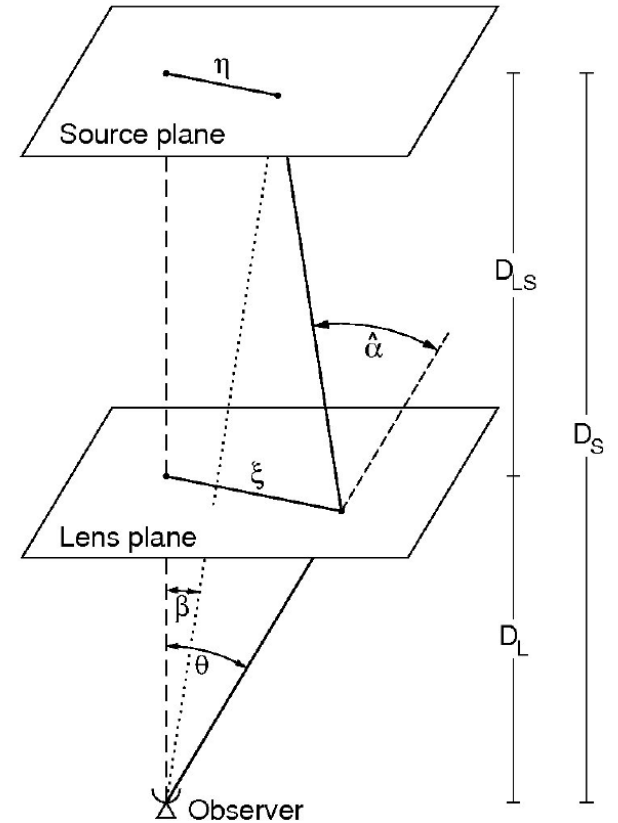
- Weak lensing

$$ds^2 = a^2 \left[-(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$

Convergence (photons follow geodesic)

$$\kappa(\vec{n}) = \int d\chi \underbrace{\frac{(\chi_s - \chi)\chi}{\chi_s}}_{\text{geometry}} \nabla_{\perp}^2 \phi_W(\eta_0 - \chi, \chi \vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

Galaxy shape is determined by shear which can be computed from convergence



Bartelmann & Schneider
[astro-ph/9912508](https://arxiv.org/abs/astro-ph/9912508)

Observations

- CMB

Integrated Sachs-Wolfe (ISW) effect

The time variation of lensing potential causes a shift of photon temperature

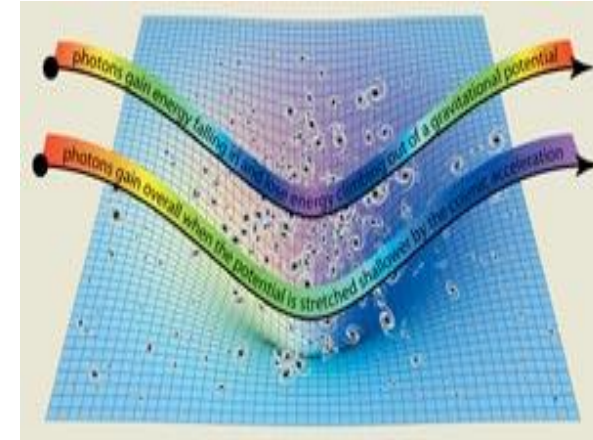
$$\Theta_{\ell}(k) = \int_0^{\eta} d\eta \frac{\partial \phi_W(k, \eta)}{\partial \eta} j_{\ell}[k(\eta_0 - \eta)] \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

lensing

CMB is also lensed

$$\Theta_{lensed}(\vec{n}) = \Theta(\vec{n} + \vec{d})$$

$$\vec{d} = \vec{\nabla} \psi, \quad \psi(\vec{n}) = -2 \int d\chi \frac{(\chi_{LSS} - \chi)\chi}{\chi_{LSS}} \phi_W(\eta_0 - \chi, \chi \vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$



<http://cmbcorrelations.pbworks.com>

Observations

- Redshift distortions

galaxies have peculiar velocities

clustering of galaxies in redshift space

is enhanced along the line of sight

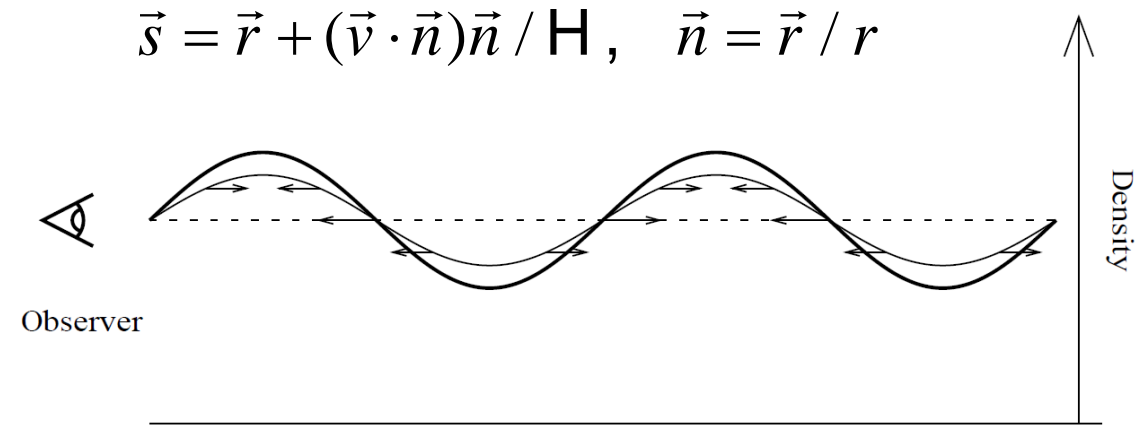
$$\delta^s(k, \mu) = \Delta_m(k) - \mu^2 \theta(k), \quad \mu^2 = \frac{(\vec{k} \cdot \vec{n})^2}{k^2}$$

If the continuity equation holds, the velocity dispersion is related to the growth rate

$$\delta^s(k, \mu) = \Delta_m(k) \left(1 - \mu^2 \frac{\theta(k)}{\Delta_m(k)} \right) = \Delta_m(k) (1 + \mu^2 f)$$

$$\Delta_m' = -H \theta_m$$

$$f = \frac{d \ln \Delta_m}{d \ln a} = \frac{a}{D_+} \frac{dD_+}{da}$$



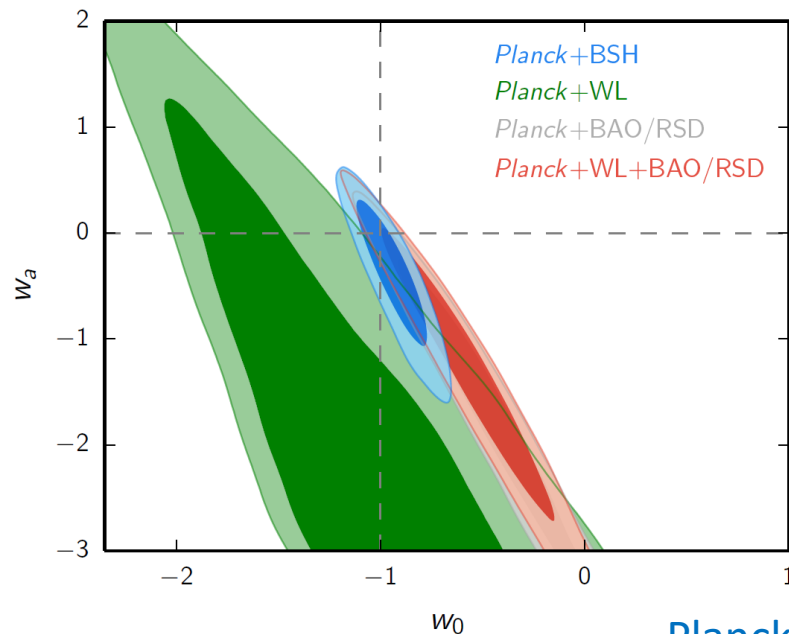
[Hamilton astro-ph/9708102](https://arxiv.org/abs/astro-ph/9708102)

Background expansion history

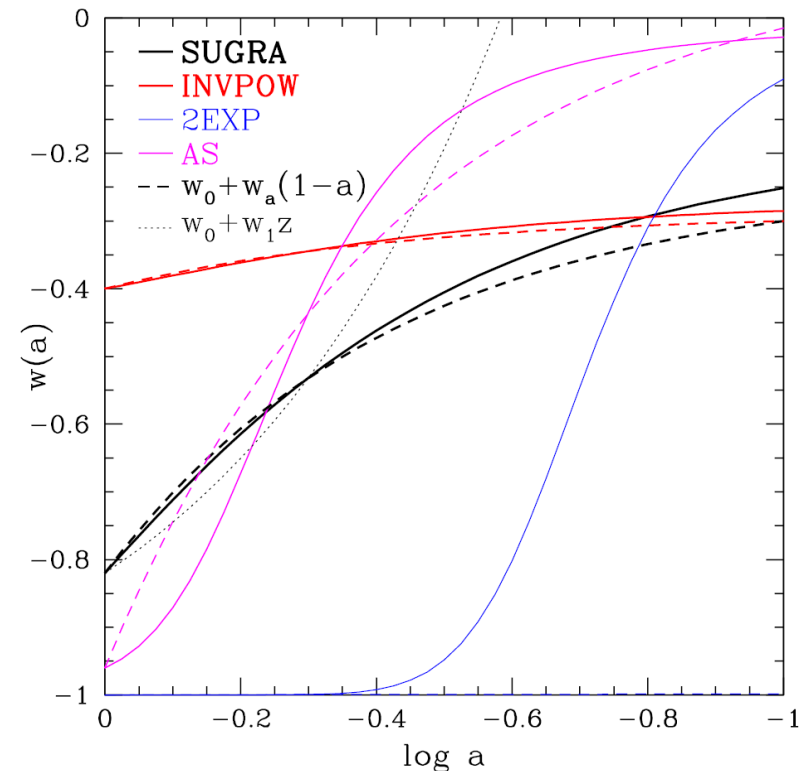
- Background expansion is determined by the equation of state

Parametrisation

$$w_{DE} = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$



Planck collaboration 1502.01590



Linder astro-ph/0311403

Model based parametrisation

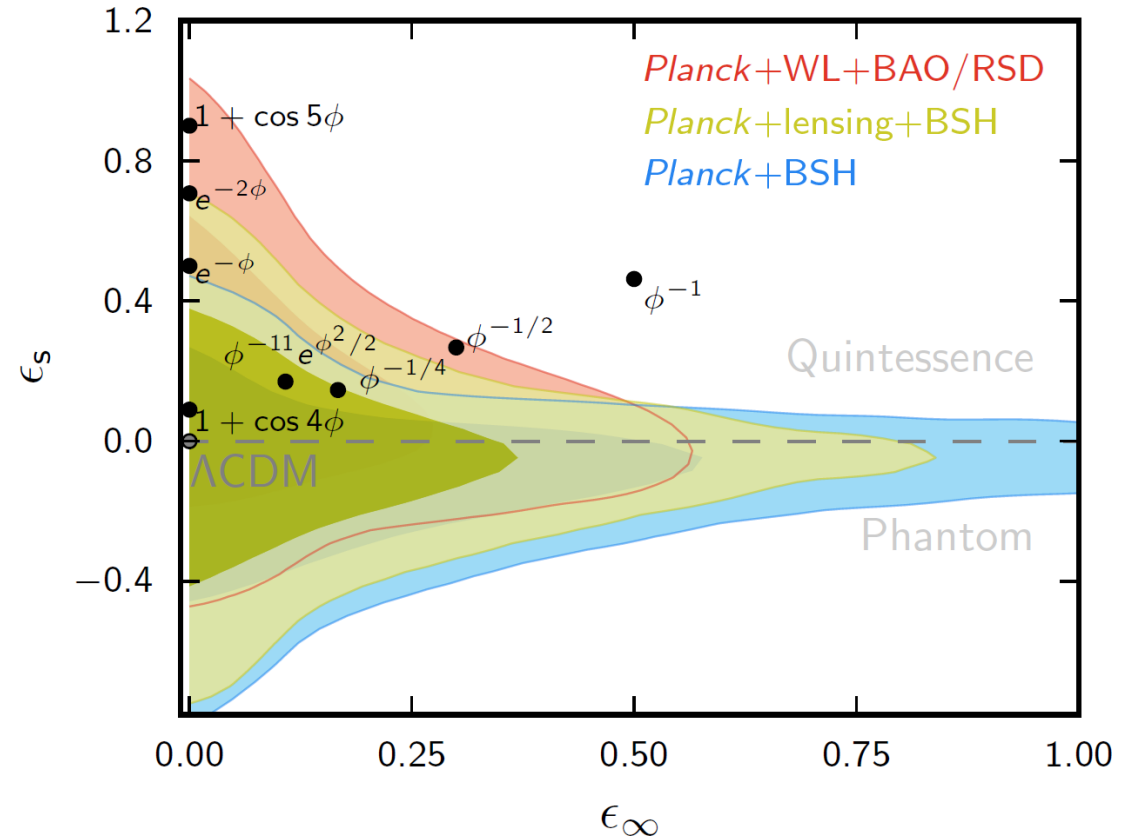
- Parametrisation

$$1 + w_\phi \approx \frac{2}{3} \varepsilon_\phi \Omega_\phi(a)$$

$$\varepsilon_\phi = \frac{1}{2\kappa^2} \left(\frac{V'}{V} \right)^2, \quad \Omega_\phi = \frac{\rho_\phi}{\rho_\phi + \rho_m}$$

$$\varepsilon_s = \varepsilon_\phi(\rho_m = \rho_{DE})$$

$$\varepsilon_\infty = \varepsilon_\phi \Omega_\phi(a \rightarrow 0)$$

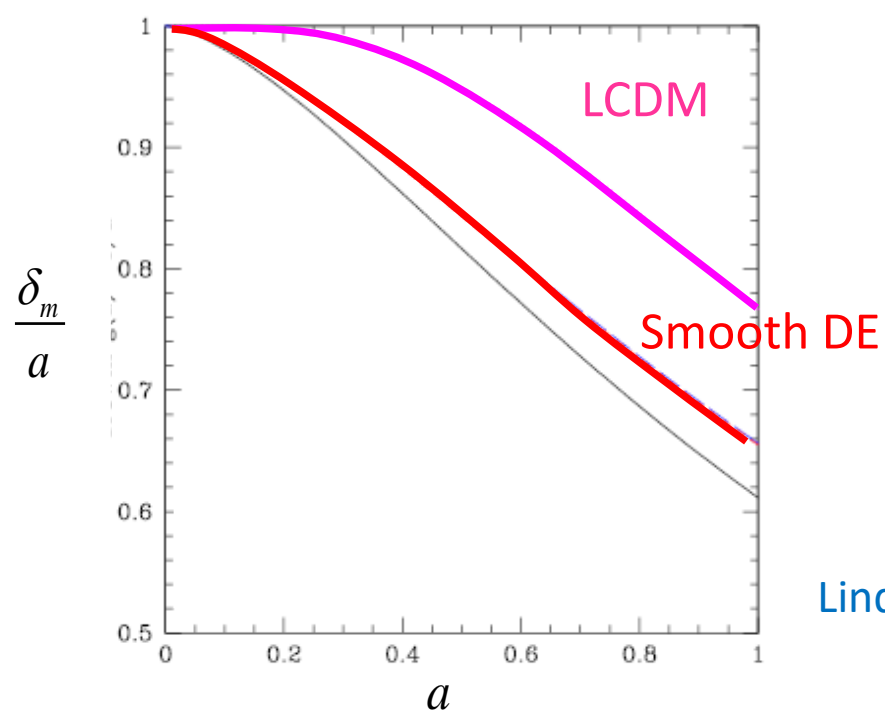
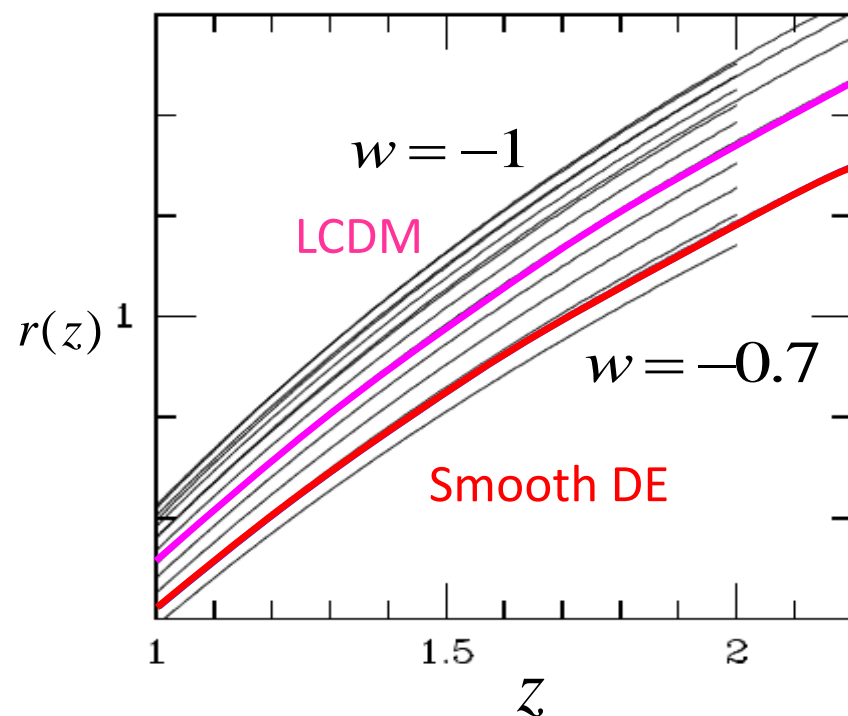


Planck 1502.01590

Expansion history v structure growth

- LCDM/Smooth DE

There is a one-to-one correspondence between background expansion history and growth of structure



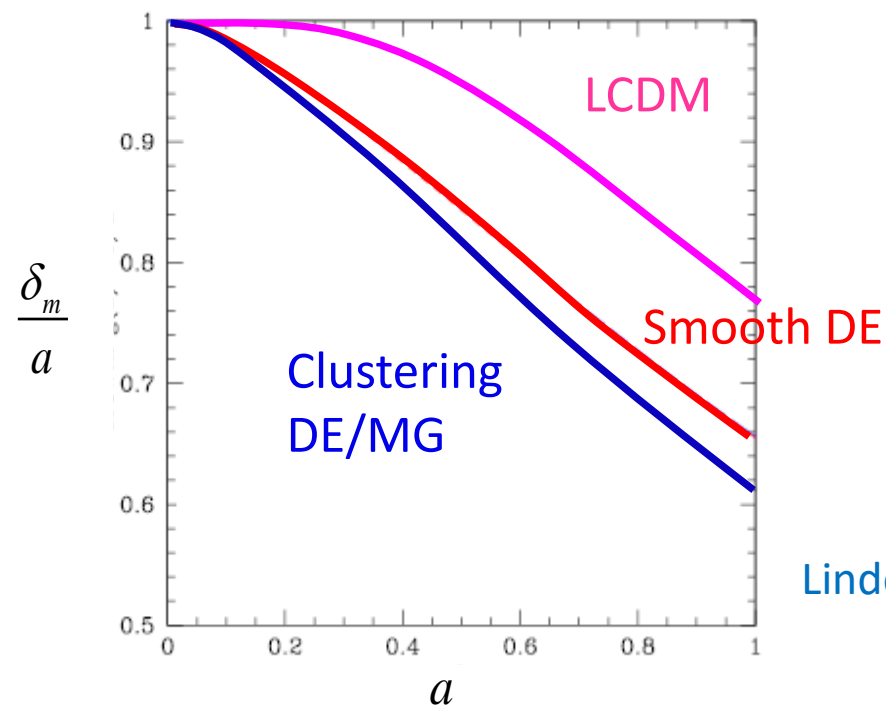
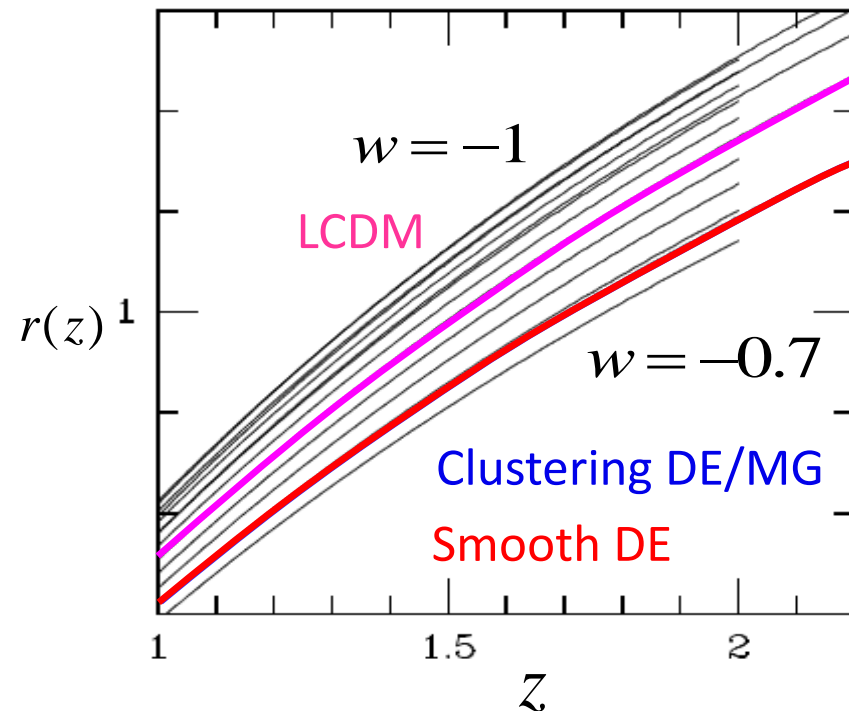
Linder astro-ph/0507263

Expansion history v structure growth

- Clustering DE/MG

structure growth is controlled also by $\delta\rho_E$

Even if it has the same expansion history as smooth DE, structure growth is different

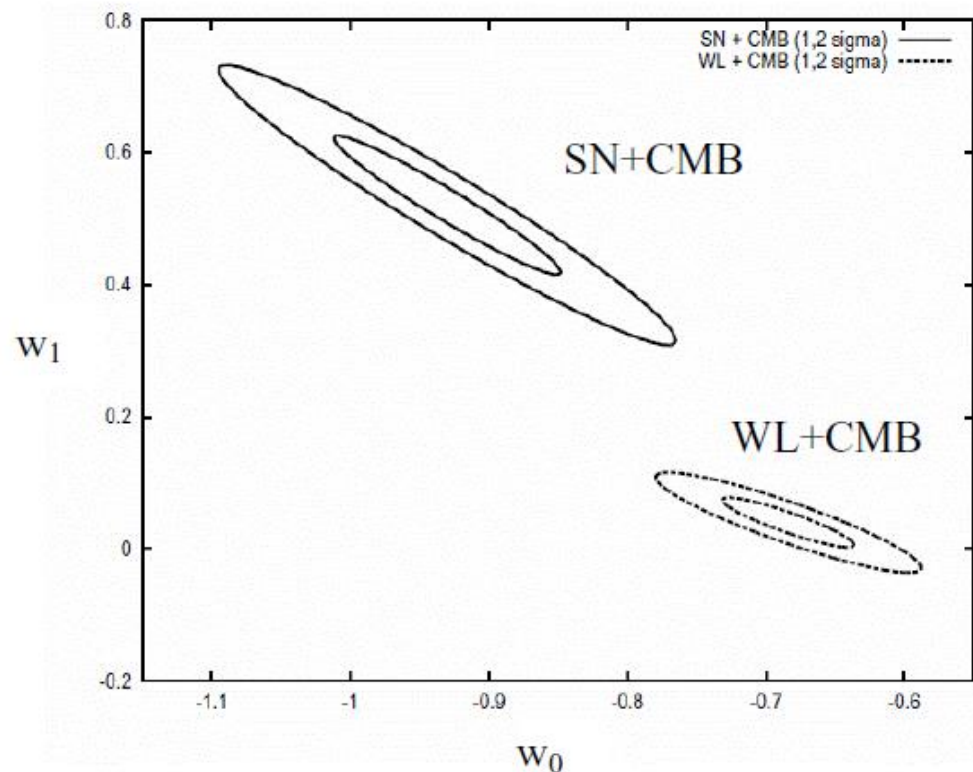


Linder astro-ph/0507263

Consistency test

Assume that the Universe is described by a clustering DE/MG model but we still try to fit the data using smooth DE

$$w(z) = w_0 + w_1 z,$$



SNe+CMB

SNe+weak lensing

Inconsistent!

[Ishak et.al. astro-ph/0507184](#)

Consistency relation

- In GR, gravitation equations are given by

$$H^2 = \frac{8\pi G}{3} \rho_T, \quad \rho_T = \sum_i \rho_i$$

$$\frac{k^2}{a^2} \Phi = 4\pi G a^2 \rho_T \delta_T, \quad \rho_T \delta_T = \sum_i \rho_i \delta_i$$

- Consistency relation

$$\alpha(k, t) = \frac{2k^2}{3a^2 \underbrace{H^2}_{\text{background}}} \frac{\underbrace{(\Phi + \Psi)}_{\text{Weak lensing}} - \underbrace{\Psi}_{\text{Peculiar velocity (Redshift distortions)}}}{\underbrace{\delta_T}_{\text{Galaxy distribution}}} = 1$$

$k^2 \Psi = \frac{d(a\theta_m)}{dt}$

$\delta_g = b_T \delta_T$

We have just enough number of observations to check the relation

Parametrisation

- Dark component

We need to specify $(\delta P_E, \pi_E)$

- Parametrisation of Einstein equations

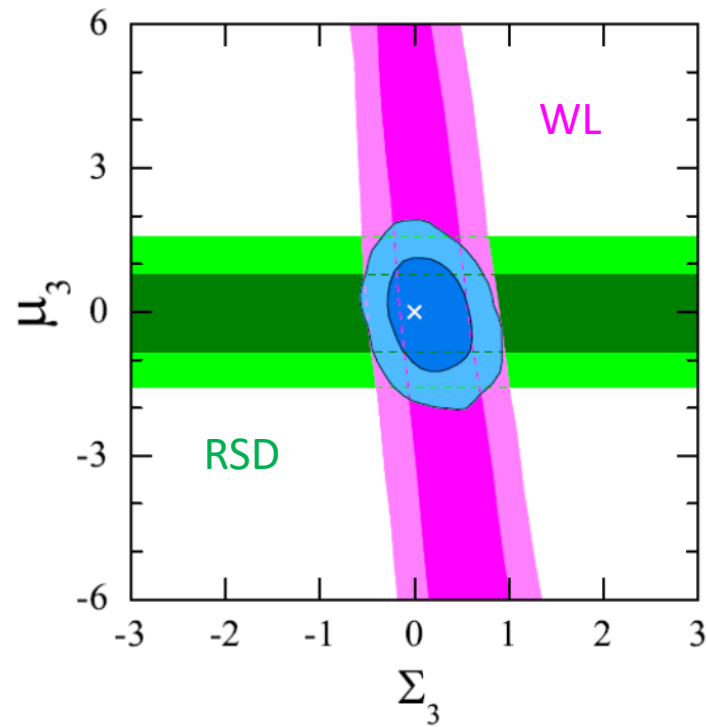
$$k^2 \Psi = -4\pi G a^2 \mu(k, a) \rho_m \Delta_m \quad ds^2 = a^2(\eta) \left[-(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$
$$\Phi = \eta(k, a) \Psi$$

equivalently, we can also parametrise the lensing potential

$$k^2 \Psi = -4\pi G a^2 \mu(k, a) \rho_m \Delta_m$$
$$k^2 \frac{(\Psi + \Phi)}{2} = -4\pi G a^2 \Sigma(k, a) \rho_m \Delta_m, \quad \Sigma = \frac{\mu(1 + \eta)}{2} \quad \mu = \eta = \Sigma = 1 \text{ for smooth DE}$$

Weak lensing and Redshift Distortions

- Combining WL ($\phi_W = (\Phi + \Psi) / 2$) and RSD (θ_m) we can break the degeneracy



$$k^2 \Psi = -4\pi G a^2 \mu(k, a) \rho_m \Delta_m$$

$$k^2 (\Psi + \Phi) = -4\pi G a^2 \Sigma(k, a) \rho_m \Delta_m, \quad \Sigma = \frac{\mu(1+\eta)}{2}$$

$$\theta_m' + H \left(1 + \frac{H'}{H^2} \right) \theta_m = \frac{k^2}{H} \Psi$$

$$\Delta_m'' + H \Delta_m' + 4\pi G a^2 \mu(k, a) \rho_m \Delta_m = 0$$

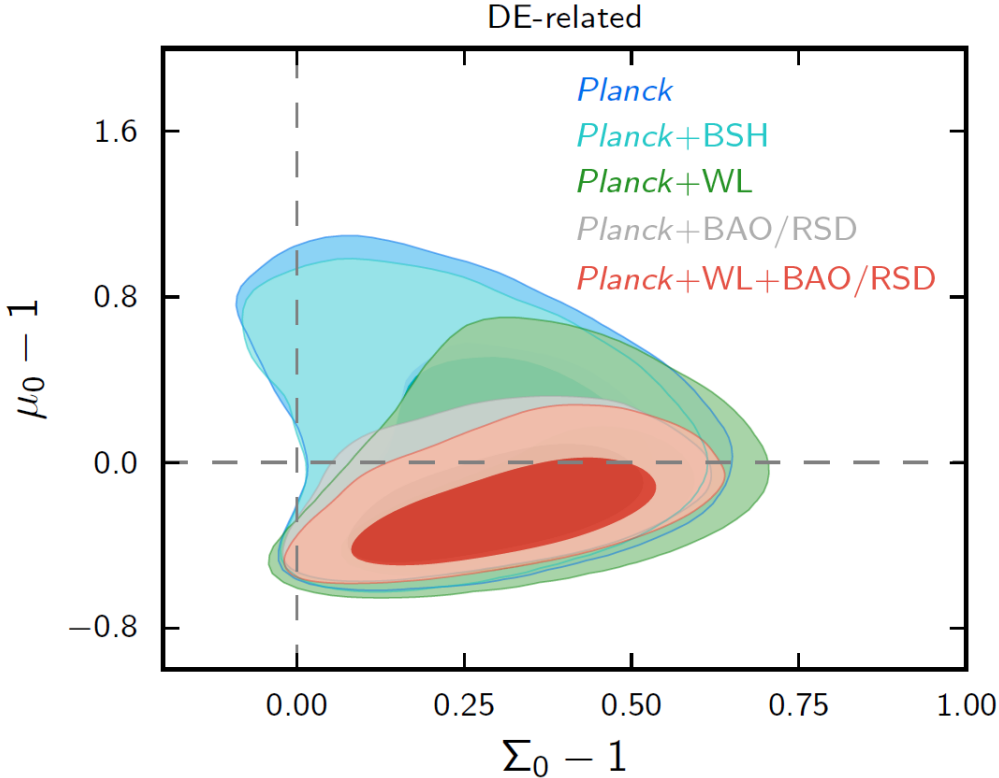
$$\mu(a) = 1 + \mu_3 a^3, \quad \Sigma(a) = 1 + \Sigma_3 a^3$$

Planck 2015 results

- Assuming LCDM background

$$\mu(a) = 1 + \bar{\mu} \Omega_{DE}(a), \quad \Sigma(a) = 1 + \bar{\Sigma} \Omega_{DE}(a)$$

$$\mu_0 = \mu(1), \quad \Sigma_0 = \Sigma(1)$$



Max. degeneracy	<i>Planck</i> TT+lowP	<i>Planck</i> TT+lowP +BSH	<i>Planck</i> TT+lowP +WL	<i>Planck</i> TT+lowP +BAO/RSD	<i>Planck</i> TT+lowP +WL+BAO/RSD
DE-related	$0.84^{+0.30}_{-0.40}$ (2.1 σ)	$0.80^{+0.28}_{-0.39}$ (2.1 σ)	$1.08^{+0.35}_{-0.42}$ (2.6 σ)	$0.90^{+0.33}_{-0.37}$ (2.4 σ)	1.03 ± 0.34 (3.0 σ)
+ CMB lensing	$0.42^{+0.18}_{-0.34}$ (1.2 σ)	$0.38^{+0.18}_{-0.28}$ (1.4 σ)	$0.58^{+0.24}_{-0.37}$ (1.6 σ)	$0.40^{+0.18}_{-0.28}$ (1.4 σ)	$0.51^{+0.21}_{-0.30}$ (1.7 σ)

tension with LCDM

Planck 1502.01590

Tension with LCDM in Planck data

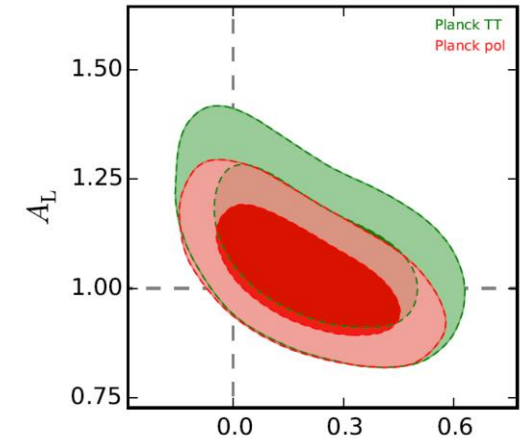
- Lensing amplitude

CMB lensing requires a larger amplitude than LCDM in the power spectrum $A_{lens} = 1.22 \pm 0.10$

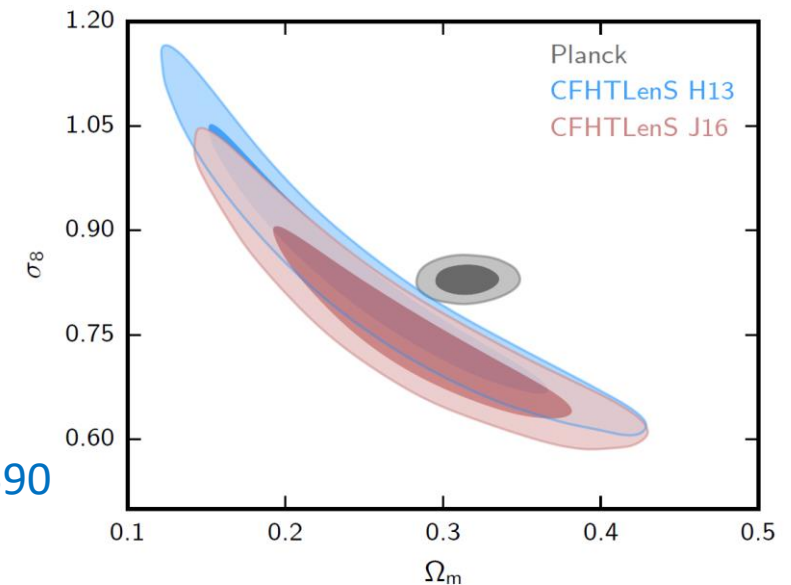
(cf. this tension does not exist for lensing measured from trispectrum)

- Amplitude of fluctuations

The late time amplitude of fluctuations in LCDM predicted from primordial amplitude measured by CMB is larger than that measured by weak lensing (CFHTLS) [Planck 1502.01590](#)

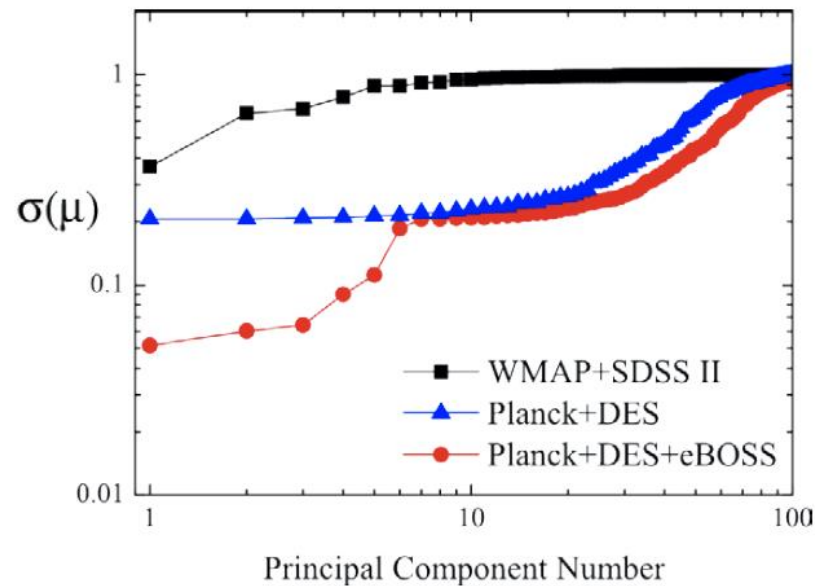


[Valentino et.al. 1509.07501](#) $\Sigma_0 - 1$



Future forecasts

Next 3-5 years



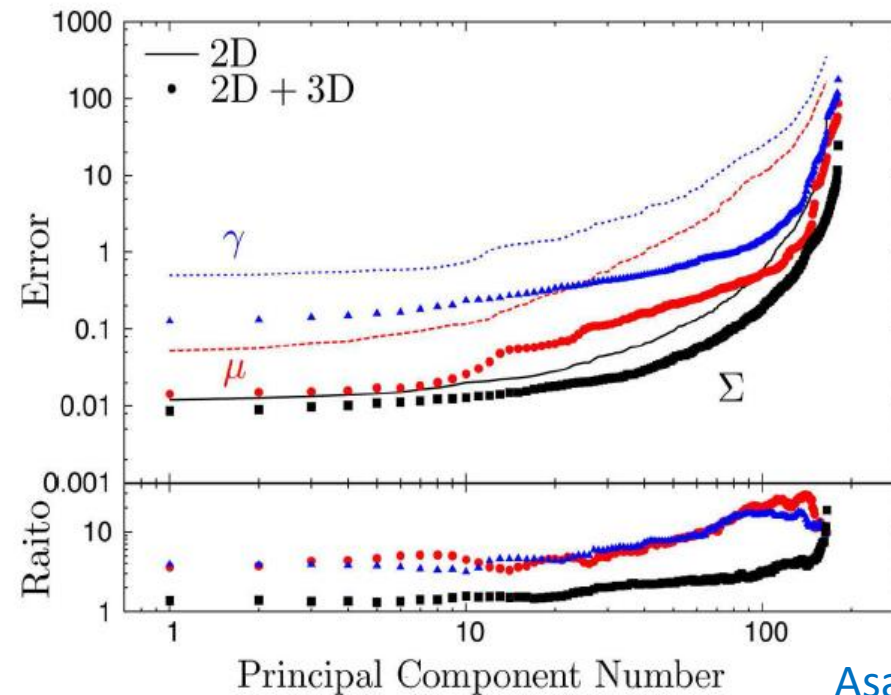
[Zhao et.al. 1510.08216](#)

DES (2012-2017) imaging

eBOSS (2014-2018) spectroscopic

Several parameters at the 5-10% level

Next 5-10 years



[Asaba et.al. 1306.2546](#)

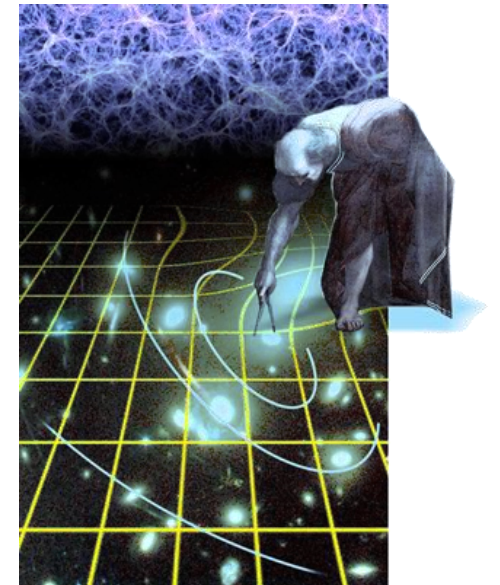
Euclid (2020-)

10 parameters at the 1% level

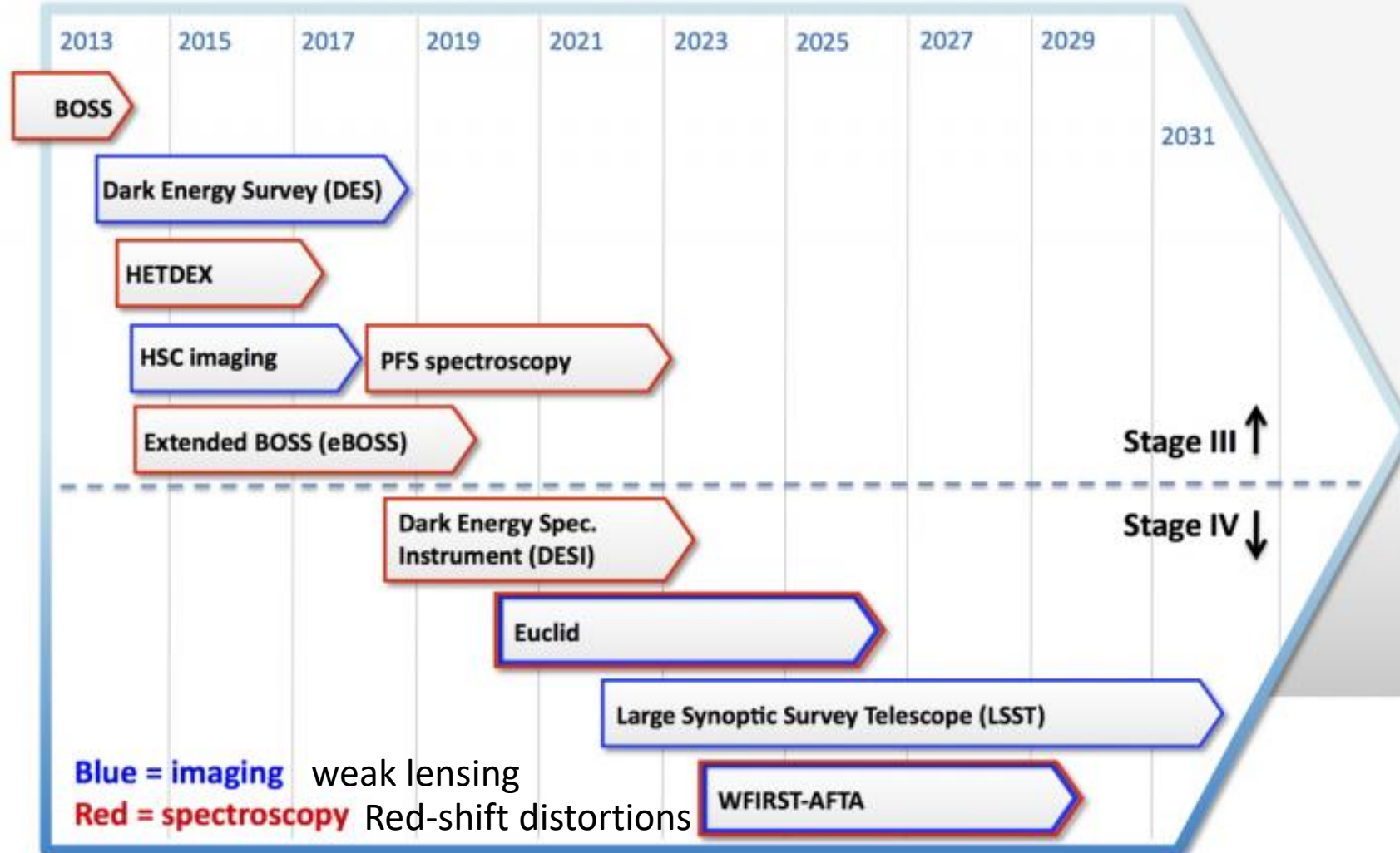
Euclid (2020-)

<http://sci.esa.int/euclid/>

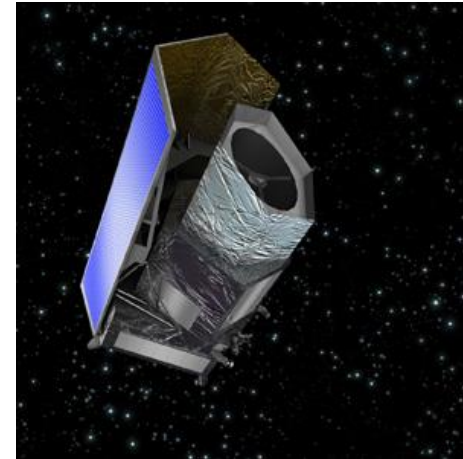
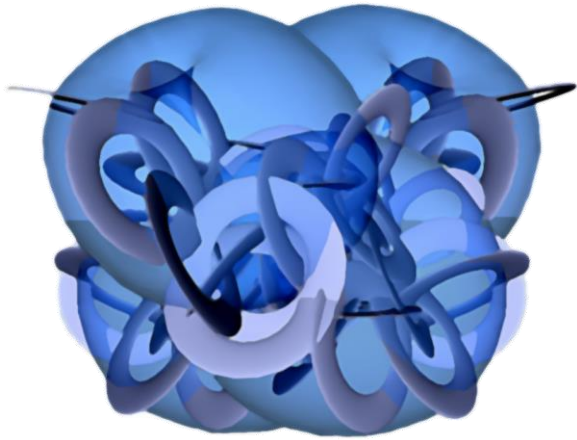
R-L0	<p>The Euclid Mission will by itself allow us to</p> <ul style="list-style-type: none">• understand the nature of the apparent acceleration of the Universe and• test gravity on cosmological scales <p>from the measurement of the cosmic expansion history and the growth rate of structures.</p>
R-L0.1	<p>To determine the nature of the apparent acceleration, Euclid will distinguish effects produced by a cosmological constant from those produced by a dynamical dark energy. This must be done by achieving a minimum $FoM > 400$ from Euclid data alone.</p>
R-L0.2	<p>To experience effects of gravity on cosmological scales, Euclid will probe the growth of structure and will separately constrain the two relativistic potentials, Ψ and Φ. This can be done by achieving an absolute 1σ precision of 0.02 on the growth index, γ, from Euclid data alone.</p>



Dark Energy Experiments: 2013 - 2031



From theory to observations



initial conditions

fluid approach
Effective Field Theory

many free functions



$$\mu(k_i, z_i), \Sigma(k_i, z_i)$$



$$C_{\ell}^{IJ}(z)$$

galaxy count
Lensing
ISW

systematics