Dark Energy

Part I: Concordance model and the cosmological constant

Part II: Alternative theories of dark energy and modified gravity

ICG PhD Lectures 2018

Emir Gümrükçüoğlu
[*partially* armed with Kazuya Koyama’s slides]

Part I: Dark Energy

1. Observational evidence
2. The cosmological constant
3. Alternative explanations

Basic assumptions (1)

Isotropy and homogeneity

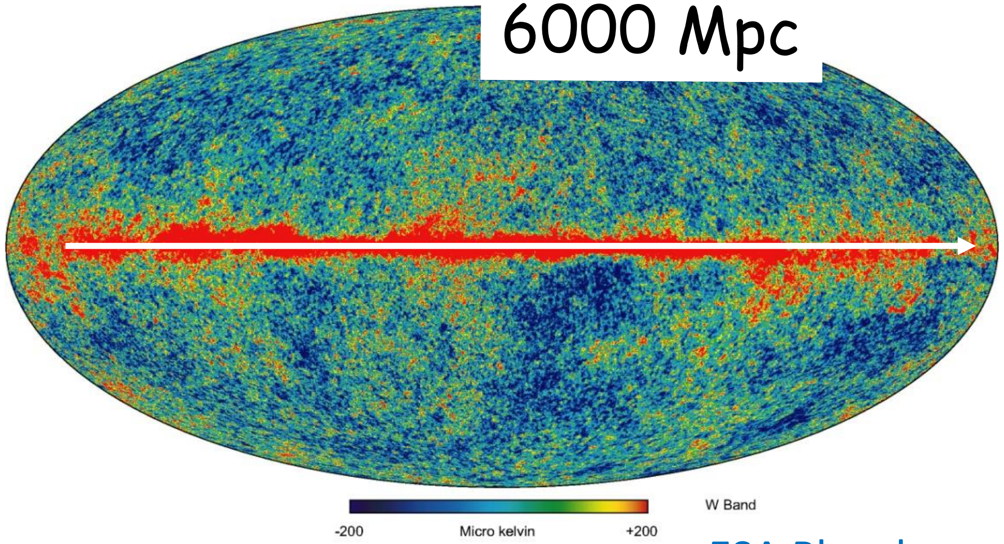
- Isotropy

  CMB fluctuations

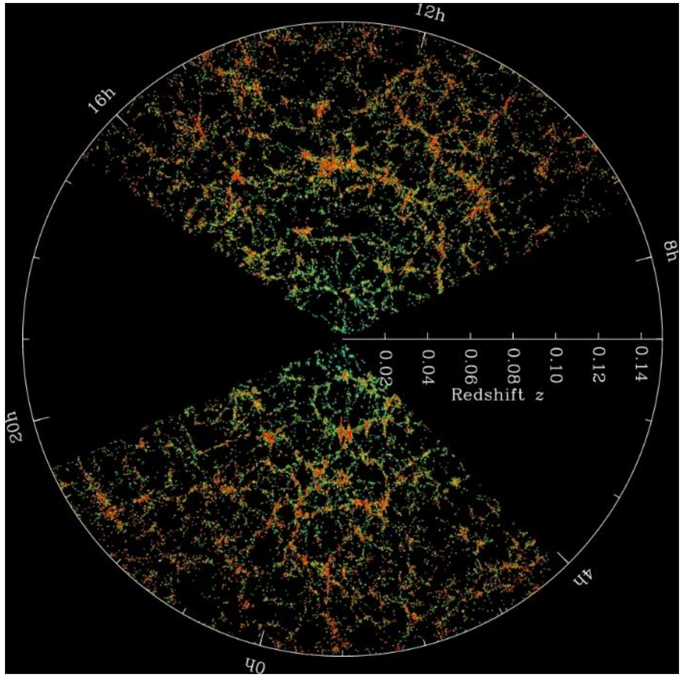
$$\frac{\Delta T}{T} \approx 10^{-5}$$

- Homogeneity

  galaxy distribution



ESA Planck



SDSSIII

Basic assumption (2)

- General Relativity (GR)

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R} = \boxed{8\pi G\, T_{\mu\nu}}$$

geometry matter

- Matter

$$T^{\mu}{}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + P\, \delta^{\mu}_{\nu} \qquad u^{\mu} = (-1, 0, 0, 0)$$

- Bianchi identity

$$\nabla^{\mu} G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^{\mu} T_{\mu\nu} = 0$$

Basic assumption (3)

- We introduce dark energy in addition to “known” matter such as baryons, cold dark matter and radiation and assume that they satisfy the conservation equation independently

$$\dot{\rho}_i + 3H(1+w_i)\rho_i = 0, \quad w_i = \frac{P_i}{\rho_i} \qquad \rho_i \propto a^{-3(1+w_i)}$$

- equation of state

$$w_r = \frac{1}{3}, \quad w_m = 0, \quad w_{DE} = ?$$

- Density parameter

$$\Omega_i = \frac{8\pi G\rho_i}{3H^2}, \quad \Omega_K = -\frac{K}{(aH)^2} \qquad \sum_i \Omega_i = 1$$

What we measure

- Distance **assumption (1)**

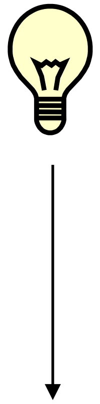
$$ds_3^{\,2} = d\chi^2 + f_K(\chi)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad f_K = \frac{1}{\sqrt{-K}}\sinh\left(\sqrt{-K}\chi\right)$$

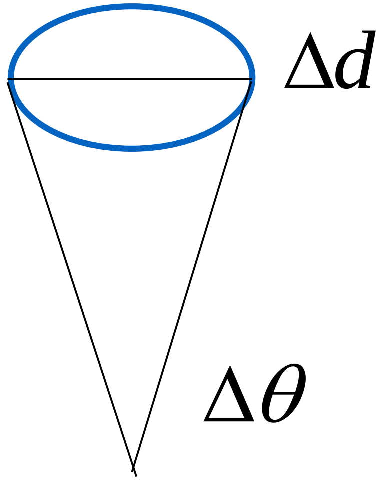
$$\chi = \int_0^t \frac{c}{a(t')}dt' = \frac{c}{a_0 H_0}\int_0^z \frac{dz'}{E(z')}, \quad E(z) = \frac{H(z)}{H_0}$$

- Luminosity distance and angular diameter distance

$$d_L = f_K(\chi)(1+z)$$ redshift $1+z = \dfrac{a_0}{a}$

$$d_A = \frac{d_L}{(1+z)^2}$$





Theoretical predictions

- Now we use **assumption (2) and (3)**

$$E(z)^2 = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{K0}(1+z)^2$$
$$+\Omega_{DE0}\exp\left[3\int_0^z dz' \frac{1+w_{DE}(z')}{1+z'}\right]$$

- ΛCDM model

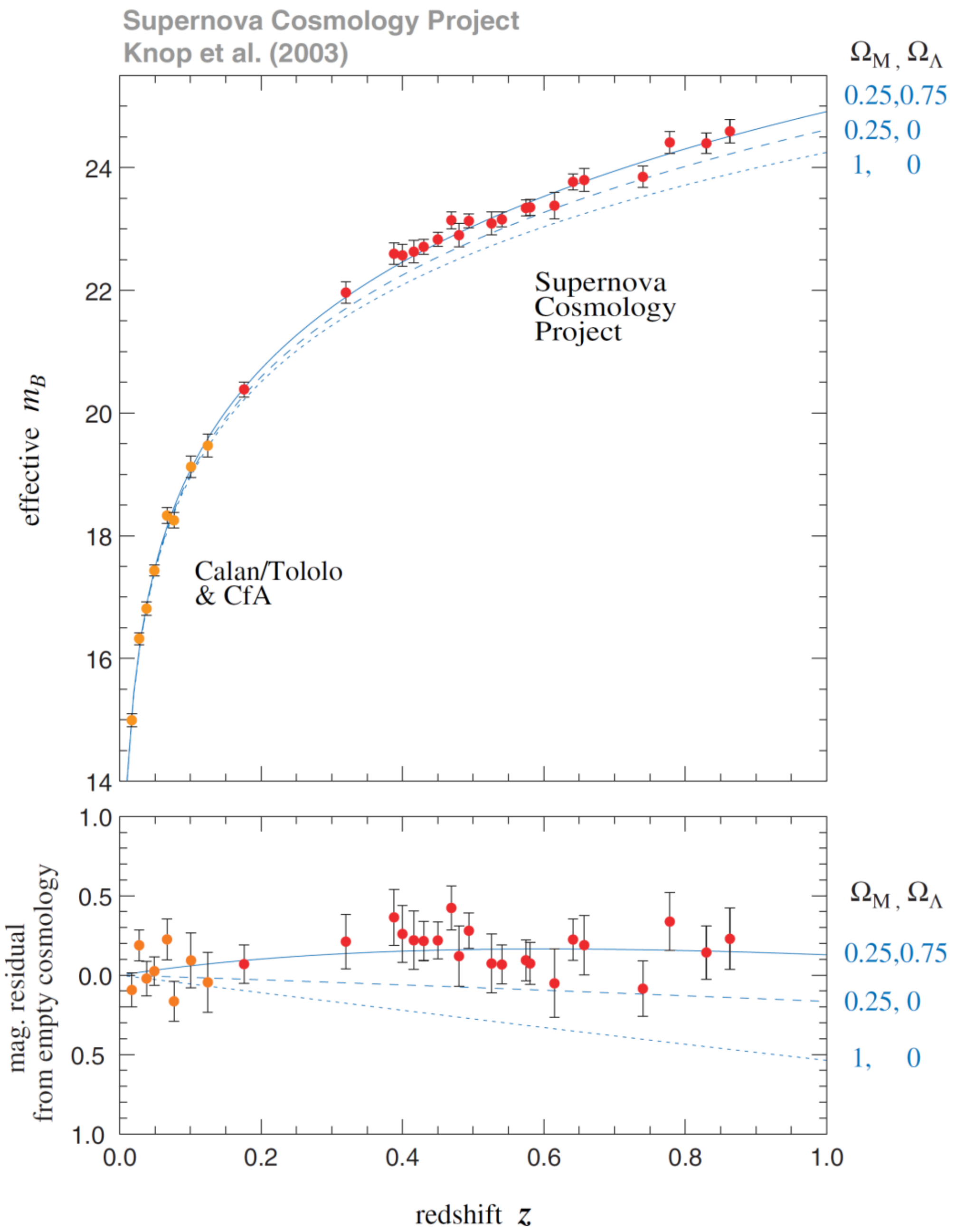
$$w_{DE} = -1, \quad \Omega_{DE0} = \Omega_\Lambda, \quad (\Omega_{r0} = 8\times 10^{-5})$$

- Distance measurements

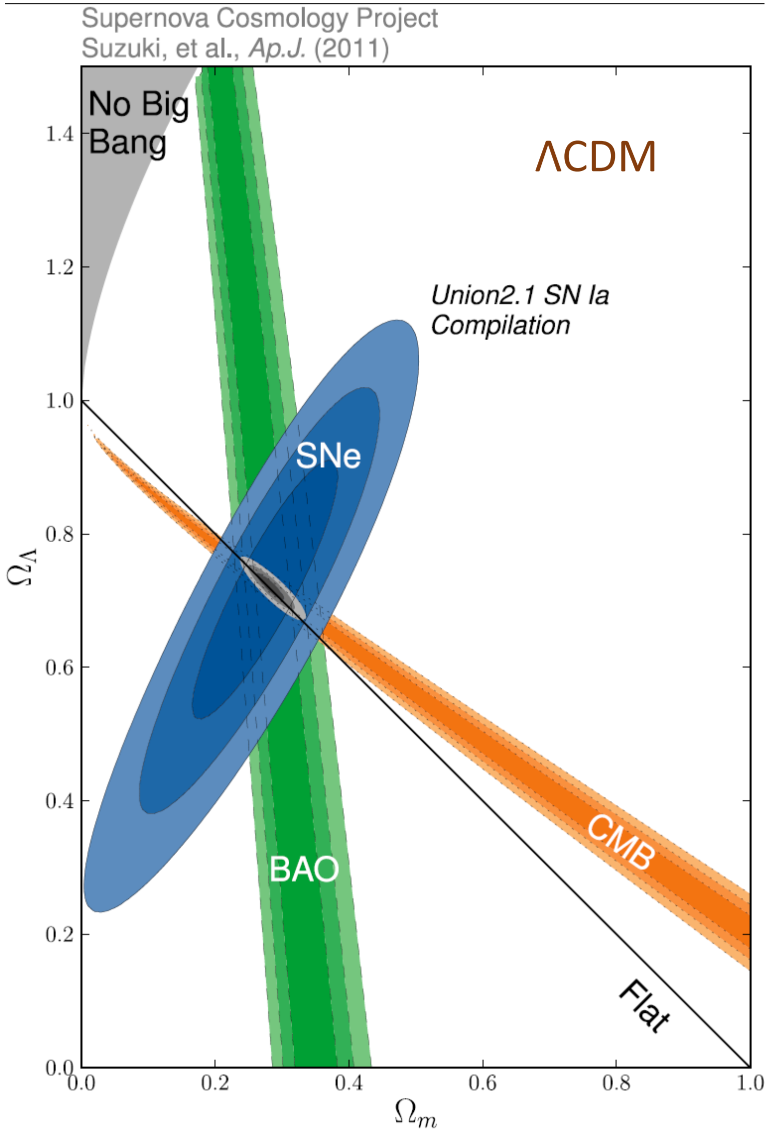
  Supernovae d_L

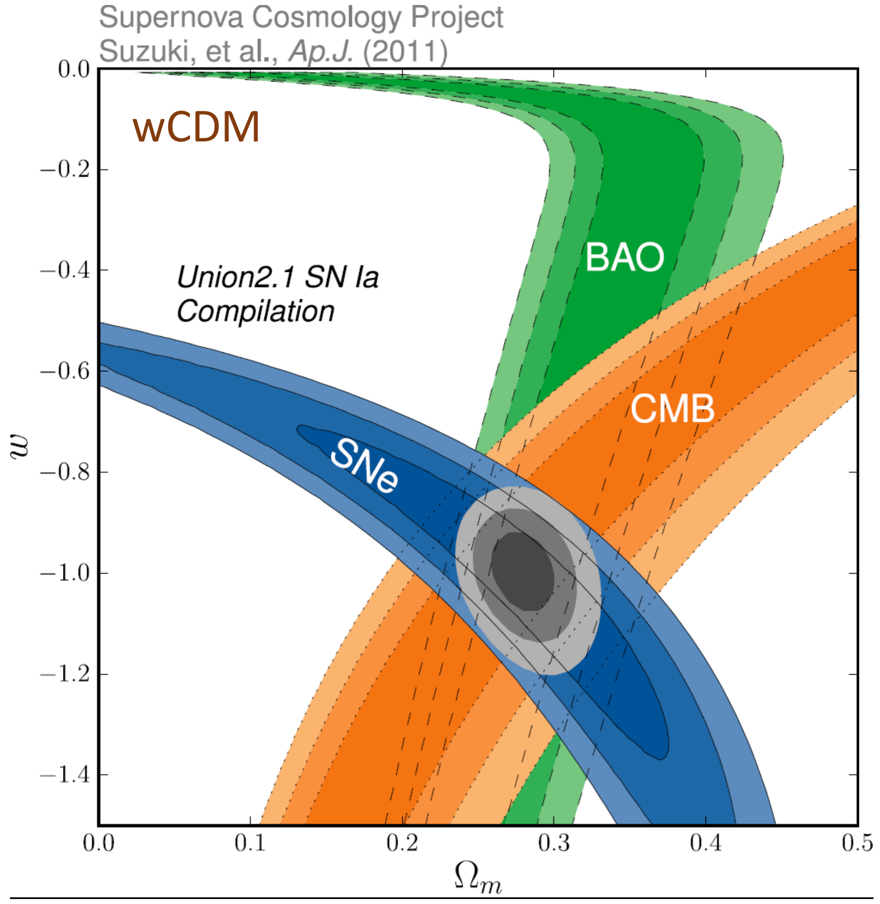
  Cosmic Microwave, d_A

  Baryon Acoustic Oscillations d_A



This is what we found





Cosmological constant

- Action $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\,\Lambda) + \int \sqrt{-g}\,\mathcal{L}_{matter}$

  Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\,T_{\mu\nu}$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G} = -P_\Lambda$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

cosmological constant does not diminish by the expansion of the universe and the expansion of the Universe accelerates

$$\ddot{a} > 0$$

Why should we bother?

- What's the problem?

  ΛCDM works well to explain observations ✓

  The cosmological constant can be included in Einstein's GR ✓

- Energy scales (natural units $\hbar = c = k_B = 1$)

$$m_{pl} = G^{-1/2} = 1.22\times 10^{19}\,\mathrm{GeV} \qquad H_0 = 2.13\times 10^{-42}\ h^{-1}\mathrm{GeV}$$

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \frac{m_{pl}^2\Lambda}{8\pi} \approx \frac{m_{pl}^2 H_0^2}{8\pi} \approx 10^{-48}\,\mathrm{GeV}^4 \approx (10^{-3}\,\mathrm{eV})^4$$

Dimensionful quantity. Is it small or large?

Vacuum energy

- Quantum fields have zero-point energy

massive fields (boson and fermion)

$$E = g_i \frac{\hbar\omega}{2} = \frac{1}{2}\sqrt{p^2 + m^2}, \quad g_{boson} = 1, \quad g_{\mathrm{fermion}} = -1$$

vacuum energy

$$\rho_{vac} = \frac{1}{2}\sum_i g_i \int_0^\infty \frac{d^3 p}{(2\pi)^3}\sqrt{p^2 + m_i^2} \qquad\qquad p_{\max}{}^2 >> m^2$$

$$\approx \sum_i \frac{g_i p_{\max}^4}{16\pi^2}\left[1 + \frac{m_i^2}{p_{\max}^2} + \frac{1}{2}\frac{m_i{}^4}{p_{\max}^4}\ln\left(\frac{m_i}{p_{\max}}\right)\right]$$

This depends on Ultra-Violet (UV) physics but it is robust that there is a contribution of order $O(m^4)$

Vacuum energy is big, but *how* big?

- A naïve approach: new physics at $p_{\max} = m_{pl}, \quad \rho_{vac} = m_{pl}^4 = 10^{124} \rho_\Lambda^{obs}$
- Lorentz breaking cut-off; stick to the physical $\propto m^4$ part e.g. Koksma, Prokopec 2011
- The observed cosmological constant

$$\rho_\Lambda^{obs} \approx (10^{-3}\mathrm{eV})^4 \quad \text{requires} \quad m < 10^{-3} \ \mathrm{eV}$$

- Contribution from e.g. electron $\Delta\rho_{vac} \approx m_e^4 = (0.5\ \mathrm{MeV})^4 = 10^{35} \rho_\Lambda^{obs}$
- Contribution from phase transitions

electroweak $\Delta\rho_{vac} \approx (200\ \mathrm{GeV})^4 = 10^{57} \rho_\Lambda^{obs}$

QCD $\Delta\rho_{vac} \approx (0.3\ \mathrm{GeV})^4 = 10^{46} \rho_\Lambda^{obs}$

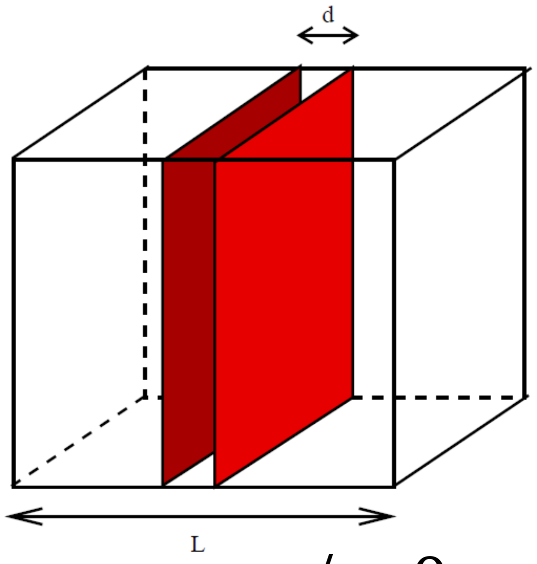
$$\boxed{\rho_{vac} = \rho_{bare} + \sum_i g_i \frac{{m_i}^4}{64\pi^2} \ln \frac{{m_i}^2}{{p_{\max}}^2} + \rho_{EW} + \rho_{QCD} + \cdots}$$

J. Martin 2012
arXiv:1205.3365

Is vacuum energy real?

- Casimir energy

$$\phi(\vec{x}) = \phi(\vec{x} + L\vec{n}), \quad \vec{p} = \left(\frac{n\pi}{d}, p_y, p_z \right), \quad n = 1, 2, 3 \ldots$$



$\phi = 0$

zero-point energy per unit area

$$\frac{E(d)}{A} = \sum_{n=1}^{\infty} \int \frac{dp_y dp_z}{(2\pi)^2} \left[\frac{1}{2} \sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2} \right] F_{reg}(a), \quad F_{reg}(a) = e^{-a\sqrt{\left(\frac{n\pi}{d}\right)^2 + p_y^2 + p_z^2}}$$

total energy $E_{tot}(d) = E(L-d) + E(d)$ depend on d and diverges as $a \to 0$ but the force between the two plates is finite

$$F = -\frac{1}{A} \frac{\partial E_{tot}(d)}{\partial d} = -\frac{\hbar c \pi^2}{240 d^4}$$

Old cosmological constant problem

- Zero-point energy is not important in quantum field theory in flat spacetime (cf. Casimir force is determined by $\partial E(d)/\partial d$ not $E(d)$)

- In GR, matter, including vacuum energy, curves spacetime.

$$\rho_{vac} \approx m_e^4 = (0.5\ \mathrm{MeV})^4 \qquad H \approx \frac{m_e^2}{m} = (10^6\,\mathrm{km})^{-1}$$

- Fine tuning

$$\Lambda_{obs} = \Lambda_{vacuum} + \Lambda \qquad S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda\right)$$

vacuum energy is very sensitive to UV physics thus tuning is not stable under radiative corrections

New cosmological constant problem

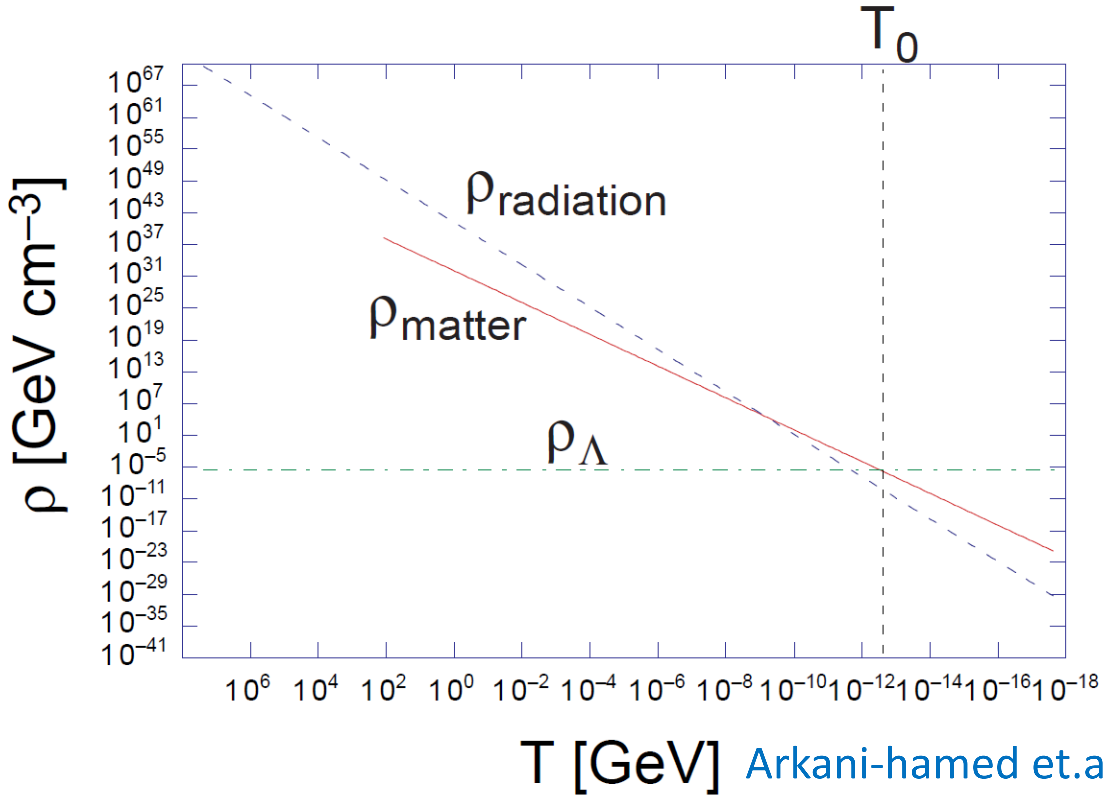
- Assume that the old cosmological constant is solved, we then need to explain why the expansion of the Universe appears to be accelerating now

- Coincidence problem

  why $\Omega_\Lambda \approx \Omega_m$

  anthropic principle?

  otherwise we don't exist



Arkani-hamed et.al. astro-ph/0005111

So, we should bother!

We know vacuum energy exists, but it does not gravitate in the way it should in GR. It is important to know whether the acceleration of the Universe is caused by the (fine-tuned) cosmological constant or not.

It is important to reconsider all the assumptions:

1. Homogeneity and Isotropy
2. General Relativity
3. Matter content of the Universe

Assumption (1)

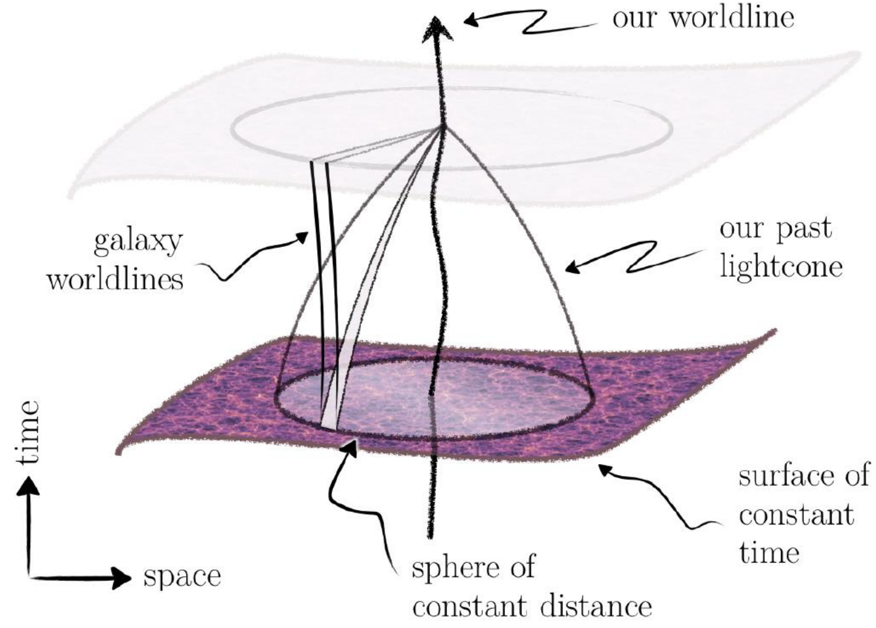
1. The Copernican principle: we are not at a special location in the universe
2. The cosmological principle: on large scales, the universe is homogeneous and isotropic

- FRW metric

If all observers measure isotropic distance-redshift relation, then the spacetime is FRW

We need the Copernican principle to show the cosmological principle but this is hard to test

Relativistic Cosmology, Ellis, Maartens, MacCallum



Clarkson 1204.5505

Assumption (1)

- Void models

If we happen to live inside a void with low densities, the expansion of the universe appears to be accelerating

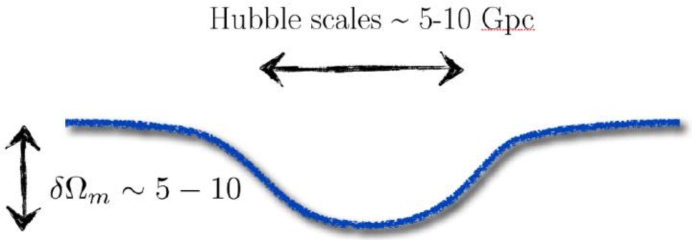
ex.) Lemaitre-Tolman-Bondi model

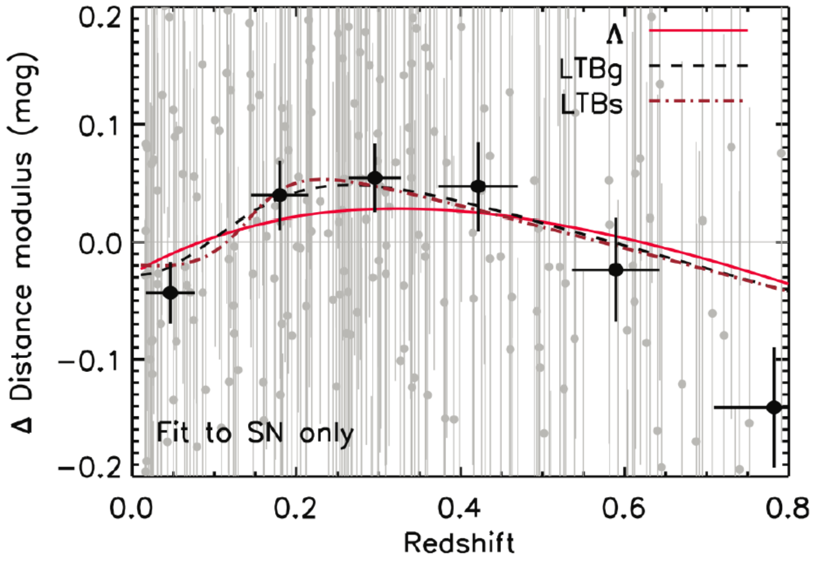
$$ds^2 = -dt^2 + \frac{a_{\parallel}{}^2(t,r)}{1-K(r)r^2}dr^2 + a_{\perp}{}^2(t,r)r^2 d\Omega^2$$

simple mode is ruled out

low H_0, radial velocities of clusters

(kinetic Sunyaev-Zeldovich effect)





Clarkson 1204.5505

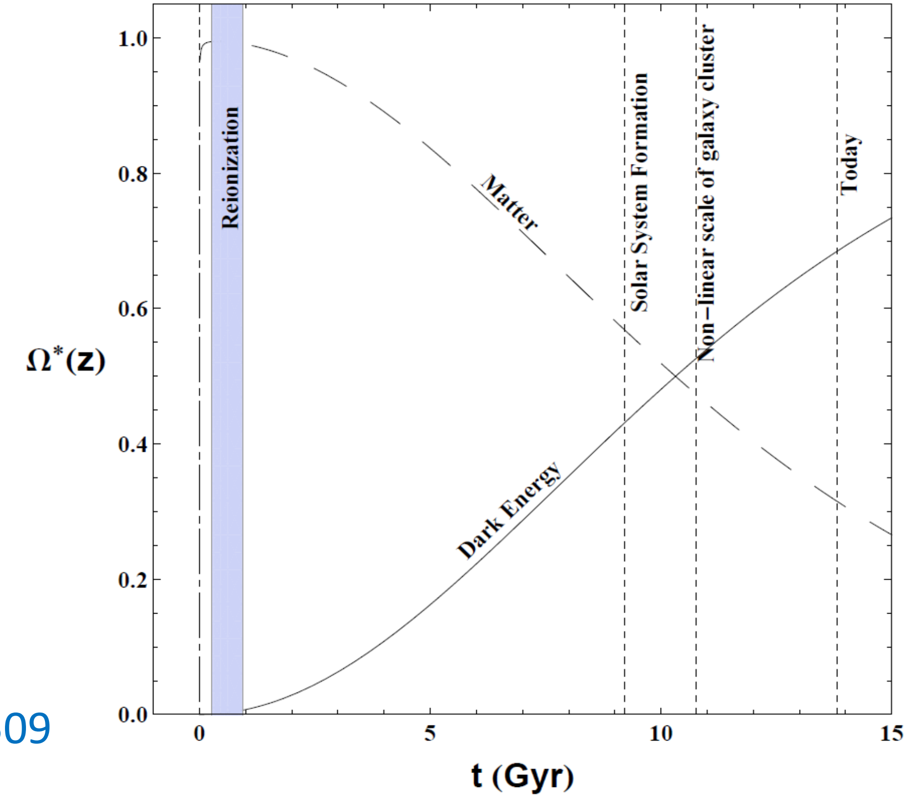
Back-reaction

- The Universe becomes inhomogeneous at late time. If the back-reaction of these imhomogeneities cause the acceleration, we can solve the coincident problem.

long-standing debates on the magnitude of the effect on the expansion of the Universe from small scale inhomogeneity.

It is difficult to explain the acceleration

Velten. et.al. 1410.2509



Assumption (2)

- Why we believe in general relativity?

  - Observational point of view

    GR is tested to very high accuracies by solar system experiments and pulsar timing measurements Will arXiv:1403.7377

  - Theoretical point of view

    GR is the unique metric theory in 4D that gives second order differential equations Lovelock 1971

Solar system tests

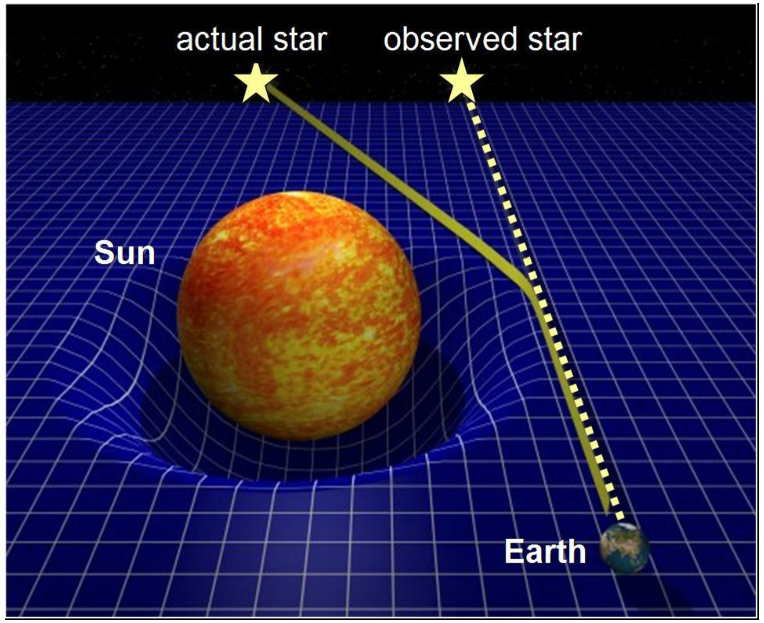
- Post-Newtonian parameter

$$g_{00} = -1 + 2GU$$
$$g_{ij} = \delta_{ij}(1 + 2\gamma GU)$$

$$\boxed{U = \frac{M}{r}}$$

γ: How much space curvature produced by unit rest mass?

- $\gamma = 0$ "Newtonian"
- $\gamma = 1$ GR



- Bending of light

$$\theta = 2(1+\gamma)\frac{M_\odot}{r} = \frac{1+\gamma}{2}\theta_{GR}$$

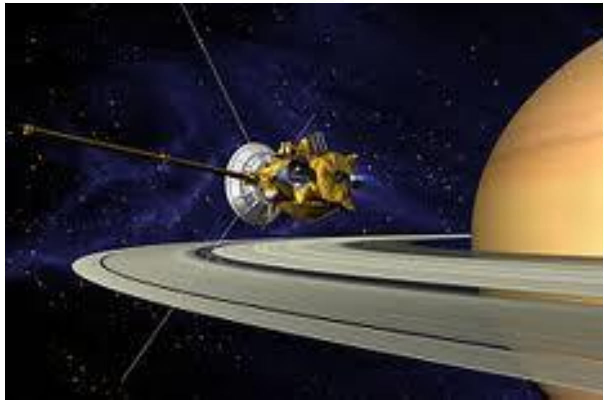
$$\theta = (0.99992 \pm 0.00023) \times 1.75''$$

$$\gamma - 1 = (-1.7 \pm 4.5) \times 10^{-4}$$

- Shapiro time delay

$$\Delta t = (1.00001 \pm 0.0001) \times \Delta t_{GR}$$

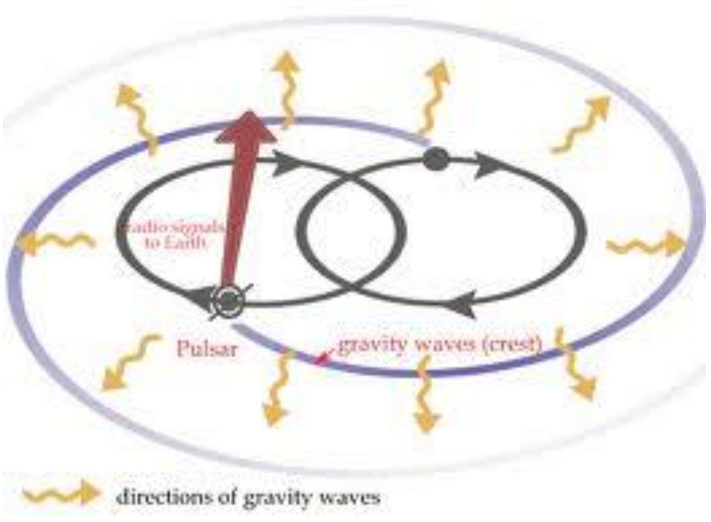
$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

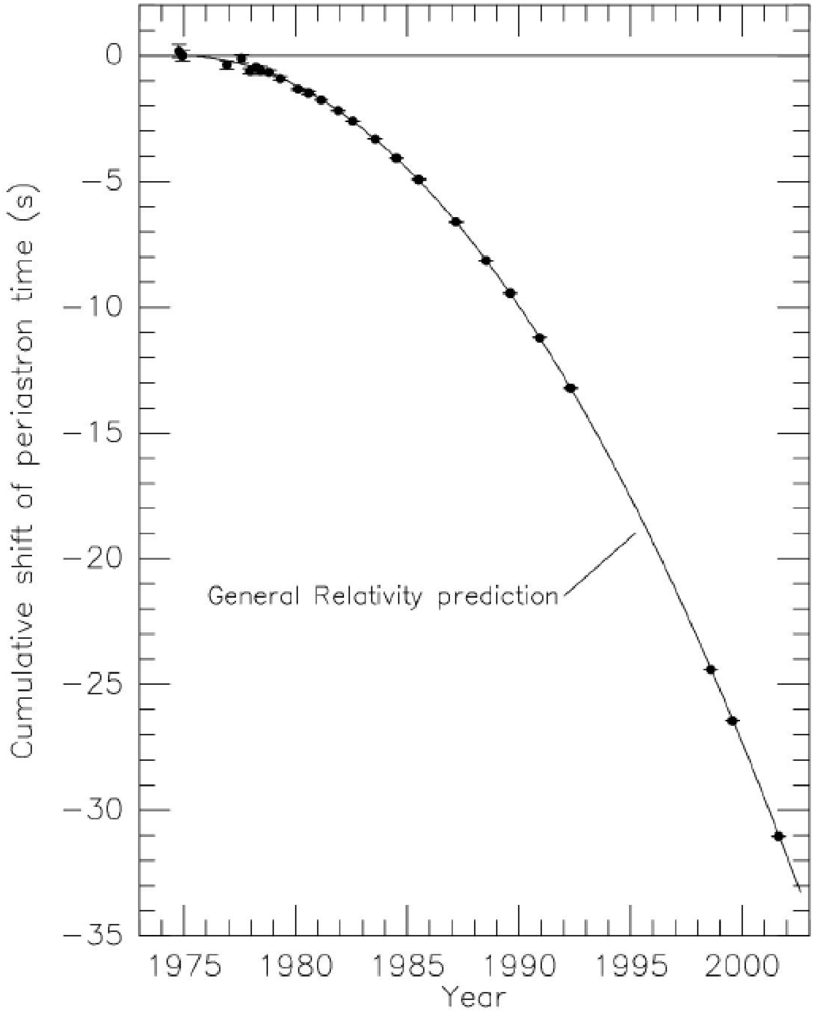


Pulsar timing

- Hulse & Taylor binary (pulsar+NS)

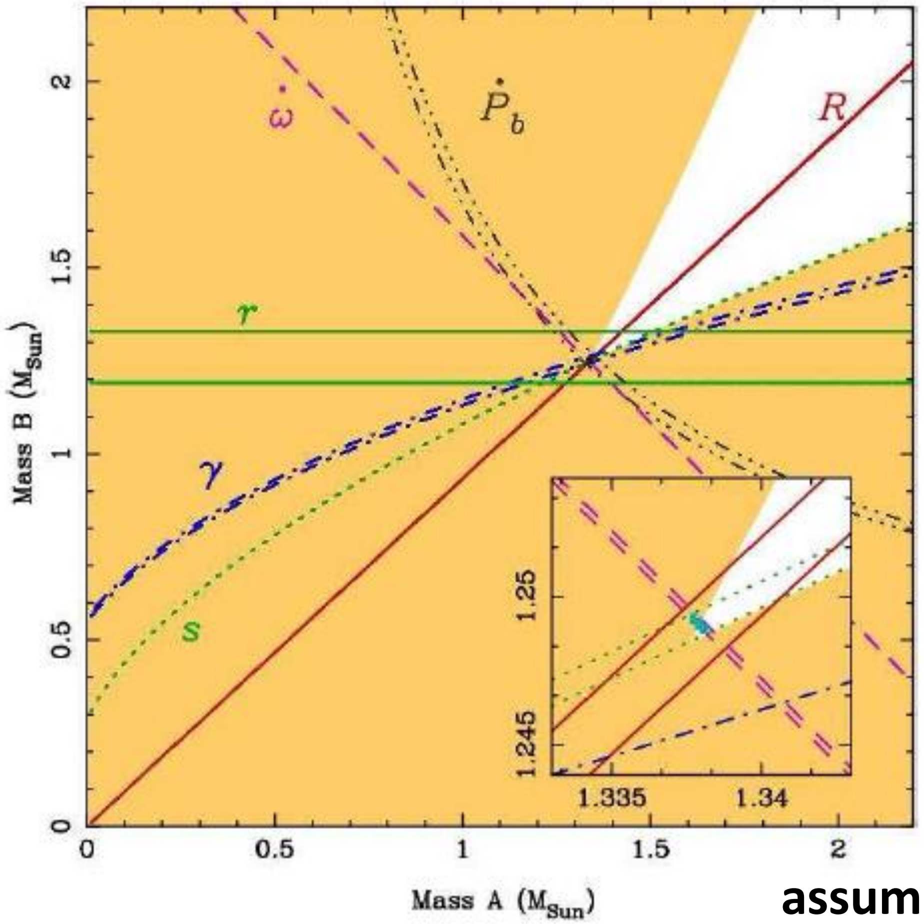
    Orbital decay due to gravitational waves perfectly agrees with GR prediction





Pulsar timing

- PSR J0737-3039A/B
  Post Keplerian parameter



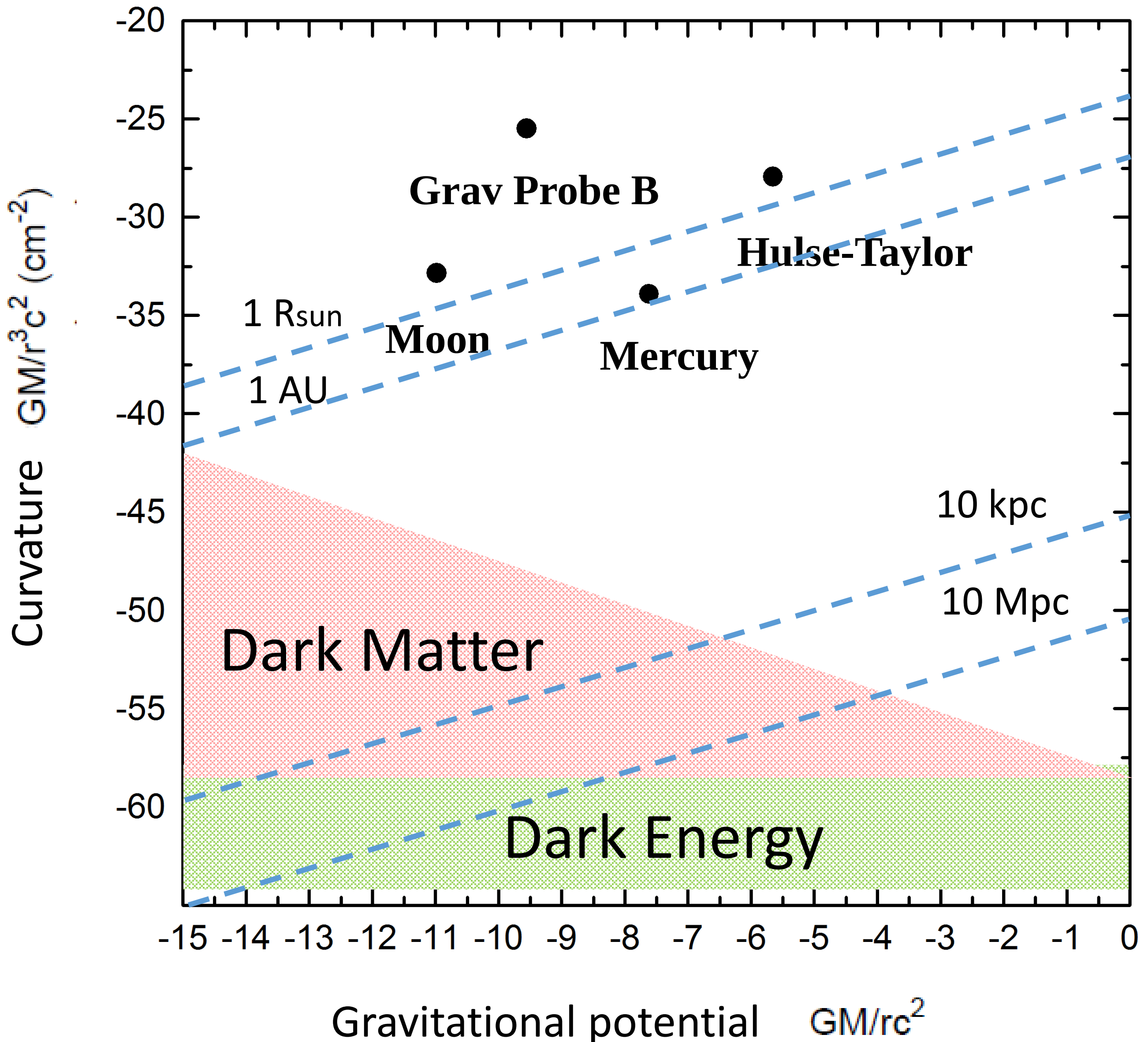
assuming GR

Science 314 (2006) 97-102

Fig. 1. The observational constraints upon the masses M_A and M_B. The colored regions are those which are excluded by the Keplerian mass functions of the two pulsars. Further constraints are shown as pairs of lines enclosing permitted regions as predicted by general relativity: (a) the measurement of the advance of periastron $\dot{\omega}$, giving the total mass M_A+M_B=2.588±0.003 $M_\odot$ (dashed line); (b) the measurement of R= $M_A/M_B = x_B/x_A = 1.069 \pm 0.006$ (solid line); (c) the measurement of the gravitational redshift/time dilation parameter γ (dot-dash line); (d) the measurement of Shapiro parameter r giving $M_B = 1.2 \pm 0.3$ $M_\odot$ (dot-dot-dot-dash line) and (e) Shapiro parameter s (dotted line). Inset is an enlarged view of the small square which encompasses the intersection of the three tightest constraints, with the scales increased by a factor of 16. The permitted regions are those between the pairs of parallel lines and we see that an area exists which is compatible with all constraints, delineated by the solid blue region.

Tests of GR

Psaltis Living Rev. Relativity 11 (2008), 9
Baker et.al. ApJ 802 63 (2015)

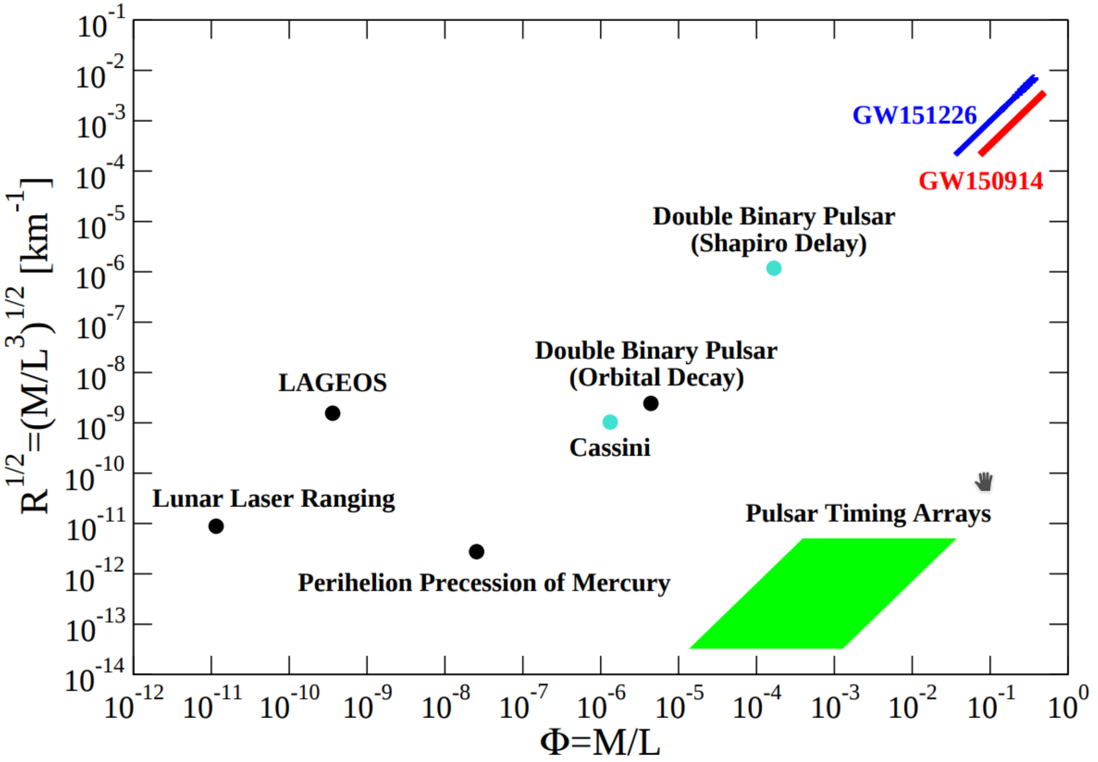


curvature

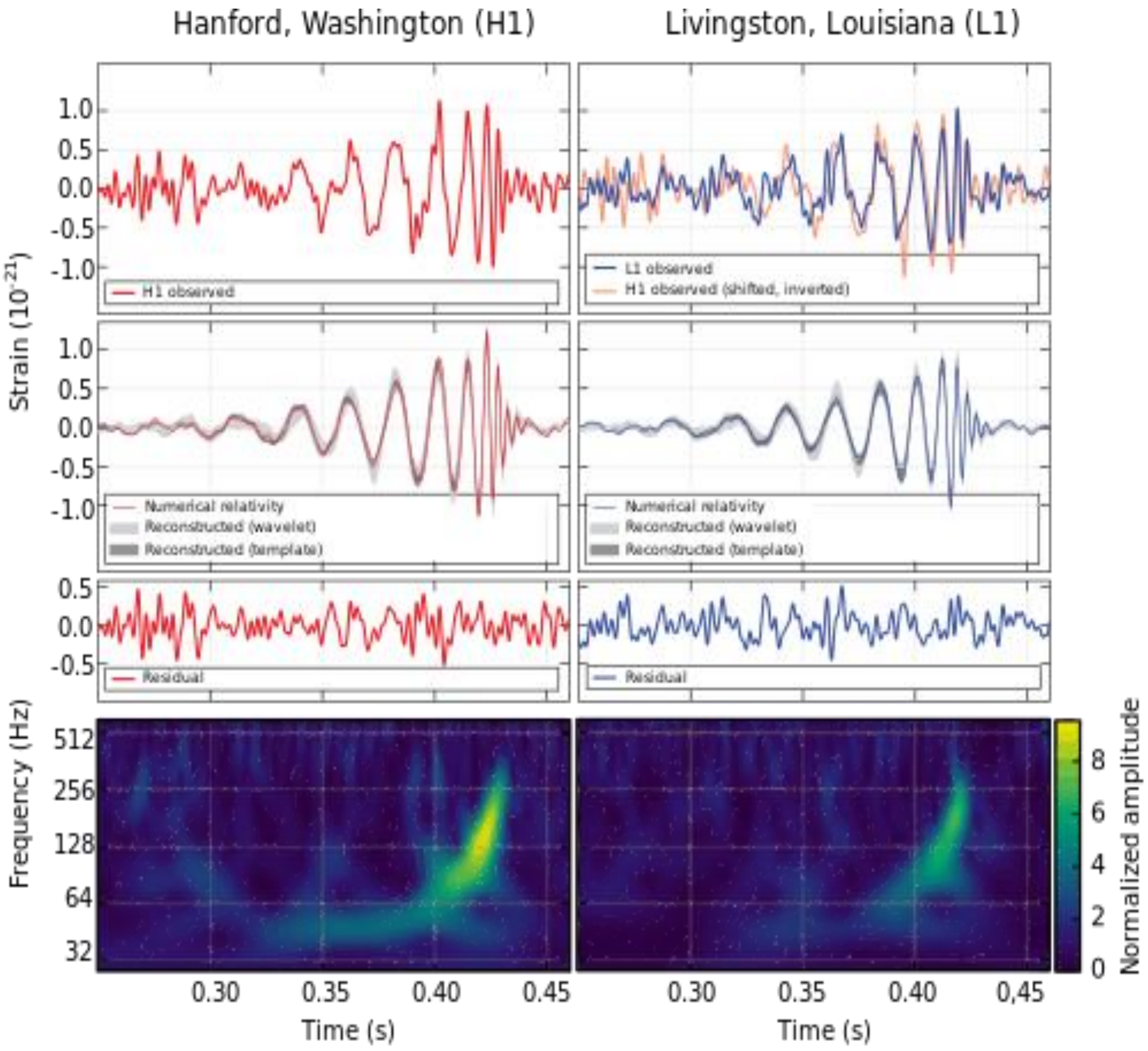
$$R = \frac{GM}{r^3c^2}$$

potential

$$\Psi = \frac{GM}{rc^2}$$



Yunes et.al. 1603.08955



LIGO collaboration

Assumption (3)

- What is dark energy?

In the background, all information is encoded in the equation of state

$$w_{DE} = \frac{P_{DE}}{\rho_{DE}}$$

what are the candidates for dynamical DE $w_{DE}(z)$

- How to distinguish between DE and modifications of gravity?

$$G_{\mu\nu} + G_{\mu\nu}^{MG} = 8\pi G\left(T_{\mu\nu} + T_{\mu\nu}^{DE}\right)$$

To be continued…