Computational Cognitive Neuroscience

Week 1 – coursework intro

University of Helsinki, 21st January 2021

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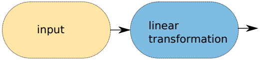
- Thursdays we cover some material useful for assignments - today neural networks

Neural networks intro

- "logistic regression in a trench coat"
- sequence of layers/modules/functions transform data and make a prediction
- layers mostly linear transformations followed by non-linearities
- parameters learned by minimizing a loss function wrt. data
- sequence of layers like composite functions $f(g(x, w))$; can use chain rule (backpropagation)

Linear layer



- Data point $\mathbf{x}_i \in \mathbb{R}^d$

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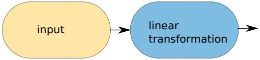

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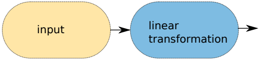

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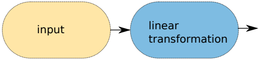

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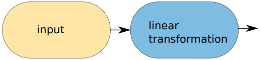

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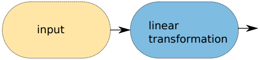

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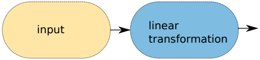

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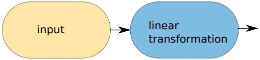

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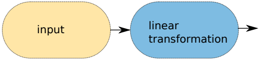

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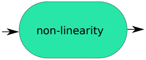
- or just $\mathbf{Z} = f_{\text{linear}}(\mathbf{X}, \mathbf{W}, \mathbf{b})$

non-linear / activation layer



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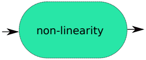

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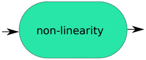

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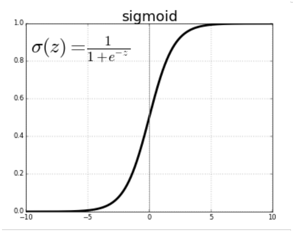

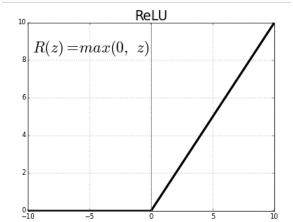

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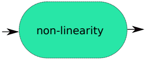

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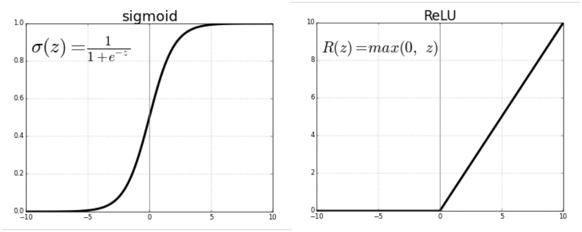

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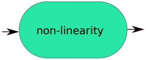

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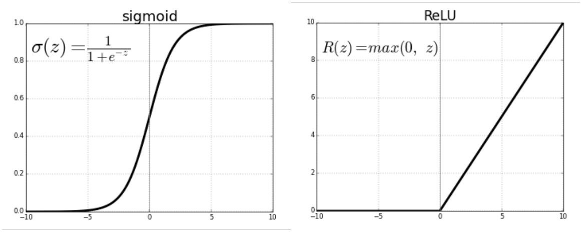

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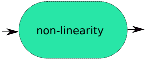

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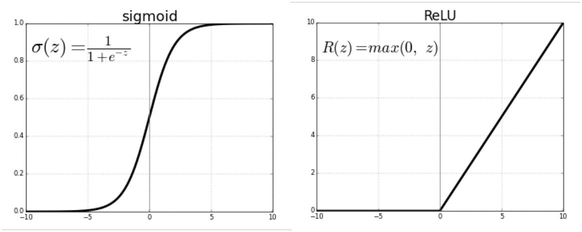

- usually no parameters

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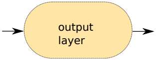

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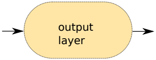

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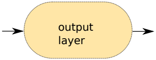

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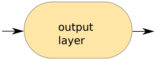

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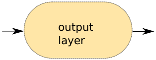

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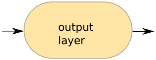

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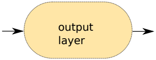

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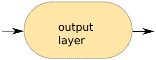

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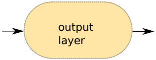
- for regression problems, no activation function on top of output layer

output layer for classification



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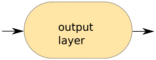

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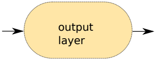

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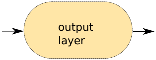

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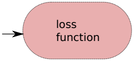
- hence $\hat{\mathbf{y}}_i = \left(\hat{p}(c_i = 1|\mathbf{z}_i), \ldots, \hat{p}(c_i = C|\mathbf{z}_i)\right)$

loss functions



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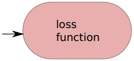

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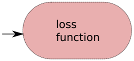

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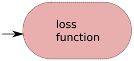

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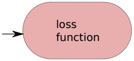

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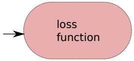

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  - here $y_{i,j} = 1$ when j is the true class, 0 for others. $\hat{y}_{i,j}$ is the predicted probability for each class

Combining layers – softmax cross-entropy

- the output softmax layer, and the cross entropy layer are often combined

$$-\sum_{i=1}^{n} \log \underbrace{\left(\frac{\exp(z_{i,j}[y_i])}{\sum_{c=1}^{10} \exp(z_{i,j}[c_j])} \right)}_{\text{softmax output}}$$

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- in softmax cross-entropy the computational properties are improved for combined layer, particularly in the backpropagation

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- thus we find out how changing parameters in specific direction would influence loss

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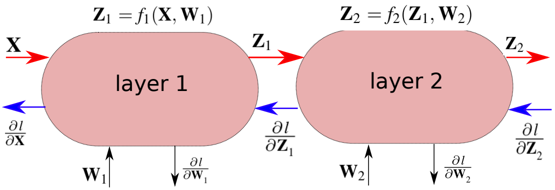
- often multiple epochs of training needed

summary



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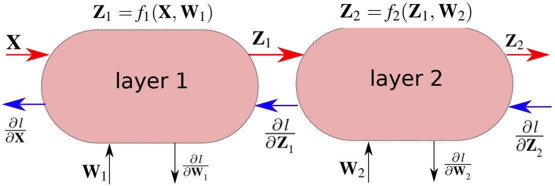

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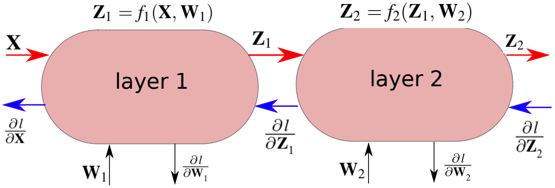

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- see github page for useful reference and reading