

propiedades $z, w \in \mathbb{C}$ 1) $|z + w| \leq |z| + |w|$ (des. triangular)
cumple tambien

$$\rightarrow \sqrt[3]{3} \text{hola} \frac{e^{\pi i}}{3} \rho_3 2 \quad (1)$$

n en el espacio euclideo R^4

$$2) |z| - |w| \leq |z - w| \quad |z| = |(z - w) + w| \leq |z - w| + |w|$$

$$\text{Cor} : |w| - |z| \leq |z - w| \Rightarrow ||z| - |w|| \leq |z - w|$$

$$z = x + iy, x, y \in R \Rightarrow \text{Real} : z = x \quad \text{Imag} : z = y \quad 3) |Rez| \leq |z|, |Imz| \leq |z|$$

$$4) |\bar{z}| = |z|, |z^n| (\forall n \in Z) \quad 5) Rez = \frac{z + \bar{z}}{2}, Imz = \frac{z - \bar{z}}{2i} = \frac{(x + iy) - (x - iy)}{2i} =$$

$$\frac{2iy}{2i} = y = Imz \in R \quad 6) |zw| = |z| * |w| \quad z = re^{i\theta}, w = \rho e^i \Rightarrow$$

$$7) arg(zw) = arg(z) + arg(w) \quad (\text{dibujito}) \quad 8) arg\left(\frac{1}{z}\right) = arg(\bar{z}) = -arg(z)$$

$$(\text{dibujito}) \quad 9) arg(z^n) = narg(z)$$

RAICES DE NUMEROS COMPLEJOS

$$z \in C, z \neq 0$$

$$W^n - z = 0$$

$$\text{nsoluciones } W_0, \dots, W_{n-1}$$

$$W^n = z \quad z = \pi e^{i*\theta} \quad 0 < \pi < \inf, \theta \in R \quad w = \rho e^{i\phi} \quad \text{Tenemos que resolver en } \rho, \phi :$$

$$(\rho e^{ie})^n = re^{i\theta}.$$

$$\rho^n e^{in\phi} = re^{i\theta}$$

$$\rho^n = r \quad n\phi = \theta + rK\pi, \text{ para alg } n \in Z$$

$$\rho = \sqrt[n]{r} > 0 \quad (\text{raiz } n)$$

$$\text{Basta coger} : k = 0, 1, \dots, n-1$$

Respuesta

$$z = re^{i\theta} \rightarrow \sqrt[n]{z} = \sqrt[n]{r} e^{i*\frac{\theta + 2k\pi}{n}} \quad (\text{raices de grado } n)$$

$$k = 0, 1, \dots, n-1$$

$$(\text{dibujito})$$

$$z = 2i \quad 3\sqrt{z} = ?$$

Obtenemos valores :

$$3\sqrt{2i} = W_k, k = 0, 1, 2.$$

$$W_k = 3\sqrt{2} * e^{i*\frac{\theta + 2k\pi}{n}}$$

$$k = 0, 1, 2.$$

$$arg W_0 = (\pi/2)/3 = \frac{\pi}{6} = 30$$

$$arg W_1 = (\pi/6) + (2\pi/3) = 150$$

$$arg W_2 = (\pi/6) + (4\pi/3) = 270$$

Las raices n -esimas de 1 :

$$n\sqrt[n]{1} = \{\epsilon \dots \epsilon_{n-1} \mid 1 = 1e^{i\theta}\}$$

$$\epsilon_k = e^{i*\frac{2\pi k}{n}}, k = 0, 1, \dots, n-1$$

$$n = 5 :$$

(dibujo)

Propiedades : 1) $f < o$ rman un grupo conmutativo respecto del producto.

$$\epsilon_k \epsilon_e = \{\epsilon_{k+e}, k+l \in n \mid \epsilon_{k+l-n}, k+l \geq n\}$$

(estegrupoesisomorfoa Z_n)

2) W es unadelasraicesenesimasdez($w_n = z$) \rightarrow todaslasraices n -esimasdezson $W_k =$

$W_{\epsilon_k}, k = 0, \dots, n - 1$

3) W_0, \dots, W_{n-1} , todassonraices n -esimasdez, $n \leq 2 \rightarrow W_0 + \dots + W_{n-1} = 0$

(dibujitodeunpentagono)

$e^{i\phi} = \cos\phi + i\sin\phi$

Def Dada una serie

$2a_n = a_0 + a_1 + \dots + a_n$

Def $\sum (de \inf an = 0) a_n = \lim_{n \rightarrow \infty} S_n$

La serie converge si $\exists \lim_{n \rightarrow \infty} S_n$

Tenemos que resolver en