Machine-Checked Proofs for AES: High-Assurance Security

v 1.0

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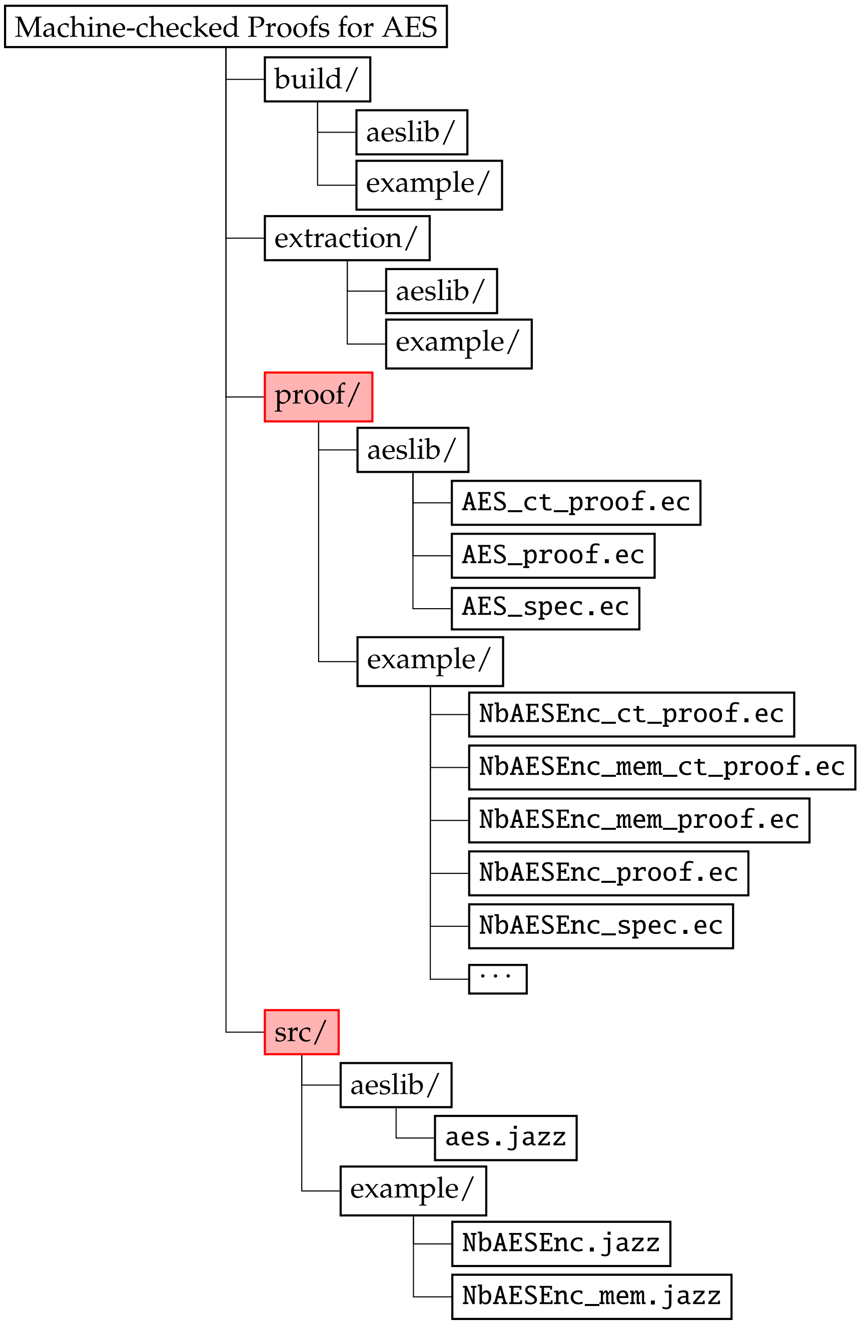
January 3, 2025

File Structure



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Changelog

v1.0	2024-12-24	Initial release:

Contents

1 Preliminaries

1.1 Cryptosystem and Encryption Scheme

- A **cryptosystem** is the abstract formal definition encompassing all components necessary for secure communication.
- An **encryption scheme** specifies the algorithms (or protocols) used to implement encryption and decryption within a cryptosystem.

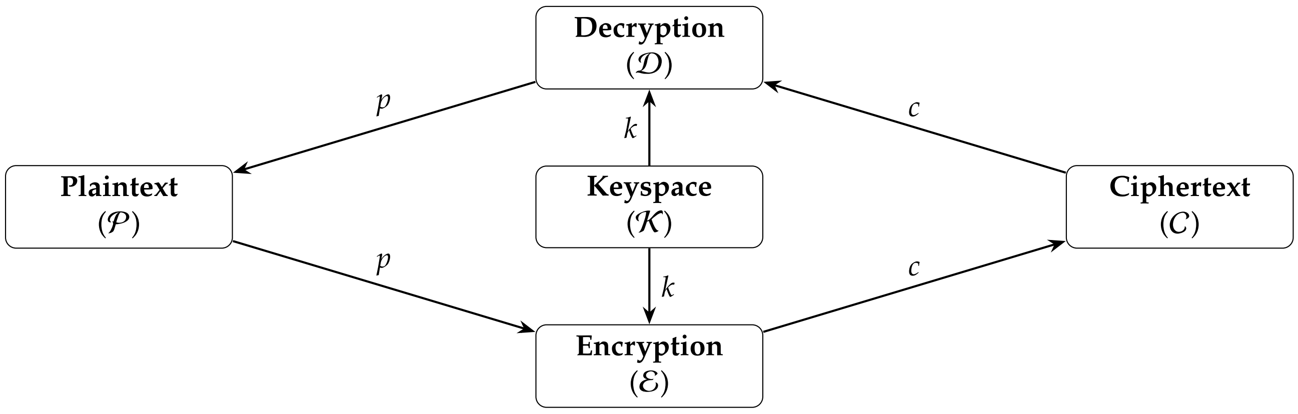


Figure 1: Cryptosystem

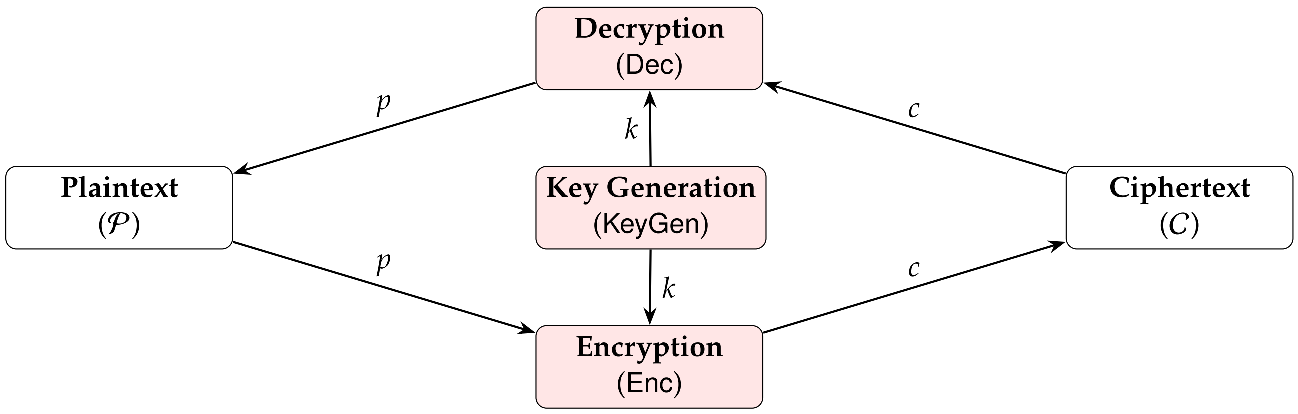


Figure 2: Encryption Scheme

Cryptosystem

Definition 1. A **cryptosystem** is a five-tuple

$$(\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D}),$$

where

(i) $\boxed{\mathcal{P}}$ is a finite set of all possible plaintexts[a].

(ii) $\boxed{\mathcal{C}}$ is a finite set of all possible ciphertexts[b].

(iii) $\boxed{\mathcal{K}}$ is a finite set of all possible keys[c].

(iv) $\boxed{\mathcal{E} : \mathcal{K} \times \mathcal{P} \to \mathcal{C}}$ is a deterministic function that maps a key $k \in \mathcal{K}$ and a plaintext $p \in \mathcal{P}$ to a ciphertext $c \in \mathcal{C}$. Formally:

$$\begin{array}{cccc} \mathcal{E} & : & \mathcal{K} \times \mathcal{P} & \longrightarrow & \mathcal{C} \\ & & (k, p) & \longmapsto & c \end{array}.$$

(v) $\boxed{\mathcal{D} : \mathcal{K} \times \mathcal{C} \to \mathcal{P}}$ is a deterministic function that maps a key $k \in \mathcal{K}$ and a ciphertext $c \in \mathcal{C}$ to a ciphertext $p \in \mathcal{P}$. Formally:

$$\begin{array}{cccc} \mathcal{D} & : & \mathcal{K} \times \mathcal{C} & \longrightarrow & \mathcal{P} \\ & & (k, c) & \longmapsto & p \end{array}.$$

[a]These are the possible inputs to the encryption algorithm and typically represent meaningful data to be protected.
[b]These are the encrypted outputs of the encryption algorithm corresponding to plaintexts in $\mathcal{P}$.
[c]Each key $k \in \mathcal{K}$ determines a specific encryption and decryption function.

Remark 1 (Correctness Property)**.** For every key $k \in \mathcal{K}$ and every plaintext $p \in \mathcal{P}$, the decryption function is the inverse of the encryption function. That is:

$$\mathcal{D}(k, \mathcal{E}(k, p)) = p.$$

Remark 2 (Security)**.** The security of the cryptosystem is defined with respect to a particular adversarial model. Informally, a cryptosystem is secure if an adversary with limited computational resources cannot distinguish between the ciphertexts of any two plaintexts, even if they know the encryption algorithm but do not know the key.

Encryption Scheme

Definition 2. An **encryption scheme** is a three-tuple

$$\Pi := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec}).$$

where

(i) KeyGen is a probabilistic algorithm that ouputs a key $k \in \mathcal{K}$, where $\mathcal{K}$ is the key space. Formally:

$$\boxed{\mathsf{KeyGen} : \{0,1\}^* \to \mathcal{K}},$$

where $\{0,1\}^*$ is the set of binary strings of arbitrary length (representing randomness or input seed). The ouput k is uniformly distributed over $\mathcal{K}$.

(ii) Enc is a (possibly probabilistic) algorithm that takes a key $k \in \mathcal{K}$ and a message $p \in \mathcal{M}$ (message space) and outpus a ciphertext $c \in \mathcal{C}$ (ciphertext sapce). Formally:

$$\boxed{\mathsf{Enc} : \mathcal{K} \times \mathcal{M} \times \{0,1\}^* \to \mathcal{C}}.$$

The algorithm may use randomness (from $\{0,1\}^*$) to ensure that repeated encryptions of the same message $m \in \mathcal{M}$ under the same key $k \in \mathcal{K}$ yield different ciphertexts c.

(iii) Dec is a deterministic algorithms that takes a key $k \in \mathcal{K}$ and a ciphertext $c \in \mathcal{C}$ and ouputs the corresponding message $m \in \mathcal{M}$. Formally:

$$\boxed{\mathsf{Dec} : \mathcal{K} \times \mathcal{C} \to \mathcal{M}}.$$

Remark 3 (Correctness Property)**.** For every $k \in \mathcal{K}$, $m \in \mathcal{M}$, and $c \in \mathcal{C}$, the scheme must satisfy

$$\mathsf{Dec}(k, \mathsf{Enc}(k, m; r)) = m$$

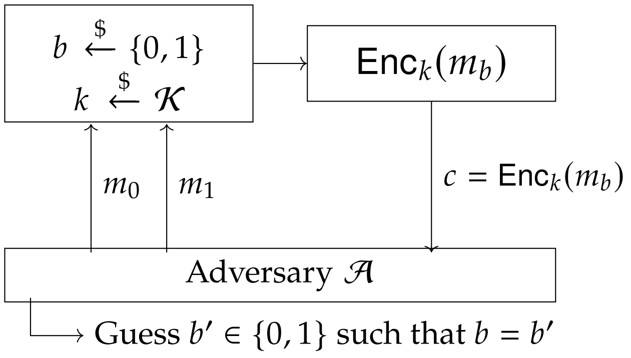
where r represents the random bits used by Enc.

Remark 4 (Security)**.** The security of an encryption scheme depends on the adversarial model. For **semantic security**, an encryption scheme must satisfy the following:

> "Given a ciphertext c, no computationally bounded adversary can distinguish between encryptions of any two messages m_0, m_1, even if they are chosen adaptively by the adversary."

Example 1 (IND-CPA)**.**
The **indistinguishability under chosen plaintext attack (IND-CPA)** model:

1. The adversary chooses two messages m_0, m_1.
2. A random bit $b \in \{0,1\}$ is chosen, and the ciphertext $c = \mathsf{Enc}(k, m_b)$ is provided to the adversary.
3. The adversary outputs a guess $b' \in \{0,1\}$.


$b \xleftarrow{\$} \{0,1\}$
$k \xleftarrow{\$} \mathcal{K}$
$\mathsf{Enc}_k(m_b)$
m_0
m_1
$c = \mathsf{Enc}_k(m_b)$
Adversary $\mathcal{A}$
Guess $b' \in \{0,1\}$ such that $b = b'$

The scheme is secure if the adversary's advantage is negligible:

$$\mathrm{Adv}_{\Pi}^{\text{IND-CPA}}(\mathcal{A}) := \left|\Pr[b' = b] - \frac{1}{2}\right| \leq \mathrm{negl}(\lambda),$$

where λ is the security parameter.

1.2 Perfect Security

Note (Measure Theory)**.** Let $(\Omega, \mathcal{F}, \Pr)$ be a probability space, where:

- Ω is the sample space representing all possible outcomes.
- $\mathcal{F}$ is a σ-algebra of subsets of Ω, representing the events.
- $\Pr : \mathcal{F} \to [0, 1]$ is a probability measure satisfying $\Pr[\Omega] = 1$.

Note (Random Variables)**.**

- A **plaintext random variable** $X : \Omega \to \mathcal{P}$ is a measurable function, i.e.,

  $$X^{-1}[A] \in \mathcal{F}, \quad \forall A \subseteq \mathcal{P}, \text{ where } A \text{ is measurable.}$$

  This means X maps outcomes in Ω to plaintexts in a way consistent with the probability structure. The distribution of X, denoted P_X, is the pushforward measure of P under X:

  $$P_X(A) = P(X \in A) = P(\{\omega \in \Omega : X(\omega) \in A\}),$$

  for all measurable subsets $A \subseteq \mathcal{P}$.

- A **ciphertext random variable** $Y : \Omega \to \mathcal{C}$ is a measurable function, i.e.,

  $$Y^{-1}[B] \in \mathcal{F}, \quad \forall B \subseteq \mathcal{C}, \text{ where } B \text{ is measurable.}$$

  This means Y maps outcomes in Ω to ciphertexts in a measurable way. The distribution of Y, denoted P_Y, is the pushforward measure of P under Y:

  $$P_Y(B) = P(Y \in B) = P(\{\omega \in \Omega : Y(\omega) \in B\}),$$

  for all measurable subsets $B \subseteq \mathcal{C}$.

- A **random variable for the key space** $K : \Omega \to \mathcal{K}$ is a measurable function, i.e.,

  $$K^{-1}[A] \in \mathcal{F}, \quad \forall A \subseteq \mathcal{K}, \text{ where } K \text{ is measurable.}$$

  This means K maps outcomes in the sample space Ω to keys in $\mathcal{K}$ in a way consistent with the probability structure. The distribution of K, denoted P_K, is the pushforward measure of P under K:

  $$P_K(A) = P(K \in A) = P(\{\omega \in \Omega : K(\omega) \in A\}),$$

  for all measurable subsets $A \subseteq \mathcal{K}$.

Note. If the key is selected uniformly from $\mathcal{K}$, then $P_K(A)$ is proportional to the size of A:

$$P_K(A) = \frac{|A|}{|\mathcal{K}|}, \quad \forall A \subseteq \mathcal{K}.$$

Perfect Security of an Encryption Scheme

Definition 3. An encryption scheme $\Pi = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ is **perfect security** if, for every $m \in \mathcal{M}$, $c \in \mathcal{C}$, and $k \in \mathcal{K}$ such that $\mathsf{Enc}(k, m) = c$, the following holds:

(i) **Ciphertext Independence**:

$$\Pr[M = m \mid C = c] = \Pr[M = m],$$

where

- M is the random variable representing the plaintext.
- C is the random variable representing the ciphertext.

(ii) **Key Uniformity**: The key K must satisfy:

$$\Pr[\mathsf{Enc}(K, m) = c] = \Pr[C = c],$$

for all $m \in \mathcal{M}$, $c \in \mathcal{C}$, and uniformly random K.

Remark 5.

$$\Pr[M = m \mid C = c] = \Pr[M = m] \iff \Pr[C = c \mid M = m] = \Pr[C = c]$$

Theorem 1. *Consider a cryptosystem* $(\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D})$ *where*

$$|\mathcal{P}| = |\mathcal{C}| = |\mathcal{K}|.$$

Then the cryptosystem (or encryption scheme) has perfect security if and only if

(i) every key is used with equal probability $1/|\mathcal{K}|$*, and*

(ii) $\forall x \in \mathcal{P}, \forall y \in \mathcal{C}, \exists! k \in \mathcal{K}$ *such that* $\mathcal{E}_k(x) = y$*.*

Proof. $(\Rightarrow)$ By definition of perfect security,

$$\Pr[X = x \mid Y = y] = \Pr[X = x], \quad \forall x \in \mathcal{P},\ y \in \mathcal{C}.$$

(i) Let K be the random variable representing the key. Since the ciphertext y is determined by the key k and the plaintext x, we have:

$$\Pr[Y = y \mid X = x] = \Pr[\exists k \in \mathcal{K} \text{ such that } \mathcal{E}_k(x) = y].$$

To achieve uniform distribution of Y for any fixed x, every key must be equality likely:

$$\Pr[K = k] = \frac{1}{|\mathcal{K}|}, \quad \forall k \in \mathcal{K}.$$

(ii) Assume that there exists two distinct keys $k_1, k_2 \in \mathcal{K}$, with $k_1 \neq k_2$, such that

$$\mathcal{E}_{k_1}(x) = y = \mathcal{E}_{k_2}(x),$$

for some $x \in \mathcal{P}$ and $y \in \mathcal{C}$.

TBA

$(\Leftarrow)$ Using Bayes' Theorem:

$$\Pr[X = x \mid Y = y] = \frac{\Pr[Y = y \mid X = x]\Pr[X = x]}{\Pr[Y = y]}.$$

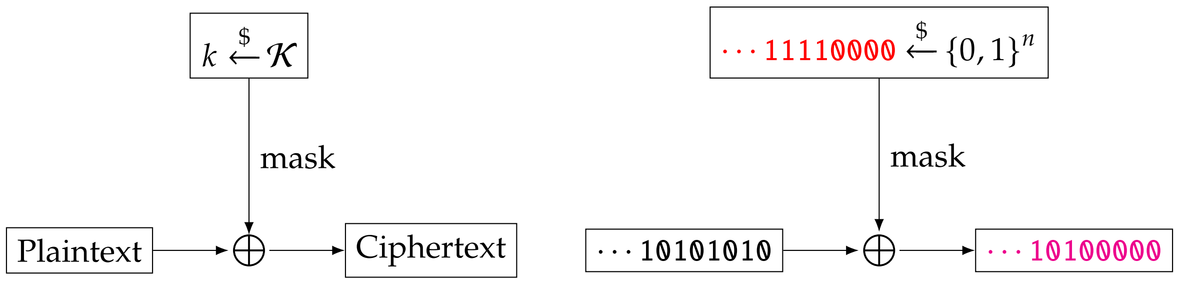
□

Example 2 (One-Time Pad)**.** The one-time pad encryption scheme is perfect security.

- $\mathcal{M} = \mathcal{C} = \mathcal{K} = \{0,1\}^n$;
- $\mathsf{Enc}(k, m) = m \oplus k$, where $\oplus$ is bitwise XOR;
- $\mathsf{Dec}(k, c) = c \oplus k$.

We must show that $\Pr[C = c \mid M = m] = \Pr[C = c]$. For $m \in \mathcal{M}$ and $c \in \mathcal{C}$,

$$\Pr[C = c \mid M = m] = \sum_{k \in \mathcal{K}} \Pr[K = k]$$


$k \overset{\$}{\leftarrow} \mathcal{K}$
mask
Plaintext
Ciphertext
$\cdots 11110000 \overset{\$}{\leftarrow} \{0,1\}^n$
mask
$\cdots 10101010$
$\cdots 10100000$

Theorem 2. *The one-time pad encryption scheme is perfectly secret.*

Proof.

$$\begin{aligned}\Pr[C = c \mid M = m] = \Pr[c = \mathsf{Enc}(K, m)] &= \Pr[c = m \oplus K] \\ &= \Pr[K = m \oplus c] \\ &= 2^{-n} \quad \text{if } K \overset{\$}{\leftarrow} \mathcal{K} = \{0,1\}^n\end{aligned}$$

Fix any distribution over $\mathcal{M}$. For any $c \in \mathcal{C}$, we have

$$\begin{aligned}\Pr[C = c] &= \sum_{m \in \mathcal{M}} \Pr[C = c \mid M = m] \cdot \Pr[M = m] \\ &= 2^{-n} \cdot \Pr[M = m]\end{aligned}$$

By Bayes' Theorem, we obtain

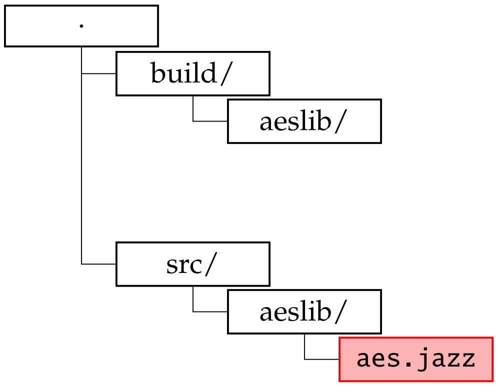
$$\begin{aligned}\Pr[M = m \mid C = c] &= \frac{\Pr[C = c \mid M = m] \cdot \Pr[M = m]}{\Pr[C = c]} \\ &= \frac{2^{-n} \cdot \Pr[M = m]}{2^{-n}} \\ &= \Pr[M = m].\end{aligned}$$

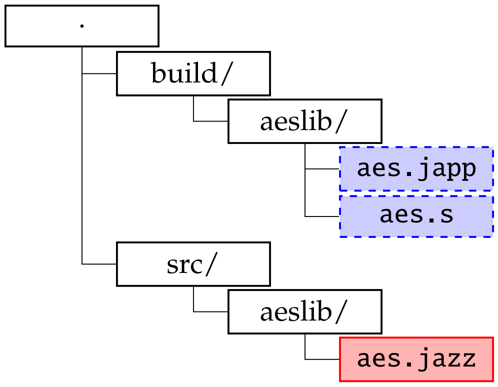
□

2 Machine-checked Proofs for AES

2.1 Cryptographic Primitive

2.1.1 Build AES Library





Code 1: aes.jazz

```
...
/* Jasmin implementation of
AES using AES-NI */
#ifdef EXPORT_TEST
export
#else
inline
#endif
fn aes(reg u128 key, reg u128 in)
→ reg u128 {
    reg u128 out;
    reg u128[11] rkeys;

    rkeys = keys_expand(key);
    out   = aes_rounds(rkeys, in);
    return out;
}
...
```

Code 2: aes.japp

```
...



export



fn aes(reg u128 key, reg u128 in)
→ reg u128 {
    reg u128 out;
    reg u128[11] rkeys;

    rkeys = keys_expand(key);
    out = aes_rounds(rkeys, in);
    return out;
}
...
```

Code 3: aes.s

```
...
    .att_syntax
.text
.p2align     5
.globl  _aes
.globl  aes

_aes:
aes:
...
vmovdqu %xmm0, %xmm12
vpxor   %xmm2, %xmm1, %xmm0
aesenc  %xmm3, %xmm0
...
aesenc  %xmm11, %xmm0
aesenclast     %xmm12, %xmm0
ret
...
```

```
@~$ cpp -nostdinc -DEXPORT_TEST src/aeslib/aes.jazz \
      | grep -v "^#" > build/aeslib/aes.japp
@~$ jasminc  build/aeslib/aes.japp -o build/aeslib/aes.s
```

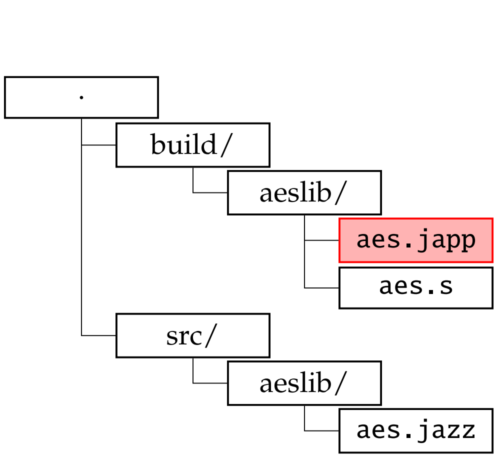
Command 1 :

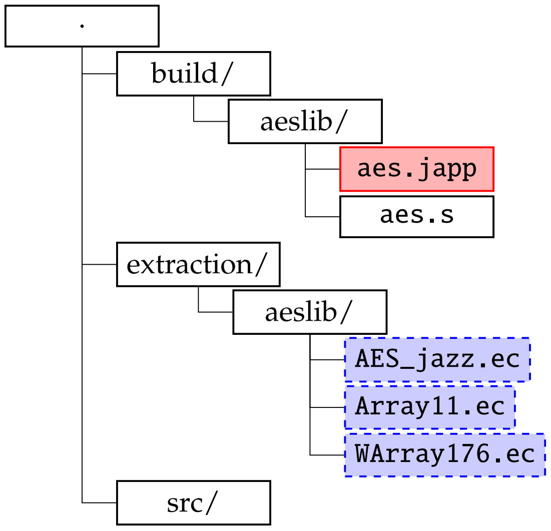
`[cpp]`	*C Preprocessor*	Preprocesses source code files by expanding macros, processing conditional compilation directives, and handling file inclusions.
`[-nostdinc]`	*No Standard Include*	Prevents the preprocessor from searching in standard system directories for include files, limiting the scope of header file processing to explicitly specified paths or local includes.
`[-DEXPORT_TEST]`	*Define Macro* `EXPORT_TEST`	Defines a preprocessor macro (`EXPORT_TEST`) that can be used for conditional compilation within the source code.
`[../aes.jazz]`	*Source File Path*	Specifies the input Jasmin source code file to be preprocessed.
`[|]`	*Pipe*	Passes the output of the cpp command as input to the next command (`grep`).
`[grep -v "^#"]`	*Global Regular Expression Print (with* `-v` *for inverse matching)*	Filters out lines starting with #, such as preprocessor directives or comments, from the preprocessed output.
`[>]`	*Output Redirection*	Redirects the filtered output from the preprocessing pipeline to a specified file.
`[../aes.japp]`	*Output File Path*	Specifies the destination file for the preprocessed Jasmin code.

Command 2 :

`[jasminc]`	*Jasmin Compiler*	Compiles Jasmin source code into optimized assembly code.
`[../aes.japp]`	*Input File Path*	Specifies the preprocessed Jasmin source file to be compiled.
`[-o]`	*Output File Option*	Indicates the output file name or path for the generated assembly code.
`[../aes.s]`	*Output File Path*	Specifies the destination file for the generated assembly code.

2.2 Extract AES Library for Correctness and Security





```
@~$ jasminc build/aeslib/aes.japp -ec aes -ec invaes -oec AES_jazz.ec
@~$ mv AES_jazz.ec Array11.ec WArray176.ec extraction/aeslib
```

[jasminc] The ‘jasminc’ command in this context extracts verification conditions specifically designed to interface with EasyCrypt, a tool used for formal reasoning about cryptographic security.

[-ec aes -ec invaes] The ‘-ec’ flag specifies functions or modules to extract formal verification constraints from the Jasmin source file.

[-oec AES_jazz.ec] The ‘-oec’ flag specifies the output file where the extracted constraints for the specified modules (aes and invaes) will be saved.

Code 4: Array11.ec

```
from Jasmin require import JArray.

clone export PolyArray as Array11  with op size <- 11.
```

Code 5: WArray176.ec

```
from Jasmin require import JArray.

clone export PolyArray as Array11  with op size <- 11.
```

Code 6: AES_jazz.ec (lines 127-134)

```
proc aes (key:W128.t, in_0:W128.t) : W128.t = {
  var out:W128.t;
  var rkeys:W128.t Array11.t;
  rkeys <- witness;
  rkeys <@ keys_expand (key);
  out <@ aes_rounds (rkeys, in_0);
  return out;
}
```

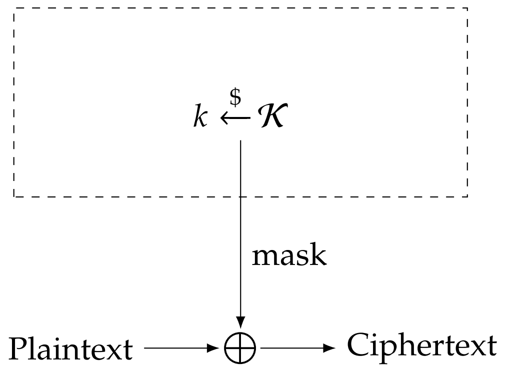
2.2.1 Extract AES Library for Constant-time

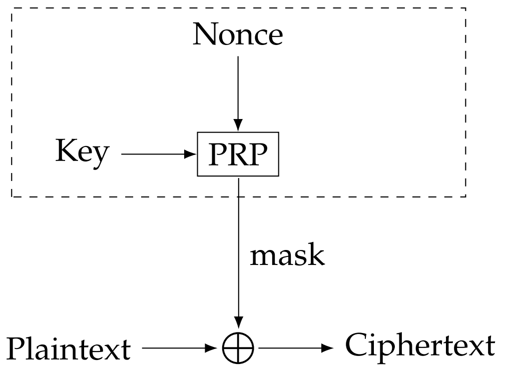
```
@~$ jasminc build/aeslib/aes.japp -CT -ec aes -ec invaes -oec AES_jazz_ct.ec
@~$ mv AES_jazz_ct.ec Array11.ec WArray176.ec extraction/aeslib
```

2.2.2 Build Nonce-based AES Encryption with register calling convension

```
@~$ cpp -nostdinc src/example/NbAESEnc.jazz  \
     | grep -v "^#" > build/example/NbAESEnc.japp
@~$ jasminc build/example/NbAESEnc.japp -o build/example/NbAESEnc.s
```

2.2.3 Implementation of Nonce-based AES Encryption Scheme





- The **one-time pad (OTP) encryption scheme** is a cryptographic construct that achieves perfect security.

$$\Pi_{\text{OTP}} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec}),$$

  where

  (i)

  (ii) $\mathsf{Enc} : \mathcal{K} \times \mathcal{M} \to \mathcal{C} : (k, m) \mapsto c = k \oplus m;$

$$\begin{array}{rccl} \mathsf{Enc} : & \mathcal{K} & \longrightarrow & \mathcal{C}^{\mathcal{M}} \\ & k & \longmapsto & c = \mathsf{Enc}_k(m) \end{array} \quad \text{where} \quad \begin{array}{rccl} \mathsf{Enc}_k : & \mathcal{M} & \longrightarrow & \mathcal{C} \\ & m & \longmapsto & c = k \oplus m \end{array}$$

- A **nonce-based PRP encryption scheme** is a cryptographic construct where a nonce (number used once) is incorporated to ensure unique ciphertexts for the same plaintext under the same key.

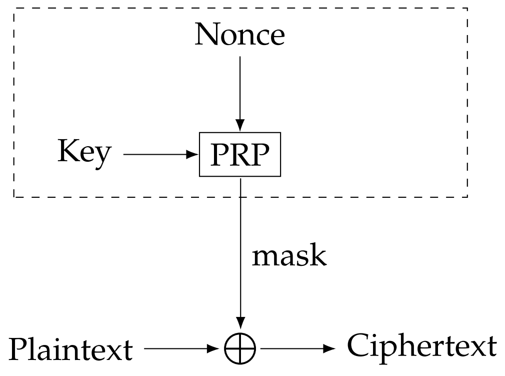
$$\Pi_{\mathcal{N}-\text{PRP}} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec}),$$

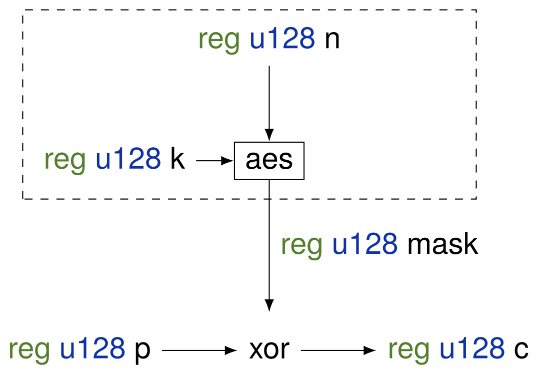
  where

  (i)

  (ii) $\mathsf{Enc} : \mathcal{K} \times \mathcal{N} \times \mathcal{M} \to \mathcal{C} : (k, n, m) \mapsto c = \mathsf{Enc}_k(n) \oplus m;$

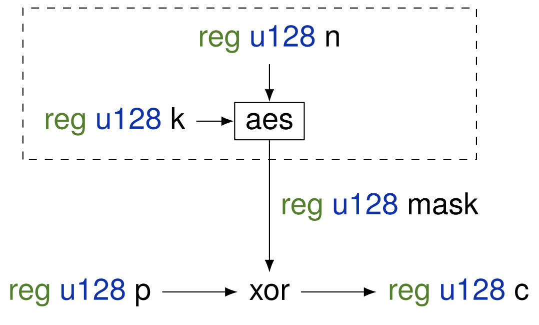
$$\begin{array}{rccl} \mathsf{Enc} : & \mathcal{K} & \longrightarrow & [\mathcal{N} \to [\mathcal{M} \to \mathcal{C}]] \\ & k & \longmapsto & c = \mathsf{Enc}_k(n) \oplus m \end{array} \quad \text{where} \quad \begin{array}{rccl} \mathsf{Enc}_k : & \mathcal{N} & \longrightarrow & [\mathcal{M} \to \mathcal{C}] \\ & n & \longmapsto & c = k \oplus m \end{array}$$

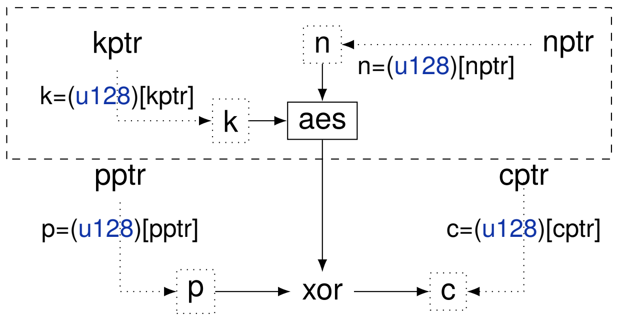




Code 7: src/example/NbAESEnc.jazz

```
/* Nonce-based symmetric encryption for 16-byte messages using AES as a PRF */

/* We use cpp to manage modules in Jasmin */
#include "../aeslib/aes.jazz"

/* We make xor into a function, but this costs nothing because
   Jasmin compiler does not include inlining moves (warning issued o/w). */
inline fn xor(reg u128 a, reg u128 b) → reg u128 {
   reg u128 r;
   r = a^b;
   return r;
}

/* These functions can be called from C for testing. */

export fn enc(reg u128 k, reg u128 n, reg u128 p) → reg u128 {
   reg u128 mask,c;
   mask = aes(k,n);
   c = xor(mask,p);
   return(c);
}

export fn dec(reg u128 k, reg u128 n, reg u128 c) → reg u128 {
   reg u128 mask,p;
   mask = aes(k,n);
   p = xor(mask,c);
   return(p);
}
```





Code 8: src/example/NbAESEnc_mem.jazz

```
/* Nonce-based symmetric encryption for 16-byte messages using AES as a PRF */

#include "../aeslib/aes.jazz"
inline fn xor(reg u128 a, reg u128 b) → reg u128 {
  reg u128 r;
  r = a^b;
  return r;
}

/* These functions can be called from C for testing.
 They receive pointers to byte arrays of length 16.
 Convention is that first pointer is writable for output. */

export fn enc(reg u64 cptr, reg u64 kptr, reg u64 nptr, reg u64 pptr) {
  reg u128 mask,k,n,p,c;
  k = (u128)[kptr];
  n = (u128)[nptr];
  mask = aes(k,n);
  p = (u128)[pptr];
  c = xor(mask,p);
  (u128)[cptr] = c;
}

export fn dec(reg u64 pptr, reg u64 kptr, reg u64 nptr, reg u64 cptr) {
  reg u128 mask,k,n,p,c;
  k = (u128)[kptr];
  n = (u128)[nptr];
  mask = aes(k,n);
  c = (u128)[cptr];
  p = xor(mask,c);
  (u128)[pptr] = p;
}
```

Code 9: test/test_NbAESEnc.c

```c
#include <smmintrin.h>
#include <immintrin.h>
#include <stdalign.h>
#include <stdio.h>

int8_t kb[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x0f };
int8_t nb[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x1f };
int8_t pb[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x2f };

extern __m128i enc(__m128i k, __m128i n,__m128i p);
extern __m128i dec(__m128i k, __m128i n,__m128i c);

int main() {
    __m128i ct, dt, pt, k, n;

    k = _mm_loadu_si128((__m128i *) kb);
    n = _mm_loadu_si128((__m128i *) nb);
    pt = _mm_loadu_si128((__m128i *) pb);

    ct = enc(k,n,p);
    dt = dec(k,n,c);

    __m128i neq = _mm_xor_si128(pt,dt);
    if(_mm_test_all_zeros(neq,neq)) printf("Verify output: OK!\n");
    else printf("Verify output: Not OK!\n");

    return 0;
}
```

```
@~$ cpp -nostdinc src/example/NbAESEnc.jazz  \
   | grep -v "^#" > build/example/NbAESEnc.japp
@~$ jasminc  build/example/NbAESEnc.japp -o build/example/NbAESEnc.s
@~$ gcc -msse4.1 -Wall build/example/NbAESEnc.s test/test_NbAESEnc.c \
-o build/example/test_NbAESEnc
```

```
************************************************
*** Testing encryption scheme (reg cc)       ***
************************************************
build/example/test_NbAESEnc
Verify output: OK!
```

Code 10: test/test_NbAESEnc_mem.c

```c
#include <stdio.h>

byte k[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x0f };
byte n[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x1f };
byte pt[] = {
    0x00, 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07,
    0x08, 0x09, 0x0a, 0x0b, 0x0c, 0x0d, 0x0e, 0x2f };

extern void enc(byte *cp, byte *kp, byte *np, byte *pp);
extern void dec(byte *pp, byte *kp, byte *np, byte *cp);

int crypto_verify(const byte *x,const byte *y) {
    unsigned int differentbits = 0;
#define F(i) differentbits |= x[i] ^ y[i];
    F(0) F(1) F(2) F(3)
    F(4) F(5) F(6) F(7)
    F(8) F(9) F(10) F(11)
    F(12) F(13) F(14) F(15)
    return (1 & ((differentbits - 1) >> 8)) - 1;
}

int main() {
    byte ct[16], dt[16];
    enc(ct,k,n,pt);
    dec(dt,k,n,ct);

    if (crypto_verify(pt,dt) == 0) printf("Verify output: OK!\n");
    else printf("Verify output: Not OK!\n");

    return 0;
}
```

3 Easycrypt Library in Jasmin

3.1 JUtils

```
require import AllCore IntDiv List Bool StdOrder.
        import IntOrder.
```

https://github.com/EasyCrypt/easycrypt
easycrypt/theories/core/AllCore.ec
easycrypt/theories/core/Bool.ec
easycrypt/theories/algebra/IntDiv.ec
easycrypt/theories/algebra/StdOrder.ec
easycrypt/theories/datatypes/List.ec

```
lemma modz_cmp m d : 0 < d => 0 <= m %% d < d.
proof. smt (edivzP). qed.
```

```
lemma divz_cmp d i n : 0 < d => 0 <= i < n * d => 0 <= i %/ d < n.
proof.
  by move=> hd [hi1 hi2]; rewrite divz_ge0 // hi1 /= ltz_divLR.
qed.
```

```
lemma mulz_cmp_r i m r : 0 < m => 0 <= i < r => 0 <= i * m < r * m.
proof.
  move=> h0m [h0i hir]; rewrite IntOrder.divr_ge0 //=; 1: by apply ltzW.
  by rewrite IntOrder.ltr_pmul2r.
qed.
```

```
lemma cmpW i d : 0 <= i < d => 0 <= i <= d.
proof. by move=> [h1 h2];split => // ?;apply ltzW. qed.
```

```
lemma le_modz m d : 0 <= m => m %% d <= m.
proof.
  move=> hm.
  have [ ->| [] hd]: d = 0 \/ d < 0 \/ 0 < d by smt().
  + by rewrite modz0.
  + by rewrite -modzN {2}(divz_eq m (-d)); smt (divz_ge0).
  by rewrite {2}(divz_eq m d); smt (divz_ge0).
qed.
```

3.2 JArray

4 EasyCrypt

4.1 AES Specification

```
require import AllCore List.
from Jasmin require import JWord AES JModel.
require import Array11.
```

- Imports the `AllCore`[1] and `List`[2] modules
  https://github.com/EasyCrypt/easycrypt
- Imports specific modules (`JWord`[3], `AES`[4], `JModel`[5]) from the `Jasmin` library.
  https://github.com/jasmin-lang/jasmin
- Imports a module `Array11`

4.1.1 Functional Style

```
(* ------------------------------------------------------------------------ *)
(* AES specification in a functional style                                  *)
```

```
op key_expand (wn1 : W128.t) (rcon : W8.t)  =
   let rcon = W4u8.pack4 [rcon; W8.zero; W8.zero; W8.zero] in
   let w0 = wn1 \bits32 0 in
   let w1 = wn1 \bits32 1 in
   let w2 = wn1 \bits32 2 in
   let w3 = wn1 \bits32 3 in

   let tmp = w3 in
   let tmp = SubWord(RotWord(tmp)) `^` rcon in
   let w4 = w0 `^` tmp in
   let w5 = w1 `^` w4 in
   let w6 = w2 `^` w5 in
   let w7 = w3 `^` w6 in
   W4u32.pack4 [w4; w5; w6; w7].
```

[1]easycrypt/theories/core/Allcore.ec
[2]easycrypt/theories/datatypes/List.ec
[3]jasmin/eclib/JWord.ec
[4]jasmin/eclib/AES.ec
[5]jasmin/eclib/JModel.ec

```
op rcon : int -> W8.t.
axiom rcon_nth i :
  0 <= i < 10 =>
  rcon (i + 1) = W8.of_int (nth 0 [1; 2; 4; 8; 16; 32; 64; 128; 27; 54] i).
```

```
op key_i (k : W128.t) i =
  iteri i (fun i ki => key_expand ki (rcon (i+1))) k
axiomatized by key_iE.
```

```
op aes (key msg : W128.t) =
  let state = AddRoundKey msg (key_i key 0) in
  let state = iteri 9 (fun i state => AESENC_ state (key_i key (i + 1))) state in
  AESENCLAST_ state (key_i key 10)
axiomatized by aesE.
```

```
op invaes (key cipher : W128.t) =
  let state = AddRoundKey cipher (key_i key 10) in
  let state = iteri 9 (fun i state => AESDEC_ state (key_i key (10 -(i + 1)))) state in
  AESDECLAST state (key_i key 0)
axiomatized by invaesE.
```

```
(* Correctness of the AES rounds : invaes_rounds k (aes_rounds k m) = m *)
lemma aux1 c k1 k2:
      AESDEC_ (AddRoundKey (AESENCLAST_ c k1) k1) k2 = InvMixColumns (c `^` k2).
proof.
  by rewrite AESDEC_E AESENCLAST_E /=
        /AddRoundKey W128.WRing.subrK InvShiftRowsK InvSubBytesK.
qed.

lemma aux2 c k1 k2 :
      AESDEC_ (InvMixColumns (AESENC_ c k1 `^` k1)) k2 = InvMixColumns (c `^` k2).
proof.
  by rewrite AESDEC_E AESENC_E
       /AddRoundKey /= W128.WRing.subrK InvMixColumnsK InvShiftRowsK InvSubBytesK.
qed.

lemma invaes_roundsK k m : invaes k (aes k m) = m.
proof.
  rewrite invaesE aesE /=.
  do 9! rewrite iteri_red 1:// /=; rewrite iteri0 1:// /=.
  do 9! rewrite iteri_red 1:// /=; rewrite iteri0 1:// /=.
  rewrite aux1 !aux2.
  rewrite AESDECLASTE /= AESENC_E /AddRoundKey /= W128.WRing.subrK.
  by rewrite InvMixColumnsK InvShiftRowsK InvSubBytesK W128.WRing.subrK.
qed.
```

4.1.2 Pseudo Code Style

```
(* ------------------------------------------------------------------------- *)
(* AES specification in a pseudo code style                                   *)
```

```
module Aes = {
  proc keyExpansion (key : W128.t) = {
    var rkeys : W128.t Array11.t;
    var round : int;

    rkeys <- witness;
    rkeys.[0] <- key;
    round    <- 1;
    while (round < 11) {
      rkeys.[round] <- key_expand rkeys.[round-1] (rcon round);
      round <- round + 1;
    }
    return rkeys;
  }

  proc aes_rounds (rkeys : W128.t Array11.t,  msg : W128.t) = {
    var state, round;
    state <- AddRoundKey msg rkeys.[0];
    round <- 1;
    while (round < 10) {
      state <- AESENC_ state rkeys.[round];
      round <- round + 1;
    }
    state <- AESENCLAST_ state rkeys.[round];
    return state;
  }

  proc aes (key msg: W128.t) = {
    var rkeys, cipher;
    rkeys  <@ keyExpansion(key);
    cipher <@ aes_rounds(rkeys, msg);
    return cipher;
  }

  proc invaes_rounds (rkeys : W128.t Array11.t, cipher : W128.t) = {
    var state, round;
    state <- AddRoundKey cipher rkeys.[10];
    round <- 9;
    while (0 < round) {
      state <- AESDEC_ state rkeys.[round];
      round <- round - 1;
    }
    state <- AESDECLAST state rkeys.[0];
    return state;
  }

  proc invaes (key cipher: W128.t) = {
    var rkeys, msg;
    rkeys  <@ keyExpansion(key);
    msg <@ invaes_rounds(rkeys, cipher);
    return msg;
  }

}.
```

References

[1] Jonathan, Katz. *Introduction to Modern Cryptography, Second Edition.*, n.d.

[2] Smart, Nigel P. *Cryptography Made Simple. Information Security and Cryptography*. Cham: Springer International Publishing, 2016. https://doi.org/10.1007/978-3-319-21936-3.