6-4LORENTZsensitivity

-

10/01/2019

This R repository is for demonstration of algorithms involved in the book Mathematical Modeling (4th Edition) written by Prof. Mark. M. Meerschaert coded, edited and tested by Hao Li during Dec. 2018 - Jan. 2019.

#Effect of different variables on the pattern of the kinetic system

#Wrap 6-4 code into functions

```
#


#Lorentz for  different conditions:
#

xLorentz_df = function(xi,param,
                       init,end,h){
  #xi -initial x value,
  #param -list of parameters
  #t Domain: from ,to,N
  t = seq(from = init, to = end,by=h)
  #Copy an paste into this function
  #from LORENTZ_plain_code.R, collapse the text using your editor to
  #tidy up
  x1p = function(x1,x2,x3,Sigma) -Sigma*x1+Sigma*x2
  x2p = function(x1,x2,x3,r) -x2+r*x1-x1*x3
  x3p = function(x1,x2,x3,b) -b*x3+x1*x2
  #Using Euler s Method
  xLorentz = function(x,dt,Sigma,r,b){
    c(x[1]+dt*x1p(x[1],x[2],x[3],Sigma = Sigma),
      x[2]+dt*x2p(x[1],x[2],x[3],r=r),
      x[3]+dt*x3p(x[1],x[2],x[3],b=b))
  }
  Sigma = param$Sigma
  b = param$b
  #Initial condition#xi=  c(7,1,2)#r = param$r
  dt = t[2] - t[1]
  xdf = matrix(NA,length(t),3)#x1,x2,x3 then cbind t to the left
  xdf[1,] = xi
  #system.time({
    for(i in seq_along(t)[-1]){
      xdf[i,] =xLorentz(xdf[i-1,],dt,Sigma,r=param$r,b)
    }
  #})
```

```
  cbind(t,xdf)
}
```

##Visualization function, default and plot3D(for non-interactive 3D plot)

```
plot.particle = function(xdf,
                         type = 'default',
                         grid =T,
                         add=F){
  if(type == 'default'){
  layout(matrix(1:4,2,2))
  plot(xdf[,1],xdf[,2],
       xlab = 't',
       ylab = 'x1',type = 'l')
  if(grid==T) grid()
  plot(xdf[,1],xdf[,3],
       xlab = 't',
       ylab ='x2',type = 'l')
  if(grid==T) grid()
  plot(xdf[,1],xdf[,4],
       xlab = 't',
       ylab ='x3',type = 'l')
  if(grid==T) grid()
  plot3D::scatter3D(x = xdf[,2],
                    y = xdf[,3],
                    z = xdf[,4],
                    colvar =xdf[,1],add = F)
  layout(matrix(1,1))
  title("Default plots of 3D particle dynamic system")
  }else if(type =='3d'){
    #require package: plot3D3d
    plot3D::scatter3D(x = xdf[,2],
                              y = xdf[,3],
                              z = xdf[,4],
                              colvar =xdf[,1],add = add)
    #if(grid ==T) 3d::grid3d(side = c('x','y','z'))
  }
}
```

#1 Figure 6-35 6-36

Compare different timestep setting: ##Case1: r=18, (x1,x2,x3) = (6.7,6.7,17),h = .005 ##Case2: r=18, (x1,x2,x3) = (6.7,6.7,17),h = .01

```
#

#Case1: r=18, (x1,x2,x3) = (6.7,6.7,17),h = .005
#Case2: r=18, (x1,x2,x3) = (6.7,6.7,17),h = .01

require(doParallel)
```

```
## Loading required package: doParallel
```

```
## Loading required package: foreach
```

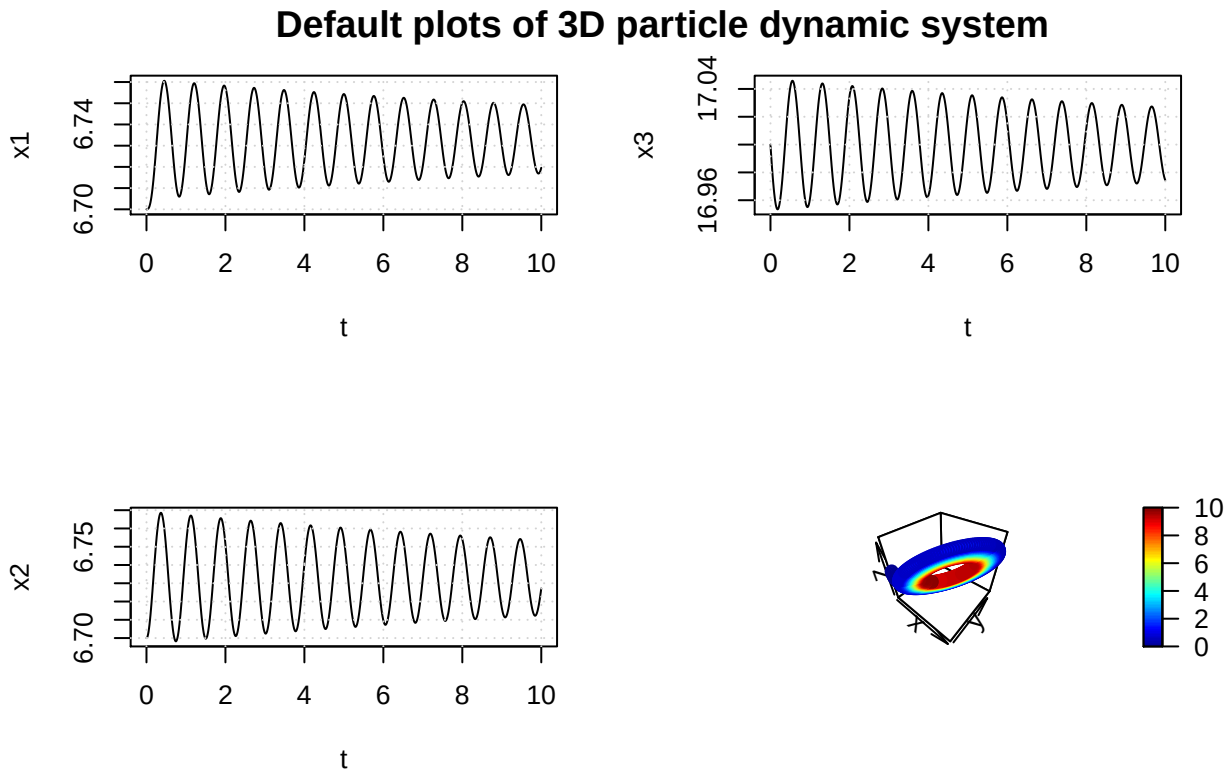
```
## Loading required package: iterators
```

```
## Loading required package: parallel
```

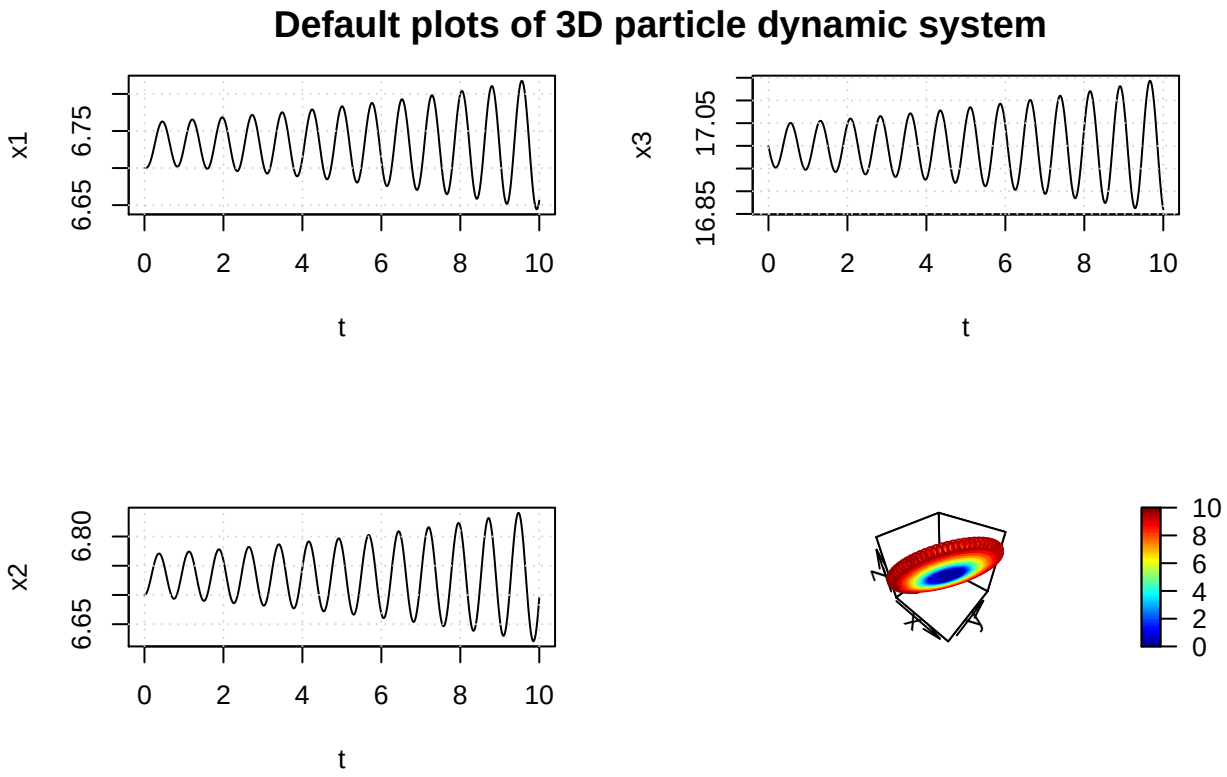
```
registerDoParallel(2)#Only 2 needed in this case

comp1 =foreach(i = c(0.005,0.01)) %dopar% {
  xLorentz_df(xi = c(6.7,6.7,17),
              param = list(Sigma = 10, b =8/3, r =18),
              init = 0,end = 10,h=i)
}

str(comp1)
```

```
## List of 2
##  $ : num [1:2001, 1:4] 0 0.005 0.01 0.015 0.02 0.025 0.03 0.035 0.04 0.045 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : NULL
##   .. ..$ : chr [1:4] "t" "" "" ""
##  $ : num [1:1001, 1:4] 0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : NULL
##   .. ..$ : chr [1:4] "t" "" "" ""
```

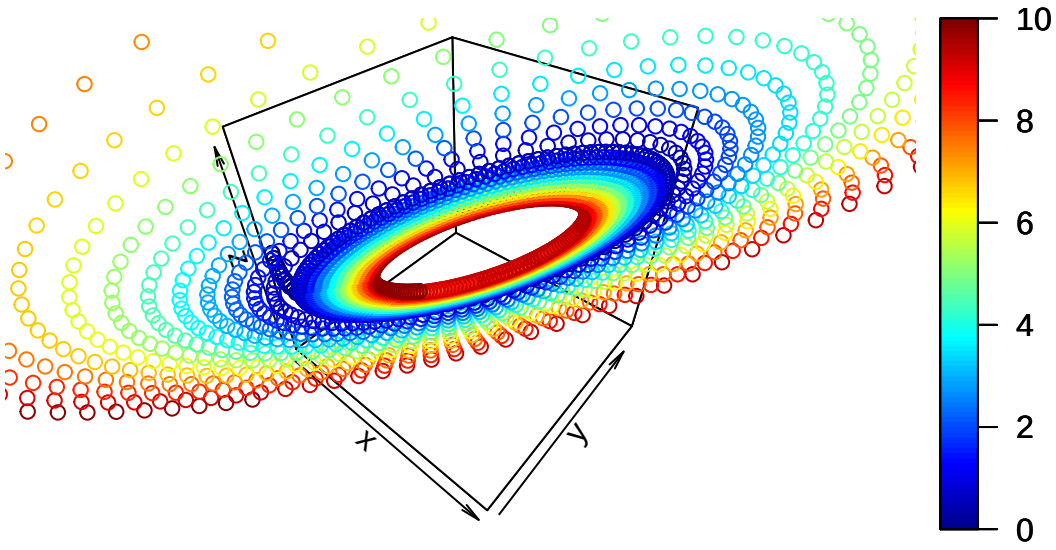
```
#Default Visualization plots defined in the previous code

plot.particle(comp1[[1]])
```



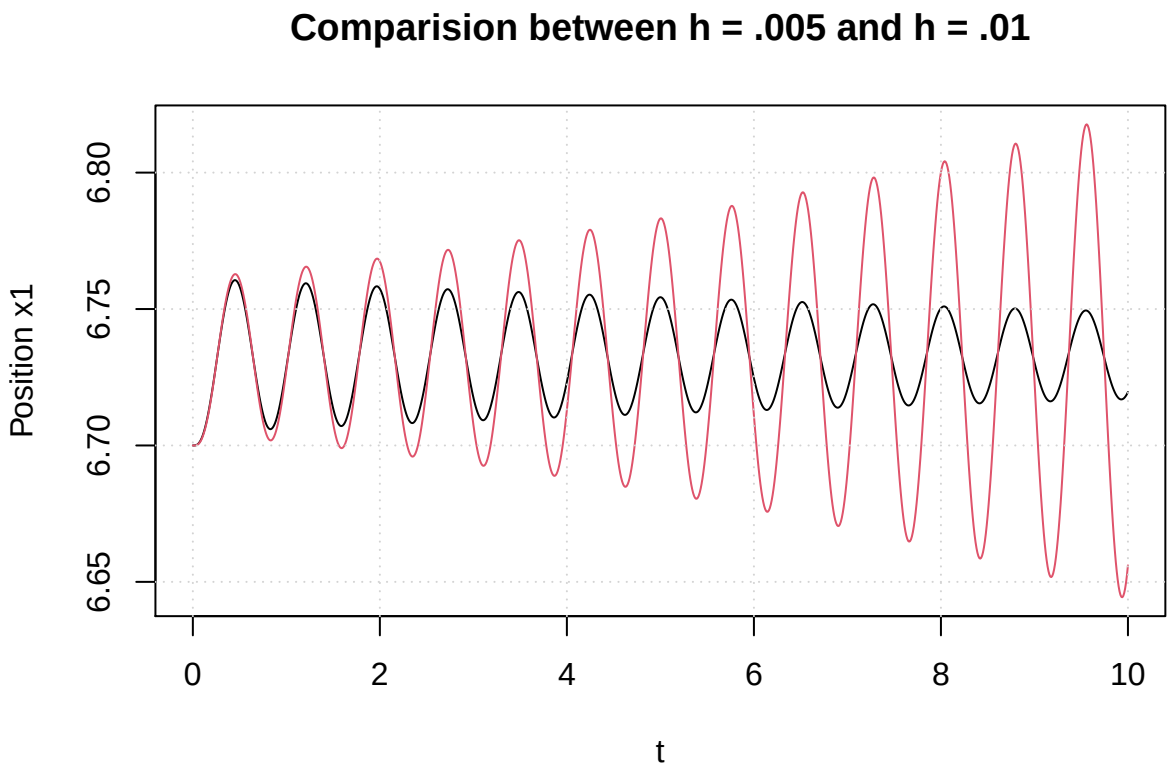

```
plot.particle(comp1[[2]])
```




```
plot.particle(comp1[[1]],type = '3d')
plot.particle(comp1[[2]],type = '3d',add = T)
```

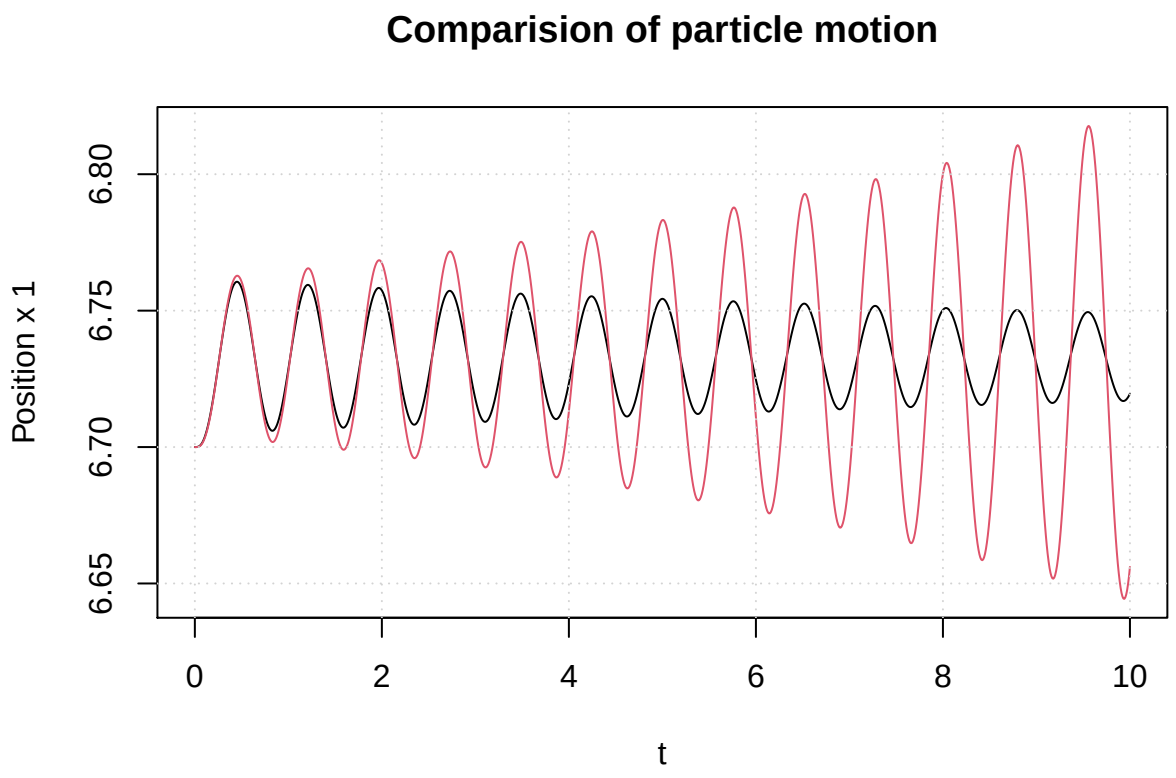



```
#This is not intuitive for comparision
#tmin = 0;tmax = 2.5

#Make a matplot with x axis: t
# y axis: Position x1
# we can later write this as a function
maxt1 = max(comp1[[1]][,1])
maxt2 = max(comp1[[2]][,1])
mint1 = min(comp1[[1]][,1])
mint2 = min(comp1[[2]][,1])
max1 = max(comp1[[1]][,2])
max2 = max(comp1[[2]][,2])
min1 = min(comp1[[1]][,2])
min2 = min(comp1[[2]][,2])

plot(c(min(mint1,mint2),max(maxt1,maxt2)),c(min(min1,min2),max(max1,max2)),
     xlab = 't',ylab = 'Position x1',type = 'n')
lines(comp1[[1]][,1],comp1[[1]][,2],col = 1)
lines(comp1[[2]][,1],comp1[[2]][,2],col = 2)
grid()
title('Comparision between h = .005 and h = .01')
```

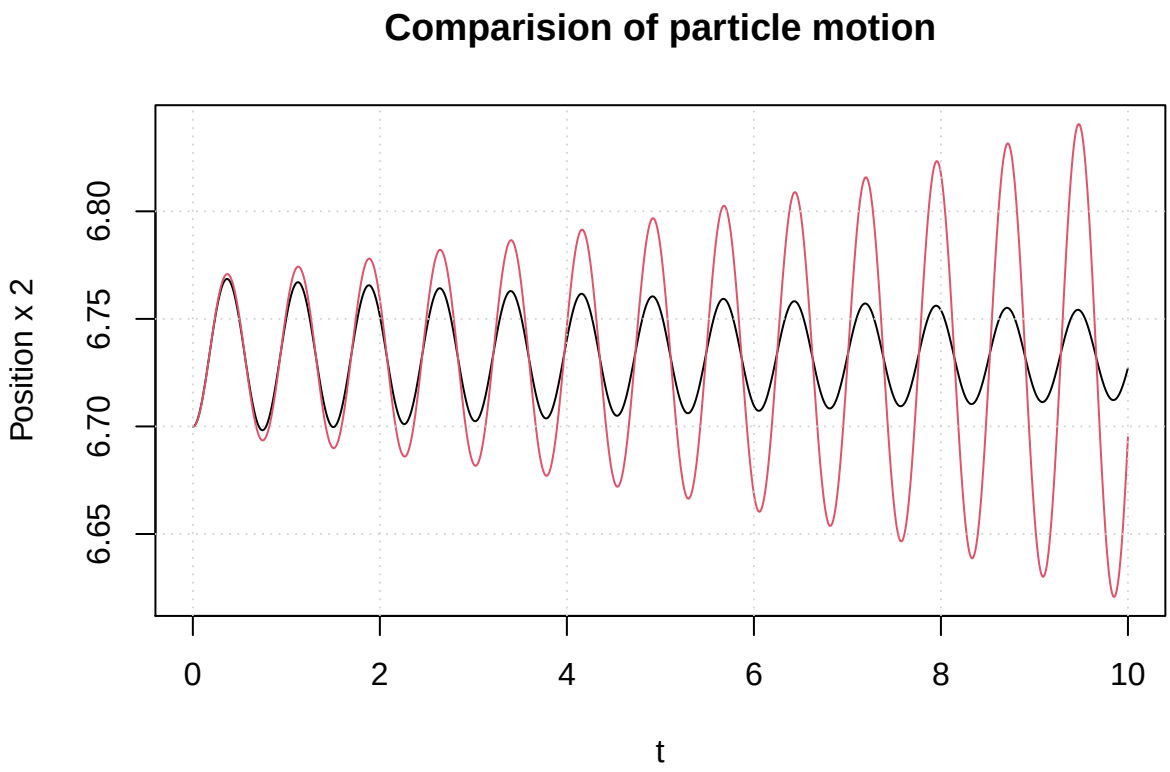



```
plot.particleTCompare = function(compList,
                                 asp1 =1,
                                 asp2= 2,
                                 tIndex =1,
                                 xIndex =2){
  maxt1 = max(compList[[asp1]][,tIndex])
  maxt2 = max(compList[[asp2]][,tIndex])
  mint1 = min(compList[[asp1]][,tIndex])
  mint2 = min(compList[[asp2]][,tIndex])
  max1 = max(compList[[asp1]][,xIndex])
  max2 = max(compList[[asp2]][,xIndex])
  min1 = min(compList[[asp1]][,xIndex])
  min2 = min(compList[[asp2]][,xIndex])

  plot(c(min(mint1,mint2),max(maxt1,maxt2)),c(min(min1,min2),max(max1,max2)),
       xlab = 't',ylab = paste("Position x",as.character(xIndex - 1)),type = 'n')
  lines(compList[[asp1]][,tIndex],compList[[asp1]][,xIndex],col = asp1)
  lines(compList[[asp2]][,tIndex],compList[[asp2]][,xIndex],col = asp2)
  grid()
  title('Comparision of particle motion')
}


plot.particleCompare = function(compList,
                                asp1 =1,
                                asp2= 2,
```

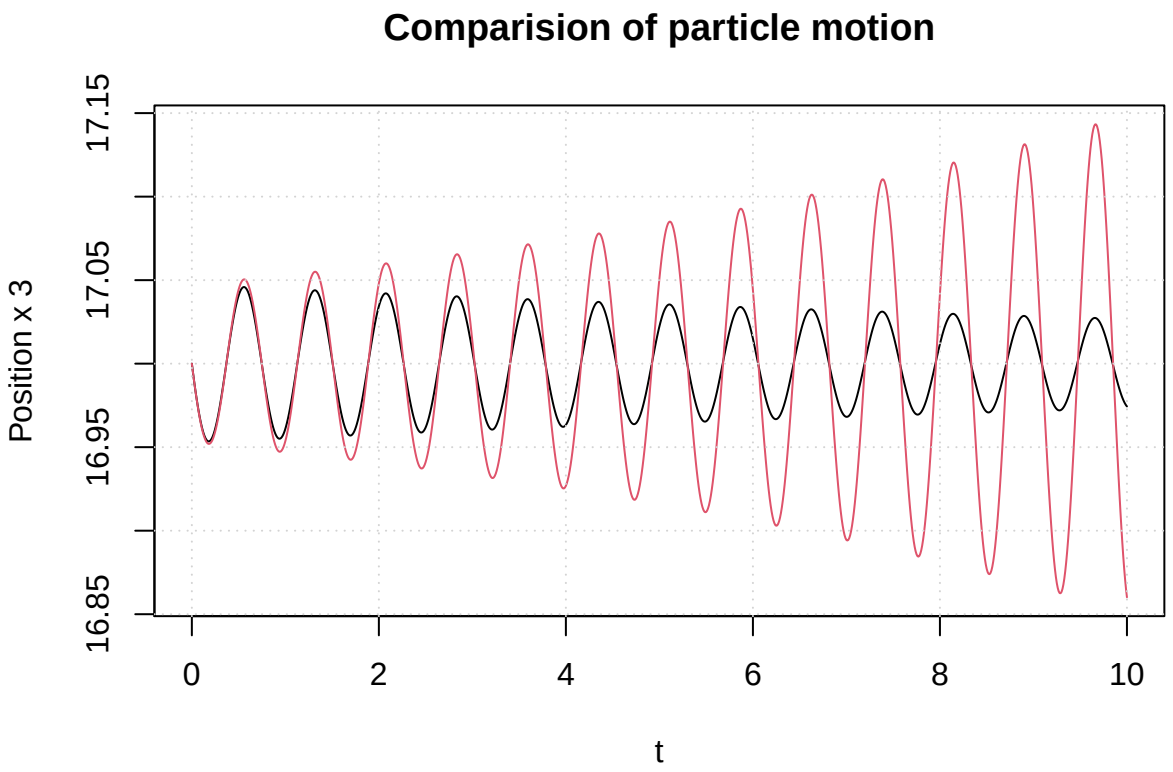
```
                                xIndex =1,
                                yIndex =2,
                                xlab = 'x',
                                ylab = 'y'){
  maxt1 = max(compList[[asp1]][,xIndex])
  maxt2 = max(compList[[asp2]][,xIndex])
  mint1 = min(compList[[asp1]][,xIndex])
  mint2 = min(compList[[asp2]][,xIndex])
  max1 = max(compList[[asp1]][,yIndex])
  max2 = max(compList[[asp2]][,yIndex])
  min1 = min(compList[[asp1]][,yIndex])
  min2 = min(compList[[asp2]][,yIndex])

  plot(c(min(mint1,mint2),max(maxt1,maxt2)),c(min(min1,min2),max(max1,max2)),
       xlab = xlab,ylab = ylab,type = 'n')
  lines(compList[[asp1]][,xIndex],compList[[asp1]][,yIndex],col = asp1)
  lines(compList[[asp2]][,xIndex],compList[[asp2]][,yIndex],col = asp2)
  grid()
  title('Comparision of particle motion')
}

#layout(matrix(1:3,3))
plot.particleTCompare(comp1,xIndex = 2)
```



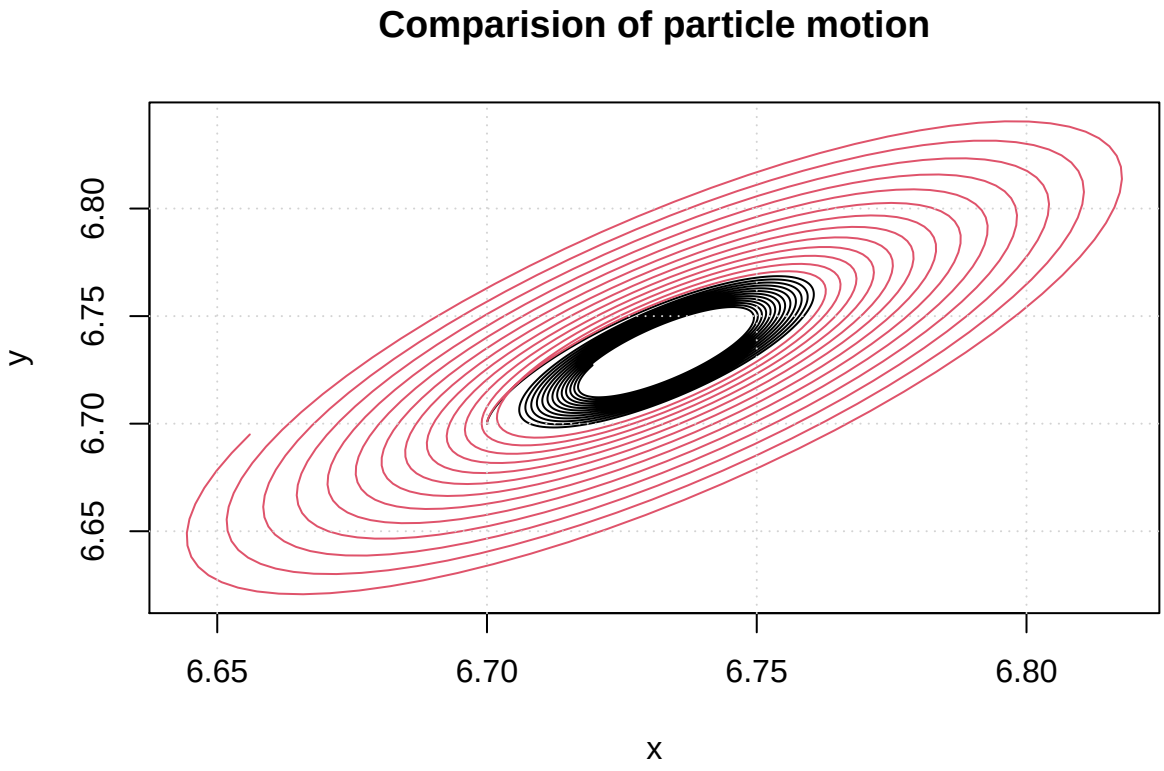
```
plot.particleTCompare(comp1,xIndex = 3)
```



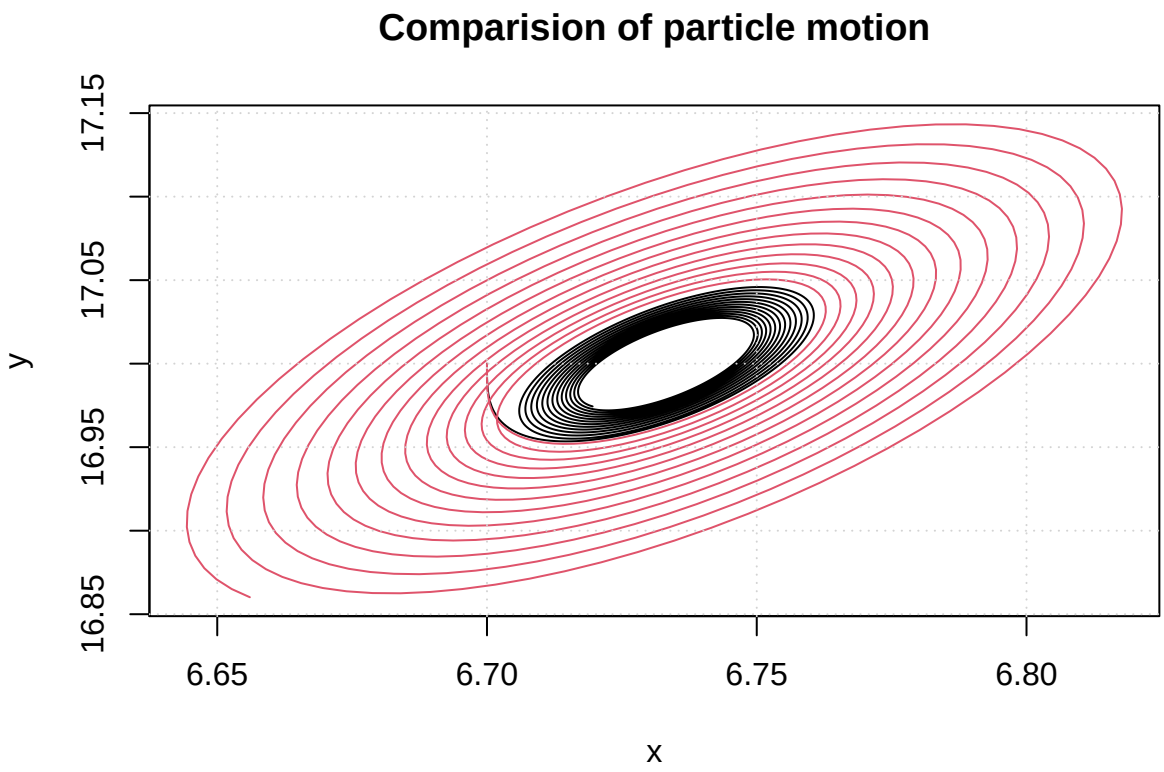

```
plot.particleTCompare(comp1,xIndex = 4)
```



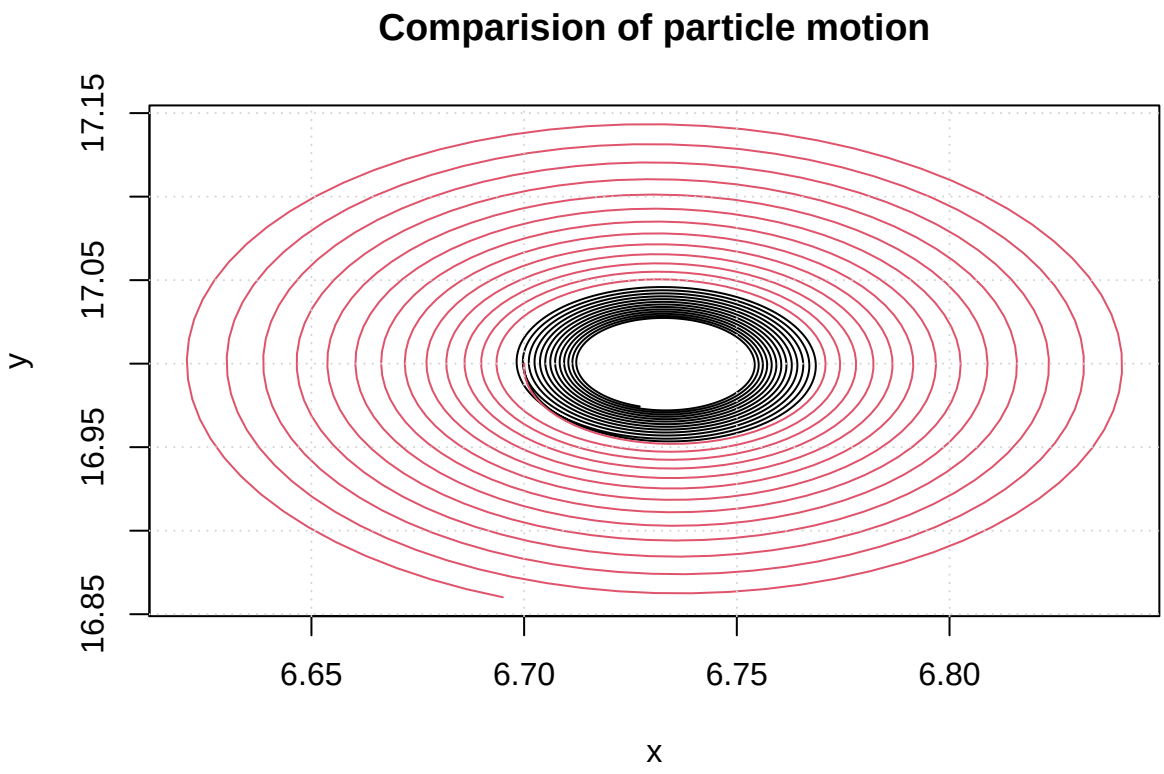

```
layout(1)
plot.particleCompare(comp1,xIndex = 2,yIndex = 3)
```




```
plot.particleCompare(comp1,xIndex = 2,yIndex = 4)
```




```
plot.particleCompare(comp1,xIndex = 3,yIndex = 4)
```




```
#I am using Dell Latitute 4250 with 16GB memory in this case

memory.limit()
```

```
## [1] 16111
```

#2 Figure 6-37 6-38 Sensitivity to the initial condition ##Case1: x1[1]=9.00 ##Case2: x1[1]=9.01

```
system.time({
comp2 = foreach(i=c(9,9.01)) %dopar% ({
  xLorentz_df(xi =c(i,8,27),param = list(Sigma = 10,b = 8/3,r = 28),
              init = 0, end = 50,h=.0005)
  })
})
```

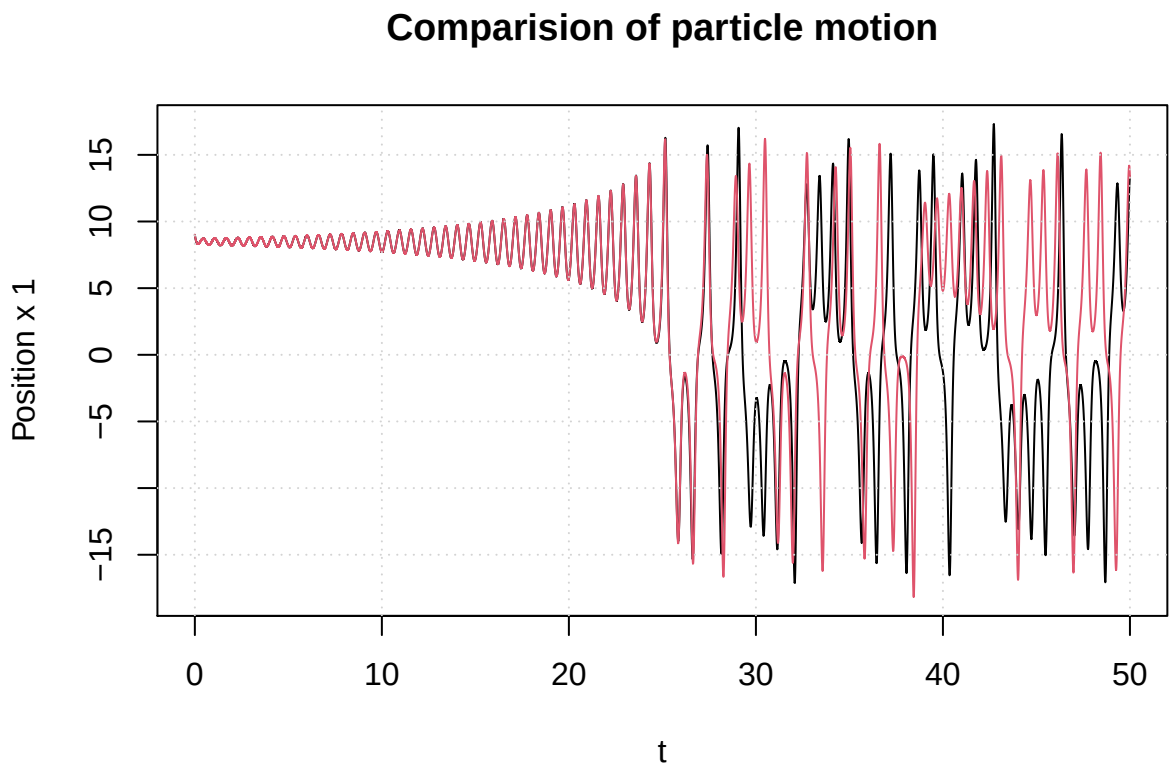
```
##    user  system elapsed
##    0.02    0.00    0.97
```

```
str(comp2)
```

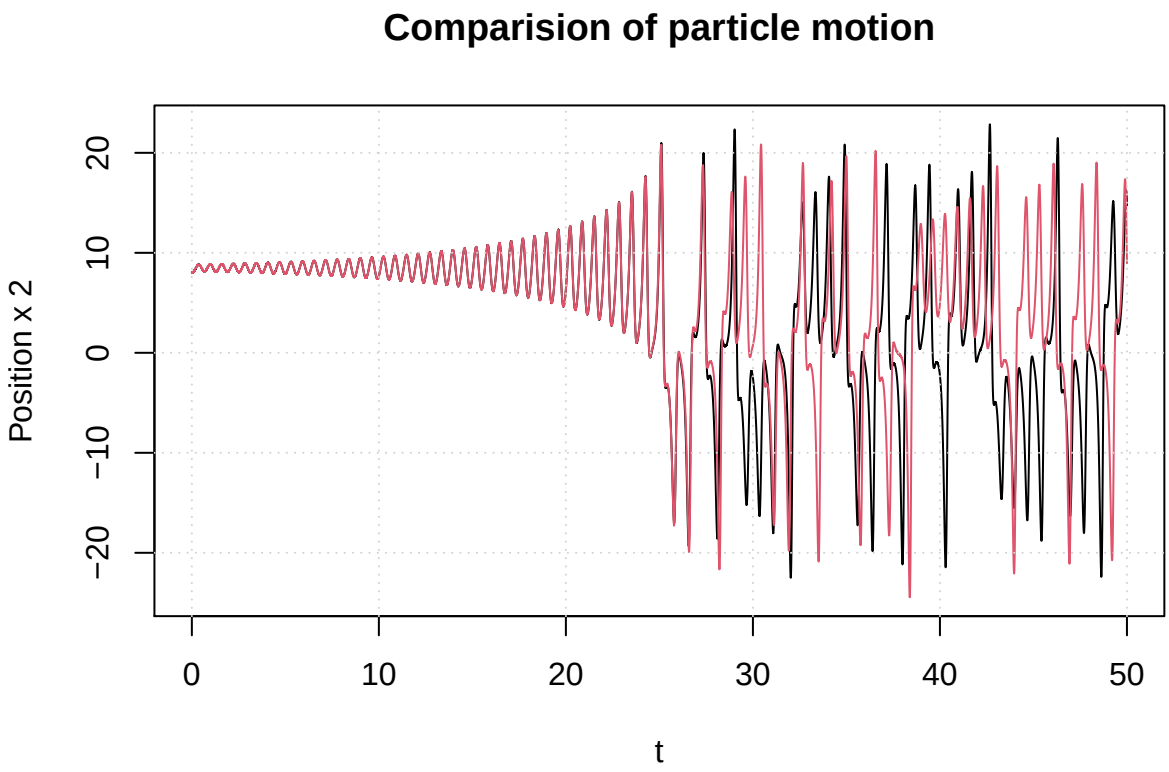
```
## List of 2
##  $ : num [1:100001, 1:4] 0 0.0005 0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004 0.0045 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : NULL
```

```
##   .. ..$ : chr [1:4] "t" "" "" ""
##  $ : num [1:100001, 1:4] 0 0.0005 0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004 0.0045 ...
##   ..- attr(*, "dimnames")=List of 2
##   .. ..$ : NULL
##   .. ..$ : chr [1:4] "t" "" "" ""
```

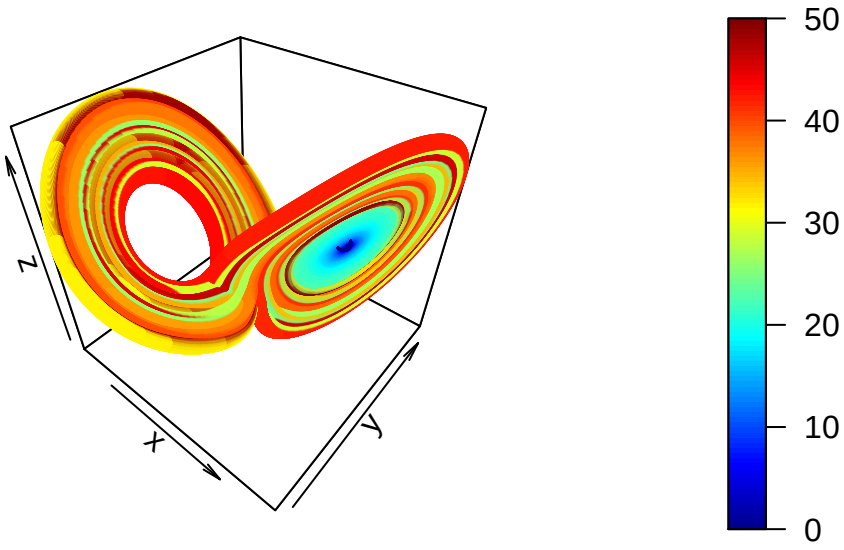
```
plot.particleTCompare(comp2,xIndex = 2)
```




```
plot.particleTCompare(comp2,xIndex = 3)
```




```
plot.particle(comp2[[1]],type ='3d')
```


z
x
y
50
40
30
20
10
0