Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
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Dr. Mark Feodoria, Ole Martin Sheet 05

Risk Management

Exercises for participants of **mathematical programmes**

C-Exercise 16

(a) Write a *scilab*-function

```
X = GARCH_11_MC (n, m, theta, sigma1),
```

that simulates m paths of n observations $X_1^m, \ldots, X_n^m$ from a GARCH(1,1)-model with parameters $\theta = (\alpha_0, \alpha_1, \beta) \in (0,\infty) \times [0,\infty) \times [0,\infty)$ and starting value σ_1.

(b) Use your function from part a) with the values

$$\alpha_0 = 3.2 \times 10^{-6},\ \alpha_1 = 0.08426,\ \beta = 0.8986,\ \sigma_1 = 0.0195$$

to simulate $n = 6561$ log-returns. Create a figure with 2×2 subplots, where you plot the log-returns of the DAX time series and their histogram as in C-Exercise 1 together with the same plots for your simulated data from the GARCH(1,1)-model. Compare the plots for the two time series!

Please give a description of your *scilab* operations in the `sce`-file.
Useful *scilab* commands: `grand`, `subplot`

C-Exercise 17

We assume that the one dimensional risk factor changes $(X_k)_{k\in\mathbb{N}}$ follow a GARCH(1,1) model.

(a) Write a *scilab*-function

```
[VaR,ES] = VaR_ES_GARCH_11_MC(k, m, l,alpha, theta, sigma1),
```

that computes the Value at Risk and Expected Shortfall estimates for the k-period loss operator $l : \mathbb{R} \to \mathbb{R}$, level $\alpha \in (0,1)$, parameters $\theta = (\alpha_0, \alpha_1, \beta)$ and starting value σ_1 using the Monte-Carlo method with $m \in \mathbb{N}$ simulations.

(b) Assume that the log returns of the DAX time series follow the GARCH(1,1) model with the values from C-Exercise 16b. Compute for the last trading day in the time series the 10-day-ahead estimates for Value at risk and Expected Shortfall of the DAX portfolio at level $\alpha = 0.95$ using $m = 100$ simulations.

Please turn over.

C-Exercise 18M

(a) Write a *scilab*-function

```
[theta_hat, sigma_1_hat] = estimates_GARCH_11(x),
```

which computes the Maximum Likelihood estimates $\hat{\theta} = (\hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta})$ and $\hat{\sigma}_1$ for given historical risk factor changes $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ of the parameters $\theta = (\alpha_0, \alpha_1, \beta)$ and starting value σ_1 in the GARCH(1,1) model.

(b) Assume that the log returns of the DAX time series follow a GARCH(1,1) model with unknown parameters. Estimate the parameters and compute for the last trading day in the time series the 10-day-ahead estimates for Value at risk and Expected Shortfall of the DAX portfolio at level $\alpha = 0.95$ using $m = 100$ simulations.

(c) For the 1-day-ahead forecasts we do not need Monte Carlo Simulation, because we know the explicit formulas for VaR and ES from Example 1.4 and Example 1.11. Compare the Value at Risk and Expected Shortfall estimates for a one period loss using the function from C-Exercise 17 with the estimates using a function, that applies the explicit formulas.

Useful *scilab* commands: `optim, list, NDcost`

P-Exercise 19

We assume that the one dimensional risk factor changes $(X_k)_{k=1\ldots,n}$ follow a AR(1) model, i.e.

$$
\begin{aligned}
X_1 &= \sigma Y_1, \\
X_k &= \beta X_{k-1} + \sigma Y_k, \quad k \in \{2, \ldots, n\}
\end{aligned}
$$

with parameters $\vartheta = (\beta, \sigma) \in \mathbb{R} \times (0, \infty)$ and independent variables $(Y_n)_{k=1\ldots,n}$ with distribution $N(0,1)$.

(a) Derive the least squares estimator for the parameter $\beta \in \mathbb{R}$ in the AR(1) model. How can you estimate σ?

(b) Assume that the GARCH(1,1) model in C-Exercises 16, 17 (and C-Exercise 18M) is replaced with an AR(1) model. Write down a brief pseudo code of the solution to the problem for the AR(1) model. Where are differences and similarities between the programmes for both models?

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 07.12.2016, 12:00
Discussion: in tutorials on Mon, 12.12.2016 and Wed, 14.12.2016