Christian-Albrechts-Universität zu Kiel **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 03

Computational Finance

Exercises for mathematical programmes

C-Exercise 8
Write a scilab function

```
V0 = Price_BS_Int (S0, r, sigma, T, f)
```

that computes the initial price $V(0)$ of a European option with payoff $f(S(T))$ at maturity $T > 0$ in a Black-Scholes model with initial stock price $S(0) > 0$, interest rate $r > 0$ and volatility $\sigma > 0$ by means of the integration formula

$$V(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f\left(S(0)\exp\left((r-\sigma^2/2)T + \sigma\sqrt{T}x\right)\right) e^{-rT} e^{-\frac{x^2}{2}}\,\mathrm{d}x.$$

Test your function for a European call option with strike K and

$$S(0) = 100,\ r = 0.05,\ \sigma = 0.2,\ T = 1,\ K = 100.$$

(The parameter `f` refers to the name of a scilab function that you can specify outside of the function `Price_BS_Int`.)

Useful scilab command: `intg`

T-Exercise 9 (Exchange rates)
Assume that the exchange rate $D(t)$ of the US-Dollar in Euro at time $t > 0$ follows the equation

$$dD(t) = D(t)\mu\,\mathrm{d}t + D(t)\sigma\,\mathrm{d}W(t)$$

with $D(0) > 0$ and $\mu,\ \sigma \in \mathbb{R}$. Hence, the exchange rate of the Euro in US-Dollar at time $t > 0$ is given by $E(t) := \frac{1}{D(t)}$. Represent the process E as Itō process, i.e. in the form

$$dE(t) = \ldots\,\mathrm{d}t + \ldots\,\mathrm{d}W(t).$$

Interpret your result economically in the case $\mu = \frac{1}{2}\sigma^2$.

Please turn over.

T-Exercise 10
Let W be a standard Brownian motion. Show that the process

$$X(t) := \mathscr{E}(W)(t)\left(1+\int_0^t \frac{1}{\mathscr{E}(W)(s)}\mathrm{d}s\right), \quad t \in \mathbb{R}_+,$$

solves the stochastic differential equation

$$\mathrm{d}X(t) = 1\mathrm{d}t + X(t)\mathrm{d}W(t), \quad X(0) = 1.$$

T-Exercise 11 (Digital option in the Black-Scholes model)
A *digital call option* with maturity $T > 0$ and strike $K > 0$ is a European option with payoff

$$V(T) = 1_{\{S(T)\geq K\}}.$$

Find a formula for the initial price of a digital call option in the Black-Scholes model, and compute the perfect hedging strategy.

Hint: To find the formula for the price, it is useful to work with the integral representation and not with the Black-Scholes PDE.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Friday, 06.05.2016, 08:30
Discussion: in tutorials on Mon, 09.05.2016