Risk Management

Exercises for participants of **mathematical programmes**

C-Exercise 20

(a) Write a *scilab*-function

```
alpha = Hill_Estimator(x, k),
```

which computes the Hill estimator $\widehat{\alpha}_{k,n}$ for $n \in \mathbb{N}$ independent observations $x = (x_1, \ldots, x_n)$ and $k \in \{1, \ldots, n-1\}$.

(b) Write a *scilab*-function

```
Hill_Plot(x),
```

which draws the corresponding Hill plot for $n \in \mathbb{N}$ independent observations $x = (x_1, \ldots, x_n)$.

(c) Generate $n = 500$ simulations for

- the t-distribution with $\nu = 3$ degrees of freedom,
- the t-distribution with $\nu = 6$ degrees of freedom,
- the exponential distribution with parameter $\lambda = 1$,

and draw the corresponding Hill plot.

(d) Write a *scilab*-function

```
[VaR,ES]=VaR_ES_Hill(x,p,k)
```

that computes the VaR and ES estimates from section 3.2.3 and 3.2.4 for $n \in \mathbb{N}$ independent observations $x = (x_1, \ldots, x_n)$, $k \in \{1, \ldots, n-1\}$ and level $p \in (0, 1)$.

(e) On the OLAT entry of this course you will find a data set with $n = 500$ iid simulations of a regularly varying random variable. Use a Hill plot for a reasonable choice of k. Compute the estimates for VaR and ES at level $p = 0.99$.

Useful *scilab* commands: `distfun_trnd` of the distfun package

Please turn over.

C-Exercise 21

(a) Write a *scilab*-function

```
e= MEF(x, u),
```

that evaluates the empirical mean excess function e_n at $u < \max\{x_i : i = 1, \ldots, n\}$ for $n \in \mathbb{N}$ observations $x = (x_1, \ldots, x_n)$.

(b) Write a *scilab*-function

```
MEP(x),
```

that draws the mean excess plot for observations $x = (x_1, \ldots, x_n)$.

(c) On the OLAT entry of this course you find a data set with $n = 500$ iid observations of a random X with cdf F. Draw the corresponding mean excess plot and find a preferably small u, such that the excess cdf F_u of X is approximately $G_{\gamma,\beta}$-distributed.

T-Exercise 22M
Let X_1 and X_2 be two nonnegative, independent and identically distributed random variables with cumulative distribution function F, such that $\overline{F} \in \mathrm{RV}_{-\alpha}$ for some $\alpha > 1$.

(a) Show that

$$\lim_{t \to \infty} \frac{P(X_1 + X_2 > t)}{P(2X_1 > t)} = 2^{1-\alpha}.$$

Hint: As a first step, show that for all $\varepsilon \in (0, \frac{1}{2})$

$$P(X_1 + X_2 > t) \leq 2P(X_2 > (1-\varepsilon)t) + P(X_1 > \varepsilon t)^2.$$

(b) Conclude with (a) that for sufficiently large $p \in (0,1)$

$$\mathrm{VaR}_p(X_1 + X_2) \leq \mathrm{VaR}_p(X_1) + \mathrm{VaR}_p(X_2).$$

P-Exercise 23
Let X be a Student's t-distributed random variable with $\nu > 0$ degrees of freedom. Show that X is regularly varying and compute the corresponding index.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 14.12.2016, 12:00
Discussion: in tutorials on Mon, 19.12.2016 and Wed, 21.12.2016