

In-tutorial exercise sheet 9

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on June 23th 2016, 2:15 p.m.)

Exercise P.19.

Let $X = M + A$ be a continuous semimartingale where M denotes a local martingale and A a continuous adapted process of bounded variation. In Definition 6.10 the quadratic variation of X was defined via $[X] = [M]$. Prove, that this definition is consistent with Theorem 4.10, i.e. show that

$$\sum_{t_i \in \pi_n} (X_{t_i} - X_{t_{i-1}})^2 \xrightarrow{\mathbb{P}} [M]_t$$

holds for a sequence of partitions $(\pi_n)_n$ with $\delta(\pi_n) \rightarrow 0$.

Exercise P.20.

Let X, Y be continuous semimartingales and $(\pi_n)_n$ a sequence of partitions of $[0, t]$ with $\delta(\pi_n) \rightarrow 0$. Prove

$$\sum_{t_i \in \pi_n} (X_{t_i} - X_{t_{i-1}})(Y_{t_i} - Y_{t_{i-1}}) \xrightarrow{\mathbb{P}} [X, Y]_t.$$