Christian-Albrechts-Universität zu Kiel **Winter term 2016/2017**
Mathematisches Seminar
Prof. Dr. M. Vetter, A. Theopold

Exercise sheet 5

supporting the lecture interest rate models

(Submission of Solutions: 9. Dezember 2016, 12:00; Discussion: 12. Dezember 2016)

Exercise 13. (4 points)

Assuming that $\sigma(t,T) = (\sigma_1(t,T), \dots, \sigma_d(t,T))^*$ are deterministic prove the following relations between instantaneous and and futures rates:

$$f(t,T) = \mathbb{E}_{\mathbb{Q}}[r(T)|\mathcal{F}_t] - \int_t^T \left(\sigma(s,T) \int_s^T \sigma(s,u)^* du \right) ds,$$

and simple forward and futures rates

$$F(t;T,S) = \mathbb{E}_{\mathbb{Q}}[F(T,S)|\mathcal{F}_t] - \frac{P(t,T)}{(S-T)P(t,S)} \left(\exp\left(\int_t^T \left(\int_T^S \sigma(s,v)dv \int_s^S \sigma(s,u)^* du \right) ds \right) - 1 \right),$$

for $t \leq T < S$.

Exercise 14. (4 points)

Consider the classical Black-Scholes model i.e. we have a risky asset S and a money-market account B with dynamics

$$dB = Brdt,\ B(0) = 1,$$
$$dS = S(\mu dt + \rho dW),\ S(0) > 0,$$

for some constants $r, \mu, \rho > 0$. Show that the European call option on the Stock S with payoff $(S(T) - K)^+$ at maturity T can be replicated by a portfolio based on the money-market account B and the futures contract on $S(T)$.

Exercise 15. (4 points)

Prove the consistency condition (Theorem 5.6) directly by using Itô's formula on equation (5.3).