

In-tutorial exercise sheet 2

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 11. November 2015)

Exercise 1.

The task is to look at times of failure of light bulbs. We model the time of failure $X \in [0, \infty)$ of a light bulb by an exponentially distributed random variable with intensity $\lambda > 0$. We observe the times of failure $X = (X_1, \dots, X_n)$ of n light bulbs.

- a) State an adequate statistical experiment.
- b) We consider the estimator $g(X) := \overline{X}_n$ for λ^{-1} . Is $g(X)$ an unbiased estimator for λ^{-1} ? Compute the quadratic risk of $g(X)$.

Hint: The density of an exponentially distributed random variable X with intensity λ is given by

$$f_X(x) = \lambda e^{-\lambda x} 1_{[0, \infty)}(x).$$

Exercise 2.

Let P_ϑ be the probability measure on \mathbb{R} with Lebesgue-density $1_{[\vartheta, \vartheta+1)}$. We consider the statistical experiment $(\mathcal{X}, \mathcal{B}, \mathcal{P})$ with $\mathcal{X} = \mathbb{R}$, $\mathcal{B} = \mathcal{B}(\mathbb{R})$ and $\mathcal{P} = \{P_\vartheta | \vartheta \in \mathbb{Z}\}$. We observe $X \sim P_\vartheta$.

- a) Find an uniformly best estimator for ϑ given the quadratic loss function.
- b) Which property of \mathcal{P} implied that it was possible to construct an uniformly best estimator?