Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
Prof. Dr. M. Vetter, O. Martin **Summer term 2016**

In-tutorial exercise sheet 1

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on 21. April 2016, 14:15 Uhr)

Exercise P.4.

Let $(B_t)_{t\geq 0}$ be a standard Brownian motion and $a \in \mathbb{R} \setminus \{0\}$.

a) Prove the *scaling property* of the Brownian motion i.e. show that the process defined via

$$X_t = \frac{1}{a} B_{a^2 t}$$

is also a standard Brownian motion.

b) Prove the *time inversion property* of the Brownian motion i.e. show that the process defined via

$$Y_t = \begin{cases} 0 & t = 0 \\ tB_{\frac{1}{t}} & t > 0 \end{cases}$$

is also a standard Brownian motion.

Exercise P.5.

Show that the discretized version

$$\tau_n = \left(\max\{m \in \mathbb{N} : m2^{-n} \leq \tau\} + 1\right) 2^{-n}$$

of an almost surely finite stopping time τ w.r.t. the filtration $(\mathcal{F}_t)_{t\geq 0}$ used in the proof of the strong Markov property is indeed a stopping time w.r.t the same filtration $(\mathcal{F}_t)_{t\geq 0}$. Is

$$\tau_n^* = \left(\max\{m \in \mathbb{N} : m2^{-n} \leq \tau\}\right) 2^{-n}$$

also a stopping time?