Stochastics II
Stochastic Processes
Winterterm 2016/17

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Sheet 3

Aufgabe 1
For all $x \in E$ let $Y_1(x), Y_2(x), \ldots$ be independent discrete identically distributed random variables, taking values in Y and $h : E \times Y \to E$ a measurable function. For some initial random variable X_0, independent of all the others, we call the stochastic process $(X_n)_{n \in \mathbb{N}_0}$ given by

$$X_n = h(X_{n-1}, Y_n(X_{n-1})),\ n \in \mathbb{N}$$

a dynamical (stochastic) system of $Y_1, Y_2, \ldots$ and h.
Show that: $(X_n)_{n \in \mathbb{N}_0}$ is a homogeneous Markov chain and its transition matrix is given by

$$P(x, y) = P(h(x, Y_1(x)) = y),\ x, y \in E.$$

Aufgabe 2
A fair die is rolled repeatedly. Which of the following processes $X = (X_n)_{n \in \mathbb{N}_0}$ are Markov chains? Give the transition matrices where applicable.

(a) X_n: biggest face that appeared up to the n-th roll.

(b) X_n: number of sixes up to the n-th roll.

(c) X_n: number of rolls since the last six appeared.

Aufgabe 3
Let $(\Omega, \mathcal{A}, (\mathcal{A}_n)_{n \in \mathbb{N}}, \mathbb{P})$ be a filtered probability space and τ a stopping time. Show that:

(a) $\mathcal{A}_\tau$ is a σ-algebra.

(b) τ is $\mathcal{A}_\tau$-measurable.

Aufgabe 4
Let σ, τ be stopping times. Show that:

(a) $\sigma \wedge \tau$ and $\sigma \vee \tau$ are also stopping times.

(b) $\sigma + \tau$ is also a stopping time.

(c) $\mathcal{A}_\sigma \subseteq \mathcal{A}_\tau$ for $\sigma \leqslant \tau$.

Hand in until friday, 18.11.2016, 12:00