Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Dr. Mark Feodoria, Ole Martin Sheet 04

Risk Management

Exercises for participants of **mathematical programmes**

C-Exercise 12

(a) Write a *scilab*-function

```
[VaR, ES] = VaR_ES_var_covar (x_data, c, w, alpha),
```

that computes the estimates $\widehat{VaR}_\alpha$ and $\widehat{ES}_\alpha$ of the variance covariance method for the linearized loss operator

$$l^\Delta(x) = -(c + w^T x),$$

$c \in \mathbb{R}$, $w \in \mathbb{R}^d$ and given historical risk factor changes $x\_data = (x_1, \ldots, x_n) \in \mathbb{R}^n$.

(b) Go to `http://www.ariva.de/dax-30` and download historical prices for the stocks of BMW, SAP, Volkswagen, Continental and ThyssenKrupp from the german DAX from the time period 1.1.2000-18.11.2016 (1.Click on "Kurse" 2.Choose historical prices ("hist. Kurse") from Xetra 3.Scroll down to csv-download 4.The closing prices ("Schlusskurs") are in the 5th column). Import the time series to *scilab* and compute the logarithmic returns $x_{2,i}, x_{3,i}, \ldots$ for $i = 1, \ldots, 5$ which we use as risk factor changes.

(c) Suppose you hold a portfolio of $\overline{\alpha} = (34, 24, 32, 54, 32)$ shares of the 5 stocks. Compute for each trading day $m \geq 254$ the estimates for *value at risk* and *expected shortfall* at level $\alpha = 0.95$ by applying the function from (a) on the last $n = 252$ risk factor changes $(x_m, x_{m-1}, \ldots, x_{m-n+1})$. Plot your results.

Hint: Have a look at Section 1.2.3 in the lecture notes.

Please give a description of your *scilab* operations in the `sce`-file. Don't forget to sent the `csv`-files with the *scilab*-files (you do not have to print the `csv`-files).
Useful *scilab* commands: `mean(x,"r")`, `cov`, `distfun_mvnpdf`, `csvRead`

C-Exercise 13

(a) Write a *scilab*-function

```
[VaR, ES] = VaR_ES_historic (x_data, l, alpha),
```

that computes the estimates $\widehat{VaR}_\alpha(L_{n+1})$ and $\widehat{ES}_\alpha(L_{n+1})$ for the one-dimensional loss operator $l : \mathbb{R} \to \mathbb{R}$, level $\alpha \in (0,1)$ and given historical risk factor changes $x\_data = (x_1, \dots, x_n) \in \mathbb{R}^n$ using the method of historical simulation.

(b) Compute the logarithmic returns $x_2, \dots, x_{6562}$ of the DAX time series, that we use as risk factor changes. Compute for each trading day $m = 254, \dots, 6562$ estimates for *value at risk* and *expected shortfall* at level $\alpha = 0.95$ by applying the function from (a) on the last $n = 252$ risk factor changes $(x_m, x_{m-1}, \dots, x_{m-n+1})$. Plot your results and compare them with the results of C-Exercise 5.

Please give a description of your *scilab* operations in the `sce`-file.
Useful *scilab* commands: `gsort`, `floor`

T-Exercise 14M
Let $(L_k)_{k \in \mathbb{N}}$ be a sequence of independent and identically distributed random variables on a probability space $(\Omega, \mathscr{F}, P)$ with $E(L_1^2) < \infty$ and strictly increasing, continuous cumulative distribution function F. Denote by F_n the empirical cumulative distribution function of $L_1, \dots, L_n$. Show that for all $\alpha \in (0,1)$ it holds

$$\mathrm{ES}_\alpha(F_n) \xrightarrow{\mathbb{P}} \mathrm{ES}_\alpha(L_1).$$

as $n \to \infty$.

P-Exercise 15M
Let $\mathscr{M} = L^p(\Omega, \mathscr{F}, \mathbb{P})$ be the space of all random variables L with $\|L\|_p = (\mathbb{E}[|L|^p])^{1/p} < \infty$. Prove that $\rho_{[p,a]}$ defined by

$$\rho_{[p,a]}(L) = \mathbb{E}[L] + a\|(L - \mathbb{E}[L])^+\|_p$$

is a coherent risk measure on $\mathscr{M}$ for $p \geq 1$ and $a \in [0,1)$.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 30.11.2016, 12:00
Discussion: in the tutorial on Mon, 5.12.2016