In-tutorial exercise sheet 2

supporting the lecture on Malliavin Calculus

(Discussion in the exercise group on May 10, 2017, 2:15 p.m.)

Exercise 3.

Let

$$f(t_1, t_2) = 1_{(0,t]\times(0,t]}(t_1, t_2)$$

and

$$g(t_1, t_2) = 1_{\{0<t_1\le t_2\le t\}}.$$

Explain the connection between $I_2(f)$ and $I_2(g)$.

Exercise 4.

Let $H = L^2((0,\tau], \mathcal{B}_{(0,\tau]}, \lambda)$, let W be the corresponding Brownian motion on $(0,\tau]$, and let $\mathcal{G}$ denote the σ-algebra generated by W. Show that for any

$$Y = \exp\Big(\int_0^\tau h(s)dW(s)\Big), \quad h \in H,$$

the martingale representation theorem holds, that is we have

$$Y = \mathbb{E}[Y] + \int_0^\tau f(s)dW(s)$$

for some process f with

$$\mathbb{E}\Big[\int_0^\tau f^2(s)ds\Big] < \infty,$$

which is adapted to the filtration generated by the Brownian motion.

Remark: The integral is to be understood as a classical Itô integral.