Stochastics II
Stochastic Processes
Winterterm 2016/17

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Sheet 7

Exercise 1
Consider a chess board. We want to model the movement of the king as a (temporally homogeneous) Markov chain X, where X_n denotes the position of the piece at time $n \geq 0$, with state space $E := \{A1, A2, \dots, H8\}$. Assume that the king chooses one of the squares he could move to at random each step.

(i) Give a sufficient model and state the transition matrix P.

(ii) Determine whether X is irreducible or aperiodic

(iii) Determine the stationary distribution π (i.e. $\pi P = \pi$)

(iv) Assume $X_0 = A1$. Determine the expected number of visits in

  (a) H1
  (b) G7

  before returning to A1.

Hint: Detailed balance condition.

Exercise 2
Let P be a double stochastic matrix, i.e.

(i) $P(i,j) \geq 0$ for all $i, j \in \{1, \dots, n\}$,

(ii) $\sum_{j=1}^{n} P(i,j) = 1$ for all $i \in \{1, \dots, n\}$,

(iii) $\sum_{i=1}^{n} P(i,j) = 1$ for all $j \in \{1, \dots, n\}$

hold true.

(a) Show that $\pi := \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$ is a stationary distribution with respect to P.

(b) Under which conditions on P is π also reversible?
(i.e. $\pi(i)P(i,j) = \pi(j)P(j,i)$)

Exercise 3 (Reflected random walk)
Let X be a Markov chain with state space $E := \{0, 1, \ldots, N\}, N \geq 1$ and transition probabilities

$$P(x,y) = \begin{cases} p & \text{if } y = x + 1 \text{ and } x = 0, 1, \ldots, N-1, \\ q & \text{if } y = x - 1 \text{ and } x = 1, 2, \ldots, N, \\ p & \text{if } y = x = N, \\ q & \text{if } y = x = 0, \end{cases}$$

where $\frac{1}{2} \leq p < 1, p + q = 1$.

1. Determine a stable distribution π.
2. Is π unique?

Hand in until friday, 16.12.2016, 12:00