Exercise sheet 5

supporting the lecture on Malliavin Calculus

(Submission of the solutions: July 7, 2017, 10:15 a.m.)

Exercise 13.

Let $T \in \mathcal{L}_{HS}(H \otimes H)$. Then

$$|\mathrm{tr}(T^2)| \leq ||T||_{HS}^2$$

holds.

Exercise 14.

Let W be an isonormal Gaussian process on H and let $u \in \mathbb{D}^{1,2}(H)$. As Du is a random variable with values in $H \otimes H \cong \mathcal{L}_{HS}(H, H)$, we may associate with Du an adjoint mapping called Du^*. Show that for any $h \in H$

$$\langle u, h \rangle_H \in \mathbb{D}^{1,2} \quad \text{and} \quad D\langle u, h \rangle_H = Du^* \cdot h.$$

Hint: As usual, start with $u \in \mathcal{S}(H)$ and extend the result to $u \in \mathbb{D}^{1,2}(H)$ afterwards.

Exercise 15.

Let $H = \mathbb{R}$, so that $L^2(\Omega, \mathcal{F}, \mathbb{P}) = L^2(\Omega; H)$. Prove that $X = 1_{\{W(1)>0\}}$ satisfies $X \notin D(\delta)$.