Christian-Albrechts-Universität zu Kiel **Winter term 2015/16**
Mathematisches Seminar
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In-tutorial exercise sheet 12

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 10. February 2015)

Exercise 1.

Let X be a random variable with Lebesgue density

$$f(x,\vartheta) = e^{-(x-\vartheta)}\mathbb{1}_{[\vartheta,\infty)}(x).$$

a) Find a statistic $T : \mathcal{X} \to \mathbb{R}$ such that the distribution family $\mathcal{P} = \{P_\vartheta \sim f(\cdot,\vartheta) | \vartheta \in \mathbb{R}\}$ has monotone likelihood ratio in T.

b) Derive a UMP test with level α for the hypotheses

$$H \ : \ \vartheta \leq \vartheta_0 \quad \text{vs.} \quad K \ : \ \vartheta > \vartheta_0.$$

Exercise 2.

Let $X_1, \ldots, X_n$ be i.i.d. $\gamma(p,b)$ distributed, i.e. their density is

$$f(x,(p,b)) = \frac{b^p}{\Gamma(p)} x^{p-1} e^{-bx}, \ x > 0,$$

for $b > 0$ and a fixed $p > 0$.

a) Compute the ML estimator for b.

b) State the likelihood-ratio test for the hypotheses

$$H \ : \ b = b_0 \quad \text{vs.} \quad K \ : \ b \neq b_0$$

for $b_0 > 0$.

Exercise 3.

X is uniformly distributed on the intervall $[-\vartheta, \vartheta]$. Compute the ML estimator for ϑ.

Exercise 4.

Examples for knwoledge questions:

a) Explain the following terms: MSE, UMVU estimator, sufficiency, completeness, Bayes estimator, ML estimator, power function, level-α test, UMP test, likelihood-ratio test

b) Use your own words to explain the following theorems: Cramer-Rao inequality, Neyman criterion, Rao-Blackwell theorem, Lehmann-Scheffe theorem, Neyman-Pearson lemma, Gauß-Markov theorem