Christian-Albrechts-Universität zu Kiel **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 07

Computational Finance

Exercises for participants of mathematical programmes

C-Exercise 24
Write a scilab function

```
V0 = EuCall_BS_MC (S0, r, sigma, T, K, M)
```

that computes the initial price of a European call option in the Black-Scholes model via the Monte-Carlo approach using $M \in \mathbb{N}$ simulations. Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 100, \quad M = 100000,$$

and compare the result to the exact value (cf. C-Exercise 16).

Hint: The initial option price is of the form $V(0) = E_Q(f(Z))$, where $Z \sim N(0,1)$ under Q and f is a suitable function.

Useful scilab commands: `grand`, `rand`

T-Exercise 25
In Section 5.1 of the lecture, we claimed that

$$\sqrt{N}\frac{\hat{V}_N - V}{\sqrt{\hat{\sigma}_N^2(f(X))}}$$

converges in distribution to a standard normal random variable as $N \to \infty$.
Hint: Slutsky's Theorem (cf. T-Exercise 19).

T-Exercise 26
Show that the following *Box-Muller algorithm* indeed simulates two independent standard normal random variables.

(i) Simulate a pair (U_1, U_2) of independent random variables $U_1, U_2 \sim \text{uniform}[0,1]$.

(ii) Set

$$Z_1 := \sqrt{-2\log(U_1)}\cos(2\pi U_2),$$
$$Z_2 := \sqrt{-2\log(U_1)}\sin(2\pi U_2).$$

(iii) Return (Z_1, Z_2).

Please turn over.

T-Exercise 27

Consider a Black-Scholes model as in C-Exercise 16 with parameters S_0, r, $\sigma > 0$ and a geometric average call option on the stock with strike $K > 0$ and monitoring times $t_k = k\frac{T}{M}$ for $T > 0$, $M \in \mathbb{N}$, $k = 0, \ldots, M$, i.e. with payoff $\left(\left(\prod_{k=0}^{M} S_{t_k} \right)^{\frac{1}{M+1}} - K \right)^{+}$ at maturity T. Show that the fair price of this option is given by

$$V_0^{\mathrm{ga}} = e^{-rT} \left(S_0 e^{a + \frac{b^2}{2}} \Phi\left(\frac{\log(S_0/K) + a}{b} + b \right) - K\Phi\left(\frac{\log(S_0/K) + a}{b} \right) \right)$$

for $a := \left(r - \frac{\sigma^2}{2} \right) \frac{T}{2}$, $b := \sqrt{\frac{2M+1}{6(M+1)}}\sigma\sqrt{T}$ and the cumulative distribution function Φ of the standard normal distribution.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Thursday, 09.06.2016, 08:30
Discussion: in the tutorial on Mon, 13.06.2016