Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Dr. Mark Feodoria, Ole Martin Sheet 08

Risk Management

Exercises for participants of **mathematical programmes**

T-Exercise 27M
Prove Theorem 4.24 in the case of continuous marginal cumulative distribution functions.

T-Exercise 28M
Let $X = (X_1, \dots, X_d)$ be a random vector with continuous marginal cumulative distribution functions $F_1, \dots, F_d$ and copula C. Show the following:

(a) $X_1, \dots, X_d$ are independent iff

$$C(u_1, \dots, u_d) = \prod_{i=1}^{d} u_i \quad \text{for all } u_1, \dots, u_d \in [0,1].$$

(b) $X_1, \dots, X_d$ are comonotone iff

$$C(u_1, \dots, u_d) = \min\{u_1, \dots, u_d\} \quad \text{for all } u_1, \dots, u_d \in [0,1].$$

C-Exercise 29

(a) Use each of the functions

```
[VaR, ES]  = VaR_ES_log_normal (s_data, alpha),
[VaR, ES]  = VaR_ES_var_covar (x_data, c, w, alpha),
[VaR, ES]  = VaR_ES_historic (x_data, l, alpha),
[VaR, ES]  = VaR_ES_Hill (s_data, alpha),
[VaR, ES]  = VaR_ES_GARCH_1step (s_data, alpha),
VaR        = VaR_t_dist (s_data, alpha),
```

which are provided on the OLAT (in the file `Functions_for_Ex_28_29.sci`) to estimate *value at risk* and *expected shortfall* at level $\alpha = 0.95$ for the Dax time series at each trading day $m = 254, \dots, 6561$ based on the last $n = 252$ risk factor changes $(x_{m-n+1}, \dots, x_m)$. Plot the estimates together with the realized losses.

Hint: Computing the values for the whole time series takes a lot of time. So we recommend you to test your code first only for the first few days, e.g. $m = 254, \dots, 260$.

(b) Compute for each method from part a) the number of days at which the loss exceeded the estimated VaR. Compare the numbers of exceedances with the expected number of violations and its 5% confidence interval which can be obtained from the function

```
[h, c1, c2] = test_binomial (v, p, beta).
```

(c) Discuss for each of the methods, whether you think they are suitable for estimating VaR and ES of the DAX time series. Explain your assessments and mention general advantages and drawbacks of the methods. Which method do you think is most suitable?

Please turn over.

P-Exercise 30

50 dwarfs live in a very dark cave. On the ground there are 25 red and 25 white hats which are randomly distributed on the floor. Each dwarf puts on one of the hats without knowing its color. The dwarfs do not talk about the color of their hats. Forming a line they leave the cave one after another. In the end all 25 dwarfs with red hats stand next to each other. The same is true for the dwarfs with white hats. How is this possible?

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until:	Wednesday, 11.01.2017, 12:00
Discussion:	in tutorials on Mon, 16.01.2017 and Wed, 18.01.2017 and in lecture on Fri, 13.01.2017 (C-Exercise 29)