Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Dr. Mark Feodoria, Ole Martin Sheet 03

Risk Management

Exercises for participants of **mathematical programmes**

T-Exercise 8M

(a) Let X be a random variable. Show that it holds

$$\mathrm{VaR}_\alpha(aX+b) = a\mathrm{VaR}_\alpha(X)+b,\ b\in\mathbb{R}, a\geq 0.$$

(b) Let X be a random variable with continuous and strictly increasing distribution function F. Prove

$$\mathrm{VaR}_\alpha(-X) = -\mathrm{VaR}_{1-\alpha}(X).$$

(c) Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$\mathrm{VaR}_\alpha(f(X)) = f(\mathrm{VaR}_\alpha(X)),\ \alpha\in(0,1),$$

holds for all random variables X.

T-Exercise 9

Denote by X the random loss of a portfolio with distribution function

$$F_X(x) = \begin{cases} 0, & \text{if } x < -\frac{1}{3}\sqrt{3}, \\ 1-\left(\frac{\sqrt{3}}{2}x+\frac{3}{2}\right)^{-3}, & \text{if } x \geq -\frac{1}{3}\sqrt{3}. \end{cases}$$

(a) Compute the expectation, variance, Value at Risk and Expected Shortfall at level $\alpha\in(0,1)$ of X.

(b) Let $\tilde{X}$ be $N(0,1)$-distributed. Compare the following functions of α graphically:

(i) $\mathrm{VaR}_\alpha(X)$ and $\mathrm{VaR}_\alpha(\tilde{X})$,

(ii) $\mathrm{ES}_\alpha(X)$ and $\mathrm{ES}_\alpha(\tilde{X})$.

You may use

$$\mathrm{ES}_\alpha(\tilde{X}) = \frac{\varphi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha}.$$

Please turn over.

T-Exercise 10
Let L be the random loss of a portfolio of the form

$$L = -s\left(e^X - 1\right),$$

where $s > 0$ is a constant and X has a normal distribution with mean $\mu \in \mathbb{R}$ and standard deviation $\sigma > 0$. Compute $\mathrm{ES}_\alpha(L)$ for $\alpha \in (0,1)$.

P-Exercise 11
Consider the situation of C-Exercise 5.

(a) Write down a brief pseudo code of your solution to C-Exercise 5.

(b) Suppose that you want to compute $\mathrm{ES}_\alpha(L_{n+1})$ instead of $\mathrm{VaR}_\alpha(L_{n+1})$. Which parts of your code have to modified and why?

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 23.11.2016, 12:00
Discussion: in tutorials on Mon, 28.11.2016 and Wed, 30.11.2016