Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
Prof. Dr. M. Vetter

Summer term 2017

In-tutorial exercise sheet 1

supporting the lecture on Malliavin Calculus

(Discussion in the exercise group on May 3, 2017, 2:15 p.m.)

Exercise 1.

Suppose $H = \mathbb{R}$ and $X = f(W(1)) \in L^2(\Omega, \mathcal{F}, \mathbb{P})$. Prove:

$$f(W(1)) = \sum_{i=0}^{\infty} a_i H_i(W(1))$$

with

$$a_i = i! \mathbb{E}[f(W(1)) H_i(W(1))].$$

Exercise 2.

Let $H = \mathbb{R}$ and let $h \in H$ be arbitrary.

(a) Prove: For $i < j$ we have

$$\mathbb{E}[H_i(W(h)) H_j(W(1))] = 0.$$

(b) Give a counter example for

$$\mathbb{E}[H_i(W(h)) H_j(W(1))] = 0$$

in the case $i > j$.