Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
Prof. Dr. Mathias Vetter
Dr. Mark Feodoria, Ole Martin Sheet 07

Risk Management

Exercises for participants of **mathematical programmes**

C-Exercise 24

(a) Write a *scilab*-function

```
[VaR, ES] = VaR_ES_historic_mult (x_data, l, alpha),
```

which computes the estimates $\widehat{VaR}_\alpha$ and $\widehat{ES}_\alpha$ of the historical simulation method for given historical risk factor changes $x\_data = (x_1, \dots, x_n) \in \mathbb{R}^{n \times d}$, a d-dimensional loss operator $l : \mathbb{R}^d \to \mathbb{R}$ and level $\alpha \in (0,1)$.

(b) Consider a portfolio with initial value of 1000 €, that always invests 50% of the current portfolio value in the BMW stock and 50% in the Continental stock. Using the time series on OLAT compute for each trading day $m = 254, \dots, 4361$ the estimates for *value at risk* and *expected shortfall* at level $\alpha = 0.99$: apply the function from (a) on the last $n = 252$ risk factor changes $(x_m, x_{m-1}, \dots, x_{m-n+1})$. Plot the estimates. Compute the number of violations, i.e. the days when the actual loss lies above the estimated VaR, and compare it with the theoretical number of violations.

Hint: T-Exercise 03

C-Exercise 25

(a) Write a *scilab*-function

```
tau = Kendall(x),
```

which estimates and returns *Kendall's tau* $\rho_\tau(X_1, X_2)$ for iid samples of a random vector $X = (X_1, X_2)$.

(b) Write a *scilab*-function

```
rho = Spearman(x),
```

which estimates and returns *Spearman's rho* $\rho_S(X_1, X_2)$ for iid samples of a random vector $X = (X_1, X_2)$.

(c) Assume that the log returns of the BMW stock and the continental stock time series on the OLAT entry of this course are iid samples from a random vector (X_1, X_2). Estimate the correlation coefficients $\rho(X_1, X_2)$, *Kendall's tau* $\rho_\tau(X_1, X_2)$ and *Spearman's rho* $\rho_S(X_1, X_2)$. Use a two-dimensional plot in order to visualize the common daily log returns.

(d) Estimate the mean μ and the covariance matrix Σ of (X_1, X_2) with appropriate estimators $\widehat{\mu}$ and $\widehat{\Sigma}$. Simulate $N = 4361$ iid samples of a $N(\widehat{\mu}, \widehat{\Sigma})$ distribution. Plot these samples and estimate *Kendall's tau* and *Spearman's rho*.

Please turn over.

T-Exercise 26M
Let $X = (X_1, X_2)$ be a random vector on a probability space $(\Omega, \mathscr{F}, P)$, such that X_1 and X_2 have strictly increasing and continuous cumulative distributions functions F_1 and F_2. Let $\widetilde{X}$ and $\widehat{X}$ be independent copies of X. Show that

$$\rho_S(X_1, X_2) = 3\left\{P\left((X_1 - \widetilde{X}_1)(X_2 - \widehat{X}_2) > 0\right) - P\left((X_1 - \widetilde{X}_1)(X_2 - \widehat{X}_2) < 0\right)\right\}.$$

Hint: First, show the assertion for $X_1, X_2 \sim \text{uniform}([0,1])$.

P-Exercise 27M
Prove Lemma 4.8 of the lecture notes.

P-Exercise 27QF
Prove assertions 3 and 4 in Lemma 4.8 of the lecture notes.

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until:	Wednesday, 21.12.2016, 12:00
Discussion:	in tutorials on Mon, 16.01.2017 and Wed, 18.01.2017