Christian-Albrechts-Universität zu Kiel **Summer term 2016**
Mathematisches Seminar
Prof. Dr. M. Vetter, O. Martin

In-tutorial exercise sheet 7

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on June 9th 2016, 2:15 p.m.)

Exercise P.15.

Let $(B_t)_{t\geq 0}$ be a standard Brownian motion, $t > 0$ and

$$H_s = B_s \mathbb{1}_{[0,t]}(s), \ s \geq 0.$$

a) Let $(\pi_n)_n$ denote a sequence of partitions of $[0,t]$ with $|\pi_n| := \max\{t_i - t_{i-1} : t_i \in \pi_n\} \to 0$. Prove, that

$$H_s^n(\omega) = \sum_{t_i \in \pi_n} \mathbb{1}_{(t_{i-1},t_i]}(s) B_{t_{i-1}}(\omega)$$

defines a sequence $(H^n)_n$ of $\mathcal{P}$-measurable processes with

$$H^n \overset{\mathcal{L}^2(B)}{\longrightarrow} H.$$

b) Use part a) to compute the stochastic integral $\int H dB$.
Hint: Have a look at Exercise 23.