Christian-Albrechts-Universität zu Kiel **Winter term 2015/16**
Mathematisches Seminar
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In-tutorial exercise sheet 5

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 2. December 2015)

Exercise 1.

Let X be poisson with parameter $\lambda \in (0, \infty)$. We consider the loss function

$$L(\lambda, a) = (\lambda - a)^2/\lambda.$$

Show that $g(X) = X$ is a Minimax estimator for λ. Proceed as following:

a) Consider a Gamma distribution $\pi_{p,b} = \gamma(p, b)$ as a prior for the parameter λ. Determine the a posteriori distribution.

b) Calculate the a posteriori risk

$$R^x_{\pi_{a,b}}(a) := \int_\Theta L(\lambda, a) Q^{\lambda|X=x}(d\lambda)$$

with $a \in \mathbb{R}$ and $p > 1$.

Hint: You may use that for $\Lambda \sim \gamma(p, b)$ we have

$$\mathbb{E}[\Lambda] = \frac{p}{b}, \qquad \mathbb{E}[1/\Lambda] = \frac{b}{p-1}.$$

c) Minimize $R^x_{\pi_{a,b}}(a)$ over $a \in \mathbb{R}$. Compute $R^x_{\pi_{a,b}}(a^*(x))$ if $a^*(x)$ is the minimizing function.

d) Calculate $R(\lambda, X)$ and thereby finish the proof.