Christian-Albrechts-Universität zu Kiel **Summer term 2016**
Mathematisches Seminar
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In-tutorial exercise sheet 0

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on 14. April 2016, 14:15 Uhr)

Let $B = (B_t)_{t\geq 0}$ be a standard Brownian motion and $(\mathcal{F}_t)_{t\geq 0}$ with $\mathcal{F}_t = \sigma(\{B_s | s \leq t\})$ be the filtration generated by B.

Exercise P.1.

Compute the conditional expectation

$$\mathbb{E}\left[(B_t)^2 \middle| \mathcal{F}_s\right], \quad t > s \geq 0.$$

Note which properties of the conditional expectation you used!

Hint: $(B_t)^2 = ((B_t - B_s) + B_s)^2$.

Exercise P.2.

Use Exercise P.1 and the Jensen inequality to show

$$\mathbb{E}\left[|B_t| | \mathcal{F}_s\right] \leq \sqrt{(B_s)^2 + (t-s)}, \quad t > s \geq 0.$$

In particular it holds $\mathbb{E}[|B_t|] \leq \sqrt{t}$ for $t \geq 0$.

Exercise P.3.

Denote $\mathcal{F} = \{\varnothing, \{B_1 > 0\}, \{B_1 \leq 0\}, \Omega\}$. Derive an expression for

$$\mathbb{E}\left[B_2 | \mathcal{F}\right]$$

using the density f of a $\mathcal{N}(0,1)$ distributed random variable.