Christian-Albrechts-Universität zu Kiel **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 02

Computational Finance

Exercises for participants of mathematical programmes

T-Exercise 4
Let W be a standard Brownian motion. Represent the process $tW(t)$ as Itō process, i.e. in the form

$$d(tW(t)) = \dots dt + \dots dW(t).$$

T-Exercise 5
Let W be a standard Brownian motion. Represent $X(t) := \frac{W(t)}{1+t}$ as Itō process.

T-Exercise 6
Let X and Y be Itō processes. Show Yor's formula:

$$\mathscr{E}(X)\,\mathscr{E}(Y) = \mathscr{E}(X + Y + [X,Y])\,.$$

T-Exercise 7
Let W be a standard Brownian motion. Consider a holomorphic function $f : \mathbb{C} \to \mathbb{C}$ and a complex-valued Itō process Z, i.e.,

$$dZ(t) = \mu(t)dt + \sigma(t)dW(t)$$

with $Z(t) = X(t) + iY(t)$ and

$$\begin{aligned} dX(t) &:= \mathrm{Re}(\mu(t))\,dt + \mathrm{Re}(\sigma(t))\,dW(t), \\ dY(t) &:= \mathrm{Im}(\mu(t))\,dt + \mathrm{Im}(\sigma(t))\,dW(t). \end{aligned}$$

Show Itō's formula for complex-valued processes:

$$df(Z(t)) = \left(f'(Z(t))\mu(t) + \frac{1}{2} f''(Z(t))\sigma^2(t) \right) dt + f'(Z(t))\sigma(t)dW(t).$$

Submit until:	Thursday, 28.04.2016, 08:30
Discussion:	in tutorials on Mon, 02.05.2016