In-tutorial exercise sheet 12

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on July 14th 2016, 2:15 p.m.)

Exercise P.24.

Let $B = (B_t)_{t \geq 0}$ be a standard Brownian motion and let $t_n = 1 - 2^{-n}$, $n \in \mathbb{N}_0$. Define a trading strategy φ recursively via

$$\varphi_0 = 2,$$
$$\varphi_{n-1} = 2^n (1 - Y_{t_{n-1}}), \ n \geq 2,$$
$$\varphi = \sum_{n=1}^{\infty} \varphi_{n-1} \mathbb{1}_{(t_{n-1}, t_n]}$$

where $Y_t = (\varphi \bullet B)_t$.

a) Make yourself familiar with the trading strategy defined above and make sure that everything is well defined.

b) Show, that the stopping time $\tau = \inf\{t_n | B_{t_n} - B_{t_{n-1}} \geq 2^{-n}\}$ is almost surely smaller than 1 and that it holds $Y_\tau \geq 1$.

Exercise P.25.

Consider a market $S = (S^0, S^1)^*$ with

$$S_t^0 = 1,$$
$$S_t^1 = \mathcal{E}(B)_t.$$

a) Use Exercise P.24 to find a self-financing strategy $\widetilde{\varphi}$ with $V_0(\widetilde{\varphi}) = 0$ and $V_1(\widetilde{\varphi}) \geq 1$ almost surely.

b) Show that $\widetilde{\varphi}$ is not admissable.