Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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In-tutorial exercise sheet 3

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on May 4th 2016, 4:15 p.m.)

Exercise P.8.

Let $(M_t)_{t\in\mathbb{N}_0}$ be an L^2-martingale i.e. a martingale with $\mathbb{E}[(M_t)^2] < \infty$ for all $t \in \mathbb{N}_0$.

a) Prove the identity

$$\mathbb{E}[(M_t)^2] = \mathbb{E}[(M_s)^2] + \mathbb{E}[(M_t - M_s)^2], \quad s, t \in \mathbb{N}_0.$$

b) Conclude

$$\sup_{t\in\mathbb{N}_0} \mathbb{E}[(M_t)^2] < \infty \Leftrightarrow \sum_{t=1}^{\infty} \mathbb{E}[(M_t - M_{t-1})^2] < \infty.$$

Exercise P.9.

Let $(M_t)_{t\in\mathbb{N}_0}$ be a martingale which is adapted to the filtration $(\mathcal{F}_t)_{t\in\mathbb{N}_0}$ and τ a stopping time. Prove that the stopped process

$$S_t = M_t^\tau = M_{t\wedge\tau}$$

is also a $(\mathcal{F}_t)_{t\in\mathbb{N}_0}$-martingale.