Christian-Albrechts-Universität zu Kiel **Summer term 2016**
Mathematisches Seminar
Prof. Dr. M. Vetter, O. Martin

In-tutorial exercise sheet 6

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on June 2nd 2016, 2:15 p.m.)

Exercise P.13.

Define the variation of a deterministic function $f : [a, b] \to \mathbb{R}$ by

$$V^f_{[a,b]} = \sup\{ \sum_{t_i \in \pi_n} |f(t_{i+1}) - f(t_i)| : \pi_n \text{ finite partition of } [a, b]\}.$$

Prove for continuously differentiable f the identity

$$V^f_{[a,b]} = \int_a^b |f'(t)|dt.$$

Hint: You may use Exercise 21.

Exercise P.14.

Let H, G be a.s. continuous and of finite variation. Prove the following calculation rule for Stieltjes integrals

$$\int_a^b H_s dG_s + \int_a^b G_s dH_s = H_b G_b - H_a G_a$$

with $0 \leq a \leq b$.

Hint: Use Remark 4.8.