Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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In-tutorial exercise sheet 5

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on May 26th 2016, 2:15 p.m.)

Exercise P.11.

Let $(B_t)_{t\geq 0}$ be a Brownian motion and $\tau_a = \inf\{t \geq 0 : B_t = a\}$. Use Wald's first lemma to prove $\mathbb{E}[\tau_a] = \infty$ for $a \neq 0$.

Hint: Reconsider Exercise 13.

Exercise P.12.

Prove the *continuous mapping theorem*: Let (M, d_M) and $(\overline{M}, d_{\overline{M}})$ be metric spaces and $f : (M, d_M) \to (\overline{M}, d_{\overline{M}})$ be a continuous function. Then we have for $(X_n)_{n\in\mathbb{N}_0}, X$ as above

$$(X_n \xrightarrow{\mathcal{L}} X) \Rightarrow (f(X_n) \xrightarrow{\mathcal{L}} f(X)).$$

Remark: Let $(X_n)_{n\in\mathbb{N}_0}, X$ be random variables on a propability space $(\Omega, \mathcal{A}, \mathbb{P})$ with values in a metric space (M, d_M). X_n *converges in law* to X (notation: $X_n \xrightarrow{\mathcal{L}} X$) if and only if

$$\mathbb{E}[h(X_n)] \to \mathbb{E}[h(X)]$$

for all continuous and bounded functions $h : (M, d_M) \to (\mathbb{R}, |\cdot|)$.