Christian-Albrechts-Universität zu Kiel **Summer term 2017**
Mathematisches Seminar
Prof. Dr. M. Vetter

Exercise sheet 2

supporting the lecture on Malliavin Calculus

(Submission of the solutions: May 19, 2017, 10:15 a.m.)

Exercise 4.

Let

$$f(t_1, t_2) = 1_{(0,t]\times(0,t]}(t_1, t_2).$$

Compute $I_2(f)$ in two ways, namely

(a) using Lemma 2.9 and Theorem 2.11,

(b) using Theorem 2.16,

and explain why both results are in fact the same.

Exercise 5.

Let $H = L^2((0,\tau], \mathcal{B}_{(0,\tau]}, \lambda)$, let W be the corresponding Brownian motion on $(0,\tau]$, and let $\mathcal{G}$ denote the σ-algebra generated by W. Prove the martingale representation theorem, that is for any $F \in L^2(\Omega, \mathcal{G}, \mathbb{P})$ there exists a process u with

$$\mathbb{E}\Big[\int_0^\tau u^2(s)ds\Big] < \infty,$$

and which is adapted to the filtration generated by the Brownian motion, such that

$$F = \mathbb{E}[F] + \int_0^\tau u(s)dW(s).$$

Remark: The integral is to be understood as a classical Itô integral.

Hint: Use the in-tutorial exercise 4.

Exercise 6.

Let $A_1 = (a_1, b_1)$ and $A_2 = (a_2, b_2)$ be disjoint. Prove

$$\int_0^\infty \int_0^{t_2} 1_{A_1}(t_1) 1_{A_2}(t_2) dW(t_1)dW(t_2) = \begin{cases} W(A_1)W(A_2), & \text{if } b_1 \le a_2, \\ 0, & \text{else.} \end{cases}$$

Remark: The integrals are to be understood as classical Itô integrals.