Christian-Albrechts-Universität zu Kiel **Summer term 2017**
Mathematisches Seminar
Prof. Dr. M. Vetter

In-tutorial exercise sheet 5

supporting the lecture on Malliavin Calculus

(Discussion in the exercise group on June 28, 2017, 2:15 p.m.)

Exercise 9.

Let $T : H \to V$ be a linear operator between separable Hilbert spaces. Prove

$$\sum_{n \in \mathbb{N}} ||T(e_n)||_V^2 = \sum_{n \in \mathbb{N}} ||T(h_n)||_V^2$$

for any two orthonormal bases $(e_n)_{n\in\mathbb{N}}$ and $(h_n)_{n\in N}$ on H.

Remark: This result shows that the notion of a Hilbert-Schmidt operator is independent of the choice of the orthonormal basis on H.

Exercise 10.

It can be shown that the operator $D : \mathbb{D}^{1,1} \to L^1(\Omega; H)$ has the following *local property*: If $X \in \mathbb{D}^{1,1}$ vanishes on a set $A \in \mathcal{F}$ almost surely, then DX vanishes on A almost surely as well.

Now let $\mathbb{D}^{1,1}_{loc}$ be the space of all operators such that there exists an increasing sequence $\Omega_n \to \Omega$ and a sequence $(X_n)_n$ with $X_n \in \mathbb{D}^{1,1}$ and $X_n = X$ on Ω_n. Then we set

$$DX = DX_n \text{ on } \Omega_n.$$

Show

(a) $D : \mathbb{D}^{1,1}_{loc} \to L^1(\Omega; H)$ is well-defined.

(b) $X = 1_{\{W(1)>0\}}$ satisfies $X \notin \mathbb{D}^{1,2}$, but $X \in \mathbb{D}^{1,1}_{loc}$.