**Christian-Albrechts-Universität zu Kiel** **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 11

# Computational Finance

Exercises for participants of mathematical programmes

**C-Exercise 40**
Write a scilab function

```
V0 = UpOutCall_BS_MC_Richardson (S0, r, sigma, T, K, B, M, m)
```

that computes the price of an up-and-out call option in the Black-Scholes model via Monte-Carlo simulation with Richardson extrapolation. Use $m$ resp. $2m$ points on the coarse resp. on the fine grid for the Euler method.

Test your function with the data of C-Exercise 31.

**C-Exercise 41**
Consider a Black-Scholes model as in C-Exercise 16. The goal is to price a forward start call option with strike $S_{T_0}$ for $0 < T_0 < T$, i.e., with payoff $(S_T - S_{T_0})^+$ at maturity $T$.
Write a Scilab function

```
V0 = BS_ForwardCall_MC (S0, r, sigma, T0, T, N)
```

that approximates the fair price $V_0$ of the forward start call option via Monte-Carlo based on $N \in \mathbb{N}$ samples of $(S_{T_0}, S_T)$.

Write a second function to approximate the price via Monte-Carlo basing only on samples of $S_{T_0}$. To this end, exploit that in the Black-Scholes model

$$e^{-r(T-T_0)}\mathrm{E}\left((S_T - S_{T_0})^+ \,\middle|\, \mathscr{F}_{T_0}\right) = C(T_0, S_{T_0}, r, \sigma, T, S_{T_0})$$

for the pricing function $C$ from C-Exercise 16. Compare the width of the confidence intervals for both approaches using the test data $S_0 = 100$, $r = 0.05$, $\sigma = 0.2$, $T_0 = 0.5$, $T = 1$, $N = 10000$.

**T-Exercise 42**
Let $A \in \mathbb{R}^{d\times d}$ be a symmetric matrix. Show that the following properties are equivalent:

(a) $\lim_{\nu\to\infty} A^\nu z = 0$ for any $z \in \mathbb{R}^d$.

(b) $\lim_{\nu\to\infty} (A^\nu)_{ij} = 0$ for any $i, j \in \{1, \dots, d\}$.

(c) The spectral radius of $A$ satisfies $\rho(A) < 1$.

Please turn over.

**C-Exercise 43**
Write a Scilab function

```
V0 = EuCall_BS_FiDi (r, sigma, a, b, m, nu_max, T, K, type)
```

that approximates the option values $v(0,x_1),\dots,v(0,x_{m-1})$ of a European call option with strike $K > 0$ and maturity $T > 0$ in the Black-Scholes model using one of the three finite difference schemes. Here, $x_i = K\exp(a + i\frac{b-a}{m})$ denote the initial stock prices and $a,b,m,\nu_{max}$ are the parameters of the algorithm presented in the course. If `type` = 0 the function is supposed to use the explicit scheme, if `type` = 1 the implicit scheme, and if `type` = 2 the Crank-Nicholson scheme. Test your function for

$$r = 0.05, \quad \sigma = 0.2, \quad a = -0.7, \quad b = 0.4, \quad m = 100, \quad \nu_{max} = 2000,$$
$$T = 1, \quad K = 100.$$

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

**Submit until:** Thursday, 07.07.2016, 08:30
**Discussion:** in the tutorial on Mon, 11.07.2016