

In-tutorial exercise sheet 1

supporting the lecture interest rate models

(Discussion in the tutorial on 15. November 2016, 14:15 Uhr)

Exercise P.2.

We work with the Hull-White extension of the Vasicek Model

$$dr(t) = (b(t) + \beta r(t))dt + \sigma d\bar{W}(t)$$

and fill in the gaps that are left in exercise 6 on sheet 2.

- a) Derive the ATS-equations implied by the model and show that these are fulfilled by

$$\begin{aligned} A(t, T) &= -\frac{\sigma^2}{2} \int_t^T B(s, T)^2 ds + \int_t^T b(s) B(s, T) ds, \\ B(t, T) &= \frac{1}{\beta} (e^{\beta(T-t)} - 1). \end{aligned}$$

- b) Use these solutions to derive a representation of $f(t, T)$ that finally gives us

$$f_0(T) = -\underbrace{\frac{\sigma^2}{2\beta} (e^{\beta T} - 1)^2}_{=:g(T)} + \underbrace{\int_0^T b(s) e^{\beta(T-s)} ds + e^{\beta T} r(0)}_{=: \phi(T)}.$$

- c) Show that ϕ satisfies

$$\partial_T \phi(T) = \beta \phi(T) + b(T), \quad \phi(0) = r(0).$$