Christian-Albrechts-Universität zu Kiel **Winter term 2015/16**
Mathematisches Seminar
Prof. Dr. M. Vetter, O. Martin/A. Theopold

Exercise sheet 6

supporting the lecture Mathematical Statistics

(Submission of Solutions: 14. December 2015, 12:00 o'clock; Discussion: 16. December 2015)

Exercise 1. (4 points)

We assume that a statistical experiment is given via a real valued random variable X, which distributions P_ϑ are symmetrically around 0 (i.e. for all $B \in \mathcal{B}(\mathbb{R})$ and all $\vartheta \in \Theta$ it holds that $P_\vartheta(B) = P_\vartheta(-B)$).

a) Show that $T(X) = |X|$ is a sufficient statistic.
b) Give a simplified proof in the case that for all $\vartheta \in \Theta$ the density f_ϑ of P_ϑ exists.

Exercise 2. (4 points)

Let $X_1, \dots, X_n$ i.i.d. pareto distributed with parameter (θ, a) i.e. they have the density

$$f_{\theta,a}(x) = \frac{\theta a^\theta}{x^{\theta+1}} 1_{(a,\infty)}(x), \quad a, \theta > 0.$$

a) Determine a sufficient statistic for (θ, a).
b) Determine a sufficient statistic for θ and a if the other parameter is known.
c) Can you derive a general concept from you calculations?

Exercise 3. (4 points)

Derive the results from example 4.17 in the lecture notes.
That means improve the method of moments estimator $g(X) := \frac{2}{n} \sum_{i=1}^n X_i$, taken from exercise 4 on exercise sheet number 4, via the sufficient statistic $T(X) := X_{(n)} = \max_{i=1}^{n} X_i$ with $X_1, \dots, X_n$ i.i.d $\sim \mathcal{U}[0, \theta]$, $X = (X_1, \dots, X_n)$.

Hint: Either follow the hints provided in the lecture notes or try to find an alternative solution using the conditional expectations $\mathbb{E}[X_i | X_{(n)} \leq x]$.

Exercise 4. (4 points)

We look at the statistical experiment $(\mathcal{X}, \mathcal{B}, \mathcal{P})$ and a sufficient statistic T for $\vartheta \in \Theta$. Show that if two parameters ϑ_1, ϑ_2 can be distinguished by their distribution they are still distinguishable by their corresponding distribution under T. That means

$$P_{\vartheta_1} \neq P_{\vartheta_2} \Rightarrow P^T_{\vartheta_1} \neq P^T_{\vartheta_2}.$$