Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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Winter term 2016/2017

Exercise sheet 6

supporting the lecture interest rate models

(Submission of Solutions: 16. Dezember 2016, 12:00; Discussion: 19. Dezember 2016)

Exercise 13. (4 points)

The aim of this exercise is to show that the independence assumption on the functions $G_1, G_2, \ldots, G_m, G_1G_1, G_1G_2, \ldots, G_mG_m$ in the Theorem 5.8 cannot be omitted.

a) Show that the time-homogeneous version of Theorem 2.12 can be derived from Theorem 5.8 for $m = 1$.

b) Consider the two-factor state process

$$\begin{aligned} dZ_1 &= Z_2 dt + \sqrt{\min\{1, Z_1\}} d\overline{W}, \\ dZ_2 &= \min\{1, Z_1\} dt \end{aligned}$$

with state Space $\mathcal{Z} = \mathbb{R}_+^2$. You can assume, without proving it, that there exists an $\mathbb{R}_+^2$-valued solution $Z = Z^z$ with $Z(0) = z$, for every $z \in \mathbb{R}_+^2$. Show that this state process is not affine but is nevertheless consistent with the ATS $\phi(x, z) = z_1 + z_2 x$.

Exercise 14. (4 points)

Consider the Nelson-Siegel family $\phi_{NS}(x, z) = z_1 + (z_2 + z_3 x) \exp(-z_4 x)$.

a) Check whether the linear independence assumption of Theorem 5.8 is satisfied.

b) Give proof of Theorem 5.19.

Exercise 15. (4 points)

Consider the Hull-White extended Vasiček short-rate model

$$dr(t) = \big(z_1 z_5 + z_3 \exp(-z_5 t) + z_4 \exp(-2 z_5 t) - z_5 r(t)\big)dt + \sqrt{z_4 z_5}\exp(-z_5 t)d\overline{W}(t).$$

which is consistent with the Svensson family given in Theorem 5.21. Show that the zero-coupon bond price equals $P(t,T) = \exp(-A(t,T) - B(t,T)r(t))$ where $r(t) = z_1 + Z_2(t)$ with

$$dZ_2(t) = \exp(-z_5 t)(z_2 + z_3 t + \frac{z_4}{z_5}(1 - \exp(-z_5 t))) + \sqrt{z_4 z_5}\exp(-z_5 t)\overline{W}(t)$$

and

$$\begin{aligned}
A(t,T) =& \frac{z_1}{z_5}(\exp(-z_5(T-t)) - 1 + z_5(T-t)) + \frac{z_3 \exp(-z_5 T)}{z_5^2}(\exp(z_5(T-t)) - 1 - z_5(T-t)) \\
&+ \frac{z_4 \exp(-2 z_5 T)}{4 z_5^2}(\exp(2 z_5(T-t)) - 1 - 2 z_5(T-t)), \\
B(t,T) =& \frac{1}{z_5}(1 - \exp(-z_5(T-t))).
\end{aligned}$$