Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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In-tutorial exercise sheet 6

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 9. December 2015)

Exercise 1.

Apply the Neyman criterion to find sufficient statistics for the distributions of $X = (X_1, \ldots, X_n)^T$, where the densities of X_1 originate from the following distribution families:

a) The family of *Weibull*-distributions with densities

$$f(x, (\theta, a)) = \theta a (\theta x)^{a-1} \exp\left(-(\theta x)^a\right) 1_{(0,\infty)}(x), \quad \theta > 0, a > 0.$$

b) The family of uniform distributions on intervals in $\mathbb{R}$

$$f(x, (\theta_1, \theta_2)) = (\theta_2 - \theta_1)^{-1} 1_{[\theta_1, \theta_2]}(x), \quad (\theta_1, \theta_2) \in \{(x, y) \in \mathbb{R}^2 | x < y\}.$$

Exercise 2.

Let $X_1, \ldots, X_n$ be i.i.d $N(\mu, 1)$-distributed. Then X_1 is an unbiased estimator for μ and $T(X) = \overline{X}_n$ is a sufficient statistic. Compute

$$\mathbb{E}\left[X_1 | T(X)\right].$$

What can you conclude using the Rao-Blackwell theorem?