**Christian-Albrechts-Universität zu Kiel** **SS 2016**
Mathematisches Seminar
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# Computational Finance

Exercises for participants of mathematical programmes

**C-Exercise 28**
Write a scilab function

```
[V0, epsilon] = EuCall_BS_MC_AV (S0, r, sigma, T, K, M)
```

that computes the initial price of a European call option in the Black-Scholes model via the Monte-Carlo approach with $M \in \mathbb{N}$ samples. Use the method of antithetic variables to reduce the variance of the estimator. In addition, the function shall return the radius $\varepsilon$ of a confidence interval that contains the true price with a probability of approximately 95% (cf. Section 5.1).

Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 100, \quad M = 100000,$$

and compare the result to the exact value (C-Exercise 16) and the plain Monte Carlo simulation (C-Exercise 24).

**C-Exercise 29**
Consider a Black-Scholes model with parameters $S(0)$, $r$, $\sigma > 0$. The goal of this exercise is to price a call option with strike $K > 0$ and maturity $T > 0$ by the importance sampling method assuming that the random variable $Y$ explained in this method has a $N(\mu, 1)$-distribution.

Write a Scilab function

```
V0 = BS_Call_MC_IS (S0, r, sigma, T, K, M, mu)
```

that approximates the fair price $V_0$ of the call option via Monte-Carlo based on $M \in \mathbb{N}$ samples. For

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1, \quad K = 200, \quad M = 1000$$

plot the approximations as a function of $\mu$ in the the range $[d - |d/2|, d + |d/2|]$, where

$$d = \frac{\log(K/S_0) - (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \,.$$

Please turn over.

**C-Exercise 30**
Consider a Black-Scholes model with parameters $S(0)$, $r$, $\sigma > 0$. The goal is to approximate the fair price $V_0$ of an Asian call option on the stock with strike $K > 0$ and monitoring times $t_k = k\frac{T}{M}$ for $T > 0$, $M \in \mathbb{N}$, $k = 0, \ldots, M$, i.e. with payoff $\left(\frac{1}{M+1}\sum_{k=0}^{M} S_{t_k} - K\right)^+$ at maturity $T$.

Write a Scilab function

```
V0 = BS_AsianCall_MC_CV (S0, r, sigma, K, T, M, N)
```

that approximates the fair price of an Asian call option in the Black-Scholes model. In order to reduce the variance, use the payoff $\left(\left(\prod_{k=0}^{M} S_{t_k}\right)^{\frac{1}{M+1}} - K\right)^+$ of a geometric average option on the stock with strike $K$ and maturity $T$ as control variate.

Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad K = 100, \quad T = 1, M = 50, \quad N = 10000.$$

*Hint:* T-Exercise 27.

**C-Exercise 31**
In the Black-Scholes model with initial stock price $S(0)$, interest rate $r$ and volatility $\sigma$, let $V(0)$ be the initial price of a European up-and-out call option on the stock $S$ with strike price $K$ and barrier $B > K$. I.e., the option pays off at maturity $T$ the amount

$$V(T) = 1_{\{S(t)<B \text{ for all } t\in[0,T]\}}(S(T) - K)^+.$$

Write a scilab function

```
V0 = UpOutCall_BS_MC (S0, r, sigma, T, K, B, M, m)
```

that computes the price of the option via the Monte-Carlo method using $M$ samples. To this end, approximate the paths with a grid of $m$ equidistant points in time.

Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 1,$$
$$K = 100, \quad B = 110, \quad M = 10000, \quad m = 250.$$

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

**Submit until:** Thursday, 16.06.2016, 08:30
**Discussion:** in the tutorial on Mon, 20.06.2016