In-tutorial exercise sheet 8

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 13. Januar 2015)

Exercise 1.

Let $(X_n)_{n\in\mathbb{N}}$ be a sequence of i.i.d. $Bin(1,p)$ distributed random variables. Use the delta method to derive a central limit theorem for the variance estimator $\overline{X}_n(1-\overline{X}_n)$.

Exercise 2.

We look at a sequence $(X_n)_{n\in\mathbb{N}}$ of i.i.d. random variables with probability density function

$$f(x,\theta) = \theta x^{\theta-1} 1_{[0,1]}(x), \quad \theta \in (0,\infty).$$

a) Determine the Maximum likelihood estimator $\hat{\theta}_{ML}$.

b) Derive the asymptotic distribution of $\hat{\theta}_{ML}$.

Hint: It holds that

$$\int_0^1 \log(x) x^{\theta-1} dx = -\frac{1}{\theta^2}, \quad \int_0^1 (\log(x))^2 x^{\theta-1} dx = \frac{2}{\theta^3}$$

for all $\theta > 0$.