Christian-Albrechts-Universität zu Kiel **WS 2016/17**
Mathematisches Seminar
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Risk Management

Exercises for participants of **mathematical programmes**

C-Exercise 5

Denote by S_n the price of a stock at day t_n, $n \in \mathbb{N}$, and by $X_n := \log\left(\frac{S_n}{S_{n-1}}\right), n \geq 2$, the log return of the stock. Assume that the conditional distribution of X_{n+1}, given the stock prices up to time t_n, is a $N(\mu_{n+1}, \sigma^2_{n+1})$-distribution with

$$\mu_{n+1} := \frac{1}{251}\sum_{k=n-250}^{n} X_k \quad \text{and} \quad \sigma^2_{n+1} := \frac{1}{250}\sum_{k=n-250}^{n}(X_k - \mu_{n+1})^2 \quad \text{for } n \geq 252,$$

i.e. the conditional distribution of the log return at time t_{n+1} is normally distributed with empirical mean and empirical variance of the log returns from the past trading year. (We ignore the days of the first trading year.)

(a) Write a *scilab* function

`VaR_log_normal(s, alpha),`

that computes for given stock prices $s = (s_{n-251}, s_{n-250}, \ldots, s_n)$ of the past trading year the Value at Risk at level α of the next trading day's loss L_{n+1} for this stock.

(b) Assume that the DAX time series data from Exercise Sheet 01 follows this model. Compute for each day after the first 252 days the $\mathrm{VaR}_{90\%}$ and the $\mathrm{VaR}_{95\%}$ of the DAX time series and visualize the violations, i.e. the days when the actual loss lies above the computed VaR. How much violations do you expect theoretically, how much do you observe? (You may write your answer directly in the script.)

Hint: In the model considered here the VaR of the loss L_{n+1} at time t_{n+1} is given by

$$\mathrm{VaR}_\alpha(L_{n+1}) = S_n\left(1 - \exp(\mu_{n+1} + \sigma_{n+1} q_{1-\alpha})\right),$$

where $q_{1-\alpha}$ denotes the $(1-\alpha)$-quantile of the standard normal distribution (compare Example 1.7 from the lecture notes).

Please give a description of your *scilab* operations in the `sce`-file.
Useful *scilab* commands: `csvRead`, `mean`, `variance`, `cdfnor`

Please turn over.

C-Exercise 6
Assume that we have a sample $v = (v_1, \dots, v_m) \in \{0,1\}^m$ of i.i.d. random variables with $\mathbb{P}(v_1 = 1) = p$. Hence their sum $\sum_{k=1}^m v_k$ follows a bin(m, p)-distribution, i.e. a binomial distribution with m experiments and success probability $p \in [0,1]$. Design a two sided statistical test at significance level $\beta \in (0,1)$ for the null hypothesis $H_0 : p = p_0$ and implement this test in a *scilab* function called

```
test_binomial(v,p0,beta)
```

This function is supposed to return the value 1, if the null hypothesis is rejected, and 0 otherwise.
We want to apply this test on the results from C-Exercise 5(b): From T-Exercise 7M (see exercise sheet 2 for participants from mathematical programmes) we know that the number of violations follows a bin$(m, 1-\alpha)$-distribution, where m is the number of considered trading days. Apply your function on the violation vectors from the DAX time series using a significance level of $\beta = 0.05$.

Hint: You may construct an exact test based on the cumulative distribution function of bin(m, p) or use a normal approximation to derive a test with asymptotic level β.

Please give a description of your *scilab* operations in the `sce`-file.

Useful *scilab* commands: `cdfbin`, `sum`

T-Exercise 7M
Let $L_1, \dots, L_n$, $n \in \mathbb{N}$, be random variables with continuous conditional cumulative distribution functions modeling the portfolio loss on the trading days $t_1, \dots, t_n$. Show that for a given $\alpha \in (0,1)$ the random variables

$$I_k := \mathbb{1}_{\{L_k > \mathrm{VaR}_\alpha(L_k)\}}, \quad k \in \{1, \dots, n\},$$

are independent and identically distributed with

$$P(I_1 = 1) = 1 - \alpha\ .$$

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until: Wednesday, 16.11.2016, 12:00
Discussion: in tutorials on Mon, 21.11.2016 and Wed, 23.11.2016