Christian-Albrechts-Universität zu Kiel **Winter term 2015/16**
Mathematisches Seminar
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In-tutorial exercise sheet 10

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 27. January 2015)

Exercise 1.

Let $X_1, \dots, X_n$ be idependent exponentially distributed with parameter $\vartheta > 0$, i.e. the density of X_i is given by

$$f_\vartheta(x) = \vartheta \exp(-\vartheta x) 1_{[0,\infty)}(x).$$

a) Show that the distribution of $X = (X_1, \dots, X_n)$ has a monotone likelihood ratio.

b) Derive a UMP test with level α for the hypotheses

$$H \ : \ \vartheta \leq \vartheta_0 \quad \text{against} \quad K \ : \ \vartheta > \vartheta_0.$$

c) Compute the critical region for the test from b) for $\vartheta_0 = 1$, $n = 10$ and $\alpha = 0.05$.
Hint: The 5% quantile of the gamma distribution $\gamma(10, 1)$ is approximately 5.43.