Christian-Albrechts-Universität zu Kiel **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 04

Computational Finance

Exercises for mathematical programmes

T-Exercise 12
In the Black-Scholes model, consider the self-financing trading strategy φ with initial value $V_\varphi(0) = 1$ that always invests half of the wealth in the stock, i.e., $\varphi_1(t) = \frac{\frac{1}{2}V_\varphi(t)}{S(t)}$. Determine the Itō process representation of V_φ. In addition, compute the expectation and variance of $V_\varphi(t)$.

T-Exercise 13
Let W_1, W_2 be independent standard Brownian motions. Consider a market with three assets S_0, S_1, S_2, which follow the equations

$$\begin{aligned} S_0(t) &= 1, \\ \mathrm{d}S_1(t) &= S_1(t)\left(3\mathrm{d}t + \mathrm{d}W_1(t) - \mathrm{d}W_2(t)\right), \\ \mathrm{d}S_2(t) &= S_2(t)\left(1\mathrm{d}t - \mathrm{d}W_1(t) + \mathrm{d}W_2(t)\right). \end{aligned}$$

Construct an arbitrage in this market.

T-Exercise 14
A *forward start option* is an option that transforms at time T_0 to a European call option with strike $S(T_0)$, i.e., it pays off at maturity $T > T_0$ the amount

$$V(T) = (S(T) - S(T_0))^+.$$

Determine the fair price and the perfect hedging strategy of the forward start option.

Please turn over.

T-Exercise 15

Let W be a standard Brownian motion and $T > 0$. Assume that the underlying filtration $(\mathscr{F}_t)_{t\geq 0}$ is generated by W. Let μ be an adapted process and Y an $\mathscr{F}_T$-measurable random variable. Show that there exist $x \in \mathbb{R}$ and a process H such that the process

$$X = x + \int_0^{\cdot} \mu(s)ds + \int_0^{\cdot} H(s)dW(s)$$

fulfills

$$X(T) = Y.$$

Determine x and H explicitly for $\mu = 0$ and

(a) $Y = (W(T))^2$,

(b) $Y = \int_0^T W(s)\mathrm{d}s$ and

(c) $Y = (W(T))^3$,

respectively.

Hint: Martingale representation theorem

Submit until:	Thursday, 19.05.2016, 08:30
Discussion:	in tutorials on Mon, 23.05.2016