Christian-Albrechts-Universität zu Kiel **Winter term 2015/16**
Mathematisches Seminar
Prof. Dr. M. Vetter, O. Martin/A. Theopold

In-tutorial exercise sheet 1

supporting the lecture Mathematical Statistics

(Discussion in the tutorial on 4. November 2015)

Exercise 1.

Let X, Y be two independent normally distributed random variables with variance 1 and X has mean 1 and Y has mean -1 on a common probability space $(\Omega, \mathcal{A}, P)$. Compute the following conditional expectations and identify their distributions:

a) $\mathbb{E}[X + Y|X]$ and $X + Y$,
b) $\mathbb{E}[XY|X]$,
c) $\mathbb{E}[XY^2|X]$.

Exercise 2.

Let X be a real valued random variable on a probability space $(\Omega, \mathcal{A}, P)$ with $\mathbb{E}[X^2] < \infty$ and let $\mathcal{F} \subset \mathcal{A}$ be a sub-σ-algebra. The *conditional variance* is defined as

$$\mathrm{Var}[X|\mathcal{F}] := \mathbb{E}\left[(X - \mathbb{E}[X|\mathcal{F}])^2 \;\middle|\mathcal{F}\right].$$

Prove the identity

$$\mathrm{Var}[X|\mathcal{F}] = \mathbb{E}[X^2|\mathcal{F}] - (\mathbb{E}[X|\mathcal{F}])^2 .$$

Exercise 3.

Let P be a probability measure on $(\mathbb{Z}, \mathcal{P}(\mathbb{Z}))$ with $P(z) = \frac{1}{3}\frac{1}{2^{|z|}}$, $B_i := \{i, -i\}$ for all $i \in \mathbb{N}$ and

$$X : (\mathbb{Z}, \mathcal{P}(\mathbb{Z}), P) \to (\mathbb{R}, \mathcal{B}) \, , X(z) = |z|.$$

Calculate the conditional expectation

$$\mathbb{E}[X|\sigma(\{B_i : i \in \mathbb{N}\})].$$