Christian-Albrechts-Universität zu Kiel **Summer term 2017**
Mathematisches Seminar
Prof. Dr. M. Vetter

Exercise sheet 1

supporting the lecture on Malliavin Calculus

(Submission of the solutions: May 5, 2017, 10:15 a.m.)

Exercise 1.

Let W be an isonormal Gaussian process on H. Show that

$$\mathcal{H}_1 = \{W(h) \mid h \in H\}$$

is a closed subspace of $L^2(\Omega, \mathcal{F}, \mathbb{P})$.

Exercise 2.

Determine the Wiener chaos decomposition of the following random variables:

(a) $H = L^2((0,\infty), \mathcal{B}|_{(0,\infty)}, \lambda)$ with $X = B_t^2$ and $B_t = W(1_{(0,t]})$;
(b) H arbitrary and $X = \exp(W(h) - ||h||^2/2)$;
(c) $H = \mathbb{R}$ and $X = 1_{(0,\infty)}(W(1))$.

Hint: Use the in-tutorial exercise 1 for part (c).

Exercise 3.

Prove Remark 2.1: An isonormal Gaussian process on $H = L^2(T, \mathcal{B}, \mu)$ is uniquely determined by

$$\{W(A) \mid A \in \mathcal{B},\ \mu(A) < \infty\}$$

with the shorthand notation $W(A) = W(1_A)$.