Stochastics II
Stochastic Processes
Winterterm 2016/17

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Sheet 5

Exercise 1
Let $X \sim \exp(\lambda), \lambda > 0$. Show that the solution of the renewal equation

$$G(t) = g(t) + \int_0^t G(t-x)\mathbb{P}^X(dx)$$

for some bounded, measurable function $g : [0, \infty) \to \mathbb{R}$ is given by

$$G(x) = g(x) + \lambda \int_0^x g(z)dz.$$

Exercise 2 (renewal sequences an Markov chains, part 1)
A sequence $(u_n)_{n\in\mathbb{N}_0}$ of real numbers is called *renewal sequence*, if there exists a sequence $(f_n)_{n\in\mathbb{N}}$ of non negative real numbers, such that $\sum_{n\in\mathbb{N}} f_n = 1$ and

$$u_0 = 1, \quad u_n = \sum_{i=1}^{n} f_i u_{n-i}, \quad n \geq 1 \tag{1}$$

holds.
Show that for a Markov chain $(X_n)_{n\in\mathbb{N}_0}$ with state space E and $x \in E$ recurrent, $u_n := \mathbb{P}_x(X_n = x), n \geq 0$ defines a renewal sequence $(u_n)_{n\in\mathbb{N}_0}$.

Exercise 3 (renewal sequences an Markov chains, part 2)
Show that for every renewal sequence $(u_n)_{n\in\mathbb{N}_0}$ there exist a Markov chain $(X_n)_{n\in\mathbb{N}_0}$ with state space $\mathbb{N}_0$ and $x \in E$, such that $(\mathbb{P}_x(X_n = x))_{n\in\mathbb{N}_0}$ satisfies the same renewal equation (1) as $(u_n)_{n\in\mathbb{N}_0}$.

Exercise 4 (renewal sequences an Markov chains, part 3)
Let $(u_n)_{n\in\mathbb{N}_0}$ and $(v_n)_{n\in\mathbb{N}_0}$ be renewal sequences. Show that $(u_n v_u)_{n\in\mathbb{N}_0}$ is also a renewal sequence.

Hand in until friday, 02.12.2016, 12:00