In-tutorial exercise sheet 11

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on July 7th 2016, 2:15 p.m.)

Exercise P.23.

Let there be a market with finite time horizon $T > 0$ and price process $S = (S^0, \ldots, S^d)^*$ where $S^0 > 0$. Prove that this market fulfills the NFLVR condition if and only if there exists a positive right-continuous adapted process $(Z_t)_{t\in[0,T]}$ such that

$$(Z_t S^i_t)_{t\in[0,T]}$$

is a martingale for all $i = 0, \ldots, d$.

Hint: Consider Theorem 8.16 and Theorem 7.3.

Remark: Z is called *state price density process* and $Z_t(\omega)$ can be interpreted as the value of one nominal unit (e.g. 1€) at time t in scenario ω.