Christian-Albrechts-Universität zu Kiel **SS 2016**
Mathematisches Seminar
Prof. Dr. Jan Kallsen
Mark Feodoria Sheet 10

Computational Finance

Exercises for participants of mathematical programmes

T-Exercise 36

A *compound Poisson process* X is a stochastic process with right-continuous paths such that there are a Poisson process N and a sequence of identically distributed random variables $(Y_n)_{n\in\mathbb{N}}$ with

$$X(t) = \sum_{k=1}^{N(t)} Y_k, \quad t \in \mathbb{R}_+,$$

and such that $N, Y_1, Y_2, \dots$ are independent.

For a compound process X, compute the characteristic function of $X(t)$ in terms of the characteristic function φ_Y of Y_1 and the intensity $\lambda \in \mathbb{R}_+$ of N.

C-Exercise 37

Write a scilab function

```
sigma = EuCall_BS_Calibrate (S0, r, T, K, V, sigma0)
```

that calibrates the Black-Scholes model to given prices of European call options. I.e., for the initial stock price $S(0)$, interest rate r, a vector of maturities T, a vector of strikes K and a vector of corresponding option prices V, the routine shall determine the volatility parameter σ that "fits as well as possible", where "fitting well" is to be understood in the sense of Formula (4.17). The parameter σ_0 is the starting value for the optimization.

Download real option prices from the web to test your routine. E.g., choose the option prices of calls on the DAX for 10 different strikes and a fixed maturity, and set $r = 0.03$ and $\sigma_0 = 0.3$.

Hint: Section 4.4 of the course.

Useful scilab commands: `leastsq`, `fminsearch`

Please turn over.

C-Exercise 38
Write a scilab function

```
V0 = EuCall_Heston_Laplace (S0, r, nu0, kappa, lambda, sigma_tilde,
                             T, K, R)
```

that computes, analogously to C-Exercise 32, the initial price of a European call option in the Heston model via the Laplace transform approach.

Test your function for

$$S(0) = 100, \quad r = 0.05, \quad \nu(0) = 0.2^2, \quad \kappa = 0.5, \quad \lambda = 2.5,$$
$$\widetilde{\sigma} = 1, \quad T = 1, \quad K = 100, \quad R = 3.$$

C-Exercise 39
Write a Scilab function

```
V0 = EuCall_BS_FiDi_Explicit (r, sigma, a, b, m, nu_max, T, K)
```

that approximates the option values $v(0,x_1),\dots,v(0,x_{m-1})$ of a European call option with strike $K > 0$ and maturity $T > 0$ in the Black-Scholes model using the explicit finite difference scheme. Here, $x_i = K\exp(a + i\frac{b-a}{m})$ denote the initial stock prices and a,b,m,ν_{max} are the parameters of the algorithm presented in the course. Test your function for

$$r = 0.05, \quad \sigma = 0.2, \quad a = -0.7, \quad b = 0.4, \quad m = 100, \quad \nu_{max} = 2000,$$
$$T = 1, \quad K = 100.$$

Please save your solution of each C-Exercise in a file named `Exercise_##.sce`, where ## denotes the number of the exercise. Please include your name(s) as comment in the beginning of the file.

Submit until:	Thursday, 30.06.2016, 08:30
Discussion:	in the tutorial on Mon, 04.07.2016