

In-tutorial exercise sheet 2

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on 22. November 2016, 14:15 Uhr)

Exercise P.3.

The goal of exercise 8 on sheet 3 is to show that parallel shifts of the forward curve create arbitrage. We consider the most simple one-period forward curve model

$$f(0, t) = 0.04, t \geq 0$$
$$f(\omega, 1, t) = \begin{cases} 0.06, t \geq 1, \omega = \omega_1, \\ 0.02, t \geq 1, \omega = \omega_2, \end{cases}$$

where $\Omega = \{\omega_1, \omega_2\}$ with $\mathbb{P}(\omega_i) > 0, i = 1, 2$.

a) Show that the matrix

$$\begin{pmatrix} P(0, 1) & P(0, 2) & P(0, 3) \\ P(\omega_1, 1, 1) & P(\omega_1, 1, 2) & P(\omega_1, 1, 3) \\ P(\omega_2, 1, 1) & P(\omega_2, 1, 2) & P(\omega_2, 1, 3) \end{pmatrix}$$

is invertible.

b) Use a) to find an arbitrage strategy with value process $V(0) = 0$ $V(\omega_i, 1) = 1$ for both ω_i .