Christian-Albrechts-Universität zu Kiel
Mathematisches Seminar
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Summer term 2016

In-tutorial exercise sheet 2

supporting the lecture Mathematical Finance and Stochastic Integration

(Discussion in the tutorial on April 28th 2016, 2:15 p.m.)

Exercise P.6.

Let $(B_t)_{t\geq 0}$ be a standard Brownian motion with respect to the filtration $(\mathcal{F}_t)_{t\geq 0}$.

a) Prove that $((B_t)^3 - 3tB_t)_{t\geq 0}$ is a martingale.

b) Prove that, although $\mathbb{E}[(B_t)^3] = 0$ for all $t \geq 0$, $((B_t)^3)_{t\geq 0}$ is not a martingale.

Exercise P.7.

Let $(B_t)_{t\geq 0}$ be a standard Brownian motion and $\tau_x = \inf\{t \geq 0 : B_t = x\}$. Show that for allmost all ω there exists a sequence of real numbers $t_n(\omega) \downarrow \tau_x(\omega)$ with $B_{t_n} = x$ for all $n \in \mathbb{N}$.
Hint: Use the strong Markov property (don't forget to check that τ_x is a.s. finite!) and Theorem 1.19.