Step-by-Step Correct Calculation for 15/28 in Base 2

Let's walk through the step-by-step correct calculation of the preperiodic integer (PPI) and periodic integer (PI) for the rational number 15/28 in base 2.

Step 1: Factor the Denominator D = 28

The number 28 factors as:

$28 = 2^2 \times 7$

Here:

- D1 = 2^2 = 4, which is the part of the denominator that shares factors with the base 2,

- D2 = 7, which is coprime with 2 and will control the periodic part of the expansion.

Step 2: Compute the Preperiodic Part (PPI)

To compute the preperiodic part, we first need to handle the division of 15 by D1 = 4 in base 2.

Let's calculate:

15 / 4 = 3.75

We now convert 3.75 into base 2:

- Integer Part: 3 in base 2 is 11_2,

- Fractional Part: 0.75 converts to base 2 as:

  $0.75 \times 2 = 1.5$ -> Take 1

  $0.5 \times 2 = 1.0$ -> Take 1

So, $0.75 = 0.11_2$.

Therefore:

15 / 4 = 3.75_{10} = 11.11_2

This fully captures the preperiodic part before the repeating cycle starts. The expansion 11.11_2 in base 2 corresponds to the preperiodic part.

Step 3: Compute the Periodic Part (PI)

Now, let's compute the periodic part governed by D2 = 7.

We need to find the smallest integer k such that:

$2^k \equiv 1 \pmod 7$

Let's compute powers of 2 modulo 7:

- $2^1 \equiv 2 \pmod 7$,

- $2^2 \equiv 4 \pmod 7$,

- $2^3 \equiv 1 \pmod 7$.

Thus, the period length is k = 3, meaning the repeating part will have 3 digits.

Step 4: Compute the Periodic Integer (PI)

Finally, the repeating part of 15/28 in base 2 is:

15 / 28 = 0.10(001)_2

This gives us the periodic integer (PI) of 001_2 = 1.

Final Answer

For 15 / 28 in base 2:

- Preperiodic integer (PPI): 2,

- Periodic integer (PI): 1.

Thus, the base 2 expansion of 15/28 is:

0.10(001)_2