# Local Linear Stability Analysis of Katabatic Prandtl Slope Flows

*Sample Assignment for a Graduate-level Fluid Mechanics Course*

In this project, you will apply the linear stability theory to a three-dimensional viscous fluid flow problem with heat transfer. The problem is much more general than textbook examples of stability analysis of inviscid two-dimensional flows. The technical approach that you will pursue in this project is general and considered the state-of-the-art in linear stability analysis.

During an in-class activity, you derived the three-dimensional governing equations for linear stability analysis. In this project, you will extend those equations to the canonical Prandtl slope flows as a generalized eigenvalue problem. A partially complete MATLAB script is provided to you which discretizes the governing equations with a spectral collocation method using Chebyshev polynomials. Because our class is not a numerical methods class, you are not expected to understand the entire details of the method, but you are encouraged to do so. At a minimum, you are expected to form the coefficient matrix A resulting from the generalized eigenvalue problem. The Matlab script/code that will be provided to you partially forms the matrix A. You are expected to understand the script and come up with the missing parts to make the script/code work for you.

The code can be downloaded at this link: [https://github.com/HiPerSimLab/PrSlopeFlow](https://github.com/HiPerSimLab/PrSlopeFlow)

At a minimum, your project report should address the following tasks and questions:

**a)** Provide a brief discussion of the Prandtl slope flow model. The following articles should be helpful:

**b)** Does the Prandtl slope flow profile satisfy the necessary condition for stability according to Rayleigh and Fjortoft criteria? Support your answer with graphs.

**c)** Use the provided linear stability analysis Matlab script to demonstrate that there are two distinct types of instabilities in the Prandtl slope flow for katabatic conditions.

**d)** Demonstrate that the one of the modes of instability satisfy the Squire's theorem and the other does not. Support your results with profiles of eigenfunctions.

**e)** Create an instability map as a function of the stratification perturbation parameter and the slope angle at a fixed Prandtl number of 0.71. The slope angle range should be from $3^{o}$ to $80^{o}$. (The map could be coarse).

**f)** Write a routine that computes at a given slope angle the wavelength and oscillation frequency of the critically stable mode which has the smallest stratification perturbation number. Plot the wavelength and frequency as a function of slope angle.

**g)** For a fixed slope angle, compute the wavelength and frequency of the most unstable mode at a given stratification number. Plot the most unstable wavelength and frequency as a function of stratification perturbation number at a given slope angle.

**h)** For a fixed slope angle, plot the growth rate contour of the transverse (longitudinal) mode as a function of the transverse (longitudinal) wave number and stratification number.

You are required to submit your project report following the manuscript requirements for the *Journal of Fluid Mechanics*. The Latex template file can be downloaded from the journal website.

Stability of Prandtl slope flows was investigated in the following papers.

Xiao, C., & Senocak, I. (2019). Stability of the Prandtl model for katabatic slope flows. Journal of Fluid Mechanics, 865, R2

Xiao, C., & Senocak, I. (2020). Stability of the anabatic Prandtl slope flow in a stably stratified medium. Journal of Fluid Mechanics, 885, A13

In your project, you are expected to reproduce some of the results presented in the above articles. You can benefit from the article to your advantage, but all your results should come from your own work and analysis.

**Note: To keep this class project original, you are NOT allowed to share the Matlab script with others.**