Digital Repository of Mathematical Formulae

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Abstract

The purpose of the Digital Repository of Mathematical Formulae (DRMF) is to create a digital knowledge base of mathematical formulae for orthogonal polynomials and special functions (OPSF) and of associated mathematical data. The DRMF addresses two separate needs of working mathematicians, physicists and engineers: providing the technical infrastructure to publish and interact with OPSF formulae on the web, as well as the editorial insight to successfully curate such a resource. Using MediaWiki, the DRMF builds on top of previous efforts, adapting their technology to support OPSF as a scientific web domain. Whereas PlanetMath and Wikipedia expose concepts or terms as first-class objects, both from a system and authoring perspective, the DRMF does that with mathematical formulae.

1 Introduction

Compendia of mathematical formulae have a long and rich history. Many scientists have developed such repositories as books and these have been extremely useful to scientists, mathematicians and engineers over the last several centuries (see [1, 4, 5, 6, 10, 12] for instance). There has been much overlap in formulae for the different compendia, but useful specific information has often been captured in single references and one often would need to be familiar with many different compendia to find to a desired formula. Online compendia of mathematical formulae exist such as voluminous Wolfram Functions Site (`http://functions.wolfram.com/`), subsets of Wikipedia (`http://en.wikipedia.org`), and the NIST Digital Library of Mathematical Functions (`http://dlmf.nist.gov/`). We hope to take advantage of the best aspects of these online efforts and to take advantage of powerful new features which a community arm of scientists should find beneficial. Ingredients which we would like the online repository to incorporate include:

- the ability to interact with a community of mathematicians and scientists who may enter formulae data relevant to their own research related to OPSF (as Wikis easily allow),
- should not be limited in its description to an "encyclopedic" viewpoint, i.e. understandable only to those who are members of the general public,

- should be internally understandable in a standalone fashion (for instance by consistently using extended LaTeXML DLMF (eDLMF) macros to define special functions to allow for easy access to definitions and to facilitate cross-repository search),
- should not be limited in size as in a printed book,
- should have a user friendly and consistent viewpoint and authoring perspective,
- should take advantage of powerful modern MathML tools for easy to read rendered mathematics at different font sizes,
- should have the ability to link to currently existing online resources.

Note that we have summarized our mission requirements in an online document (`http://drmf.instance-proxy.wmflabs.org/wiki/DRMF_Requirements`). It is the desire of our group to build a tool which attempts to bring the above features together to be a tool for mathematicians and scientists to bridge together currently existing compendia and to publish new orthogonal polynomial and special function (OPSF) formulae, and their corresponding mathematical data, online and on the web. We will refer to this web tool as a Digital Repository of Mathematical Formulae (DRMF).

An origination of this concept can be ascertained by viewing a Society for Industrial and Applied Mathematics activity group OPSF-Net post by Dmitry Karp (2011) [2, Topic #5]. In that OPSF-Net edition, there were two related posts [2, Topic #6, Topic#7] with a follow-up post in [3, Topic #3].

2 Implementation

On the technical front, we have built on the experience of the DLMF project [11, see also `http://dlmf.nist.gov`], as well as the Planetary (`http://planetmath.org`) and MediaWiki (`http://www.mediawiki.org`) publishing platforms. Each feature of the publishing process is modeled as a separate general-purpose MediaWiki extension, and published on GitHub (`https://github.com`) for redistribution. The first demonstration provides experimental extensions for (1) formula interactivity, (2) formula home pages, (3) centralized bibliography and (4) mathematical search. At its foundation, DRMF shares the core technologies of the DLMF, based on community-recognized open standards (TeX, HTML+MathML), as stringed together by the LaTeXML (`http://dlmf.nist.gov/LaTeXML/`) converter. We incorporate some of the high-quality components used in Planetary: the JOBAD interactivity framework (`https://github.com/KWARC/jobad`), MathWebSearch search engine (`http://search.mathweb.org/`), as well as those designed for MediaWiki: the MathSearch (`http://www.mediawiki.org/wiki/Extension:MathSearch`) and Math extension (`http://www.mediawiki.org/wiki/Extension:Math`). We will investigate the use of MathJax and JOBAD for menues and formula interactivity mechanisms that we hope to incorporate.

Examples of formula interactivity include:

- a clipboard for mathematics, allowing easy copy/paste of a formula's source, presentation or content representations;
- on-demand summary of constraints and substitutions applicable to a formula;
- on-demand consulting with external web services (e.g., WolframAlpha (`https://www.wolframalpha.com/`)) and local computer algebra systems (e.g., Mathematica [7], Maple (`http://www.maplesoft.com`), Sage (`http://www.sagemath.org/`)).

The DRMF's first vision of a repository for mathematical formulae is to treat any notable expression as a primary object of interest, describing it in its own formula home page. Currently, formula home pages contain: (1) a description of the formula itself, (2) open section for proofs, (3) bibliographic citations, and (4) a glossary of special extended DLMF symbols and LaTeXML extended DLMF (eDLMF) macros used in the formula with links to definitions, and (5) a link to the formula in the DLMF, whenever applicable. Optionally, one may also enter relevant constraints, substitutions, notes, the formula name, as well as links to formula generalizations and specializations. For each formula home page there is a corresponding talk page where discussions about the formula and its page may take place. We are incorporating a strategy for handling the insertion of formula errata.

We are also exploring a variety of search strategies within our MediaWiki deployment. A key asset in our development of search capabilities is the use of eDLMF macros in writing the TeX markup of formulae. LaTeXML uniquely translates the macros for specific special functions, orthogonal polynomials and general mathematical objects into Content MathML symbols, which in turn fuel structural search engines, such as MathWebSearch, to return results with very high accuracy.

Next, we present an early overview of the shortlisted seed resources which we plan to incorporate. We have been given permission to seed the DRMF with data from the NIST DLMF [11]. We would also like to extend the DLMF list of formulae by including all relevant formulae which are cited within the DLMF. We have also been given permission to include formulae data from Koekoek, Lesky and Swarttouw's (KLS) book [8]. We also plan to incorporate Tom Koornwinder's additions to KLS which are given in [9]. We have also been given permission to incorporate seed formula data from the Bateman Manuscript Project (BMP) Higher Transcendental Functions and Tables of Integral Transforms [4, 5]. Efforts to upload BMP data are known to be extremely difficult since this effort will rely on the use of mathematical optical character recognition software to produce LaTeX source for these formulae. This effort is currently under consideration for feasibility of use. We are in active communication with other publishers to get permission for other proven sources for mathematical formulae.

Our current focus is on seeding the DRMF with DLMF data, and we have completed this for Chapter 25 in the DLMF entitled Zeta and Related Functions. Future near-term efforts will focus on seeding the rest of the DLMF data as well as the KLS data with eDLMF macros incorporated.

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