We start from the distance equations, where d_N is the distance between the landmark (x_N, y_N, z_N) and the point to localize (x, y, z):

$$
\begin{aligned}
{d_A}^2 &= (x_A - x)^2 + (y_A - y)^2 + (z_A - z)^2 \\
{d_B}^2 &= (x_B - x)^2 + (y_B - y)^2 + (z_B - z)^2 \\
{d_C}^2 &= (x_C - x)^2 + (y_C - y)^2 + (z_C - z)^2
\end{aligned}
$$

By expanding the squares,

$$
\begin{aligned}
{d_A}^2 &= \left({x_A}^2 - 2x_Ax + x^2\right) + \left({y_A}^2 - 2y_Ay + y^2\right) + \left({z_A}^2 - 2z_Az + z^2\right) \\
{d_B}^2 &= \left({x_B}^2 - 2x_Bx + x^2\right) + \left({y_B}^2 - 2y_By + y^2\right) + \left({z_B}^2 - 2z_Bz + z^2\right) \\
{d_C}^2 &= \left({x_C}^2 - 2x_Cx + x^2\right) + \left({y_C}^2 - 2y_Cy + y^2\right) + \left({z_C}^2 - 2z_Cz + z^2\right)
\end{aligned}
$$

And by grouping some terms together,

$$
\begin{aligned}
{d_A}^2 - \left({x_A}^2 + {y_A}^2 + {z_A}^2\right) &= -2\left(x_Ax + y_Ay + z_Az\right) + x^2 + y^2 + z^2 \\
{d_B}^2 - \left({x_B}^2 + {y_B}^2 + {z_B}^2\right) &= -2\left(x_Bx + y_By + z_Bz\right) + x^2 + y^2 + z^2 \\
{d_C}^2 - \left({x_C}^2 + {y_C}^2 + {z_C}^2\right) &= -2\left(x_Cx + y_Cy + z_Cz\right) + x^2 + y^2 + z^2
\end{aligned}
$$

The left term of each equation does not depend on the point (x, y, z), we can therefore simplify the equation by creating a new notation d_N :

$$
\begin{bmatrix} d'_A \\ d'_B \\ d'_C \end{bmatrix} = \begin{bmatrix} {d_A}^2 - \left({x_A}^2 + {y_A}^2 + {z_A}^2\right) \\ {d_B}^2 - \left({x_B}^2 + {y_B}^2 + {z_B}^2\right) \\ {d_C}^2 - \left({x_C}^2 + {y_C}^2 + {z_C}^2\right) \end{bmatrix}
$$

$$
\begin{aligned}
d'_A &= -2\left(x_Ax + y_Ay + z_Az\right) + x^2 + y^2 + z^2 && (1) \\
d'_B &= -2\left(x_Bx + y_By + z_Bz\right) + x^2 + y^2 + z^2 && (2) \\
d'_C &= -2\left(x_Cx + y_Cy + z_Cz\right) + x^2 + y^2 + z^2 && (3)
\end{aligned}
$$

We want to cancel $x^2 + y^2 + z^2$ from (1), (2), and (3). For that, we will do the following differences : (1) - (2), (2) - (3), (3) - (1).

$$
\begin{aligned}
d'_A - d'_B &= -2\left[\left(x_Ax + y_Ay + z_Az\right) - \left(x_Bx + y_By + z_Bz\right)\right] \\
d'_B - d'_A &= -2\left[\left(x_Bx + y_By + z_Bz\right) - \left(x_Ax + y_Ay + z_Az\right)\right] \\
d'_C - d'_C &= -2\left[\left(x_Cx + y_Cy + z_Cz\right) - \left(x_Cx + y_Cy + z_Cz\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
d'_A - d'_B &= -2\left[\left(x_A - x_B\right)x + \left(y_A - y_B\right)y + \left(z_A - z_B\right)z\right] \\
d'_B - d'_C &= -2\left[\left(x_B - x_C\right)x + \left(y_B - y_C\right)y + \left(z_B - z_C\right)z\right] \\
d'_C - d'_A &= -2\left[\left(x_C - x_A\right)x + \left(y_A - y_B\right)y + \left(z_C - z_A\right)z\right]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{d'_A - d'_B}{2} &= \left(x_A - x_B\right)x + \left(y_A - y_B\right)y + \left(z_A - z_B\right)z \\
-\frac{d'_B - d'_C}{2} &= \left(x_B - x_C\right)x + \left(y_B - y_C\right)y + \left(z_B - z_C\right)z \\
-\frac{d'_C - d'_A}{2} &= \left(x_C - x_A\right)x + \left(y_A - y_B\right)y + \left(z_C - z_A\right)z
\end{aligned}
$$

This notation can be rewritten with matrices:

$$
\begin{bmatrix} x_A - x_B & y_A - y_B & z_A - z_B \\ x_B - x_C & y_B - y_C & z_B - z_C \\ x_C - x_A & y_A - y_B & z_C - z_A \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} d'_A - d'_B \\ d'_B - d'_C \\ d'_C - d'_A \end{bmatrix}
$$

$$
\mathbf{A}x = \mathbf{B}
$$

If the rank of the first matrix **A** is at least 3, we can solve this system of equations with simple matrix solving techniques.