Deep Learning Seminar
Chapter 9-2. Convolutional Networks

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Chapter 9. Convolutional Networks

Chapter 9. Convolutional Networks

- **Part 2**

  9.6 Structured Outputs

  9.7 Data Types

  9.8 Efficient Convolution Algorithms

  9.9 Random or Unsupervised Features

  9.10 The Neuroscientific Basis for Convolutional Networks

  9.11 Convolutional Networks and the History of Deep Learning

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Efficient Convolution Algorithms

- Modern CNN compute more than one million units
- Exploiting parallel computation resources are essential
- Selecting an appropriate convolution algorithm is important
- If the kernel is separable, naive convolution is inefficient
- Naive d-dimensional convolution requires $O(\boldsymbol{W}^{\boldsymbol{d}})$ runtime
  - W is wide element's number in each dimension
- Separable convolution requires $O(\boldsymbol{W} \times \boldsymbol{d})$ runtime

Separable kernel convolution definition

- **Definition of convolution 2D**

$$y[m,n] = h[m,n] * x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h[i,j] \cdot x[m-i, n-j]$$

- $h[m,n]$ **is separable**

$$h[m,n] = h_1[m] \cdot h_2[n]$$

y : input image tensor

m : column index of y

n : row index of y

h : tensor of convolution kernels

source : http://www.songho.ca/dsp/convolution/convolution2d_separable.html

Separable kernel convolution definition

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$$y[m,n] = h[m,n] * x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h[i,j] \cdot x[m-i, n-j]$$

- $h[m,n]$ **is separable**

$$h[m,n] = h_1[m] \cdot h_2[n]$$

- **Substitute** $h[m,n]$ **into the equation**

$$\begin{aligned} y[m,n] = h[m,n] * x[m,n] &= \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h[i,j] \cdot x[m-i, n-j] \\ &= \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h_1[i] \cdot h_2[j] \cdot x[m-i, n-j] \\ &= \sum_{j=-\infty}^{\infty} h_2[j] \cdot \left[\sum_{i=-\infty}^{\infty} h_1[i] \cdot x[m-i, n-j] \right] \end{aligned}$$

source : https://www.slideshare.net/viisonartificial2012/grupo-2-convolution-separable

Separable kernel definition

- **Definition of convolution 1D**

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

- **Separable 2D convolution: twice of 1D convolution**

$$\begin{aligned} y[m,n] = \left(h_1[m] \cdot h_2[n]\right) * x[m,n] &= h_2[n] * \left(h_1[m] * x[m,n]\right) \\ &= h_1[m] * \left(h_2[n] * x[m,n]\right) \end{aligned}$$

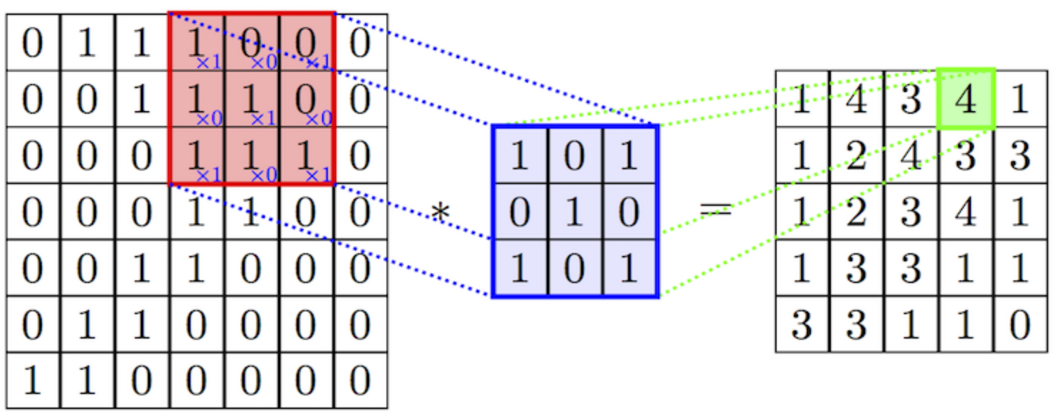
Separable kernel

- **Concept**

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} \qquad \begin{bmatrix}1 & 2 & 1\\2 & 4 & 2\\1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \cdot \begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

Input Separable Kernel

- **Convolution process image**



source : https://cambridgespark.com/content/tutorials/convolutional-neural-networks-with-keras/index.html

Computation costs

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} \qquad \begin{bmatrix}1 & 2 & 1\\2 & 4 & 2\\1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix}\cdot\begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

Input Separable Kernel

- **A: Convolution 2D**

$$\boxed{1\cdot 1}+\boxed{2\cdot 2}+\boxed{3\cdot 1}+\boxed{4\cdot 2}+\boxed{5\cdot 4}+\boxed{6\cdot 2}+\boxed{7\cdot 1}+\boxed{8\cdot 2}+\boxed{9\cdot 1}=80$$

- **B: Separable convolution 2D**

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} * \begin{bmatrix}1\\2\\1\end{bmatrix} = \begin{bmatrix}\boxed{1\cdot 1}+\boxed{4\cdot 2}+\boxed{7\cdot 1} & \boxed{2\cdot 1}+\boxed{5\cdot 2}+\boxed{8\cdot 1} & \boxed{3\cdot 1}+\boxed{6\cdot 2}+\boxed{9\cdot 1}\end{bmatrix} = \begin{bmatrix}16 & 20 & 24\end{bmatrix}$$

$$\begin{bmatrix}16 & 20 & 24\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\end{bmatrix} = \boxed{16\cdot 1}+\boxed{20\cdot 2}+\boxed{24\cdot 1}=80$$

- **Computation cost**
  - A: $O(W^d)$ vs B: $O(W \times d)$

Strategy for pixel-wise labeling of images

Convolution in 1D

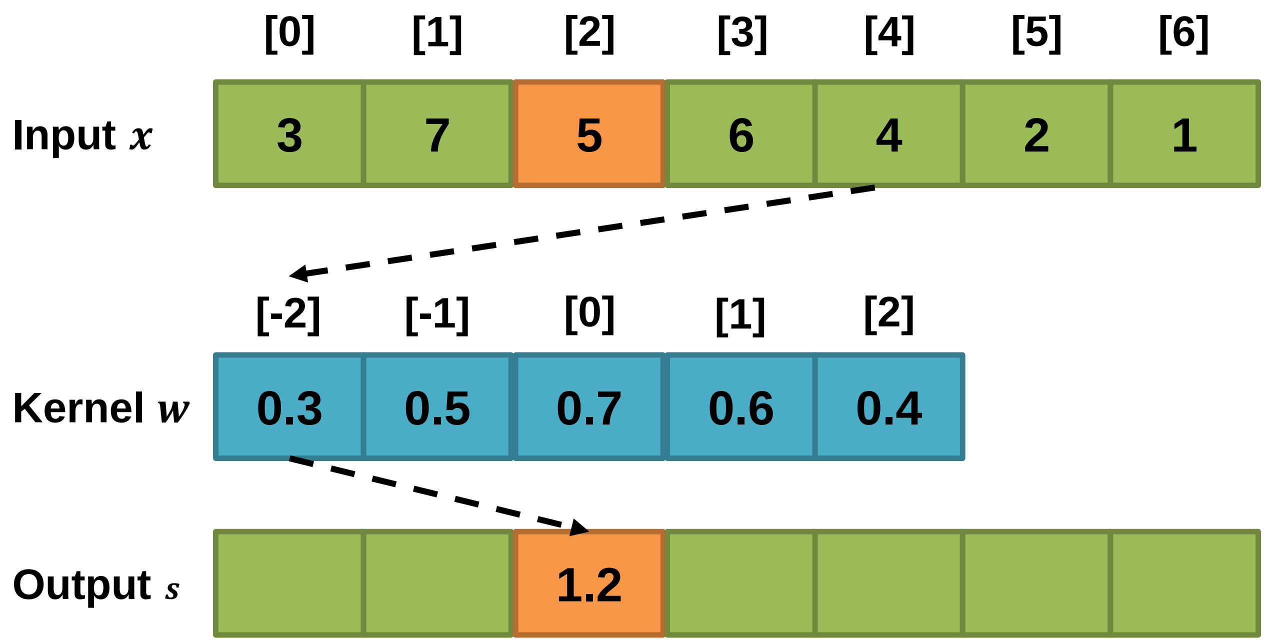
- **Definition**

$$y[k] = h[n] * x[n] = \sum_{i=-\infty} x[i]h[k-i] \quad k = 0, 1, \ldots, 6$$

$$\begin{aligned}
y[0] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[-i] &&= x[0]h[0] + 0 + 0 \\
y[1] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[1-i] &&= x[0]h[1] + x[1]h[0] + 0 \\
y[2] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[2-i] &&= x[0]h[2] + x[1]h[1] + x[2]h[0] \\
y[3] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[3-i] &&= x[0]h[3] + x[1]h[2] + x[2]h[1] \\
y[4] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[4-i] &&= x[1]h[3] + x[2]h[1] + 0 \\
y[5] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[5-i] &&= x[2]h[3] + 0 + 0
\end{aligned}$$

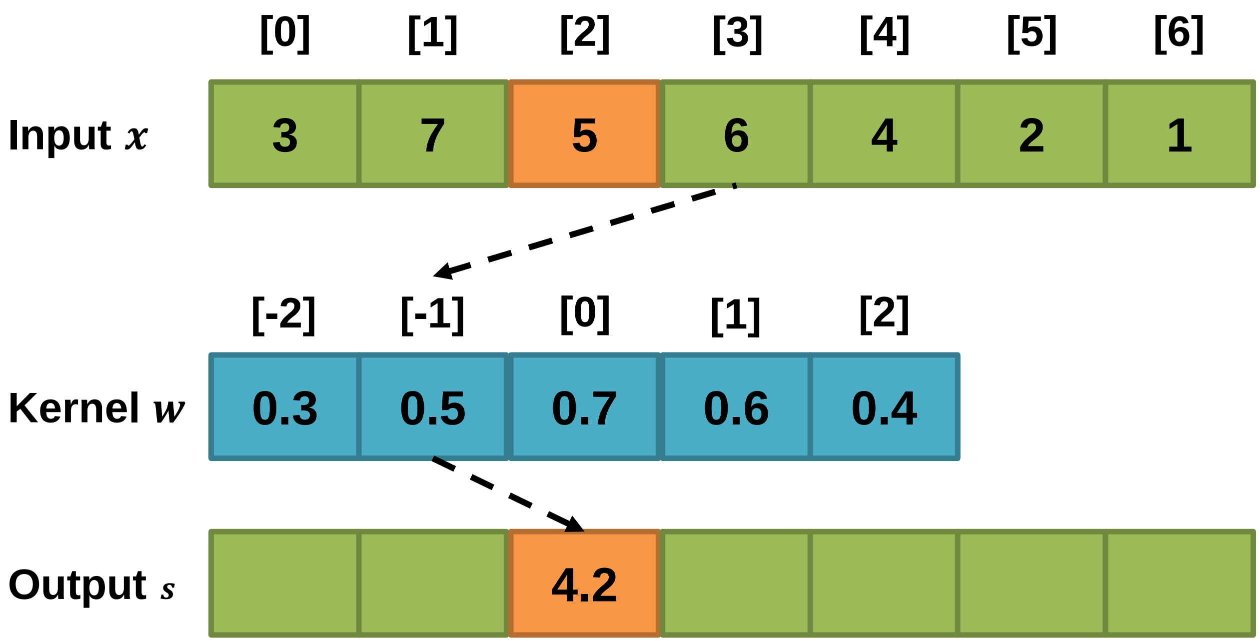
Convolution in 1D

- Discrete 1D convolution in computer



Convolution in 1D

- **Discrete 1D convolution in computer**



Convolution in 1D

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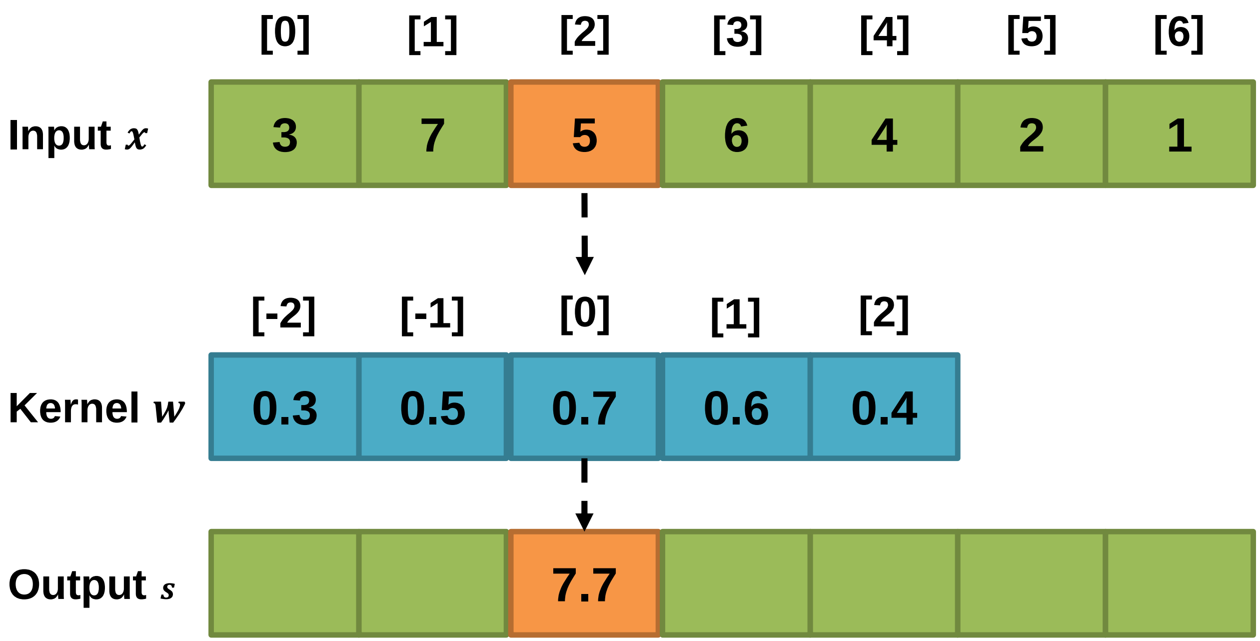
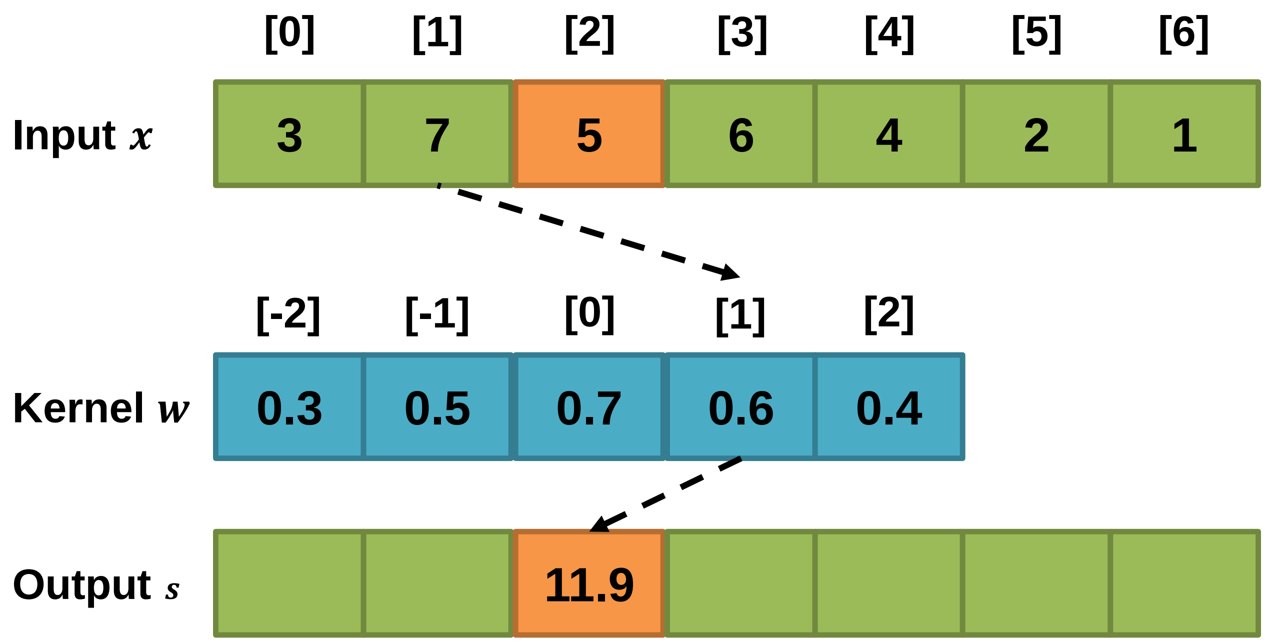


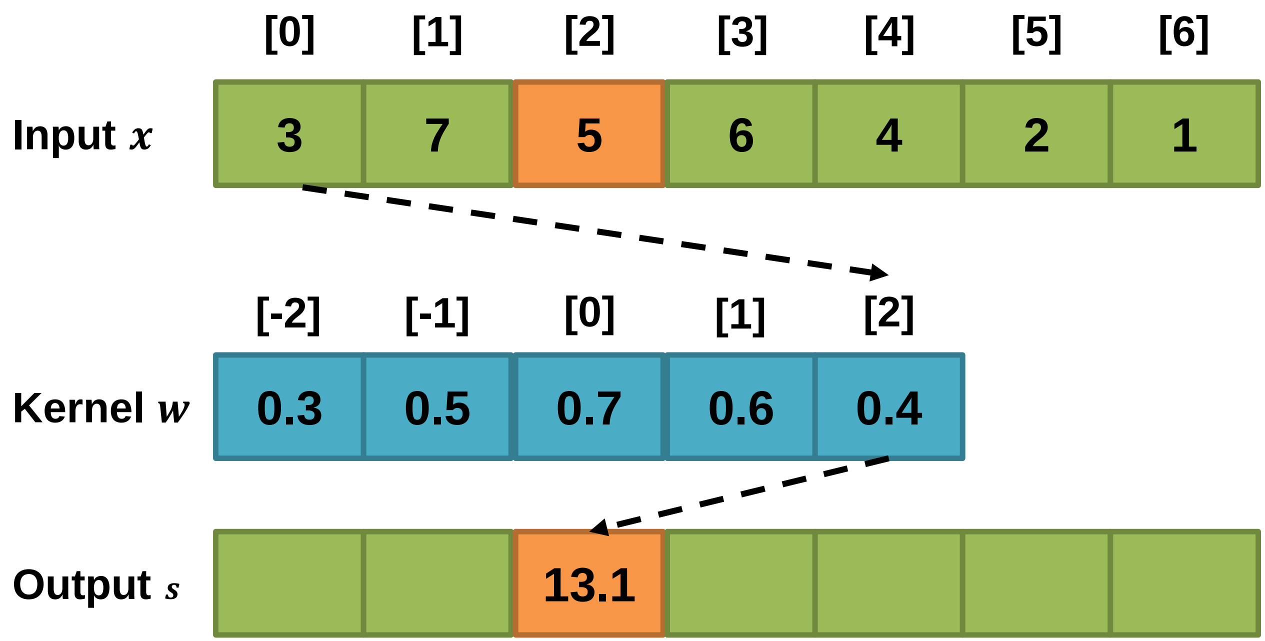
Convolution in 1D

- **Discrete 1D convolution in computer**



Convolution in 1D

- **Discrete 1D convolution in computer**



Convolution code in 2D

```
// find center position of kernel (half of kernel size)
kCenterX = kCols / 2;
kCenterY = kRows / 2;

for(i=0; i < rows; ++i)              // rows
{
    for(j=0; j < cols; ++j)          // columns
    {
        for(m=0; m < kRows; ++m)     // kernel rows
        {
            mm = kRows - 1 - m;      // row index of flipped kernel

            for(n=0; n < kCols; ++n) // kernel columns
            {
                nn = kCols - 1 - n;  // column index of flipped kernel

                // index of input signal, used for checking boundary
                ii = i + (m - kCenterY);
                jj = j + (n - kCenterX);

                // ignore input samples which are out of bound
                if( ii >= 0 && ii < rows && jj >= 0 && jj < cols )
                    out[i][j] += in[ii][jj] * kernel[mm][nn];
            }
        }
    }
}
```

Computation costs

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Input

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Separable Kernel

- **A: Process using convolution 2D**

$$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 + 4 \cdot 2 + 5 \cdot 4 + 6 \cdot 2 + 7 \cdot 1 + 8 \cdot 2 + 9 \cdot 1 = 80$$

- **Computation cost ($W = 3, d = 2$)**
  - $O(\boldsymbol{W^d})$
  - $\mathbf{9 \times 9 = 81,\ 81 \div 9 = 9}$

Computation costs

y : input image tensor

m : column index of y (0 ~ 2)

n : row index of y (0 ~ 2)

h : tensor of convolution kernels

I, j : (-1 ~ 1)

Definition of B

$$y[m,n] = \sum_{j=-\infty}^{\infty} h_2[j] \cdot \left[\sum_{i=-\infty}^{\infty} h_1[i] \cdot x[m-i, n-j] \right]$$

B: Process using separable convolution 2D

m: 0~2, n: 1

i: -1~1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot1+4\cdot2+7\cdot1 & 2\cdot1+5\cdot2+8\cdot1 & 3\cdot1+6\cdot2+9\cdot1 \end{bmatrix} = \boxed{\begin{bmatrix} 16 & 20 & 24 \end{bmatrix}}$$

j: -1~1

$$\begin{bmatrix} 16 & 20 & 24 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

reuse

Computation cost ($W = 3, d = 2$)

i: -1~1

i: -1~1

n: 0~2

- $3 \times 3 + 3 \times 3 = 18$ in 1 row and 3 row $18 \times 3 = 54$, $54 \div 9 = 6$

m: 0~2

j: -1~1

Computation costs

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Input — Separable Kernel

- **A: Process using convolution 2D**
- **B: Process using separable convolution 2D**
- **Computation cost ($W = 3, d = 2$)**
  - **A** $\rightarrow O(W^d) \rightarrow 9 \times 9 = 81,\ 81 \div 9 = 9$
  - **B** $\rightarrow O(W \times d) \rightarrow$
    $3 \times 3 + 3 \times 3 = 18$ **in** 1 **row and** 3 **row** $18 \times 3 = 54,\ 54 \div 9 = 6$

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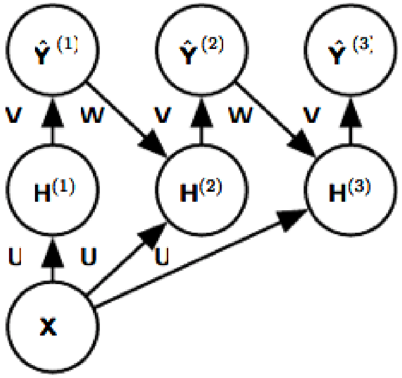
Structured outputs

- **Pixel-wise labeling of images**
- **Tensor $S_{i,j,k}$**
  - The **probability** that pixel (i, j) of the input belongs to class k
  - Allows the model to **label every pixel**

Strategy for pixel-wise labeling of images

- Produce an **initial guess** of the image labels
- **Refine** this using the interactions between neighboring pixels
- **Repeating** this refinement step several times
- **Sharing** weights between the last layers of the deep net



X : Input image tensor

Y : Probability distribution over labels for each pixel

H : Hidden representation

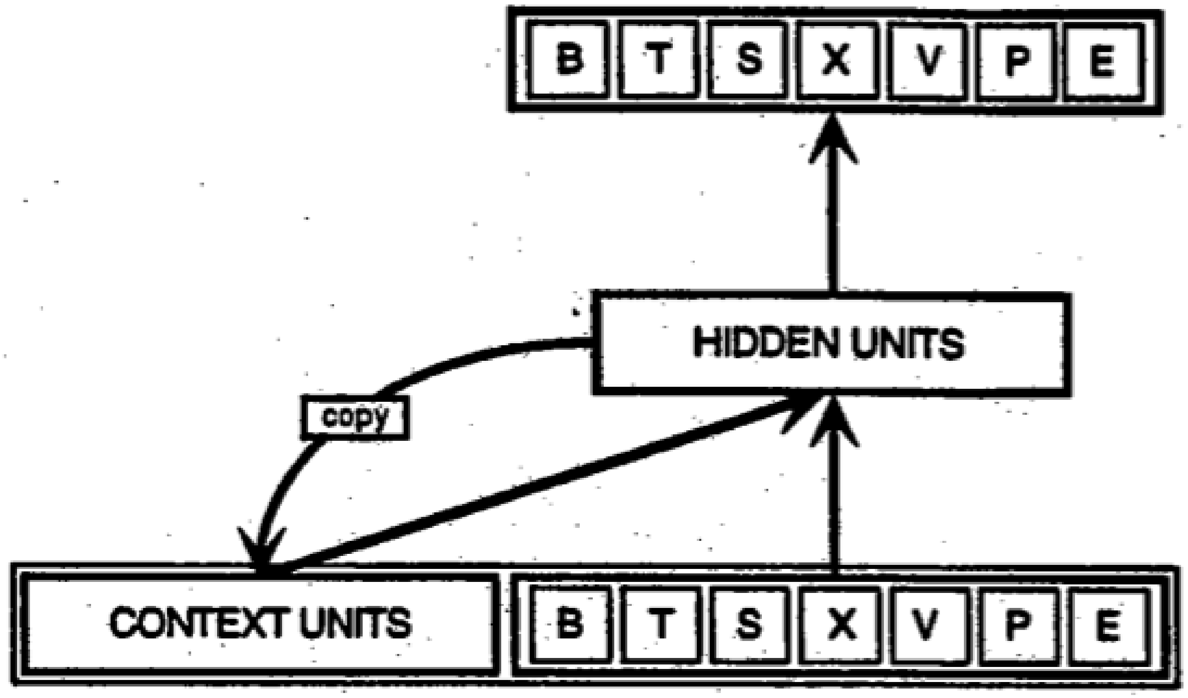
U : Tensor of convolution kernels

V : Tensor of kernels to produce an estimate of the labels

W : Kernel tensor to convolve over Y to provide input to H

Strategy for pixel-wise labeling of images

- **Basic RNN structure introduced by Elman**
- **Receiving input data and context unit in hidden layer**
- **Feedback structure that seems like memory**

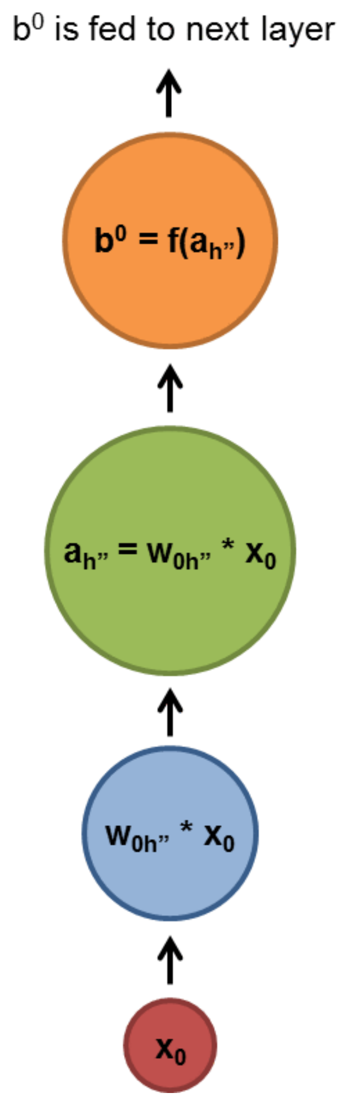


source : Elman, J. L. Finding structure in time. In *Cognitive Sciences*, 1990

Strategy for pixel-wise labeling of images

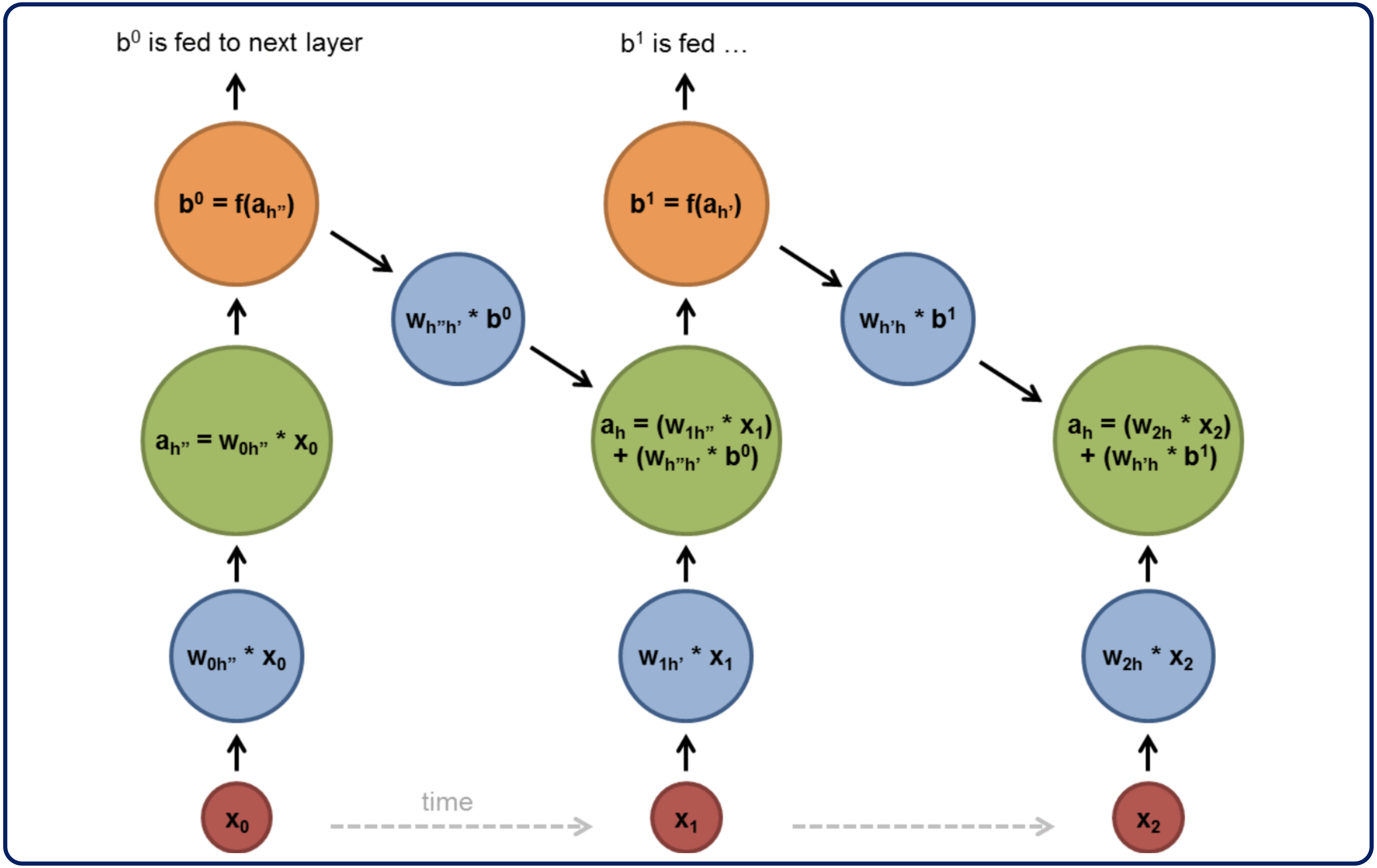
?

Strategy for pixel-wise labeling of images



source : https://imgur.com/kpZBDfV

Strategy for pixel-wise labeling of images



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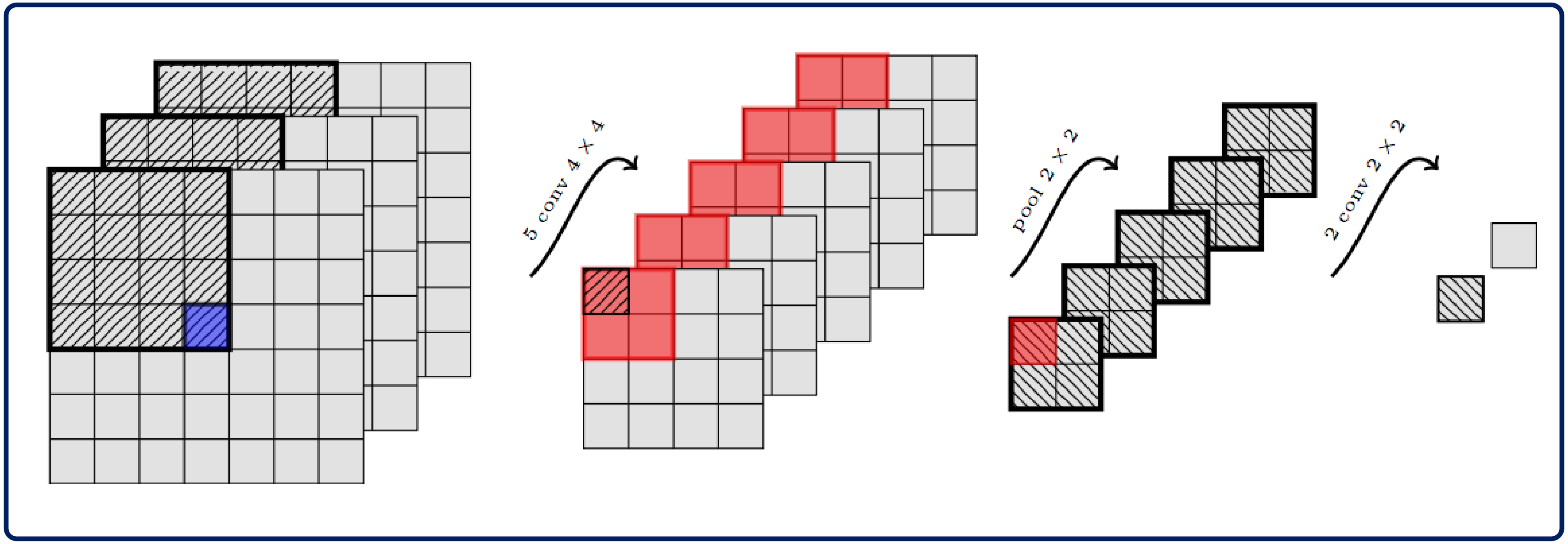
Recurrent Convolutional Neural Networks for Scene Labeling

- The goal is to assign a class label to each pixel
- To ensure a high class accuracy,
  it is essential to capture long range (pixel) label dependencies
- We propose a recurrent convolutional neural network

Recurrent Convolutional Neural Networks for Scene Labeling

- A simple convolutional network
- 4 × 4 convolutions, followed by one 2 × 2 pooling, followed by two 2 × 2 convolutions
- Each 1 × 1 output plane : A score for a given class



source : 2014 ICML--Pinheiro--Recurrent convolutional neural networks for scene labeling

Recurrent Network Approach

- $\mathbf{F}^p = [f(\mathbf{F}^{p-1}), I^p_{i,j,k}], \qquad \mathbf{F}^1 = [\mathbf{0}, I_{i,j,k}]$
- $L(f) + L(f \circ f) + \ldots + L(f \boxed{\circ^P} f)$ $\quad \boxed{(f \circ g)(x) = f(g(x))}$
- $f(I_{i,j,k}; (\mathbf{W}, \mathbf{b})) = \mathbf{W}_M \mathbf{H}_{M-1}$
- $\mathbf{H}_m = \tanh(\mathrm{pool}(\mathbf{W}_m \mathbf{H}_{m-1} + \mathbf{b}_m))$
- $$p(c|I_{i,j,k}; (\mathbf{W}, \mathbf{b})) = \frac{e^{f_c(I_{i,j,k};(\mathbf{W},\mathbf{b}))}}{\sum_{d \in \{1,\ldots,N\}} e^{f_d(I_{i,j,k};(\mathbf{W},\mathbf{b}))}}$$

I : Input image data
(i, j) : Location (i, j) of the training image k
k : Training image channel number
p : Instance number of the network (1 ≤ p ≤ P)
L : Maximum likelihood

W : weight parameter (toeplitz matrix)
b : bias vector
tanh : point-wise hyperbolic tangent function
pool : max-pooling function
m : {1,…,M} and M is number of stages

source : 2014 ICML--Pinheiro--Recurrent convolutional neural networks for scene labeling

Toeplitz matrix

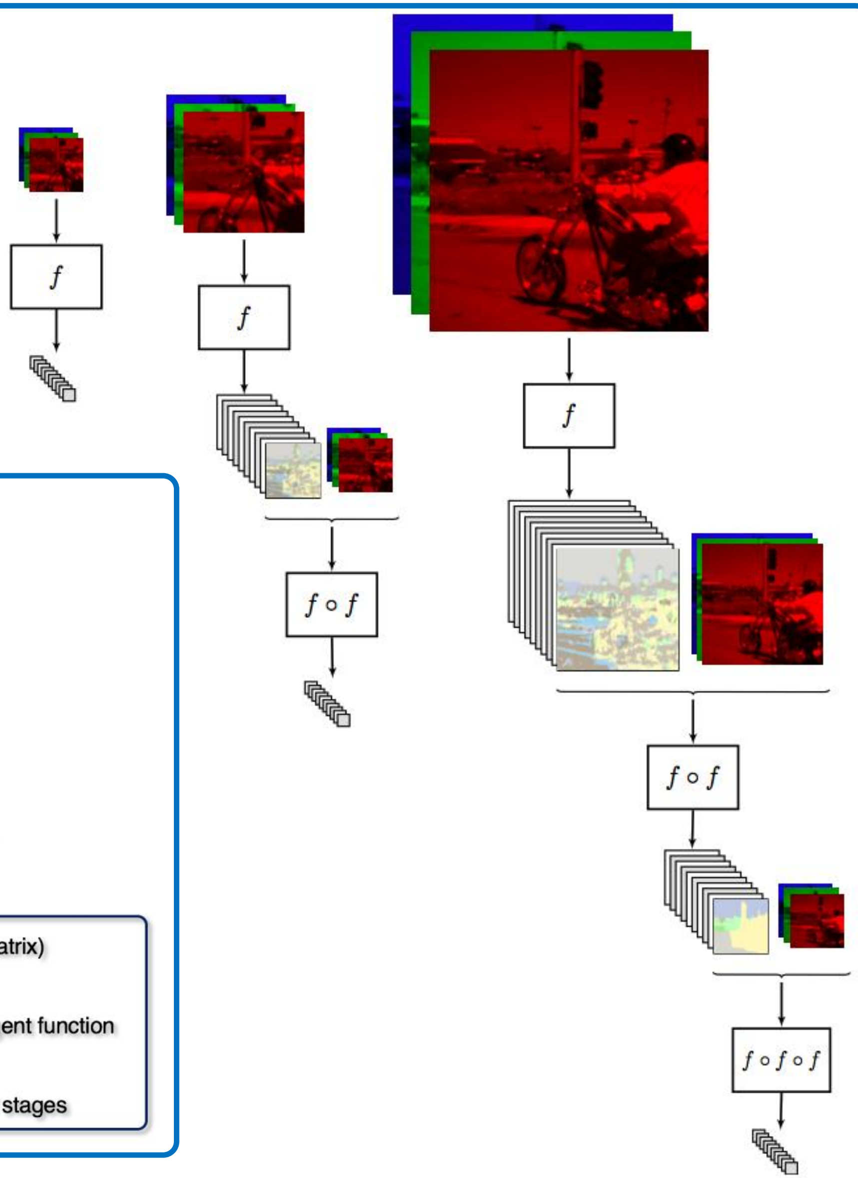
- $y[k] = h[n] * x[n] = \sum_{i=-\infty}^{} x[i]h[k-i] \quad k = 0, 1, \ldots, 6$

- $$\begin{aligned} y[0] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[-i] &&= x[0]h[0] + 0 + 0 \\ y[1] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[1-i] &&= x[0]h[1] + x[1]h[0] + 0 \\ y[2] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[2-i] &&= x[0]h[2] + x[1]h[1] + x[2]h[0] \\ y[3] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[3-i] &&= x[0]h[3] + x[1]h[2] + x[2]h[1] \\ y[4] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[4-i] &&= x[1]h[3] + x[2]h[1] + 0 \\ y[5] &= \textstyle\sum_{i=-\infty}^{\infty} x[i]h[5-i] &&= x[2]h[3] + 0 + 0 \end{aligned}$$

- $$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \end{bmatrix} = \boxed{\begin{bmatrix} h[0] & 0 & 0 \\ h[1] & h[0] & 0 \\ h[2] & h[1] & h[0] \\ h[3] & h[2] & h[1] \\ 0 & h[3] & h[2] \\ 0 & 0 & h[3] \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix}}$$

source : http://www.gaussianwaves.com/2014/02/polynomials-convolution-and-toeplitz-matrices-connecting-the-dots/
http://www.purplemath.com/modules/fcncomp4.htm

Recurrent Network Approach



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- $L(f) + L(f \circ f) + \ldots + L(f \circ^P f)$
- $f(I_{i,j,k}; (\mathbf{W}, \mathbf{b})) = \mathbf{W}_M \mathbf{H}_{M-1}$
- $\mathbf{H}_m = \tanh(\text{pool}(\mathbf{W}_m \mathbf{H}_{m-1} + \mathbf{b}_m))$
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L : Maximum likelihood

W : weight parameter(toeplitz matrix)
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tanh : point-wise hyperbolic tangent function
pool : max-pooling function
m : {1,…,M} and M is number of stages

Strategy for pixel-wise labeling of images

- Once a prediction for each pixel is made, various methods can be used to further process
- Various graphical models can describe that
- Refer to
  - 2005 IEEE, F Ning:
    Toward automatic phenotyping of developing embryos from videos
  - 2014 NIPS, Jonathan Tompson:
    Joint training of a convolutional network and a graphical model for human pose estimation

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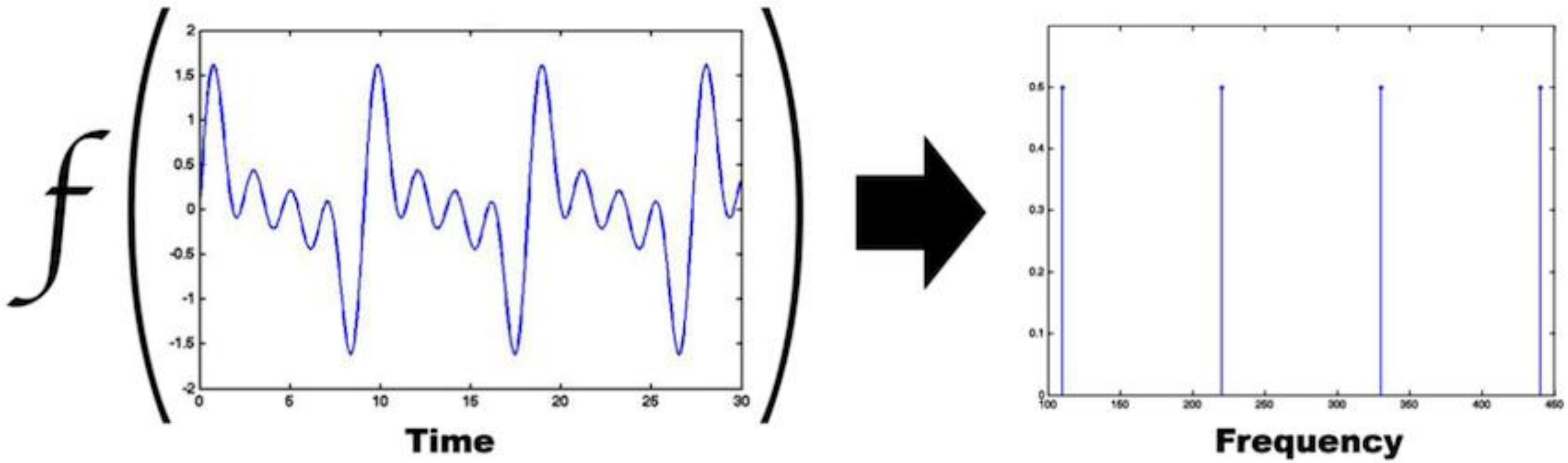
Data types

- The output of the network is allowed to have **variable size** as well as the input
- The data consists of several channels
  - Dimension: 1, 2, 3, ...
  - Channel: 1, 2, 3, …

Dimension	Single channel	Multi channel
1 D	Audio waveform	Skeleton animation data
2 D	Audio data that has been preprocessed with a Fourier transform	Color image data
3 D	Volumetric data	Color video data

Single channel

- **Fourier transform**



source : http://devonbryant.github.io/blog/2013/03/02/fourier-transforms/

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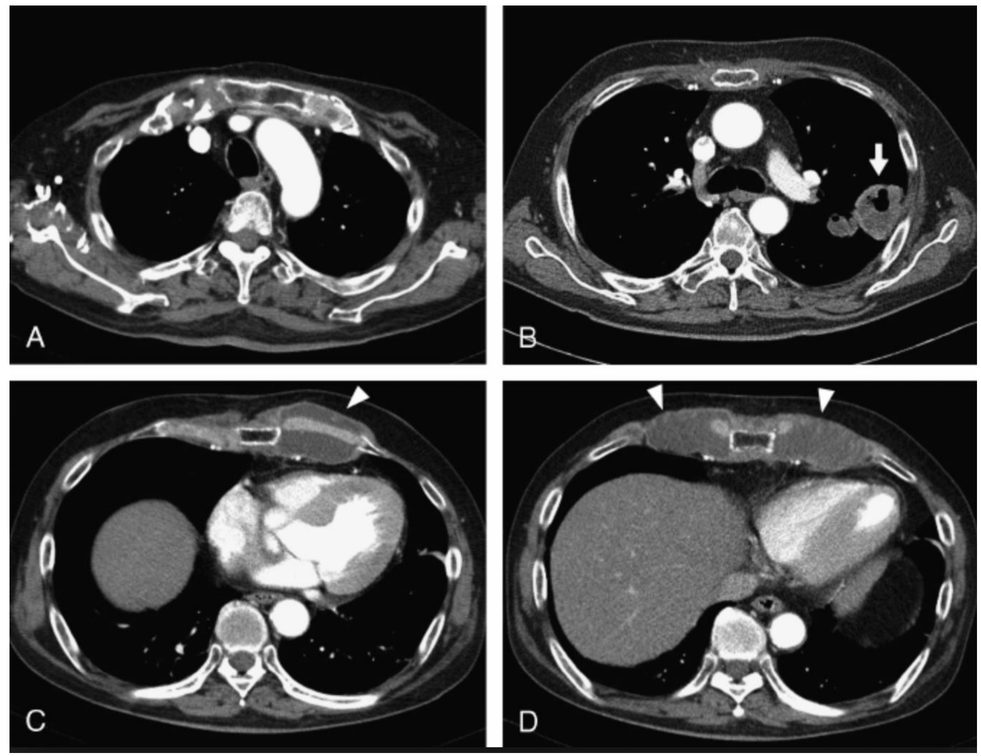
3 D	$\boxed{\text{Volumetric data}}$	Color video data

Single channel

- **Volumetric data**



source : google image

Data types

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  - Dimension : 1, 2, 3, ...
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Dimension	Single channel	Multi channel
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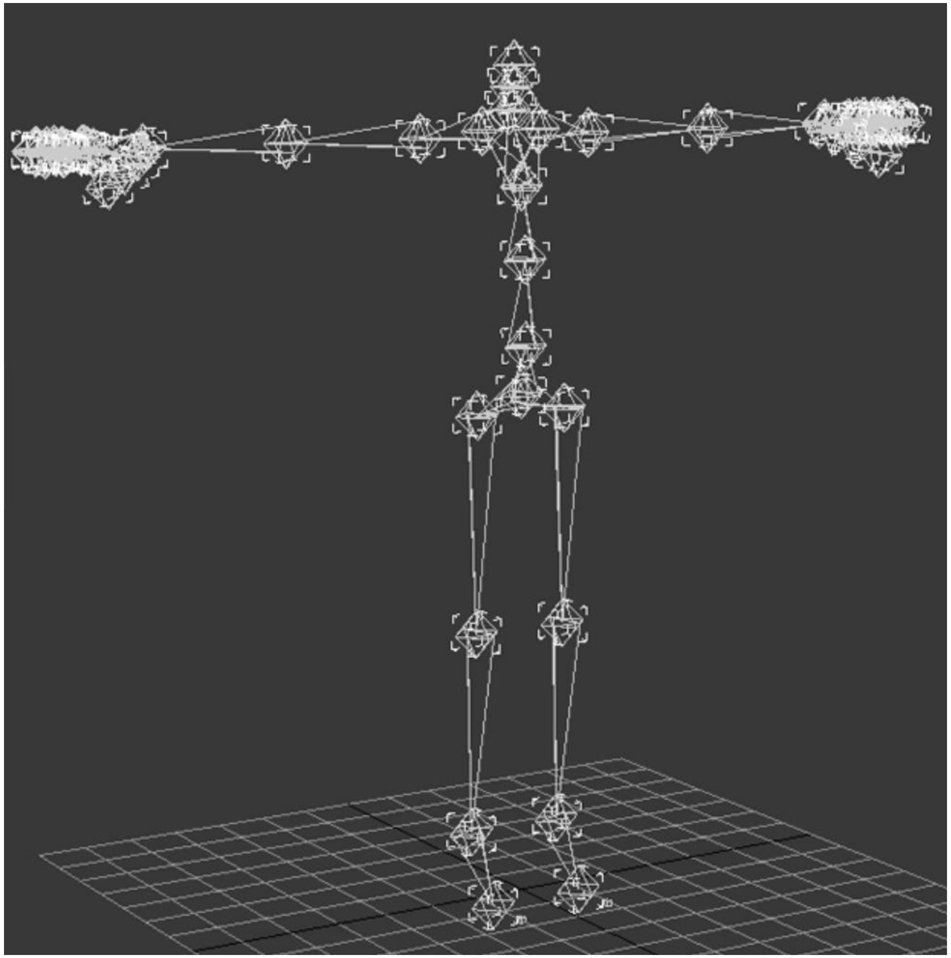
2 D	Audio data that has been preprocessed with a Fourier transform	Color image data
3 D	Volumetric data	Color video data

Multi-channel

- **Skeleton animation data**



source : google image

Data types

- **The output of the network is allowed to have variable size as well as the input**
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1 D	Audio waveform	Skeleton animation data
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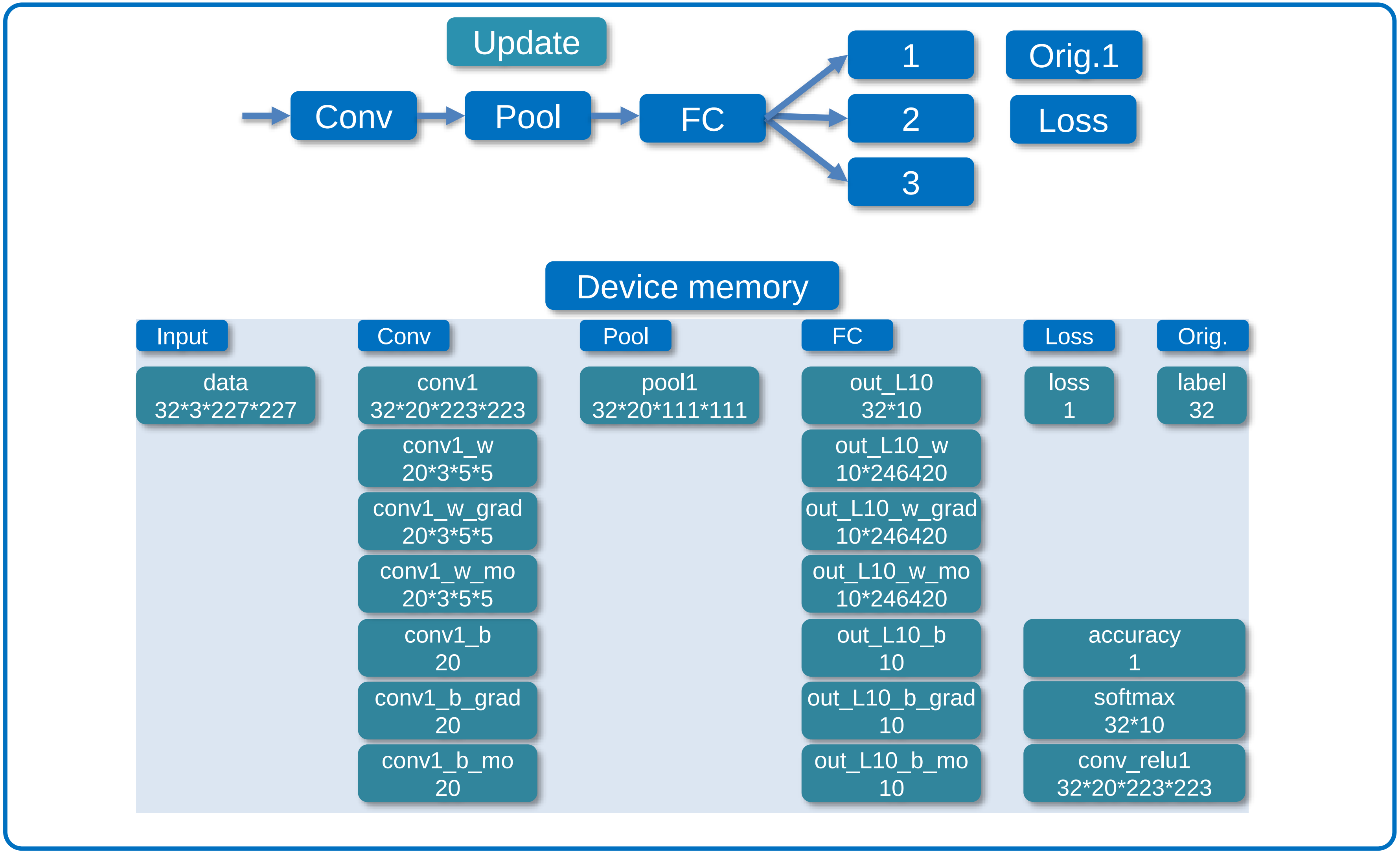
3 D	Volumetric data	Color video data

Data types

- CNN can also process inputs with varying spatial extents
- Variable size of input
  - To assign a class label to each pixel of the input
- Fixed-size of input
  - To assign a single class label to the entire image

Dimension	Single channel	Multi channel
1 D	Audio waveform	Skeleton animation data
2 D	Audio data that has been preprocessed with a Fourier transform	Color image data
3 D	Volumetric data	Color video data

Produce some fixed-size output



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Random or Unsupervised Features

- This approach was popular from roughly 2007–2013
- Today, CNN is trained in a purely supervised fashion
- Initializing kernel
  - Simply initialize them randomly.
  - Design them by hand
  - Learn the kernels with an unsupervised criterion
- Refer to
  - 2011 AISTATS, Coates Adam:
    An analysis of single-layer networks in unsupervised feature learning
  - 2009 ICML, Honglak Lee Andrew y ng:
    Convolutional deep belief networks for scalable unsupervised learning of hierarchical representations
  - 2009 ICCV, Kevin Jarrett:
    What is the best multi-stage architecture for object recognition

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The Neuroscientific Basis for Convolutional Networks

- Neural networks that were drawn from neuroscience
- Neurophysiologists David Hubel and Torsten Wiesel
- Greatest influence on deep learning models
- Brain function that are beyond the scope of this book

TDNN and CNN

- **Lang and Hinton (1988)**
  - Time delay neural networks (TDNNs)
- **LeCun (1989)**
  - Developing the modern convolutional network

The Neuroscientific Basis for Convolutional Networks

- On a simplified, cartoon view of brain function
- A part of the brain called V1 (primary visual cortex)
- V1: First area of the brain processing of visual input
- Process
  - Light -> retina -> neuron in retina -> optic nerve -> lateral geniculate nucleus -> V1 at the back of the head

Three properties of V1

- 2-D spatial map
- Many simple cells
- Many complex cells
  - This inspires the pooling units of convolutional networks

Grandmother cells in medial temporal lobe

- Cells that respond to some specific concept
- Regardless of
  - Whether she appears in the left or right side of the image
  - Whether the image is a close-up or zoomed out shot of her entire body
  - Whether she is brightly lit, or in shadow, etc.
- An individual neuron that is activated by certain human
- More general than modern convolutional networks

The inferotemporal cortex

- When viewing an object, information flows
  - Retina -> LGN -> V1 -> V2 -> V4 -> IT
- This happens within the first 100ms of glimpsing an object
- If we interrupt the person's gaze only the firing rates, then IT proves to be similar to a CNN

Differences between CNN and the mammalian vision system

- **The human eye**
  - Very low resolution as input
  - Integrating many other senses
  - Understanding entire scenes including many objects and relationships between objects
  - Processing rich 3-D geometric information

- **CNN**
  - Large full resolution photographs as input
  - Purely visual

Gabor function

- **Response of a simple cell formula**

$$s(I) = \sum_{x \in \mathbb{X}} \sum_{y \in \mathbb{Y}} w(x,y) I(x,y)$$

- **Definition of gabor function about $w(x,y)$ in book**

$$w(x,y;\alpha,\beta_x,\beta_y,f,\phi,x_0,y_0,\tau) = \alpha \exp\left(-\beta_x x'^2 - \beta_y y'^2\right) \boxed{\cos(fx' + \phi)}$$

$$x' = (x - x_0)\cos(\tau) + (y - y_0)\sin(\tau)$$

$$y' = -(x - x_0)\sin(\tau) + (y - y_0)\cos(\tau)$$

Gabor function

- **Gerneral definition**

$$g(x,y;\lambda,\theta,\psi,\sigma,\gamma) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\boxed{\exp\left(i\left(2\pi\frac{x'}{\lambda}+\psi\right)\right)}$$

- **Euler's formula**

$$e^{ix} = \cos x + i \sin x$$

- **Gabor function in real space**

$$g(x,y;\lambda,\theta,\psi,\sigma,\gamma) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\boxed{\cos\left(2\pi\frac{x'}{\lambda}+\psi\right)}$$

source : https://en.wikipedia.org/wiki/Gabor_filter

Gabor function

- **Definition of gabor function about $w(x, y)$ in book**

$$w(x, y; \alpha, \beta_x, \beta_y, f, \phi, x_0, y_0, \tau) = \alpha \boxed{\exp\left(-\beta_x x'^2 - \beta_y y'^2\right)} \cos(f x' + \phi)$$

$$x' = (x - x_0)\cos(\tau) + (y - y_0)\sin(\tau)$$

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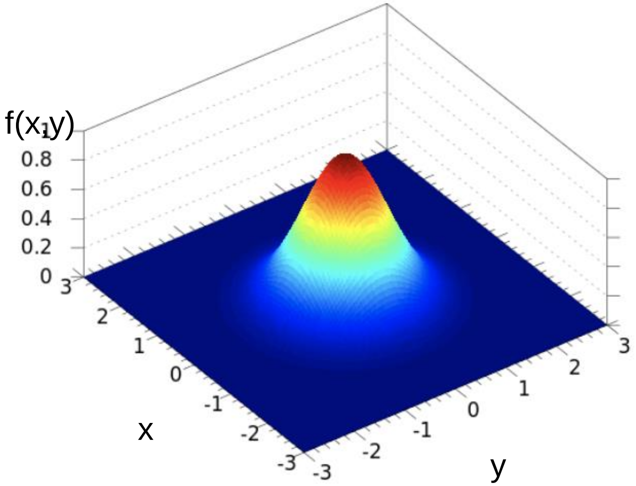
- **Gaussian function**
  - **1-demension**

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

  - **2-demension**

$$f(x, y) = A\exp\left(-\left(\frac{(x - x_o)^2}{2\sigma_x^2} + \frac{(y - y_o)^2}{2\sigma_y^2}\right)\right)$$



source : https://en.wikipedia.org/wiki/Gaussian_function

Gabor function

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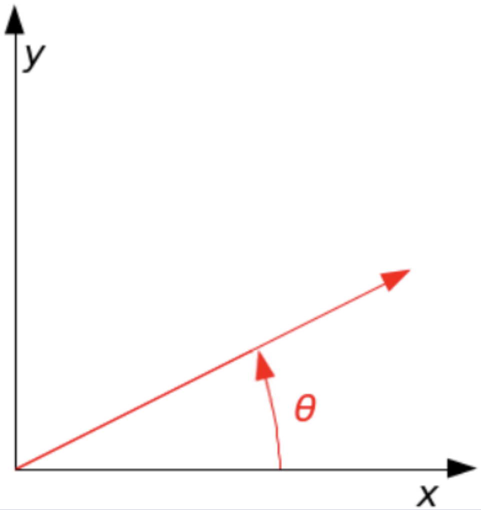
$$\boxed{\begin{aligned} x' &= (x - x_0)\cos(\tau) + (y - y_0)\sin(\tau) \\ y' &= -(x - x_0)\sin(\tau) + (y - y_0)\cos(\tau) \end{aligned}}$$

- **Rotation maxrix**

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



source : https://en.wikipedia.org/wiki/Rotation_matrix

Gabor function

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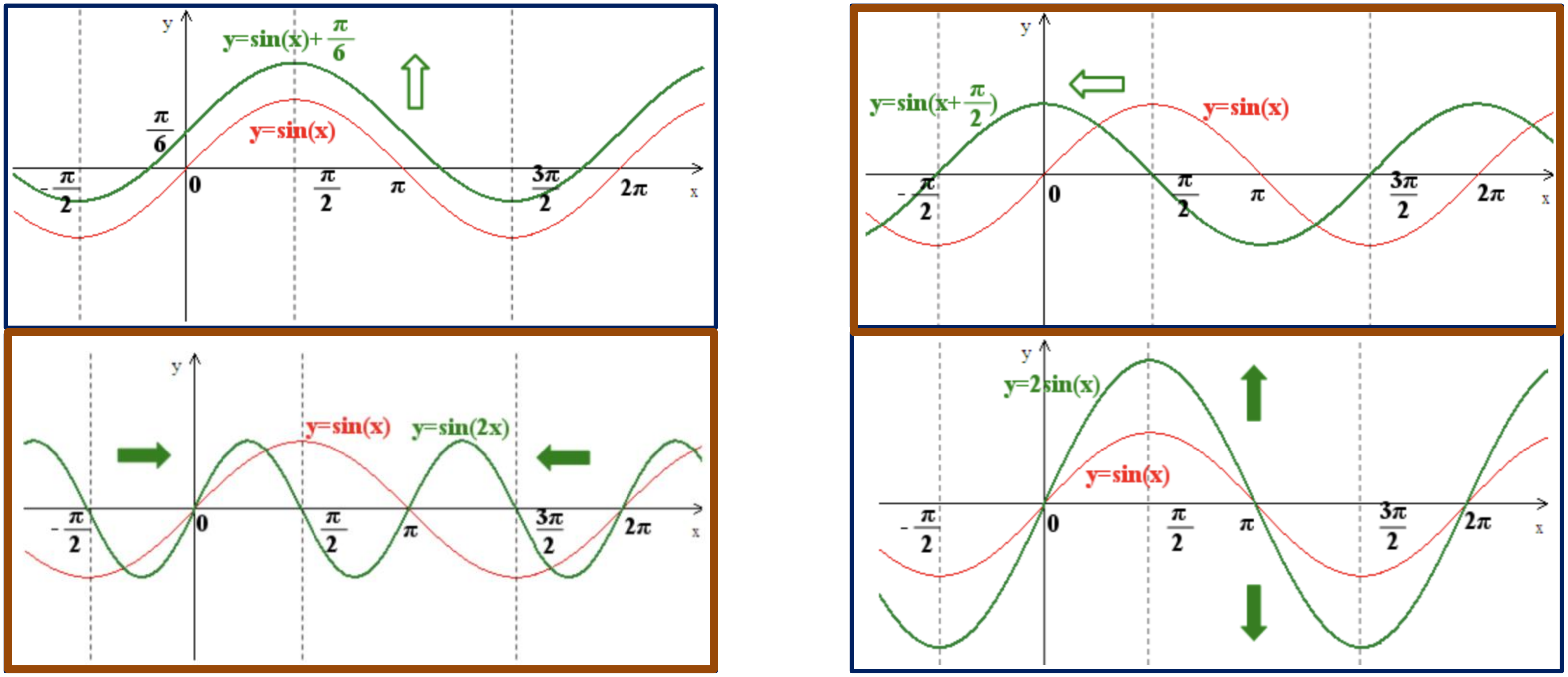
- **Trigonometrical function**



source : http://mathbang.net/529

Gabor function

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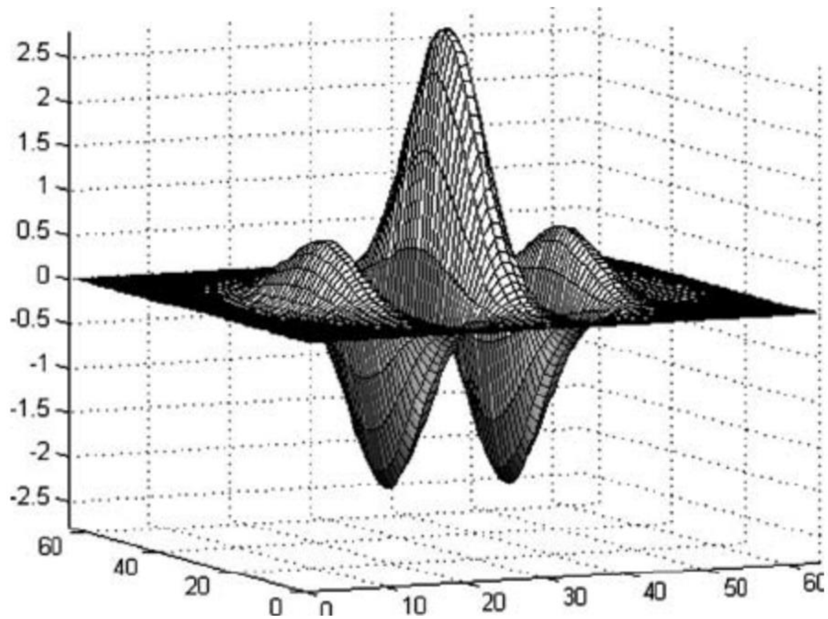

source : https://www.researchgate.net/figure/250147548_fig1_Fig-1-Perspective-view-of-real-Gabor-function-in-spatial-domain

Thank you