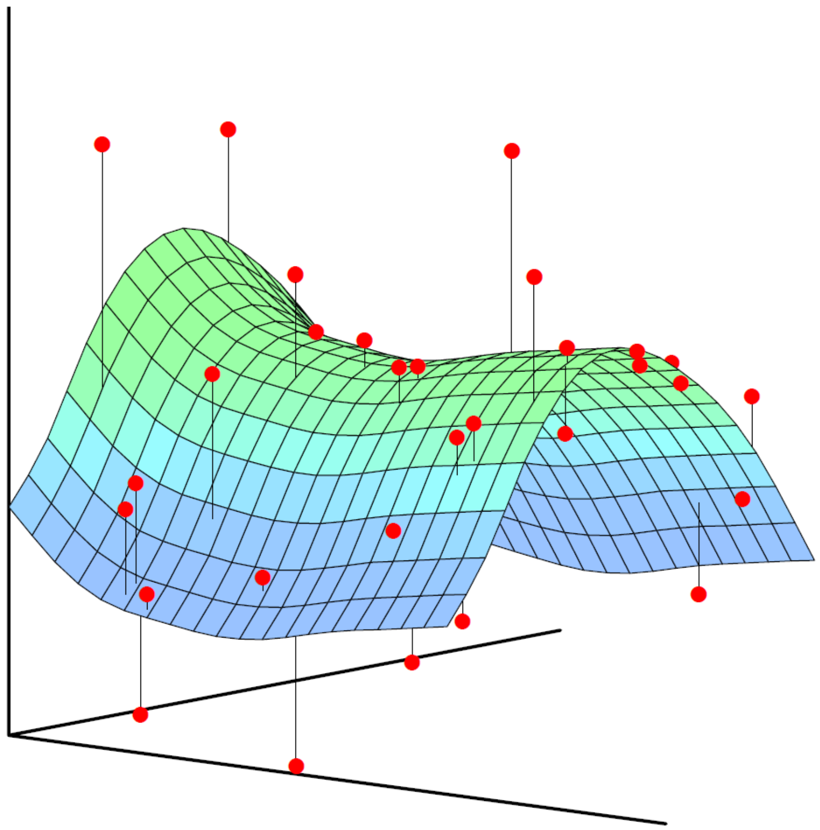
Machine Learning



第11讲 深度学习
Deep Learning

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11.1 Perceptron

感知机

- 美国学者Rosenblatt在1957年首次提出感知机概念
  ※ 感知机学习算法是Rosenblatt在1958年提出的
- IEEE设立IEEE Frank Rosenblatt Award[1]
- 感知机模型
  ※ 包含一个突触权值可调的神经元
  ※ 属于前向神经网络类型
  ※ 只能区分线性可分的模式，属于线性分类模型

1. https://www.ieee.org/about/awards/tfas/rosenblatt.html

单层感知机

- 单层感知机：Perceptron Learning Algorithm (PLA)

  ※ 是具有单层处理单元的神经网络

  ※ 模拟神经元接受环境信息，通过神经冲动（激活函数）进行信息传递

  ※ 用于求解输入空间的二分类问题：求解分类超平面

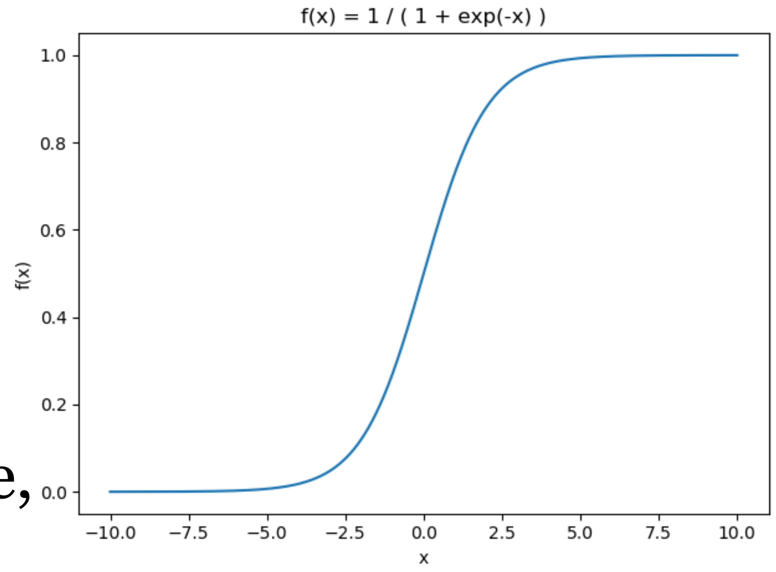
- 设输入向量 $\mathbf{x} \in R^n$，类别标记 $y \in \{-1, +1\}$，则感知机模型表示为：

$$f(\mathbf{x}) = sign(\mathbf{w} \cdot \mathbf{x} + b)$$

- 思考：$\mathbf{w} \cdot \mathbf{x} + b = 0$ 的几何意义是什么?
- 思考：$\mathbf{w}$ 的几何意义是什么?
- 思考：感知机与逻辑斯蒂回归的联系与区别在哪里?

Logistic Regression

- Let's write $p(X) = \Pr(Y = 1|X)$ for short and consider using balance to predict default. Logistic regression uses the form



$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$$

(e ≈ 2.71828 is a mathematical constant [Euler's number.])

- It is easy to see that no matter what values β_0, β_1 or X take,

※ $p(X)$ will have values between 0 and 1.

- A bit of rearrangement gives

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

- This monotone transformation is called the *log odds* or *logit* transformation of $p(X)$

单层感知机的求解

- 思考：若发生误分类的情况，误分类点到超平面的距离是?

$$d = -\frac{1}{\|\mathbf{w}\|_2} y^i(\mathbf{w} \cdot \mathbf{x}^i + b)$$

- 思考：若发生误分类的情况，误分类点到超平面的距离总和是?

$$\mathcal{L}(\mathbf{w}, b) = -\sum_{\mathbf{x}^i \in D'} y^i(\mathbf{w} \cdot \mathbf{x}^i + b)$$

- 其中: D' 表示模型当前误分类的点的集合
- 思考：如何最小化该损失函数?

$$\nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}, b) = -\sum_{\mathbf{x}^i \in D'} y^i \mathbf{x}^i \qquad \mathbf{w} = \mathbf{w} + \eta \sum_{\mathbf{x}^i \in D'} y^i \mathbf{x}^i$$

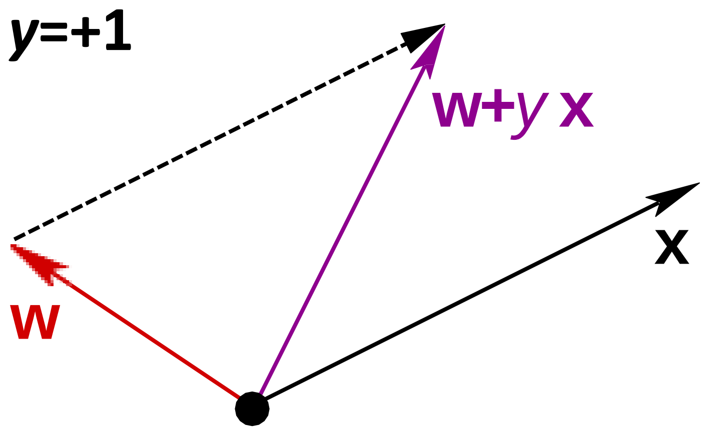
$$\nabla_b \mathcal{L}(\mathbf{w}, b) = -\sum_{\mathbf{x}^i \in D'} y^i \qquad b = b + \eta \sum_{\mathbf{x}^i \in D'} y^i$$

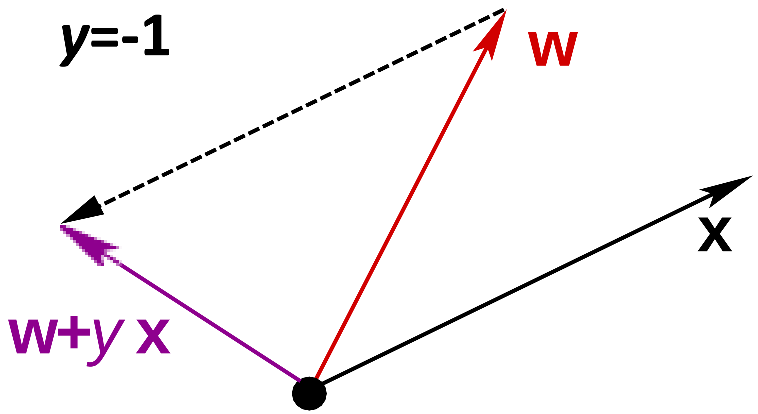
Batch Gradient Descent

单层感知机的求解

- 随机梯度下降（Stochastic Gradient Descent）

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \eta y^i \mathbf{x}^i \qquad b_{t+1} = b_t + \eta y^i$$





Practical Implementation of PLA

- Start from some w_o (say, 0), and 'correct' its mistakes on D
- For t = 0, 1, . . .

- find the next mistake of w_t called (x_t, y_t)

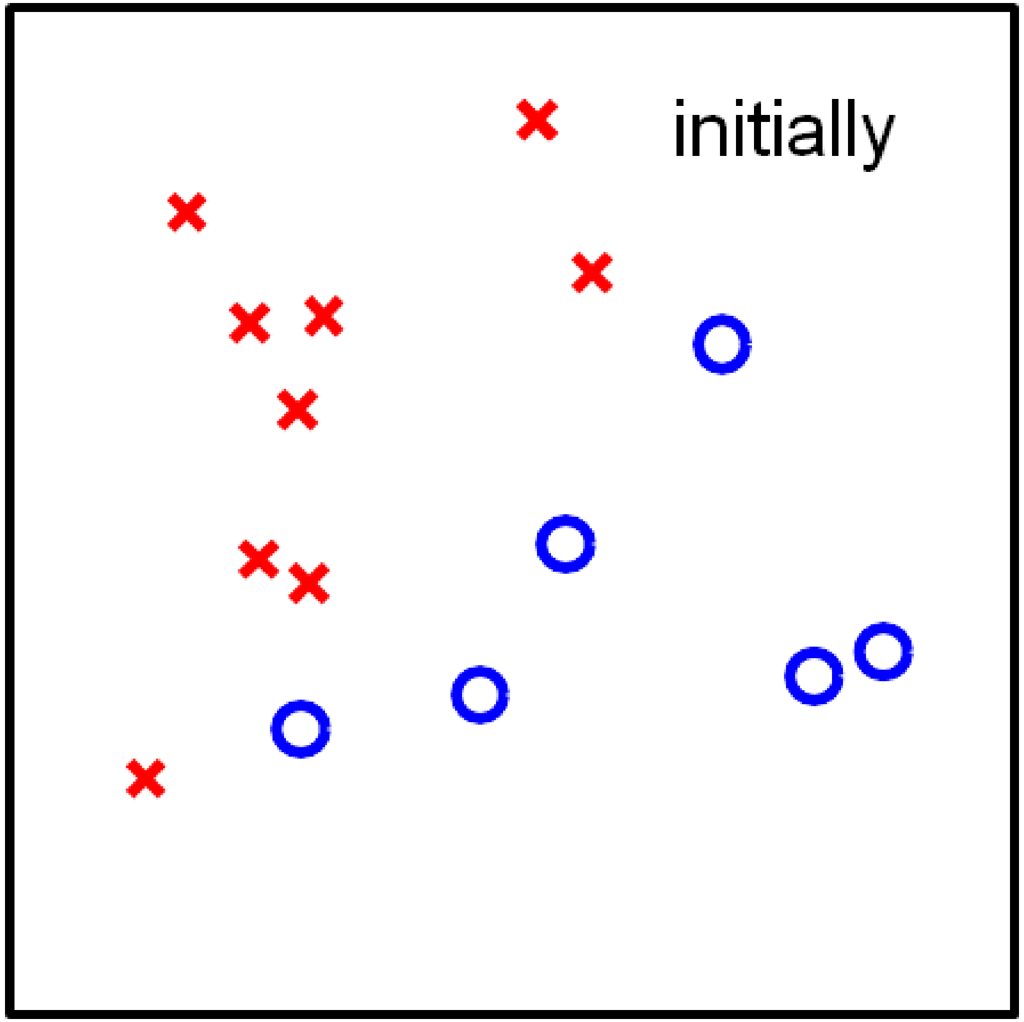
$$sign(\mathbf{w}_t \cdot \mathbf{x}_t + b) \neq y_t$$

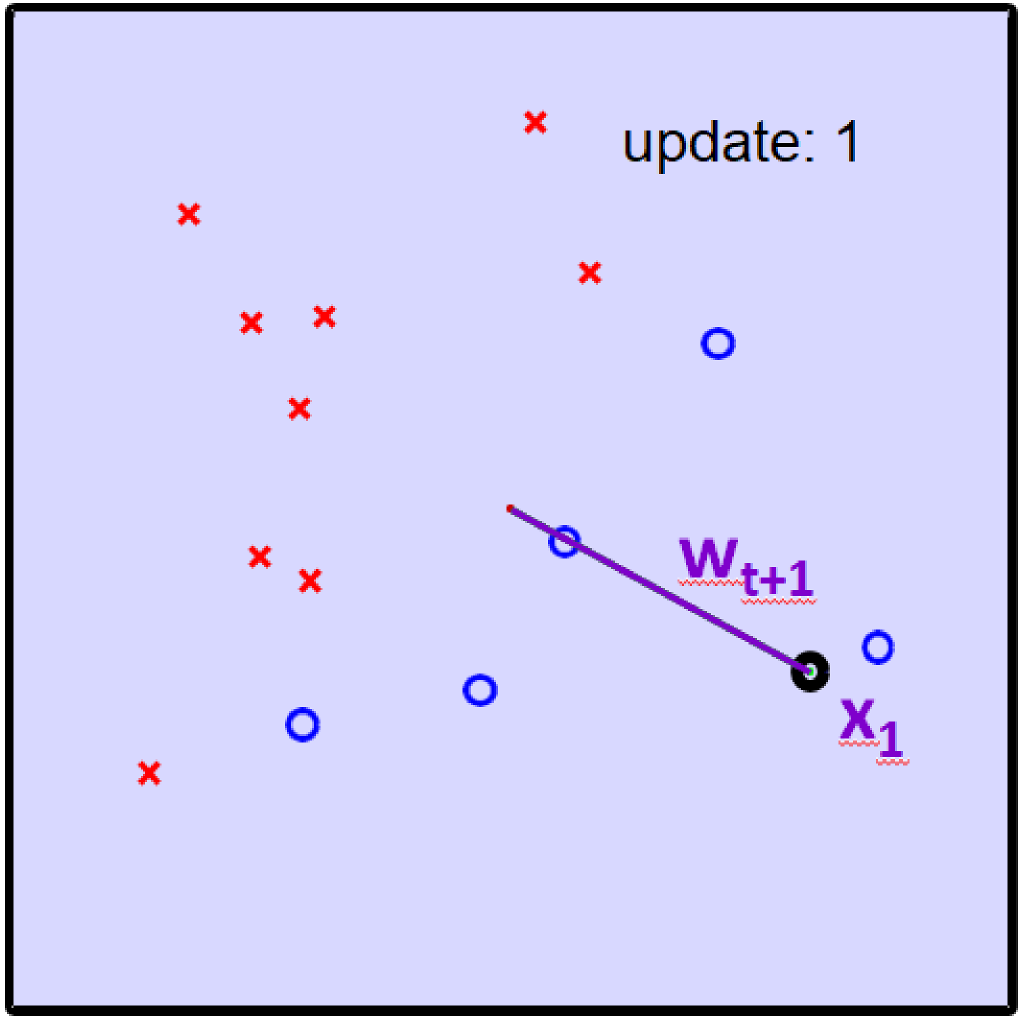
- correct the mistake by

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + y_t \mathbf{x}_t$$

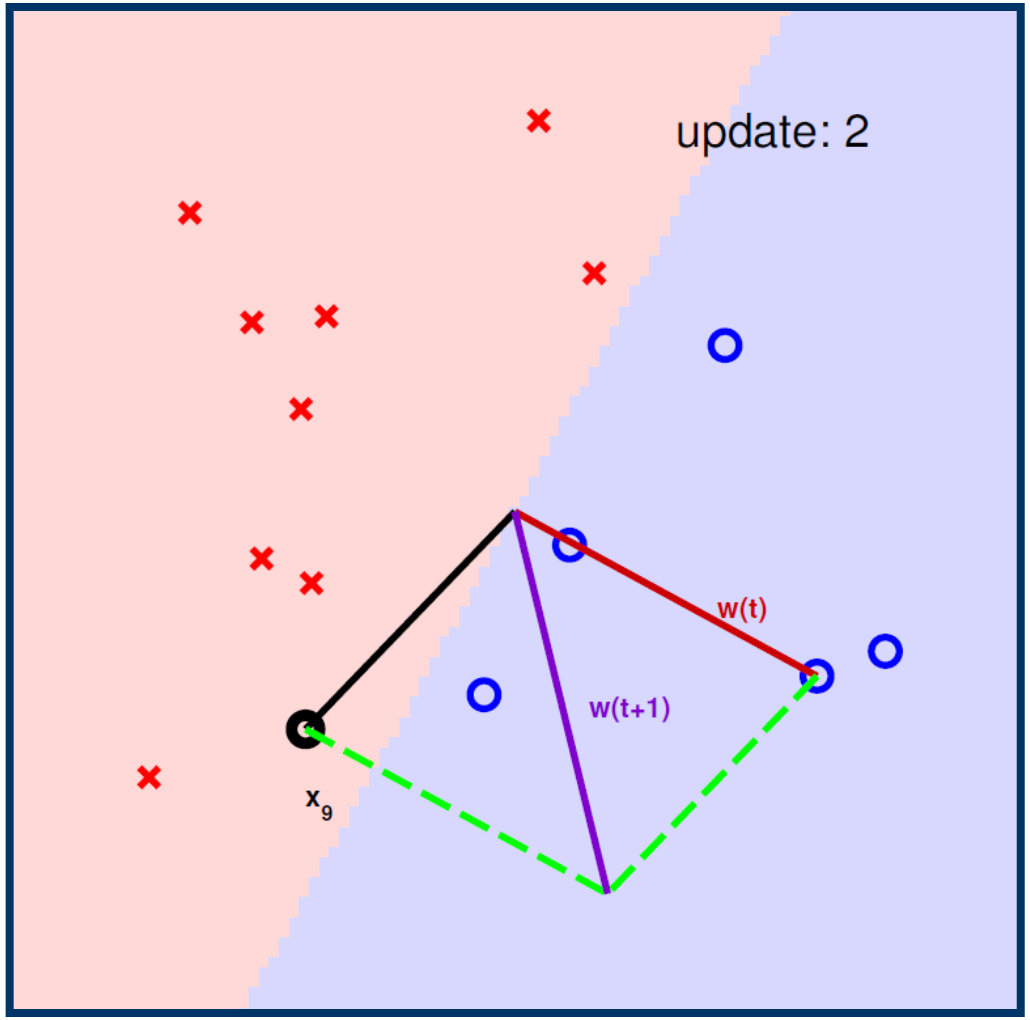
- … until a full cycle of not encountering mistakes

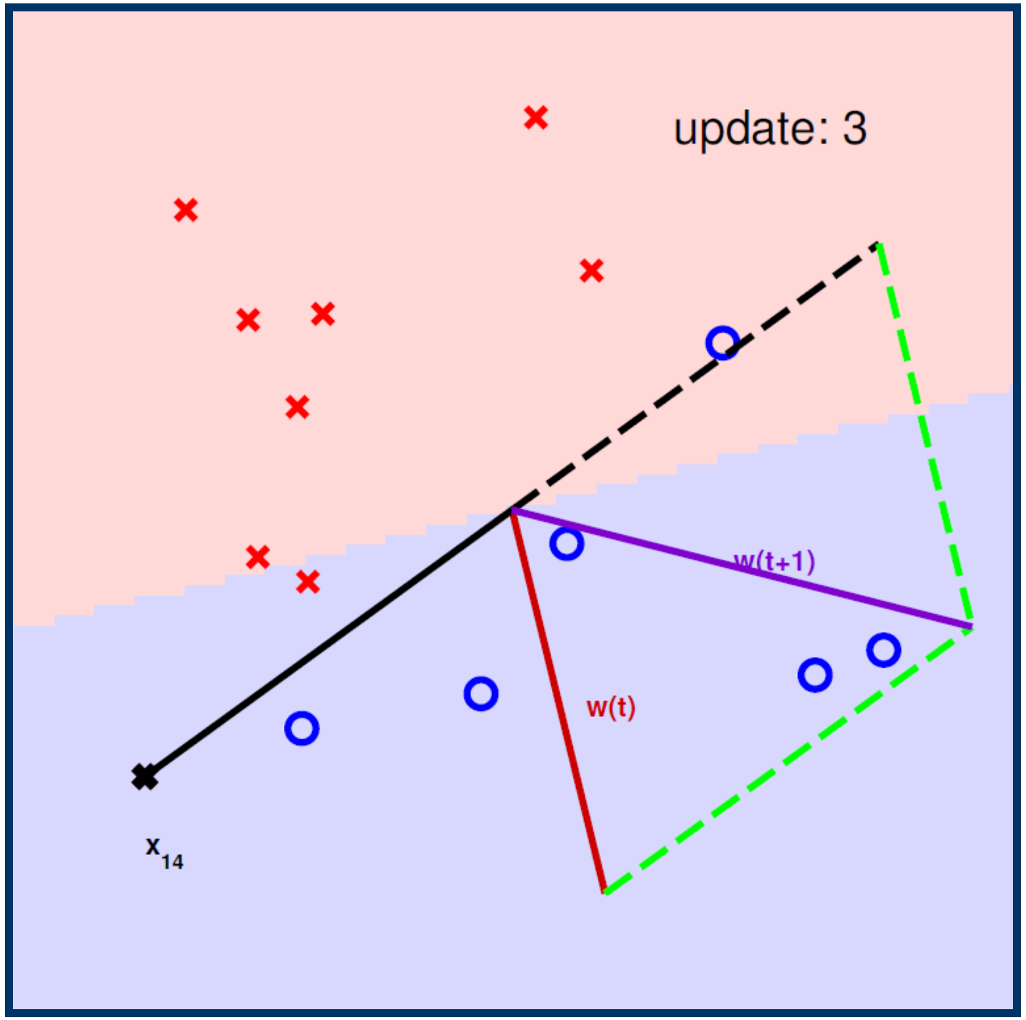
Seeing is Believing



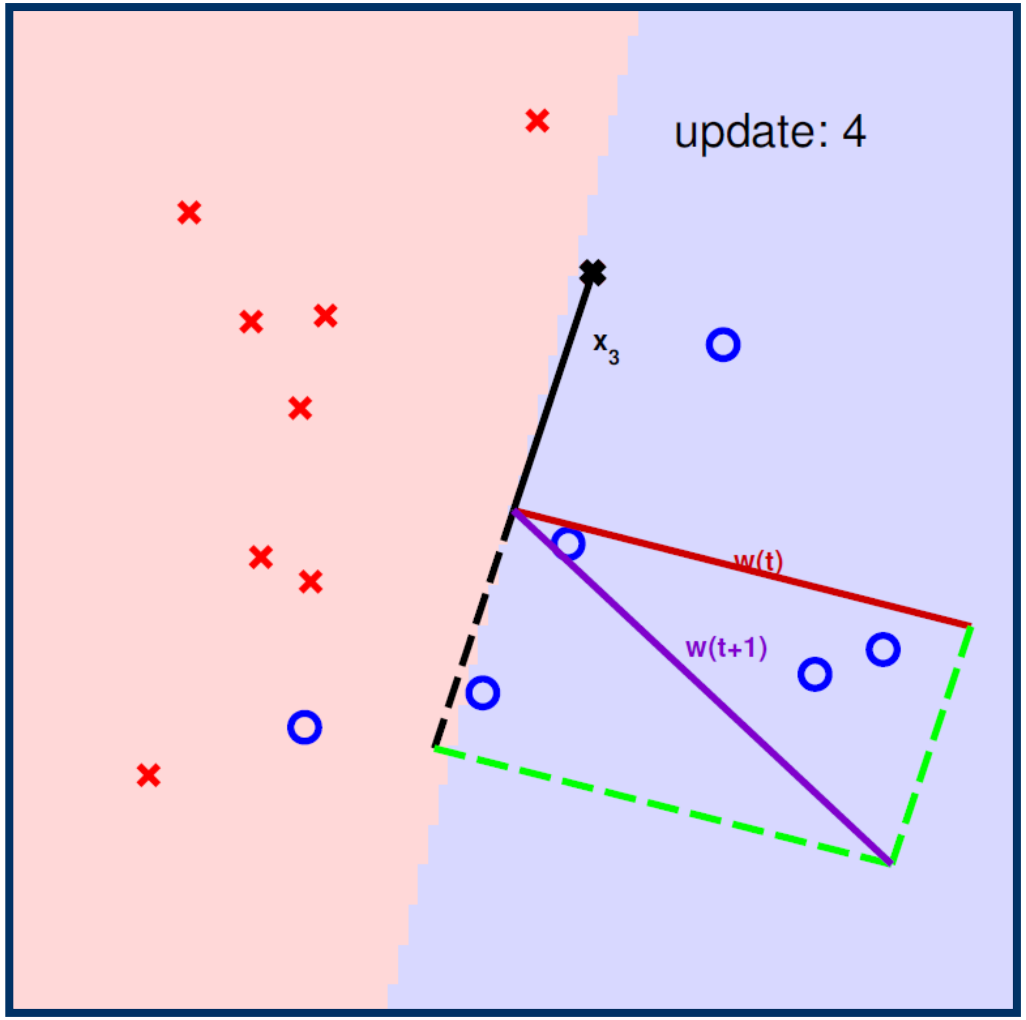


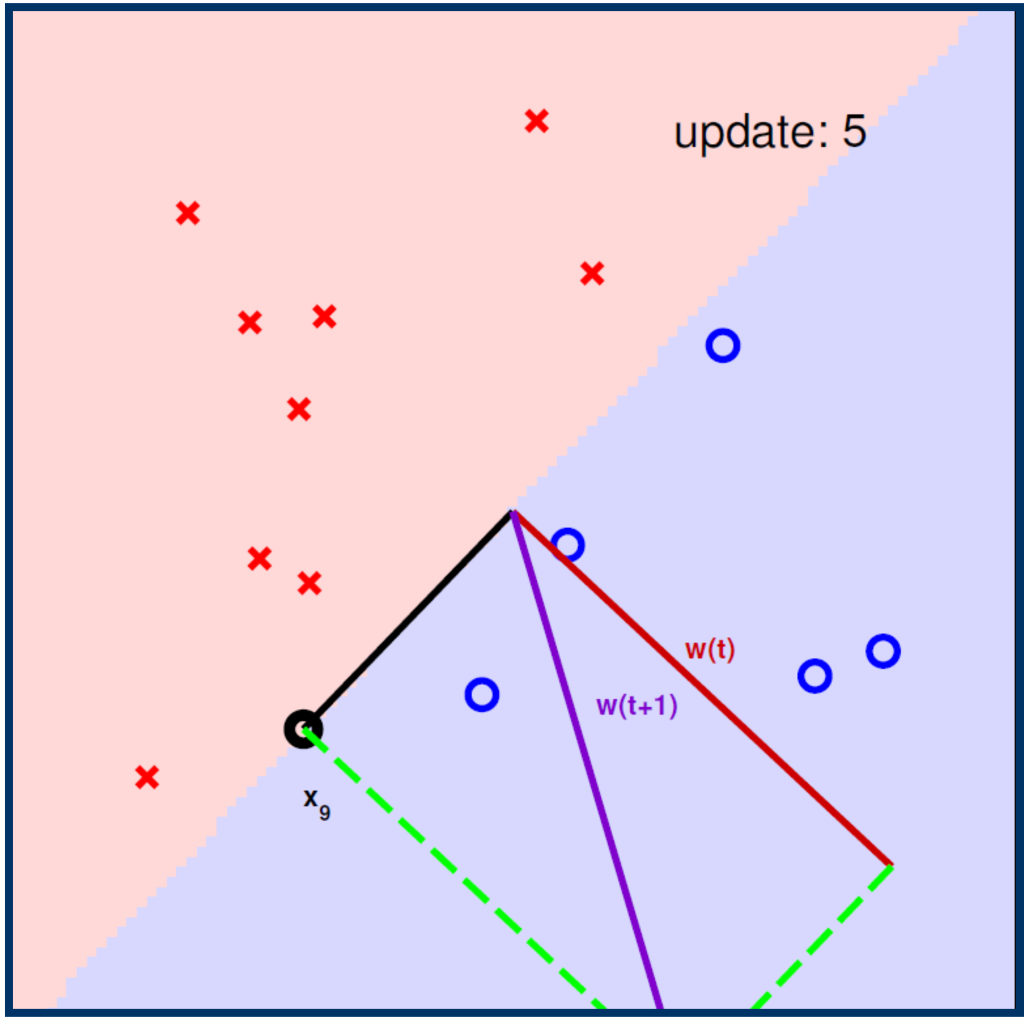
Seeing is Believing





Seeing is Believing





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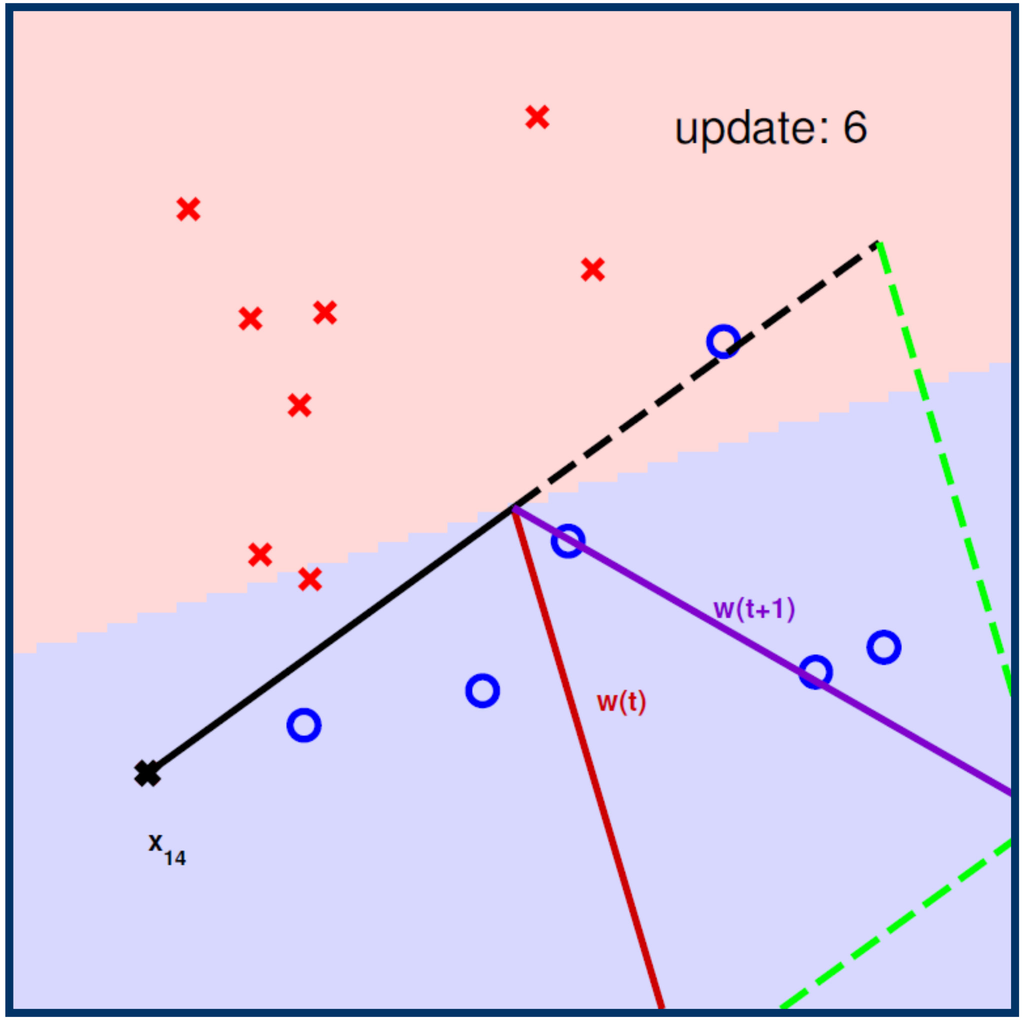

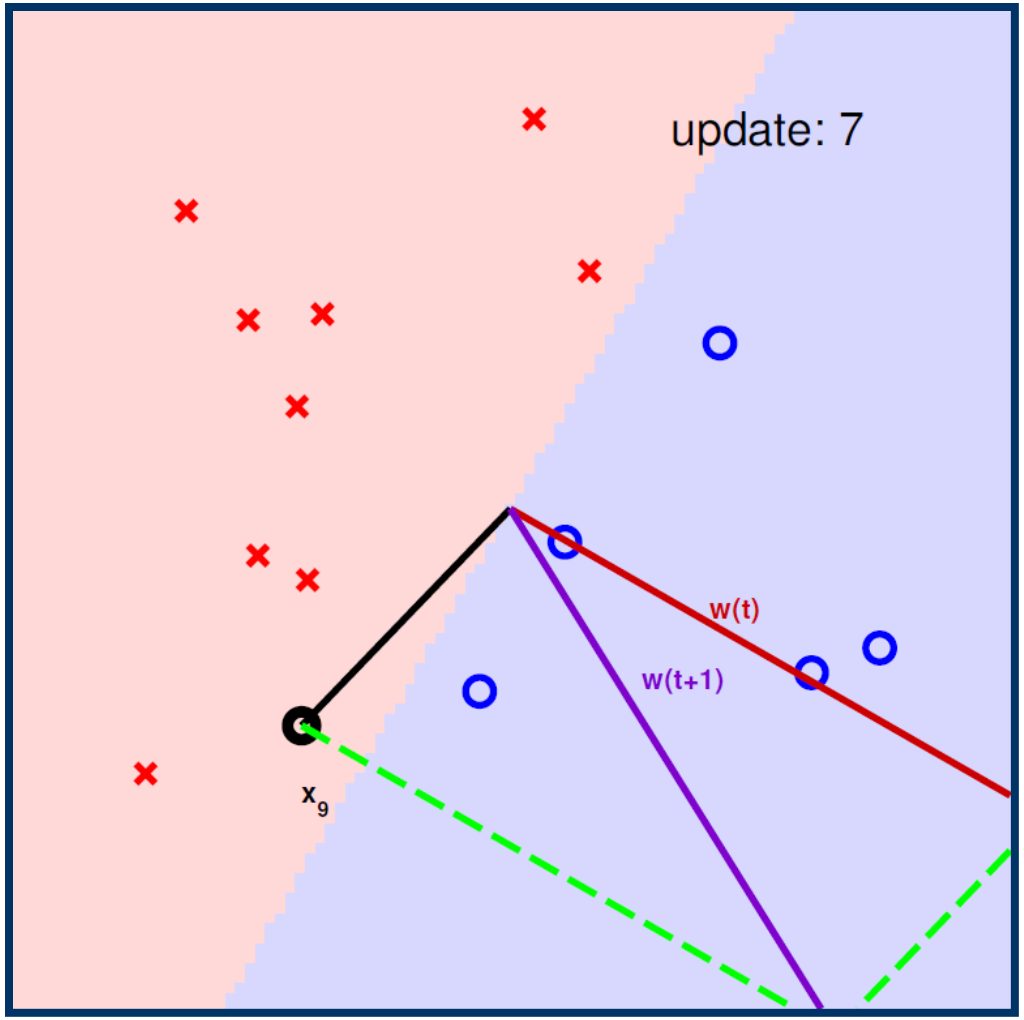

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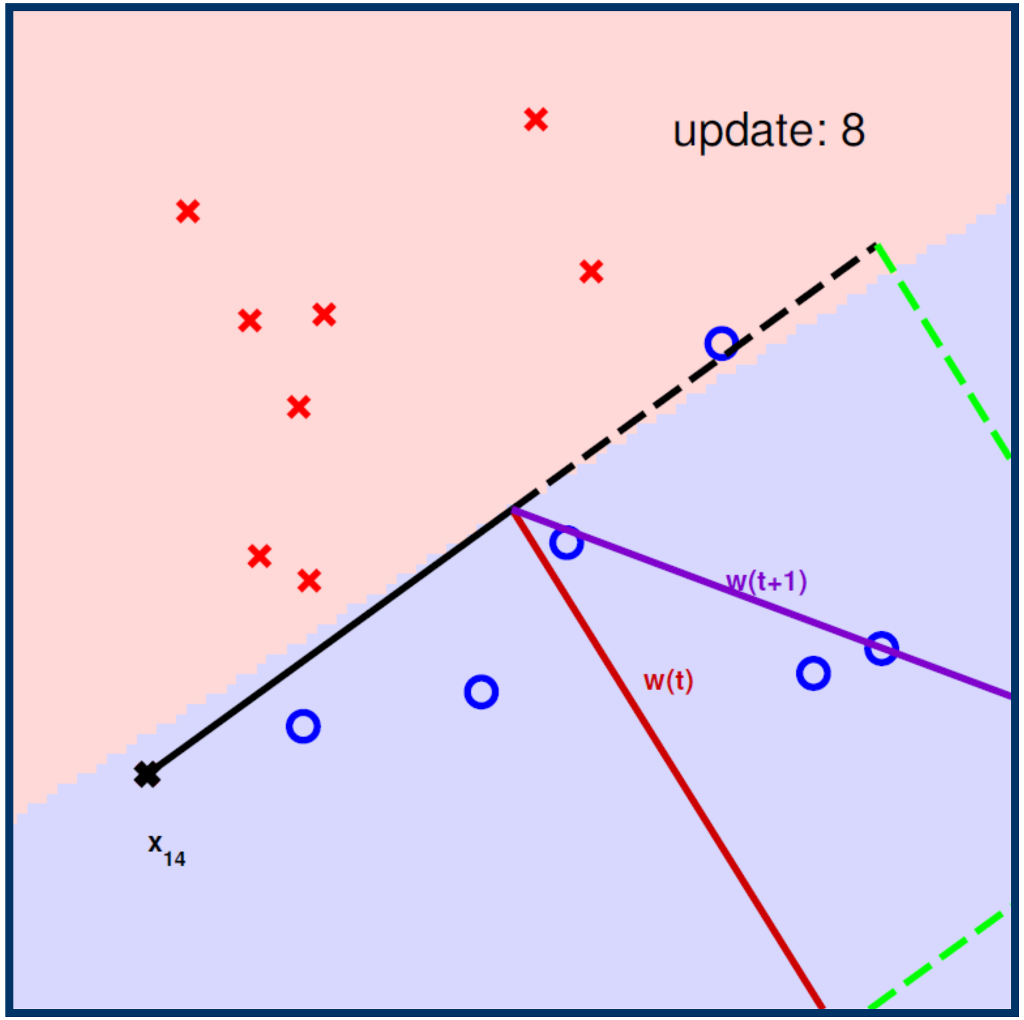

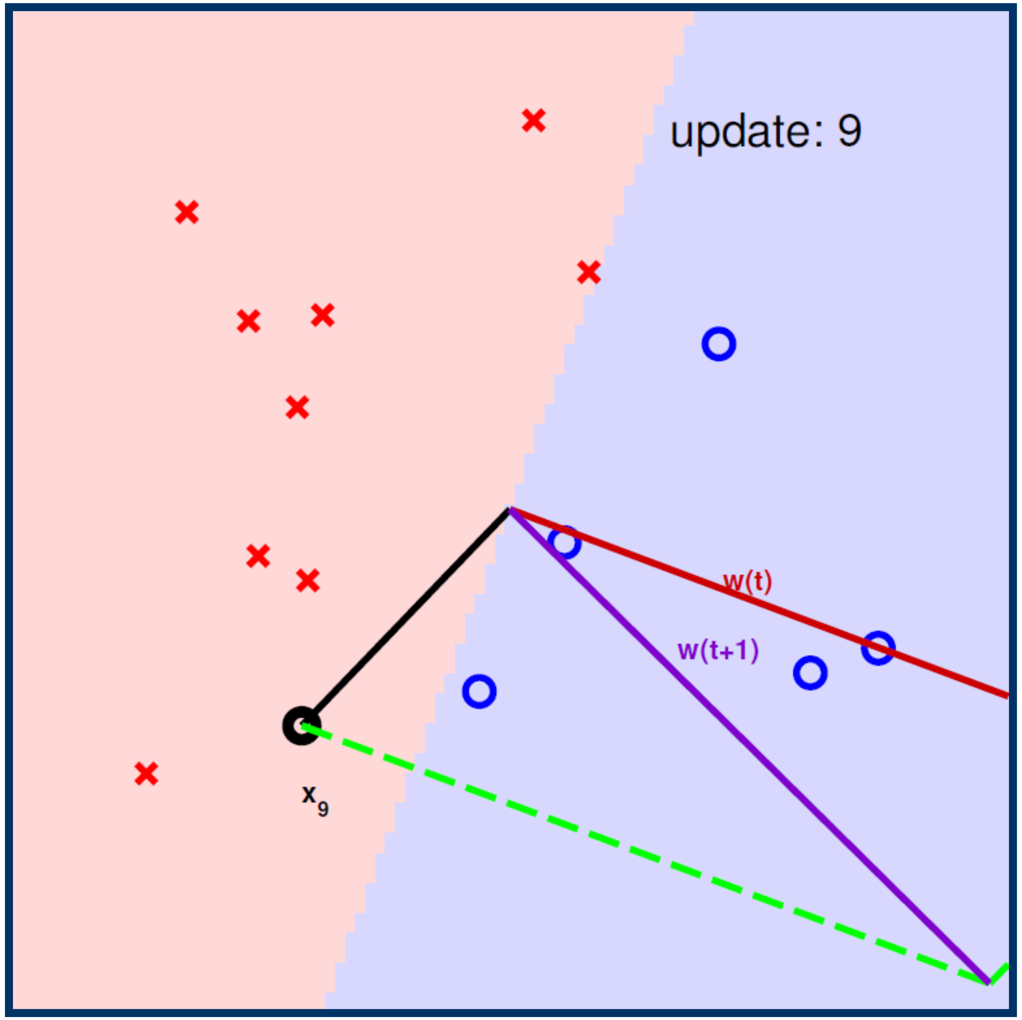

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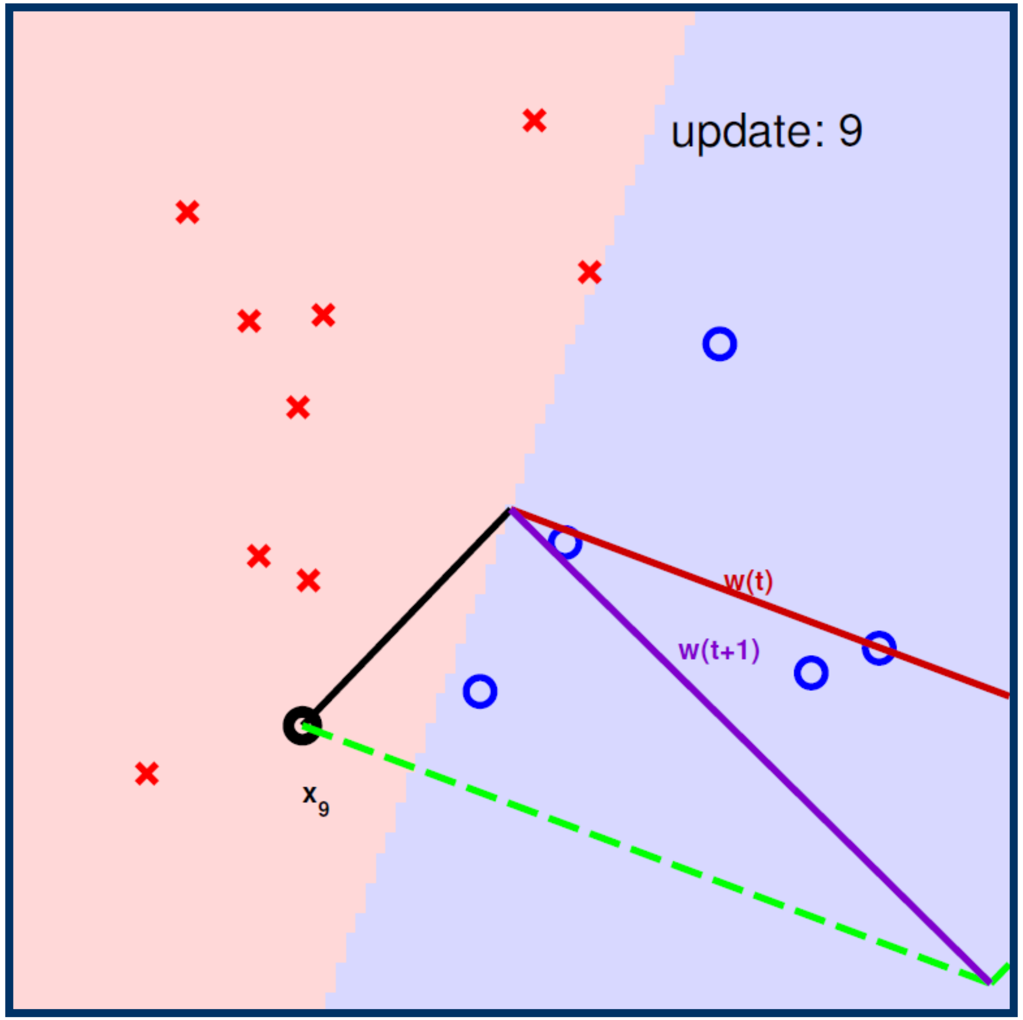


11.2 Deep Learning

Ups and downs of Deep Learning

- 1958: Perceptron (linear model)
- 1969: Perceptron has limitation
- 1980s: Multi-layer perceptron

  ※ Do not have significant difference from DNN today
- 1986: **Backpropagation**

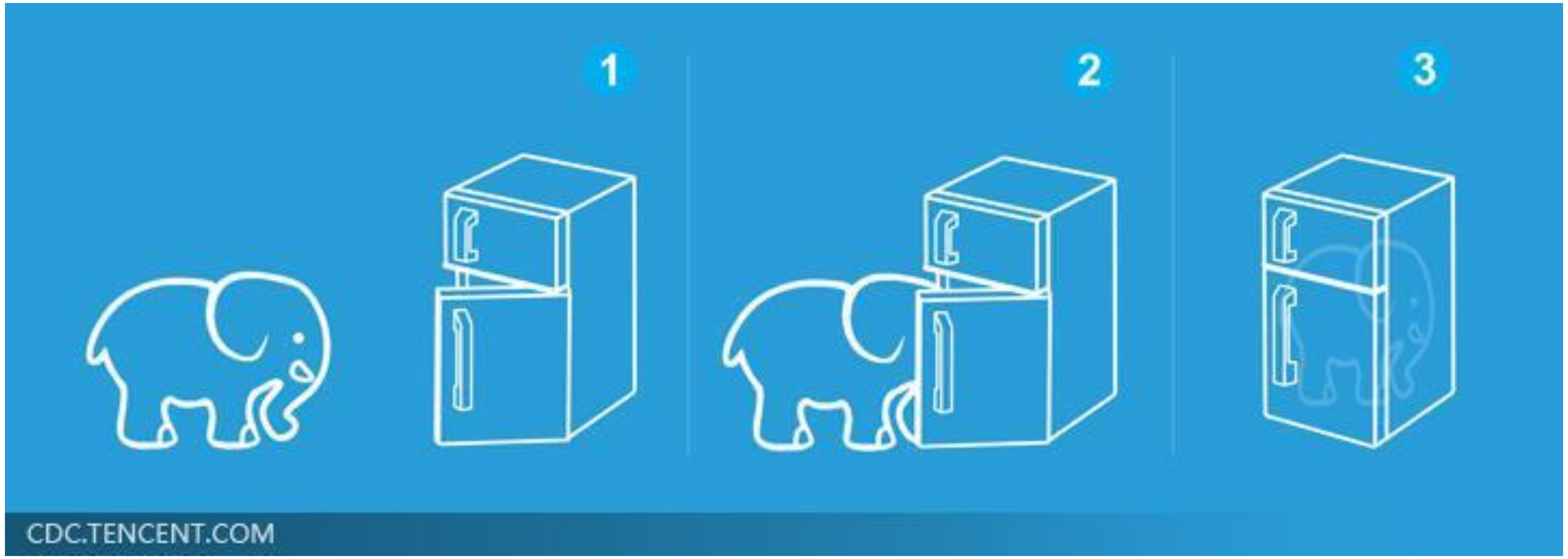
  ※ Usually more than 3 hidden layers is not helpful
- 1989: 1 hidden layer is "good enough", why deep?
- 2006: RBM initialization (breakthrough)
- 2009: GPU
- 2011: Start to be popular in speech recognition
- 2012: win ILSVRC image competition

Three Steps for Deep Learning

- Deep Learning is so simple ……

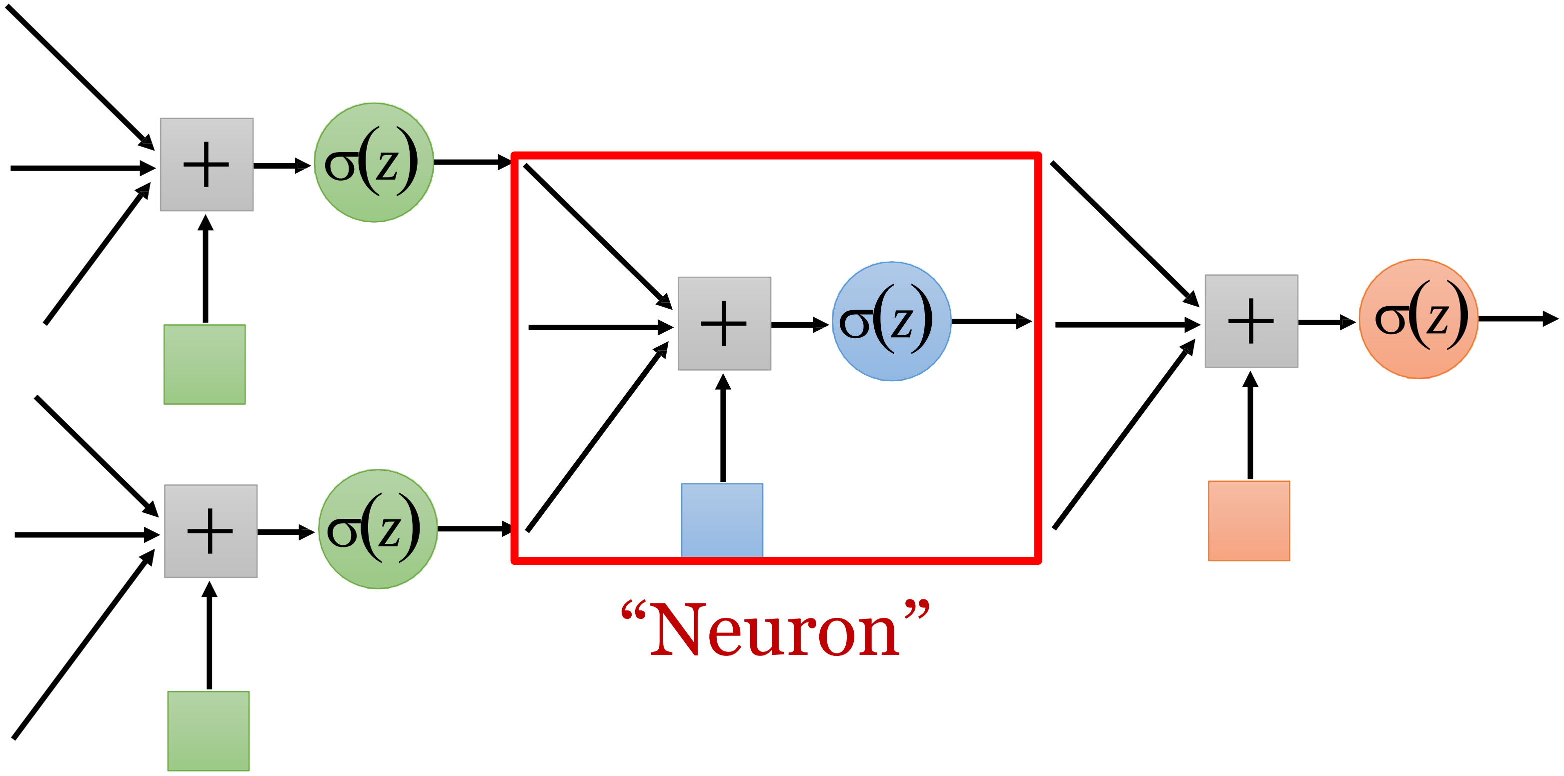
Step 1: define a set of function → **Step 2: goodness of function** → **Step 3: pick the best function**

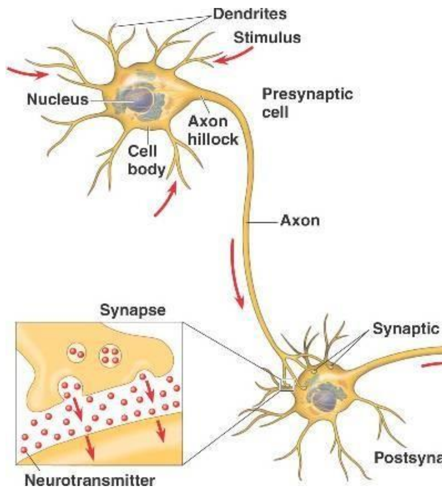
Neural Network



Neural Network

- Neural Network: Different connection leads to different network structures



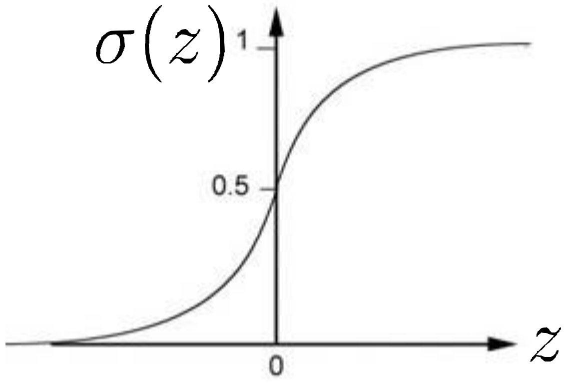


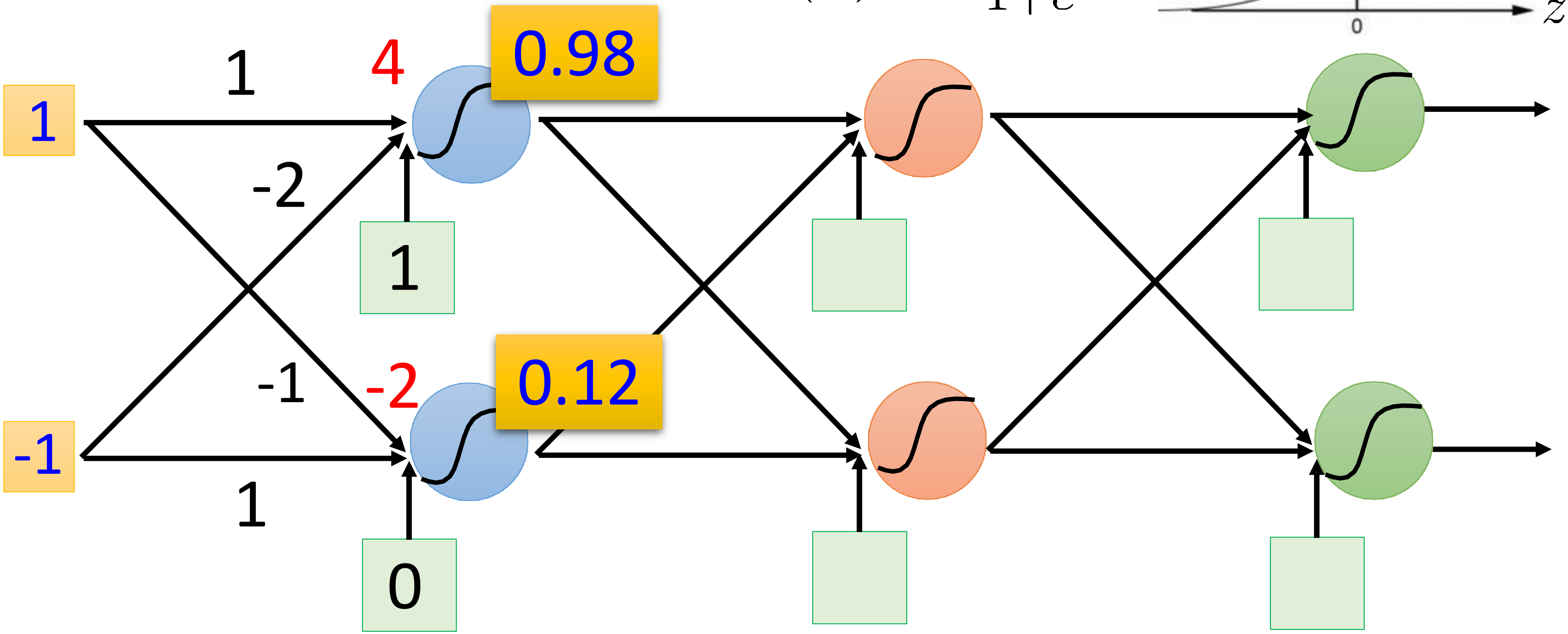
- Network parameter θ: all the weights and biases in the "neurons"

Fully Connect Feedforward Network

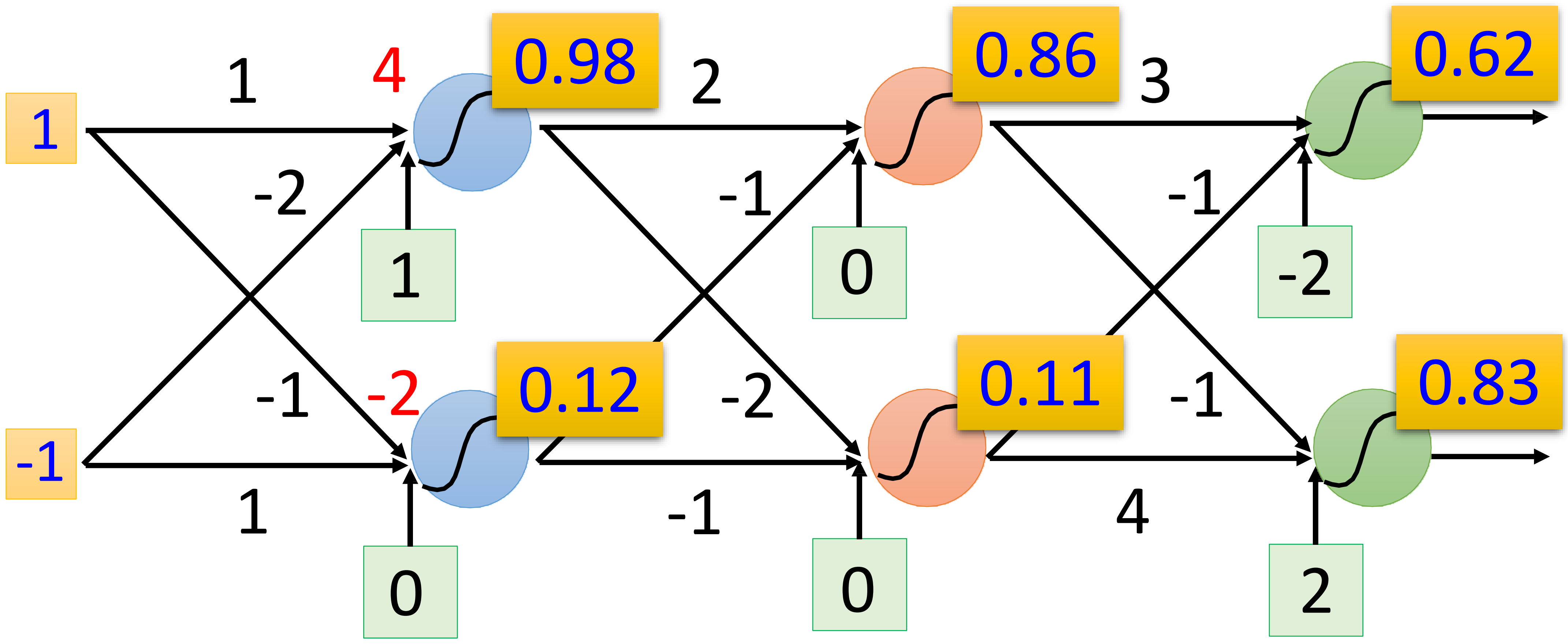
Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{z}}$$

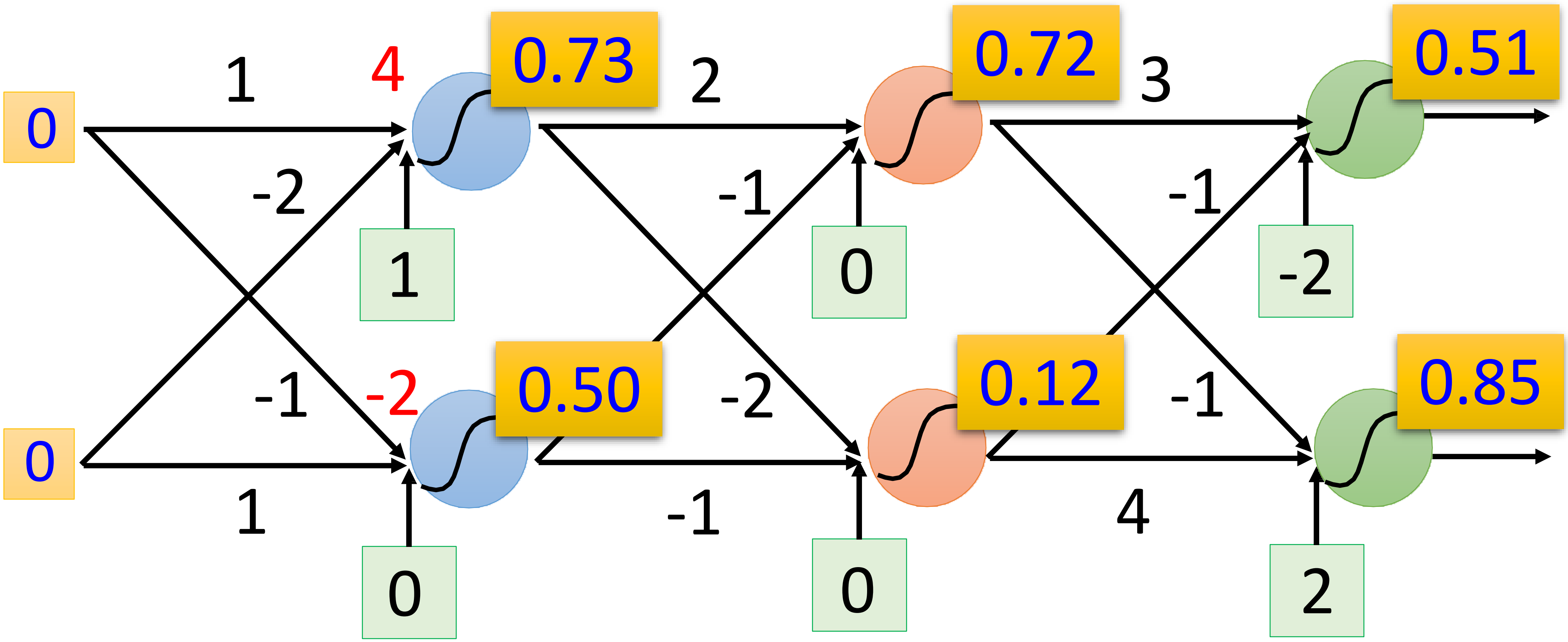




Fully Connect Feedforward Network



Fully Connect Feedforward Network



This is a function： Input vector, output vector

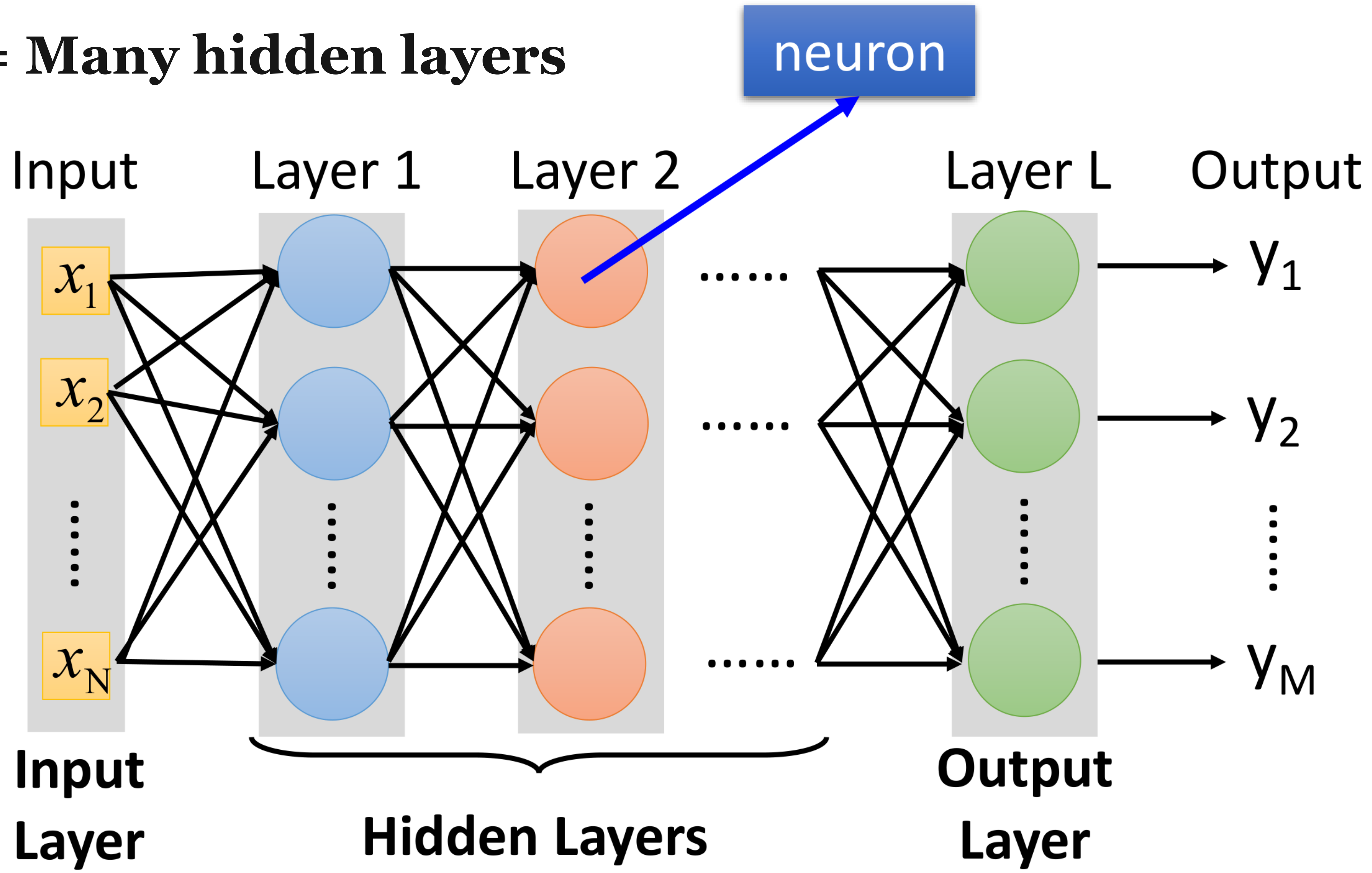
$$f\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}0.62\\0.83\end{bmatrix}$$

Given network structure, define a function set

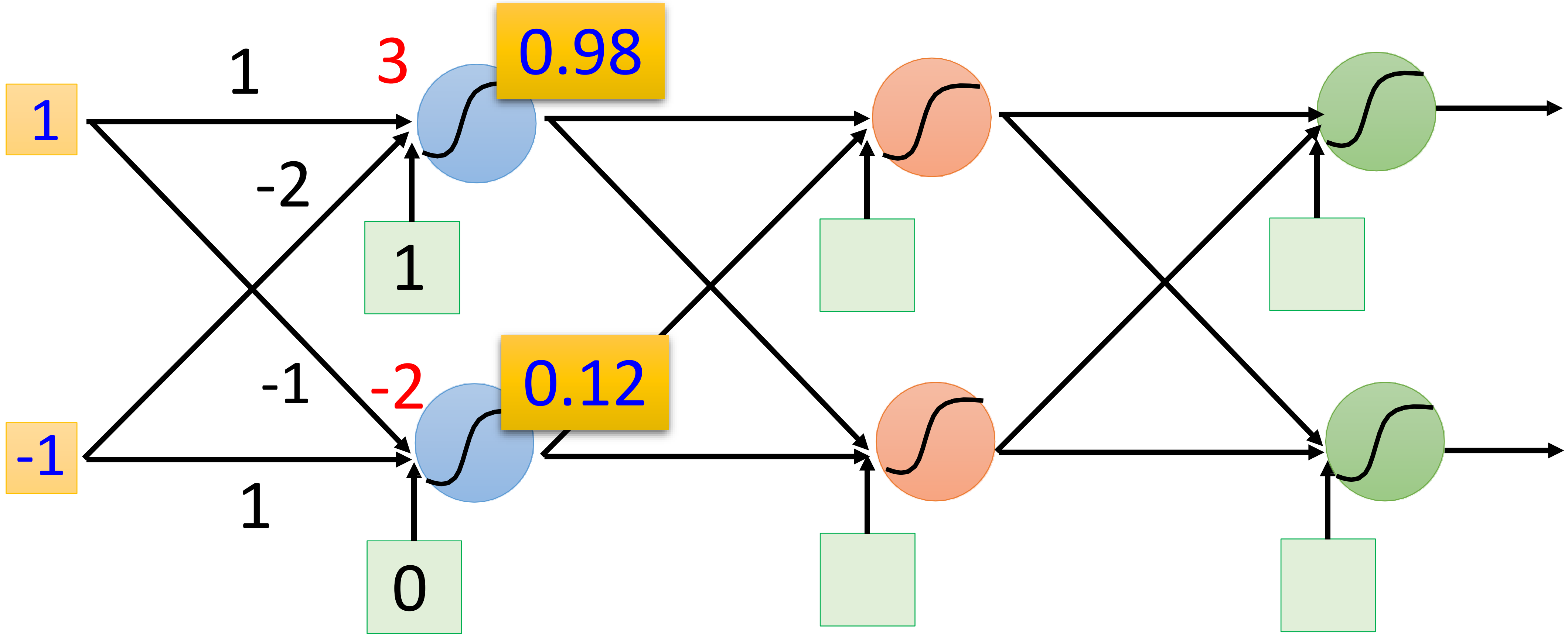
$$f\left(\begin{bmatrix}0\\0\end{bmatrix}\right)=\begin{bmatrix}0.51\\0.85\end{bmatrix}$$

Fully Connect Feedforward Network

Deep = Many hidden layers



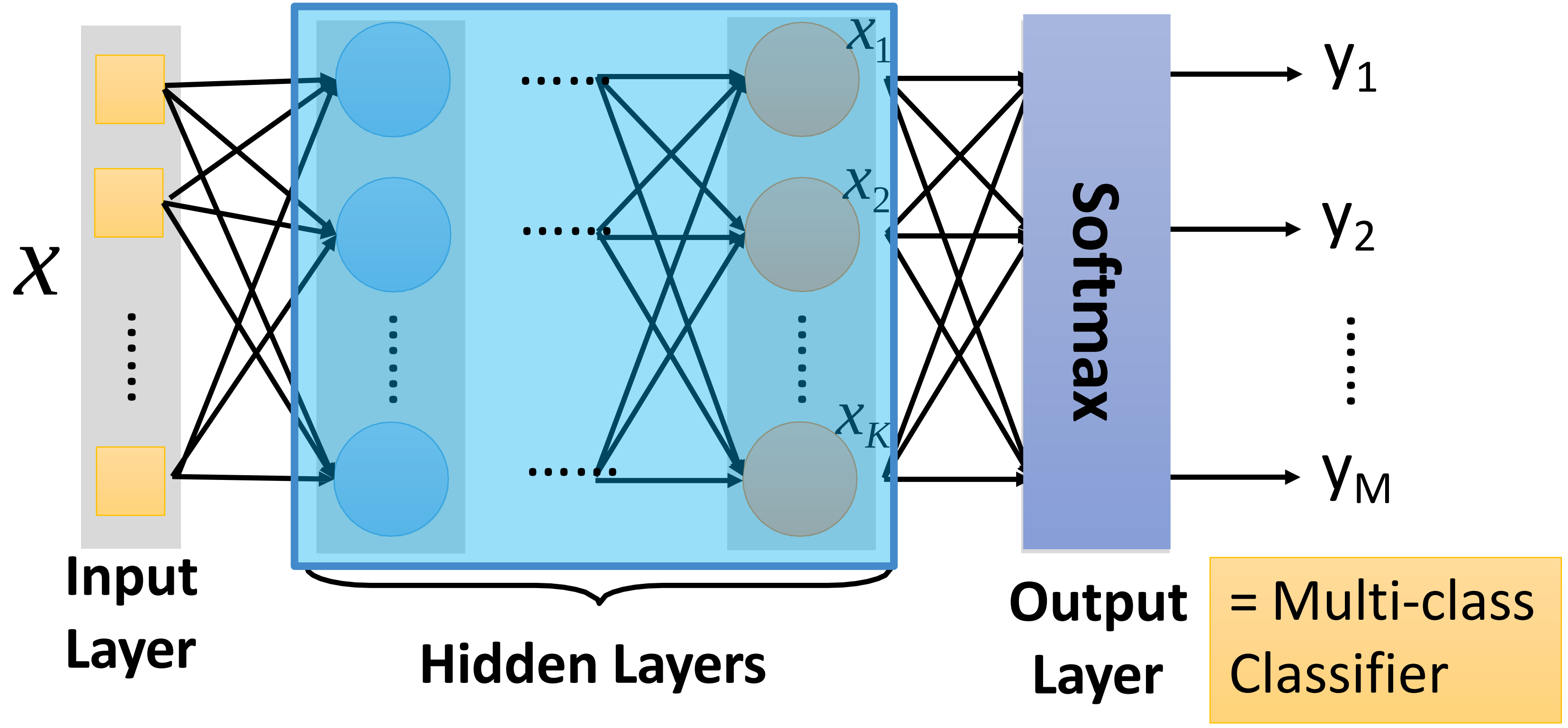
Matrix Operation



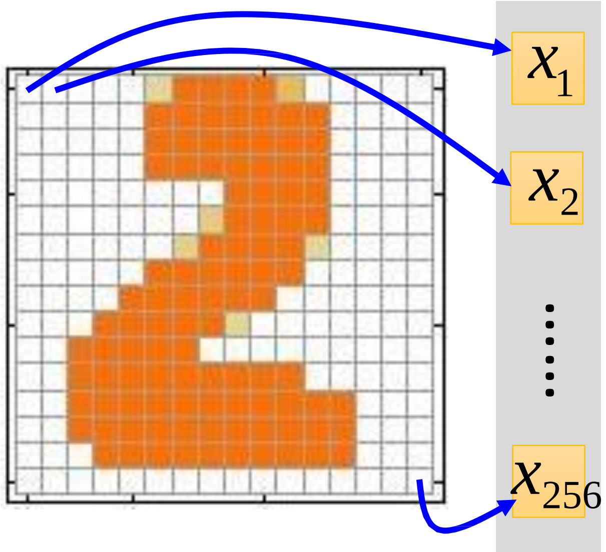
$$\sigma\left(\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

Output Layer

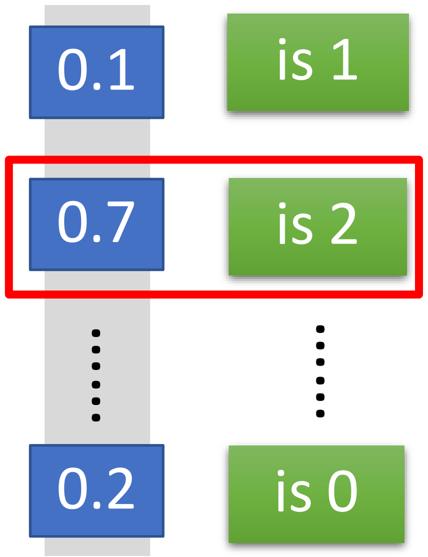
Feature extractor replacing feature engineering



Input

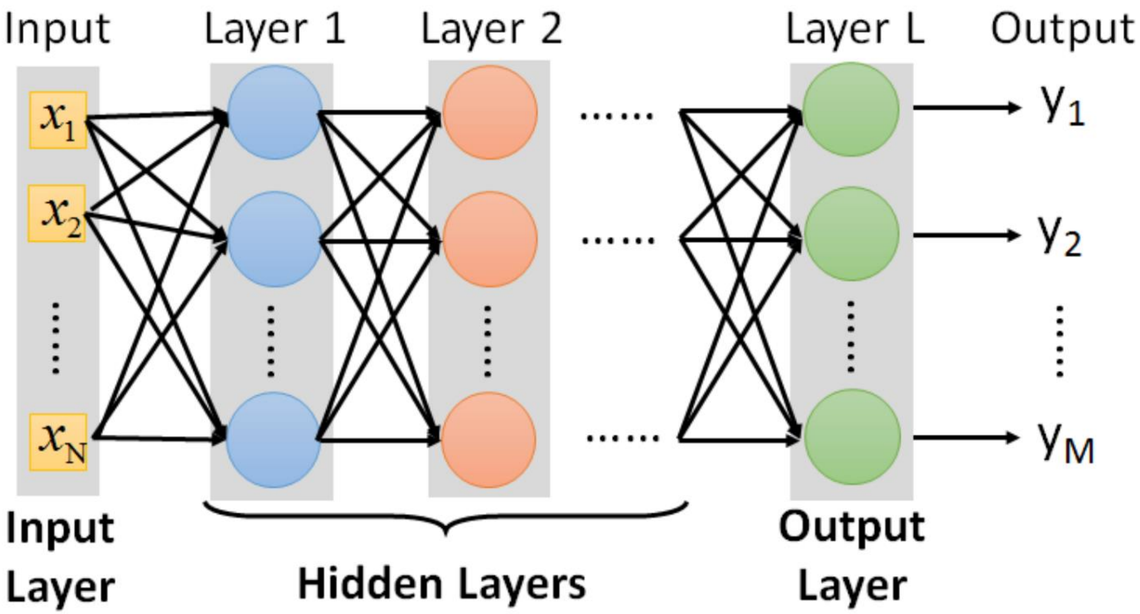


Output



Each dimension represents the confidence of a digit.

FAQ



- Q: How many layers? How many neurons for each layer?

  ※ Trial and Error + Intuition

- Q: Can the structure be automatically determined?

  ※ E.g. Evolutionary Artificial Neural Networks

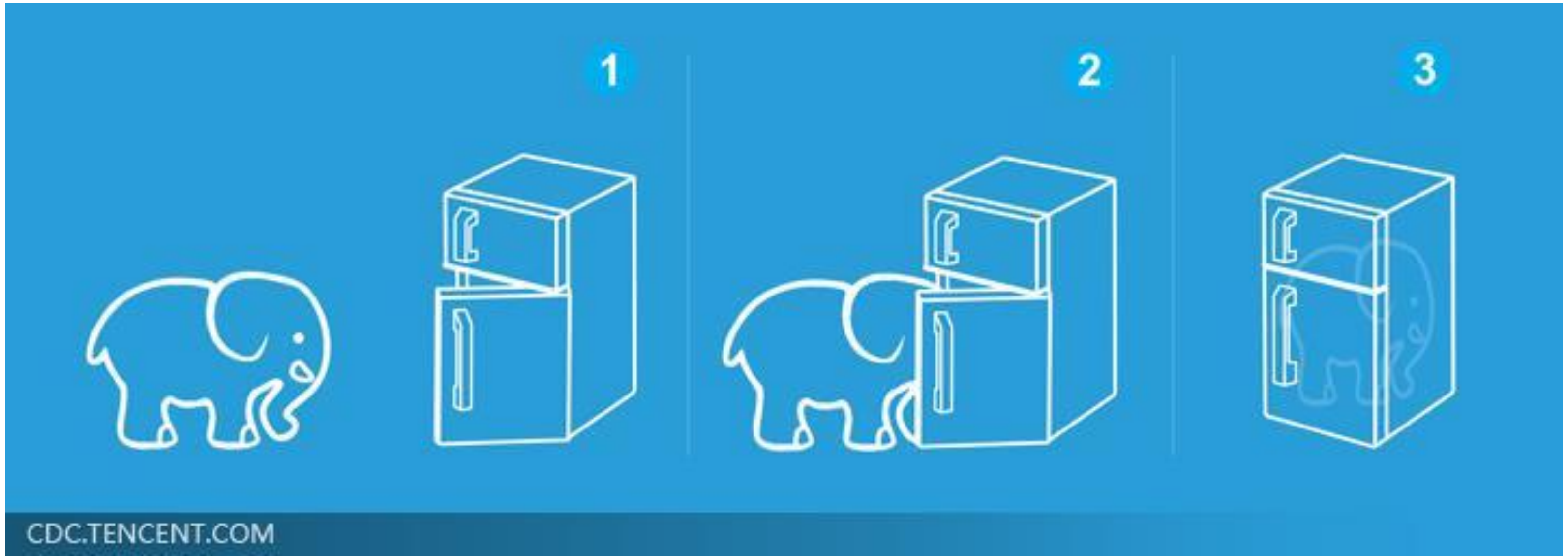
- Q: Can we design the network structure?

Three Steps for Deep Learning

- Deep Learning is so simple ……

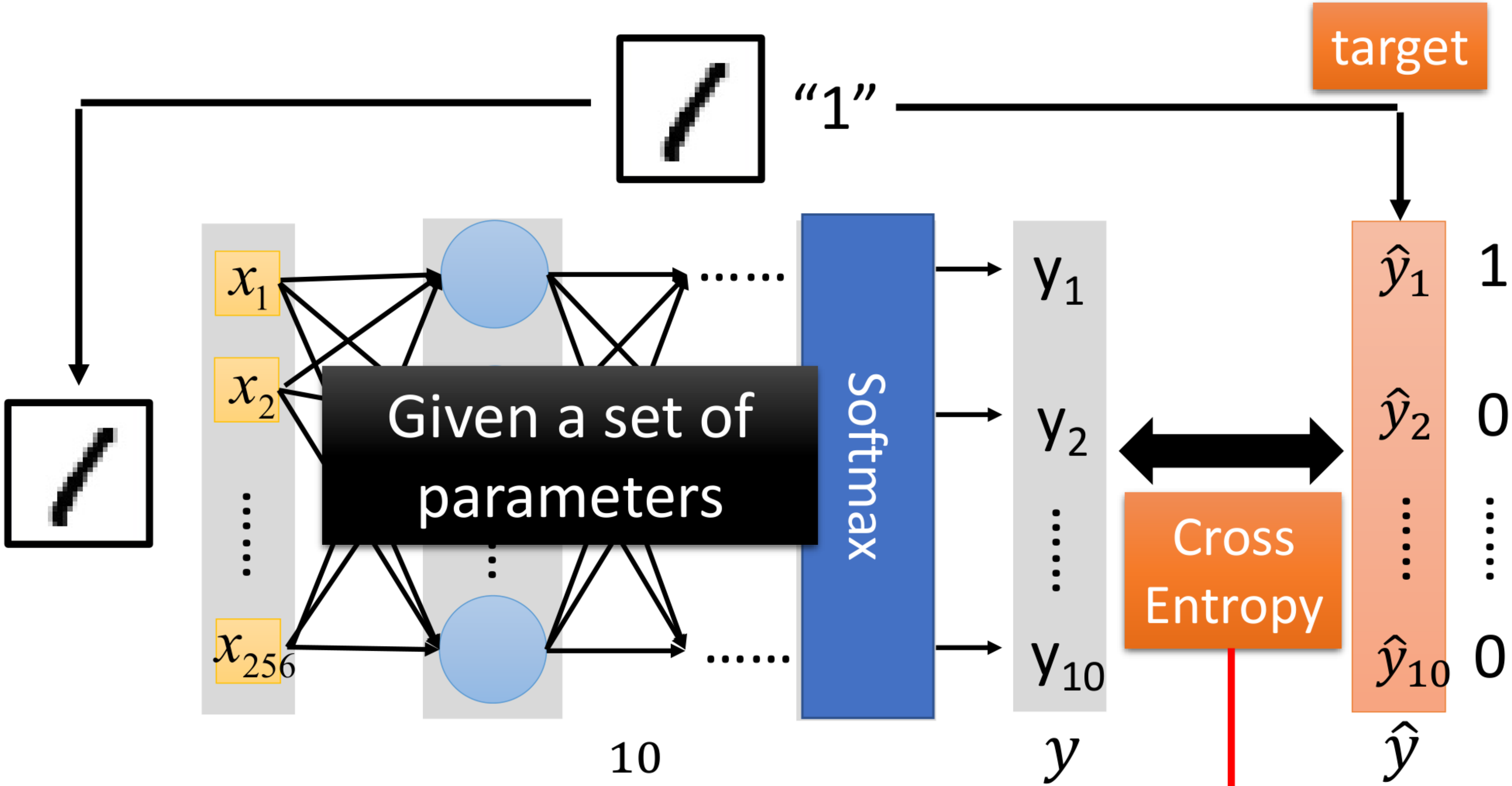
Step 1: define a set of function → **Step 2: goodness of function** → **Step 3: pick the best function**

Neural Network



11.3 The Loss Function

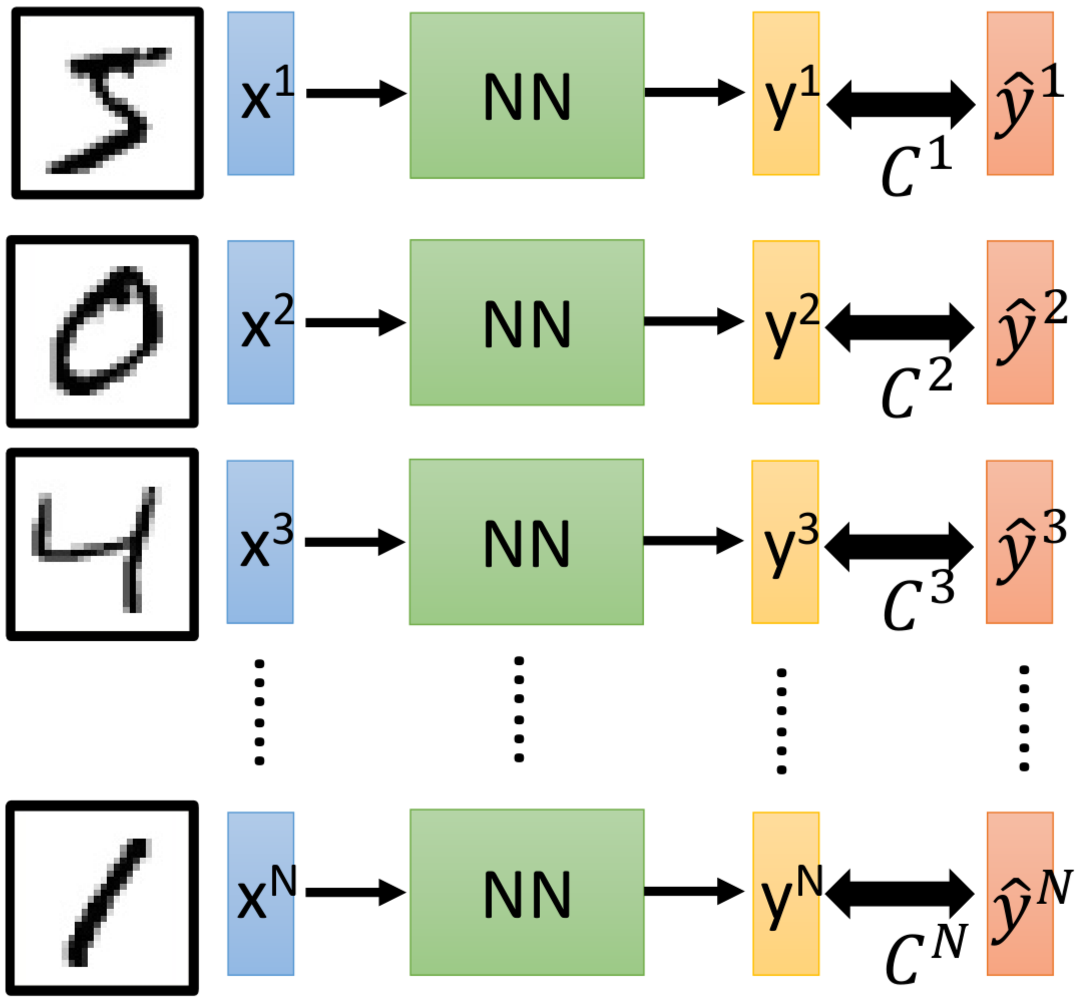
Loss for an Example



$$C(y, \hat{y}) = -\sum_{i=1}^{10} \hat{y}_i \ln y_i$$

Total Loss

- For all training data …



- Total Loss:

$$L = \sum_{n=1}^{N} C^n$$

$\Downarrow$

Find a function in function set that minimizes total loss L

$\Downarrow$

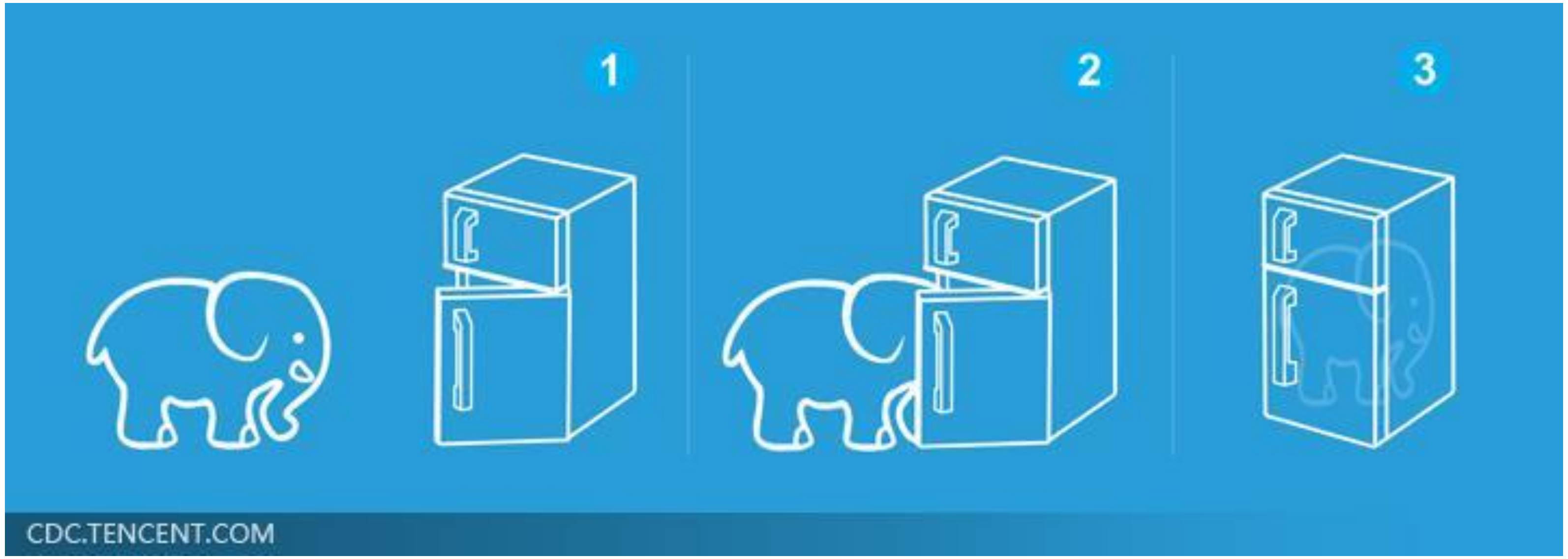
Find the network parameters $\boldsymbol{\theta}*$ that minimize total loss L

Three Steps for Deep Learning

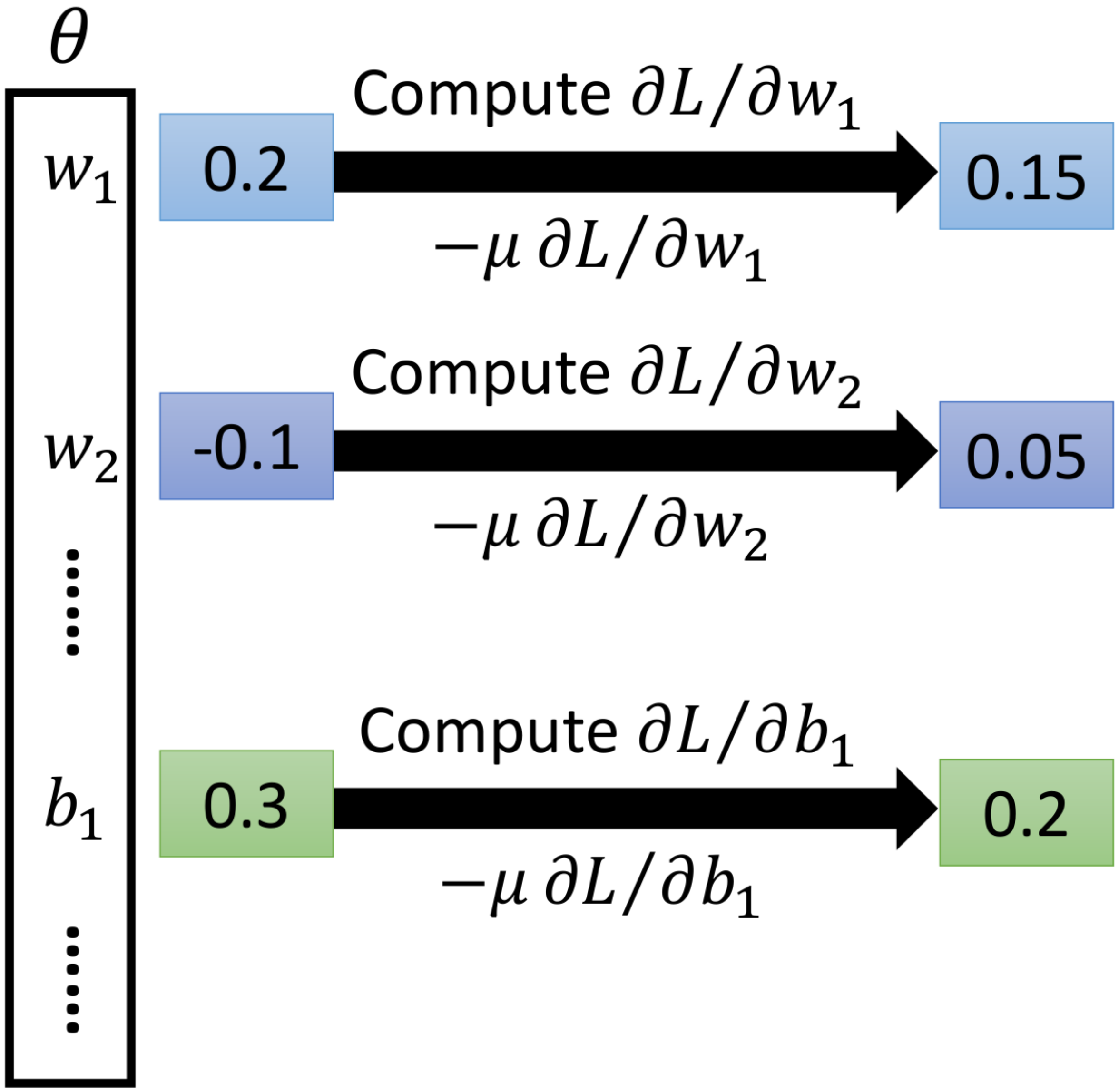
- Deep Learning is so simple ……

Step 1: define a set of function → Step 2: goodness of function → Step 3: pick the best function

Neural Network

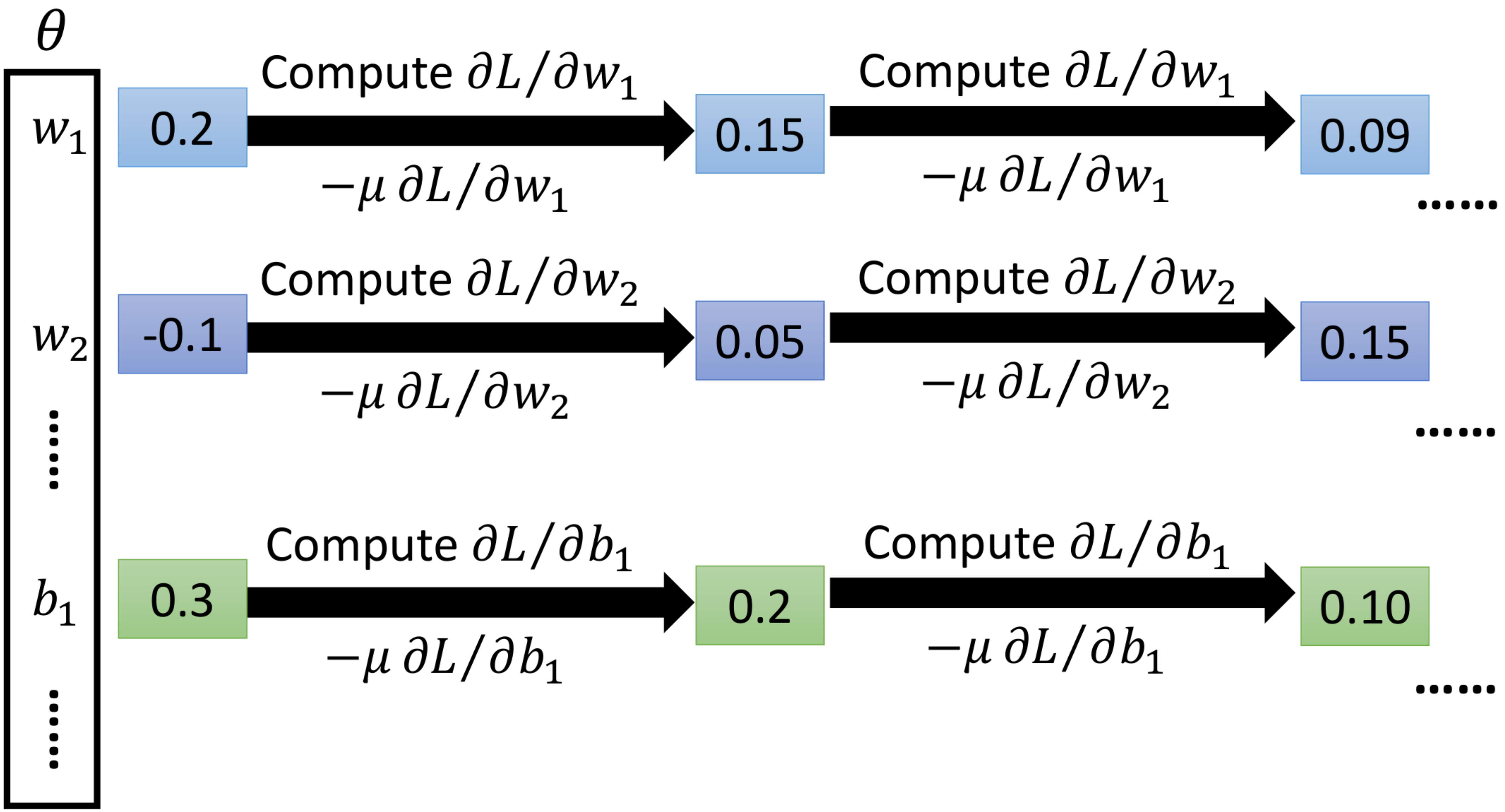


Gradient Descent



gradient $$\nabla L = \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial w_2} \\ \vdots \\ \frac{\partial L}{\partial b_1} \\ \vdots \end{bmatrix}$$

Gradient Descent



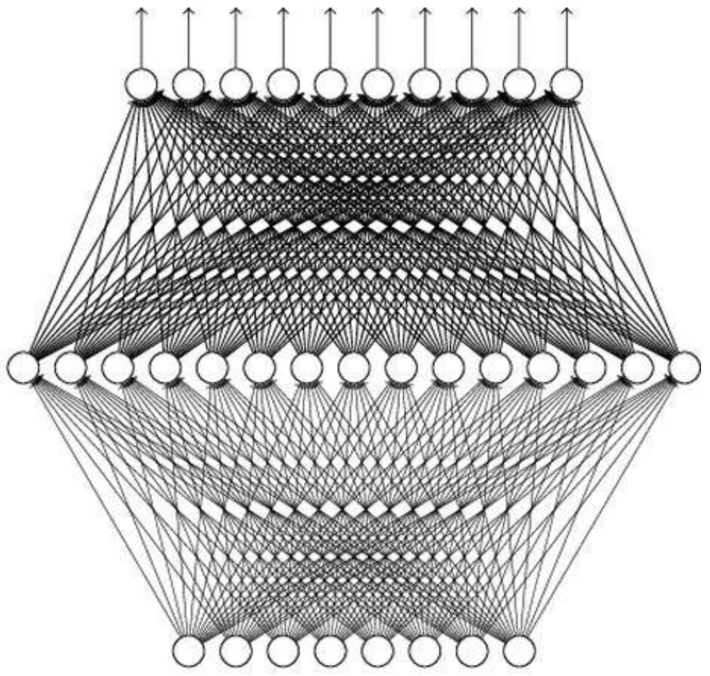
Universality Theorem

- Any continuous function f

$$f : \mathbb{R}^N \Rightarrow \mathbb{R}^M$$

- Can be realized by a network with one hidden layer

  ※ given enough hidden neurons



- Reference for the reason:

  ※ http://neuralnetworksanddeeplearning.com/chap4.html

11.4 Backpropagation

Gradient Descent

- Network parameters: $\theta = \{w_1, w_2, \cdots, b_1, b_2, \cdots\}$
- Starting Parameters: $\theta^0 \rightarrow \theta^1 \rightarrow \theta^2 \rightarrow \cdots$

$$\nabla L(\theta) = \begin{bmatrix} \partial L(\theta)/\partial w_1 \\ \partial L(\theta)/\partial w_2 \\ \vdots \\ \partial L(\theta)/\partial b_1 \\ \partial L(\theta)/\partial b_2 \\ \vdots \end{bmatrix}$$

$Compute\ \nabla L(\theta^0)$ $\qquad \theta^1 = \theta^0 - \eta \nabla L(\theta^0)$

$Compute\ \nabla L(\theta^1)$ $\qquad \theta^2 = \theta^1 - \eta \nabla L(\theta^1)$

Millions of parameters ......

To compute the gradients efficiently, we use:

backpropagation.

Chain Rule

- Case 1: $y = g(x), \quad z = h(y)$

$$\Delta x \to \Delta y \to \Delta z \qquad\qquad \frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

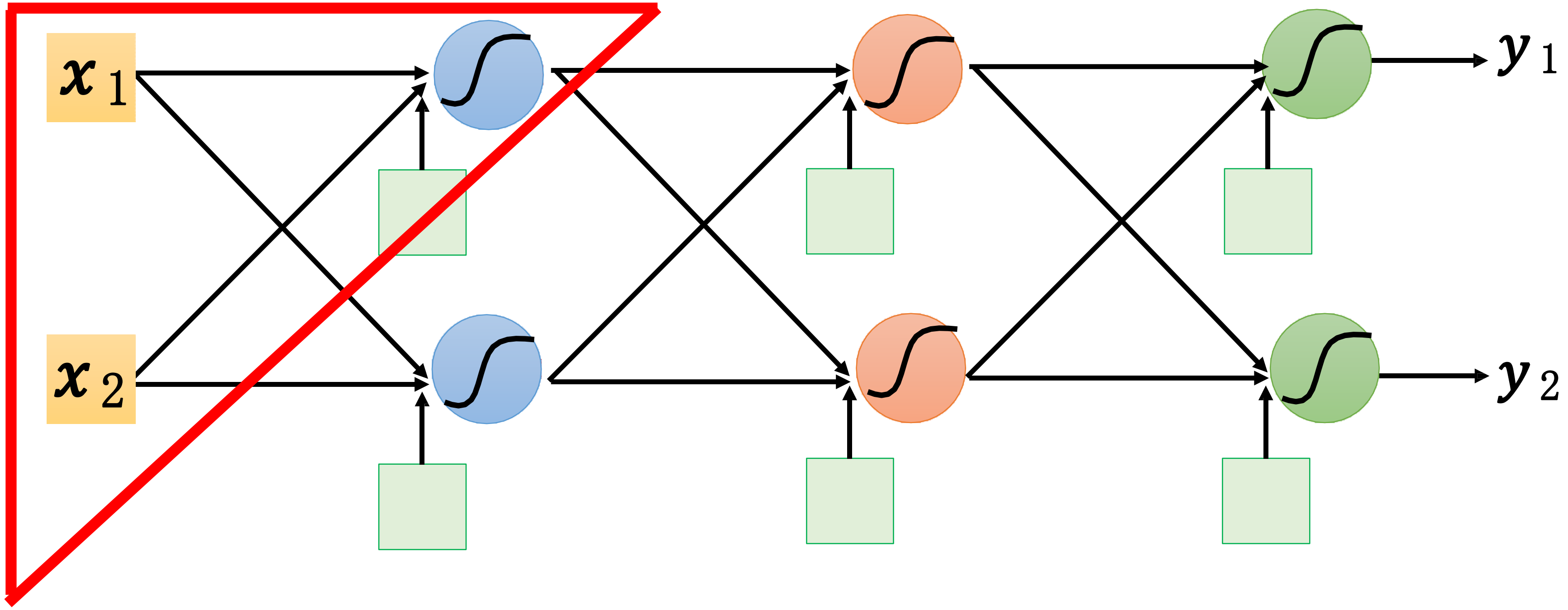
- Case 2: $x = g(s), \quad y = h(s), \quad z = k(x, y)$

$$\begin{tikzcd}
 & \Delta x \arrow[dr] & \\
\Delta s \arrow[ur] \arrow[dr] & & \Delta z \\
 & \Delta y \arrow[ur] &
\end{tikzcd}$$

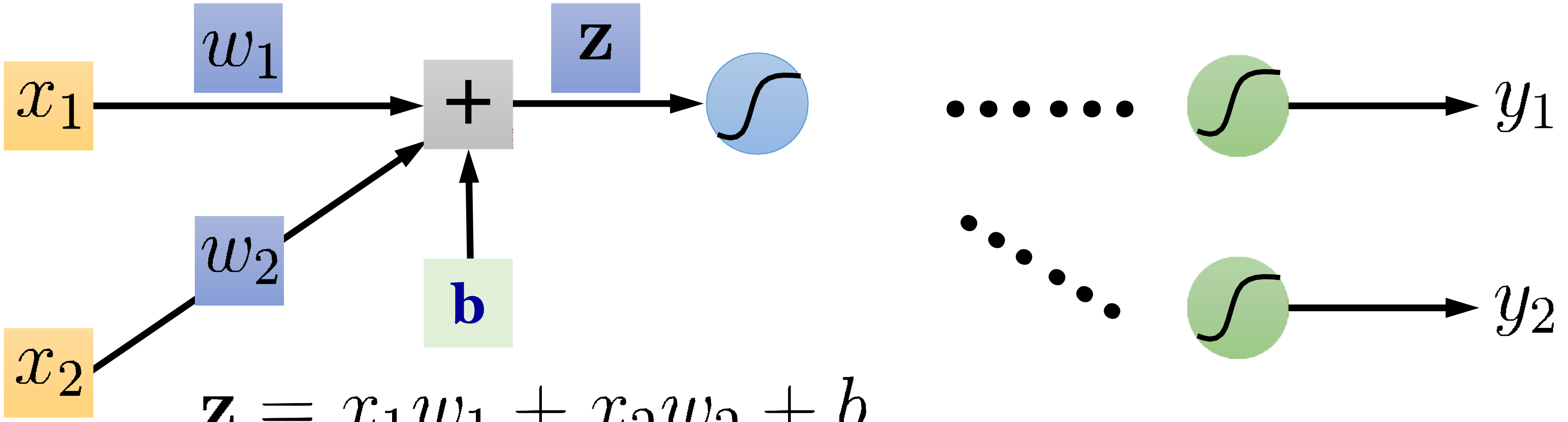
$$\frac{dz}{ds} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

Backpropagation

$$L(\theta) = \sum_{n=1}^{N} C^n(\theta) \quad \Rightarrow \quad \frac{\partial L(\theta)}{\partial w} = \sum_{n=1}^{N} \frac{\partial C^n(\theta)}{\partial w}$$



Backpropagation



$$\mathbf{z} = x_1 w_1 + x_2 w_2 + b$$

$$\frac{\partial C}{\partial w} = ? \qquad \frac{\partial z}{\partial w}\frac{\partial C}{\partial z}$$

(Chain rule)

- Forward pass:

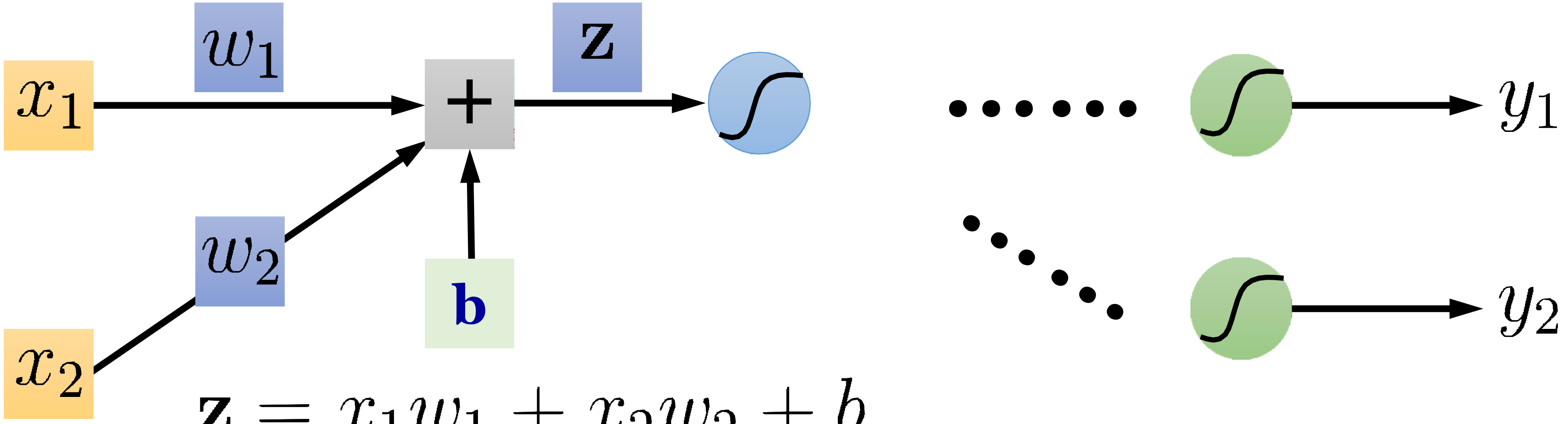
  ※ Compute $\partial z/\partial w$ for all parameters

- Backward pass:

  ※ Compute $\partial C/\partial z$ for all activation function inputs z

Backpropagation – Forward pass

- Compute $\partial z/\partial w$ for all parameters

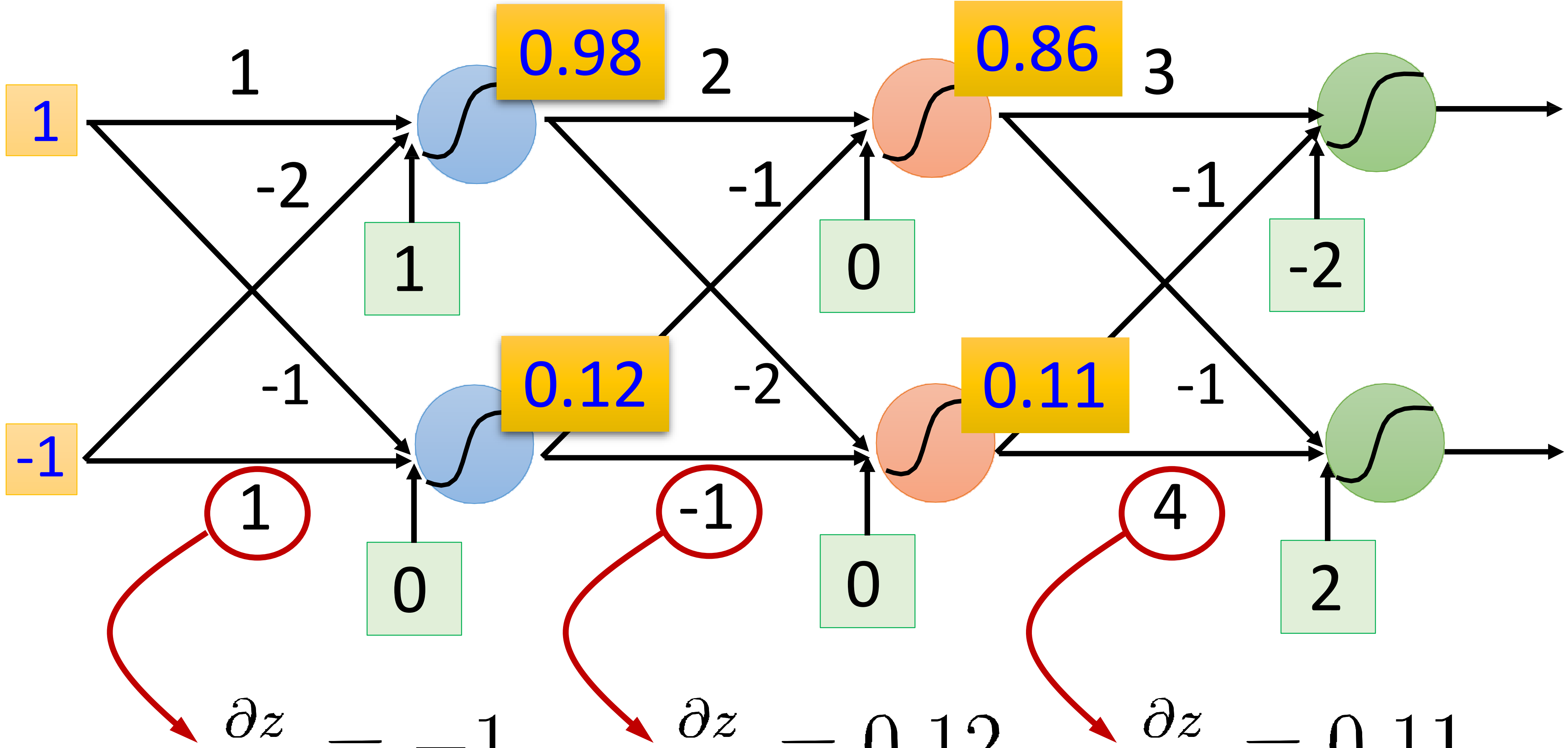


$$\mathbf{z} = x_1 w_1 + x_2 w_2 + b$$

$$\left.\begin{aligned} \frac{\partial z}{\partial w_1} &= ? \quad x_1 \\ \frac{\partial z}{\partial w_2} &= ? \quad x_2 \end{aligned}\right\} \text{The value of the input connected by the weight}$$

Backpropagation – Forward pass

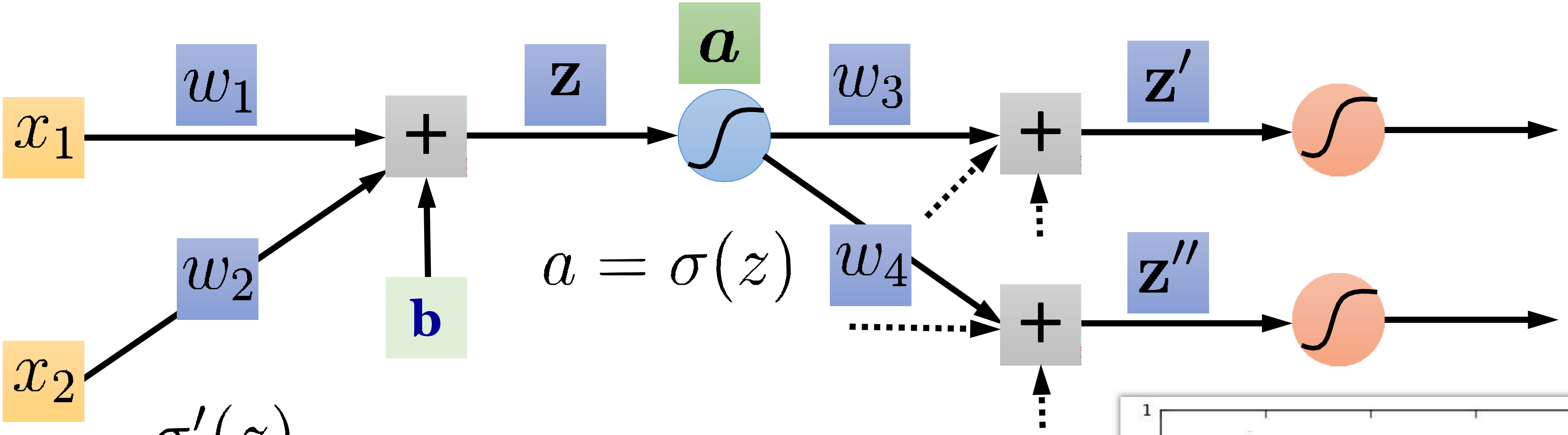
- Compute $\partial z / \partial w$ for all parameters



$$\frac{\partial z}{\partial w} = -1 \qquad \frac{\partial z}{\partial w} = 0.12 \qquad \frac{\partial z}{\partial w} = 0.11$$

Backpropagation – Backward pass

- Compute $\partial C / \partial z$ for all activation function inputs z

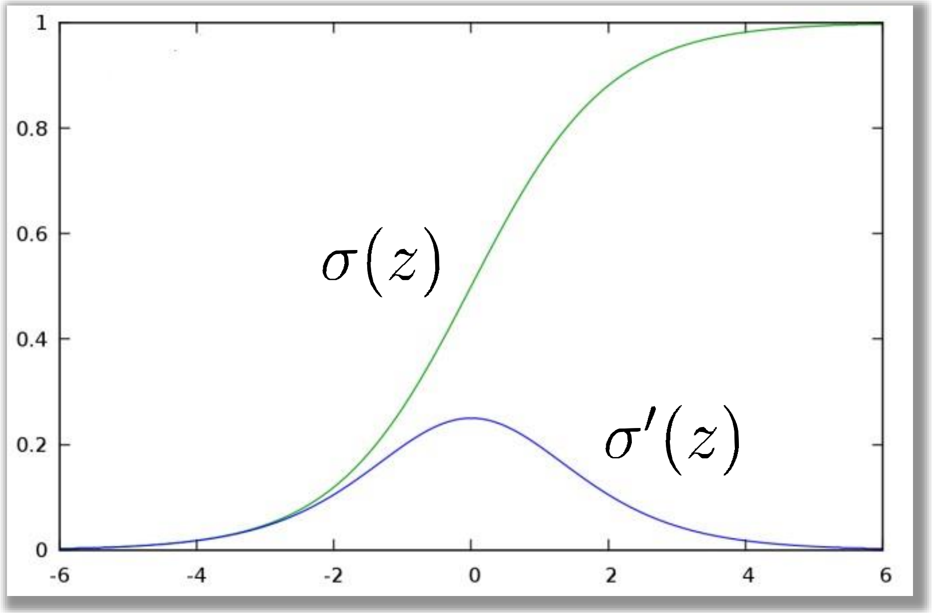


$\sigma'(z)$

$$\frac{\partial C}{\partial z} = \frac{\partial a}{\partial z} \frac{\partial C}{\partial a}$$

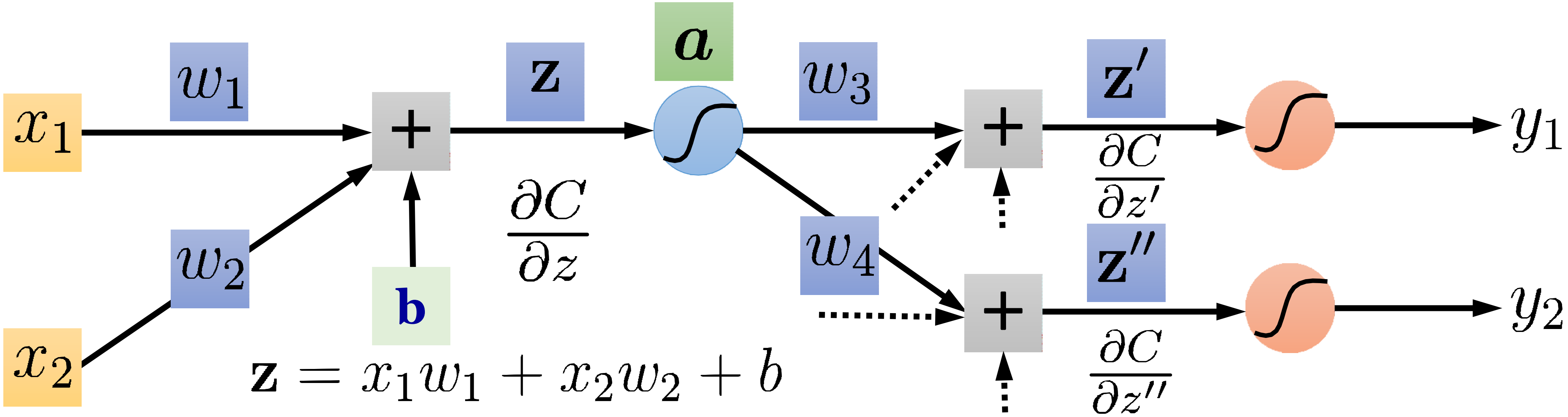
Assumed it's known

Chain rule: $\frac{\partial C}{\partial a} = \underbrace{\frac{\partial z'}{\partial a}}_{w_3} \frac{\partial C}{\partial z'} + \underbrace{\frac{\partial z''}{\partial a}}_{w_4} \frac{\partial C}{\partial z''}$



Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z

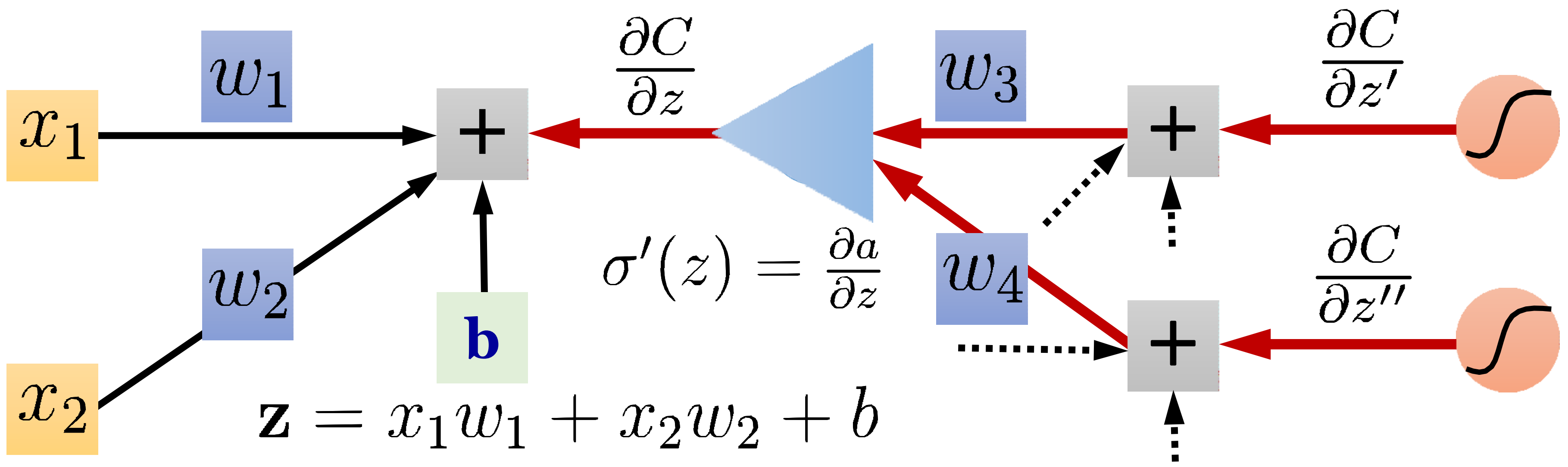


$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3 \frac{\partial C}{\partial z'} + w_4 \frac{\partial C}{\partial z''}\right]$$

$$\sigma'(z) = \frac{\partial a}{\partial z}$$

Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z

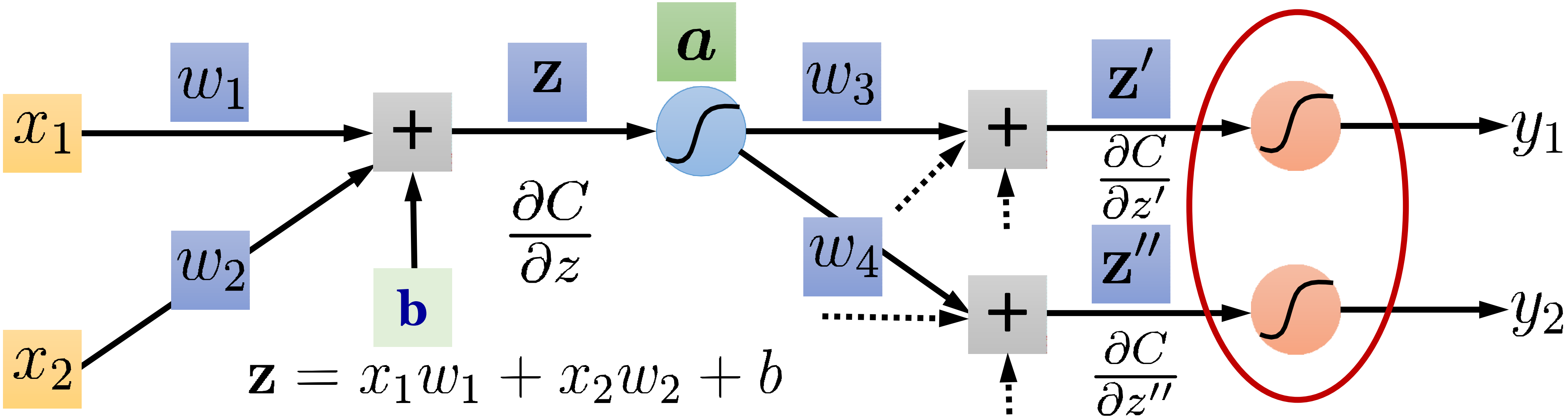


- $\sigma'(z)$ is a constant because z is already determined in the forward pass.

$$\frac{\partial C}{\partial z} = \sigma'(z) \left[w_3 \frac{\partial C}{\partial z'} + w_4 \frac{\partial C}{\partial z''} \right]$$

Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z



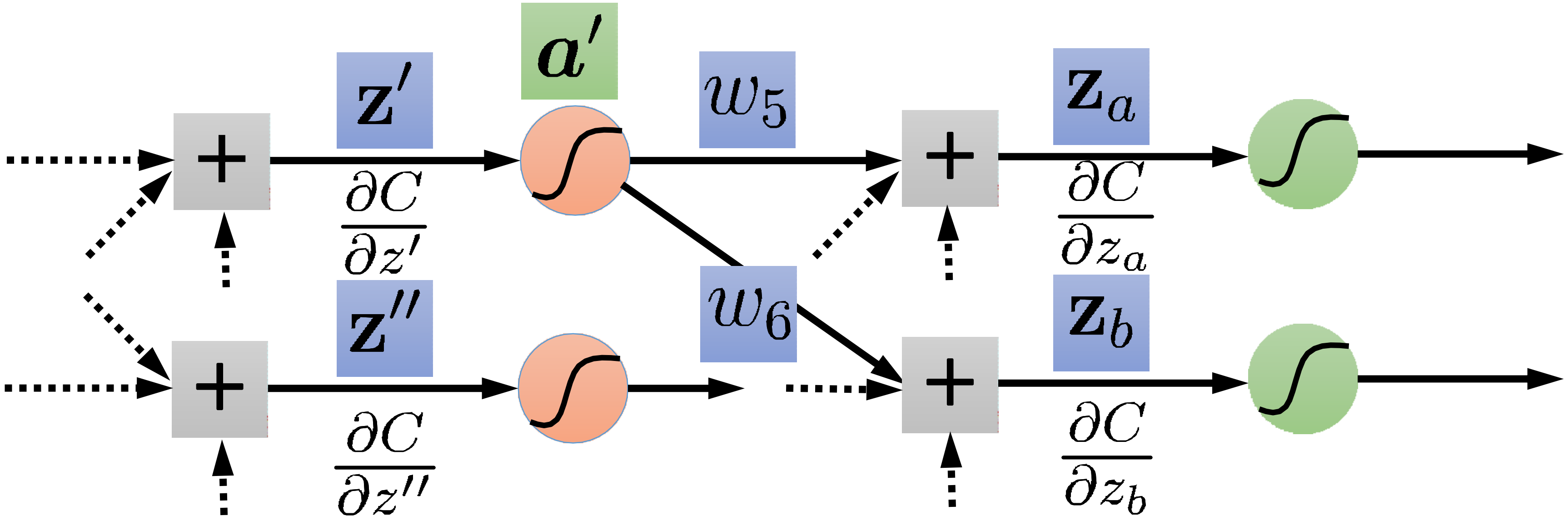
- Case 1. Output Layer $\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3 \frac{\partial C}{\partial z'} + w_4 \frac{\partial C}{\partial z''}\right]$

$$\frac{\partial C}{\partial z'} = \frac{\partial y_1}{\partial z'} \frac{\partial C}{\partial y_1} \qquad \frac{\partial C}{\partial z''} = \frac{\partial y_2}{\partial z''} \frac{\partial C}{\partial y_2}$$

Done!

Backpropagation – Backward pass

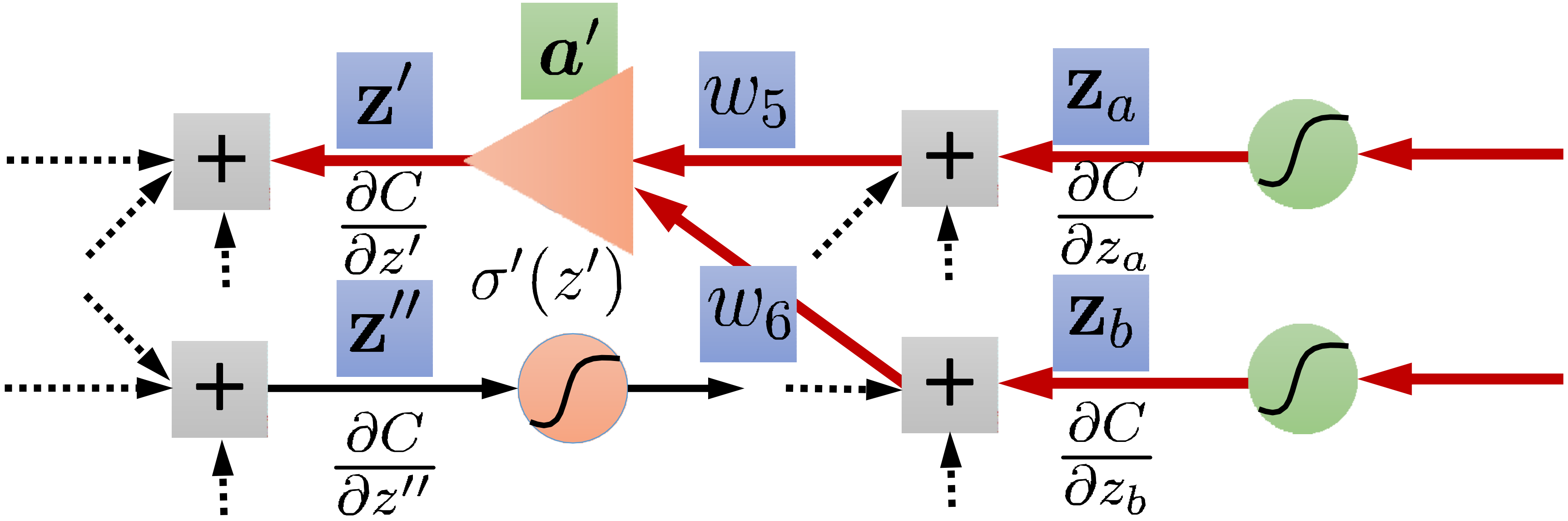
- Compute $\partial C/\partial z$ for all activation function inputs z
- Case 2. Not Output Layer



$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3 \frac{\partial C}{\partial z'} + w_4 \frac{\partial C}{\partial z''}\right]$$

Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z
- Case 2. Not Output Layer

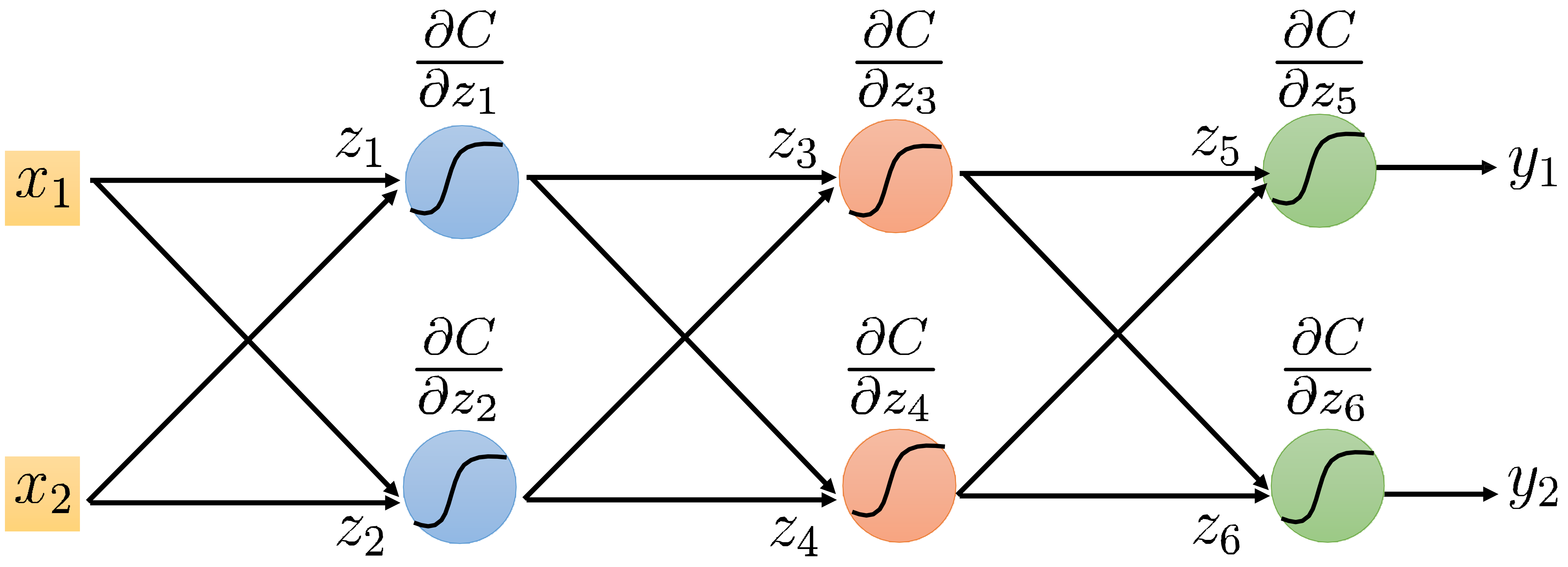


- Compute $\partial C/\partial z$ recursively

※ Until we reach the output layer ……

Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z
- Compute $\partial C/\partial z$ from the output layer

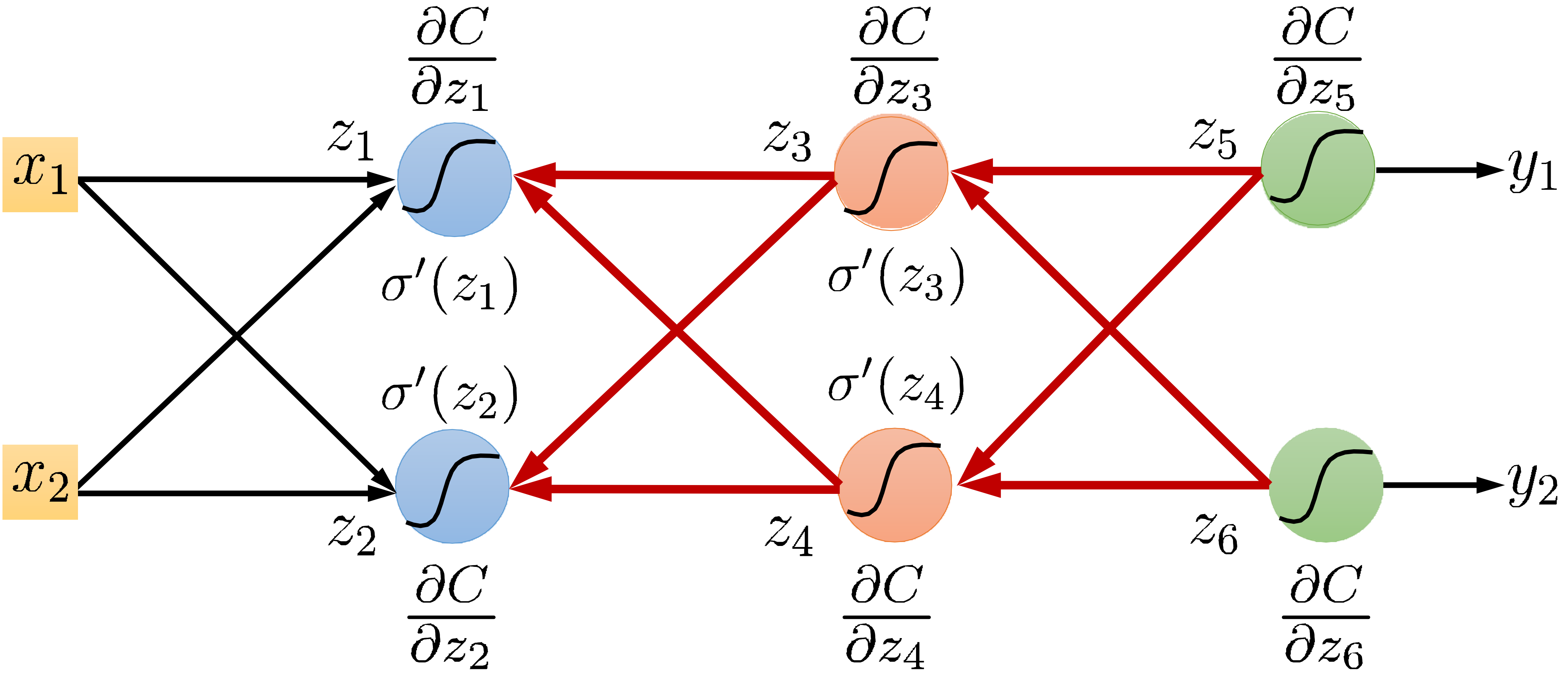


$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3\frac{\partial C}{\partial z'} + w_4\frac{\partial C}{\partial z''}\right]$$

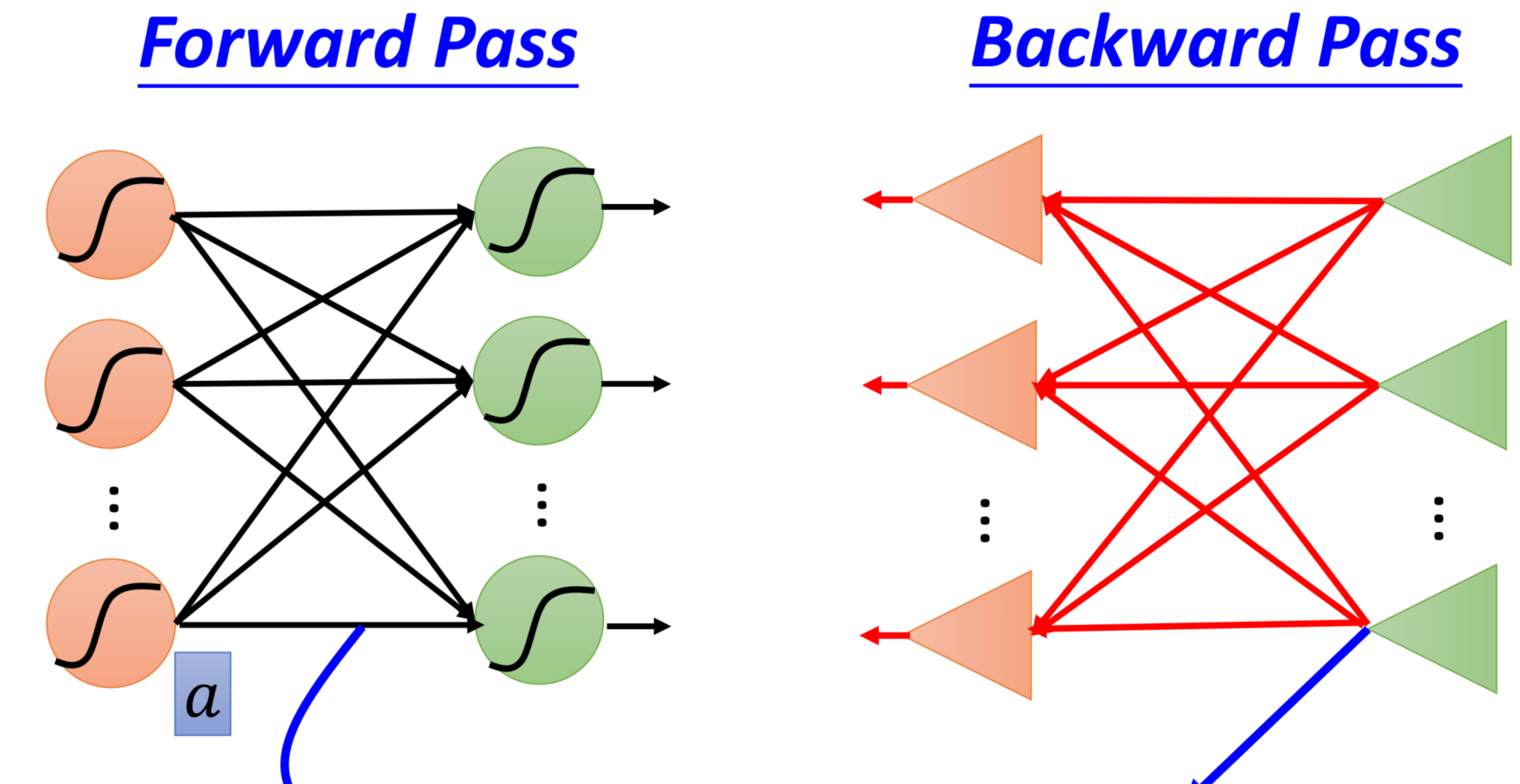
Backpropagation – Backward pass

- Compute $\partial C/\partial z$ for all activation function inputs z
- Compute $\partial C/\partial z$ from the output layer

$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3\frac{\partial C}{\partial z'} + w_4\frac{\partial C}{\partial z''}\right]$$



Backpropagation – Summary



$$\frac{\partial z}{\partial w} = a \quad \times \quad \frac{\partial C}{\partial z} \quad = \frac{\partial C}{\partial w}$$

for all w

References

- 《 Neural Networks and Deep Learning 》

  ※ written by Michael Nielsen

  ※ http://neuralnetworksanddeeplearning.com/

- 《 Deep Learning 》

  ※ written by Yoshua Bengio, Ian J. Goodfellow and Aaron Courville

  ※ http://www.deeplearningbook.org

- 《 Machine learning 》

  ※ https://speech.ee.ntu.edu.tw/~hylee/ml/2022-spring.php

Next chapter: Reinforcement Learning