Einstein's riddle

The situation:
* There are 5 houses in five different colors.
* In each house lives a person with a different nationality.
* These five owners drink a certain type of beverage, smoke a certain brand of cigar and keep a
  certain pet.
* No owners have the same pet, smoke the same brand of cigar or drink the same beverage.

The question is: Who owns the fish?

Hints:
* the *Brit* lives in the red house
* the *Swede* keeps dogs as pets
* the *Dane* drinks tea
* the green house is on the left of the white house
* the green house's owner drinks coffee
* the person who smokes Pall Mall rears birds
* the owner of the yellow house smokes *Dunhill*
* the man living in the center house drinks milk
* the *Norwegian* lives in the first house
* the man who smokes blends lives next to the one who keeps cats
* the man who keeps horses lives next to the man who smokes *Dunhill*
* the owner who smokes *BlueMaster* drinks beer
* the *German* smokes *Prince*
* the *Norwegian* lives next to the blue house
* the man who smokes blend has a neighbor who drinks water

34 EXTENDS *Integers*, *FiniteSets*, *Sequences*

Define the possible values for each of the categories

39 $Colors \triangleq \{$ "red", "white", "green", "yellow", "blue" $\}$
40 $Nationalities \triangleq \{$ "Brit", "Swede", "Dane", "Norwegian", "German" $\}$
41 $Beverages \triangleq \{$ "tea", "coffee", "milk", "beer", "water" $\}$
42 $Cigars \triangleq \{$ "PallMall", "Dunhill", "blends", "BlueMaster", "Prince" $\}$
43 $Pets \triangleq \{$ "dog", "birds", "cats", "horses", "fish" $\}$
44 $Houses \triangleq 1 .. Cardinality(Colors)$

The answer will be sequences of each of the categories. The owner of the fish will be *Nationality*[i]
where $Pet[i] = $ "fish".

All the permutations of a given set (as a set of sequences)

54 $Perm(set) \triangleq \{s \in [1 .. Cardinality(set) \rightarrow set] : \forall x, y \in 1 .. Cardinality(set) : s[x] \neq s[y] \vee x = y\}$

Define the "universe" of possible configurations.

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65  Universe ≜ [
66       color : {P ∈ Perm(Colors) :
67                    ∧ ∃ a, b ∈ Houses :
68                         ∧ a = b − 1
69                         ∧ P[a] = "green"
70                         ∧ P[b] = "white"
71                    ∧ P[2] = "blue"
72                    ∧ P[3] ≠ "green"
73       },
74       nationality : {P ∈ Perm(Nationalities) :
75                         ∧ P[1] = "Norwegian"
76                         ∧ P[3] ≠ "Dane"
77       },
78       beverage : {P ∈ Perm(Beverages) : P[3] = "milk"},
79       cigar : {P ∈ Perm(Cigars) :
80                    ∧ P[3] ≠ "BlueMaster"
81       },
82       pet : Perm(Pets)
83  ]

85  Answer ≜ CHOOSE ans ∈ Universe :
86           the Brit lives in the red house
87       ∧ ∃ h ∈ Houses : ∧ ans.nationality[h] = "Brit"
88                         ∧ ans.color[h] = "red"
89           the Swede keeps dogs as pets
90       ∧ ∃ h ∈ Houses : ∧ ans.nationality[h] = "Swede"
91                         ∧ ans.pet[h] = "dog"
92           the Dane drinks tea
93       ∧ ∃ h ∈ Houses : ∧ ans.nationality[h] = "Dane"
94                         ∧ ans.beverage[h] = "tea"
95           the green house is on the left of the white house (opt)
96       ∧ ∃ g, w ∈ Houses : ∧ ans.color[g] = "green"
97                           ∧ ans.color[w] = "white"
98                           ∧ g = w − 1
99           the green house's owner drinks coffee
100      ∧ ∃ h ∈ Houses : ∧ ans.color[h] = "green"
101                        ∧ ans.beverage[h] = "coffee"
102          the person who smokes Pall Mall rears birds
103      ∧ ∃ h ∈ Houses : ∧ ans.cigar[h] = "PallMall"
104                        ∧ ans.pet[h] = "birds"
105          the owner of the yellow house smokes Dunhill
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106        ∧ ∃ h ∈ Houses :  ∧ ans.color[h] = "yellow"
107                           ∧ ans.cigar[h] = "Dunhill"
108        the man living in the center house drinks milk (opt)
109        ∧ ans.beverage[3] = "milk"
110        the Norwegian lives in the first house (opt)
111        ∧ ans.nationality[1] = "Norwegian"
112        the man who smokes blends lives next to the one who keeps cats
113        ∧ ∃ b, c ∈ Houses :  ∧ ans.cigar[b] = "blends"
114                             ∧ ans.pet[c] = "cats"
115                             ∧ ∨ b = c − 1
116                               ∨ b = c + 1
117        the man who keeps horses lives next to the man who smokes Dunhill
118        ∧ ∃ h, d ∈ Houses :  ∧ ans.pet[h] = "horses"
119                             ∧ ans.cigar[d] = "Dunhill"
120                             ∧ ∨ h = d − 1
121                               ∨ h = d + 1
122        the owner who smokes BlueMaster drinks beer
123        ∧ ∃ h ∈ Houses :  ∧ ans.beverage[h] = "beer"
124                          ∧ ans.cigar[h] = "BlueMaster"
125        the German smokes Prince
126        ∧ ∃ h ∈ Houses :  ∧ ans.nationality[h] = "German"
127                          ∧ ans.cigar[h] = "Prince"
128        the Norwegian lives next to the blue house (opt)
129        ∧ ∃ n, b ∈ Houses :  ∧ ans.nationality[n] = "Norwegian"
130                             ∧ ans.color[b] = "blue"
131                             ∧ ∨ n = b − 1
132                               ∨ n = b + 1
133        the man who smokes blend has a neighbor who drinks water
134        ∧ ∃ b, w ∈ Houses :  ∧ ans.cigar[b] = "blends"
135                             ∧ ans.beverage[w] = "water"
136                             ∧ ∨ b = w − 1
137                               ∨ b = w + 1
138   FishOwner ≜ LET h ≜ CHOOSE h ∈ Houses : Answer.pet[h] = "fish"
139               IN   Answer.nationality[h]

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