MODULE *KnightsBridge*

Four knights must cross a norrow bridge . . .

It is night, and they will need the light from *Archibald*'s torch to cross. Sadly, the bridge will hold only two knights at a time, and the torch will only burn for another fifteen minutes. Now, *Archibald* is quick and nimble, he can make it in just one minute. Baldur is strong and can make it in two minutes. Now *Charleston* is old and will need five minutes to cross. And lastly *David* is trimit and frail, he will need eight minutes to make the trip.

Can all four knights cross the bridge before the torch burns out?

https://*twitter.com*/*MCLMouritzen*/status/1362929925083308032

EXTENDS *Integers*, *FiniteSets*

VARIABLES *Knights*, The set of knights
TorchSide, The side of the river the torch is on
ElapsedTime The amount of time that has passed

The sides of the river.

This is used for type checking (*TypeOk*)

$Side \triangleq \{\text{“Near”}, \text{“Far”}\}$

$Max(S)$ returns the maximum element in a set (of numbers)

$Max(S) \triangleq \text{CHOOSE } m \in S : (\forall\, n \in S : m \geq n)$

The type invariant for this specification.

$TypeOk \triangleq \land \forall\, k \in Knights : k.side \in Side$ Every knight must be on a side
$\land\ TorchSide \in Side$ The torch must be on a side
$\land\ ElapsedTime \in Nat$

The starting configuration:
- All knights start on the *Near* side
- Each knight takes a certain amount of time to cross (constant)
- The torch starts on the *Near* side
- Time starts at zero

$Init \triangleq \land\ Knights = \{$
$[side \mapsto \text{“Near”}, time \mapsto 1],$ Archibald
$[side \mapsto \text{“Near”}, time \mapsto 2],$ Baldur
$[side \mapsto \text{“Near”}, time \mapsto 5],$ Charleston
$[side \mapsto \text{“Near”}, time \mapsto 8]$ David
$\}$
$\land\ TorchSide = \text{“Near”}$ Torch starts on the *Near* side
$\land\ ElapsedTime = 0$ Time starts from zero

The allowed steps are to move one or two (max the bridge will hold) knights, with the torch from one side to the other.

For a step to be valid, the torch must be on the starting side, and there must be one or more knights also on that side.

The time that elapses for the crossing is the time it takes the slowest knight to cross.

```
NearToFar ≜ ∧ TorchSide = "Near"
            ∧ ∃ travelers ∈ SUBSET Knights :
                ∧ Cardinality(travelers) = 1 ∨ Cardinality(travelers) = 2
                ∧ ∀ k ∈ travelers : k.side = "Near"
                ∧ Knights' = {IF k ∈ travelers
                              THEN [k EXCEPT !.side = "Far"]
                              ELSE k
                              : k ∈ Knights}
                ∧ ElapsedTime' = ElapsedTime + Max({k.time : k ∈ travelers})
            ∧ TorchSide' = "Far"

FarToNear ≜ ∧ TorchSide = "Far"
            ∧ ∃ travelers ∈ SUBSET Knights :
                ∧ Cardinality(travelers) = 1 ∨ Cardinality(travelers) = 2
                ∧ ∀ k ∈ travelers : k.side = "Far"
                ∧ Knights' = {IF k ∈ travelers
                              THEN [k EXCEPT !.side = "Near"]
                              ELSE k
                              : k ∈ Knights}
                ∧ ElapsedTime' = ElapsedTime + Max({k.time : k ∈ travelers})
            ∧ TorchSide' = "Near"

Next ≜ ∨ NearToFar
       ∨ FarToNear

Spec ≜ Init ∧ □[Next]⟨Knights, TorchSide, ElapsedTime⟩
```

The riddle is solved by having all knights on the far side of the river within the 15 minutes that the torch has left.

By specifying *AllAcross* = FALSE as invariant, the model checker will alert us to any execution traces that result in a solution.

```
AllAcross ≜ ∧ ∀ k ∈ Knights : k.side = "Far"
            ∧ ElapsedTime ≤ 15
```

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* Modification History
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