Ann Oper Res (2016) 242:303–320
DOI 10.1007/s10479-015-1792-x

An improved particle swarm optimization for carton heterogeneous vehicle routing problem with a collection depot

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Published online: 5 March 2015

Abstract In this paper, a carton heterogeneous vehicle routing problem with a collection depot is presented, which can collaboratively pick the cartons from several carton factories to a collection depot and then from the depot to serve their corresponding customers by using of heterogeneous fleet. Since the carton heterogeneous vehicle routing problem with a collection depot is a very complex problem, particle swarm optimization (PSO) is used to solve the problem in this paper. To improve the performance of the PSO, a self-adaptive inertia weight and a local search strategy are used. At last, the model and the algorithm are illustrated with two test examples. The results show that the proposed PSO is an effective method to solve the multi-depot vehicle routing problem, and the carton heterogeneous vehicle routing problem with a collection depot. Moreover, the proposed model is feasible with a saving of about 28 % in total delivery cost and could obviously reduce the required number of vehicles when comparing to the actual instance.

Keywords Carton · Heterogeneous vehicle routing problem with a collection depot · Particle swarm optimization · Local search · Self-adaptive inertia weight

1 Introduction

1.1 Background

Carton is a special product whose production cycle is short. Moreover, carton is generally used as the outer packaging. However the leaflet on the cartons may frequently revise according

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to the market demand or the note describing the revised product design. Therefore, some former cartons are always forced to be waste products when the leaflets on the cartons or the size of cartons have been revised. To reduce the customers' inventory cost, they want to be served with a small quantity of cartons every day. As for carton factories, they also attempt to attain the maximum profit while meeting the demands of customers. Due to that the delivery cost for serving customers with a small quantity of cartons every day is too high to endure, carton factories attempt to decrease their operating costs by assigning the delivery task to a third-party logistics company. Similarly, the pursuit of the maximum profit is also a constant challenge for a third-party logistics company.

In fact, the profit for the third-party logistics company comes from the difference between the delivery cost from carton factories and the delivery cost from the third-party logistics company. Thus, a least-cost delivery route has attracted the interests of a third-party logistics company. Moreover, the fixed cost from vehicles is also considered by the third-party logistics company. In the classical vehicle routing problem (VRP), the vehicles with a same capacity and fixed cost are not suitable for real-life cases. It is due to that a feasible combination with heterogeneous vehicles can be more flexible than the fix cost from vehicles. Therefore, the third-party logistics company attempts to seek the least cost, in which case the least-length vehicle routes and the least fixed cost for several vehicle models are simultaneously considered while completing the carton delivery task. This paper deals with a real-life VRP (that is the carton heterogeneous VRP with a collection depot in this paper) for a third-party logistics company. Thus, a major objective of this study is to acquire appropriate strategy to look for the least-cost vehicle routes and optimal vehicle combination for the third-party logistics company.

1.2 Problem description

To describe the carton heterogeneous vehicle delivery problem with a collection depot proposed in this paper, we generated a special example of three carton factories (A,B,C) and 8 customers (1,2,3,4,5,6,7,8) in which all the demands of customers cannot be beyond the capacity of one vehicle. And there are some customers which belong to several factories. For example, customer 3 is served by factory A and B; Customer 6 is served by factory B and C. Then Fig. 1 describes the difference between a current carton VRP and a carton heterogeneous VRP with a collection depot. In Fig. 1, the routes for the carton heterogeneous VRP with a collection depot are obviously less than the current carton vehicle routing problem. Moreover, the current carton VRP requires three vehicles while the carton VRP with a collection depot requires one vehicle. It is due to that, as a third-party logistics company, to acquire the maximize profits for itself, the cartons from different factories may be placed in a bigger vehicle. Thus, one big vehicle can contain the cartons from some factories, or even from all the factories in a special case. Thus, a carton VRP with a collection depot is proposed in this paper.

From Fig. 1, it can be found that there are some differences compared with the current situation. Firstly, there are two VRPs. One is the pickup process, which is that the vehicles start from the collection depot (third-party logistics company) O to pick up the cartons from these factories without exceeding the capacity constraints of each vehicle, and lastly return to the collection depot. The other is the delivery process, which is that the vehicles start from the collection depot O to send the cartons to the corresponding customers accordingly, and lastly return to the collection depot. Secondly, in the current situation, each carton factory uses a fixed fleet of unified vehicles whose capacity is relatively small due to minor demands of these customers every day. While in the carton VRP with a collection depot based on

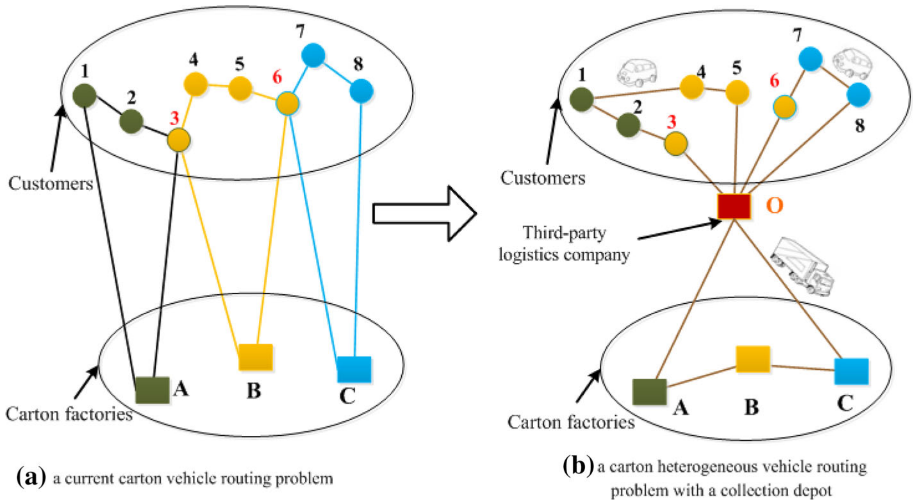

Fig. 1 Comparison between a current carton vehicle routing problem and a carton heterogeneous vehicle routing problem with a collection depot

heterogeneous fleet, the vehicles may be heterogeneous due to the dependence on the total cost. The concept of the VRP includes determining vehicle routes from a depot to a number of customers without exceeding vehicle capacity constraints.

1.3 Literature review

Finding efficient vehicle routes have been studied for the last 50 years (Yu et al. 2009; Crevier et al. 2007; Athanasopoulos and Minis 2013; Goncalves et al. 2014; Mu and Eglese 2013). In classical VRP or multi-depot vehicle routing problem (MDVRP), customer demands/locations are generally assumed to be known and deterministic. The objective of VRP/MDVRP is to find a set of tours for several vehicles from a depot/depots to a lot of customers and return to the depot without exceeding the capacity constraints of each vehicle at minimum cost. Therefore, most of literatures paid attentions on the least-cost vehicle routes in a single-load delivery method like VRP and MDVRP. However, some aspects that arise in real-life applications have received many attentions. For example, a good mixed-load delivery method, in which several types of goods are collected in one vehicle, can shorten the length of delivery routes and decrease the cost of delivery goods. In recent years, a few researchers have studied the integrated vehicle to decrease the length of vehicle delivery. Liu and Chen (2008) attempted to solve the profit allocation of integrated delivery. In their study, a modified method based on compensation is introduced to solve the problem. Zhang and Luo (2008) proposed shapely value method to solve the benefit assignment problems of integrated delivery. There are few literatures on integrated delivery. It is due to that in the integrated delivery, different sizes goods should be collected in one vehicle, which increases the complexity of problem. However, lots of literatures have paid attentions on a special integrated delivery, which is the VRP with simultaneous delivery and pick-up. Zachariadis et al. (2010) proposed an adaptive memory algorithm to solve the VRP with simultaneous pick-ups and deliveries. Zhang et al. (2012) attempted to solve the VRP with simultaneous pick-ups and deliveries by a scatter search.

Since the customer combination is not restricted to the selection of vehicle routes, besides considering the constraints of a classic VRP, the carton VRP with a collection depot like VRP is also considered as a combinatorial optimization problem where the number of feasible solutions for the problem increases exponentially with the number of customers increasing Bell and McMullen (2004). Many literatures suggested that heuristic algorithm was often a first choice to solve this kind of complicated problems (Chen et al. 2012, 2013a, b; Yao et al. 2014a, c, 2013; Yu and Yang 2011a; Yu et al. 2011a, 2013). Among heuristic algorithms, particle swarm optimization (PSO) is a heuristic which is a population-based search method developed by Kennedy and Eberhart (1995). PSO is derived from the social behaviors of bee swarm, fish school or bird flock. Thus, in PSO, each solution of optimization problem is corresponding to the position of one particle in the searching space. Then, PSO attains the search for optimum by the physical movement of the individuals which is based on the experiences from the best particle in the swarm and itself. Every particle moves iteratively until the number of iteration reaches the maximum number of generations. At the end, the position of best particle is the best solution of the solving problem.

The PSO is a very popular method which has been successfully used to solve many complex problems. Mohemmed et al. (2008) presented particle swarm optimization to solve the shortest path routing problems, in the PSO, the authors used a modified priority-based encoding with a heuristic operator to reduce the possibility of loop-formation in the path construction process. Lin et al. (2010) attempted to solve the job shop scheduling problem by using of PSO. Ai and Kachitvichyanukul (2009) proposed a PSO with multiple social structures to solve the VRP with simultaneous pickup and delivery. These successful experiences suggested that PSO is easier to be implemented and few parameters need to be adjusted (Eberhart and Shi 2001; Chatterjee and Siarry 2006). Therefore, PSO is also adopted to solve the carton VRP with a collection depot in this paper. However, considering the weaker local search ability of PSO, some corresponding improvement strategies are introduced to improve the searching performance of this algorithm.

Therefore, our aim is to adapt to PSO for the carton heterogeneous VRP with a collection depot. The rest of the paper is organized as follows. Section 2 introduces the carton heterogeneous VRP with a collection depot. In Sect. 3, PSO and some improved strategies are presented. The computational results of some benchmark instances on MDVRP and a carton heterogeneous VRP with a collection depot are discussed in Sect. 4 and lastly, the conclusions are provided in Sect. 5.

2 The carton heterogeneous vehicle routing problem with a collection depot

Firstly, we will describe a simple generalization about the carton heterogeneous VRP with a collection depot. In the carton heterogeneous VRP with a collection depot, due to that there are two VRPs in the problem, one VRP is for the pickup process and the other VRP is for the delivery process. In the pickup process, it is assumed that the third-logistics company (or a stop from the third-logistics company) is viewed as its original depot and the carton factories are viewed as the customers. The process of the VRP is that vehicles start from the depot, then go to serve customers assigned by collecting the cartons and upon completion of their routes return to the depot. In the delivery process, it is also assumed that the collection depot is viewed as its original depot and the customers are still as the customers. The process of the VRP is that vehicles start from the depot, then go to serve customers, and upon completion of their routes return to the depot. In the two VRPs, there is a common depot, and the customers (carton factories) in the pickup process have some customers (in the delivery process). Due to that the vehicle capacities are different in the two VRPs, a mathematical model for the proposed carton VRP with a collection depot is formulated below, aiming to minimize the total delivery cost.

There is a third-party logistics company O, H carton factories and N customers to be served. The pickup demands of carton factory H are q_h and the delivery demands of customer i are q_i $(i = 1, \ldots, n)$, $q_h < q_k$ and $q_i < q_k(q_k$ is the limit load of vehicle k). The distance between two nodes is described as the vertex set C which is partitioned into two subsets: $C_f = \{c_0, c_1, \ldots c_g, \ldots, c_H\}$ is the set of carton factories and $Cc = \{c_0, c_1, \ldots c_i, \ldots, c_N\}$ is the set of customers, respectively. The distance matrix is a real symmetric one satisfying the triangle inequality principle, that is, $c_{ik} \leq c_{ij} + c_{jk}$. Each vehicle can serve for several carton factories (or several customers) whose pickup demands (delivery demands) must not overpass the transportation capacity of this vehicle. Since different vehicle types have different transportation cost, a cost factor is introduced to represent the transportation cost per kilometer of each vehicle type. The cost factor λ_r is considered to be related with the cost of petrol, labor and so on in this paper. Thus, λ_{r1} and λ_{r2} denote the cost factor of the vehicle during the pickup process and the delivery process, respectively. Thus, the carton heterogeneous VRP with a collection depot consists of two parts. One is the transportation cost and the other is the fixed cost. Since the objective of the carton vehicle routing problem with a collection depot is to minimize the total delivery cost during the process of carton pickup and delivery while meeting the following constraints. Thus, the formulation of the carton heterogeneous VRP with a collection depot is described as follows.

$$\min \left(\sum_{g=0}^{H} \sum_{h=0,h\neq g}^{H} \sum_{k=1}^{K} d_{gh} x_{gh}^{k} \right) \times \lambda_{r1} + \left(\sum_{i=0}^{N} \sum_{j=0,j\neq i}^{N} \sum_{k'=1}^{K'} d_{ij} x_{ij}^{k'} \right) \times \lambda_{r2} + \sum_{k=1}^{K} F_k x_k + \sum_{k'=1}^{K'} F_{k'} x_{k'} \tag{1}$$

$$\sum_{g=1}^{H} x_{gh}^{k} = \sum_{g=1}^{H} x_{hg}^{k} \leq 1 \; for \quad g = 0, k \in \{1, 2, \ldots, K\}$$

$$and \quad \sum_{i=1}^{N} x_{gh}^{k} = \sum_{j=1}^{N} x_{hg}^{k} \leq 1 \; for \quad i = 0, k' \in \{1, 2, \ldots, K'\} \tag{1a}$$

$$\sum_{k'\in K} \sum_{j\in C_c ? j\neq i} x_{ij}^{k'} = 1 \quad and \quad \sum_{k\in K} \sum_{h\in C_z ? h\neq g} x_{gh}^{k} = 1 \quad i \in C_C, \; g \in C_f \tag{1b}$$

$$\sum_{k'\in K} \sum_{i\in C_c ? i\neq j} x_{ij}^{k'} = 1 \quad and \quad \sum_{k\in K} \sum_{g\in C_f ? g\neq h} x_{gh}^{k} = 1 \quad j \in C_C, \; h \in C_f \tag{1c}$$

$$\sum_{i\in C_c} q_i \sum_{i\in C_c ? i\neq j} x_{ij}^{k'} \leq q_{k'} \; and \quad \sum_{g\in C_f} q_h \sum_{g\in C_f ? g\neq h} x_{gh}^{k} \leq q_k k \in K, k' \in K' \tag{1d}$$

where,

$$x_{ij}^{k'} = \begin{cases} 1 & \text{the link from customer i to j is visited by vehicle k}' \\ 0 & \text{otherwise} \end{cases}$$

$$x_{gh}^{k} = \begin{cases} 1 & \text{the link from factory g to h is visited by vehicle k} \\ 0 & \text{otherwise} \end{cases}$$

$$x_k = \begin{cases} 1 & \text{vehicle k is selected for delivery} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{k'} = \begin{cases} 1 & \text{vehicle k}' \text{ is selected for delivery} \\ 0 & \text{otherwise} \end{cases}$$

K, K' are the number of smaller vehicle and bigger vehicle from the third-party logistics company, respectively.

d_{gh} is the length on the link from factory g to h.
d_{ij} is the length on the link from customer i to j.
q_g is the quantity of the goods to be picked up at factory g.
q_i is the quantity of the goods to be delivered at customer i.
q_k *and* $q_{k'}$ are the capacity of vehicle k and the capacity of vehicle k', respectively.
(1a) makes sure every vehicle starts and ends at the third-party logistics company O.
(1b)–(1c) assume that every factory (customer) node can be visited exactly once by one vehicle.
(1d) assumes the quantity of a vehicle cannot exceed its capacity.

3 IPSO for the carton heterogeneous vehicle routing problem with a collection depot

3.1 Particle swarm optimization

PSO is an optimization method which was developed by Kennedy and Eberhart (1995). In PSO, each individual in their population is as a particle. Each particle flies at a certain speed in the searching space and adjust its position dynamically according to not only its own experience, but also the experiences of other particles. That is, each particle constantly adjusts its way and speed based on a positive feedback mechanism by using of the optimal value of itself and the population. The basic algorithm for a global continuous optimization problem, corresponding to the fitness of each particle to the environment, uses a population of m particles; each particle i of the population is associated with a position and a fitness. Thus, the principles of the algorithm can be described as follow (Nickabadi et al. 2011; Mohemmed et al. 2008).

In PSO, the problem solution space is formulated as a search space. Each position of each particle is a solution of the problem. Particles cooperate to find the best solution in search space. The particle movement is mainly affected by three factors: inertia, the best position of the particle and the global best position of the whole population. The inertia is the velocity of the particle in the latest iteration, which is controlled by inertia weight. The objective of the inertia is to prevent particles from moving back to their current positions. The best position and the global best position are the best solutions found by each particle itself and the whole population so far, respectively.

If PSO is adopted to solve a continuous optimization problem with$d$ variables, the solution space can be formulated as a d-dimensional search space. The position (X_i) and the speed (V_i) of particle i can be denoted as $(X_{i1}, X_{i2}, \ldots, X_{id})$ and $(V_{i1}, V_{i2}, \ldots, V_{id})$, respectively. At the iteration t, if the best fitness of particle i is p_i^t and the best fitness of the whole population is p_g^t. Then, the velocity and the position of the particle i are updated using the following formula:

$$v_i^{t+1} = wV_i^t + c_1 rand1(p_i^t - X_i) + c_2 rand2(p_g^t - X_i) \tag{2}$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{3}$$

where t is the iteration counter; c_1 and c_2 are the acceleration coefficients which determine the effect of the experiences from the particle and the population. w is the inertia weight which is used to control the impact of previous histories of velocities on current velocity.

3.2 Self-adaptive inertia weight

We can see that the inertia weight w is the key element affecting the performance of the algorithm according to Eq. (2). It can be found that if the value of the inertia weight w is larger, the particle will have stronger global searching ability (Nickabadi et al. 2011; Shi and Eberhart 2001). While the value of the inertia weight w is smaller, the local searching ability of the particle will be enhanced. The balance between global and local search throughout the searching process is critical to the success of an optimization algorithm. If the inertia weight can be changed dynamically, the search capability can be dynamically adjusted.

Most successful applications on the inertia weight are to use time-varying inertia weight strategies, in which the value of the inertia weight is determined based on the iteration number (Nickabadi et al. 2011). Based on the Formula (2) and (3), it can be attained that, in the early stage of the iteration, w with larger value would lead to particles searching in a large space. In the late of the iteration, w with smaller value would lead to particles carefully searching in a small space. But, if a general linear model for determining the value of the inertia weight w, it would lead to that the inertia weight w decreases rapidly and couldn't keep a larger value in the early algorithm for a long time. Therefore, a self-adaptive inertia weight w is introduced to control the convergence behavior of PSO (Yue and Sun 2011).

$$w_c = w_0 + \sum_{k=1}^{N} \left[\frac{\left| g' - \bar{g} \right|}{(g_{\max} - \bar{g})} \right]^{[(t\times 2)/N]+1} \tag{4}$$

where, w_o is the initial value of the inertia weight w. In general, $w_o = 1$. g_{max} and $\bar{g}$ represent the best fitness and the average fitness of the population until now. g' is the better fitness between the best fitness of particle i (p_i^t) and the best fitness of the whole population p_g^t. t is the current iteration and N is the maximum number of iterations. Equation (4) provides a self-adaptive non-linear model for optimizing the value of the inertia weight w which is used to balance the global and local searching ability of the algorithm.

3.3 Local search optimization

From the basic concept of PSO, it can be found that the search process of PSO is based on a positive feedback mechanism among the whole population. However, a particle itself has not a variation mechanism for evolution. Thus, when a single particle traps into a certain local optimum, it is difficult for the particle to get out. Therefore, it needs the successful discoveries of other particle to help get out the local optimum. In fact, the optimizing ability of PSO mainly depends on the mutual interaction and influence among particles. To enlarge the interaction and influence from other particles, a new search strategy is also used in this paper (Duan et al. 1993; Nelder and Mead 1965) which has been successfully used to solve some complicated problems (Yao et al. 2014b). The process of the search strategy can be described as follows:

(1) Randomly initialize the position and speed of all the particle groups within the allowable range. Compute the fitness of each particle (solution).
(2) Select s points randomly from the feasible particle groups and sort the s points in an increasing order.

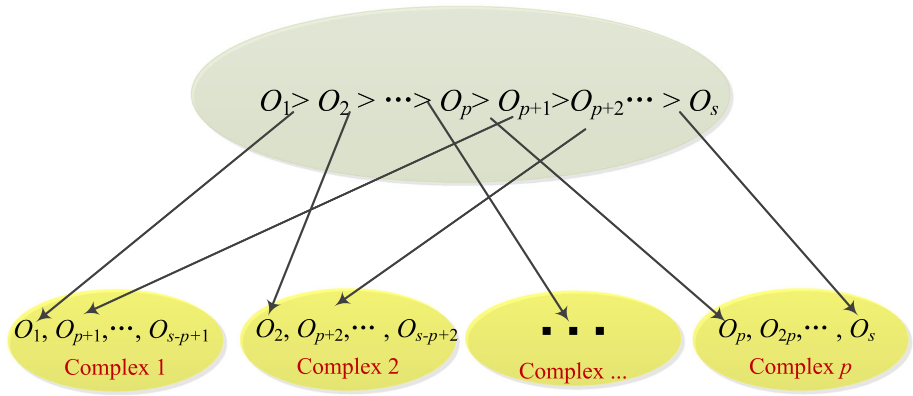

Fig. 2 The description of the local optimization based on complexes

(3) Partition the s points into p complexes, each containing v points. Therefore, the complexes are partitioned such that the first complex contains every $p(j-1)+1$ ranked point, the hth complex contains every $p(j-1)+h$ ranked point, and so on, where $j = 1, 2, \ldots h \ldots v$ (See Fig. 2).
(4) To effectively describe the process, the best fitness of each particle up to now is denoted as “individual extremum”. Then, If the current fitness of a certain particle is better than its individual extremum, the individual extremum will updated.
(5) To effectively describe the process, the best fitness of each complex up to now is denoted as “global extremum”. If the current global fitness of each complex is better than its global extremum, the global extremum will updated.
(6) When the search reaches a certain number of generations (N_j), combine the points in all evolved complex into a single sample population; sort the population in an increasing order and shuffle (i.e. re-partition) them into p complexes according to procedure specified in Step (3).
(7) If the convergence criteria is satisfied, then stop the calculation, otherwise, continue.
(8) Update the position and flying speed of each particle based on Formula (2) and (3).

4 Numerical analysis

The integrated vehicle routing model and the improved PSO (IPSO) are tested with two VRP. The first one is some well-known benchmark problems of MDVRP. The second one aims to test the performance of the model and the algorithm for optimizing a real carton VRP with a collection depot in the city of Dalian, China. Then, the IPSO were coded in Visual studio. NET 2003: C++ and executed on a PC equipped with the Pentium IV 2.93 GHz processor and 3 GB for the Windows platform. Large numbers of experiments to determine the IPSO parameters are set as that: the number of particle is 52 which are set into four complexes with 13 points each complex. The number of iteration is 1,000. c_1 and c_2 is equal to 2. The following will describe the two examples, respectively.

4.1 The benchmark problems of MDVRP

The first is some benchmark problems of MDVRP problems, as presented in Christofides and Eilon (1969), Gillett and Johnson (1976) and Chao et al. (1993), which can be

Table 1 The information of the test problems

No.	H	n	Q	No.	H	n	Q
1	4	50	80	13	2	80	60
2	4	50	160	14	2	80	60
3	5	75	140	15	4	160	60
4	2	100	100	16	4	160	60
5	2	100	200	17	4	160	60
6	3	100	100	18	6	240	60
7	4	100	100	19	6	240	60
8	2	249	500	20	6	240	60
9	3	249	500	21	9	360	60
10	4	249	500	22	9	360	60
11	5	249	500	23	9	360	60
12	2	80	60				

downloaded from http://neo.lcc.uma.es/radi-aeb/WebVRP//index.html?/ProblemInstances/MDVRPInstances.html., are also used to in this paper. The main characteristics of the benchmark problems are shown in Table 1. In Table 1, H, n, Q denotes the number of depots, the number of customers and the capacity of a vehicle, respectively.

In order to examine the performance of the IPSO, the results from FIND algorithm (Renaud et al. 1996), CGL method (Cordeau and GendreauMand Laporte 1997) and PIACO (Yu et al. 2011a) are used for comparison with the ones of the IPSO. Table 2 is the results from these algorithms for solving the MDVRP.

From Table 2, it can be seen that the number in bold from IPSO are the best solutions among several algorithms. It can be found that most of the results from IPSO are close to the well-known best solutions. Thus, the IPSO is suitable for solving MDVRP. Due to the similarity between MDVRP and the carton VRP with a collection depot, the paper attempts to use this proposed PSO to solve the carton VRP with a collection depot.

To further validate the performance of a self-adaptive inertia weight and a local search strategy, a standard PSO and IPSO are compared in this paper. The computational results are as shown in Table 3.

It can be observed from Table 3 that the solution quality of IPSO is obviously better than the standard PSO, especially for some relatively larger problems. It can be explained that the a self-adaptive inertia weight and a local search strategy can large the search space and improve the performance of PSO. Therefore, this is just as expected as the incorporation of the adaptive strategy and a local search strategy for PSO can ensure the better optimization.

4.2 The carton heterogeneous vehicle routing problem with a collection depot

The above chapter verifies the IPSO is effective for solving vehicle routing problem by the benchmark problem of MDVRP. Then, a carton heterogeneous VRP with a collection depot which is encountered by a third-party logistics company is studied in this paper. In the carton heterogeneous VRP with a collection depot, some information is considered as a given condition while the problems are created, such as the locations of the third-party company, factory nodes and customer nodes, the number of the factory and customer nodes, the real route distance between two nodes. The schematic view of the carton heterogeneous VRP with a collection depot can be described in Fig. 3. In Fig. 3, there are eight factories: *A*,

Table 2 Computational results using IPSO and some well-known published results

No.	Best-known results	FIND (Renaud et al. 1996)	CGL (Cordeau and GendreauMand Laporte 1997)	PIACO (Yu et al. 2011b)	IPSO
1	576.86	576.86	576.86	576.86	576.86
2	473.53	473.53	473.87	473.53	473.53
3	641.18	641.18	645.15	641.18	641.18
4	1,001.49	1,003.86	1,006.66	1,001.49	1,001.49
5	750.26	750.26	753.4	750.26	750.26
6	876.5	876.5	877.84	876.5	876.5
7	885.69	892.58	891.95	885.69	885.69
8	4,437.58	4,485.08	4,482.44	4,482.38	4,485.09
9	3,900.13	3,937.81	3,920.85	3,912.23	3,937.82
10	3,663.00	3,669.38	3,714.65	3,663.00	3,663.00
11	3,554.08	3,648.94	3,580.84	3,554.08	3,648.95
12	1,318.95	1,318.95	1,318.95	1,318.95	1,318.95
13	1,318.95	1,318.95	1,318.95	1,318.95	1,318.95
14	1,360.12	1,365.68	1,360.12	1,365.68	1,365.68
15	2,505.29	2,551.45	2,534.13	2,551.45	**2,505.29**
16	2,572.23	2,572.23	2,572.23	2,572.23	2,587.87
17	2,708.99	2,731.37	2,720.23	2,708.99	2,708.99
18	3,702.75	3,781.03	3,710.49	3,781.03	3,781.04
19	3,827.06	3,827.06	3,827.06	3,827.06	3,827.06
20	4,058.00	4,097.06	4,058.07	4,097.06	4,058.07
21	5,474.74	5,656.46	5,535.99	5,474.74	5,474.74
22	5,702.06	5,718	5,716.01	5,772.23	**5,702.06**
23	6,095.36	6,145.58	6,139.73	6,125.58	6,145.58

Table 3 The comparison results between a standard PSO and IPSO

No.	PSO	IPSO	No.	PSO	IPSO
1	576.86	576.86	13	1,318.95	1,318.95
2	484.28	473.53	14	1,365.68	1,365.68
3	645.16	641.18	15	2,551.45	2,505.29
4	1,001.49	1,001.49	16	2,587.87	2,587.87
5	750.26	750.26	17	2,708.99	2,708.99
6	876.5	876.5	18	3,781.04	3,781.04
7	887.11	885.69	19	3,827.06	3,827.06
8	4,500.15	4,485.09	20	4,097.06	4,058.07
9	3,913	3,937.82	21	5,495.54	5,474.74
10	3,693.4	3,663.00	22	5,772.23	5,702.06
11	3,648.95	3,648.95	23	6,183.13	6,145.58
12	1,318.95	1,318.95			

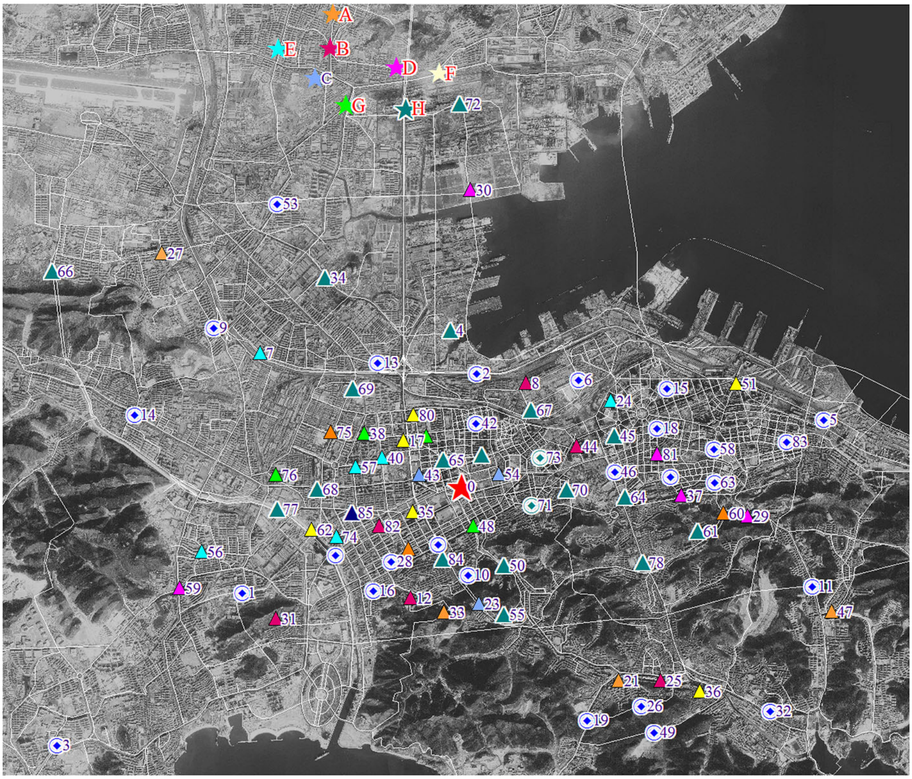

Fig. 3 The location information of the carton heterogeneous vehicle routing problem with a collection depot

B, C, D, E, F, G and *H* which are denoted with *star*. There are 85 customers who are denoted with *triangle* and the third-party logistics company *O* which is denoted with a bigger *star*. To distinguish the customers that belong to their factories, the factory and the corresponding customers are denoted in the same color. However, there are many customers who need serving by several factories simultaneously. Thus, the common customers are denoted with *circle.*

As Fig. 4 shows, in the carton heterogeneous vehicle routing problem with a collection depot, there are eight carton factories and 85 customers in Dalian city. The problem is how the vehicle of the third-party logistics company picks the cartons from the eight factories and delivers the cartons to the corresponding customers with the least delivery cost. As shown in Fig. 4, the eight carton factories, the number represents the geographical position of customers and the *O* denotes the third-party logistics company. In addition, since the only one difference between different cartons for delivery is the carton size, different cartons can be delivered in one vehicle by changing into the standard one by quantity, in order to unify the cartons from different factories, the number of different cartons is converted into the number of the standard carton. Thus, the demands from customers are also adjusted accordingly. The demand information of the carton heterogeneous VRP with a collection depot is shown in Table 4.

As is the case in practice, the distance traveled by vehicles should be measured based on the road network. Thus, a method is used to measure the actual distance among all the points

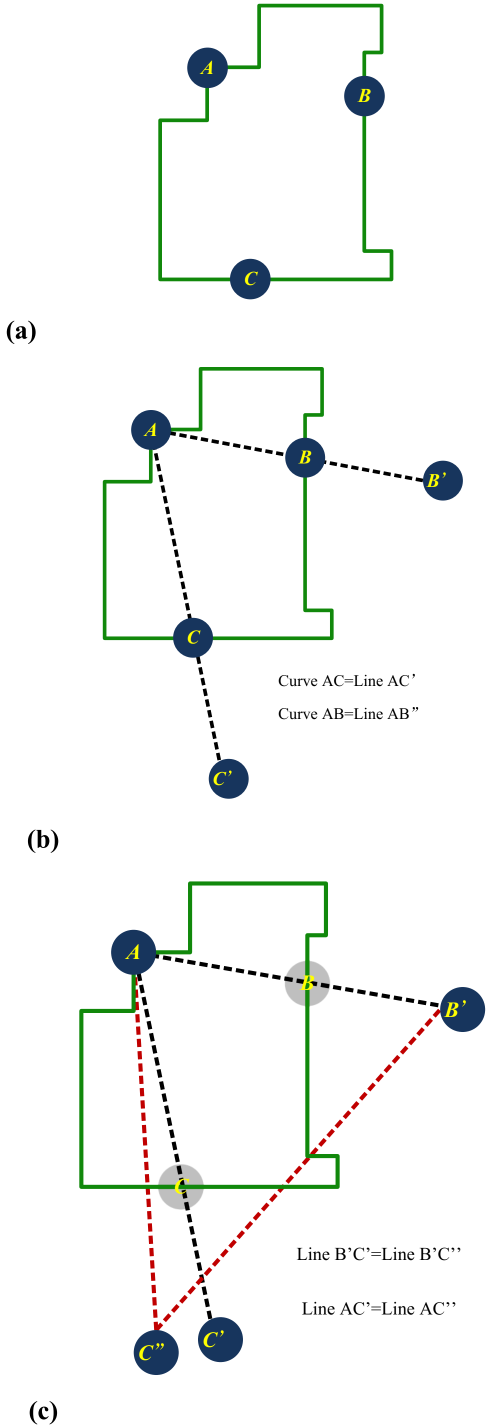

Fig. 4 The construction of the distance matrix based on road network. **a** The location information based on route network of three points. **b** The conversion of the actual route length between two points. **c**The conversion of the actual route length among three points

Table 4 The demand information of the carton heterogeneous vehicle routing problem with a collection depot

n	A	B	C	D	E	F	G	H	n	A	B	C	D	E	F	G	H
1	–	2	2	–	2	–	2	–	44	–	3.2	–	–	–	–	–	–
2	–	3.7	–	–	–	2	–	–	45	–	–	–	–	–	–	–	4.7
3	2	–	–	2.6	–	–	–	–	46	–	–	–	7	2	–	–	–
4	–	–	–	-	–	–	–	3.8	47	3	–	-	–	–	–	–	–
5	2.4	–	2.2	–	–	–	–	–	48	–	–	–	–	–	–	3.5	–
6	–	–	–	3	–	–	6	–	49	–	–	2	–	–	–	2	–
7	–	–	–	–	3.2	–	–	–	50	–	–	–	–	–	–	–	4
8	–	5.9	–	–	–	–	–	–	51	–	–	–	–	–	2	–	–
9	–	–	–	2.5	–	2.5	–	–	52	–	–	–	–	–	–	–	3.6
10	2	–	2	–	–	–	–	–	53	2.5	–	–	3	–	–	–	–
11	–	–	–	–	2.9	–	–	3	54	–	–	2.7	–	–	–	–	–
12	–	2	–	–	–	–	–	–	55	–	-	–	–	–	–	–	5
13	–	–	–	3	–	–	4	–	56	–	–	–	–	2	–	–	–
14	–	–	–	–	2	–	–	4.6	57	–	–	–	–	2	–	–	–
15	–	2	2	–	–	–	–	–	58	–	4	–	–	–	–	3.5	–
16	2	–	–	–	2	–	–	–	59	–	–	–	2	–	–	–	–
17	–	–	–	–	–	3.5	–	–	60	2.9	–	–	–	–	–	–	–
18	–	–	–	4.4	–	–	–	6	61	–	–	–	–	–	–	–	7
19	–	6.3	2	–	–	–	–	–	62	–	–	–	–	–	2.8	–	4
20	–	–	–	4	–	–	6.2	–	63	–	–	2	2	–	–	–	–
21	6	–	–	–	–	–	–	–	64	–	–	–	–	–	–	–	2
22	–	–	–	–	–	3.4	–	4	65	–	–	–	–	–	–	–	4.7
23	–	–	2	–	–	–	–	–	66	–	–	–	–	–	–	–	2.9
24	–	–	–	–	3.4	–	–	–	67	–	–	–	–	–	–	–	2
25	–	4.9	–	–	–	–	–	–	68	–	–	–	–	–	–	–	2
26	–	–	2.5	–	–	3.2	–	–	69	–	–	–	–	–	–	–	2.7
27	4.3	–	–	–	–	–	–	–	70	–	–	–	–	–	–	–	2.7
28	–	–	–	–	–	2.3	7.2	4	71	–	4	2	–	–	–	–	–
29	–	–	–	4	–	–	–	–	72	–	–	–	–	–	–	–	4
30	–	–	–	2	–	–	–	–	73	–	–	–	–	–	–	–	2
31	–	2	–	–	–	–	–	–	74	–	–	–	–	3.6	–	–	–
32	–	–	3.6	–	2	–	–	–	75	5	–	–	–	–	–	–	–
33	2	–	–	–	–	–	–	–	76	–	–	–	–	–	–	6	–
34	–	–	–	–	–	–	–	3.5	77	–	–	–	–	–	–	–	2.2
35	–	–	–	–	–	2	–	–	78	–	–	–	–	–	–	–	2
36	–	–	–	–	–	2	–	–	79	–	–	–	–	–	–	–	2.2
37	–	–	–	3.4	–	–	–	–	80	–	–	–	–	–	3.1	–	–
38	–	2	2	–	–	–	2	–	81	–	–	–	2	–	–	–	–
39	5.8	–	–	–	–	–	–	5	82	–	3.2	–	–	–	–	–	–
40	–	–	–	–	3.1	–	–	–	83	–	6	–	–	–	–	–	2
41	–	2	–	–	2	–	–	–	84	–	–	–	–	–	–	–	2
42	–	–	–	2	–	–	3	–	85	–	–	–	–	–	–	–	8
43	2.8	–	2	–	–	–	–	–									

Table 5 The cost information of different vehicle type

	A	B
Vehicle size	100	200
Unit running cost	1	1.2
Fixed cost	100	180

(the third-party logistics company, factories and customers), and then a distance matrix can be attained based on the rectilinear distances. The following will describe the change process from the actual distance based on the road network into the rectilinear distance between two points.

Figure 4a describes the “initial” scenario of an example. There are three points A, B, C. The three actual distances based on route network are shown in green lines. Then, we firstly need to describe the rectilinear distance between two points (for example point A and B; point A and C). By moving B to B’, the rectilinear distance between point A and point B’ is equal to the route length between point A and point B. Similarly, the rectilinear distance between point A and point C’ can be equal to the route length between point A and point C, by moving point C to point C’. Figure 4b shows the change process of the actual distances based on route network to the rectilinear distance between two points. Thirdly, how to deal with the actual distances based on route network between point B and point C can be described as follows. Since the rectilinear distance between point A and B’ is equal to the actual distance between point A and B, and the rectilinear distance between A and C’ is equal to the actual distance between point A and C, C” is determined by assuring the actual distance between BC equal to the rectilinear distance between point B’C” while the AC” is ensured to be equal to the rectilinear distance between AC’ (see Fig. 4c). That is to say, In this paper, point C” is the crossover point of the two circles. One circle is with center point B’ and a radius of B’C’. The other circle is with center point A and a radius AC’. Therefore, AC’ = AC” and B’C’ = B’C”. Thus, Fig. 4 shows the process of the method to transfer the actual distance into the symmetric distance matrix.

(a) The location information based on route network of three points
(b) The conversion of the actual route length between two points
(c) The conversion of the actual route length among three points

In the carton heterogeneous VRP with a collection depot, there are two types of vehicles. One vehicle size is 200 which is used for the process of picking up the cartons from factories. And the other vehicle size is 100 which is used for the process of carton delivery to customers. Thus, different vehicle types have different fixed costs and variable costs per unit distance. Based on the experiences of Imran et al. (2009), the fixed cost, variable cost can be described in Table 5.

Then, the IPSO will continue computing 10 times, and the results are shown in Fig. 5. From Fig. 5, it can be found that the results are very stable and the difference between the optimum and the worst result is $<8\,\%$. Therefore, the IPSO has an excellent convergence performance. In addition, the computing time, from 283 to 369 s, is relatively short to solve a real-life problem. A comprehension of the optimization quality and the computing time of this algorithm show that the IPSO can solve the carton VRP with a collection depot effectively. The optimized routes can be shown in Table 6.

From the point of the third-party logistics company, the difference between the total delivery cost from all the factories and the delivery cost from the third-party logistics company is the profit space for the third-party logistics company. Thus, the comparison to the current

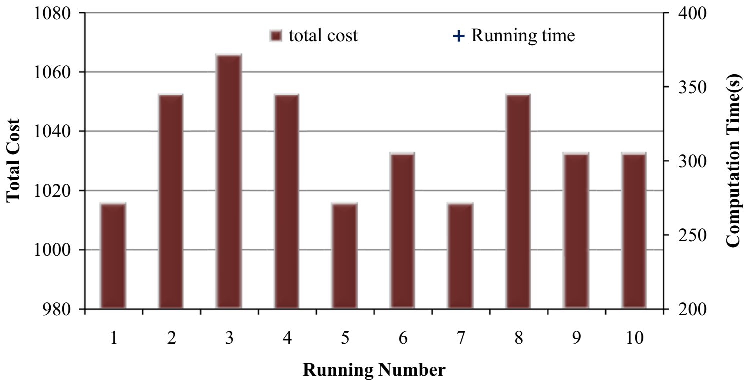

Fig. 5 Computing results of IPSO after running 10 times

Table 6 The solution of the carton heterogeneous vehicle routing problem with a collection depot

	Tour number	Sequence of customers
Pickup process	1	O-C-E-F-H-O
	2	O-G-A-B-D-O
Delivery process	1	0-17-79-80-69-13-30-72-53-34-7-9-37-66-14-56-59-3-1-31-16-28-0
	2	0-65-42-22-39-43-17-38-40-52-57-75-76-77-68-62-85-74-82-35-0
	3	0-54-73-70-71-48-84-12-33-23-10-50-55-78-25-21-19-26-49-36-32-47-11-0
	4	0-4-2-8-67-6-24-15-51-5-83-29-61-60-63-58-37-81-18-45-46-64-44-41-0

situation is also executed in this paper. In the current situation, the carton delivery tasks can be completed based on the idea from the original VRP with each factory to serve its customer respectively. That is, there are several VRPs. While in the carton heterogeneous VRP with a collection depot, there are two VRPs. The routes of the current situation can be shown in Table 7. In fact, the vehicles from the current carton factories are relatively smaller than the one from a third-party logistics company to reduce the fixed cost of vehicle. In the current situation, the vehicle capacity is 100, while in the carton heterogeneous VRP with a collection depot, there are two kinds of vehicles, one capacity is 100 and the other capacity is 200. The cost parameters about the two kinds of vehicles can be seen in Table 5. According to the solutions of the current situation and the optimized solutions of the carton heterogeneous VRP with a collection depot in this paper, the total cost can be acquired in Table 8.

From Table 8, it can be found that the total cost is smaller than that of the current situation. This can be explained that the proposed carton integrated vehicle routing model can integrate the services from several different factories to their customers while the current method only deals with the service from one factory to its customers. The results also show that the total delivery routes from the current situation are longer than the one of the proposed model. It can be due to that the proposed delivery routing model can provide a new idea for solving the delivery problem simultaneously, which can greatly shorten the delivery routes to complete the delivery tasks. Furthermore, in the situation where some customers belong to more than one factory, one delivery route may be suitable for providing services for the customers while there must be several routes, in the current situation, to deal with the services of the

Table 7 The solutions of the current situation

Depot	Sequence of customers
A	A-53-27-75-43-39-16-33-10-60-5-47-3-21-A
B	B-2-8-44-71-82-12-31-1-38-15-58-41-83-25-19-B
C	C-38-43-54-71-10-23-19-26-49-32-63-15-5-1-C
D	D-30-53-9-13-20-42-46-18-81-37-63-29-6-59-3-D
E	E-7-14-56-1-74-16-57-40-46-24-41-11-32-E
F	F-2-80-17-35-28-22-62-9-51-36-26-F
G	G-13-38-20-28-42-48-6-58-49-1-76-G
H	H-72-34-4-67-73-79-52-85-68-62-22-28-39-84-77-14-69-H H-66-50-55-70-64-78-83-61-18-45-11-65-H

Table 8 The total cost of the current situation and the carton heterogeneous vehicle routing problem with a collection depot

	The number of vehicle (Q = 100)	The number of vehicle (Q = 200)	Total distance (KM)	Total cost
The current situation	9	0	622.23	1422.23
The pickup process	0	2	42.13	1020.42
The delivery process	4	0	218.39	

customers. In addition, the number of vehicle is obviously less than the one of the current situation. It is due to that in the current situation, each factory needs to serve its customer respectively. In the situation, to meet the demands from customers, each vehicle is often with a partly empty container which greatly wastes the fixed cost for keeping vehicles. Therefore, it is feasible to solve the carton heterogeneous VRP with a collection depot by the proposed model and the IPSO is an effective method to solve the carton heterogeneous VRP with a collection depot.

5 Conclusions

This paper introduces a carton heterogeneous VRP with a collection depot. In this problem, the location of the third-party logistics company carton factories and customers is given in advance. The vehicle firstly starts from the third-party logistics company, and then runs to the factories to pick up cartons until the vehicle is full. Thus, the vehicle then serves the corresponding customers. After delivering all the cartons in the vehicle, the vehicle returns to the third-party-logistics company. The problem will be solved until all demands of customers are met. In addition, due to minor differences between different cartons, the paper attempts to attain the distribution of cartons by converting different types of cartons into one type of standard carton. Since the carton heterogeneous VRP with a collection depot is a very complex problem, PSO is used to solve the problem in this paper. To improve the performance of the proposed PSO, a self-adaptive inertia weight and a local search strategy are proposed in this paper. At last, the proposed PSO is examined with two cases. The first case is some benchmark instances on MDVRP, the comparison results between several well-known algorithms and the proposed algorithm indicate the PSO is effective to solve the complicated problem. The second case is a carton heterogeneous VRP with a collection depot in Dalian city. The results also show that the proposed PSO is effective with a saving of 28 % in total delivery cost when comparing to the current situation. Furthermore, the proposed method can obviously reduce the number of vehicles.

This paper give the third-party-logistics company a instruction about how to pickup and delivery carton using heterogeneous vehicle to minimize the costs. And, with empirical analysis, we can draw a conclusion that IPSO is more suitable for solving the carton heterogeneous VRP due to its quickness and accuracy.

Acknowledgments This work was supported by the National Natural Science Foundation of China 51208079 and 51108053, the Trans-Century Training Program Foundation for Talents from the Ministry of Education of China NCET-12-0752, Ministry of Housing and Urban-Rural Development K520136 and the Fundamental Research Funds for the Central Universities 3013-852019.

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