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Adaptive Particle Swarm Optimization Based on Population Entropy for MDVRPTW

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***Abstract*—Multi-depots vehicle routing problem with time windows (MDVRPTW) is a kind of NP combination problem which possesses important practical value. In order to overcome PSO's premature convergence and slow astringe, an adaptive particle swarm optimization based on population entropy is put forward, it uses the population entropy to makes a quantitative description about the diversity of population, in the meanwhile cellular is introduced to PSO, and adaptively adjusts the cellular structure according to the change of population entropy to have an effective balance between the global exploration and local exploitation, so enhance the performance of the algorithm. In this paper, the algorithm is used to solve MDVRPTW, a kind of new particles coding method is constructed and the solution algorithm is developed. The simulation results of example indicate that the algorithm has better capability of jumping out of local optimum than GA and PSO.**

Keywords-component; multi-depot vehicle routing problem; time windows; (PSO); population entropy; cellular

I. INTRODUCTION

Vehicle routing problem was proposed by Dantzig Z and Ramser J in 1959[1] , it refers to the design of a set of minimum-cost vehicle routes, originating and terminating at a central depot, for a fleet of vehicles that services a set of customers with known demands. Each customer is serviced exactly once, furthermore, all customers must be assigned to vehicles without exceeding vehicle capacities.

In our life，many optimization problems in logistics vehicle can be attributed to the MDVRPTW. Although the problem has been widely applied, it is not still good solutions in the theory field. The constraints of multi-depots and time window in terms of space and time makes the solution to the problem more complicated. When the scale of the problem is small, it is only possible to get exact solution, especially for large dimension problems, it is difficult to get the optimal solution. In recent years, GA, ACO and other heuristic optimization algorithms have been applied in solving these problems [2-4], but these algorithms have a long search time and easily fall into local optimal solution. Therefore, based on the characteristics of the multi-depot vehicle routing problem, it is of great significance to construct heuristic optimization algorithm with simple operation and excellent results for those possibly converted to multi-depot vehicle routing problem.

PSO is a newly-emerged bionic algorithm imitating birds to find food which is characterized by less individual number, simple calculation, fast convergence and easy to implement, etc[5-6]. The kind of algorithm has been applied to VRP [7-8] and achieved very good results, but it exists such problems as being easy to stagnate and easy to fall into local optimum. Therefore, in this paper, an adaptive particle swarm optimization based on population entropy is put forward by adaptively adjusting the cellular structure according to the change of population entropy and applied to solve MDVRPTW. The experiment proves that the algorithm has achieved very good results in terms of reducing iteration number and avoiding premature convergence.

II. DESCRIPTION ABOUT MDVRPTW AND MATHEMATICAL MODEL

The MDVRPTW can be described as follows：There are M depots and N customers, each depot has K_m $(m = 1,..., M)$ vehicles of capacity q , d_{ij} is transport distance from customer i to customer j , v is the speed of the vehicle, g_i $(i = 1,..., N)$ is demand goods for customer i and g_i is less than q . Each vehicle route must start and finish at the same depot, now seek a minimum route that each customer i is served by exactly one vehicle with the time is not later than l_i $(i = 1,..., N)$ and the total distance traveled is minimized.

Set customers ID is $1,..., N$, depots ID is $N+1$ ， …， $N+M$, t_{ik} is the moment to arrive at customer i of vehicle k , t_{ijk} is the travel time from customer i to customer j , define x_{ij}^{mk} as follows in formula (1).

$$x_{ij}^{mk} = \begin{cases} 1 & \text{vehicle k of depot m from i to j} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

So the formulation of MDVRP is as follows from formula (2) to in formula (11).

$$\text{Minimize} \sum_{i=1}^{N+M}\sum_{j=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} d_{ij}x_{ij}^{mk} \quad (2)$$

Subject to

$$\sum_{i=1}^{N}\sum_{k=1}^{K_m} x_{ij}^{mk} \le K_m \quad i = m \in \{N+1,...,.N+M\} \quad (3)$$

$$\sum_{i=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} x_{ij}^{mk} = 1 \quad i \in \{1,...,N\} \quad (4)$$

$$\sum_{j=1}^{N+M}\sum_{m=1}^{M}\sum_{k=1}^{K_m} x_{ij}^{mk} = 1 \quad j \in \{1,...,N\} \quad (5)$$

$$\sum_{i=1}^{N} x_{ij}^{mk} = \sum_{j=1}^{N} x_{ij}^{mk} \le 1 \quad i = m \in \{N+1,...,.N+M\},$$
$$k \in \{1,..., K_m\} \quad (6)$$

$$t_{ijk} = d_{ij}/v \quad i, j \in \{1,..., N\} \quad (7)$$

$$t_{ik} = t_{jk} + t_{ijk}x_{ij}^{mk},\ i, j \in \{1,..., N\},\ k \in \{1,..., K_m\} \quad (8)$$

$$t_{ik} \le l_i, i, j \in \{1,..., N\},\ k \in \{1,..., K_m\} \quad (9)$$

$$\sum_{i=1}^{N} g_i \sum_{j=1}^{N+M} x_{ij}^{mk} \le q \ m \in \{N+1,...,.N+M\},$$
$$k \in \{1,..., K_m\} \quad (10)$$

$$\sum_{i=N+1}^{N+M} x_{ij}^{mk} = \sum_{j=N+1}^{N+M} x_{ij}^{mk} = 0 \quad i = m \in \{N+1,...,.N+M\},$$
$$k \in \{1,..., K_m\} \quad (11)$$

In the model, formulation (2) is the target function; formulation (3) limit that the total number of vehicles dispatched from depot does not exceed the total number of the depot has; formulation (4) and formulation (5) guarantee that each customer will be visited exactly once; formulation (6) ensure that each vehicle is depart from the respective depot and return to the depot; formulation (7) , formulation (8)and formulation (5) are time constraint conditions of vehicle, formulation (10) express each vehicle load does not exceed its carrying capacity; formulation (11) ensure that the vehicle does not travel from a depot to another.

III. Adaptive Particle Swarm Optimization Based on Population Entropy

A. Standard particle swarm optimization algorithm

Particle swarm optimization (PSO) is a set of new intelligent optimization algorithms developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space which are associated with the best solution it has achieved so far. This value is called $pbest$. When a particle takes all the population as its topological neighbors, the best value is a global best and is called $gbest$. The particle swarm optimization concept consists of, at each time step, changing the velocity of each particle toward its $pbest$ and $gbest$ locations. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward $pbest$ and $gbest$ locations.

After finding the two best values, the particle updates its velocity and positions with following equation (12) and (13)

$$v_{id}^{k+1} = wv_{id}^{k} + c_1 * r_1(pbest - x_{id}^{k}) + c_2 * r_2(gbest - x_{id}^{k}) \quad (12)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \quad (13)$$

In the formula: v_{id}^{k} is the velocity of particle i, $v_{id}^{k} \in [V_{\min}, V_{\max}]$, $V_{\min}$ and $V_{\max}$ defined by the user are two constants, x_{id}^{k} is the position of particle i, w is inertia weighting factor, r_1 and r_2 are two random number between (0, 1), c1, c2 are learning factors，usually c1 = c2 = 2.

B. Population entropy

Selecting proper A、B to meet any particle fitness values are interval $[A，B]$, the interval $[A，B]$ is divided into N equal parts，the particle numbers of N subintervals are S_1， S_2,..., S_N, the population entropy are defined as shown in equation (14).

$$E = -\sum_{i=1}^{N} \frac{S_i}{N} \lg \frac{S_i}{N} \quad (14)$$

C. Adaptive adjustment strategy based on population entropy

In the paper, adaptive particle swarm optimization based on population entropy is designed by using two-dimensional annular cellular structure, all particles are placed inside and each particle has 5 neighbors that is the particle itself and its four neighbors. When the population entropy decreases too quickly, the cellular structure is narrowed to strengthen the global searching ability of the algorithm; otherwise, the cellular structure is widened to strengthen the local search ability of the algorithm. Because each particle only learns the best particle information of their neighbors, the speed and location of new algorithm is updated with equation (15) and equation (16).

$$v_{id}(t+1) = wv_{id}(t) + c_1r_1(pbest_{id}(t) - x_{id}(t)) + c_2r_2(gbest_{id}(t) - x_{id}(t)) \quad (15)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (16)$$

In the paper, only 2 different cellular structures respectively written as $r \times c$ and $r' \times c'$ is adopted in order to facilitate the description, $r \times c$ is corresponding to wide cellular structure, $r' \times c'$ is corresponding to wide cellular structure and $r \times c = r' \times c'$,among of them, r and r' express line number , c and c' express column number, so the particle position of conversing cellular structure is defined with equation (17)[9] .

$$(i, j) \to ([i \times c + j] div c', [i \times c + j] \bmod c') \tag{17}$$

IV. THE SOLUTION TO MDVRPTW BASED ON THE ALGORITHM PROPOSED IN THE PAPER

A. Coding Strategy of Particles

The position of particle corresponding with the answer to the question is the key idea of PSO. In the paper, a new particle encoding of 3 dimensional vectors X on the basis of reference [10] is constructed to MDVRPTW. In the vector X, the first dimension X_r of particles is depot information; the second dimension X_s of particles is vehicle information; the third dimension X_t of particles is traveling distance of the vehicle.

B. Decoding Strategy of Particles

To get to the travel path of the vehicle order, X_t must be adjusted. Adjustment function can be got according to the size sequence of vector X_t, that is to say, finding out X_t of the vehicle for customer i first, and then sorted from small to large in accordance with X_t, thus the driving path order of vehicle i is determined. For example, if there are 10 customers, 2 depots, one of depots has 2 vehicles, the other has 3 vehicles. If the position vector X of a particle is shown as table 1, then vector X of the position after adjustment is shown as table 2. So the corresponding vehicle routings are as follows:

depot 1: vehicle 1: 1→2
vehicle 2: 4→8
depot 2: vehicle 3: 3
vehicle 4: 6→7→5
vehicle 5: 10→9

TABLE I. VECTOR X BEFORE ADJUSTING

customer	1	2	3	4	5	6	7	8	9	10
X_r	1	1	2	1	2	2	2	1	2	2
X_s	1	1	3	2	4	4	4	2	5	5
X_t	0.3	1.2	0.6	2.3	4.8	1.1	2.9	3.9	2.8	1.7

TABLE II. VECTOR X AFTER ADJUSTING

customer	1	2	3	4	5	6	7	8	9	10
X_r	1	1	2	1	2	2	2	1	2	2
X_s	1	1	3	2	4	4	4	2	5	5
X_t	1	2	1	1	3	1	2	2	2	1

C. Process Description of Algorithm

Step1: initialize species: set the population size N, inertia weight coefficient w, accelerating factor c_1 、 c_2, maximum number of iterations $N_{\max}$, constant ε. The current iteration number t is set to 1, random generation initial position $x_1, x_2, \ldots x_N$ and initial velocity $v_1, v_2, \ldots, v_N$ of N particles; the N particles are placed in the cellular of $r \times c$ according to the order of initialization.

Step2: Calculate the fitness value f_i of i particle, and seek the best particle information $gbest_i$ $(i = 1,2,\ldots,N)$ of their neighbors.

Step3: For all particles, if $fitness_i > fitness_{pbest_i}$, then $fitness_{pbest_i} = fitness_i$, $x_{pbest_i} = x_i$, update $gbest_i$ according to the new particle fitness value.

Step4: Calculate population entropy E_t of t iteration, update each particle's speed and position with the formula (16) and formula (17).

Step5: Separately calculate $\Delta E_t = E_t - E_{t-1}$ and $\Delta E_{t-1} = E_{t-1} - E_{t-2}$, if satisfying $\Delta E_t > (2-\varepsilon) \times \Delta E_{t-1}$, the structure of cellular is converted to $r' \times c'$, if satisfying $\Delta E_t < (1+\varepsilon) \times \Delta E_{t-1}$, the structure of cellular is converted to $r \times c$.

Step6: Judge whether the current iteration number t is equal to or greater than the iteration number $N_{\max}$, if not satisfied, $t = t+1$ return Step2, otherwise, the current optimal solutions is output.

V. EXPERIMENTAL EXAMPLE

To verify the viability of algorithm, randomly generated MDVRPTW through the computer within the range of 10×10(unit: km), the speed of vehicles is 25(unit: km/h),, the location(unit: km) and demand(unit: t) of customers and the latest time of vehicles (unit: min)is shown as table 3, the location(unit: km) and vehicles number of depots is shown as table 4, all of the vehicle's maximum capacity is 20.

The equation is calculated respectively by GA, PSO and the algorithm proposed in the paper. Through calculation, the optimal scheduling schemes of vehicle are shown as table 5, the objective function changes as number of iterations is shown as figure 1.

From the simulation result, it could quickly and accurately find the optimal solution to MDVRPTW using the algorithm proposed in the paper and the time and efficiency are superior to GA and PSO, which provides a new idea for MDVRPTW.

TABLE III. CUSTOMER INFORMATION

customers	X coordinates	Y coordinates	demand	delay time
1	4.3	8.2	7	40
2	3.4	1.1	6	10
3	9.5	8.3	9	60
4	2.2	5.3	5	10
5	7.3	4.1	7	10
6	5.1	4.8	4	30
7	5.9	6.8	5	15
8	4	0.2	7	40
9	1.8	9.9	6	20
10	6.7	2	3	20
11	0.6	2.7	3	30
12	7.8	10	4	40
13	3.9	5.1	7	10
14	0.3	7.7	6	20
15	6	0.9	2	30
16	1.1	0.9	4	15
17	9.5	5.2	8	50
18	0.4	6.1	3	20
19	1.9	4.7	5	60
20	6.3	2.6	8	10
21	8.8	1.6	3	20
22	2	7.9	5	50
23	6.2	3.1	8	40
24	0.5	2.3	1	20
25	1.2	7	4	30

TABLE IV. DEPOT INFORMATION

depot	**26**	**27**	**28**
location	**(3,7)**	**(4,3)**	**(8,6)**
number of vehicles	**5**	**5**	**5**

TABLE V. THE OPTIMAL SCHEDULING SCHEME OF EXAMPLE

depot	**driving route**	**driving distance**	**depot**
26	**26→4→18→25→22→26**	**7.6**	**22.8**
26	**26→14→9→1→26**	**10.24**	**30.72**
27	**27→2→16→24→11→19→27**	**11.32**	**33.96**
27	**27→20→10→15→8→27**	**9.28**	**27.84**
27	**27→13→6→23→27**	**7.57**	**22.71**
28	**28→5→21→17→28**	**10.31**	**30.93**
28	**28→7→12→3→28**	**11.12**	**33.36**
total		**67.44**	**202.32**

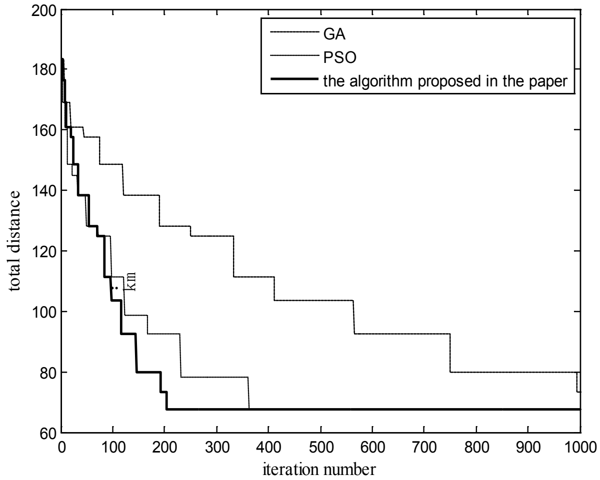

Figure 1. Performance comparison of GA, PSO and the algorithm proposed in the paper

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