Matrixrekenen II

I

i

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 8 & 10 & 0 \\ 3 & 6 & 11 & 17 & 0 \end{array}\right)$$

coefficient matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 8 & 10 \\ 3 & 6 & 11 & 17 \end{pmatrix}$$

$R_2 = R_2 - 2 \cdot R_1$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 2 \\ 3 & 6 & 11 & 17 \end{pmatrix}$$

$R_3 = R_3 - 3 \cdot R_1$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 5 \end{pmatrix}$$

$R_3 = R_3 - R_2$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

Echelon vorm

ii $\#$ basisoplossingen $= \cancel{3} \; 4 - 3 = 1$

$$\begin{array}{cccc} \boxed{x_1} & x_2 & \boxed{x_3} & \boxed{x_4} \end{array} \longrightarrow \text{pivot variabelen}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$\begin{cases} x_1 = -2x_2 - 3x_3 - 4x_4 \\ x_3 = -x_4 \\ x_4 = \cancel{x_4}\, 0 \end{cases} \longrightarrow \begin{cases} x_1 = -2x_2 \\ x_3 = 0 \\ x_4 = 0 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2x_2 \\ x_2 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is een basisoplossing.

iii General solution:

$$(x_1, x_2, x_3, x_4) = x_2 \cdot \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

II

i

$$\left(\begin{array}{ccccc|c} 2 & 3 & 1 & 2 & 1 & 1 \\ 0 & 0 & 4 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array}\right)$$

basic solutions = # variables − # pivots
= 5 − 3
= 2

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$
$$\begin{pmatrix} 2 & 3 & 1 & 2 & 1 \\ 0 & 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_5 \end{array}$$
$$\begin{pmatrix} 2 & 3 & 1 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$
$$\downarrow$$
$$(-3, -2, 0, 0, 0)$$

$$\begin{array}{cccc} x_1 & x_3 & x_4 & x_5 \end{array}$$
$$\begin{pmatrix} 2 & 1 & 2 & 1 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$
$$\downarrow$$
$$(3\tfrac{1}{2}, 0, 1, -4, 0)$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{basic solutions}}$$

ii

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 4 & 1 & 2 \\ 0 & 0 & 2 & 6 \end{array}\right)$$
$$\downarrow$$
$$(-1\tfrac{3}{8}, 0, -\tfrac{1}{4}, 0, 3) = (x_1, x_2, x_3, x_4, x_5)$$

iii

$$\underbrace{\begin{pmatrix} -1\frac{3}{8} \\ 0 \\ -\frac{1}{4} \\ 0 \\ 3 \end{pmatrix} + c_1 \cdot \begin{pmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c_2 \cdot \begin{pmatrix} 3\frac{1}{2} \\ 0 \\ 1 \\ -4 \\ 0 \end{pmatrix}}_{}$$

↳ general solution

3

i A unique solution is found when each row has a pivot.

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 2 & 1 & 2a+1 & 5 \\ 3 & a-1 & 2 & b+2 \end{array}\right)$$

$R_3 := R_3 - R_2 - R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 2 & 1 & 2a+1 & 1 \\ 0 & a-3 & 3-3a & b-5 \end{array}\right)$$

$R_2 := R_2 - 2 \cdot R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 0 & -1 & 1 & 1 \\ 0 & a-3 & 3-3a & b-5 \end{array}\right)$$

$a = 3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -6 & b-5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -6 & b-5 \end{array}\right)$$

$b = -1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -6 & -6 \end{array}\right) \rightarrow x_3 = 1,\ x_2 = -5,\ x_1 = 1$$

$a = 3$ and $b = -1$ gives a unique solution.

ii

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 2 & 1 & 2a+1 & 5 \\ 3 & a-1 & 2 & b+2 \end{array}\right)$$

$R_2 := R_2 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 0 & -1 & 1 & 1 \\ 3 & a-1 & 2 & b+2 \end{array}\right)$$

$R_3 := R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 0 & -1 & 1 & 1 \\ 0 & a-4 & 2-3a & b-4 \end{array}\right)$$

$R_3 := R_3 + (a-4)R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & a & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -2-2a & a+b-8 \end{array}\right)$$

We want no solution so: ~~Ga~~

$$-2-2a=0$$
$$-2=2a$$
$$a=-1$$

and

$$a+b-8\neq 0$$
$$a+b\neq 8$$
$$-1+b\neq 8$$
$$b\neq 9$$

iii ~~b~~ For a system to have multiple solutions, there need to be less pivots than variables so we can make the last row $0\ 0\ 0\ |\ \cancel{0}\ \cancel{\neq 0}\ 0$

$a=-1$, $b=9$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The system has $3-2=1$ basic solution.

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

with basic solution:

$$(x_1, x_2, x_3) = (0, 1, 1)$$

Then for the particular solution:

$$\begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right) \end{array}$$

with particular solution:

$$(3, -1, 0)$$

The general solution is:

$$\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + c_1 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

4

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & -1 & -1 \end{array}\right)$$

$R_2 := R_2 - R_1$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & 7 & -3 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & -1 & -1 \end{array}\right)$$

$R_4 := R_4 - R_3$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & 7 & -3 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \end{array}\right)$$

$R_1 := R_1 - R_3$

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 1 & \cancel{5}4 \\ 0 & 1 & 3 & 7 & -3 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \end{array}\right)$$

$R_2 := R_2 + R_4$

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 4 & 6 & -5 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \end{array}\right)$$

$R_1 := R_1 + R_4$

$$\left(\begin{array}{cccc|c} 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 4 & 6 & -5 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \end{array}\right)$$

$R_2 := R_2 - 2 \cdot R_1$

$$\left(\begin{array}{cccc|c} 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 6 & -9 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & -2 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 6 & -9 \end{array}\right)$$

$x_4 = -1\frac{1}{2}$

$x_3 = 1$

$x_2 = \cancel{0}\ 4\frac{1}{2}$

$x_1 = 1$

$f(x) = -1\frac{1}{2}x^3 + 0x^2 + 4\frac{1}{2}x + 1$