Matrixrekenen assignment 6

1.

$$\begin{pmatrix} R \\ C \\ S \end{pmatrix} \cdot \begin{pmatrix} 0,7 & 0,2 & 0,2 \\ 0,2 & 0,6 & 0,4 \\ 0,1 & 0,2 & 0,4 \end{pmatrix}$$

$$\begin{pmatrix} 0,7 & 0,2 & 0,2 \\ 0,2 & 0,6 & 0,4 \\ 0,1 & 0,2 & 0,4 \end{pmatrix} \cdot \begin{pmatrix} 0,5 \\ 0,4 \\ 0,1 \end{pmatrix} = \begin{pmatrix} 0,45 \\ 0,38 \\ 0,17 \end{pmatrix}$$

$$\begin{pmatrix} 0,7 & 0,2 & 0,2 \\ 0,2 & 0,6 & 0,4 \\ 0,1 & 0,2 & 0,4 \end{pmatrix} \cdot \begin{pmatrix} 0,45 \\ 0,38 \\ 0,17 \end{pmatrix} = \begin{pmatrix} \cancel{0,45}\ 0,425 \\ 0,386 \\ 0,189 \end{pmatrix}$$

There is a 39 % chance of a cloudy day. ~~the da~~

2.

a. $A - \lambda \cdot I$ ~~= eigenvalue vector~~

$$\begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \cancel{\begin{pmatrix} 0.7-\lambda & 0.1 \\ 0.3 & 0.9-\lambda \end{pmatrix}}$$

$$\begin{pmatrix} 0.7-\lambda & 0.1 \\ 0.3 & 0.9-\lambda \end{pmatrix}$$

$$\begin{vmatrix} 0.7-x & 0.1 \\ 0.3 & 0.9-\lambda \end{vmatrix} = 0$$

$$ad - bc = 0$$

$$(0.7-\lambda)(0.9-\lambda) - (0.1)(0.3) = 0$$

$$\frac{1}{10}\left((7-10\lambda)(9-10\lambda) - (1)(3)\right) = 0$$

$$\frac{1}{10}\left((7-10\lambda)(9-10\lambda) - 3\right) = 0$$

$$\frac{1}{10}\left(\cancel{46}3 - 90\lambda - 70\lambda + 100\lambda^2 - 3\right) = 0$$

$$\frac{1}{10}\left(100\lambda^2 - 160\lambda + \overset{60}{\cancel{66}}\right) = 0$$

$$\Leftrightarrow \cancel{\lambda^2}\ \lambda^2 - 1.6\lambda + 0.6 = 0$$

$$(\lambda - 1)(\lambda - 0.6) = 0$$

$$\lambda = 1 \ \wedge \ \lambda = 0.6$$

$$\begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix} - \lambda \cdot I = \begin{pmatrix} -0.3 & 0.1 \\ 0.3 & -0.1 \end{pmatrix}$$

$(\lambda = 1)$

$$\cancel{-0{,}3x + 0{,}1y = 0}$$
$$\cancel{0{,}3x - 0{,}1y = 0}$$

$$-0{,}3x + 0{,}1y = 0$$
$$0{,}3x - 0{,}1y = 0$$

$x = 1$

$y = 3$

eigenvector $= \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix} - \lambda \cdot I = \begin{pmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{pmatrix}$$

$(\lambda = 0{,}6)$

$$0{,}1x + 0{,}1y = 0$$
$$0{,}3x + 0{,}3y = 0$$

$x = 1$

$y = -1$

eigenvector $= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

b. $B = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0{,}6 \end{pmatrix}_B$$

c. ~~inverse of D:~~

$$\cancel{\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 0.6 & 0 & 1 \end{array}\right)}$$

$$\cancel{\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{0.6} \end{array}\right)}$$

$$\cancel{\begin{pmatrix} 1 & 0 \\ 0 & 1\frac{2}{3} \end{pmatrix}}$$

~~B⇒S~~ $T_{B \Rightarrow S} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

$$T_{S \Rightarrow B} = T_{B \Rightarrow S}^{-1}$$

$$T_{S \Rightarrow B} = \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -4 & -3 & 1 \end{array}\right)$$

$$= \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{3}{4} & -\frac{1}{4} \end{array}\right)$$

$$= \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{3}{4} & -\frac{1}{4} \end{array}\right)$$

$$= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} \end{pmatrix}$$

$$S = T_{B \Rightarrow S} \cdot D \cdot T_{S \Rightarrow B}$$

$$= \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0{,}6 \end{pmatrix} \cdot T_{S \Rightarrow B}$$

$$= \begin{pmatrix} 1 & 0{,}6 \\ 3 & -0{,}6 \end{pmatrix} \cdot T_{S \Rightarrow B}$$

$$= \begin{pmatrix} 1 & 0{,}6 \\ 3 & -0{,}6 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} \end{pmatrix}$$

$$= \begin{pmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{pmatrix}$$

$$= S$$

$D \parallel D$

d.

$$S^2 = T_{B \Rightarrow S} D^2 \cdot \cancel{T_{B \Rightarrow S}} T_{S \Rightarrow B}$$

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0{,}6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0{,}6 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 0{,}36 \end{pmatrix}$$

$$S^2 = \cancel{\begin{pmatrix} 1 & 0 \\ 0 & 0{,}36 \end{pmatrix}} \cdot \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0{,}36 \end{pmatrix} \cdot T_{S \Rightarrow B}$$

$$= \begin{pmatrix} 1 & 0{,}36 \\ 3 & -0{,}36 \end{pmatrix} \cdot T_{S \Rightarrow B}$$

$$= \begin{pmatrix} 1 & 0{,}36 \\ 3 & -0{,}36 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & -\frac{1}{4} \end{pmatrix}$$

$$= \begin{pmatrix} 0.52 & 0.16 \\ 0.48 & 0.84 \end{pmatrix}$$

e. ~~Every~~ Every ~~inte~~ number ~~la~~ smaller than 1 can be set to 0, every number larger than 1 ~~a~~ goes to infinity

3.

a.

$$F = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix}$$

$$A - \lambda \cdot I = 0$$

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 & 1 \\ -1 & 2-\lambda & 1 \\ -1 & -1 & 4-\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ -1 & 2-\lambda & 1 \\ -1 & -1 & 4-\lambda \end{vmatrix} = -\lambda \cdot \begin{vmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & 4-\lambda \end{vmatrix}$$

$$-1 \cdot \begin{vmatrix} 1 & 1 \\ 2-\lambda & 1 \end{vmatrix}$$

$$= -\lambda \cdot \left((2-\lambda)(4-\lambda) + 1 \right) + \left(4 - \cancel{x}\lambda + 1 \right)$$

$$- \cancel{x} \left(1 - (2 - \lambda) \right)$$

$$= -\lambda \cdot \left(\cancel{6-\lambda}\ 8 - 6\lambda + \lambda^2 + 1 \right) + \cancel{4-\lambda+1}\ (5-\lambda)$$

$$- (\lambda - 1)$$

$$= \lambda^3 - 6\lambda^2 \cancel{+8}\ \overset{9}{-}\lambda + 5 - \lambda - \lambda + 1$$

$$\cancel{= -\lambda^3 + 6\lambda^2 - 8\lambda + 6}$$

$$= -\lambda^3 + 6\lambda^2 - 11\lambda + 6$$

b. $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

~~de 1 is an~~ $\lambda = 1$

$$\cancel{\lambda \cdot (\lambda^2 - 6\lambda + 11) - 6 = 0}$$

$$\cancel{\lambda \cdot (\lambda^2 - 6\lambda + 11) = 6}$$

$$\lambda \cdot ((\lambda$$

$$(\lambda - 1)(\lambda^2 - 5\lambda + 6)$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$(\lambda - 2)(\lambda - 3) = 0 \longrightarrow \lambda = 2, \lambda = 3$$

eigenvalues: 1, 2 & 3

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 3 \end{pmatrix}$$

$$-x + y + z = 0$$
$$-x + y + z = 0$$
$$-x - y + 3z = 0$$

$$x = 2$$
$$y = 1$$
$$z = 1$$

eigenvector : $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$$

$$\begin{pmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$-2x + y + z = 0$$
$$-x + z = 0$$
$$-x - y + 2z = 0$$

$$x = 1$$
$$y = 1$$
$$z = 1$$

eigenvector: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ -1 & -1 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} =$$

$$\begin{pmatrix} -3 & 1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$-3x + y + z = 0$$
$$-x - y + z = 0$$
$$-x - y + z = 0$$

$$x = 1$$
$$y = 1$$
$$z = 2$$

eigenvector : $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$