Aufgaben Differentialquotient 1

a.) $\lim\limits_{h\to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

$\lim\limits_{h\to 0} \dfrac{3(1+h)^2 - 3}{h} = \lim\limits_{h\to 0} \dfrac{3(1+2h+h^2)-3}{h}$

$\lim\limits_{h\to 0} \dfrac{3+6h+3h^2-3}{h} = \lim\limits_{h\to 0} \dfrac{h(6+h)}{h} \underline{\underline{= 6}}$

b.) $\lim\limits_{h\to 0} \dfrac{\frac{1}{4}(-1+h)^2 - (-1+h) - \frac{5}{4}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{4}(1-2h+h^2)+1-h-\frac{5}{4}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\frac{1}{4} - \frac{1}{2}h + \frac{1}{4}h^2 - h - \frac{1}{4}}{h} = \lim\limits_{h\to 0} \dfrac{h(-\frac{1}{2}+\frac{1}{4}h-1)}{h} = -$

c.) $\lim\limits_{h\to 0} \dfrac{1-(3+h)^2+8}{h} = \lim\limits_{h\to 0} \dfrac{1-9-6h-h^2+8}{h}$

$= \lim\limits_{h\to 0} \dfrac{h(-6-h)}{h} \underline{\underline{= -6}}$

d.) $\lim\limits_{h\to 0} \dfrac{(1-2(2+h))^2-9}{h} = \lim\limits_{h\to 0} \dfrac{(1-4-2h)^2-9}{h}$

$= \lim\limits_{h\to 0} \dfrac{(-3-2h)^2-9}{h} = \lim\limits_{h\to 0} \dfrac{9+12h+4h^2-9}{h}$

$= \lim\limits_{h\to 0} \dfrac{h(12+4h)}{h} \underline{\underline{= 12}}$

e.) $\lim\limits_{h\to 0} \dfrac{\sqrt{3+h}-\sqrt{3}}{h} = \lim\limits_{h\to 0} \dfrac{(\sqrt{3+h}-\sqrt{3})(\sqrt{3+h}+\sqrt{3})}{h(\sqrt{3+h}+\sqrt{3})}$

$= \lim\limits_{h\to 0} \dfrac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})} = \lim\limits_{h\to 0} \dfrac{h}{h(\sqrt{3+h}+\sqrt{3})}$

$\underline{\underline{= \dfrac{1}{2\sqrt{3}}}}$

f) $\lim\limits_{h\to 0} \dfrac{\dfrac{2}{(-1+h)} + 2}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{2+2(-1+h)}{(-1+h)}}{h}$

$= \lim\limits_{h\to 0} \dfrac{2-2+2h}{h(-1+h)} = \lim\limits_{h\to 0} \dfrac{2h}{h(-1+h)}$

g.) $\lim\limits_{h\to 0} \dfrac{\sqrt{2(4+h)+1} - \sqrt{9}}{h}$

$= \lim\limits_{h\to 0} \dfrac{\sqrt{9+2h} - \sqrt{9}}{h}$

$= \lim\limits_{h\to 0} \dfrac{(\sqrt{9+2h} - \sqrt{9})(\sqrt{9+2h} + \sqrt{9})}{h(\sqrt{9+2h} + \sqrt{9})}$

$= \lim\limits_{h\to 0} \dfrac{9+2h-9}{h(\sqrt{9+2h} + \sqrt{9})} = \lim\limits_{h\to 0} \dfrac{2h}{h(\sqrt{9+2h} + \sqrt{9})}$

$= \dfrac{2}{2\sqrt{9}} = \dfrac{1}{\sqrt{9}} = \underline{\underline{\dfrac{1}{3}}}$