Date

Page No.

Numerical Methods (JRS)

1. Solution;

~~For~~ Function: $x*\sin(x)+\cos(x)=0$.

Linear Search:

x	0	1	2	3	4	5	6	7
f(x)	1.000	1.382	1.402	-0.567	-3.681	-4.511	-0.716	5.353

Since $f(6) = -ve$ & $f(7) = +ve$;
and the function seems to be continuous in any interval,
there is atleast one real root between $x=2$ and $x=3$.

Now, let initial interval be $a=6$ and $b=7$.

Calculation Table

$f(a) = -ve$ $f(b) = +ve$

Iteration	a	b	$c=(a+b)/2$	f(c)
1	6	7	6.5	2.37486
2	6	6.5	6.25	0.79207
3.	6	6.25	6.125	0.02266
4.	6	6.125	6.0625	-0.35132
5.	6.0625	6.125	6.0938	-0.16506
6	6.0938	6.125	6.1094	-0.07145
7.	6.1094	6.125	6.1172	-0.02455
8.	6.1172	6.125	6.1211	-0.00090
9.	6.1211	6.125	6.12305	0.010874
10.	6.1211	6.12305	6.12207	0.00498
11.	6.1211	6.12305	6.12207	0.00249
12.	6.1211	6.12207	6.12158	0.00056
13.	6.1211	6.12158	6.12134	-0.00017
14.	6.1211	6.12134	6.12122	0.00019
15.	6.1212	6.12134	6.12128	0.00001

From table;

+ve real root correct to 2 decimal places = 6.12

-ve real root correct to 4 decimal places = 6.1212

2. Solution;

Error tolerance (ε) = 0.0005

Interval ($|a-b|$) = 1

Let approx. no. of iterations be n.

We know,

$$\frac{|a-b|}{2^n} \le \varepsilon$$

$$\Rightarrow 2^n \ge \frac{|a-b|}{\varepsilon}$$

$$\Rightarrow \log(2^n) \ge \log(|a-b|) - \log \varepsilon$$

$$\therefore n \ge \frac{\log(|a-b|) - \log \varepsilon}{\log(2)}$$

So; For given values;

$n \ge 10.96$

$\therefore$ ~~Max~~ Approx. no. of iterations would be 11.

For ~~ε~~ $|a-b| = 0.1$

$$n \ge \frac{\log(0.1) - \log(0.0005)}{\log(2)}$$

$\therefore n \ge 7.64$

$\therefore$ Approx. no. of iterations would be 8

3. Solution;

For cube root of 7; let the root be x.

Eqn is: $x^3 - 7 = 0 = f(x)$

Now;

Linear Search:

x	1.8	1.9	2	2.1
$f(x)$	-1.168	-0.141	1.000	2.261

Since $f(1.9) = -ve$ and $f(2) = +ve$;

~~and the f~~ there is at least one real root between ~~x = 2 and x~~ $x = 1.9$ and $x = 2$.

So; let initial interval be $a = 1.9$ and $b = 2.0$

Iteration	a	b	$c = (a+b)/2$	f(c)
1	1.9	2.0	1.95	0.41487
2	1.9	1.95	1.925	0.13333
3	1.9	1.925	1.9125	-0.00473
4	1.9125	1.925	1.9188	0.06463
5	1.9125	1.9188	1.91565	0.02989
6	1.9125	1.91565	1.914075	0.01256
7	1.9125	1.914075	1.913288	0.00392
8	1.9125	1.913288	1.912894	-0.00041
9	1.912894	1.913288	1.913091	0.00175
10	1.912894	1.913091	1.912993	0.00068
11	1.912894	1.912993	1.912944	0.00014
12	1.912894	1.912944	1.912919	-0.00013
13	1.912919	1.912944	1.912932	0.00001

14

15 Here; tolerance = 0.00005 $\therefore f(1.912932) < 0.00005$

$\therefore$ Root = 1.91293

4. Algorithm:

1. Start
2. Input values of interval start and end, allowed error and maximum iterations.
3. Find mean of interval
4. Check if mean and interval start or end have opposite signs in equation.
5. Set new start and end as opposite signed values from above.
6. Check if error is less than tolerance
7. If yes print mean as answer
8. Else, check if max iterations are reached
9. If no, repeat from 3
10. Else, end the program

Program:

```
/* Coded in C, */ ~~expression sam~~
#include<stdio.h>
#include <math.h>
float f(float x)
{
  /* return expression using x as root */
  return (x * cos(x) + sin(x));
}
void bisect(float *x, float *a, float b, int *itr)
/* performs and prints results of one iteration */
{
  *x = (a+b)/2;
  ++(*itr);
  printf("Iteration no. %3d X = %7.5f\n", *itr,
  *x);
```

```
}
void main()
{
int itr=0; maximumItr;
float x, a, b, allowed Err, newX;
printf("\Enter ~~a, b~~, start and end value of interval,
  allowed error and max iterations :\n");
scanf("%f %f %f %d, &a, &b, &allowed Err,
     &maximum Itr);
bisect (&x, a, b, &itr);
do
{
if (f(a)*f(x) <0)
     b=x;
else ~~fff~~
     a=x;
bisect(&newX, a, b, &itr);
if (fabs(newX-x) < allowed Err)
 {
   printf("After %d iterations, root=%5.5f\n",
    itr, newX);
   return 0;
 }
 x=newX;
}
while(itr<maximum Itr);
printf("The solution doesn't converge or insufficient
 iterations);
 return 1;
}
```