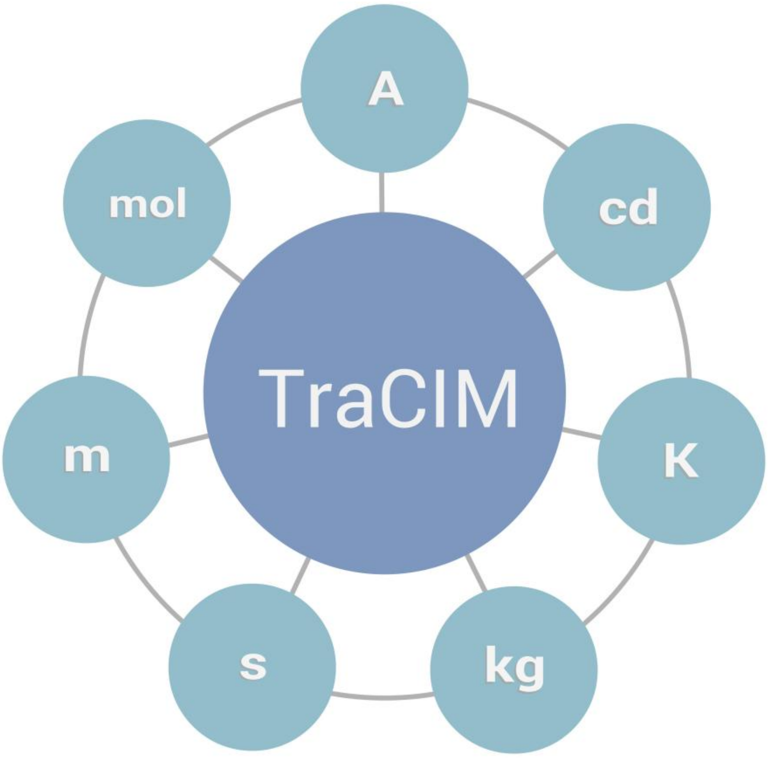
Traceability for Computationally-Intensive Metrology

Validation of Gaussian Algorithm Implementation



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Change History

Date	*Revision*	*Rationale*
2015-02-13	1.0	First non-draft version
2015-02-17	1.1	1) Changed element IDs in tables 1 and 2 and in sections 1.2, 4.2 and 4.3 2) Changed 'cylinder axis' to 'cone axis' in table 2 (element 'CONE', parameter 'Orientation') 3) Changed 'HTTP' to 'HTTPS' in section 3.1 4) Added notification on validity of order key at end of section 3.1 (p. 10) 5) Changed 'certificate' to 'test report' in sections 4.4 and 5 (new description of test report on p. 9)
2015-02-18	1.1	1) 'rad' was replace by 'grad' for cone angle α (p. 27) 2) 'apexAngle' was replaced by 'angle' for 'Cone'-ID (p. 12)
2015-03-10	1.2	1) Adaption of URL's for XML-schema
2015-04-17	1.3	1) In 'constraints: $0 < \alpha < \pi$' π was replace by 180 for cone angle α (p. 27**)**
2015-07-06		1) 'grad' was replaced by 'degree' for cone angle α (p. 27) 2) changes on page 7 3) Tracim support email address changed to 'support.tracim@ptb.de'

Please note: the 'Revision' consists of a'major number' – 'dot' – 'minor number' . Work completed regarding major grammar or spelling errors will be marked with a lowercase letter after the revision number.

Content

1. Introduction

PTB’s TraCIM system offers testing of gauss minimization algorithms for the following basic geometric elements:

Line 3D, Plane, Circle 3D, Cylinder, Cone and Sphere

The PTB test was provided within the frame of a project named TraCIM, founded by the European Union (EMRP project NEW06 TraCIM – www.ptb.de/emrp/tracim.html) and is under strict quality control of TraCIM e.V. Association.

The test is provided by the PTB TraCIM Online System. For the test a customer will get an XML file with data sets containing points in space for the elements listed above. The points are both randomly and systematically distributed on fragments of these elements mainly representing full features with different random form deviation components. A few datasets represent partial features.

Task of the customer is to evaluate the parameters of the best-fit geometric elements regarding the gaussian-criterion and send them back to the TraCIM system. The TraCIM system automatically compares the customer results with its own reference values and sends a report with the test result. Evaluation procedures are subject to section 2.

To order a test it is necessary to provide a client application for communication with the TraCIM Server. In section 3 support for developing a client can be found. The XML data schemata used for data exchange are described in section 4.

Placing an order requires access to the web shop for buying single tests and test packages (web shop: future extension; use e-mail contact until then). After successful purchase of a test a customer will get an order key that allows him to request test data from the TraCIM system with his client application.

In order to check the functionality of a TraCIM client, a charge free test suite with public sample data including results as well is provided. A registered customer is allowed to request a test with these data sets from the TraCIM system at any time in order to evaluate the correct function of his client-server communication. See section 3.2 on how to request these test data sets. Additional support will be charged.

1.1. Basic data sets

The gaussian elements test contains 44 data sets with following basic data set IDs and associated geometric element IDs:

Table 1: Data/Element IDs

Data set ID	Element ID
b01 – b04	LINE_3D
b05 – b08	PLANE
b10 – b15	CIRCLE_3D
b16 – b25	CYLINDER
b26 – b37	CONE
b38 – b44	SPHERE

- Each data set consists of points in 3-dimensional Cartesian space.
- A point is represented by the three coordinates (x, y, z).
- The number of points per data set varies between 8 and 50.
- Each point coordinate value is delivered as decimal number with 16 digits (scientific e-format).
- The values refer to the unit mm (millimeter) [1)]
- All point coordinates are within the value range [-500 mm, +500 mm] [2)].
- Section 4.2 shows the xml data structure that contains the test data sets.

1) subject to future change
2) preliminary; newer software version may accept customer defined range values.

1.2. Result parameters

For each element the following result parameters must be computed by the customer:

Table 2: Result parameter list

Element ID	Parameter ID	Parameter	Description
LINE_3D	Position	Coordinates of a point on the line	Best fit of a Gaussian line to 3D point coordinates in coordinate metrology
	Orientation	Vector pointing along the line	
PLANE	Position	Coordinates of a point on the plane	Best fit of a Gaussian plane to 3D point coordinates in coordinate metrology
	Orientation	Direction cosines of the normal to the plane	
CIRCLE_3D	Position	Coordinates of a point on the circle	Best fit of a Gaussian circle to 3D point coordinates in coordinate metrology
	Orientation	Direction cosines of the normal to the circle	
	Radius	Radius of the circle	
CYLINDER	Position	Coordinates of a point on the cylinder axis	Best fit of a Gaussian cylinder to 3D point coordinates in coordinate metrology
	Orientation	Vector pointing along the cylinder axis	
	Radius	Radius of the cylinder	
CONE	Position	Coordinates of a point on the cone axis, preferably the projection of the centroid of the input coordinates on the cone axis, i.e. X_0 is the foot point of (x_M, y_M, z_M) on the cone axis)	Best fit of a Gaussian cone to 3D point coordinates in coordinate metrology
	Orientation	Vector pointing along the cone axis in the direction of decreasing radius	
	Radius	Radius of the cone perpendicular to the cone axis at X_0 (r = 0, if X_0 marks the apex of the cone)	
	Apex Angle	Full angle at the apex	
SPHERE	Position	Coordinates of the centre of the sphere	Best fit of a Gaussian sphere to 3D point coordinates in coordinate metrology
	Radius	Radius of the sphere	

- For all elements the position parameter is a vector in 3-dimensional Cartesian space with components x, y, and z. The values refer to the unit mm (millimeter) - see section 1.1 (Basic Data Sets), note 1 and 2 for ‘point coordinates’.
- For ‘LINE_3D’, ‘PLANE’, ‘CIRCLE_3D’, ‘CYLINDER’, ‘CONE’ and ‘SPHERE’ elements the orientation parameter is a vector in 3-dimensional Cartesian space with components x, y and z. Each vector must be scaled to length value one (direction cosines).
- Radiuses must be positive values. These refer to the unit mm (millimeter). (see note 1 and 2 in section 1.1)
- All parameter values must be in decimal number format (fixed point or floating point e-format are applicable).
- Section 4.3 shows how to write the test results in a file consistent with the TraCIM system test result data structure.

2. Test result evaluation 1)

During validation the test results of customer software are compared to the associated reference results in the TraCIM system data base. For this purpose deviations regarding position, orientation and size of the geometric elements are evaluated and verified to the specified maximum permissible error values (MPE). The following MPEs are the defaults:

Table 3: Default MPEs

Parameter	Specification of deviation	Default MPE value
Position	Orthogonal distance of reference Position to test result line, plane, axis or center point	0.1 µm
Orientation	Arc between test result Orientation vector and reference Orientation vector	0.1 µrad
Apex Angle	Deviation of the cone apex angle	0.1 µrad
Radius	Linear deviation between test result Radius and reference value	0.1 µm

The test is passed successfully, if all deviations are less than the default MPE values. After the evaluation the TraCIM system will issue a test report containing a certificate for tests passed or a table listing the results of the comparison for each of the elements where the test failed. The test report is send back to the customer by the client-server connection.

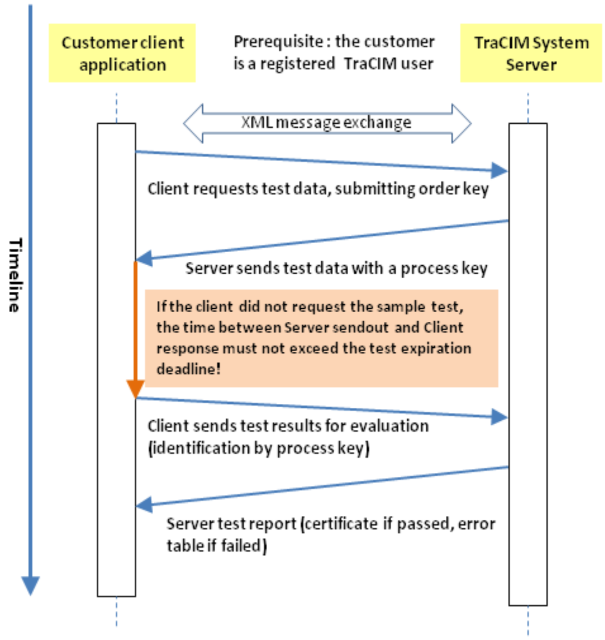
The customer may specify MPE values larger than the default’s. The results are then analysed with reference to these values. These values are also documented in the test report.
If a customer specifies MPE values smaller than the default’s, the data are not accepted and an error message is issued, except all MPE values are exactly given as 0.0. Then the default values are taken over.
If customers do not explicitly specify the MPEs, the default values are used.

1) ‘test expiration’ : When a test was requested and the server submitted the data, a 180 day deadline for the customer will start. After this date, test results will not be processed any more. 7 days before the expiration date and time, a notification mail will be sent to the customer’s e-mail address (as submitted with the registration data). One day before the expiration date and time, a final notification will be sent.
Additional note: public sample data are available without an expiration date.

3. Client-server communication

The test procedure is highly automated using internet based data exchange by client-server communication. Through a client application the user is able to obtain test data and send back the calculated results. The TraCIM system server automatically handles requests. The necessary communication between client and server to perform a single test is shown in the following figure:

Picture 1: Message exchange



The data exchanged between client and server is encapsulated within XML. A proper specification of the applied XML data schemata is subject to section 4. During processing messages are treated as plain character strings send between the client application and the TraCIM system server.

3.1. Development of a client application

Configuration of the HTTPS connection:

For communication with the TraCIM server a client application must use an HTTPS (Hypertext Transfer Protocol Secure = encrypted HTTPS) connection that allows to send and receive content in the form of character strings containing messages in XML format. Each HTTPS connection is created from a specific URL (Uniform Resource Locator) regarding requests for test data or requests for test result evaluation.

After opening the connection according to the URL the following configurations must be done:

- Enable output and input operations for the connection
- Set the request method “POST” (request comprising input and output)
- Set connection property “content-Type” to “application/xml”
- Set connection property “accept” to “application/xml”
- Set connection property “content-length” to the amount of characters of the content that is send to the TraCIM system.

Packages for creation and configuration of an HTTPS connection are available for different programming languages such as for (examples):

- Java: java.net API (HttpURLConnection)
- C/C++: Microsoft C++ REST SDK (“Casablanca”) or similar
- C#: System.Net (.NET 4.5 : System.Net.Http) Assembly

POST request for obtaining test data sets:

The URL of an HTTPS connection for POST request in order to obtain gaussian test data sets is

https://tracim.ptb.de/tracim/api/order/<GAUSS_ORDER_KEY>/test

where <GAUSS_ORDER_KEY> has to be replaced by the gaussian test order key purchased at the TraCIM web shop (future extension). After creation and configuration of a connection as described above the client application has to send the request message. When the TraCIM system receives the client message it will start creating test data sets and a unique process key. The test data and process key is returned to the client.

For cases that a customer is not properly registered for the TraCIM System or incorrect order keys were submitted the TraCIM server will send an error message.

Client content and TraCIM system content for retrieving test data are presented in sections 4.1 and 4.2. Error messages are presented in section 4.5.

POST request for sending test results and obtaining the certificate:

The URL of an HTTPS connection for a POST request in order to send test results for evaluation and get the certificate for gaussian testing is

https://www.tracim.ptb.de/tracim/api/test/<PROCESS_KEY>

where <PROCESS_KEY> has to be replaced by the individual process key that was returned by the TraCIM system at the POST request for test data sets.

The client will prepare the XML content (test result parameters, process key, software identification, ….) to be attached to the POST request and will send it to the TraCIM system.
The TraCIM system will receive and evaluate the content and generate a test report. It states whether the test is passed (user results sufficiently accurate) or gives a table with an error outline for all elements ‘b01-b44’ according to each parameter. In case of a sample test request the test report is not countersigned by PTB seal. The data sent to the client contains the test report as a PDF document encoded in XML.

For the case of an incorrect process key the TraCIM content is an error message.

Client content and TraCIM system content for retrieving the test evaluation and PDF report are presented in sections 4.3 and 4.4. Error messages are presented in section 4.5.

!	TraCIM service for testing is limited in time. After receiving an order key a customer has a total of 200 days for performing the gaussian test. After 20, 120 and 180 days the TraCIM system will automatically send warning messages to the customer e-mail address stating the remaining time for testing. When the validity of a test expires a final information message is send to the customer.

3.2. Public sample data for client testing

Sample data denotes a complete gaussian test for backtracking errors within the client application that could compromise a commercial test. For any registered customer the data is free of charge with unlimited request amount. In comparison to a test with commercial test data the certificate returned by the server is not countersigned by PTB as legally valid certificate.

The sample order key is available for registered customers. Please check the website (www.tracim.ptb.de) or send a mail to info.tracim@ptb.de for further information.

4. XML message content

4.1. Test request

The XML schema below shows the complete message for the initial test data request by the client application. The client has to specify the fields [TEST_CLIENT] with its proper name and its version in the field [TEST_CLIENT_VERSION]. All entries must be UTF-8 formatted character strings with at least one and a maximum of 20 characters.

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
xmlns:gauss="http://tracim.ptb.de/gauss/test"
xmlns:tracim="http://tracim.ptb.de/tracim">

         <product>[TEST_CLIENT]</product>
         <version>[TEST_CLIENT_VERSION]</version>

</tracim:tracim>
```

4.2. Test data

Test data returned by the TraCIM system are composed according to an XML schema with three major elements for order identification, process identification and the test data sets. The order element contains the order key, the date of the creation of the test data and a date for the expiration of the test data (deadline for sending test results for evaluation). The process element contains the process key associated with the test data request. The test element contains several point clouds with vector elements that give the x, y and z coordinates of the test data. For each point cloud the field “basicID” contains the unique data set ID (b01 – b44) and the field (XML tag) “computationObject” contains the corresponding geometrical element (LINE_3D, PLANE, CIRCLE_3D, CYLINDER, CONE, SPHERE).

```xml
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
xmlns:gauss="http://tracim.ptb.de/gauss/test"
xmlns:tracim="http://tracim.ptb.de/tracim">
  <tracim:order>
    <tracim:key>[ORDER_KEY]</tracim:key>
    <tracim:creationDate>[…]</tracim:creationDate>
    <tracim:expiration Date>[…]</tracim:expirationData>
  </tracim:order>
  <tracim:process>
          <tracim:key>[PROCESS_KEY]</tracim:key>
  </tracim:process>
  <tracim: tests xsi:type="gauss:gaussTestPackage">
    <gauss:package>
      <gauss:testElement>
        <gauss:basicID>b01</gauss:basicID>
 <gauss:computationObject>Line 2D</gauss:computationObject>
    <gauss:pointCloud>
      <gauss:vectors>
        <gauss:x>-44.7893023434000000</gauss:x>
        <gauss:y>58.3432222322222200</gauss:y>
        <gauss:z>0.000000000000000000</gauss:z>
      </gauss:vectors>
      <gauss:vectors>
        <gauss:x>-44.7893023434000000</gauss:x>
        <gauss:y>58.3432222322222200</gauss:y>
        <gauss:z>0.000000000000000000</gauss:z>
      </gauss:vectors>
        […]
   </gauss:pointCloud>
    </gauss:testElement>
    <gauss:testElement>
      <gauss:basicID>b16</gauss:basicID>
      <gauss:computationObject>Cylinder</gauss:computationObject>
        […]
    </gauss:testElement>
      […]
      </gauss:package>
    </tracim:tests>
  </tracim:tracim>
```

4.3. Test results

For sending the calculated results of software under test to the TraCIM system the XML schema on page 11 must be used. The client has to fill in enter the following information:

- [PROCESS_KEY] key received in message from section 4.2
- [CUSTOMER_NAME] customer name from TraCIM system registration
- [SOFTWARE_NAME] name of software that computed the test results
- [SOFTWARE_VERSION] version of software that computed the test results
- [SOFTWARE_REVISION] revision of software that computed the test results

This information is followed by elements that contain the result parameters of the software under test. For each test data set the client has to specify a single results element set containing the basic data set ID (b01 – b44) and the geometrical element ID (LINE_3D, PLANE, CIRCLE_3D, CYLINDER, CONE, SPHERE).

```xml
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<gauss:gaussResultPackage
xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
xmlns:gauss="http://tracim.ptb.de/gauss/test"
xmlns:tracim="http://tracim.ptb.de/tracim">
  <gauss:processKey>[PROCESS_KEY]</gauss:processKey>
  <gauss:customer>[CUSTOMER_NAME]</gauss:customer>
  <gauss:softwareName>[SOFTWARE_NAME]</gauss:softwareName>
  <gauss:softwareVersion>[SOFTWARE_VERSION]</gauss:softwareVersion>
  <gauss:softwareRev>[SOFTWARE_REVISION]</gauss:softwareRev>
  <gauss:resultPackage>
    <gauss:results>
      <gauss:basicID>b01</gauss:basicID>
             <gauss:computationObject>Line 3D</gauss:computationObject>
         <gauss:refParameter>
           <gauss:positionX>0.000001</gauss:positionX>
           <gauss:positionY>0.000005</gauss:positionY>
           <gauss:positionZ>0.000000</gauss:positionZ>
           <gauss:orientationX>1.00000000021</gauss:orientationX>
           <gauss:orientationY>-0.00000000005</gauss:orientationY>
           <gauss:orientationZ>0.0000000000</gauss:orientationZ>
         </gauss:refParameter>
    </gauss:results>
    <gauss:results>
      <gauss:basicID>b16</gauss:basicID>
             <gauss:computationObject>Cylinder</gauss:computationObject>
         […]
    </gauss:results>
      […]
  </gauss:resultPackage>
</gauss:gaussResultPackage>
```

According to the result parameter list in section 1.2., a client has to provide the following types of parameter elements:

- positionX x-coordinate of the position parameter
- positionY y- coordinate of the position parameter
- positionZ z- coordinate of the position parameter
- orientationY y- coordinate of the orientation vector
- orientationX x- coordinate of the orientation vector
- orientationZ z- coordinate of the orientation vector
- radius radius for ‘CIRCLE 3D’, ‘CYLINDER’, ‘CONE’ and ‘SPHERE’ ID
- apexAngle apex angle for ‘CONE’ ID

Entries for parameters that are not part of the geometrical element parameter specification have not to be written in the schema (in the example above this applies for the radius and apex angle parameters in the context of LINE_3D result parameters). The client must fill in these with value zero. All values that are entered into the schema must apply with the requirements in section 1.2. In addition these values have to be character strings compatible with UTF-8 format.

4.4. Test report

After evaluation the TraCIM system returns XML data according to a XML schema with the validation result. It comprises the three elements

- passed — true, if the software passed the test successfully, else false
- report — character string with short report on the test evaluation
- reportPDF — character string with the test report PDF

In order to create the test report PDF document as PDF file, the "reportPDF" character string ('base64' encoded – check programming language objects/methods) can be converted and written to a new file with the proper ".pdf" file name extension.

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
xmlns:xsi=”http://www.w3.org/2001/XMLSchema-instance”
xmlns:gauss="http://tracim.ptb.de/gauss/test"
xmlns:tracim="http://tracim.ptb.de/tracim">
         <tracim:validation>
           <tracim:passed>true</tracim:passed>
           <tracim:report></tracim:report>
           <tracim:reportPDF>1)JKD5iuDUD098IHh[…]</tracim:reportPDF>
         </tracim:validation>
</tracim:tracim>

Note: 1) 'base64' encoded
```

4.5. TraCIM Error Messages

The following XML code is sent by the server in case of errors, where [error code] and [error description] are replaced with values according to table 4 (after the XML excerpt).

```
<?xml version="1.0" encoding="UTF-8" standalone="true"?>
<tracim:tracim
       xmlns:xsi=”http://www.w3.org/2001/XMLSchema-instance”
       xmlns:gauss="http://tracim.ptb.de/gauss/test"
       xmlns:tracim="http://tracim.ptb.de/tracim">
  <tracim:error>
    <tracim:code>[error code]</tracim:code>
    <tracim:description>[error description]</tracim:description>
  </tracim:error>
</tracim:tracim>
```

Table 4 : Error messages [1)]

[error code]	[error description]
403	No more tests left for this test process.
403	Test process already closed.
403	Test Data already used.
404	Unknown order number.
422 [2)]	Unknown request type. Consult specific message.
422 [2)]	Process key does not match!
422 [2)]	Customer element is blank!
422 [2)]	Software vendor element is blank!
422 [2)]	Software name element is blank!
422 [2)]	Software version element is blank!
422 [2)]	MPE-elements not present. You are allowed to set MPE's to zero but the elements have to be present!
422 [2)]	Check your MPEs! If you set one MPE you have to set it all!
422 [2)]	Check your MPEs! Negative MPE's are not allowed.
500	Internal Server error.

Notes to table 4
[1)] Column [error code]: these codes are identical with the HTTP error codes. As one code can describe more than one reason, multiple lines with the same code are present; details can be found in the next column ([error description]).
Column [error description]: a textual explanation of the error reason.
[2)] For this code, a 'frame message' will be generated – '*Sorry, but your request could not be interpreted. Please check your message or contact the support. The system says:* '[error description]
Please note: if, due to specific reasons, the server can not generate a proper XML error message, please check the HTML response of the server. This can be done i.e. with programming the client for processing both XML and HTML messages.

4.6 TraCIM schemata

The XML schemata can be obtained from the following URL's:

https://tracim.ptb.de/tracim/api/schema/PTB_MATH_GAUSS_v1_test.xsd [1)]
https://tracim.ptb.de/tracim/api/schema/PTB_MATH_GAUSS_v1_result.xsd [2)]
https://tracim.ptb.de/tracim/api/schema/tracim.xsd [3)]

Notes:
1) Test data schema
2) Result data schema
3) Report schema

It is strongly recommended, that a customer in order to test client functions uses these schemata during client development.

5. The test report

The certificate can be extracted from the string stream ('base64' encoded; see chapter 4.4, XML tag 'tracim: reportPDF') into a PDF file ready for printing.

6. How to get support

- technical support for client software : support.tracim@ptb.de
- support regarding computational aims : info.tracim@ptb.de
- Provider support (contact, fees, …) : info.tracim@ptb.de

7. Legal and copyright information

Appendix: Computational Aims

The Gaussian substitute elements (section 1.1) are calculated such, that the sum of the squared distances from a point (x,y,z) to the geometric element is minimized.

- **Gaussian 3D line**
- **Gaussian plane**
- **Gaussian 3D circle**
- **Gaussian cylinder**
- **Gaussian sphere**
- **Gaussian cone**

Gaussian 3D line

Title

Best fit of a Gaussian line to 3D point coordinates in coordinate metrology

Subtitle

Determine a line to data points in space such, that the sum of the squared distances from a point (xi, yi, zi,) to the line is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 2$
Dimension or reference (refinement):
Values (refinement):

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints:
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of a point on the line
Type: real
Shape: vector of length 3
Constraints:
Dimension or reference (refinement): mm
Values (refinement):

Output parameter 2

Symbol: $A = (a, b, c)$
Description: vector pointing along the line
Type: real
Shape: vector of length 3
Constraints: $a^2 + b^2 + c^2 = 1$
Dimension or reference (refinement): dimensionless

Mathematical model (mandatory)

Given (x_i, y_i, z_i), i = 1, ..., n, determine values (x_0, y_0, z_0) and (a,b,c) to solve $\sum_1^n d_i^2 = \min$, where

$$d_i = \sqrt{\left(u_i^2 + v_i^2 + w_i^2\right)}$$

and

$$\begin{aligned} u_i &= c(y_i - y_0) - b(z_i - z_0) \\ v_i &= a(z_i - z_0) - c(x_i - x_0) \\ w_i &= b(x_i - x_0) - a(y_i - y_0) \end{aligned}$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991

Gaussian plane

Title

Best fit of a Gaussian plane to 3D point coordinates in coordinate metrology

Subtitle

Determine a plane to data points in space such, that the sum of the squared distances from a point (x_i, y_i, z_i,) to the plane is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 3$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of a point on the plane
Type: real
Shape: vector of length 3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): none

Output parameter 2

Symbol: $A = (a, b, c)$
Description: direction cosines of the normal to the plane
Type: real
Shape: vector of length 3
Constraints: $a^2 + b^2 + c^2 = 1$
Dimension or reference (refinement): dimensionless

Values (refinement): **none**

Mathematical model (mandatory)

Given (x_i, y_i, z_i,), i = 1, ..., n, determine values (x_0, y_0, z_0) and (a,b,c) to solve $\sum_1^n d_i^2 = \min$, where

$$d_i = a(x_i - x_0) + b(y_i - y_0) + c(z_i - z_0)$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991

Gaussian 3D circle

Title

Best fit of a Gaussian circle to 3D point coordinates in coordinate metrology

Subtitle

Determine a circle to data points in space such, that the sum of the squared distances from a point (x_i, y_i, z_i,) to the circle is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 3$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of the centre of the circle
Type: real
Shape: vector of length 3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): none

Output parameter 2

Symbol: $A = (a, b, c)$
Description: direction cosines of the plane containing the circle
Type: real
Shape: vector of length 3
Constraints: $a^2 + b^2 + c^2 = 1$
Dimension or reference (refinement): dimensionless

Values (refinement): none

Output parameter 3

Symbol: *r*

Description: radius of the circle

Type: real

Shape: scalar

Constraints: $r > 0$

Dimension or reference (refinement): mm

Values (refinement): none

Mathematical model (mandatory)

Given (x_i, y_i, z_i,), i = 1, ..., n, determine values (x_0, y_0, z_0), (a,b,c) and r to solve $\sum_1^n d_i^2 = \min$, where

$$d_i^2 = e_i^2 + f_i^2$$

and

$$e_i = a(x_i - x_0) + b(y_i - y_0) + c(z_i - z_0)$$
$$f_i = \sqrt[2]{u_i^2 + v_i^2 + w_i^2} - r$$

with

$$u_i = c(y_i - y_0) - b(z_i - z_0)$$
$$v_i = a(z_i - z_0) - c(x_i - x_0)$$
$$w_i = b(x_i - x_0) - a(y_i - y_0)$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991

Gaussian cylinder

Title

Best fit of a Gaussian cylinder to 3D point coordinates in coordinate metrology

Subtitle

Determine a cylinder to data points in space such, that the sum of the squared distances from a point (x_i, y_i, z_i,) to the cylinder is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 5$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of a point on the cylinder axis
Type: real
Shape: vector of length 3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): none

Output parameter 2

Symbol: $A = (a, b, c)$
Description: vector pointing along the cylinder axis
Type: real
Shape: vector of length 3
Constraints: $a^2 + b^2 + c^2 = 1$

Dimension or reference (refinement): **dimensionless**
Values (refinement): **one**

Output parameter 3

Symbol: ***r***
Description: **radius of cylinder**
Type: **real**
Shape: **scalar**
Constraints: $r > 0$
Dimension or reference (refinement): **mm**
Values (refinement): **none**

Mathematical model (mandatory)

Given (x_i, y_i, z_i), i = 1, ..., n, determine values (x_0, y_0, z_0), (a,b,c) and r to solve $\sum_1^n d_i^2 = \min$, where

$$d_i = r_i - r$$

and

$$r_i = \sqrt{\frac{u_i^2 + v_i^2 + w_i^2}{a^2 + b^2 + c^2}}$$

with

$$u_i = c(y_i - y_0) - b(z_i - z_0)$$
$$v_i = a(z_i - z_0) - c(x_i - x_0)$$
$$w_i = b(x_i - x_0) - a(y_i - y_0)$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991

Gaussian sphere

Title

Best fit of a Gaussian sphere to 3D point coordinates in coordinate metrology

Subtitle

Determine a sphere to data points such, that the sum of the squared distances from a point (x_i, y_i, z_i,) to the sphere is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 4$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of the centre of the sphere
Type: real
Shape: vector of length 3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): none

Output parameter 2

Symbol: r
Description: radius of the sphere
Type: real
Shape: scalar
Constraints: $r > 0$

Dimension or reference (refinement): **mm**
Values (refinement): **none**

Mathematical model (mandatory)

Given (x_i, y_i, z_i,), i = 1, ..., n, determine values (x_0, y_0, z_0) and r to solve $\sum_1^n d_i^2 = \min$, where

$$d_i = r_i - r$$

and

$$r_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2}$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991

Gaussian cone

Title

Best fit of a Gaussian cone to 3D point coordinates in coordinate metrology

Subtitle

Determine a cone to data points in space such, that the sum of the squared distances from a point (x_i, y_i, z_i,) to the cone is minimized.

Input parameters

Input parameter 1

Symbol: n
Description: number of data points
Type: unsigned integer
Shape: scalar
Constraints: $n \geq 6$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Input parameter 2

Symbol: X, with $X_{i,1} = x_i$, $X_{i,2} = y_i$, $X_{i,3} = z_i$,
Description: coordinates of data points
Type: real
Shape: matrix of dimension n×3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): $-10{,}000.0 \leq x_i \leq +10{,}000.0$
$-10{,}000.0 \leq y_i \leq +10{,}000.0$
$-10{,}000.0 \leq z_i \leq +10{,}000.0$

Output parameters

Output parameter 1

Symbol: $X_0 = (x_0, y_0, z_0)$
Description: coordinates of a point on the cone axis, preferably the projection of the centroid of the input parameters 2 on the cone axis, i.e. X_0 is the foot point of (x_M, y_M, z_M) on the cone axis

$$x_M = \frac{1}{n}\sum_{1}^{n} x_i \quad ; \qquad y_M = \frac{1}{n}\sum_{1}^{n} y_i \quad ; \qquad z_M = \frac{1}{n}\sum_{1}^{n} z_i$$

Type: real
Shape: vector of length 3
Constraints: none
Dimension or reference (refinement): mm
Values (refinement): none

Output parameter 2

Symbol: $A = (a, b, c)$
Description: vector pointing along the cone axis in the direction of decreasing radius
Type: real
Shape: vector of length 3
Constraints: $a^2 + b^2 + c^2 = 1$
Dimension or reference (refinement): dimensionless
Values (refinement): none

Output parameter 3

Symbol: α
Description: full angle at the apex
Type: real
Shape: scalar
Constraints: $0 < \alpha < 180$
Dimension or reference (refinement): degree
Values (refinement): none

Output parameter 4

Symbol: r
Description: radius of the cone perpendicular to the cone axis at X_0
($r = 0$, if X_0 marks the apex of the cone)
Type: real
Shape: scalar
Constraints: $r \geq 0$
Dimension or reference (refinement): mm
Values (refinement): none

Mathematical model (mandatory)

Given (x_i, y_i, z_i,), $i = 1, \ldots, n$, determine values (x_0, y_0, z_0), (a,b,c), α and r to solve $\sum_1^n d_i^2 = \min$, where

$$d_i = e_i \cos(\frac{\alpha}{2}) + f_i \sin(\frac{\alpha}{2}) - r \cos(\frac{\alpha}{2})$$

and

$$\boldsymbol{e_i} = \sqrt{\boldsymbol{u_i^2} + \boldsymbol{v_i^2} + \boldsymbol{w_i^2}}$$

and

$$f_i = a(x_i - x_0) + b(y_i - y_0) + c(z_i - z_0)$$

with

$$u_i = c(y_i - y_0) - b(z_i - z_0)$$
$$v_i = a(z_i - z_0) - c(x_i - x_0)$$
$$w_i = b(x_i - x_0) - a(y_i - y_0)$$

References

1. Forbes, Alistair B.: Least-squares best fit geometric elements, NPL Report DITC 140/89 , ISSN 0262-5369, 40 pages, revised edition, 1991