Project 1

Zilong Li
Student ID: 518070910095

February 22, 2021

Warmups

1 All horses are the same color; we can prove this by induction on the number of horses in a given set. Here's how: If there's just one horse then it's the same color as itself, so the basis is trivial. For the induction step, assume that there are n horses numbered 1 to n. By the induction hypothesis, horses 1 through $n-1$ are the same color, and similarly horses 2 through n are the same color. But the middle horses, 2 through $n-1$, can't change color when they're in different groups; these are horses, not chameleons. So horses 1 and n must be the same color as well, by transitivity. Thus all n horses are the same color; QED." What, if anything, is wrong with this reasoning?

Solution. It is wrong. In fact, the definition of *same* is not exact for $n = 1$ senario. The *same* should describe the relationship between two *different* objects. The mathematical induction should start from $n = 2$.

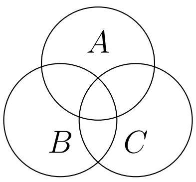
For $n = 2$, *the middle horses* are not existed (from 2 through $n - 1 = 1$). The basis step does not holds. □

2 Find the shortest sequence of moves that transfers a tower of n disks from the left peg A to the right peg B, if direct moves between A and B are disallowed. (Each move must be to or from the middle peg. As usual, a larger disk must never appear above a smaller one.)

3 Show that, in the process of transferring a tower under the restrictions of the preceding exercise, we will actually encounter every properly stacked arrangement of n disks on three pegs.

4 Are there any starting and ending congurations of n disks on three pegs that are more than $2n - 1$ moves apart, under Lucas's original rules?

5 A "Venn diagram" with three overlapping circles is often used to illustrate the eight possible subsets associated with three given sets:



Can the sixteen possibilities that arise with four given sets be illustrated by four overlapping circles?