Mathematical Foundations of Computer Science

Project 13

Zilong Li

Student ID: 518070910095

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Warmups

7 Is (5.34) true also when $k < 0$?

8 Evaluate

$$\sum_k \binom{n}{k}(-1)^k(1-\frac{k}{n})^n$$

What is the approximate value of this sum, when n is very large? Hint: The sum is $\Delta^n f(0)$ for some function f.

9 Show that the generalized exponentials of (5.58) obey the law

$$\mathcal{E}_t(z) = \mathcal{E}(tz)^{1/t}, \text{if } t \neq 0,$$

where $\mathcal{E}(z)$ is an abbreviation for $\mathcal{E}_1(z)$.

Basics

14 Prove identity (5.25) by negating the upper index in Vandermonde's con-volution (5.22). Then show that another negation yields (5.26).

15 What is $\sum_k \binom{n}{k}^3(-1)^k$?Hint:See (5.29).

16 Evaluate the sum

$$\sum_k \binom{2a}{a+k}\binom{2b}{b+k}\binom{2c}{c+k}(-1)^k$$

when a, b, c are nonnegative integers.

17 Find a simple relation between $\binom{2n-1/2}{n}$ and $\binom{2n-1/2}{2n}$.

18 Find an alternative form analogous to (5.35) for the product

$$\binom{r}{k}\binom{r-1/3}{k}\binom{r-2/3}{k}$$

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