Project 2

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Warmups

6 Some of the regions defined by n lines in the plane are infinite, while others are bounded. What's the maximum possible number of bounded regions?

Solution. The answer. □

7 Let $H(n) = J(n+1) - J(n)$. Equation (1.8) tells us that $H(2n) = 2$, and $H(2n+1) = J(2n+2) - J(2n+1) = (2J(n+1) - 1) - (2J(n) + 1) = 2H(n) - 2$, for all $n > 1$. Therefore it seems possible to prove that $H(n) = 2$ for all n, by induction on n. What's wrong here?

Solution. □

Homework

8 Solve the recurrence $Q_0 = \alpha; Q_1 = \beta; Q_n = (1 + Q_n - 1) = Q_n - 2; \text{for} n > 1$. Assume that $Q_n \neq 0$ for all $n \geq 0$. *Hint:* $Q_4 = (1+\alpha) = \beta$.

10 Let Q_n be the minimum number of moves needed to transfer a tower of n disks from A to B if all moves must be clockwise —that is, from A to B, or from B to the other peg, or from the other peg to A. Also let Rn be the minimum number of moves needed to go from B back to A under this restriction. Prove that

$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1} + 1, & \text{if } n > 0; \end{cases} R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n = Q_{n-1} + 1, & \text{if } n > 0 \end{cases}$$

(You need not solve these recurrences; we'll see how to do that in Chapter 7.)