Mathematical Foundations of Computer Science

Project 4

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Warmups

1 What does the notation

$$\sum_{i=4}^{0} q_i$$

mean?

Solution. In a programmer's perspective, this notation could be interpreted as the decreasing order on the index:

$$\sum_{i=4}^{0} q_i = q_4 + q_3 + q_2 + q_1 + q_0 = \sum_{i=0}^{4} q_k$$

But in mathematics, the formula also holds:

$$\sum_{i=4}^{0} q_i = \sum_{4 \le i \le 0} q_i = \sum_{\varnothing} q_i = 0$$

However, if $\sum_{k \le n} q_k$ and $\sum_{k<m} q_k$ are finite sums, the following interpretation also holds if the summation is regarded generally:

$$\sum_{i=4}^{0} q_i = \sum_{i \le 0} q_i - \sum_{i<4} q_i = -q_1 - q_2 - q_3$$

The interpretation is different when the definition differs, one should always use an increasing order on index to avoid misinterpretations. □

2 Simplify the expression $x \cdot ([x > 0] - [x < 0])$.

Solution. According to Iverson's convention,

$$x \cdot ([x > 0] - [x < 0]) = \begin{cases} x \cdot 1, & x > 0, \\ 0 \cdot 0, & x = 0, \\ x \cdot (-1), & x < 0 \end{cases} = |x|$$

□

3 Demonstrate your understanding of $\sum$-notation by writing out the sums

$$\sum_{0 \le k \le 5} a_k \text{ and } \sum_{0 \le k^2 \le 5} a_{k^2}$$

in full. (Watch out the second sum is a bit tricky.)

Solution.

$$\sum_{0 \le k \le 5} a_k = a_0 + a_1 + a_2 + a_3 + a_4 + a_5$$

The second notation here impiles:

$$0 \le k^2 \le 5 \Rightarrow k = \{0, 1, 2, -1, -2\}$$

So, the summation follows:

$$\sum_{0 \le k^2 \le 5} a_{k^2} = a_0 + 2a_1 + 2a_2$$

□

4 Express the triple sum

$$\sum_{1 \le i < j < k \le 4} a_{ijk}$$

as a three-fold summation (with three $\sum$'s),

a summing first on k, then j, then i;

b summing first on i, then j, then k.

Also write your triple sums out in full without the $\sum$-notation, using parentheses to show what is being added together first.

Solution. (This solution is broken.) □

5 What's wrong with the following derivation?

$$\left(\sum_{j=1}^{n} a_j\right)\left(\sum_{k=1}^{n} \frac{1}{a_k}\right) = \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{a_j}{a_k} = \sum_{k=1}^{n}\sum_{k=1}^{n} \frac{a_k}{a_k} = \sum_{k=1}^{n} n = n^2$$

Solution.

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{a_j}{a_k} \ne \sum_{k=1}^{n}\sum_{k=1}^{n} \frac{a_k}{a_k}$$

Because

$$\begin{aligned}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{a_j}{a_k} &= \sum_{j=1}^{n}\left(\frac{a_j}{a_1} + \cdots + \frac{a_j}{a_n}\right) \\ &= \left(\frac{a_1}{a_1} + \cdots + \frac{a_1}{a_n}\right) + \cdots + \left(\frac{a_n}{a_1} + \cdots + \frac{a_n}{a_n}\right)\end{aligned}$$

is not equal to

$$\sum_{k=1}^{n}\sum_{k=1}^{n}\frac{a_k}{a_k}=\sum_{k=1}^{n}\sum_{k=1}^{n}\frac{a_k}{a_k}=\sum_{k=1}^{n}n=n^2$$

□