Mathematical Foundations of Computer Science

Project 9

Zilong Li

Student ID: 518070910095

April 29, 2021

Warmups

1 What is the smallest positive integer that has exactly k divisors, for $1 \leq k \leq 6$?

2 Prove that $\gcd(m,n) \cdot \operatorname{lcm}(m,n) = m \cdot n$, and use this identity to express $\operatorname{lcm}(m,n)$ in terms of $\operatorname{lcm}(n \bmod m, m)$, when $n \bmod m \neq 0$. Hint: Use (4.12), (4.14), and (4.15).

3 Let $\pi(x)$ be the number of primes not exceeding x. Prove or disprove: $\pi(x) - \pi(x-1) = [x \text{ is prime}]$.

4 What would happen if the Stern-Brocot construction started with the five fractions $\left(\frac{0}{1}, \frac{1}{0}, \frac{0}{-1}, \frac{-1}{0}, \frac{0}{1}\right)$ instead of with $\left(\frac{0}{1}, \frac{1}{0}\right)$?

5 Find simple formulas for L^k and R^k, when L and R are the 2×2 matrices of (4.33).

6 What does '$a \equiv b \pmod 0$' mean?

7 Ten people numbered 1 to 10 are lined up in a circle as in the Josephus problem, and every mth person is executed. (The value of m may bemuch larger than 10.) Prove that the first three people to go cannot be 10,k, and $k+1$(in this order), for any k.

Basics

14 Prove or disprove:

a $\gcd(km, kn) = k\gcd(m,n)$

b $\operatorname{lcm}(km, kn) = k\operatorname{lcm}(m,n)$

15 Does every prime occur as a factor of some Euclid number e_n?