

## Project 4

Log Creative

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### Warmups

- 1 What does the notation

$$\sum_{i=4}^0 q_i$$

mean?

**Solution.** In a programmer's perspective, this notation could be interpreted as the decreasing order on the index:

$$\sum_{i=4}^0 q_i = q_4 + q_3 + q_2 + q_1 + q_0 = \sum_{i=0}^4 q_k$$

But in mathematics, the formula also holds:

$$\sum_{i=4}^0 q_i = \sum_{4 \leq i \leq 0} q_i = \sum_{\emptyset} q_i = 0$$

However, if  $\sum_{k \leq n} q_k$  and  $\sum_{k < m} q_k$  are finite sums, the following interpretation also holds if the summation is regarded generally:

$$\sum_{i=4}^0 q_i = \sum_{i \leq 0} q_i - \sum_{i < 4} q_i = -q_1 - q_2 - q_3$$

The interpretation is different when the definition differs, one should always use an increasing order on index to avoid misinterpretations.  $\square$

- 2 Simplify the expression  $x \cdot ([x > 0] - [x < 0])$ .

**Solution.** According to Iverson's convention,

$$x \cdot ([x > 0] - [x < 0]) = \begin{cases} x \cdot 1, & x > 0, \\ 0 \cdot 0, & x = 0, \\ x \cdot (-1), & x < 0 \end{cases} = |x|$$

$\square$

- 3 Demonstrate your understanding of  $\sum$ -notation by writing out the sums

$$\sum_{0 \leq k \leq 5} a_k \text{ and } \sum_{0 \leq k^2 \leq 5} a_{k^2}$$

in full. (Watch out the second sum is a bit tricky.)

**Solution.**

$$\sum_{0 \leq k \leq 5} a_k = a_0 + a_1 + a_2 + a_3 + a_4 + a_5$$

The second notation here implies:

$$0 \leq k^2 \leq 5 \Rightarrow k = \{0, 1, 2, -1, -2\}$$

So, the summation follows:

$$\sum_{0 \leq k^2 \leq 5} a_{k^2} = a_0 + 2a_1 + 2a_2$$

□

- 4 Express the triple sum

$$\sum_{1 \leq i < j < k \leq 4} a_{ijk}$$

as a three-fold summation (with three  $\sum$ 's),

- a summing first on  $k$ , then  $j$ , then  $i$ ;
- b summing first on  $i$ , then  $j$ , then  $k$ .

Also write your triple sums out in full without the  $\sum$ -notation, using parentheses to show what is being added together first.

**Solution.** (This solution is broken.)

□

- 5 What's wrong with the following derivation?

$$\left( \sum_{j=1}^n a_j \right) \left( \sum_{k=1}^n \frac{1}{a_k} \right) = \sum_{j=1}^n \sum_{k=1}^n \frac{a_j}{a_k} = \sum_{k=1}^n \sum_{j=1}^n \frac{a_k}{a_k} = \sum_{k=1}^n n = n^2$$

**Solution.**

$$\sum_{j=1}^n \sum_{k=1}^n \frac{a_j}{a_k} \neq \sum_{k=1}^n \sum_{j=1}^n \frac{a_k}{a_k}$$

Because

$$\begin{aligned} \sum_{j=1}^n \sum_{k=1}^n \frac{a_j}{a_k} &= \sum_{j=1}^n \left( \frac{a_j}{a_1} + \cdots + \frac{a_j}{a_n} \right) \\ &= \left( \frac{a_1}{a_1} + \cdots + \frac{a_1}{a_n} \right) + \cdots + \left( \frac{a_n}{a_1} + \cdots + \frac{a_n}{a_n} \right) \end{aligned}$$

is not equal to

$$\sum_{k=1}^n \sum_{k=1}^n \frac{a_k}{a_k} = \sum_{k=1}^n \sum_{k=1}^n \frac{a_k}{a_k} = \sum_{k=1}^n n = n^2$$

□