Mathematical Foundations of Computer Science

Project 10

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Warmups

8 The residue number system $(x \bmod 3, x \bmod 5)$ considered in the text hasthe curious property that 13 corresponds to (1, 3), which looks almost thesame. Explain how to nd all instances of such a coincidence, without calculating all fifteen pairs of residues. In other words, find all solutions to the congruences

$$10x + y \equiv x \pmod 3, \quad 10x + y \equiv y \pmod 5.$$

Hint: Use the facts that $10u + 6v \equiv u \pmod 3$ and $10u + 6v \equiv v \pmod 5$

9 Show that $(3^{77} - 1)/2$ is odd and composite. Hint: What is $3^{77} \bmod 4$?

10 Compute $\varphi(999)$.

11 Find a function $\sigma(n)$ with the property that

$$g(n) = \sum_{0 \le k \le n} f(k) \Leftrightarrow f(n) = \sum_{0 \le k \le n} \sigma(k) g(n-k).$$

(This is analogous to the Möbius function; see (4.56).)

12 Simplify the formula $\sum_{d \backslash m} \sum_{k \backslash d} \mu(k) g(d/k)$.

13 A positive integernis called *squarefree* if it is not divisible by m^2 for any $m > 1$. Find a necessary and sufficient condition that n is squarefree,

- **a** in terms of the prime-exponent representation (4.11) of n;
- **b** in terms of $\mu(n)$.

Basics

16 What is the sum of the reciprocals of the first n Euclid numbers?

17 Let f_n be the "Fermat number" $2^{2^n} + 1$. Prove that $f_m \perp f_n$ if $m < n$.

18 Show that if $2n + 1$ is prime then n is a power of 2.