Application of Optimization Techniques to Hydro-Québec

MGSC 662: Decision Analytics

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1 Introduction

The Power Balance

The electricity that runs the appliances in our homes are generated by large-scale power plants, very often located in a faraway land...

This electricity is consumed by large-scale enterprises, often called "load-serving entities" (LSE for short), who take upon the responsibility of distributing this electricity downstream to various parties, including end-consumers such as ourselves. The load-serving entity responsible for the distribution of electricity in Montréal (as well as the province of Québec) is Hydro Québec.

In accordance with the law of conservation of energy, the amount of electric power produced must exactly equal the amount of electric power consumed. This is termed "the power balance" and happens to be an absolute non-negotiable.

The reason as to why the power balance must be adhered to at all times is because whenever the grid enters an unbalanced state, the operating frequency of the power grid will deviate (lower or higher) from the standard 60 Hz it operates at. Operating the power grid at the wrong frequency results in serious damages to the power generators. This is why generators prefer to completely shut down than try and keep operating, causing blackouts.

A recent example of the power grid shutting down owing to a power imbalance was observed during the 2021 Texas Power Crisis[1], causing the grid to fail due to improper winterization and overwhelming demand at the onset of a surprise blizzard.

Pricing of Electricity

Electric power is usually always supplied by the generator that minimizes the total cost to the system. During times of lower demand, the cheapest generators will supply all of the electricity and the resulting price of electricity is lower. On the other hand, in times of high demand, more generators will need to be roped into supply power thereby increasing the cost of generation which in turn leads to a higher price for electric power.

The cost of producing electricity also depends on the type of fuel used to generate it. It is more expensive to generate an additional unit of power with gas or coal as compared to sources with lower marginal costs such as hydro-electricity, solar or nuclear.

A large, central organization usually is in charge of ensuring that electricity is produced in a way that the total cost to the system is minimized. However, during the Quiet Revolution, the Government of Québec nationalized Hydro-Québec, thereby ensuring that it is the single organization that regulates and handles all electricity generation in the province. In the United States, large private organizations such as PJM Interconnection and NYISO handle this, depending on the region.

Hydro-Québec

Hydro-Québec is a state-owned enterprise in charge of all generation, transmission and distribution of electricity in Québec. It operates 62 generating stations across the province with a combined output capacity of 37, 370 megawatts. With 40% of Canada's water resources being in Québec, Hydro-Québec is the fourth largest producer of hydropower in the world. In fact, Hydro-Québec is capable of generating a quarter of the electricity needed by the entire world!

Financially, the Government of Québec has massively benefited from nationalizing and operating Hydro-Québec: as the sole shareholder of the enterprise, it was entitled to $2.39 billion CAD in 2018 in dividends. In addition, the very high output capacity of the company enables it to export electricity to the North-eastern United States.

2 Problem Description and Formulation

Power distribution is a complex problem involving supply of electricity from generators with generating capacity ranging from 4-5,616 kWh in Québec, supplying to 379 municipalities at different rates every hour in a single day. Hydro Québec's mandate is to meet electricity demand and power municipalities at an affordable cost. To achieve this, production costs and electricity loss during transmission must be minimized.

Data Collection and Preparation

List of all generating stations operated by Hydro-Québec

Information relating to the 62 generating stations as well as their maximum generation capacities (in MW) operated by Hydro-Québec was obtained by scraping the associated webpage on Hydro Québec's website.

Municipality-wise Demand Data

To get a list of all distinct neighborhoods serviced by the entity, consumption data from 2016-2021 was collected from the website and unique municipalities were extracted from the data. The latitudes and longitudes of each municipality was obtained using the Google Maps API.

Power Grid Data

The above two data sources in conjunction with the Google Maps API Toolkit (particularly the Places and Geocoding APIs) were used to retrieve the latitude and longitude of every location as precisely as possible. Later, the GeoPy open-source Python library was used to calculate the distances between each generator and each municipality to facilitate the building of the model.

These are the most important sources of data. Other data sources used are listed below with their exact sources cited in the references:

- Demand Projection - 2.1x from 2020 to 2050, linear growth ("Energy Fact Book 2023- 2024", 2023)
- Operating and Maintenance costs per MWh – (U.S. Department of Energy, 2020, p. 28-29)

- CO_2 emission/MWh for each type of plant - (Hydro Québec, 2023)
- Installation cost of each new type of power plant for capacity (in MW) - (U.S. Department of Energy, 2020, p. 28-29)
- Hydro-Québec Investment estimation ("Action Plan 2035", 2023)
- Maximum proportion of demand that can be met with wind power plants - 15% ("Action Plan 2035", 2023)
- Minimum proportion of demand that should be met with existing hydro power plants - 70% - ("Action Plan 2035", 2023)

3 Model Implementation and Results

Modeling Overview

The aim of this model is to optimize the hydro-powered electricity grid using Gurobi, with a focus on the efficient operation of power stations and the distribution of electricity. It takes into account targets, data sources, grid details and key components, with a strong emphasis on minimizing costs, distribution losses and a sustainability factor.

The mathematical model for optimizing Hydro Québec's power grid is a complex linear programming problem designed to minimize costs and optimize power distribution. Below described is a detailed representation of the model:

Decision Variables

The decision variables used in this model are:

$$X_{i,j,k} = \text{Power transmitted (in MWh) from Plant } i \text{ to Municipality } j \text{ in Month } k$$

$$P_{i,k} = \text{Power generated (in MWh) at Plant } i \text{ in Month } k$$

The data on the hydro-electricity generating stations, their capacities (in MW) and the list of municipalities serviced are publicly available on Hydro-Québec's website as mentioned in the previous section. In total, there are 62 generating stations/plants (i), 379 municipalities (j) and 12 months (k) being taken into account.

Objective Function

The problem objective is to minimize cost. In this model the cost is comprised of the cost of power generation and the lost earning from electricity lost in the electricity transmission process. The cost of generation per MWh is estimated as $3.456 (U.S. Department of Energy, 2020). 8-15% is the range of electricity lost when transmitting electricity (Hydro-Québec, 2003). The distance from plant i to municipality j is calculated using geodesic distance using the latitude and longitude of the coordinates of where the plants are located from the destination municipalities. Distances are normalized and inserted as a two-dimensional matrix. It is assumed that the cost of energy loss is the same as the cost of generation, \$3.456.

Sustainability Factor

A sustainability factor is added into the model to minimize the sustainability impact cost, by considering the age of the generators. The operational cost is presumed to increase by 1.5% for every additional year of generator age. Adding this secondary objective ensures that Gurobi prioritizes newer generators. This is not an objective, but in the code, it is set as a secondary objective to represent the age factor.

Model Formulation

Taking all of this into account, the overall objective function for this model can be written as:

$$\text{Cost of Power Generation} = \sum_{i=1}^{I}\sum_{k=1}^{K}(\text{production\_cost\_val} \times P_{i,k}) \tag{1}$$

where *production_cost_val* refers to the cost of generation per MWh (\$3.456) and $P_{i,k}$ refers to the power generated by generating station i at time k

$$\text{Cost of Transmission Loss} = \text{production\_cost\_val} \times \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} X_{i,j,k} \times (0.08 + 0.07 \times \text{distance\_matrix}_{i,j}) \tag{2}$$

where $X_{i,j,k}$ refers to the power transmitted from generating station i to municipality j at time k, and $distance\_matrix_{i,j}$ refers to the distance between generating station i and municipality j

$$\text{Sustainability Factor} = \sum_{i=1}^{I}\sum_{k=1}^{K}((2024 - \text{year\_of\_construction}_i) \times 0.015 \times P_{i,k}) \tag{3}$$

Constraints

In the below listed constraints, I and K denote the range of generating stations and time periods respectively whereas J refers to the range of nodes in the power grid. $P_{i,k}$, $X_{i,j,k}$, $capacity_i$ and $demand\_matrix_{j,k}$ represent power generation, power transmission, capacity of the generating station and demand respectively.

Constraint 1 Considering that plants need maintenance and are not operational at full capacity, each plant is set to operate under 80% of capacity.

$$P_{i,k} \leq 0.8 \times \text{capacity}_i \quad \forall i \in I, \forall k \in K \tag{4}$$

Constraint 2 Total Production must be at least 20% higher than the demand in each month to ensure no power outages occur for underdelivering.

$$\sum_{i=1}^{I} P_{i,k} \geq 1.2 \times \sum_{j=1}^{J} \text{demand\_matrix}_{j,k} \quad \forall k \in K \tag{5}$$

Constraint 3 Hydro Québec cannot distribute more than what is produced each month.

$$\sum_{j=1}^{J} X_{i,j,k} \leq P_{i,k} \quad \forall i \in I, \forall k \in K \tag{6}$$

Constraint 4 Assuming that all electricity generated is consumed, distribution losses is equal to demand.

$$\sum_{i=1}^{I} (X_{i,j,k} - (X_{i,j,k} \times (0.08 + 0.07 \times \text{distance\_matrix}_{i,j}))) = \text{demand\_matrix}_{j,k} \quad \forall j \in J, \forall k \in K \tag{7}$$

Constraint 5 Electricity supply is consistent. For instance, electricity generation cannot be at capacity in one month and be 0 in the following month. Therefore, a range of ±20% is applied on the electricity supply in one month to the following month.

$$P_{i,k} - P_{i,k-1} \leq 0.2 \times \text{capacity}_i \quad \forall i \in I, \forall k \in \{2, \ldots, K\} \tag{8}$$

$$P_{i,k} - P_{i,k-1} \geq -0.2 \times \text{capacity}_i \quad \forall i \in I, \forall k \in \{2, \ldots, K\} \tag{9}$$

Analysis of Results

The results from the technical implementation of the above model provides a wealth of quantitative data that can be analyzed to assess the efficiency and effectiveness of the optimization model. Below shown is the utilization of generators based on their operational capacity in 2024.

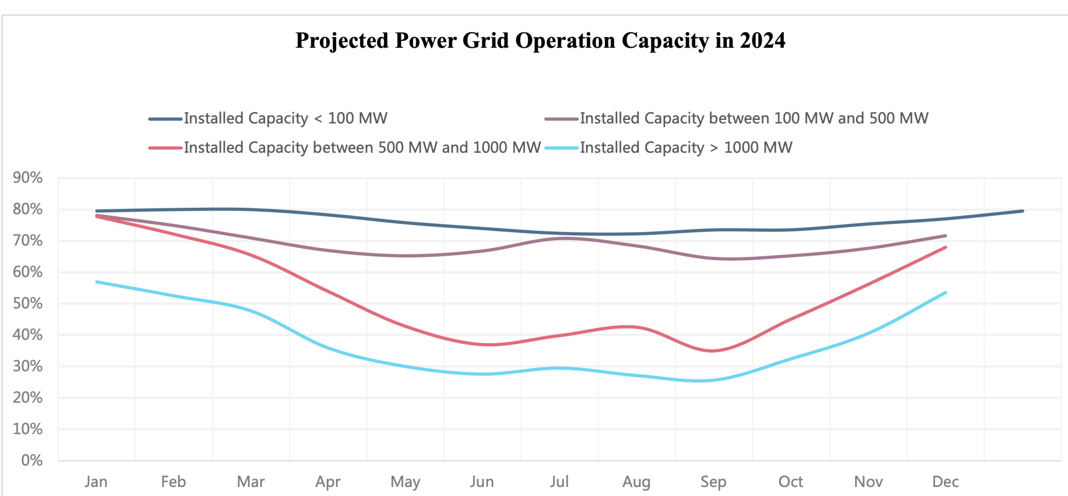

Figure 2: Projected Power Grid Operation by Capacity in 2024

From the graph we can see that the operating capacity of large generating stations is smaller than small generating stations, especially in summer. For the full operation schedule in 2024, please refer to Appendix 2. Some of the salient results from the model are listed as follows:

Operating Capacity and Power Generation

Beauharnois: A constant generation of 1,073,664 MWh each month, operating at 80% capacity. This consistent output indicates its critical role in the grid for stable energy supply.

Bernard-Landry (Eastmain-1-A): Generation varies from 442,368 MWh in January to 0 MWh in June, and then increases to 331,776 MWh by December. This fluctuation could be due to seasonal factors or strategic operational decisions.

The operating capacities of the plants, mostly at or below 80%, suggest an emphasis on not overburdening the facilities, possibly extending their operational life and reducing maintenance costs.

Demand Fulfilment and Distribution Plan

The distribution plan shows specific monthly distribution values for each plant to various cities. For example, Bersimis-2 distributes 14,036 MWh to Acton Vale in February, indicating a precise calibration of supply to meet demand.

Distribution values also reflect the consideration of transmission losses, as they are adjusted for the distance factor, ensuring efficient delivery of power.

The demand fulfillment percentages reveal how each plant's output is aligned with the demand of different municipalities. For instance, Bersimis-2 fulfills 100% of Acton Vale's demand from February to December, showing responsiveness to regional energy needs.

Environmental and Economic Balancing

The incorporation of a sustainability factor in the model likely leads to a preference for newer plants. This could indirectly contribute to lower emissions, as newer plants are generally more efficient and environmentally friendly.

Cost Considerations

The model includes a production cost value of 3.456 per MWh. The total cost of generation and distribution is minimized in the objective function, indicating an economically optimized operation plan.

Seasonal and Monthly Variations

The model's capacity to adjust power generation according to seasonal demands is evident. For example, the increase in power generation in colder months by certain plants indicates a response to higher energy demands typical of these periods.

Modeling Conclusions

The project showcases a comprehensive approach to optimizing a complex power grid. The results indicate a successful balancing act between cost-efficiency, environmental sustainability, and demand fulfillment. Future iterations of the model could potentially integrate real-time data analytics for even more dynamic optimization and include more explicit environmental metrics. Overall, the project serves as a potent example of using advanced optimization techniques in utility management, paving the way for more sustainable and efficient energy distribution.

4 Problem Extension: Greenhouse Gas Emissions and Exploring Alternative Clean Electricity Sources

In June 2021, the *Canadian Net-zero Emissions Accountability Act* was passed. This shows Canada's commitment to keep transparency of its actions. Furthermore, through growing investments in clean energy, Hydro Québec should begin to differentiate its power generation sources ("*Energy Fact Book 2023-2024*", 2023). Adding these considerations into the basic model, the emissions and operational costs from hydropower, wind, nuclear, geothermal, biomass, and solar are explored (Appendix 4).

Source	**Emission (g of CO2 eq./MWh)**	**Variable cost ($/MWh/year)**	**Setup cost per unit ($/kW)**
Hydro	28 g	3.456	5,316
Wind	14 g	3.049	1,265
Nuclear	8 g	13.995	6,191
Geothermal	38 g	16.037	2,521
Biomass	230 g	19.381	4,097
Solar	64 g	1.765	1,313

Table 1: GHG Emission and Operational Cost by Electricity Source (Sources: US Energy Information Administration (2022), Hydro Québec (2023), U.S. Department of Energy. (2020).

Modeling

The primary objective is to minimize greenhouse gas emission for each MWh of electricity supplied. The secondary objective is to minimize the set up cost of new plants and operating cost. Additional constraints in the model include: 1) Maximum investment per year is $4 billion; 2) Demand from wind is kept below 15%, as per Hydro-Québec's Action Plan 2035; 3) At least 70% of total supply needs to come from existing hydro plants. The mathematical formulations with all seven constraints are in Appendix 3.

In the 2023 Budget, the Government of Canada projected the generation of electricity to increase 2.1 times from 2020. This model uses linear growth for the electricity generation from 2024 to 2050. This model also assumes market equilibrium, where all electricity generated is consumed. Detailed assumptions are in Appendix 1.

Results

To minimize emission and operational cost for each electricity source, the proportion of electricity should be the highest for existing hydro power generators, followed by wind until 2030. Then, from 2031 until 2050, more nuclear plants will be built, taking more share of the total electricity supply by source. Figure 1 illustrates the power generation by source type until 2050.

The optimal minimum cost is $45.65 billion from 2024 to 2050. This number aligns with the budget provided in Hydro-Québec's 2035 Action Plan.

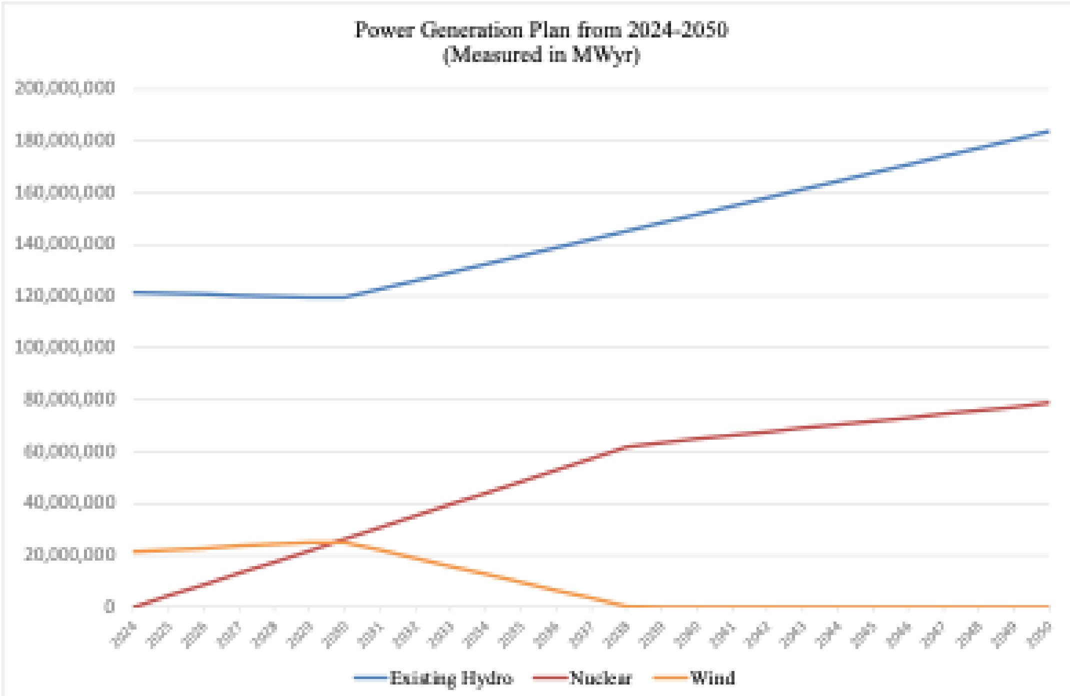

Figure 1: Power Generation Plan from 2024-2050 to Minimize Emission and Cost

Using the electricity generation proportions in 2022 as a benchmark, it is recommended that in 2035 Hydro-Québec generates 70% of electricity from existing hydro plants, 25% from newly built nuclear plants, and 5% from wind plants.

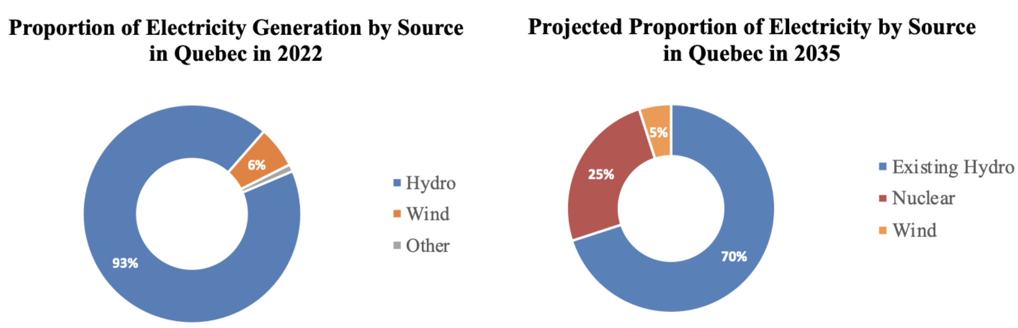

Figure 2: Electricity Generation Proportion in 2022 against Recommended Proportion by 2035

Generating electricity emits relatively less carbon emissions compared to large pollutant industries like transportation, construction, and manufacturing (Government of Canada, 2023). With the electrification of the economy in fields like automobile, industrial processes, and residential energy consumption, electricity will replace traditional fuel sources that required carbon-heavy activities, thereby contributing to a more sustainable and environmentally friendly future.

5 Limitations

Data Collection

Amidst the vast expanse of data, numerous sources presented disparate information, varying in terms of demands, supply figures, carbon emissions, and estimations of demand growth. Demands for certain municipalities were incomplete, therefore when estimating their demand projection in 2024, the average of existing data is taken. To find accurate costs for different electricity sources, US data is used due to lack of centralized Canadian data on electricity costs by type.

In addition, a more deregulated market open to outside players implies the availability of more granular power grid specific data, down to the transmission capacity of each power line and how much it costs to transport energy from one node to another (locational marginal pricing). However, this data for Hydro-Québec seems to be unavailable to the public. On the other hand, the complexity of the model would also increase exponentially and possibly cannot be solved with a typical computer.

Demand and Supply Constraints

To model the problem, we make the utopic assumption that all demand is fulfilled. In practical terms, real-world electricity distribution data is gathered hourly due to the dynamic nature of energy demand and supply patterns, necessitating a fine-grained temporal resolution for effective monitoring and management. Therefore, for a more detailed analysis of electricity distribution, a zoomed-in approach could have been employed. Nevertheless, this comes with the trade-off that results may lack accuracy unless actual data on hourly demand and supply is utilized.

6 Recommendations and Conclusions

In aligning with the dynamic and ever-evolving energy landscape, our analysis underscores the imperative need for Hydro-Québec to optimize its electricity distribution strategy. This strategy is not only crucial for powering millions of homes and businesses but is also a key component in addressing the multifaceted challenges posed by the changing macroenvironment.

Our model, underpinned by rigorous analytical constraints, suggests that electricity generation should ideally remain below the 80% capacity threshold to ensure efficiency and longevity of resources. Notably, stations such as La Grande and Robert Bourassa exhibit higher utilization rates compared to their counterparts. This discrepancy, as highlighted in the heatmap in Appendix 2, signals a need for Hydro-Québec to implement enhanced operational measures. These include intensified quality inspections and precautionary steps at these specific plants, ensuring their sustainable operation and mitigating potential risks.

Diversifying power sources emerges as a pivotal strategy in reducing emissions and bolstering sustainability. In this context, the integration of wind and nuclear power, as supplementary to our hydroelectric backbone, aligns seamlessly with Hydro-Québec's Action Plan 2035. This strategy not only diversifies our energy portfolio but also positions us to achieve a significant reduction in greenhouse gas emissions—projected at 21.43% by 2050. Such diversification is not just an environmental imperative but also a strategic leverage in the broader energy market.

Emphasizing fiscal prudence, our proposed investment strategy advocates for a targeted annual investment of $3 billion. This approach, which is 25% lower than the initial $4 billion plan, is designed to optimize resource allocation without compromising environmental or operational goals. This strategic investment achieves a remarkable balance, maintaining Hydro-Québec's leadership in sustainable energy practices while ensuring financial viability and stewardship.

Central to this plan is the reliance on hydropower, a resource abundantly available in Québec and crucial for meeting consistent electricity demand. This reliance is not only a testament to Québec's natural endowments but also a strategic choice in harnessing a stable and controllable energy source. Complementing this with wind and solar sources, we embrace a holistic approach that acknowledges the limits of intermittent renewables and leverages the reliability of hydropower.

Looking ahead, Québec is poised to contribute significantly to the global electricity supply, with the potential to fulfill 25-30% of the world's demand. Our projected capacity utilization for 2024 stands at an average of 62.63% as showed in Appendix 5, peaking in January at 74.45% and tapering in September. This pattern presents opportunities for heightened participation in North American electricity trading markets, expanding our footprint and influence in the energy sector.

In conclusion, Hydro-Québec's strategic approach to electricity distribution is integral to the socio-economic fabric of Québec. By harnessing the power of analytics and strategic planning, we are not just solving immediate operational challenges but are also paving the way for a sustainable, responsible, and prosperous energy future. This strategic orientation not only aligns with Hydro-Québec's foundational goals of social responsibility and environmental stewardship but also reinforces its critical role in shaping Québec's energy landscape for generations to come.

7 Appendices

Appendix 1

Basic Model Assumptions

1. All generated electricity is consumed
2. Demand Projection for 2024 is 11% higher from consumption in base year 2020
3. The operational cost is presumed to increase by 1.5% for every additional year of generator age (Government of British Columbia, 2014)
4. Electricity loss during transmission is 8%-15% (Hydro-Québec, 2003)
5. Cost of each unit of electricity lost is the same as its cost of generation ($3.456)

Extended Model Assumptions

1. All generated electricity is consumed
2. Supply will **always** be met by demand
3. Supply growth from 2020 to 2050 is linear
4. Capacity per generator is equal among the six different sources
5. Due to maintenance and inspection needs, a power generator cannot operate more than 80% of its capacity
6. Variable costs are an addition of fixed operational costs per MW and other maintenance costs
7. Setup costs incur in the year the plants are built
8. Electricity generation is 35% higher than the demand each month to prepare for demand surges and offset lost electricity during transmission
9. The maximum investment in a year is estimated to be $4 billion ("Action Plan 2035", 2023)
10. Time taken to build a plant is not considered, hence it is assumed that a plant is ready to use as soon as it is built

Appendix 2

Power Plant Operation Plan in 2024 (electricity generated in MWh)

Plant	Jan	Feb	Mar	Apr	May	Jun
Beauharnois	805248	805248	805248	805248	805248	805248
Beaumont	116640	116640	116640	116640	116640	116640
Bernard-Landry	331776	331776	331776	331776	0	0
Bersimis (2)	873936	873936	873936	873936	873936	873936
Brisay	202608	202608	202608	202608	202608	202608
Bryson	26352	26352	26352	26352	26352	26352
Carillon	325296	325296	325296	325296	325296	325296
Chelsea	65664	65664	65664	65664	65664	65664
Chute (4)	83376	83376	83376	83376	83376	83376
Drummondville	6912	6912	6912	6912	6912	6912
Eastmain-1	207360	207360	207360	207360	85049	61312
Hart-Jaune	22032	22032	22032	22032	203	182
Jean-Lesage	530928	530928	530928	530928	530928	530928
La Gabelle	56592	56592	56592	56592	56592	56592
La Grande (4)	3774816	3774816	2861710	431424	55539	6967
La Tuque	127008	127008	127008	127008	127008	127008
Lac-Robertson5	9504	9504	9504	8655	7113	6227
Laforge (2)	517104	517104	517104	517104	517104	503391
Les Cèdres	48816	48816	48816	48816	48816	48816
Manic (3)	1228608	1228608	1228608	1228608	1085735	667869
McCormick4	101520	101520	101520	101520	101520	101520
Mercier	23760	23760	23760	23760	23760	23760
Mitis	4320	4320	4320	4320	4320	4320
Outardes (3)	1008288	1008288	1008288	1008288	1008288	1008288
Paugan	93312	93312	93312	93312	93312	93312
Péribonka	166320	166320	166320	166320	166320	166320
Première-Chute	56592	56592	56592	56592	56592	56592
Rapide (7)	341712	341712	341712	341712	341712	341712
René-Lévesque	572832	572832	572832	572832	572832	572832
Rivière-des-Prairies	23328	23328	23328	23328	23328	23328
Robert-Bourassa	1500760	241118	4163	3945	3669	3407
Rocher-de-Grand-Mère	99360	99360	99360	99360	99360	99360
Romaine (4)	669600	669600	669600	669600	669600	669600
Sainte-Marguerite-3	381024	381024	381024	381024	187	149
Sarcelle	64800	64800	64800	64800	56499	12829
Sept-Chutes	9504	9504	9504	9504	9504	9504
Shawinigan (2)	170208	170208	170208	170208	170208	170208
Toulnustouc	227232	227232	227232	227232	227232	227232
Trenche	130464	130464	130464	130464	130464	130464

Figure 3: Electricity Generation by Source

Plant	Jul	Aug	Sep	Oct	Nov	Dec
Beauharnois	805248	805248	805248	805248	805248	805248
Beaumont	116640	116640	116640	116640	116640	116640
Bernard-Landry	0	0	0	77265	331776	331776
Bersimis (2)	873936	873936	873936	873936	873936	873936
Brisay	202608	202608	202608	202608	202608	202608
Bryson	26352	26352	26352	26352	26352	26352
Carillon	325296	325296	325296	325296	325296	325296
Chelsea	65664	65664	65664	65664	65664	65664
Chute (4)	83376	83376	83376	83376	83376	83376
Drummondville	6912	6912	6912	6912	6912	6912
Eastmain-1	63462	64909	36008	207360	207360	207360
Hart-Jaune	182	183	180	1307	22032	22032
Jean-Lesage	530928	530928	530928	530928	530928	530928
La Gabelle	56592	56592	56592	56592	56592	56592
La Grande (4)	20860	36080	7077	110175	1171697	3774816
La Tuque	127008	127008	127008	127008	127008	127008
Lac-Robertson5	5857	5880	6022	7628	9504	9504
Laforge (2)	517104	517104	494296	517104	517104	517104
Les Cèdres	48816	48816	48816	48816	48816	48816
Manic (3)	1178859	922457	347775	1228608	1228608	1228608
McCormick4	101520	101520	101520	101520	101520	101520
Mercier	23760	23760	23760	23760	23760	23760
Mitis	4320	4320	4320	4320	4320	4320
Outardes (3)	1008288	1008288	1008288	1008288	1008288	1008288
Paugan	93312	93312	93312	93312	93312	93312
Péribonka	166320	166320	166320	166320	166320	166320
Première-Chute	56592	56592	56592	56592	56592	56592
Rapide (7)	341712	341712	341712	341712	341712	341712
René-Lévesque	572832	572832	572832	572832	572832	572832
Rivière-des-Prairies	23328	23328	23328	23328	23328	23328
Robert-Bourassa	3462	3447	3369	3862	4015	251049
Rocher-de-Grand-Mère	99360	99360	99360	99360	99360	99360
Romaine (4)	669600	669600	654759	669600	669600	669600
Sainte-Marguerite-3	147	143	147	50965	381024	381024
Sarcelle	54262	35512	0	64800	64800	64800
Sept-Chutes	9504	9504	9504	9504	9504	9504
Shawinigan (2)	170208	170208	170208	170208	170208	170208
Toulnustouc	227232	227232	227232	227232	227232	227232
Trenche	130464	130464	130464	130464	130464	130464

* Numbers in parenthesis indicate the plants that have been grouped

Figure 4: Electricity Generation by Source

Appendix 3

The decision variables for the extended problem are:

$$e_h[k] = \text{Electricity generated by the existing hydro power plant in year } k$$

$$c[i,j] = \text{Capacity of plant of type } i \text{ built in year } j$$

$$x[i,j,k] = \text{Electricity built using source } i \text{ built in year } j \text{ in year of production } k$$

$$e[k] = \text{Emission of existing plant in year } k$$

The objective function for this extended problem can therefore be expressed as:

Minimize the total cost of yearly power generation:

$$Z_1 = \sum_{k=1}^{K} e_h[k] \cdot \text{var\_cost\_mwh}[0] + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x[i,j,k] \cdot (\text{var\_cost\_mwh}[i] + \text{var\_cost\_mwh}[i]) \tag{10}$$

where *var_cost_mwh* is the sum of fixed and variable MWh/yr costs

Cost of setting up new plants:

$$Z_2 = \sum_{i=1}^{I}\sum_{j=1}^{J} (\text{setup\_cost}[i] \cdot 1000) \cdot \frac{c[i,j]}{24 \cdot 30 \cdot 12} \tag{11}$$

The total cost function:

$$\text{Total Cost} = Z_1 + Z_2 \tag{12}$$

Minimize the total CO2 emission:

$$Z_3 = \sum_{k=1}^{K} e[k] \tag{13}$$

The multi-objective optimization problem is set as:

$$\text{Minimize} \quad \text{Total Cost}$$

$$\text{Minimize} \quad \text{Total Emission}$$

Subject to constraints:

The power plants cannot run at more than 80% capacity:

$$x_{ijk} \leq 0.8 \cdot c_{ij} \quad \forall i \in I,\, j \in J,\, k \in K \tag{14}$$

$$e_h[k] \leq 0.8 \cdot \text{total\_capacity\_existing} \quad \forall k \in K \tag{15}$$

Total production each year must be at least 35% higher than the demand in each month:

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} + e_h[k] = 1.35 \cdot \text{demand\_projection}[k] \quad \forall k \in K \tag{16}$$

A power plant can only generate electricity after it is built:

$$x_{ijk} \leq c_{ij} \quad \text{if } j \leq k \tag{17}$$

$$x_{ijk} = 0 \quad \text{if } j > k \tag{18}$$

Definition of the emission variable:

$$e_k = \sum_{i \in I} \sum_{j \in J} x_{ijk} \cdot \text{emission}[i] + e_h[k] \cdot \text{emission}[0] \quad \forall k \in K \tag{19}$$

Max investment that can be added in a year (assuming it is 4 billion dollars):

$$\sum_{i \in I} \left(\text{setup\_cost}[i] \cdot 1000 \cdot \frac{c_{ij}}{24 \cdot 30 \cdot 12} \right) \leq 4 \times 10^9 \quad \forall j \in J \tag{20}$$

Maximum demand that can be met with wind should be less than 15% of the total demand:

$$\sum_{j \in J} x_{1jk} \leq 0.15 \cdot 1.35 \cdot \text{demand\_projection}[k] \quad \forall k \in K \tag{21}$$

At least 70% of the total demand should be met with existing hydro plants:

$$e_h[k] \geq 0.7 \cdot 1.35 \cdot \text{demand\_projection}[k] \quad \forall k \in K \tag{22}$$

Appendix 4

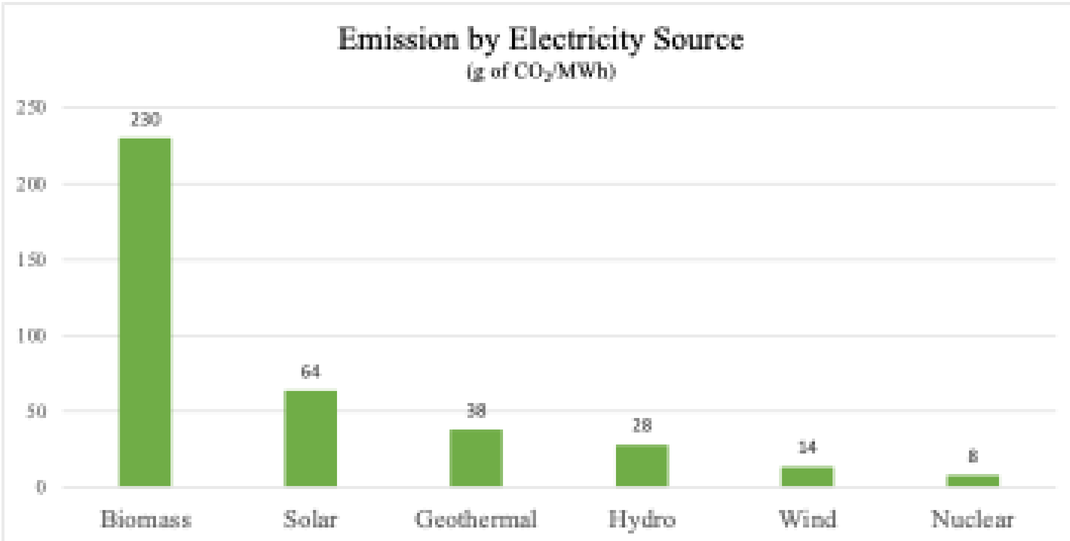

Figure 5: Emission by Electricity Source

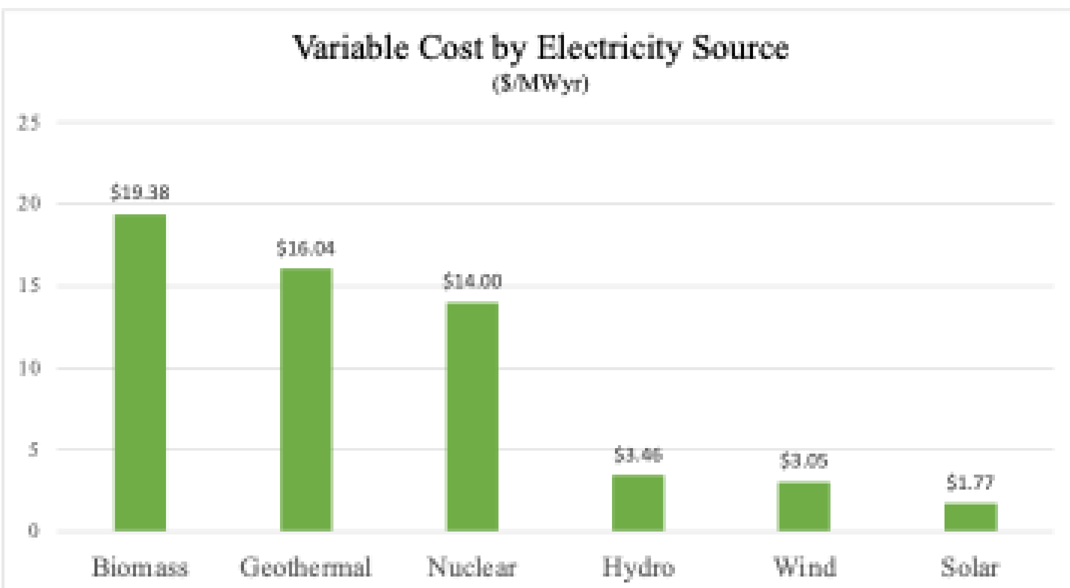

Figure 7: Variable Cost by Electricity Source ($ per MW yr).

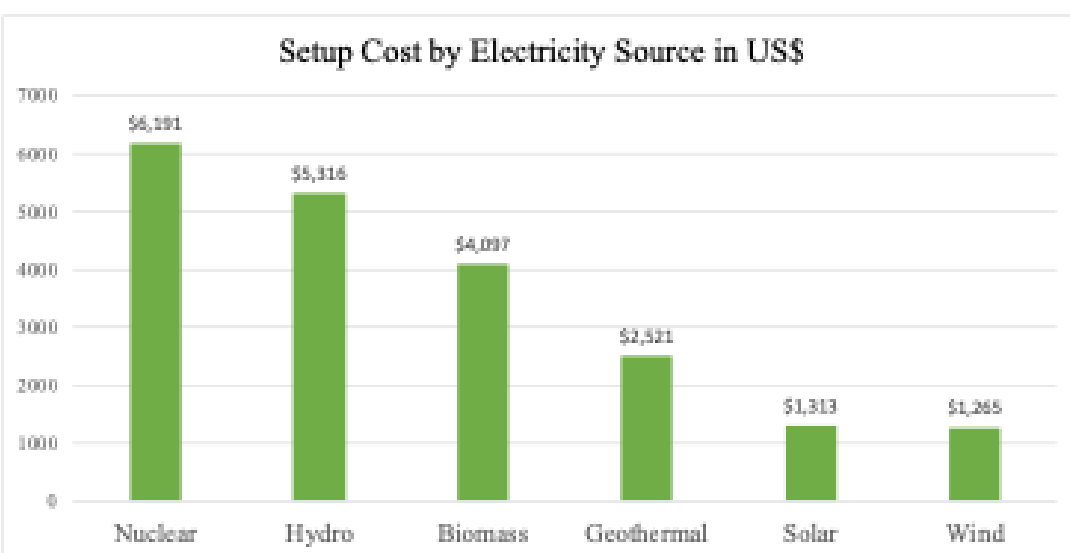

Figure 8: Set Up Cost by Electricity Source (in U.S. $). Courtesy: U.S. Department of Energy and Hydro-Québec

Appendix 5

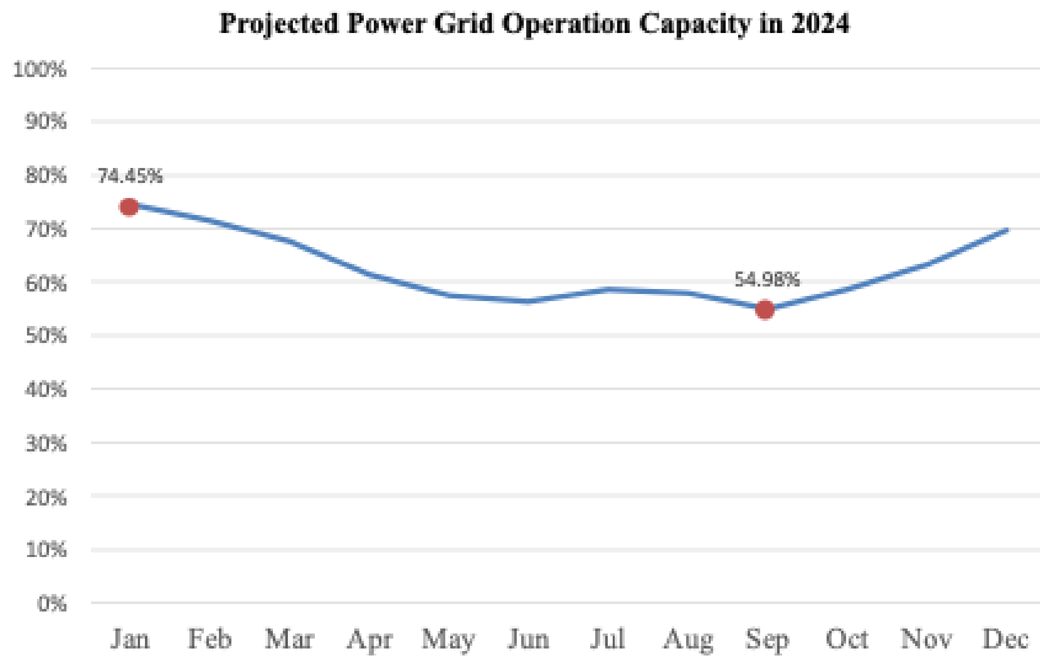

Figure 9: Projected Power Grid Operation Capacity in 2024

The average operational capacity is at 62.63%. This implies that, Hydro-Québec will supply 10.2 million MWh worth of electricity per month in 2024.

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