

# MPNG

## MATPOWER-Natural Gas

### User's Manual

Version 0.99a

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# Introduction

## 1.1 Background

MPNG is a MATPOWER-based [1,2] package for solving optimal power and natural gas flow problems. MPNG uses the general user nonlinear constraints capability of MATPOWER to model the gas network taking into account: gas-fired power generators, storage units, wells, power-and-gas-driven compressors, and nodes with stratified demand (different market segments get different priorities). The MPNG source code forms part of the MATPOWER project and can be found at:

<https://github.com/MATPOWER/mpng.git>

MPNG was developed by Sergio García-Marín <sup>1</sup> and Wilson González-Vanegas <sup>2</sup> under the direction of professor Carlos E. Murillo-Sánchez <sup>1</sup>. The initial need for a MATPOWER-based power and natural gas optimal flow package was born out of a project aimed to analyze the integrated operation of the Colombian power and natural gas systems.<sup>3</sup>

## 1.2 License and Terms of Use

As a MATPOWER-based package, MPNG is distributed under the 3-clause BSD license [3]. The full text of the license can be found in the LICENSE file at the top level of the distribution or at <https://github.com/MATPOWER/mpng/blob/master/LICENSE> and reads as follows.

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<sup>2</sup> Universidad Tecnológica de Pereira.

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# Getting started

## 2.1 System Requirements

To use MPNG you will need the following system requirements:

- ✓ MATLAB® version 7.3 (R2016b) or later.<sup>1</sup>
- ✓ MATPOWER version 7.0 or later.<sup>2</sup>

## 2.2 Getting MPNG

You can obtain the *current development version* from the MATPOWER Github repository: <https://github.com/MATPOWER/mpng.git>.

## 2.3 Installation

Installation and use of MPNG requires familiarity with basic operations of MATLAB. In short, installing MPNG is as simple as adding all the distribution files to the MATLAB path. The user could either proceed manually with such an addition, or run the quick installer released with the package by opening MATLAB at the <MPNG> directory and typing:

```
install_mpng
```

A succeeded installation of a distribution located at the directory <E:\GITHUB\MPNG> looks like:

---

<sup>1</sup>MATLAB is available from The MathWorks, Inc. (<https://www.mathworks.com/>). An R2016b or later MATLAB version is required as the MPNG code uses MATLAB-files with multiple function declarations.

<sup>2</sup>MATPOWER is available thanks to the Power Systems Engineering Research Center (PSERC) (<https://matpower.org>)



```
>> install_mpng

----- MPNG installation routine -----

Adding to the path: E:\GITHUB\MPNG\Functions
Adding to the path: E:\GITHUB\MPNG\Cases
Adding to the path: E:\GITHUB\MPNG\Examples

MPNG has been successfully installed!
```

## 2.4 Running a Simulation

The primary functionality of MPNG is to solve optimal power and natural gas flow problems. Running a simulation using MPNG requires (1) preparing the natural gas input data, (2) specifying the interconnection input data to couple the gas network to the power system, (3) invoking the function to run the integrated simulation and (4) accessing and viewing the results.

The classical MATPOWER input data is a “MATPOWER-case” struct denoted by the variable `mpc` [4]. To integrate the power and natural gas systems we use the extended Optimal Power Flow (OPF) capability of MATPOWER. Namely, we model the natural gas system and its connection to the power system via general user nonlinear constraints. Then, MPNG uses an extended “MATPOWER-gas case” struct denoted by the variable `mpgc`. In particular, `mpgc` is a traditional MATPOWER-case struct with two additional fields, `mpgc.mgc` and `mpgc.connect` standing for the natural gas case and interconnection case, respectively.

### 2.4.1 Preparing the Natural Gas Case

The input data of the natural gas system are specified in a set of matrices arranged in a MATLAB struct that we refer to as the “gas case” (`mpgc.mgc`). The structure of such a case is formatted in a similar way to the MATPOWER-case but holding the natural gas information that comprise gas bases, nodes, wells, pipelines, compressors, and storage units. See Appendix A for more details about the gas case structure.

### 2.4.2 Connecting the Gas Case to the MATPOWER Case

The input data regarding the connection between the power and natural gas systems are declared in a set of matrices packaged as a MATLAB struct which we

call “interconnection case” (`mpgc.connect`). The structure of this case contains specific information about coupling elements like gas-fired power generators and power-and-gas-driven compressors, according to the optimization model described in section 4. See Appendix B for more details about the interconnection case structure.

### 2.4.3 Solving the Optimal Power&Gas Flow

Once the MATPOWER-gas case is properly formatted, the solver can be invoked using the (mandatory) `mpgc` struct and the traditional (optional) MATPOWER options struct `mpopt`. The calling syntax at the MATLAB prompt could be one of the following:

```
>> mpng(mpgc);
>> mpng(mpgc,mpopt);
>> results = mpng(mpgc);
>> results = mpng(mpgc,mpopt);
```

We have included a description for all MPNG’s functions to work properly with the built-in `help` command. For instance, to get the help for `mpng`, type:

```
>> help mpng
```

### 2.4.4 Accessing the Results

By default, the results of the optimization run are pretty-printed on the screen, displaying the traditional MATPOWER results for the power system<sup>3</sup> along with a gas system summary, node data, pipeline data, compressor data, storage data, and the interconnection results concerning gas-fired generators data.

The optimal results are also stored in a `results` struct packaged as the default MATPOWER superset of the input case struct `mpgc`. Table 2.1 shows the solution values included in the `results`.

---

<sup>3</sup>Including the non-supplied power demand as described in the formulation introduced in section 4.

Table 2.1: Power&amp;Gas Flow Results

name	description
<code>results.success</code>	success flag, 1 = succeeded, 0 = failed
<code>results.et</code>	computation time required for solution
<code>results.iterations</code>	number of iterations required for solution
<code>results.order</code>	see <code>ext2int</code> help for details on this field
<code>results.bus(:, VM)</code> <sup>§</sup>	bus voltage magnitudes
<code>results.bus(:, VA)</code>	bus voltage angles
<code>results.gen(:, PG)</code>	generator real power injections
<code>results.gen(:, QG)</code> <sup>§</sup>	generator reactive power injections
<code>results.branch(:, PF)</code>	real power injected into “from” end of branch
<code>results.branch(:, PT)</code>	real power injected into “to” end of branch
<code>results.branch(:, QF)</code> <sup>§</sup>	reactive power injected into “from” end of branch
<code>results.branch(:, QT)</code> <sup>§</sup>	reactive power injected into “to” end of branch
<code>results.f</code>	final objective function value
<code>results.x</code>	final value of optimization variables (internal order)
<code>results.om</code>	OPF model object <sup>†</sup>
<code>results.bus(:, LAM_P)</code>	Lagrange multiplier on real power mismatch
<code>results.bus(:, LAM_Q)</code>	Lagrange multiplier on reactive power mismatch
<code>results.bus(:, MU_VMAX)</code>	Kuhn-Tucker multiplier on upper voltage limit
<code>results.bus(:, MU_VMIN)</code>	Kuhn-Tucker multiplier on lower voltage limit
<code>results.gen(:, MU_PMAX)</code>	Kuhn-Tucker multiplier on upper $P_g$ limit
<code>results.gen(:, MU_PMIN)</code>	Kuhn-Tucker multiplier on lower $P_g$ limit
<code>results.gen(:, MU_QMAX)</code>	Kuhn-Tucker multiplier on upper $Q_g$ limit
<code>results.gen(:, MU_QMIN)</code>	Kuhn-Tucker multiplier on lower $Q_g$ limit
<code>results.branch(:, MU_SF)</code>	Kuhn-Tucker multiplier on flow limit at “from” bus
<code>results.branch(:, MU_ST)</code>	Kuhn-Tucker multiplier on flow limit at “to” bus
<code>results.mu</code>	shadow prices of constraints <sup>‡</sup>
<code>results.g</code>	(optional) constraint values
<code>results.dg</code>	(optional) constraint 1st derivatives
<code>results.raw</code>	raw solver output in form returned by MINOS, and more <sup>‡</sup>
<code>results.var.val</code>	final value of optimization variables, by named subset <sup>‡</sup>
<code>results.var.mu</code>	shadow prices on variable bounds, by named subset <sup>‡</sup>
<code>results.nle</code>	shadow prices on nonlinear equality constraints, by named subset <sup>‡</sup>
<code>results.nli</code>	shadow prices on nonlinear inequality constraints, by named subset <sup>‡</sup>
<code>results.lin</code>	shadow prices on linear constraints, by named subset <sup>‡</sup>
<code>results.cost</code>	final value of user-defined costs, by named subset <sup>‡</sup>

<sup>§</sup> AC power flow only.

<sup>†</sup> See help for `opf_model` and `opt_model` for more details.

<sup>‡</sup> See help for `opf` for more details.

# Fundamentals of Natural Gas Flow

The steady-state Natural Gas Flow (NGF) problem for transmission networks aims to find the value for a set of state-variables that satisfy the flow balance in all nodes. We show how the NGF can be derived in a similar way as the Power Flow (PF) problem is introduced for power systems. In particular, a set of nonlinear equations must be solved where the definition of the state-variables depends on the selected models for all the elements of the system. In this section, we derive the NGF problem and introduce the modeling for the main elements considered in MPNG: nodes, wells, pipelines, compressors, and storage units.

## 3.1 Modeling

An exact description of the natural gas flow in transmission networks requires applying the laws of fluid mechanics and thermodynamics [1]. Complex analyzes provide an accurate description for variables such as temperature, pressure, flow, adiabatic head, among others, for all time instants. However, as the primary concern of MATPOWER (and so does MPNG) is the system operation in steady-state, we define some models to describe the main elements of the default natural gas network, as explained below.

### 3.1.1 Nodes

By definition, a node is the location of a natural gas system where one or more elements are connected. Users are commonly associated with a node where a stratified demand is modeled as different market segments that get different priorities. Figure 1 shows the  $i$ -th node of a gas network with some traditional markets connected to form the nodal demand  $f_{dem}$ . The primary variable for a node is pressure  $p_i$ .

**3.1.2 Wells**

**3.1.3 Pipelines**

**3.1.4 Compressors**

**3.1.5 Storage Units**

**3.2 Derivation of the Natural Gas Flow Problem**

# Formulation

## Nomenclature

### Indexes

$i, j$	Gas nodes.
$m, n$	Electric nodes (buses).
$o$	Gas pipeline.
$c$	Compressor.
$l$	Transmission line.
$w$	Gas well.
$e$	Power generator.
$ref$	Reference bus.
$r$	Spinning reserve.
$\sigma$	Type of gas load.

### Parameters

$\alpha_{\pi+}^i, \alpha_{\pi-}^i$	Penalties for over-pressure and under-pressure at node $i$ .
$\alpha_{\gamma}$	Penalties for non-supplied gas.
$\alpha_{\epsilon}$	Penalties for non-supplied electricity.
$C_G^w$	Gas cost at the well $w$ .
$C_O^{oij}$	Transport cost of pipeline $o$ , from node $i$ to node $j$ .

$C_C^{cij}$	Compression cost of compressor $c$ , from node $i$ to node $j$ .
$C_S^i$	Storage cost at node $i$ .
$C_{S+}^i$	Storage outflow price at node $i$ .
$C_{S-}^i$	Storage inflow price at node $i$ .
$C_E^e$	Power cost generation (excluding gas cost).
$\eta_e^q$	Thermal efficiency at generator $q$ [MMSCF/MW].
$D_g^{i\sigma}$	Gas demand of type $\sigma$ at node $i$ .
$D_e^{tm}$	Electricity demand in the bus $m$ at time $t$ .
$\bar{g}^w, \underline{g}^w$	Gas production limits.
$\bar{\pi}^i, \underline{\pi}^i$	Quadratic pressure limits at node $i$ .
$S_0^i$	Initial stored gas at node $i$ .
$\bar{S}^i, \underline{S}^i$	Storage limits at node $i$ .
$\kappa^{oj}$	Weymouth constant of pipeline $o$ .
$\delta^{oj}$	Width for gas flow capacities.
$\beta^{cij}$	Compression ratio of compressor $c$ .
$Z^c$	Ratio parameter of compressor $c$ .
$B^c$	Compressor design parameter of compressor $c$ .
$x, y, z$	Gas consumption parameters of gas-fired compressors.
$\bar{f}_g^{oj}$	Gas transport capacity of pipeline $o$ , from node $i$ to node $j$ .
$\bar{f}_g^{cij}$	Gas flow capacity of compressor $c$ , from node $i$ to node $j$ .
$\bar{f}_s^i, \underline{f}_s^i$	Storage outflow capacities at node $i$ .
$\bar{p}_g^e, \underline{p}_g^e$	Active power generation limits of generator $e$ .
$\bar{q}_g^e, \underline{q}_g^e$	Reactive power generation limits of generator $e$ .
$\bar{V}^{tm}, \underline{V}^{tm}$	Voltage limits for every bus $m$ at time $t$ .

$S^l$	Transmission capacity of power line $l$ .
$R^{tr}$	Spinning reserve in the $r$ -th spinning reserve zone at time $t$ .
$M$	Generators assignment matrix.
$L$	Compressors assignment matrix.
$u^{te}$	Unit commitment state for generator $q$ at time $t$ .
$\tau^t$	Energy weight related to period of time $t$ .
$E^e$	Available energy for hydroelectric generator $e$ , during the total analysis window.

## Sets

$\mathcal{N}$	Gas nodes, $ \mathcal{N}  = n_{\mathcal{N}}$ .
$\mathcal{N}_S$	Gas nodes with storage, $\mathcal{N}_S \subset \mathcal{N}$ , $ \mathcal{N}_S  = n_S$ .
$\mathcal{O}$	Gas pipelines, $ \mathcal{O}  = n_{\mathcal{O}}$
$\mathcal{C}$	Compressors, $ \mathcal{C}  = n_{\mathcal{C}}$
$\mathcal{C}_G$	Compressors based on natural gas, $\mathcal{C}_G \subseteq \mathcal{C}$ , $ \mathcal{C}_G  = n_{\mathcal{C}_G}$
$\mathcal{C}_E$	Compressors based on electric power, $\mathcal{C}_E \subseteq \mathcal{C}$ , $ \mathcal{C}_E  = n_{\mathcal{C}_E}$
$\mathcal{W}$	Gas wells, $ \mathcal{W}  = n_{\mathcal{W}}$ .
$\mathcal{W}^i$	Gas wells at node $i$ , $\mathcal{W}^i \subset \mathcal{W}$ , $ \mathcal{W}^i  = n_{\mathcal{W}^i}$ .
$\mathcal{B}$	Power buses, $ \mathcal{B}  = n_{\mathcal{B}}$ .
$\mathcal{L}$	Power lines, $ \mathcal{L}  = n_{\mathcal{L}}$ .
$\mathcal{E}$	Power unit generators, $ \mathcal{E}  = n_{\mathcal{E}}$ .
$\mathcal{E}_H$	Hydroelectric power units, $\mathcal{E}_H \subseteq \mathcal{E}$ , $ \mathcal{E}_H  = n_{\mathcal{E}_H}$ .
$\mathcal{E}_G^i$	Gas-fired power units connected to gas node $i$ , $\mathcal{E}_G^i \subseteq \mathcal{E}$ , $ \mathcal{E}_G^i  = n_{\mathcal{E}_G^i}$ .
$\mathcal{Z}_r$	Spinning reserve zones.
$\mathcal{F}_G^i, \mathcal{T}_G^i$	Connected pipelines to node $i$ at side <i>From</i> or <i>To</i> .



$\mathcal{F}_C^i, \mathcal{T}_C^i$	Connected compressors to node $i$ at side <i>From</i> or <i>To</i> .
$\mathcal{F}_E^m, \mathcal{T}_E^m$	Connected power lines to bus $m$ at side <i>From</i> or <i>To</i> .
$\mathcal{T}$	Total periods of analysis.
$\Sigma$	Different types of gas loads.

## Variables

$f_g^{oj}$	Gas flow in pipeline $o$ , from node $i$ to node $j$ .
$f_{g+}^{oj}, f_{g-}^{oj}$	Positive and negative gas flow in pipeline $o$ .
$f_g^{cij}$	Gas flow in compressor $c$ , from node $i$ to node $j$ .
$\psi^c$	Power consumed by compressor $c$ .
$\phi^c$	Gas consumed by compressor $c$ , connected to node $i$ at side <i>From</i> .
$\gamma^{i\sigma}$	Non-served gas of type $\sigma$ at node $i$ .
$\pi^i$	Quadratic pressure.
$\pi_+^i, \pi_-^i$	Over/Under quadratic pressures at node $i$ .
$g^w$	Gas production at well $w$ .
$f_s^i$	Storage outflow difference.
$f_{s+}^i, f_{s-}^i$	Storage outflow and inflow.
$p_g^{te}$	Active power production at generator $q$ at time $t$ .
$q_g^{te}$	Reactive power production at generator $q$ at time $t$ .
$V^{tm}$	Voltage magnitude at bus $m$ at time $t$ .
$\theta^{tm}$	Voltage angle at bus $m$ at time $t$ .
$\epsilon^{tm}$	Non-served active power at bus $m$ at time $t$ .

## 4.1 Objective function

$$\begin{aligned}
C(x) = & \sum_{w \in \mathcal{W}} C_G^w g^w + \sum_{t \in \mathcal{T}} \tau^t \sum_{e \in \mathcal{E}} C_E^e p_g^{te} \\
& + \sum_{i \in \mathcal{N}_S} (C_{S_+}^i f_{s_+}^i - C_{S_-}^i f_{s_-}^i) \\
& + \sum_{i \in \mathcal{N}_S} C_S^i (S_0^i - f_s^i) \\
& + \sum_{o \in \mathcal{O}} C_O^{oij} f_{g_+}^{oij} - \sum_{o \in \mathcal{O}} C_O^{oij} f_{g_-}^{oij} \\
& + \sum_{c \in \mathcal{C}} C_C^{cij} f_g^{cij} \\
& + \sum_{i \in \mathcal{N}} \alpha_{\pi_+}^i \pi_+^i + \sum_{i \in \mathcal{N}} \alpha_{\pi_-}^i \pi_-^i \\
& + \sum_{i \in \mathcal{N}} \sum_{\sigma \in \Sigma} \alpha_\gamma^{i\sigma} \gamma^{i\sigma} + \alpha_\epsilon \sum_{t \in \mathcal{T}} \tau^t \sum_{m \in \mathcal{B}} \epsilon^{tm}
\end{aligned} \tag{4.1}$$

## 4.2 Constraints

### 4.2.1 Gas network

$$\begin{aligned}
& \sum_{o \in \mathcal{T}_G^k} f_g^{oij} - \sum_{o \in \mathcal{F}_G^k} f_g^{oij} + \sum_{c \in \mathcal{T}_G^k} f_g^{cij} - \sum_{c \in \mathcal{F}_G^k} (f_g^{cij} + \phi^c) \\
& + \sum_{w \in \mathcal{W}^k} g^w + f_s^k - \sum_{t \in \mathcal{T}} \tau^t \sum_{e \in \mathcal{E}_G^k} (\eta_e^q \cdot p_g^{te}) = \sum_{\sigma \in \Sigma} (D_g^{\sigma k} - \gamma^{\sigma k}) \\
& \forall k \in \mathcal{N}
\end{aligned} \tag{4.2}$$

The non-supply gas demand in every node of the system can only be as most as the total demand at the same node. This constraint is represented as follows:

$$0 \leq \gamma^{\sigma k} \leq D_g^{\sigma k} \quad \forall \sigma \in \Sigma \quad \forall k \in \mathcal{N} \tag{4.3}$$

#### Storage

$$f_s^k = f_{s_+}^k - f_{s_-}^k \quad \forall k \in \mathcal{N} \tag{4.4}$$

$$\underline{f}_s^k \leq f_s^k \leq \bar{f}_s^k \quad \forall k \in \mathcal{N} \tag{4.5}$$

$$0 \leq f_{s_+}^i \leq S_0^k - \underline{S}^k \quad \forall k \in \mathcal{N} \tag{4.6}$$

$$0 \leq f_{s_-}^i \leq \bar{S}^k - S_0^k \quad \forall k \in \mathcal{N} \quad (4.7)$$

### Wells

The constraints related to the gas wells production depends on each well specific characteristics, these constraints are represented by:

$$\underline{g}^w \leq g^w \leq \bar{g}^w \quad \forall w \in \mathcal{W} \quad (4.8)$$

### Pipelines:

$$f_g^{oj} = \kappa^{oj} \text{sgn}(\pi^i - \pi^j) \sqrt{|\pi^i - \pi^j|} \quad \forall o \in \mathcal{O} \quad (4.9)$$

And the gas flow limits for every pipeline:

$$f_g^{oj} = f_{g_+}^{oj} + f_{g_-}^{oj} \quad \forall o \in \mathcal{O} \quad (4.10)$$

$$-\bar{f}_g^{oj} \leq f_g^{oj} \leq \bar{f}_g^{oj} \quad \forall o \in \mathcal{O} \quad (4.11)$$

$$0 \leq f_{g_+}^{oj} \leq \delta^{oj} \cdot \bar{f}_g^{oj} \quad \forall o \in \mathcal{O} \quad (4.12)$$

$$-\delta^{oj} \cdot \bar{f}_g^{oj} \leq f_{g_-}^{oj} \leq 0 \quad \forall o \in \mathcal{O} \quad (4.13)$$

### Compressors:

The power consumed by the compressors depend on its gas flow:

$$\psi^c = B^c f_g^{cij} \cdot \left( \left( \frac{\pi^j}{\pi^i} \right)^{Z^c/2} - 1 \right) \quad \forall c \in \mathcal{C} \quad (4.14)$$

$$\phi^c = x + y\psi^c + z\psi^{c2} \quad \forall c \in \mathcal{C}_G \quad (4.15)$$

$$0 \leq f_g^{cij} \leq \bar{f}_g^{cij} \quad \forall c \in \mathcal{C} \quad (4.16)$$

$$\begin{aligned} \pi^i &\leq \pi^j \leq \beta^{cij} \pi^i \\ \beta^{cij} &\geq 1 \end{aligned} \quad \forall i, j \in \mathcal{N} \quad \forall c \in \mathcal{C} \quad (4.17)$$

The equations 4.18 and 4.19 are the constraints that characterize the quadratic overpressure and underpressure at every node of the system, respectively.

$$\begin{aligned} \pi^k &\leq \bar{\pi}^k + \pi_+^k \\ 0 &\leq \pi_+^k \end{aligned} \quad \forall k \in \mathcal{N} \quad (4.18)$$

$$\begin{aligned} \frac{\pi^k}{0} - \pi_-^k &\leq \pi^k \quad \forall k \in \mathcal{N} \end{aligned} \quad (4.19)$$

### 4.2.2 Power network

Equation 4.20 could be explained in an appendix section.

$$\begin{aligned} g_{p_m}(\theta^{tm}, V^{tm}, p_g^{te}, \epsilon^{te}, \psi^c) &= 0 \\ g_{q_m}(\theta^{tm}, V^{tm}, q_g^{te}) &= 0 \end{aligned} \quad (4.20)$$

$$\forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad \forall c \in \mathcal{C}_E$$

Variables limits:

$$\begin{aligned} \theta^{tref} &= 0 \\ \underline{V}^{tm} &\leq V^{tm} \leq \bar{V}^{tm} \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \end{aligned} \quad (4.21)$$

$$\begin{aligned} \underline{p}_g^e &\leq p_g^{te} \leq \bar{p}_g^e \\ \underline{q}_g^e &\leq q_g^{te} \leq \bar{q}_g^e \end{aligned} \quad \forall q \in \mathcal{E} \quad \forall t \in \mathcal{T} \quad (4.22)$$

Power flow limits:

$$\begin{aligned} |\mathbb{S}_{fl}(\theta, V)| &\leq \bar{\mathbb{S}}_{fl} \\ |\mathbb{S}_{tl}(\theta, V)| &\leq \bar{\mathbb{S}}_{tl} \end{aligned} \quad \forall l \in \mathcal{L} \quad (4.23)$$

Non-supplied active power limits:

$$0 \leq \epsilon^{tm} \leq D_e^{tm} \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad (4.24)$$

Reserve constraint:

$$\begin{aligned} \sum_{e \in \mathcal{Z}_r} u^{te} (\bar{p}_g^e - p_g^{te}) &\geq R^{tr} \\ \forall r \in \mathcal{Z}_r \quad \forall t \in \mathcal{T} \end{aligned} \quad (4.25)$$

Hydro-energy constraint:

$$\sum_{t \in \mathcal{T}} \tau^t p_g^{te} \leq E^e \quad \forall e \in \mathcal{E}_H \quad (4.26)$$

# Examples

In this section, we provide some examples to show the main capabilities of **MPNG** for simulating the operation of power and natural gas networks. We have included the folder *<MPNG/Cases>* in the distribution, which contains the gas and inter-connection cases used for testing. Moreover, the folder *<MPNG/Examples>* contains the files used in the examples. In particular, we explore two examples: (1) the integrated operation of a nine-bus power system (**case9\_new**) and an eight-node natural gas grid (**mgc\_case8**); and (2) the single operation of a 48-node looped natural gas network.

## 5.1 9-bus 8-node Power and Gas System

## 5.2 48-node Looped Natural Gas Network

# Appendix

# Appendix A: Gas Case Data File Format

All details about the gas case (`mgc`) format are provided in the tables below. For the sake of convenience and code portability, `idx_node` defines a set of constants (positive integers) to be used as named indices into the columns of the `node.info` matrix. Similarly, `idx_well`, `idx_pipe`, `idx_comp`, and `idx_sto` defines names for the columns in `well`, `pipe`, `comp`, and `sto`, respectively. On the other hand, `mgc_PU` converts from real to per-unit (P.U) quantities, while `mgc_REAL` converts from P.U to real values. Moreover, the `pbase`, `fbase`, and `wbase` fields are simple scalar values to define the gas system pressure, flow and power bases, respectively.

Table A.1: Node Information Data (`mgc.node.info`)

name	column	description
<code>NODE_I</code>	1	node number (positive integer)
<code>NODE_TYPE</code>	2	node type (1 = demand node, 2 = extraction node)
<code>PR</code>	3	pressure [psia]
<code>PRMAX</code>	4	maximum pressure [psia]
<code>PRMIN</code>	5	minimum pressure [psia]
<code>OVP</code>	6	over-pressure [psia]
<code>UNP</code>	7	under-pressure [psia]
<code>COST_OVP</code>	8	over-pressure cost [\$ / psia <sup>2</sup> ]
<code>COST_UNP</code>	9	under-pressure cost [\$ / psia <sup>2</sup> ]
<code>GD</code>	10	full nodal demand [MSCFD] <sup>†</sup>
<code>NGD</code>	11	number of different nodal users (positive integer)

<sup>†</sup> MSCFD: Million Standard Cubic Feet Per Day.

Table A.2: Well Information Data (`mgc.well`)

name	column	description
WELL_NODE	1	well number (positive integer)
G	2	well gas production [MSCFD]
PW	3	known well pressure [psia]
GMAX	4	maximum gas injection [MSCFD]
GMIN	5	minimum gas injection [MSCFD]
WELL_STATUS	6	well status (0 = disable, 1 = enable)
COST_G	7	well production cost [\$/MSCFD]

Table A.3: Pipeline Information Data (`mgc.pipe`)

name	column	description
F_NODE	1	from node number (positive integer)
T_NODE	2	to node number (positive integer)
FG_O	3	known gas pipeline flow [MSCFD]
K_O	4	Weymouth constant [MSCFD/psia]
DIAM	5	diameter [inches]
LNG	6	longitude [km]
FMAX_O	7	maximum flow [MSCFD]
FMIN_O	8	minimum flow [MSCFD]
COST_O	9	pipeline transportation cost [\$/MSCFD]

Table A.4: Compressor Information Data (`mgc.comp`)

name	column	description
F_NODE	1	from node number (positive integer)
T_NODE	2	to node number (positive integer)
TYPE_C	3	compressor type (1 = power-driven, 2 = gas-driven)
FG_C	4	gas flow through compressor [MSCFD]
PC_C	5	consumed compressor power [MVA]
GC_C	6	gas consumed by the compressor [MSCFD] <sup>†</sup>
RATIO_C	7	compressor ratio
B_C	8	compressor-dependent constant [MVA/MSCFD]
Z_C	9	compressibility factor
X	10	independent approximation coefficient [MSCFD]
Y	11	linear approximation coefficient [MSCFD/MVA]
Z	12	quadratic approximation coefficient [MSCFD/MVA <sup>2</sup> ]
FMAX_C	13	maximum flow through compressor [MSCFD]
COST_C	14	compressor cost [\$/MSCFD]

<sup>†</sup> Only relevant for a gas-driven compressor.



Table A.5: Storage Information Data (`mgc.sto`)

name	column	description
STO_NODE	1	node number (positive integer)
STO	2	end of day storage level [MSCF] <sup>†</sup>
STO_0	3	initial storage level [MSCF]
STOMAX	4	maximum storage [MSCF]
STOMIN	5	minimum storage [MSCF]
FSTO	6	storage outflow difference [MSCFD] <sup>‡</sup>
FSTO_OUT	7	storage outflow [MSCFD]
FSTO_IN	8	storage inflow [MSCFD]
FSTOMAX	9	maximum storage outflow difference [MSCFD]
FSTOMIN	10	minimum storage outflow difference [MSCFD]
S_STATUS	11	storage status
COST_STO	12	storage cost [\$/MSCF]
COST_OUT	13	storage outflow cost [\$/MSCFD]
COST_IN	14	storage inflow cost [\$/MSCFD]

<sup>†</sup> Volume in Million Standard Cubic Feet (MSCF).

<sup>‡</sup> Storage outflow minus storage inflow. See Section 4 for more details.

# Appendix B: Interconnection Case Data File Format

A detailed description about the interconnection case (`connect`) is provided in Table B.1. As seen, some additional information is required for the power system besides the input data given in the MATPOWER-case. For the sake of clarity and readability, we decided to include such an additional information in the interconnection case rather than the MATPOWER-case. In short, different periods that are modeled using an island-based approach are allowed for the power system, where each island define the network conditions at each period. On the other hand, the power-driven compressors and the gas-fired generator units set the coupling features between the power and natural gas systems. The user could define any of these two coupling options as empty arrays whether they are not considered for a specific analysis. See Section 5 for details.

Table B.1: Connection Data (`mpgc.connect`)

name	domain	description
<code>.power.time</code>	$\mathbb{R}^{n_t}$	vector to define the number of $n_t$ periods to be considered in the power system. Each component in the vector represents the number of hours for each period such that $\text{sum}(\text{power.time})=24^\dagger$
<code>.power.demands</code>		
<code>.pd</code>	$\mathbb{R}^{n_b \times n_t}$	matrix to define the active power demand for $n_b$ buses over $n_t$ periods of time.
<code>.qd</code>	$\mathbb{R}^{n_b \times n_t}$	matrix to define the reactive power demand for $n_b$ buses over $n_t$ periods of time.
<code>.power.cost</code>	$\mathbb{R}^+$	non-supplied power demand cost.
<code>.power.sr</code>	$\mathbb{R}^{n_a \times n_t}$	matrix to define the spinning reserve of $n_a$ areas over $n_t$ periods.
<code>.power.energy</code>	$\mathbb{R}^{n_{g_h} \times 2}$	matrix to define the maximum energy available for the $n_{g_h} \subseteq n_g$ hydroelectric power generators, holding columns as follows: <sup>‡</sup> column 1 – generator number (positive integer) column 2 – maximum energy for hydroelectric unit [MW·h]
<code>.interc.comp</code>	$\mathbb{R}^{n_{c_p} \times 2}$	index matrix to locate the $n_{c_p} \subseteq n_c$ power-driven compressors at some specific buses, holding columns as below: <sup>§</sup> column 1 – compressor number (positive integer) column 2 – bus number to locate the power-driven compressor (positive integer)
<code>.interc.term</code>	$\mathbb{R}^{n_{g_g} \times 3}$	matrix to locate the $n_{g_g} \subseteq n_g$ gas-fired generators at some specific nodes and buses, holding the following columns: <sup>‡</sup> column 1 – bus number to locate the gas-fired unit as generator (positive integer) column 2 – node number to locate the gas-fired unit as demand (positive integer) column 3 – thermal efficiency of the gas-fired unit (positive real)

<sup>†</sup> The gas temporal resolution is one *day*, while the electrical resolution is in terms of *hours*. See Section 4 for more details.

<sup>‡</sup>  $n_g$  is the number of all power generator units installed in the power system.

<sup>§</sup>  $n_c$  is the number of all compressors installed in the gas system.

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