MPNG

Matpower-Natural Gas

User's Manual

Version 0.99a

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Introduction

Background

MPNG is a MATPOWER-based [1, 2] package for solving optimal power and natural gas flow problems. MPNG uses the general user nonlinear constraints capability of MATPOWER to model the gas network taking into account: gas-fired power generators, storage units, wells, power-and-gas-driven compressors, and nodes with stratified demand (different market segments get different priorities). The MPNG source code forms part of the MATPOWER project and can be found at:

https://github.com/MATPOWER/mpng.git

MPNG was developed by Sergio García-Marín [1] and Wilson González-Vanegas [2] under the direction of professor Carlos E. Murillo-Sánchez [1]. The initial need for a MATPOWER-based power and natural gas optimal flow package was born out of a project aimed to analyze the integrated operation of the Colombian power and natural gas systems.[3]

License and Terms of Use

As a MATPOWER-based package, MPNG is distributed under the 3-clause BSD license [3]. The full text of the license can be found in the LICENSE file at the top level of the distribution or at https://github.com/MATPOWER/mpng/blob/master/LICENSE and reads as follows.

[1] Universidad Nacional de Colombia - sede Manizales.
[2] Universidad Tecnológica de Pereira.
[3] Project number 1110-745-58696 funded by Colciencias, Colombia.

Getting started

System Requirements

To use `MPNG` you will need the following system requirements:

- ✓ MATLAB® version 7.3 (R2016b) or later.[1]
- ✓ MATPOWER version 7.0 or later.[2]

Getting MPNG

You can obtain the *current development version* from the MATPOWER Github repository: https://github.com/MATPOWER/mpng.git.

Installation

Installation and use of `MPNG` requires familiarity with basic operations of MATLAB. In short, installing `MPNG` is as simple as adding all the distribution files to the MATLAB path. The user could either proceed manually with such an addition, or run the quick installer released with the package by opening MATLAB at the <*MPNG*> directory and typing:

```
install_mpng
```

A succeeded installation of a distribution located at the directory <`E:\GITHUB\MPNG`> looks like:

[1] MATLAB is available from The MathWorks, Inc. (https://www.mathworks.com/). An R2016b or later MATLAB version is required as the `MPNG` code uses MATLAB-files with multiple function declarations.

[2] MATPOWER is available thanks to the Power Systems Engineering Research Center (PSERC) (https://matpower.org)

```
>> install_mpng

---------- MPNG installation routine ----------

Adding to the path: E:\GITHUB\MPNG\Functions
Adding to the path: E:\GITHUB\MPNG\Cases
Adding to the path: E:\GITHUB\MPNG\Examples

MPNG has been successfully installed!
```

Running a Simulation

The primary functionality of `MPNG` is to solve optimal power and natural gas flow problems. Running a simulation using `MPNG` requires (1) preparing the natural gas input data, (2) specifying the interconnection input data to couple the gas network to the power system, (3) invoking the function to run the integrated simulation and (4) accessing and viewing the results.

The classical MATPOWER input data is a "MATPOWER-case" struct denoted by the variable `mpc` [4]. To integrate the power and natural gas systems we use the extended Optimal Power Flow (OPF) capability of MATPOWER. Namely, we model the natural gas system and its connection to the power system via general user nonlinear constrains. Then, `MPNG` uses an extended "MATPOWER-gas case" struct denoted by the variable `mpgc`. In particular, `mpgc` is a traditional MATPOWER-case struct with two additional fields, `mpgc.mgc` and `mpgc.connect` standing for the natural gas case and interconnection case, respectively.

Preparing the Natural Gas Case

The input data of the natural gas system are specified in a set of matrices arranged in a MATLAB struct that we refere to as the "gas case" (`mpgc.mgc`). The structure of such a case is formatted in a similar way to the MATPOWER-case but holding the natural gas information that comprise gas bases, nodes, wells, pipelines, compressors, and storage units. See Appendix A for more details about the gas case structure.

Connecting the Gas Case to the MATPOWER Case

The input data regarding the connection between the power and natural gas systems are declared in a set of matrices packaged as a MATLAB struct which we call “interconnection case” (`mpgc.connect`). The structure of this case contains specific information about coupling elements like gas-fired power generators and power-and-gas-driven compressors, according to the optimization model described in section 4. See Appendix B for more details about the interconnection case structure.

Solving the Optimal Power&Gas Flow

Once the MATPOWER-gas case is properly formatted, the solver can be invoked using the (mandatory) `mpgc` struct and the traditional (optional) MATPOWER options struct `mpopt`. The calling syntax at the MATLAB prompt could be one of the following:

```
>> mpng(mpgc);
>> mpng(mpgc,mopt);
>> results = mpng(mpgc);
>> results = mpng(mpgc,mpopt);
```

We have included a description for all `MPNG`'s functions to work properly with the built-in `help` command. For instance, to get the help for `mpng`, type:

```
>> help mpng
```

Accessing the Results

By default, the results of the optimization run are pretty-printed on the screen, displaying the traditional MATPOWER results for the power system[3] along with a gas system summary, node data, pipeline data, compressor data, storage data, and the interconnection results concerning gas-fired generators data.

The optimal results are also stored in a `results` struct packaged as the default MATPOWER superset of the input case struct `mpgc`. Table 2.1 shows the solution values included in the `results`.

[3]Including the non-supplied power demand as described in the formulation introduced in section 4.

Table 2.1: Power and Gas Flow Results

name	description
`results.success`	success flag, 1 = succeeded, 0 = failed
`results.et`	computation time required for solution
`results.iterations`	number of iterations required for solution
`results.order`	see `ext2int` help for details on this field
`results.bus(:, VM)`§	bus voltage magnitudes
`results.bus(:, VA)`	bus voltage angles
`results.gen(:, PG)`	generator real power injections
`results.gen(:, QG)`§	generator reactive power injections
`results.branch(:, PF)`	real power injected into "from" end of branch
`results.branch(:, PT)`	real power injected into "to" end of branch
`results.branch(:, QF)`§	reactive power injected into "from" end of branch
`results.branch(:, QT)`§	reactive power injected into "to" end of branch
`results.f`	final objective function value
`results.x`	final value of optimization variables (internal order)
`results.om`	OPF model object†
`results.bus(:, LAM_P)`	Lagrange multiplier on real power mismatch
`results.bus(:, LAM_Q)`	Lagrange multiplier on reactive power mismatch
`results.bus(:, MU_VMAX)`	Kuhn-Tucker multiplier on upper voltage limit
`results.bus(:, MU_VMIN)`	Kuhn-Tucker multiplier on lower voltage limit
`results.gen(:, MU_PMAX)`	Kuhn-Tucker multiplier on upper P_g limit
`results.gen(:, MU_PMIN)`	Kuhn-Tucker multiplier on lower P_g limit
`results.gen(:, MU_QMAX)`	Kuhn-Tucker multiplier on upper Q_g limit
`results.gen(:, MU_QMIN)`	Kuhn-Tucker multiplier on lower Q_g limit
`results.branch(:, MU_SF)`	Kuhn-Tucker multiplier on flow limit at "from" bus
`results.branch(:, MU_ST)`	Kuhn-Tucker multiplier on flow limit at "to" bus
`results.mu`	shadow prices of constraints‡
`results.g`	(optional) constraint values
`results.dg`	(optional) constraint 1st derivatives
`results.raw`	raw solver output in form returned by MINOS, and more‡
`results.var.val`	final value of optimization variables, by named subset‡
`results.var.mu`	shadow prices on variable bounds, by named subset‡
`results.nle`	shadow prices on nonlinear equality constraints, by named subset‡
`results.nli`	shadow prices on nonlinear inequality constraints, by named subset‡
`results.lin`	shadow prices on linear constraints, by named subset‡
`results.cost`	final value of user-defined costs, by named subset‡

§ AC power flow only.
† See help for `opf_model` and `opt_model` for more details.
‡ See help for `opf` for more details.

Natural Gas Flow

The steady-state Natural Gas Flow (NGF) problem for transmission networks aims to find the value for a set of state-variables that satisfy the flow balance in all nodes. We show how the NGF can be derived in a similar way as the Power Flow (PF) problem is introduced for power systems. In particular, a set of nonlinear equations must be solved where the definition of the state-variables depends on the selected models for all the elements of the system. In this section, we derive the NGF problem and introduce the modeling for the main elements considered in `MPNG`: nodes, wells, pipelines, compressors, and storage units.

Modeling

An exact description of the natural gas flow in transmission networks requires applying the laws of fluid mechanics and thermodynamics [6]. Complex analyzes provide an accurate description for variables such as temperature, pressure, flow, adiabatic head, among others, for all time instants. However, as the primary concern of MATPOWER (and so does `MPNG`) is the system operation in steady-state, we define some models to describe the main elements of the default natural gas network, as explained below.

Nodes

By definition, a node is the location of a natural gas system where one or more elements are connected. Users are commonly associated with a node where a stratified demand is modeled as different market segments that get different priorities. Figure 3.1 shows the i-th node of a gas network with some traditional markets connected to form the nodal demand $f_{dem} = \sum_j f_{\mathrm{dem}_j}$. The primary variable related to a node is pressure p_i.

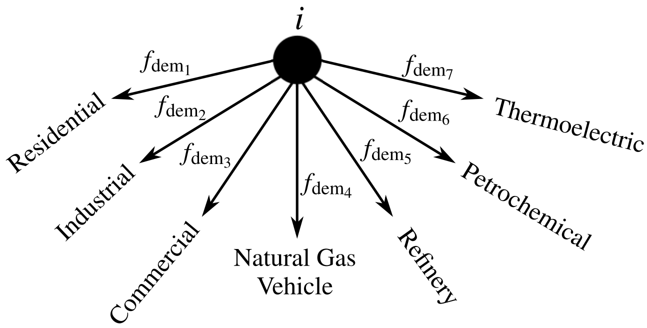

Figure 3.1: A natural gas node and some traditional users.

Wells

Natural gas is extracted from deep underground and injected into the system in wells. Depending on the well capacity, injection could be made either at constant pressure, where a control system regulates the amount of gas flow such that pressure behaves constant, or at constant flow, where pressure is adjusted such that injected flow remains constant. Figure 3.2 shows a well connected to the i-th node whose operation depends on two principal variables, the injected gas flow f_{inj}^{w}, and the nodal pressure p_i.

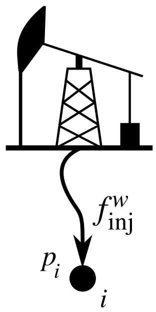

Figure 3.2: A natural gas well.

Pipelines

In general, the flow of gas through pipes is studied using the energy equation of fluid mechanics [7]. However, in practice, the relationship between the gas flow in the pipe and the upstream and downstream pressures can be described by various equations. The Weymouth's general flow equation is the frequent option in gas industry applications to model the steady-state flow in pipes in transmission networks [5]. Figure 3.3 shows a pipeline where the gas flow from the node i to the node j is represented by f_{ij}. The Weymouth's equations states the relationship between f_{ij}, p_i, and p_j in the following form:

$$\operatorname{sgn}(f_{ij})f_{ij}^2 = K_{ij}(p_i^2 - p_j^2). \tag{3.1}$$

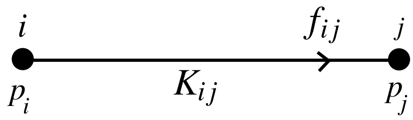

Figure 3.3: A natural gas pipeline.

In Equation 3.2, $\operatorname{sgn}(\cdot)$ represents the sign function, and K_{ij} is the Weymouth constant[1] of the pipeline defined in terms of the pipe length and diameter as below [8]:

$$K_{ij} = \sqrt{5.695756510 \times 10^{-13} \, \frac{D^5}{\lambda Z T L \delta}} \quad \left[\frac{\text{MSCFD}}{\text{psia}}\right], \tag{3.2}$$

where:

$$\frac{1}{\lambda} = \left[2 \log\left(\frac{3.7D}{\varepsilon}\right)\right]^2, \tag{3.3}$$

with:

D	Diameter [in].
L	Length [km].
T	Gas temperature [K].
ε	Absolute rugosity [mm].
δ	Gas density relative to air [-].
Z	Gas compressibility factor [-].

For mathematical convenience, we rewrite Equation 3.1 as follows:

$$f_{ij} = K_{ij} \operatorname{sgn}(\pi_i - \pi_j)\sqrt{|\pi_i - \pi_j|}, \tag{3.4}$$

where $\pi = p^2$ is defined as the quadratic pressure.

As seen, the gas flow through a pipeline is a nonlinear function of the quadratic pressures of the initial and final nodes, that is, $f_{ij} = g(\pi_i, \pi_j)$.

[1] Measured in Million Standard Cubic Feet per Day (MSCFD) over psia. Different expressions for K_{ij} can be derived depending on the parameters used in the flow equations. See [5] and reference therein for details.

Compressors

As seen in Equation 3.4, there exists a downstream pressure drop when transporting large flows through pipes caused by energy losses. Analogous to the transformer in power systems, compressors are installed in the gas network to compensate pressure drops. Figure 3.4 shows a compressor that increases the discharge pressure p_j with respect to the suction pressure p_i by compressing gas in a way that a flow f_{ij} passes through it. The power demanded by the compressor, ψ_c, states the relationship between the flow and the suction and discharge pressures in the following way [9]:

$$\psi_c = B_c\, f_{ij} \left[\left(\frac{\pi_j}{\pi_i} \right)^{\frac{Z_c}{2}} - 1 \right], \tag{3.5}$$

where B_c is the compressor constant that describe its construction features, and Z_c is the compressibility factor.

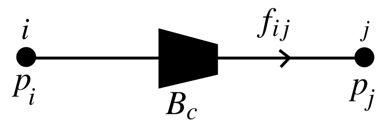

Figure 3.4: A natural gas compressor.

Moreover, the compressor ratio, R_c, is defined as below:

$$R_c = \frac{\pi_j}{\pi_i}, \quad R_c \geq 1. \tag{3.6}$$

In general, there exists two types of compressors: the power-driven compressors, whose demanded energy is supplied from the power system, and the gas-driven compressors, that requires additional gas to operate. In the latter, the additional gas demanded at the suction node, ϕ_c, can be expressed as a quadratic function of the power as [10]:

$$\phi_c = x + y\psi_c + z\psi_c^2, \tag{3.7}$$

where $x, y, z \in \mathbb{R}$.

Notice that the gas flow through a compressor (and so does the consumed flow for a gas-driven compressor) is a function of the consumed power and the quadratic suction and discharge pressures, that is, $f_{ij} = h(\pi_i, \pi_j, \psi_c)$.

Storage Units

The possibility of storing natural gas provides flexibility with regards to production and transportation decisions [11]. A storage unit is a reservoir that allows both storing and injection of gas. Figure 3.5 shows a storage unit located at node i with an associated gas flow f_s that could be either an *outflow* in the case of injection to the system or an *inflow* in the case of storing operation. Then, in a node with a specific demand and injection, the (known) value of f_s could be added to the nodal demand when it is a storage inflow or could be summed to the injection flow of a constant-flow well when it is a storage outflow.

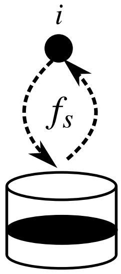

Figure 3.5: A natural gas storage unit.

Deriving the Natural Gas Flow Problem

Let us consider the transmission natural gas network shown in Figure 3.6. According to the principle of conservation of mass, the balance equation applied to the node i states:

$$f_{inj} - f_{dem} \pm f_s = \sum_{\substack{j=1 \\ j \neq i}}^{m} f_{ij} \tag{3.8}$$

where f_{dem} is the known demand flow, f_s is either the inflow (negative) or outflow (positive) of the storage unit, and f_{inj} is the injected flow of the well.

Notice in equation 3.8 that the left hand side is a known value if injection is produced by a constant-flow well. However, for a constant-pressure well, the value of f_{iny} must be determined. Besides, the algebraic sum of the right hand side depends on the nature of the elements connected between nodes i and j, such that f_{ij} is $g(\pi_i, \pi_j)$, $h(\pi_i, \pi_j, \psi_c)$, or $h(\pi_i, \pi_j, \psi_c) + \phi_c(\psi_c)$ for a pipeline, a power-driven compressor, and a gas-driven-compressor, respectively.

As a consequence, we can rewrite the balance equation applied at node i in a functional form as follows:

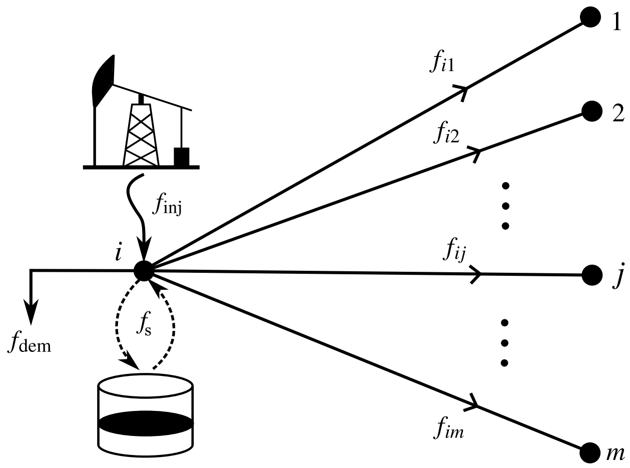

Figure 3.6: Nodal balance in a transmission natural gas network.

$$\mathbb{F}_i \left(\pi, \psi_c, f_{iny}^w \right) = 0. \tag{3.9}$$

In practice, for a given gas network with n_n nodes, n_c compressors, and n_{w_p} constant-pressure wells, the application of the balance equation for all nodes will produce a set of n_n nonlinear equations with $(n_n - n_{w_p}) + n_c + n_{w_g} = n_n + n_c$ unknown variables. To get a squared system with the same number of equations and variables, the n_c missing equations are obtained from the compressor ratios of all compressors. Then, the NGF problem can be formulated as follows:

$$\mathbb{F}_i \left(\pi, \psi_c, f_{iny}^w \right) = 0, \quad \forall i \in \mathcal{N}, c \in \mathcal{C}; w \in \mathcal{W}_p, \tag{3.10}$$

$$R_c = \frac{\pi_j}{\pi_i}, \quad \forall c \in \mathcal{C}, \ i, j \in \mathcal{N}, \tag{3.11}$$

where:

$\mathcal{N}$	Set of gas nodes, $\lvert\mathcal{N}\rvert = n_n$.
$\mathcal{C}$	Set of compressors, $\lvert\mathcal{C}\rvert = n_c$.
$\mathcal{W}_p$	Set of constant-pressure wells, $\lvert\mathcal{W}_p\rvert = n_{w_p}$.

Optimal Power and Natural Gas Flow

Nomenclature

Indexes

i, j	Gas nodes.
m, n	Electric nodes (buses).
o	Gas pipeline.
c	Compressor.
l	Transmission line.
w	Gas well.
e	Power generator.
ref	Reference bus.
r	Spinning reserve.
σ	Type of gas load.

Parameters

$\alpha^i_{\pi_+}, \alpha^i_{\pi_-}$	Penalties for over-pressure and under-pressure at node i.
α_γ	Penalties for non-supplied gas.
α_ϵ	Penalties for non-supplied electricity.
C^w_G	Gas cost at the well w.
C^{oij}_O	Transport cost of pipeline o, from node i to node j.
C^{cij}_C	Compression cost of compressor c, from node i to node j.
C^i_S	Storage cost at node i.
$C^i_{S_+}$	Storage outflow price at node i.
$C^i_{S_-}$	Storage inflow price at node i.
C^e_E	Power cost generation (excluding gas cost).
η^q_e	Thermal efficiency at generator q *[MMSCF/MW]*.

$D_g^{i\sigma}$	Gas demand of type σ at node i.
D_e^{tm}	Electricity demand in the bus m at time t.
$\bar{g}^w$, $\underline{g}^w$	Gas production limits.
$\overline{\pi}^i$, $\underline{\pi}^i$	Quadratic pressure limits at node i.
S_0^i	Initial stored gas at node i.
$\overline{S}^i$, $\underline{S}^i$	Storage limits at node i.
κ^{oij}	Weymouth constant of pipeline o.
δ^{oij}	Width for gas flow capacities.
β^{cij}	Compression ratio of compressor c.
Z^c	Ratio parameter of compressor c.
B^c	Compressor design parameter of compressor c.
x, y, z	Gas consumption parameters of gas-fired compressors.
$\overline{f}_g^{oij}$	Gas transport capacity of pipeline o, from node i to node j.
$\overline{f}_g^{cij}$	Gas flow capacity of compressor c, from node i to node j.
$\overline{f}_s^i$,$\underline{f}_s^i$	Storage outflow capacities at node i.
$\overline{p}_g^e$, $\underline{p}_g^e$	Active power generation limits of generator e.
$\overline{q}_g^e$, $\underline{q}_g^e$	Reactive power generation limits of generator e.
$\overline{V}^{tm}\underline{V}^{tm}$	Voltage limits for every bus m at time t.
$\mathbb{S}^l$	Transmission capacity of power line l.
R^{tr}	Spinning reserve in the r-th spinning reserve zone at time t.
M	Generators assignment matrix.
L	Compressors assignment matrix.
u^{te}	Unit commitment state for generator q at time t.
τ^t	Energy weight related to period of time t.
E^e	Available energy for hydroelectric generator e, during the total analysis window.

Sets

$\mathcal{N}$	Gas nodes, $\|\mathcal{N}\| = n_{\mathcal{N}}$.
$\mathcal{N}_{\mathcal{S}}$	Gas nodes with storage, $\mathcal{N}_{\mathcal{S}} \subset \mathcal{N}$, $\|\mathcal{N}_{\mathcal{S}}\| = n_{\mathcal{S}}$.
$\mathcal{O}$	Gas pipelines, $\|\mathcal{O}\| = n_{\mathcal{O}}$
$\mathcal{C}$	Compressors, $\|\mathcal{C}\| = n_{\mathcal{C}}$
$\mathcal{C}_G$	Compressors based on natural gas, $\mathcal{C}_G \subseteq \mathcal{C}$, $\|\mathcal{C}_G\| = n_{\mathcal{C}_G}$
$\mathcal{C}_E$	Compressors based on electric power, $\mathcal{C}_E \subseteq \mathcal{C}$, $\|\mathcal{C}_E\| = n_{\mathcal{C}_P}$
$\mathcal{W}$	Gas wells, $\|\mathcal{W}\| = n_{\mathcal{W}}$.
$\mathcal{W}^i$	Gas wells at node i, $\mathcal{W}^i \subset \mathcal{W}$, $\|\mathcal{W}^i\| = n_{\mathcal{W}^i}$.

$\mathcal{B}$	Power buses, $\lvert\mathcal{B}\rvert = n_{\mathcal{B}}$.
$\mathcal{L}$	Power lines, $\lvert\mathcal{L}\rvert = n_{\mathcal{L}}$.
$\mathcal{E}$	Power unit generators, $\lvert\mathcal{E}\rvert = n_{\mathcal{E}}$.
$\mathcal{E}_H$	Hydroelectric power units, $\mathcal{E}_H \subseteq \mathcal{E}$, $\lvert\mathcal{E}_H\rvert = n_{\mathcal{E}_H}$.
$\mathcal{E}_G^i$	Gas-fired power units connected to gas node i, $\mathcal{E}_G^i \subseteq \mathcal{E}$, $\lvert\mathcal{E}_G^i\rvert = n_{\mathcal{E}_G}$.
$\mathcal{Z}_r$	Spinning reserve zones.
$\mathcal{F}_G^i$, $\mathcal{T}_G^i$	Connected pipelines to node i at side *From* or *To*.
$\mathcal{F}_C^i$, $\mathcal{T}_C^i$	Connected compressors to node i at side *From* or *To*.
$\mathcal{F}_E^m$, $\mathcal{T}_E^m$	Connected power lines to bus m at side *From* or *To*.
$\mathcal{T}$	Total periods of analysis.
Σ	Different types of gas loads.

Variables

f_g^{oij}	Gas flow in pipeline o, from node i to node j.
f_{g+}^{oij} f_{g-}^{oij}	Positive and negative gas flow in pipeline o.
f_g^{cij}	Gas flow in compressor c, from node i to node j.
ψ^c	Power consumed by compressor c.
ϕ^c	Gas consumed by compressor c, connected to node i at side *From*.
$\gamma^{i\sigma}$	Non-served gas of type σ at node i.
π^i	Quadratic pressure.
π_+^i, π_-^i	Over/Under quadratic pressures at node i.
g^w	Gas production at well w.
f_s^i	Storage outflow difference.
$f_{s_+}^i$, $f_{s_-}^i$	Storage outflow and inflow.
p_g^{te}	Active power production at generator q at time t.
q_g^{te}	Reactive power production at generator q at time t.
V^{tm}	Voltage magnitude at bus m at time t.
θ^{tm}	Voltage angle at bus m at time t.
ϵ^{tm}	Non-served active power at bus m at time t.

Objective function

The cost function represented in equation 4.1 consists of several linear components, both from the power and the gas networks. It minimizes the sum of the operation cost of the interdependent system. In the case of the natural gas network, it includes the natural gas extraction cost, the transportation cost, the storage cost, and the penalties associates with quadratic over/sub pressures, and non-supplied natural gas. In the case of the power network it includes the generation cost and the penalties associates with non-supplied power demand.

$$
\begin{aligned}
C(x) = & \sum_{w \in \mathcal{W}} C_G^w g^w + \sum_{t \in \mathcal{T}} \tau^t \sum_{e \in \mathcal{E}} C_E^e p_g^{te} \\
& + \sum_{i \in \mathcal{N}_S} \left(C_{S_+}^i f_{s_+}^i - C_{S_-}^i f_{s_-}^i \right) \\
& + \sum_{i \in \mathcal{N}_S} C_S^i \left(S_0^i - f_s^i \right) \\
& + \sum_{o \in \mathcal{O}} C_O^{oij} f_{g+}^{oij} - \sum_{o \in \mathcal{O}} C_O^{oij} f_{g-}^{oij} \\
& + \sum_{c \in \mathcal{C}} C_C^{cij} f_g^{cij} \\
& + \sum_{i \in \mathcal{N}} \alpha_{\pi_+}^i \pi_+^i + \sum_{i \in \mathcal{N}} \alpha_{\pi_-}^i \pi_-^i \\
& + \sum_{i \in \mathcal{N}} \sum_{\sigma \in \Sigma} \alpha_\gamma^{i\sigma} \gamma^{i\sigma} + \alpha_\epsilon \sum_{t \in \mathcal{T}} \tau^t \sum_{m \in \mathcal{B}} \epsilon^{tm}
\end{aligned}
\tag{4.1}
$$

Constraints

Gas network

The equation 4.2 shows the gas balance for a specific node k during a day. This gas balance is composed by the incoming and outgoing flows at the node k, coming from pipelines and compressors, the related generation to that node, the outgoing stored flow in the available storage, and the total gas demand. The gas demand is composed by the required gas of the gas-fired power plants and compressors, as well as the total gas demand of the rest of the consumers, excluding the non-supplied natural gas.

$$
\begin{aligned}
& \sum_{o \in \mathcal{T}_G^k} f_g^{oij} - \sum_{o \in \mathcal{F}_G^k} f_g^{oij} + \sum_{c \in \mathcal{T}_C^k} f_g^{cij} - \sum_{c \in \mathcal{F}_C^k} \left(f_g^{cij} + \phi^c \right) \\
& + \sum_{w \in \mathcal{W}^k} g^w + f_s^k - \sum_{t \in \mathcal{T}} \tau^t \sum_{e \in \mathcal{E}_G^k} \left(\eta_e^q \cdot p_g^{te} \right) = \sum_{\sigma \in \Sigma} \left(D_g^{\sigma k} - \gamma^{\sigma k} \right) \\
& \forall k \in \mathcal{N}
\end{aligned}
\tag{4.2}
$$

Nodes

Constraints related to node k are those which involves variables of non-supplied gas demands and its quadratic pressures. The non-supplied natural gas demand at node k for a specific type σ can not be bigger than the total demand of this type at this node. This constraint is represented by equation 4.3.

$$0 \leq \gamma^{\sigma k} \leq D_g^{\sigma k} \quad \forall \sigma \in \Sigma \quad \forall k \in \mathcal{N} \tag{4.3}$$

The equations 4.4 and 4.5 are the constraints that characterize the quadratic over-pressure and under-pressure at every node of the system, respectively.

$$\begin{array}{ll} \pi^k \leq \overline{\pi}^k + \pi_+^k & \\ 0 \leq \pi_+^k & \end{array} \quad \forall k \in \mathcal{N} \tag{4.4}$$

$$\begin{array}{ll} \underline{\pi}^k - \pi_-^k \leq \pi^k & \\ 0 \leq \pi_-^k & \end{array} \quad \forall k \in \mathcal{N} \tag{4.5}$$

Wells

The constraints related to the gas wells injection depends on each well specific characteristics. The injection limits are represented as follows:

$$\underline{g}^w \leq g^w \leq \overline{g}^w \quad \forall w \in \mathcal{W} \tag{4.6}$$

Pipelines

The gas flow in pipeline o, connecting nodes i and j, depends of the quadratic pressure difference between such nodes. This behavior is given by Weymouth equation 4.7 and the flow is allowed to be bidirectional within a physical limit for maximum daily transportation, the gas flow limits are represented by equation 4.8. As the transport cost is always positive no matter the direction, variables f_{g+}^{oij} and f_{g-}^{oij} were created to represent the bidirectional flow. Equation 4.9 shows the sum of both directional flow to determine the actual flow in the direction *from - to*. These flow variables are constrained by equations 4.10 and 4.11, where the positive gas flow is bigger than zero, and lower than the maximum transport capacity multiplied by a width factor as a extra margin for the flow. Analogously, the negative gas flow has the same bounds in the negative side.

$$f_g^{oij} = \kappa^{oij} sgn\left(\pi^i - \pi^j\right)\sqrt{|\pi^i - \pi^j|} \quad \forall o \in \mathcal{O} \tag{4.7}$$

$$-\overline{f}_g^{oij} \leq f_g^{oij} \leq \overline{f}_g^{oij} \quad \forall o \in \mathcal{O} \tag{4.8}$$

$$f_g^{oij} = f_{g+}^{oij} + f_{g-}^{oij} \quad \forall o \in \mathcal{O} \tag{4.9}$$

$$0 \leq f_{g+}^{oij} \leq \delta^{oij} \cdot \overline{f}_g^{oij} \quad \forall o \in \mathcal{O} \tag{4.10}$$

$$-\delta^{oij} \cdot \overline{f}_g^{oij} \leq f_{g-}^{oij} \leq 0 \quad \forall o \in \mathcal{O} \tag{4.11}$$

Compressors

Compressors allows to recover pressure loss through gas network, but this process demands energy. The power consumption of compressor c, between node i of suction and node j of discharge, is given by equation 4.12. It depends of the quadratic pressure ratio of nodes i and j, and the gas flow through compressor. Then the gas consumed by gas-fired compressors rely on the power consumed and it is shown in equation 4.13. As the gas flow into compressors is restricted to flow in only one way, the gas flow limits of compressor c is given by equation 4.14. Finally the quadratic pressure at suction and discharge nodes must be inside acceptable margins, as shown in equation 4.15, where β is the maximum compressor ratio.

$$\psi^c = B^c f_g^{cij} \cdot \left(\left(\frac{\pi^j}{\pi^i} \right)^{Z^c/2} - 1 \right) \quad \forall c \in \mathcal{C} \tag{4.12}$$

$$\phi^c = x + y\psi^c + z\psi^{c2} \quad \forall c \in \mathcal{C}_G \tag{4.13}$$

$$0 \leq f_g^{cij} \leq \overline{f}_g^{cij} \quad \forall c \in \mathcal{C} \tag{4.14}$$

$$\begin{matrix} \pi^i \leq \pi^j \leq \beta^{cij}\pi^i \\ \beta^{cij} \geq 1 \end{matrix} \quad \forall i, j \in \mathcal{N} \quad \forall c \in \mathcal{C} \tag{4.15}$$

Storage

The storage outflow difference is the subtraction between the storage outflow and the storage inflow at the storage nodes, this relationship is represented by equation 4.16. Additionally, the outflow storage difference is restricted by the maximum and minimum amount of gas that is allowed to be injected into the network in every storage node, which is formulated in equation 4.17. As the storage can be either an injection or a demand for the network, equations 4.18 and 4.19 represent the behavior of the fluxes as follows. The maximum amount of natural gas that can be injected into the network by the storage, is the difference between the available natural gas and the minimum possible remaining gas. In the same sense, the maximum inflow of natural gas to storage units, is the difference between the maximum remaining natural and the previously available natural gas.

$$f_s^k = f_{s_+}^k - f_{s_-}^k \quad \forall k \in \mathcal{N} \tag{4.16}$$

$$\underline{f}_s^k \leq f_s^k \leq \overline{f}_s^k \quad \forall k \in \mathcal{N} \tag{4.17}$$

$$0 \leq f_{s_+}^i \leq S_0^k - \underline{S}^k \quad \forall k \in \mathcal{N} \tag{4.18}$$

$$0 \leq f_{s_-}^i \leq \overline{S}^k - S_0^k \quad \forall k \in \mathcal{N} \tag{4.19}$$

Power network

The power network balance equations of active and reactive power are given by equation 4.20. The model also takes into consideration the non-supplied power demand and the power consumed by compressors connected to the power network.

$$\begin{gathered} g_{p_m}\left(\theta^{tm}, V^{tm}, p_g^{te}, \epsilon^{te}, \psi^c\right) = 0 \\ g_{q_m}\left(\theta^{tm}, V^{tm}, q_g^{te}\right) = 0 \\ \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad \forall c \in \mathcal{C}_E \end{gathered} \tag{4.20}$$

The main variables of the power system are the voltage angle θ^{tm} and voltage magnitude V^{tm} at every bus m for every period of time t, as well as the active generation p_g^{te} and reactive generation g_g^{te} at every generator e for every period of time t. The voltage limits are represented by equation 4.21, and the generation limits are shown by equation 4.22.

$$\left.\begin{aligned} &\theta^{tref} = 0 \\ &\underline{V}^{tm} \leq V^{tm} \leq \overline{V}^{tm} \end{aligned}\right. \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \tag{4.21}$$

$$\left.\begin{aligned} &\underline{p}_g^e \leq p_g^{te} \leq \overline{p}_g^e \\ &\underline{q}_g^e \leq q_g^{te} \leq \overline{q}_g^e \end{aligned}\right. \quad \forall e \in \mathcal{E} \quad \forall t \in \mathcal{T} \tag{4.22}$$

The power flow limits are bidirectional and are represented by equation 4.23, where $\mathbb{S}_{fl}$ and $\mathbb{S}_{tl}$ are the power injections at side $from$ and *to* of line l, respectively.

$$\left.\begin{aligned} &|\mathbb{S}_{fl}\left(\theta, V\right)| \leq \overline{\mathbb{S}}_{fl} \\ &|\mathbb{S}_{tl}\left(\theta, V\right)| \leq \overline{\mathbb{S}}_{tl} \end{aligned}\right. \quad \forall l \in \mathcal{L} \tag{4.23}$$

The non-supplied active power demand at bus m can not be bigger than the total demand of this bus, as is shown as follows:

$$0 \leq \epsilon^{tm} \leq D_e^{tm} \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \tag{4.24}$$

The model also considers the required spinning reserve for all zones r at every time t. This constraint is given by equation 4.25.

$$\sum_{e \in \mathcal{Z}_r} u^{te} \left(\overline{p}_g^e - p_g^{te} \right) \geq R^{tr} \quad \forall r \in \mathcal{Z}_r \quad \forall t \in \mathcal{T} \tag{4.25}$$

Finally, the model takes into consideration the maximum available energy during a day for certain generators, especially the energy stored in the dams for hydro-power plants. Equation 4.26 represents such constraint.

$$\sum_{t \in \mathcal{T}} \tau^t p_g^{te} \leq E^e \quad \forall e \in \mathcal{E}_H \tag{4.26}$$

Examples

In this section, we provide some examples to show the main capabilities of MPNG for simulating the operation of power and natural gas networks. We have included the folder <*MPNG/Cases*> in the distribution, which contains the gas and interconnection cases used for testing. Moreover, the folder <*MPNG/Examples*> contains the files used in the examples. In particular, we explore two examples: (1) the integrated operation of a nine-bus power system (`case9_new`) and an eight-node natural gas grid (`mgc_case8`); and (2) the single operation of a 48-node looped natural gas network.

9-bus 8-node Power&Gas System

48-node Looped Natural Gas Network

Appendix

Appendix A: Gas Case Data File Format

All details about the gas case (`mgc`) format are provided in the tables below. For the sake of convenience and code portability, `idx_node` defines a set of constants (positive integers) to be used as named indices into the columns of the `node.info` matrix. Similarly, `idx_well`, `idx_pipe`, `idx_comp`, and `idx_sto` defines names for the columns in `well`, `pipe`, `comp`, and `sto`, respectively. On the other hand, `mgc_PU` converts from real to per-unit (P.U) quantities, while `mgc_REAL` converts from P.U to real values. Moreover, the `pbase`, `fbase`, and `wbase` fields are simple scalar values to define the gas system pressure, flow and power bases, respectively.

Table A.1: Node Information Data (`mgc.node.info`)

name	column	description
`NODE_I`	1	node number (positive integer)
`NODE_TYPE`	2	node type (1 = demand node, 2 = extraction node)
`PR`	3	pressure [psia]
`PRMAX`	4	maximum pressure [psia]
`PRMIN`	5	minimum pressure [psia]
`OVP`	6	over-pressure [psia]
`UNP`	7	under-pressure [psia]
`COST_OVP`	8	over-pressure cost [$/psia^2]
`COST_UNP`	9	under-pressure cost [$/psia^2]
`GD`	10	full nodal demand [MSCFD]†
`NGD`	11	number of different nodal users (positive integer)

† MSCFD: Million Standard Cubic Feet Per Day.

Table A.2: Well Information Data (`mgc.well`)

name	column	description
WELL_NODE	1	well number (positive integer)
G	2	well gas production [MSCFD]
PW	3	known well pressure [psia]
GMAX	4	maximum gas injection [MSCFD]
GMIN	5	minimum gas injection [MSCFD]
WELL_STATUS	6	well status (0 = disable, 1 = enable)
COST_G	7	well production cost [$/MSCFD]

Table A.3: Pipeline Information Data (`mgc.pipe`)

name	column	description
F_NODE	1	from node number (positive integer)
T_NODE	2	to node number (positive integer)
FG_O	3	known gas pipeline flow [MSCFD]
K_O	4	Weymouth constant [MSCFD/psia]
DIAM	5	diameter [inches]
LNG	6	longitude [km]
FMAX_O	7	maximum flow [MSCFD]
FMIN_O	8	minimum flow [MSCFD]
COST_O	9	pipeline transportation cost [$/MSCFD]

Table A.4: Compressor Information Data (`mgc.comp`)

name	column	description
F_NODE	1	from node number (positive integer)
T_NODE	2	to node number (positive integer)
TYPE_C	3	compressor type (1 = power-driven, 2 = gas-driven)
FG_C	4	gas flow through compressor [MSCFD]
PC_C	5	consumed compressor power [MVA]
GC_C	6	gas consumed by the compressor [MSCFD]†
RATIO_C	7	compressor ratio
B_C	8	compressor-dependent constant [MVA/MSCFD]
Z_C	9	compresibility factor
X	10	independent approximation coefficient [MSCFD]
Y	11	linear approximation coefficient [MSCFD/MVA]
Z	12	quadratic approximation coefficient [MSCFD/MVA^2]
FMAX_C	13	maximum flow through compressor [MSCFD]
COST_C	14	compressor cost [$/MSCFD]

† Only relevant for a gas-driven compressor.

Table A.5: Storage Information Data (`mgc.sto`)

name	column	description
`STO_NODE`	1	node number (positive integer)
`STO`	2	end of day storage level [MSCF]†
`STO_0`	3	initial storage level [MSCF]
`STOMAX`	4	maximum storage [MSCF]
`STOMIN`	5	minimum storage [MSCF]
`FSTO`	6	storage outflow difference [MSCFD]‡
`FSTO_OUT`	7	storage outflow [MSCFD]
`FSTO_IN`	8	storage inflow [MSCFD]
`FSTOMAX`	9	maximum storage outflow difference [MSCFD]
`FSTOMIN`	10	minimum storage outflow difference [MSCFD]
`S_STATUS`	11	storage status
`COST_STO`	12	storage cost [$/MSCF]
`COST_OUT`	13	storage outflow cost [$/MSCFD]
`COST_IN`	14	storage inflow cost [$/MSCFD]

† Volume in Million Standard Cubic Feet (MSCF).
‡ Storage outflow minus storage inflow. See Section 4 for more details.

Appendix B: Interconnection Case Data File Format

A detailed description about the interconnection case (`connect`) is provided in Table B.1. As seen, some additional information is required for the power system besides the input data given in the Matpower-case. For the sake of clarity and readability, we decided to include such an additional information in the interconnection case rather than the Matpower-case. In short, different periods that are modeled using an island-based approach are allowed for the power system, where each island define the network conditions at each period. On the other hand, the power-driven compressors and the gas-fired generator units set the coupling features between the power and natural gas systems. The user could define any of these two coupling options as empty arrays whether they are not considered for a specific analysis. See Section 5 for details.

Table B.1: Connection Data (`mpgc.connect`)

name	domain	description
`.power.time`	$\mathbb{R}^{n_t}$	vector to define the number of n_t periods to be considered in the power system. Each component in the vector represents the number of hours for each period such that `sum(power.time)=24`[†]
`.power.demands`		
`.pd`	$\mathbb{R}^{n_b \times n_t}$	matrix to define the active power demand for n_b buses over n_t periods of time.
`.qd`	$\mathbb{R}^{n_b \times n_t}$	matrix to define the reactive power demand for n_b buses over n_t periods of time.
`.power.cost`	$\mathbb{R}^{+}$	non-supplied power demand cost.
`.power.sr`	$\mathbb{R}^{n_a \times n_t}$	matrix to define the spinning reserve of n_a areas over n_t periods.
`.power.energy`	$\mathbb{R}^{n_{g_h} \times 2}$	matrix to define the maximum energy available for the $n_{g_h} \subseteq n_g$ hydroelectric power generators, holding columns as follows:[‡] column 1 – generator number (positive integer) column 2 – maximum energy for hydroelectric unit [MW·h]
`.interc.comp`	$\mathbb{R}^{n_{c_p} \times 2}$	index matrix to locate the $n_{c_p} \subseteq n_c$ power-driven compressors at some specific buses, holding columns as below:[§] column 1 – compressor number (positive integer) column 2 – bus number to locate the power-driven compressor (positive integer)
`.interc.term`	$\mathbb{R}^{n_{g_g} \times 3}$	matrix to locate the $n_{g_g} \subseteq n_g$ gas-fired generators at some specific nodes and buses, holding the following columns:[‡] column 1 – bus number to locate the gas-fired unit as generator (positive integer) column 2 – node number to locate the gas-fired unit as demand (positive integer) column 3 – thermal efficiency of the gas-fired unit (positive real)

[†] The gas temporal resolution is one *day*, while the electrical resolution is in terms of *hours*. See Section 4 for more details.
[‡] n_g is the number of all power generator units installed in the power system.
[§] n_c is the number of all compressors installed in the gas system.

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