

### User's Manual

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 $\ \, \bigcirc \,$  Define final affiliation (ask C.Murillo)  $\mbox{All Rights Reserved}$ 

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### Introduction

#### 1.1 Background

MPNG is a MATPOWER-based [1,2] package for solving optimal power and natural gas flow problems. MPNG uses the general user nonlinear constraints capability of MATPOWER to model the gas network taking into account: gas-fired power generators, storage units, wells, power-and-gas-driven compressors, and nodes with stratified demand (different market segments get different priorities). The MPNG source code forms part of the MATPOWER project and can be found at:

https://github.com/MATPOWER/mpng.git

MPNG was developed by Sergio García-Marín <sup>1</sup> and Wilson González-Vanegas <sup>2</sup> under the direction of professor Carlos E. Murillo-Sánchez <sup>1</sup>. The initial need for a MATPOWER-based power and natural gas optimal flow package was born out of a project aimed to analyze the integrated operation of the Colombian power and natural gas systems.<sup>3</sup>

#### 1.2 License and Terms of Use

As a Matpower-based package, MPNG is distributed under the 3-clause BSD license [3]. The full text of the license can be found in the LICENSE file at the top level of the distribution or at URLTOMPNGLICENSE and reads as follows.

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## Getting started

#### 2.1 System Requirements

To use MPNG you will need the following system requirements:

- ✓ Matlab<sup>®</sup> version 7.3 (R2016b) or later. <sup>1</sup>
- ✓ Matpower version 7.0 or later.<sup>2</sup>

#### 2.2 Getting MPNG

You can obtain the *current development version* from the MATPOWER Github repository: https://github.com/MATPOWER/mpng.git.

#### 2.3 Running a Simulation

The primary functionality of MPNG is to solve optimal power and natural gas flow problems. Running a simulation using MPNG requires (1) preparing the natural gas input data, (2) specifying the interconnection input data to couple the gas network to the power system, (3) invoking the function to run the integrated simulation and (4) accessing and viewing the results.

The classical MATPOWER input data is a "MATPOWER-case" struct denoted by the variable mpc [4]. To integrate the power and natural gas systems we use the extended Optimal Power Flow (OPF) capability of MATPOWER. Namely, we model the natural gas system and its connection to the power system via

<sup>&</sup>lt;sup>1</sup>Matlab is available from The MathWorks, Inc. (https://www.mathworks.com/). An R2016b or later Matlab version is required as the MPNG code uses Matlab-files with multiple function declarations.

<sup>&</sup>lt;sup>2</sup>MATPOWER is available thanks to the Power Systems Engineering Research Center (PSERC) (https://matpower.org)

general user nonlinear constrains. Then, MPNG uses an extended "MATPOWER-gas case" struct denoted by the variable mpgc. In particular, mpgc is a traditional MATPOWER-case struct with two additional fields, mpgc.mgc and mpgc.connect standing for the natural gas case and interconnection case, respectively.

#### 2.3.1 Preparing the Natural Gas Case

The input data of the natural gas system are specified in a set of matrices arranged in a MATLAB struct that we refere to as the "gas case" (mpgc.mgc). The structure of such a gas case is formatted in a similar way to the MATPOWER-case but holding the natural gas information that comprise gas bases, nodes, wells, pipelines, compressors, and storage units. See Appendix A for more details about the gas case structure.

#### 2.3.2 Connecting the Gas Case to the Matpower Case

The input data regarding the connection between the power and natural gas systems are declared in a set of matrices packaged as a MATLAB struct which we call "interconnection case" (mpgc.connect). The structure of this case contains specific information about coupling elements as gas-fired power generators and power-and-gas-driven compressors, according to the optimization model described in section 3. See appendix B for more details about the interconnection case structure.

#### 2.3.3 Solving the Optimal Power-Gas Flow

Once the MATPOWER-gas case is properly formatted, one can invoke the solver using the (mandatory) mpgc struct and the traditional (optional) MATPOWER options struct mpopt. The calling syntax at the MATLAB prompt is as follows:

```
>> results = mpng(mpgc,mpopt);
```

For more details, type:

```
>> help mpng
```

## **Formulation**

#### Nomenclature

#### **Indexes**

i, j Gas nodes.

m, n Electric nodes (buses).

o Gas pipeline.

c Compressor.

l Transmission line.

w Gas well.

e Power generator.

ref Reference bus.

r Spinning reserve.

 $\sigma$  Type of gas load.

#### **Parameters**

 $\alpha^i_{\pi_+}, \alpha^i_{\pi_-}$  Penalties for over-pressure and under-pressure at node i.

 $\alpha_{\gamma}$  Penalties for non-supplied gas.

 $\alpha_{\epsilon}$  Penalties for non-supplied electricity.

 $C_G^w$  Gas cost at the well w.

 $C_O^{oij}$  Transport cost of pipeline o, from node i to node j.

 $C_C^{cij}$  Compression cost of compressor c, from node i to node j.

 $C_S^i$  Storage cost at node i.

 $C_{S_{+}}^{i}$  Storage outflow price at node i.

 $C_{S_{-}}^{i}$  Storage inflow price at node i.

 $C_E^e$  Power cost generation (excluding gas cost).

 $\eta_e^q$  Thermal efficiency at generator q [MMSCF/MW].

 $D_q^{i\sigma}$  Gas demand of type  $\sigma$  at node i.

 $D_e^{tm}$  Electricity demand in the bus m at time t.

 $\bar{g}^w, g^w$  Gas production limits.

 $\overline{\pi}^i, \, \underline{\pi}^i$  Quadratic pressure limits at node i.

 $S_0^i$  Initial stored gas at node i.

 $\overline{S}^i$ ,  $\underline{S}^i$  Storage limits at node i.

 $\kappa^{oij}$  Weymouth constant of pipeline o.

 $\delta^{oij}$  Width for gas flow capacities.

 $\beta^{cij}$  Compression ratio of compressor c.

 $Z^c$  Ratio parameter of compressor c.

 $B^c$  Compressor design parameter of compressor c.

 $x,\,y,\,z$  Gas consumption parameters of gas-fired compressors.

 $\overline{f}_g^{oij}$  Gas transport capacity of pipeline o, from node i to node j.

 $\overline{f}_g^{cij}$  Gas flow capacity of compressor c, from node i to node j.

 $\overline{f}_s^i, f_s^i$  Storage outflow capacities at node *i*.

 $\overline{p}_g^e,\,\underline{\underline{p}}_g^e\quad \text{ Active power generation limits of generator }e.$ 

 $\overline{q}_{g}^{e}, \, \underline{q}_{q}^{e}$  Reactive power generation limits of generator e.

 $\overline{V}^{tm}\underline{V}^{tm}$  Voltage limits for every bus m at time t.

 $\mathbb{S}^l$  Transmission capacity of power line l.

 $R^{tr}$  Spinning reserve in the r-th spinning reserve zone at time t.

M Generators assignment matrix.

L Compressors assignment matrix.

 $u^{te}$  Unit commitment state for generator q at time t.

 $\tau^t$  Energy weight related to period of time t.

 $E^e$  Available energy for hydroelectric generator e, during the total analysis window.

#### Sets

 $\mathcal{N}$  Gas nodes,  $|\mathcal{N}| = n_{\mathcal{N}}$ .

 $\mathcal{N}_{\mathcal{S}}$  Gas nodes with storage,  $\mathcal{N}_{\mathcal{S}} \subset \mathcal{N}$ ,  $|\mathcal{N}_{\mathcal{S}}| = n_{\mathcal{S}}$ .

 $\mathcal{O}$  Gas pipelines,  $|\mathcal{O}| = n_{\mathcal{O}}$ 

C Compressors,  $|C| = n_C$ 

 $C_G$  Compressors based on natural gas,  $C_G \subseteq C$ ,  $|C_G| = n_{C_G}$ 

 $\mathcal{C}_E$  Compressors based on electric power,  $\mathcal{C}_E \subseteq \mathcal{C}$ ,  $|\mathcal{C}_E| = n_{\mathcal{C}_P}$ 

 $\mathcal{W}$  Gas wells,  $|\mathcal{W}| = n_{\mathcal{W}}$ .

 $\mathcal{W}^i$  Gas wells at node  $i, \mathcal{W}^i \subset \mathcal{W}, |\mathcal{W}^i| = n_{\mathcal{W}^i}$ .

 $\mathcal{B}$  Power buses,  $|\mathcal{B}| = n_{\mathcal{B}}$ .

 $\mathcal{L}$  Power lines,  $|\mathcal{L}| = n_{\mathcal{L}}$ .

 $\mathcal{E}$  Power unit generators,  $|\mathcal{E}| = n_{\mathcal{E}}$ .

 $\mathcal{E}_H$  Hydroelectric power units,  $\mathcal{E}_H \subseteq \mathcal{E}$ ,  $|\mathcal{E}_H| = n_{\mathcal{E}_H}$ .

 $\mathcal{E}_G^i$  Gas-fired power units connected to gas node i,  $\mathcal{E}_G^i \subseteq \mathcal{E}$ ,  $|\mathcal{E}_G^i| = n_{\mathcal{E}_G}$ .

 $\mathcal{Z}_r$  Spinning reserve zones.

 $\mathcal{F}_G^i$ ,  $\mathcal{T}_G^i$  Connected pipelines to node i at side From or To.

 $\mathcal{F}_C^i$ ,  $\mathcal{T}_C^i$  Connected compressors to node *i* at side *From* or *To*.

 $\mathcal{F}_{E}^{m}$ ,  $\mathcal{T}_{E}^{m}$  Connected power lines to bus m at side From or To.

 $\mathcal{T}$  Total periods of analysis.

 $\Sigma$  Different types of gas loads.

#### Variables

 $f_g^{oij}$  Gas flow in pipeline o, from node i to node j.

 $f_{g_+}^{oij}$   $f_{g_-}^{oij}$  Positive and negative gas flow in pipeline o.

 $f_g^{cij}$  Gas flow in compressor c, from node i to node j.

 $\psi^c$  Power consumed by compressor c.

 $\phi^c$  Gas consumed by compressor c, connected to node i at side From.

 $\gamma^{i\sigma}$  Non-served gas of type  $\sigma$  at node i.

 $\pi^i$  Quadratic pressure.

 $\pi_+^i$ ,  $\pi_-^i$  Over/Under quadratic pressures at node *i*.

 $g^w$  Gas production at well w.

 $f_s^i$  Storage outflow difference.

 $f_{s_{+}}^{i}, f_{s_{-}}^{i}$  Storage outflow and inflow.

 $p_g^{te}$  Active power production at generator q at time t.

 $q_q^{te}$  Reactive power production at generator q at time t.

 $V^{tm}$  Voltage magnitude at bus m at time t.

 $\theta^{tm}$  Voltage angle at bus m at time t.

 $e^{tm}$  Non-served active power at bus m at time t.

We made a toolbox and we want to explain how it works.

#### 3.1 Objective function

$$C(x) = \sum_{w \in \mathcal{W}} C_G^w g^w + \sum_{t \in \mathcal{T}} \tau^t \sum_{e \in \mathcal{E}} C_E^e p_g^{te}$$

$$+ \sum_{i \in \mathcal{N}_S} \left( C_{S_+}^i f_{s_+}^i - C_{S_-}^i f_{s_-}^i \right)$$

$$+ \sum_{i \in \mathcal{N}_S} C_S^i \left( S_0^i - f_s^i \right)$$

$$+ \sum_{o \in \mathcal{O}} C_O^{oij} f_{g_+}^{oij} - \sum_{o \in \mathcal{O}} C_O^{oij} f_{g_-}^{oij}$$

$$+ \sum_{c \in \mathcal{C}} C_C^{cij} f_g^{cij}$$

$$+ \sum_{i \in \mathcal{N}} \alpha_{\pi_+}^i \pi_+^i + \sum_{i \in \mathcal{N}} \alpha_{\pi_-}^i \pi_-^i$$

$$+ \sum_{i \in \mathcal{N}} \sum_{\sigma \in \Sigma} \alpha_{\gamma}^{i\sigma} \gamma^{i\sigma} + \alpha_{\epsilon} \sum_{t \in \mathcal{T}} \tau^t \sum_{m \in \mathcal{B}} \epsilon^{tm}$$

$$(3.1)$$

#### 3.2 Constraints

#### 3.2.1 Gas network

$$\sum_{o \in \mathcal{T}_{G}^{k}} f_{g}^{oij} - \sum_{o \in \mathcal{F}_{G}^{k}} f_{g}^{oij} + \sum_{c \in \mathcal{T}_{C}^{k}} f_{g}^{cij} - \sum_{c \in \mathcal{F}_{C}^{k}} \left( f_{g}^{cij} + \phi^{c} \right) + \sum_{w \in \mathcal{W}^{k}} g^{w} + f_{s}^{k} - \sum_{t \in \mathcal{T}} \tau^{t} \sum_{e \in \mathcal{E}_{G}^{k}} \left( \eta_{e}^{q} \cdot p_{g}^{te} \right) = \sum_{\sigma \in \Sigma} \left( D_{g}^{\sigma k} - \gamma^{\sigma k} \right)$$

$$\forall k \in \mathcal{N}$$

$$(3.2)$$

The non-supply gas demand in every node of the system can only be as most as the total demand at the same node. This constraint is represented as follows:

$$0 \le \gamma^{\sigma k} \le D_g^{\sigma k} \quad \forall \sigma \in \Sigma \quad \forall k \in \mathcal{N}$$
 (3.3)

Storage

$$f_s^k = f_{s_+}^k - f_{s_-}^k \quad \forall k \in \mathcal{N}$$
 (3.4)

$$\underline{f}_{s}^{k} \le f_{s}^{k} \le \overline{f}_{s}^{k} \quad \forall k \in \mathcal{N}$$

$$(3.5)$$

$$0 \le f_{s_+}^i \le S_0^k - \underline{S}^k \quad \forall k \in \mathcal{N}$$
 (3.6)

$$0 \le f_{s_{-}}^{i} \le \overline{S}^{k} - S_{0}^{k} \quad \forall k \in \mathcal{N}$$

$$(3.7)$$

Wells

The constraints related to the gas wells production depends on each well specific characteristics, these constraints are represented by:

$$g^w \le g^w \le \overline{g}^w \quad \forall w \in \mathcal{W} \tag{3.8}$$

#### Pipelines:

$$f_g^{oij} = \kappa^{oij} sgn\left(\pi^i - \pi^j\right) \sqrt{|\pi^i - \pi^j|} \quad \forall o \in \mathcal{O}$$
 (3.9)

And the gas flow limits for every pipeline:

$$f_g^{oij} = f_{g_+}^{oij} + f_{g_-}^{oij} \quad \forall o \in \mathcal{O}$$

$$(3.10)$$

$$-\overline{f}_g^{oij} \le f_g^{oij} \le \overline{f}_g^{oij} \quad \forall o \in \mathcal{O}$$
 (3.11)

$$0 \le f_{g_+}^{oij} \le \delta^{oij} \cdot \overline{f}_g^{oij} \quad \forall o \in \mathcal{O}$$
 (3.12)

$$-\delta^{oij} \cdot \overline{f}_{a}^{oij} \le f_{a-}^{oij} \le 0 \quad \forall o \in \mathcal{O}$$
 (3.13)

#### Compressors:

The power consumed by the compressors depend on its gas flow:

$$\psi^c = B^c f_g^{cij} \cdot \left( \left( \frac{\pi^j}{\pi^i} \right)^{Z^c/2} - 1 \right) \quad \forall c \in \mathcal{C}$$
 (3.14)

$$\phi^c = x + y\psi^c + z\psi^{c2} \quad \forall c \in \mathcal{C}_G$$
 (3.15)

$$0 \le f_g^{cij} \le \overline{f}_g^{cij} \quad \forall c \in \mathcal{C} \tag{3.16}$$

$$\frac{\pi^{i} \leq \pi^{j} \leq \beta^{cij}\pi^{i}}{\beta^{cij} \geq 1} \quad \forall i, j \in \mathcal{N} \quad \forall c \in \mathcal{C}$$
(3.17)

The equations 3.18 and 3.19 are the constraints that characterize the quadratic overpressure and underpressure at every node of the system, respectively.

$$\begin{array}{ll}
\pi^k \le \overline{\pi}^k + \pi_+^k \\
0 \le \pi_+^k
\end{array} \quad \forall k \in \mathcal{N}$$
(3.18)

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$$\frac{\pi^k - \pi_-^k \le \pi^k}{0 < \pi_-^k} \quad \forall k \in \mathcal{N}$$
(3.19)

#### 3.2.2 Power network

Equation 3.20 could be explained in an appendix section.

$$g_{p_m} \left( \theta^{tm}, V^{tm}, p_g^{te}, \epsilon^{te}, \psi^c \right) = 0$$

$$g_{q_m} \left( \theta^{tm}, V^{tm}, q_g^{te} \right) = 0$$
(3.20)

$$\forall m \in \mathcal{B} \quad \forall t \in \mathcal{T} \quad \forall c \in \mathcal{C}_E$$

Variables limits:

$$\theta^{tref} = 0$$

$$\underline{V}^{tm} \le V^{tm} \le \overline{V}^{tm} \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T}$$
(3.21)

$$\underline{p}_{g}^{e} \leq p_{g}^{te} \leq \overline{p}_{g}^{e} 
\underline{q}_{g}^{e} \leq q_{g}^{te} \leq \overline{q}_{g}^{e} \quad \forall q \in \mathcal{E} \quad \forall t \in \mathcal{T}$$
(3.22)

Power flow limits:

$$|\mathbb{S}_{fl}(\theta, V) \leq \overline{\mathbb{S}}_{fl} \\ |\mathbb{S}_{tl}(\theta, V)| \leq \overline{\mathbb{S}}_{tl} \quad \forall l \in \mathcal{L}$$
(3.23)

Non-supplied active power limits:

$$0 \le \epsilon^{tm} \le D_e^{tm} \quad \forall m \in \mathcal{B} \quad \forall t \in \mathcal{T}$$
 (3.24)

Reserve constraint:

$$\sum_{e \in \mathcal{Z}_r} u^{te} \left( \overline{p}_g^e - p_g^{te} \right) \ge R^{tr}$$

$$\forall r \in \mathcal{Z}_r \quad \forall t \in \mathcal{T}$$
(3.25)

Hydro-energy constraint:

$$\sum_{t \in \mathcal{T}} \tau^t p_g^{te} \le E^e \quad \forall e \in \mathcal{E}_H \tag{3.26}$$

## Examples

Here the examples section goes...  $\,$ 

# Appendixes

# Appendix A: Gas Case Data File Format

Table A.1: Node Information Data (mgc.node.info)

name	column	description
NODE_I	1	node number (positive integer)
$NODE_TYPE$	2	node type $(1 = demand node, 2 = extraction node)$
PR	3	pressure [psia]
PRMAX	4	maximum pressure [psia]
PRMIN	5	minimum pressure [psia]
OVP	6	over-pressure [psia]
UNP	7	under-pressure [psia]
$COST_OVP$	8	over-pressure cost [\$/psia <sup>2</sup> ]
COST_UNP	9	under-pressure cost $[\$/psia^2]$
GD	10	full nodal demand [MSCFD]
NGD	11	Number of different nodal users (positive integer)

Table A.2: Node Demand Data (mgc.node.dem)

name	column	description
RES	1	residential demand [MSCFD]
IND	2	industrial demand [MSCFD]
COM	3	commercial demand [MSCFD]
NGV	4	natural gas vehicle demand [MSCFD]
REF	5	refinery demand [MSCFD]
PET	6	petrochemical demand [MSCFD]

Table A.3: Node Non-supplied Demand Data (mgc.node.demcost)

name	column	description
AL_RES	1	residential non-supplied demand cost [\$/MMSCFD]
$\mathtt{AL}_{-}\mathtt{IND}$	2	industrial non-supplied demand cost [\$/MMSCFD]
$AL\_COM$	3	commercial non-supplied demand cost [\$/MMSCFD]
$\mathtt{AL\_NGV}$	4	natural gas vehicle non-supplied demand cost [\$/MMSCFD]
$AL_REF$	5	refinery non-supplied demand cost [\$/MMSCFD]
$AL\_PET$	6	petrochemical non-supplied demand cost $[\$/\mathrm{MMSCFD}]$

Table A.4: Well Data (mgc.well)

name	column	description
WELL_NODE	1	well id-node (positive integer)
G	2	well injection [MMSCFD]
PW	3	known well pressure [psia]
GMAX	4	maximum gas injection [MMSCFD]
GMIN	5	minimum gas injection [MMSCFD]
$WELL\_STATUS$	6	well status ( $0 = \text{disable}, 1 = \text{enable}$ )
$COST\_G$	7	production cost [\$/MMSCFD]

# Appendix B: Interconnection Case Data File Format

Table B.1: Node Demand Data (mgc.node.dem)

name	column	description
NODE_I	1	node number (positive integer)
$NODE_TYPE$	2	node type $(1 = demand node, 2 = extraction node)$
PR	3	pressure []
PRMAX	4	maximum pressure []
PRMIN	5	minimum pressure
OVP	6	over-pressure []
UNP	7	under-pressure []
COST_OVP	8	over-pressure cost (positive integer)
COST_UNP	9	under-pressure cost (positive integer)
GD	10	full nodal demand [MSCFD]
NGD	11	Number of different nodal users (positive integer)

 $<sup>^\</sup>dagger$  Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units u.

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