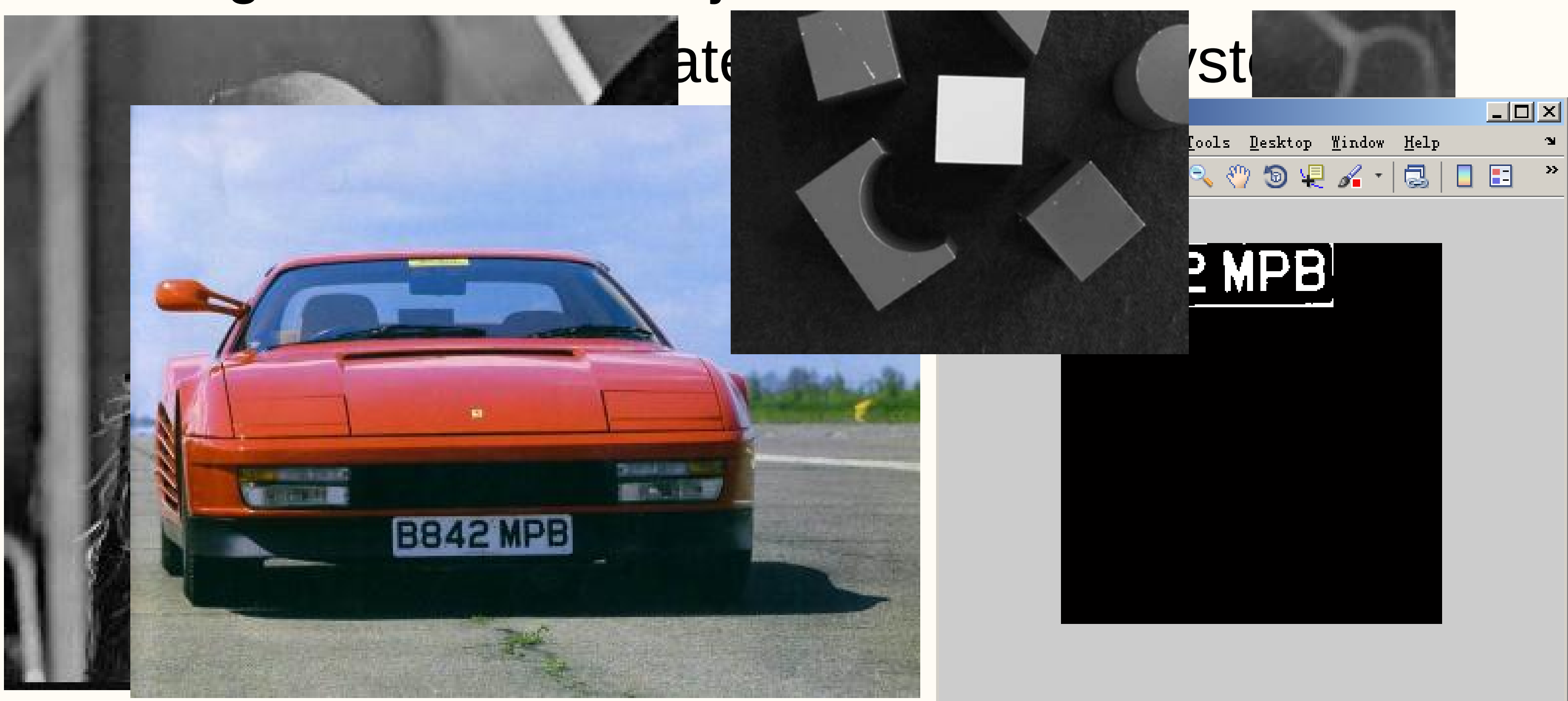
Chapter 4 Image Segmentation

Course Arrangement

– Theory: 32 hours

– Project & Practice: 16 hours

  • Segmentation of Objects



Why we need to learn “image segmentation” ?

Video of “cow tracking ”

Image capture

↓

Preprocessing

↓

Segmentation → Standard segmentation method; Advanced segmentation method

↓

Model fitting

↓

Model prediction

↓

Conclusion (understanding)

Fig. Flowchart of “cow tracking”

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Outline

- 4.1 Standard segmentation method

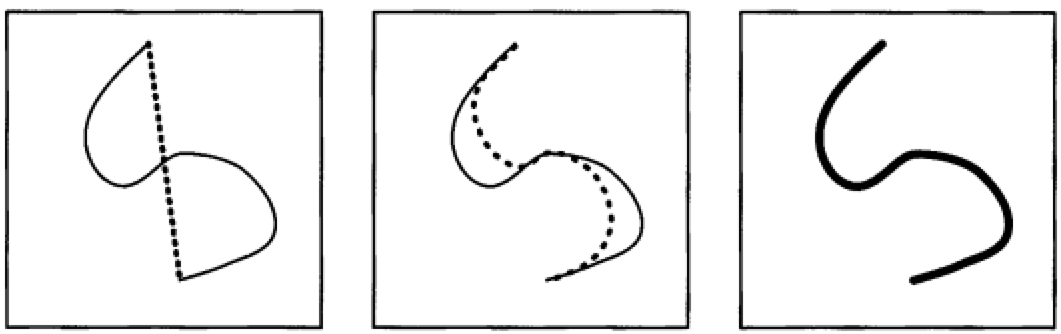
- 4.2 Advanced segmentation method

4.2 Advanced segmentation method

- 4.2.1 Active contour models-snakes
- 4.2.2 Geometric deformable models-level sets and geodesic active contours
- 4.2.3 Fuzzy connectivity

4.2.1 Active contour models-snakes

Examples



(a) (b) (c)

Figure 7.6: Active contour model—snake. (a) Initial snake position (dotted) defined interactively near the true contour. (b), (c) Iteration steps of snake energy minimization: the snake is pulled toward the true contour.

4.2.1 Active contour models-snakes

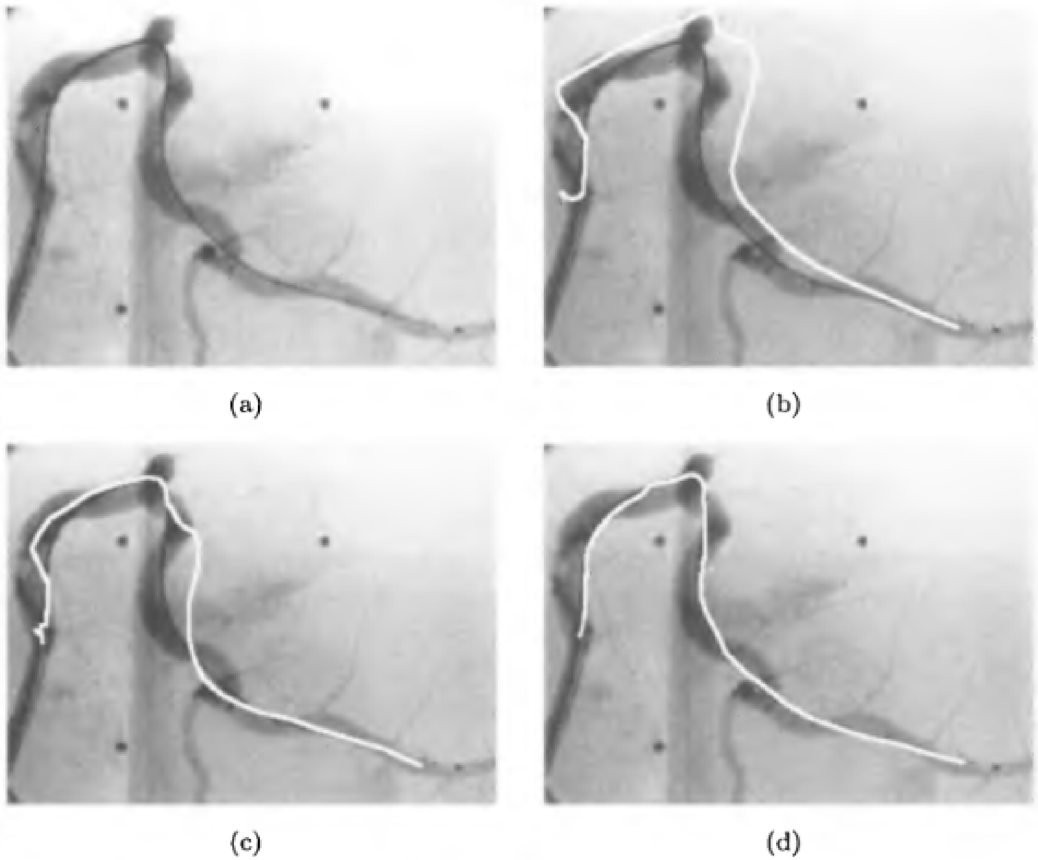
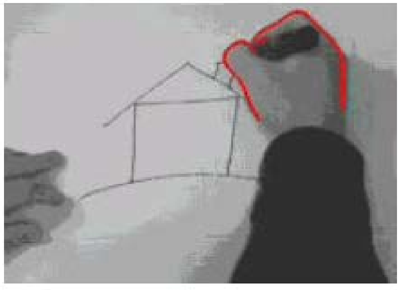


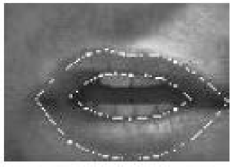
Figure 7.7: Snake-based detection of the intravascular ultrasound catheter (dark line positioned inside the coronary artery lumen) in an angiographic X-ray image of a pig heart. (a) Original angiogram. (b) Initial position of the snake. (c) Snake deformation after 4 iterations. (d) Final position of the snake after 10 iterations.

4.2.1 Active contour models - snakes

- Applications

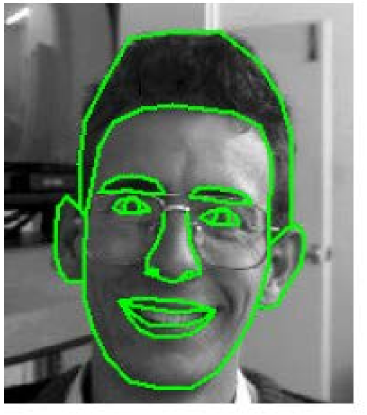


Human-computer interaction



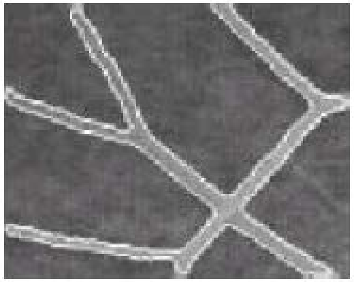
Lip-reading

Surveillance/speed control

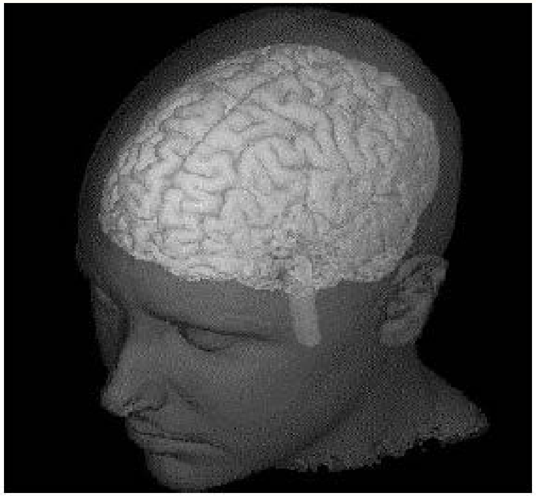


Face recognition

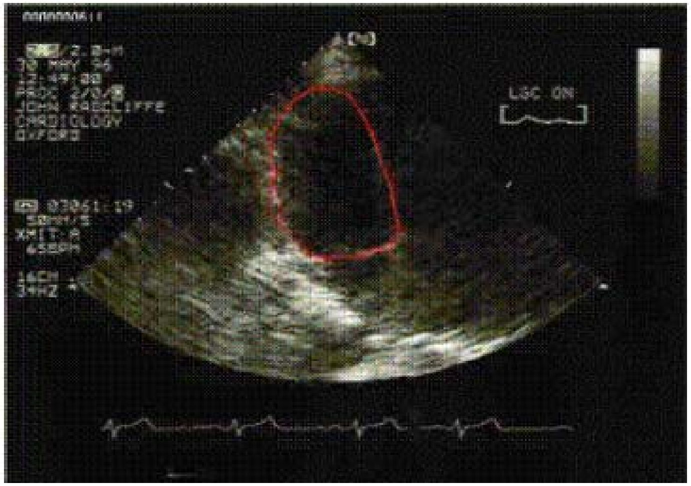
Applications -- Medical



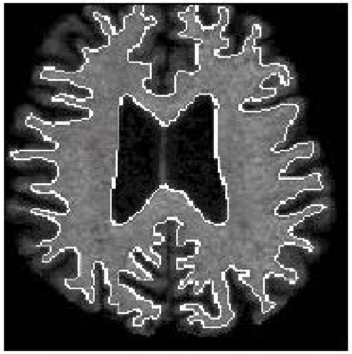
Blood vessels



3D brain



Heart in ultrasound



brain in MR

4.2.1 Active contour models -snakes

- First introduced in 1987 by Kass et al, and gained popularity since then.

- Represents an object boundary or some other salient image feature as a **parametric curve.**

- An energy functional E is associated with the curve.

- The problem of finding object boundary is cast as an energy minimization problem.

4.2.1 Active contour models-snakes

- Parametric active contour model

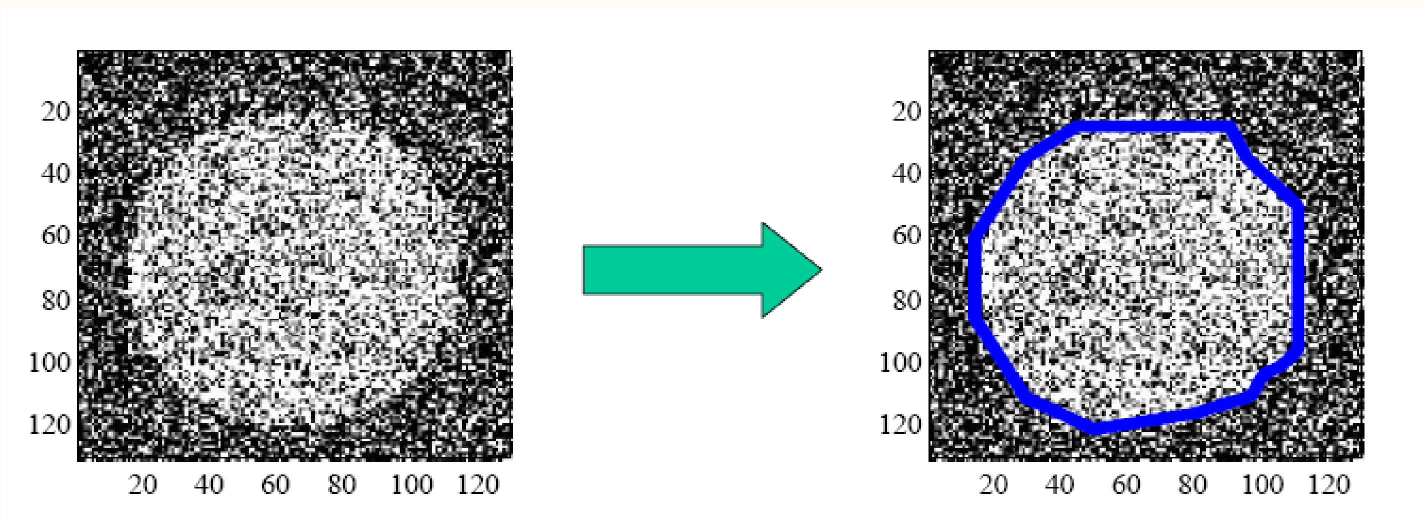
  snake

  balloon model

  GVF snake model

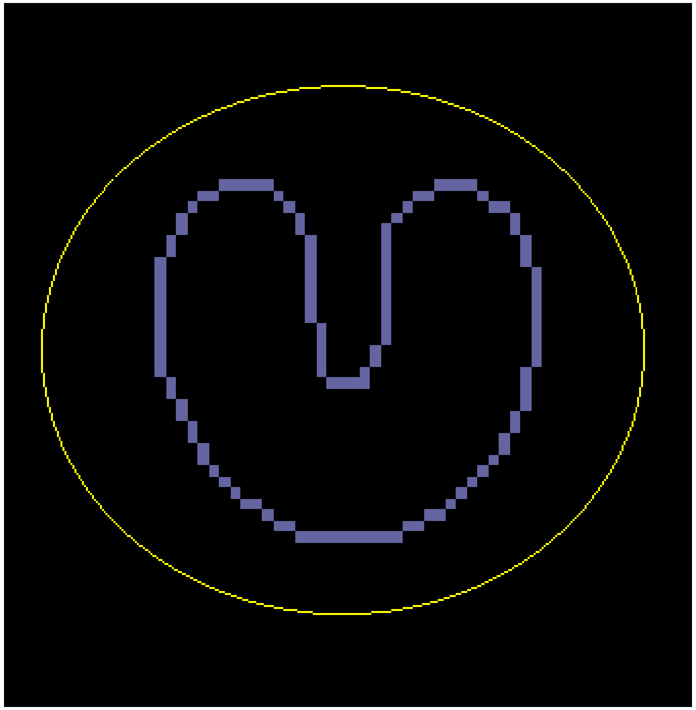
- Geometric active contour model

  Level set



Framework for snakes

- A higher level process or a user initializes any curve *close to the object boundary*.
- The snake then starts *deforming* and moving towards the desired object boundary.
- In the end it completely “shrink-wraps” around the object.



(Diagram courtesy “Snakes, shapes, gradient vector flow”, Xu, Prince)

Modeling

- The contour is defined in the (x, y) plane of an image as a parametric curve

$$\mathbf{v}(s)=(x(s), y(s))$$

- Contour is said to **possess an energy** (E_{snake}) which is defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- The energy terms are defined ***cleverly*** in a way such that the final position of the contour will have a minimum energy (E_{min})

- Therefore our problem of detecting objects reduces to an energy minimization problem.

Modeling

- Contour is said to **possess an energy** (***E_{snake}***) which is defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- The energy terms are defined ***cleverly*** in a way such that the final position of the contour will have a minimum energy (*E_{min}*)

- The energy functional which is minimized is a weighted combination of internal and external forces.

- The internal forces emanate from the shape of the snake

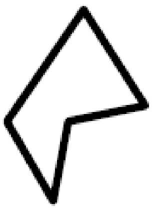
- While the external forces come from the image and/or from higher-level image understanding processes.

Contour Energy - Internal

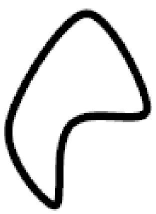
Assign low energy values to ‘good’ contours and high energy to ‘bad’ ones

Internal energy

In natural objects we usually require smooth contours



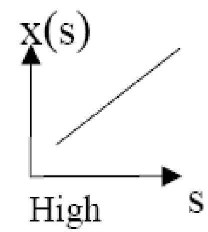
High energy

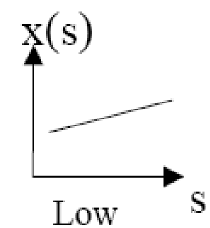


Low energy

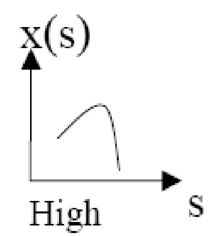
Internal Energy depends on the shape of the contour itself.

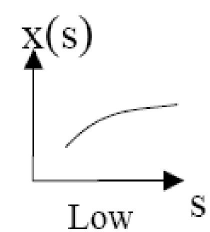
small $\dfrac{\partial \mathbf{v}}{\partial s}$ $\Rightarrow$ tension in the contour, low internal energy





small $\dfrac{\partial^2 \mathbf{v}}{\partial s^2}$ $\Rightarrow$ No bending in the contour, low internal energy



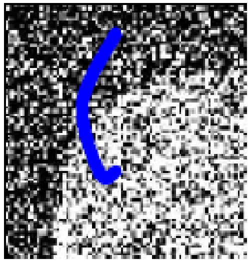


Contour Energy - External

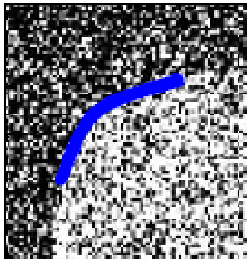
Assign low energy values to 'good' contours and high energy to 'bad' ones

External energy

When locating an object in an image...



High energy



Low energy

External Energy is derived from the image data

Look for high intensity gradient
(watch out for noise - gradient of smoothed image)

high $\nabla[G_\sigma * I(x,y)]$ ⇨ Low external energy

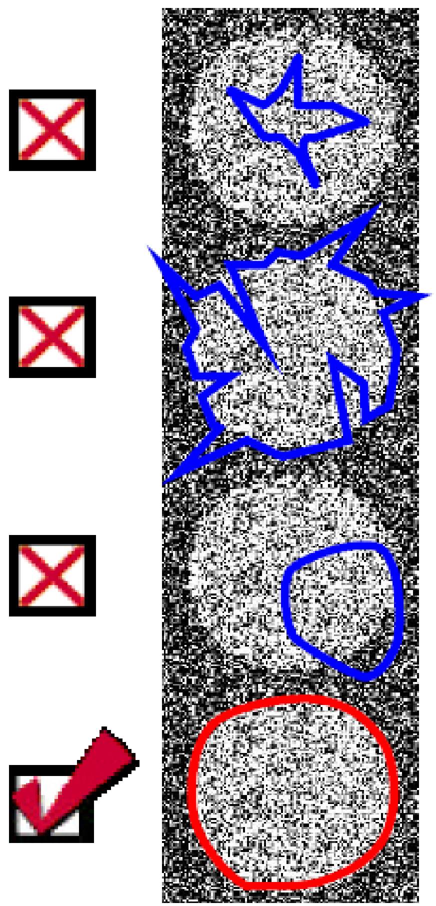
$G_\sigma * I$ image convolved with a smoothing (ex. Gaussian) filter.

$I(x,y)$ image intensity

σ parameter controlling the extent of the smoothing (ex. variance of Gaussian).

could have other than gradient e.g. intensity, termination, corners...

Contour Energy - Examples



high-internal energy
high-external energy
(not smooth and away from target)

high-internal energy
low-external energy
(not smooth and close to target)

low-internal energy
high-external energy
(smooth and away from target)

low-internal energy
low-external energy
(smooth and very close to target)

Energy and force equations

- The problem at hand is to find a contour v(s) that minimize the energy functional

$$E_{snake} = \int_s \frac{1}{2}(\alpha(s)|v_s|^2 + \beta(s)|v_{ss}|^2) + E_{image}(v(s))ds$$

- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation

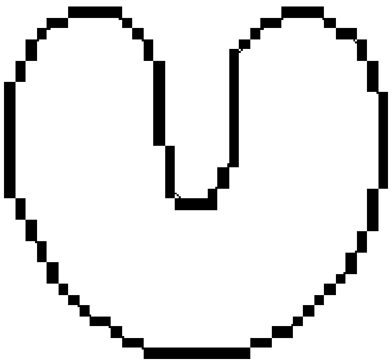
$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

- Equation can be interpreted as a force balance equation.
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

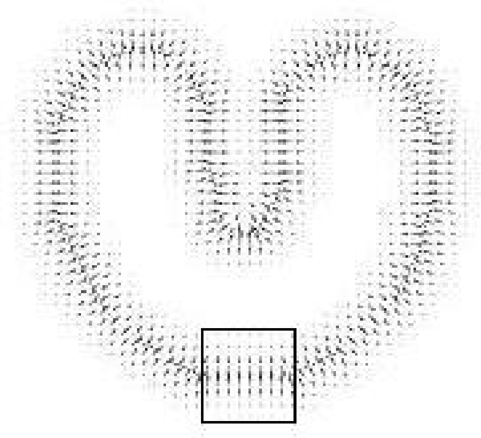
External force

$$F_{ext} = -\nabla E_{image}$$

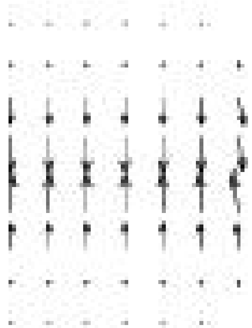
- It acts in the direction so as to minimize E_{ext}



Image



External force



Zoomed in

Discretizing

- the contour v(s) is represented by a set of control points

  $v_0, v_1, \ldots\ldots, v_{n-1}$
- The curve is piecewise linear obtained by joining each control point.

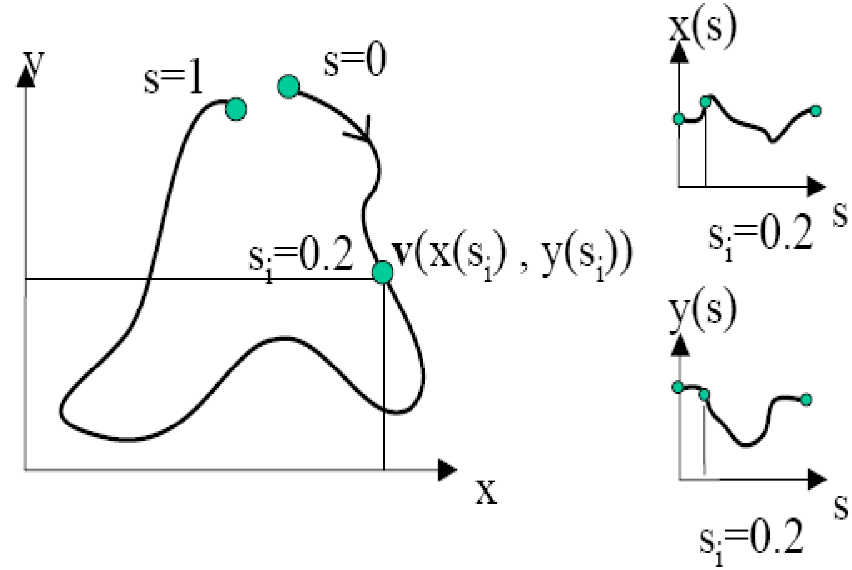
When working with complex contours usually implicit and parametric representations are used

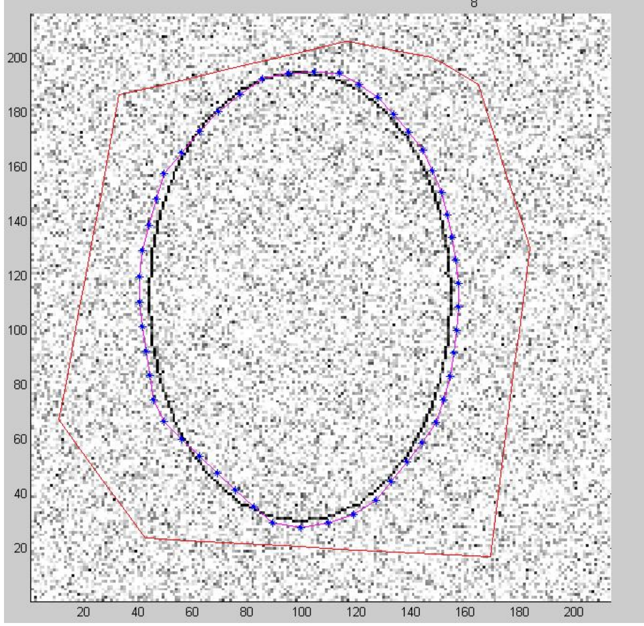
$x = x(s); y = y(s)$ for a parameter s.

E.g. an ellipse for example is represented by

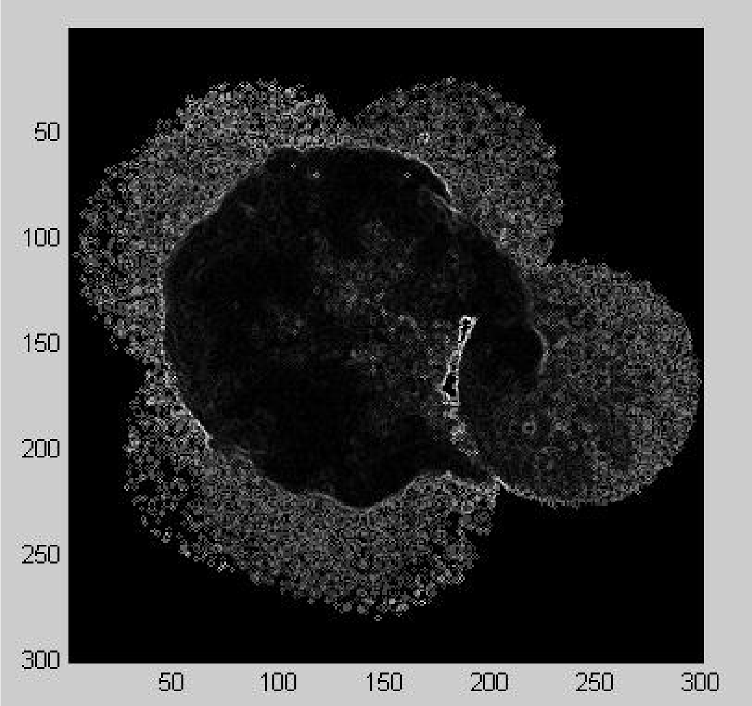
$x = a\cos(s); y = b\sin(s), s \in [-\pi, \pi]$

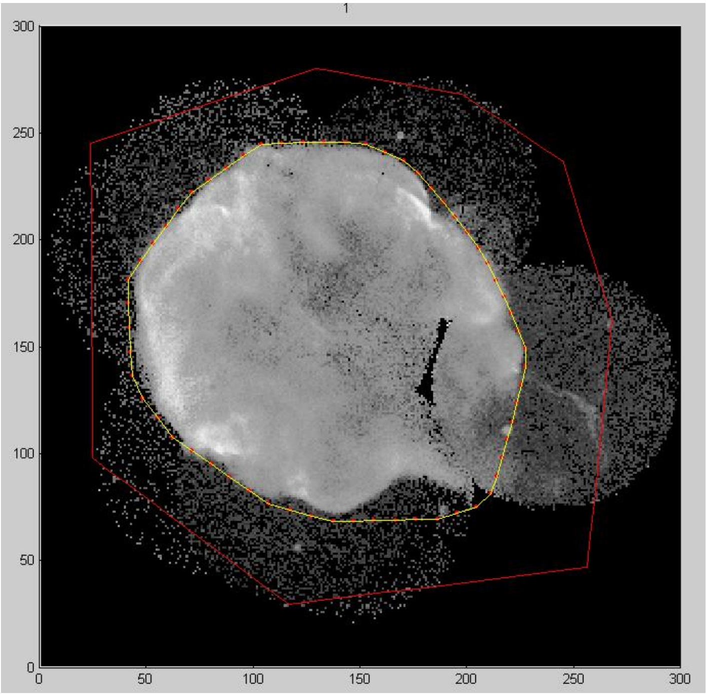
$\mathbf{v}(s) = [x(s) \quad y(s)]^T, s \in [0,1]$





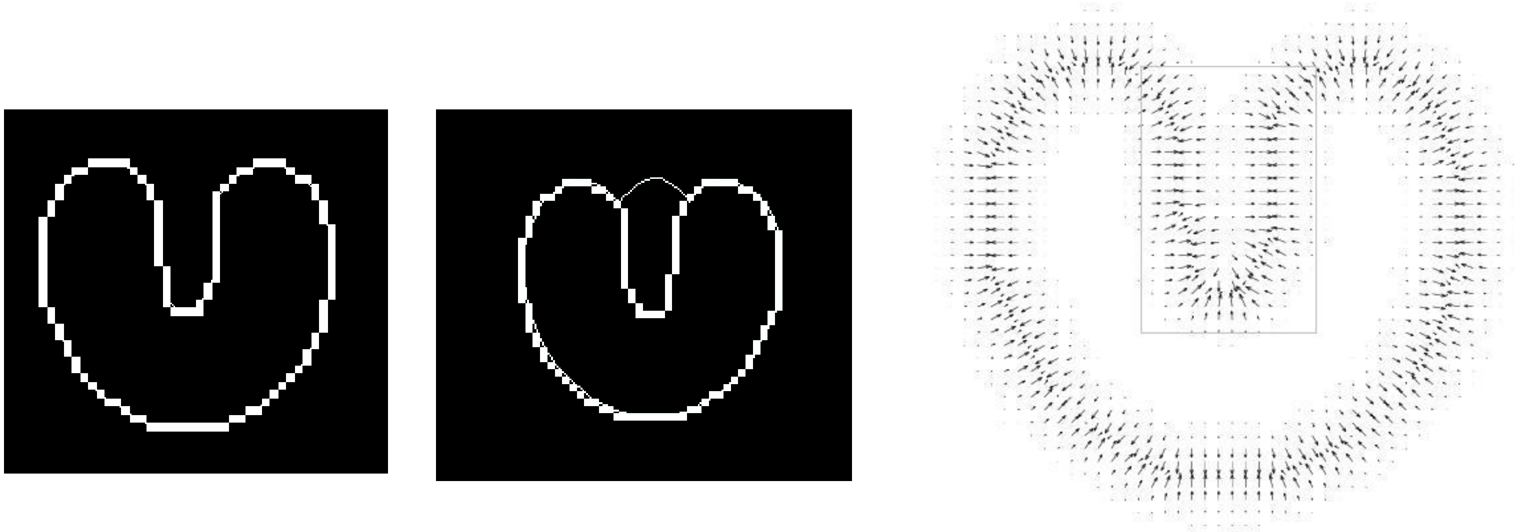
•Noisy image with many local minimas
•WGN sigma=0.1
•Threshold=15





Weakness of traditional snakes (Kass model)

- Extremely sensitive to parameters.
- Small capture range.



- No external force acts on points which are far away from the boundary.
- Convergence is dependent on initial position.

4.2.1 -Balloon

- Additional force applied to give stable results. Why?
- A snake which is not close enough to contours is not attracted by them.
- Add an inflation force which makes the curve behave well in this case.
- The curve behaves like a balloon which is inflated. When it passes by edges, will not be trapped by spurious edges and only is stopped when the edge is strong.
- The initial guess of the curve not necessarily is close to the desired solution.

- Pressure force is added to the internal and external forces

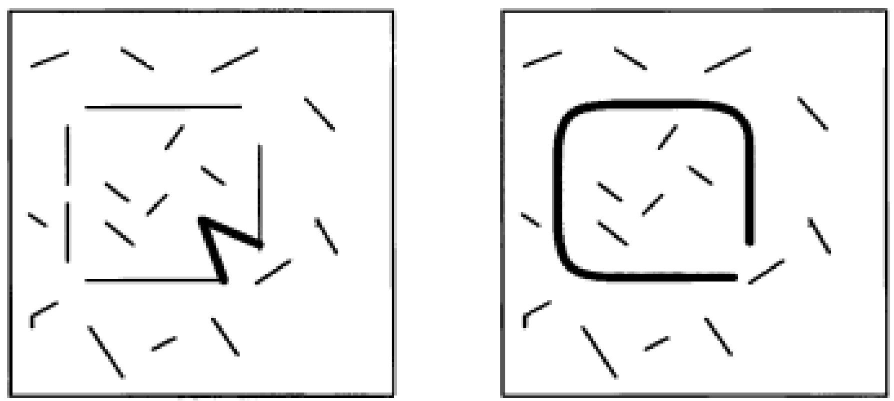
$$F = k_1 \vec{n}(s) - k \frac{\nabla P}{\|\nabla P\|}(v(s)),$$

- Increase the capture range of an active contour

➢ Require the balloon initialized to shrink or grow

➢ Strength of the force may be difficult to set
  - Large enough to overcome weak edges and forces
  - Small enough not to overwhelm legitimate edge forces

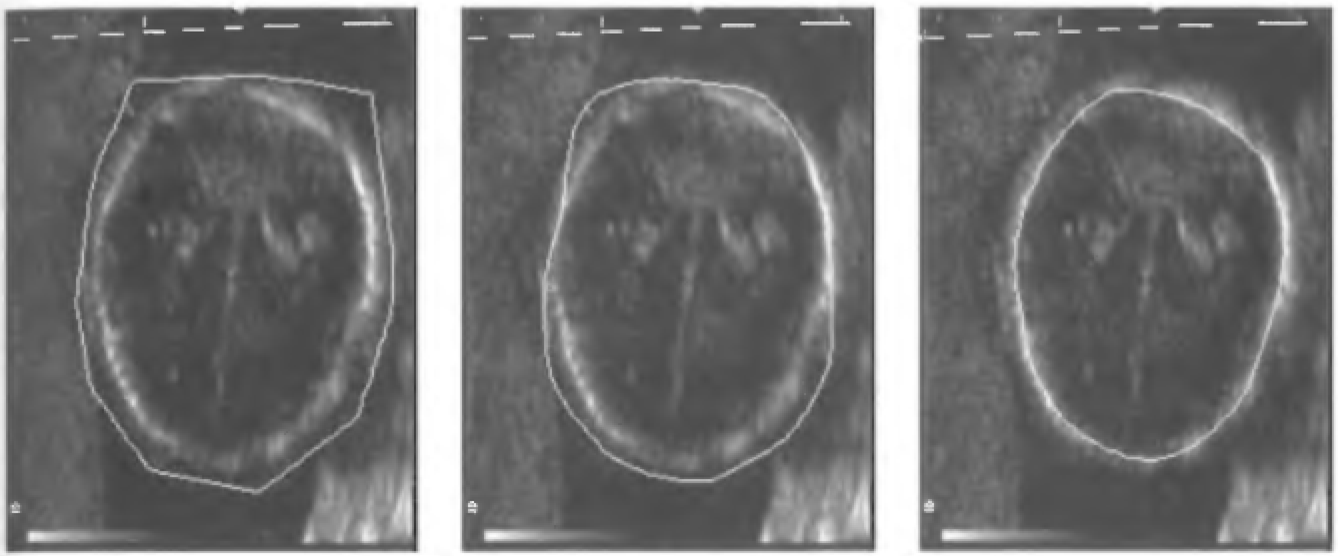
4.2.1 -Balloon



(a) (b)

Figure 7.9: Active contour model—balloon. (a) Initial contour. (b) Final contour after inflation and energy minimization. Adapted from [Cohen and Cohen, 1992].

4.2.1 -Balloon



(a) (b) (c)

Figure 7.10: Balloon-based image segmentation of an ultrasound image of a fetal head. (a) Initial position of the balloon. (b) Balloon deformation after 10 iterations. (c) Final position of the balloon after 25 iterations. *Courtesy of V. Chalana.*

4.2.1 -Gradient Vector Flow (GVF)

- A new external force for snakes
- Detects shapes with boundary concavities.
- Large capture range.

Model for GVF snake

- The GVF field is defined to be a vector field
  V(x,y) = $(u(x,y), v(x,y))$
- Force equation of GVF snake

$$\alpha v_{ss} - \beta v_{ssss} + V = 0$$

- V(x,y) is defined such that it minimizes the energy functional

$$E = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \, dxdy$$

f(x,y) is the edge map of the image.

- $|\nabla f|$ is small, energy dominated by first term
  - (smoothing)
- $|\nabla f|$ is large, second term dominates
  - minimal when $v = \nabla f$
- μ is tradeoff parameter, increase with noise

$$E = \iint \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \, dxdy$$

- GVF field can be obtained by solving following Euler equations

$$\mu\nabla^2 u-(u-f_x)(f_x^2+f_y^2)=0$$

$$\mu\nabla^2 v-(v-f_y)(f_x^2+f_y^2)=0$$

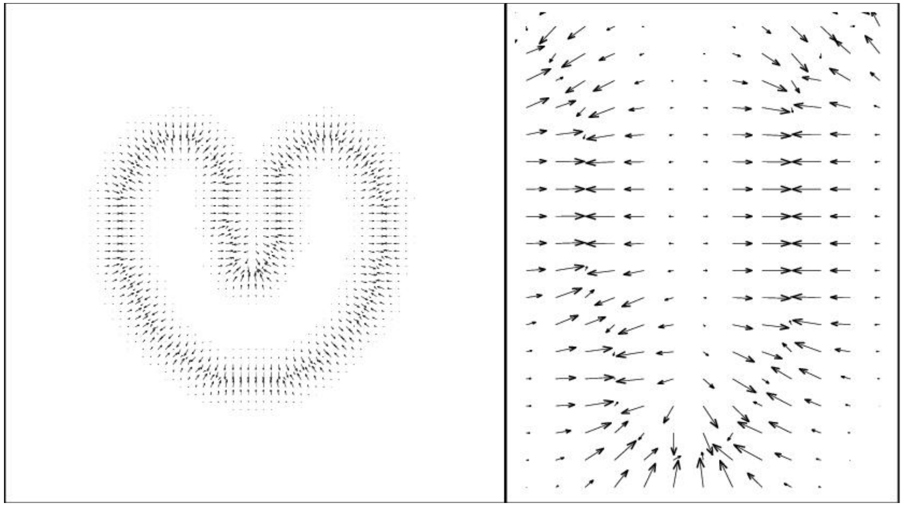
∇^2 Is the Laplacian operator.

- Reason for detecting boundary concavities.

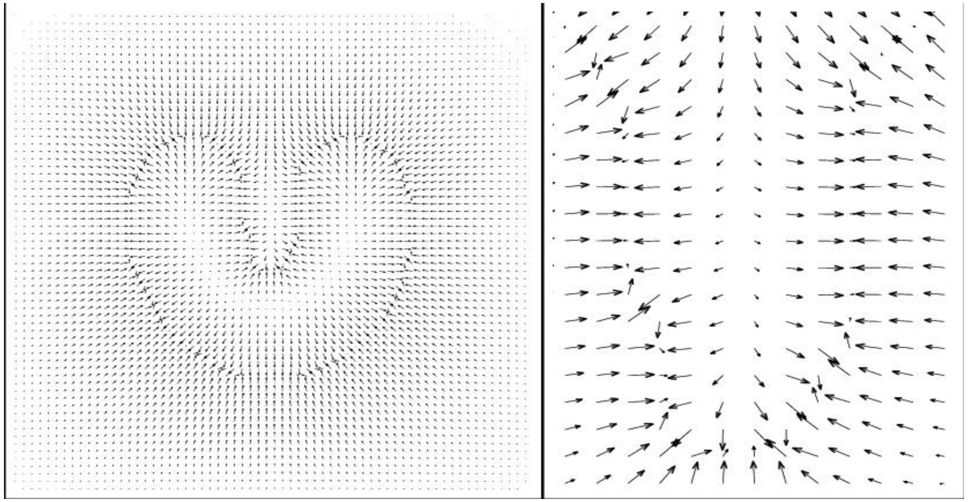
- The above equations are solved iteratively using time derivative of u and v.

Traditional external force field v/s GVF field

Traditional force

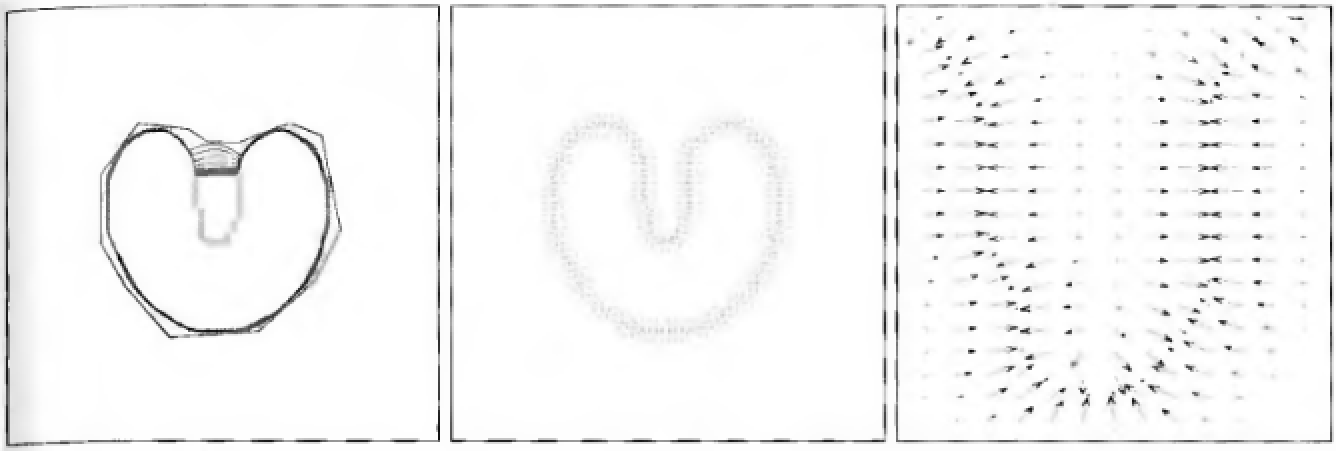


GVF force



(Diagrams courtesy “Snakes, shapes, gradient vector flow”, Xu, Prince)

4.2.1 Active contour models-snakes



(a) (b) (c)

Figure 7.11: Classic snake convergence. (a) Convergent sequence of snake locations. Note that the snake fails segmenting the concave boundary. (b) Classic external forces. (c) Close-up of the concave object region. No forces exist capable of pulling the snake inside the bay. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

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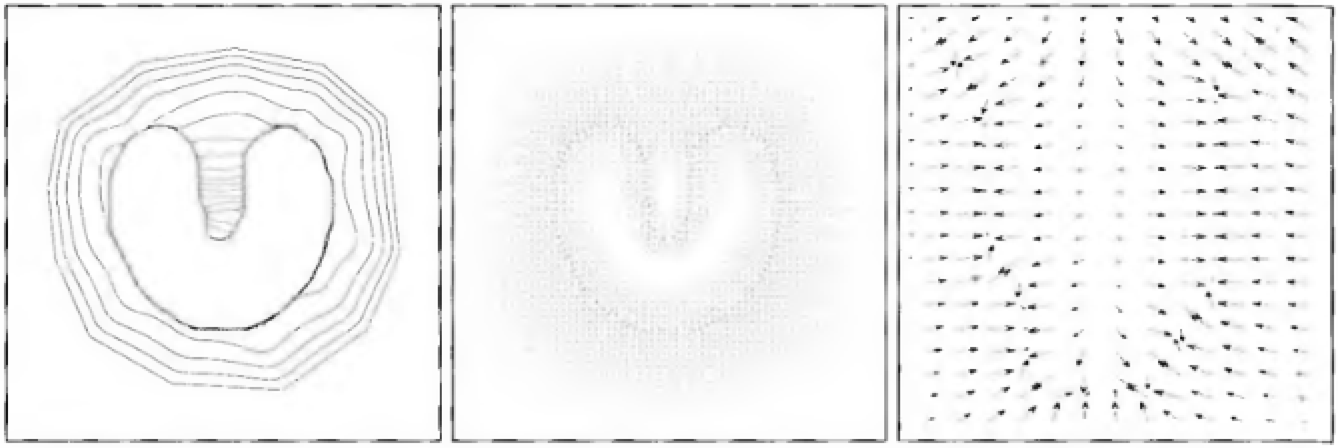
4.2.1 Active contour models-snakes



(a) (b) (c)

Figure 7.12: GVF snake convergence. (a) Convergent sequence of snake locations. Note that the snake succeeded in segmenting the concave boundary. (b) GVF external forces. (c) Close-up of the concave object region. Forces exist capable of pulling the snake inside the bay. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

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4.2.1 Active contour models-snakes

(a) (b) (c)

Figure 7.13: Streamlines originating in a regular 32×32 grid of points. (a) Classic potential force field–only locations very close to the border can be attracted to the object. (b) Distance-based external force field. Note that there is insufficient pull of the locations inside the bay to correctly segment the concave region. However, the snake can be initialized at a distance from the object. (c) GVF force field demonstrating the ability to correctly segment the concave region and maintaining the ability of a large-distance initialization. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

4.2.1 Active contour models-snakes

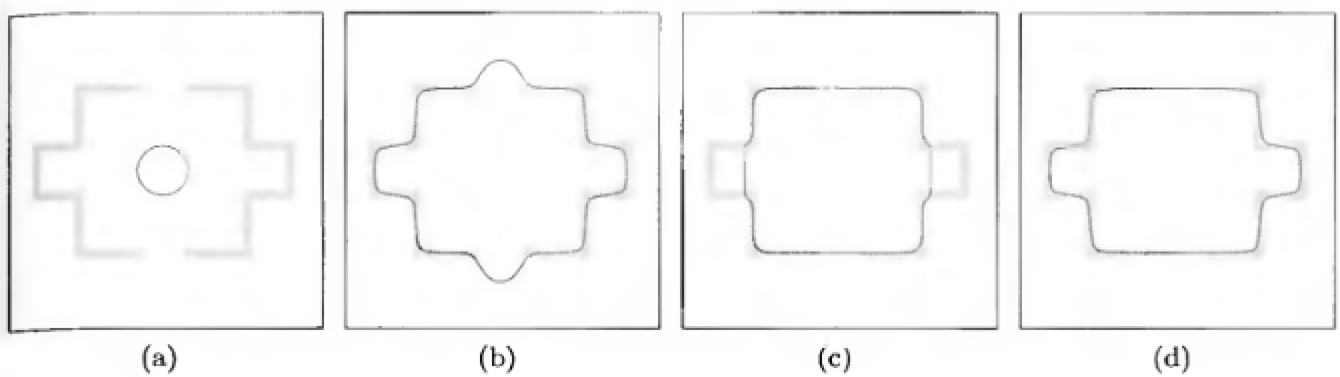
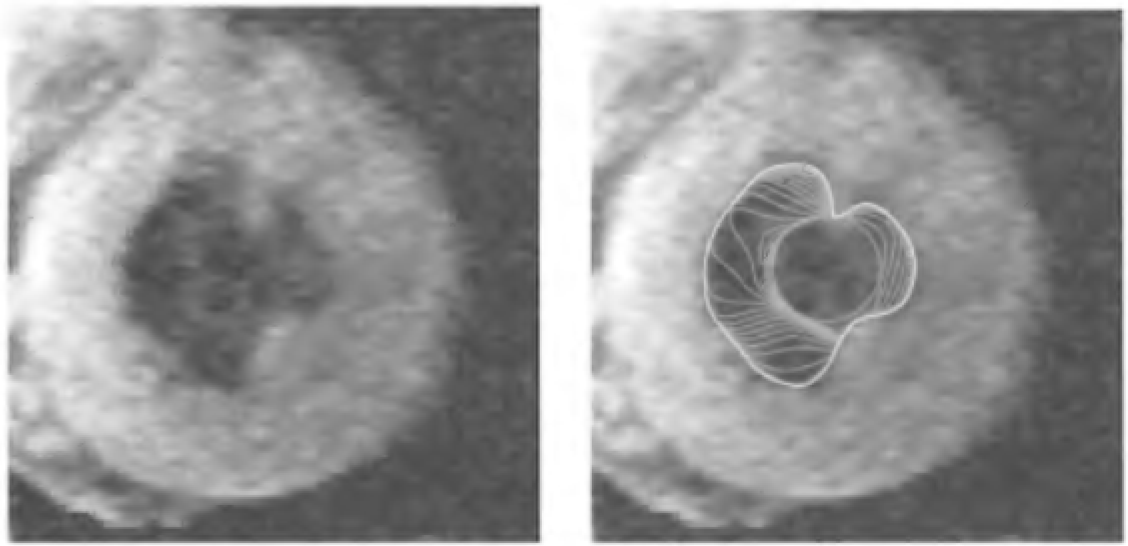


Figure 7.14: Snake behavior. (a) Initialization–common to a balloon, distance potential, and GVF snakes. (b) Balloon with an outward pressure force. (c) Distance potential force snake. (d) GVF snake. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

4.2.1 Active contour models-snakes



(a) (b)

Figure 7.15: Cardiac MR segmentation using GVF snakes. (a) Original short axis MR image of the left cardiac ventricle. (b) GVF snake segmentation showing the convergence process. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

Result

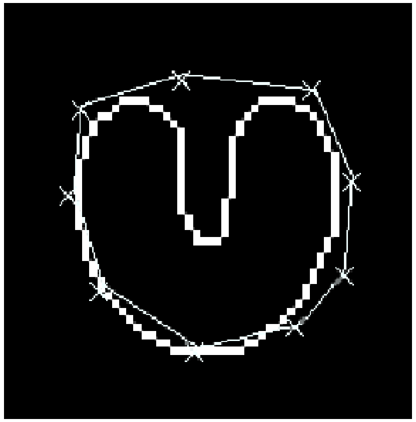
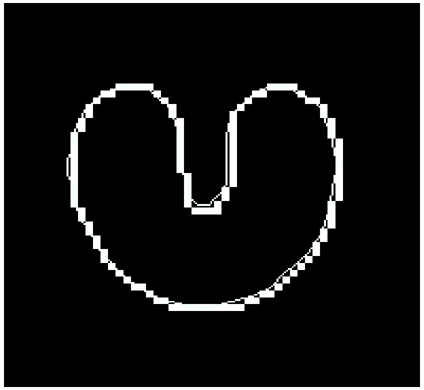
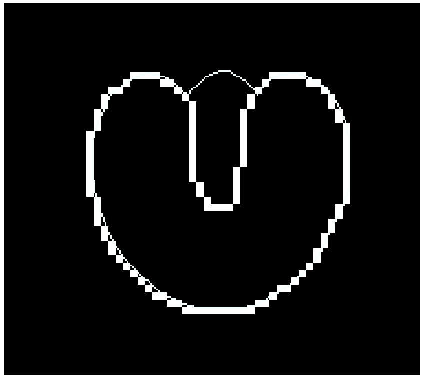


Image with initial contour



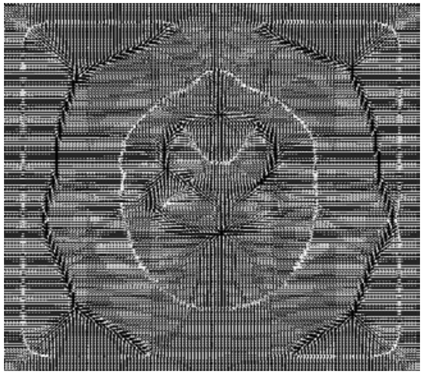
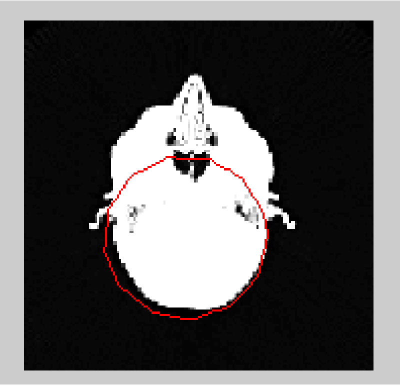


GVF snake

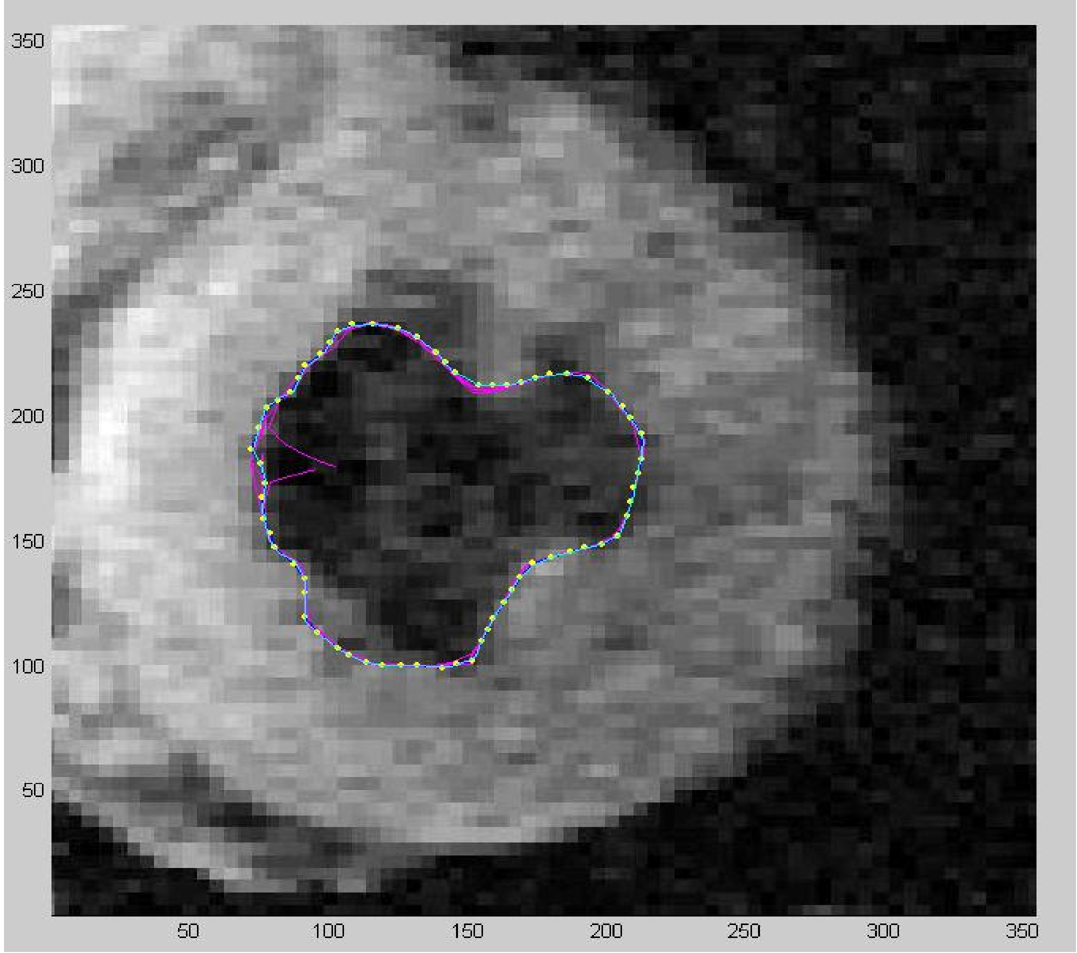
Traditional snake

Problem with GVF snake

- Very sensitive to parameters.
- Initial location dependent.
- Slow. Finding GVF field is computationally expensive.


Medical Imaging



Magnetic resonance image of the left ventricle of human heart

Notice that the image is poor quality with sampling artifacts

4.2.1 Active contour models-snakes

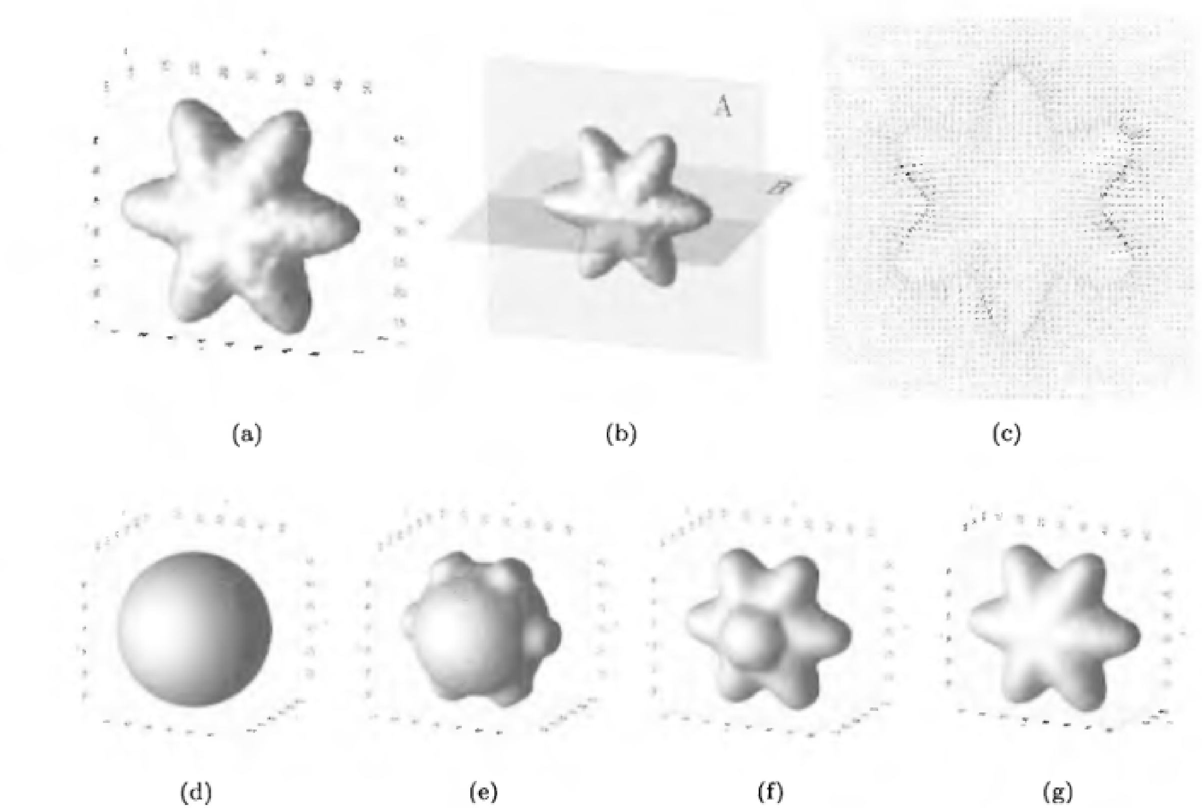


Figure 7.16: GVF snake segmentation in 3D. (a) Isosurface of a 3D object defined on a 64^3 grid. (b) Position of plane A on which the 3-D GVF vectors are depicted in (c). (d) The initial configuration of a deformable surface using GVF and its positions after (e) 10, (f) 40, and (g) 100 iterations. *Courtesy of J. L. Prince and C. Xu, Johns Hopkins University, ©1998 IEEE [Xu and Prince, 1998].*

4.2.1 Active contour models-snakes

- 4.2.2 Active contour models-snakes

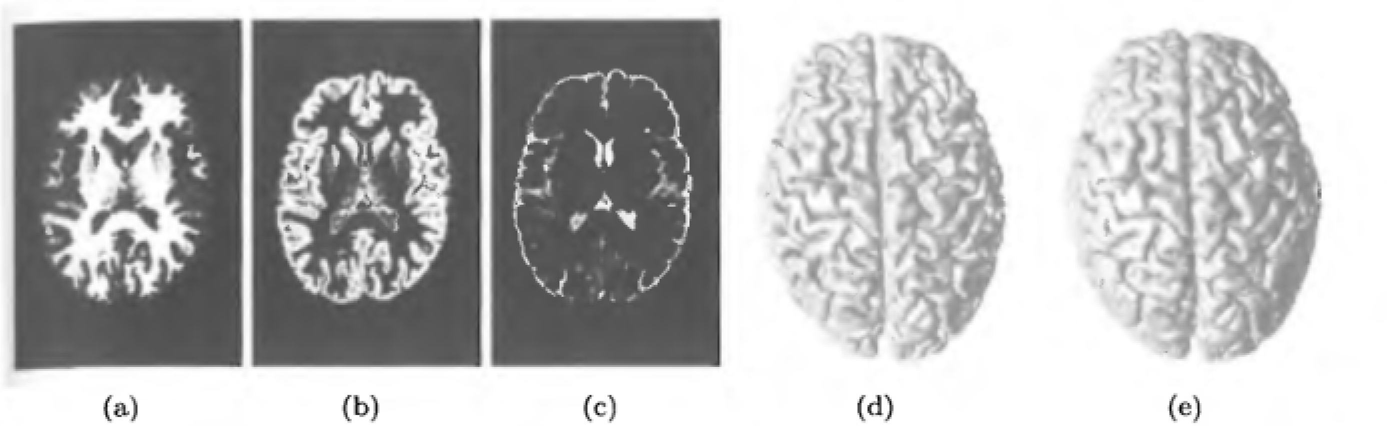


Figure 7.17: 3D segmentation of the MR brain image using GVF snakes. (a-c) Fuzzy classification of the white matter (a), gray matter (b), and cerebrospinal fluid (c) in the cerebrum. A fuzzy classification yielded three membership functions, cross-sections of which are shown. (d,e) GVF snake segmentation was used to obtain anatomically feasible surfaces representing the central (d), and pial (e) surfaces bounding the cortex. *Courtesy of J. L. Prince, Johns Hopkins University.*

Applications of snakes

- Image segmentation particularly medical imaging community (tremendous help).

- Motion tracking.

- Stereo matching (Kass, Witkin).

- Shape recognition.

References

- M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models.“, International Journal of Computer Vision. v. 1, n. 4, pp. 321-331, 1987.
- Laurent D.Cohen , “Note On Active Contour Models and Balloons“, CVGIP: Image Understanding, Vol53, No.2, pp211-218, Mar. 1991.
- C. Xu and J.L. Prince, “Gradient Vector Flow: A New External Force for Snakes”, Proc. IEEE Conf. on Comp. Vis. Patt. Recog. (CVPR), Los Alamitos: Comp. Soc. Press, pp. 66-71, June 1997.

4.2.2 Geometric deformable models-level sets and geodesic active contours

- ACM vs. Level set

- Initial location sensitive
  – GVF snake still require the initial contour close enough
- Parameterization of Curve
- Topological change
- Parameters selection

- Initial curve selection or reinitialization

- Ref: C. Xu, A. Yezzi, and J. L. Prince, “On the relationship between Parametric and Geometric Active Contours”, TR JHU/ECE 99-14

4.2.2 Geometric deformable models-level sets and geodesic active contours

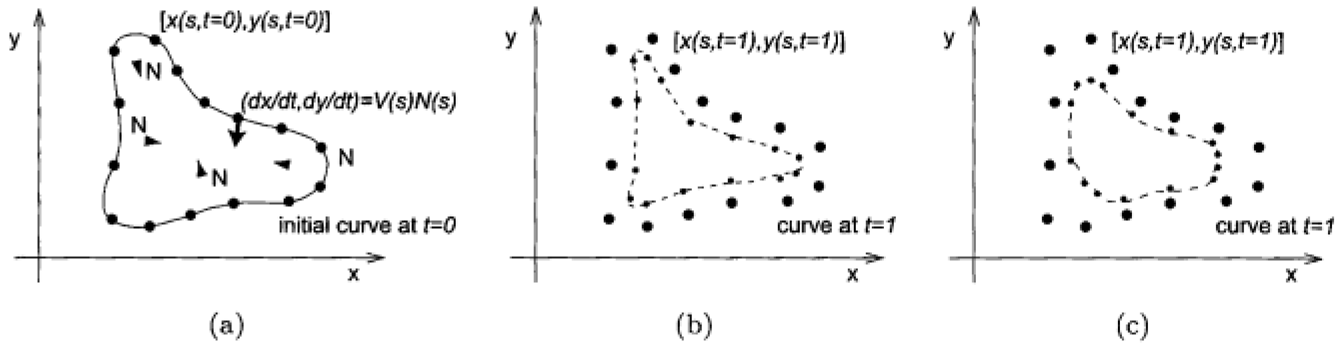
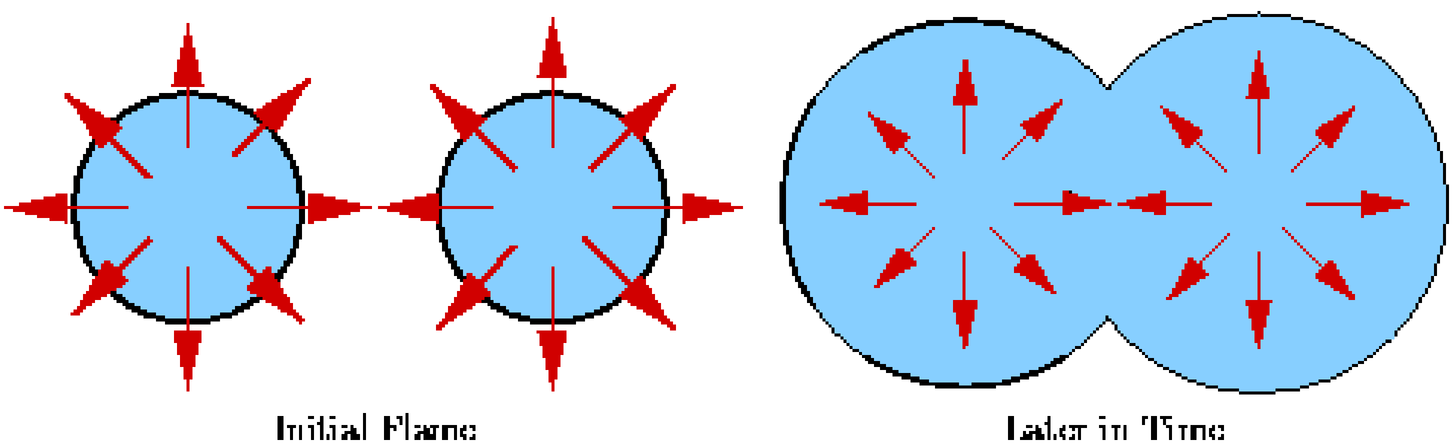


Figure 7.18: Concept of front evolution. (a) Initial curve at $t = 0$. (b) Curve at $t = 1$. Note that each curve point moved in direction of $\mathbf{N}$ by distance given by velocity V. (c) Curve at $t = 1$ assuming the velocity $V(c)$ is a function of curvature.

Level Set Methods

- Contour evolution method due to J. Sethian and S. Osher, 1988
- www.math.berkeley.edu/~sethian/level_set.html
- Difficulties with snake-type methods
  - Hard to keep track of contour if it self-intersects during its evolution
  - Hard to deal with changes in topology



- The level set approach:

  – Define problem in 1 higher dimension

  – Define level set function $\boldsymbol{z} = \phi(\boldsymbol{x,y,t} = 0)$

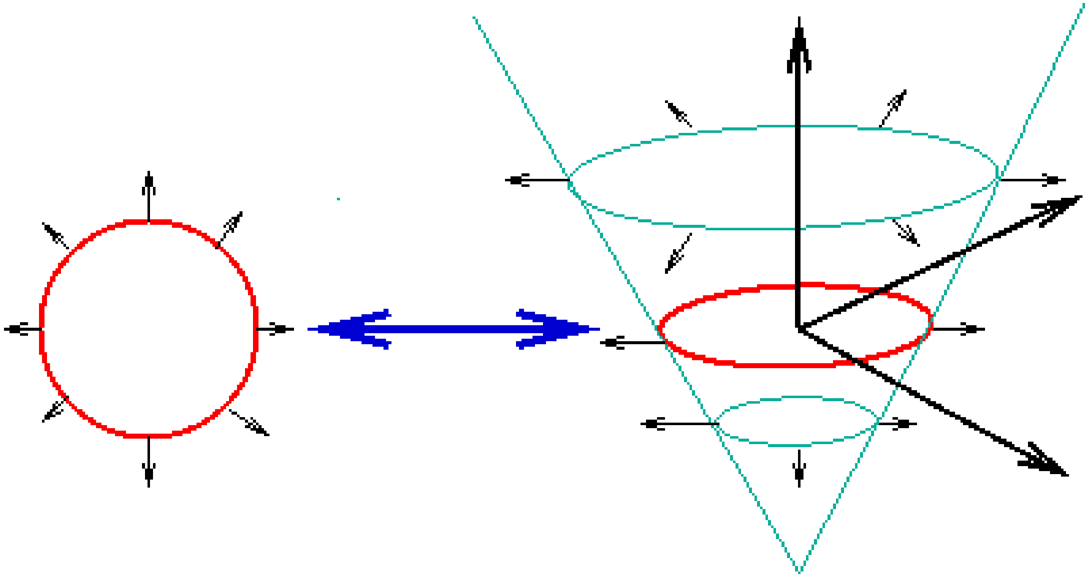
    where the ($\boldsymbol{x,y}$) plane contains the contour, and

    $\boldsymbol{z}$ = signed Euclidean distance transform value (negative means inside closed contour, positive means outside contour)

How to Move the Contour?

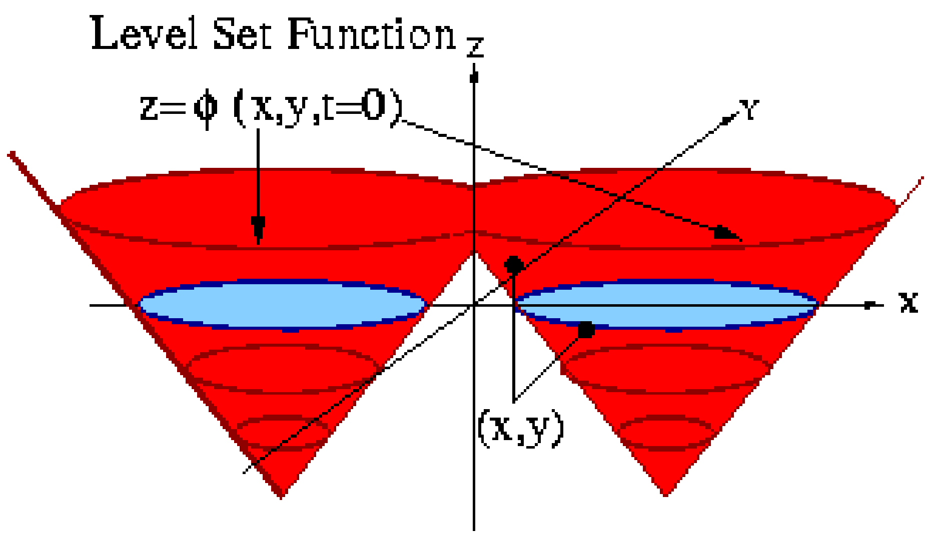
- Move the level set function, $\phi(\mathbf{x,y,t})$, so that it rises, falls, expands, etc.
- Contour = cross section at **z** = 0, i.e.,

$$\{(x,y) \mid \phi(x,y,t) = 0\}$$



Level Set Surface

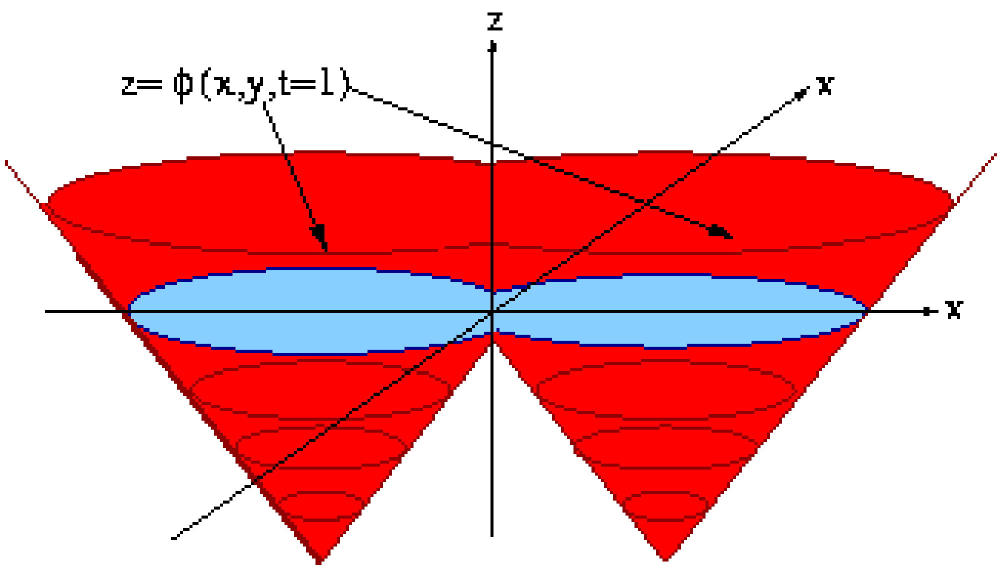
- The zero level set (in blue) at one point in time as a slice of the level set surface (in red)



The Level Set Surface (in red) plots the distance from each point (x, y) to the Interface (in blue)

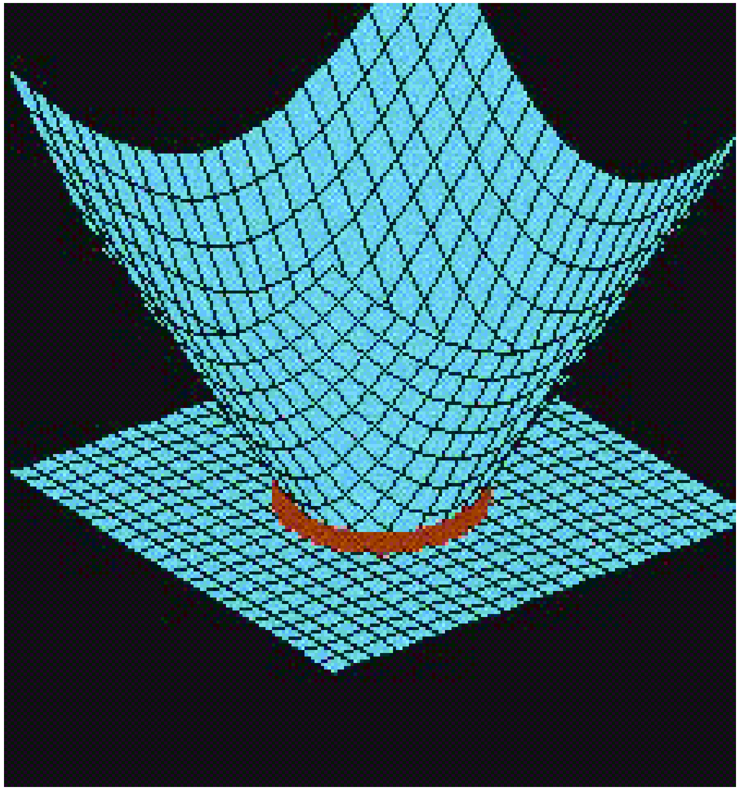
Level Set Surface

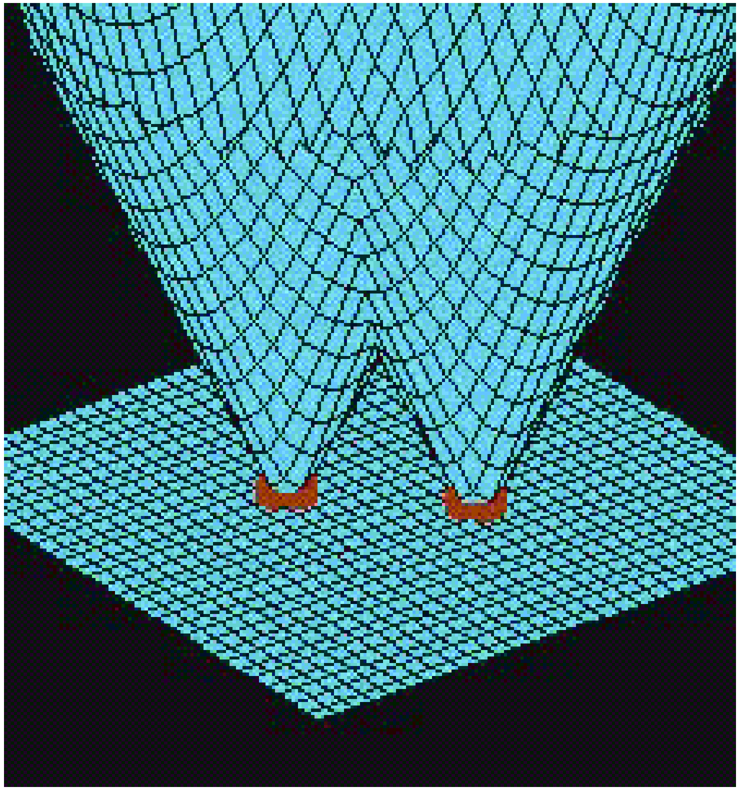
- Later in time the level set surface (red) has moved and the new zero level set (blue) defines the new contour



Later in Time: Red Level Set Surface has moved, yielding new Blue Interface

Level Set Surface





How to Move the Level Set Surface?

1. Define a velocity field, **F**, that specifies how contour points move in time
   – Based on application-specific physics such as time, position, normal, curvature, image gradient magnitude
2. Build an initial value for the level set function, **ϕ*(x,y,t=0)***, based on the initial contour position
3. Adjust ϕ over time; contour at time **t** defined by **ϕ*(x(t), y(t), t) = 0***

$$\frac{\partial \mathbf{\Phi}}{\partial \mathbf{t}} + \vec{\mathbf{F}} \cdot \nabla \mathbf{\Phi} = \mathbf{0}$$

Hamilton-Jacobi equation

$$\frac{\partial \mathbf{\Phi}}{\partial \mathbf{t}} + \mathbf{F}\left(\left(\frac{\partial \mathbf{\Phi}}{\partial \mathbf{x}} \right)^2 + \left(\frac{\partial \mathbf{\Phi}}{\partial \mathbf{y}} \right)^2 \right)^{1/2} = \mathbf{0}$$

Level Set Formulation

- Constraint: level set value of a point on the contour with motion **x**(**t**) must always be 0

  $$\boldsymbol{\phi(x(t), t) = 0}$$

- By the chain rule

  $$\boldsymbol{\phi_t + \nabla\phi(x(t), t) \cdot x'(t) = 0}$$

- Since F supplies the speed in the outward normal direction

  $x'(t) \cdot n = F$, where $n = \nabla\phi \,/\, |\nabla\phi|$

- Hence evolution equation for ϕ is

  $$\phi_t + F|\nabla\phi| = 0$$

Speed Function

$$F(k) = F_0 + F_1(k) = \left(1 - \varepsilon k\right)$$

$$F(k) = k_I\left(x,y\right) * \left(1 - \varepsilon k\right)$$

$$\mathbf{k_I} = \frac{\mathbf{1}}{\mathbf{1} + \left|\nabla \mathbf{G_\sigma} * \mathbf{I}\left(\mathbf{x,y}\right)\right|}$$

$$\mathbf{k_I} = \mathbf{e}^{-\left|\nabla \mathbf{G_\sigma} * \mathbf{I}\left(\mathbf{x,y}\right)\right|}$$

4.2.2 Geometric deformable models-level sets and geodesic active contours

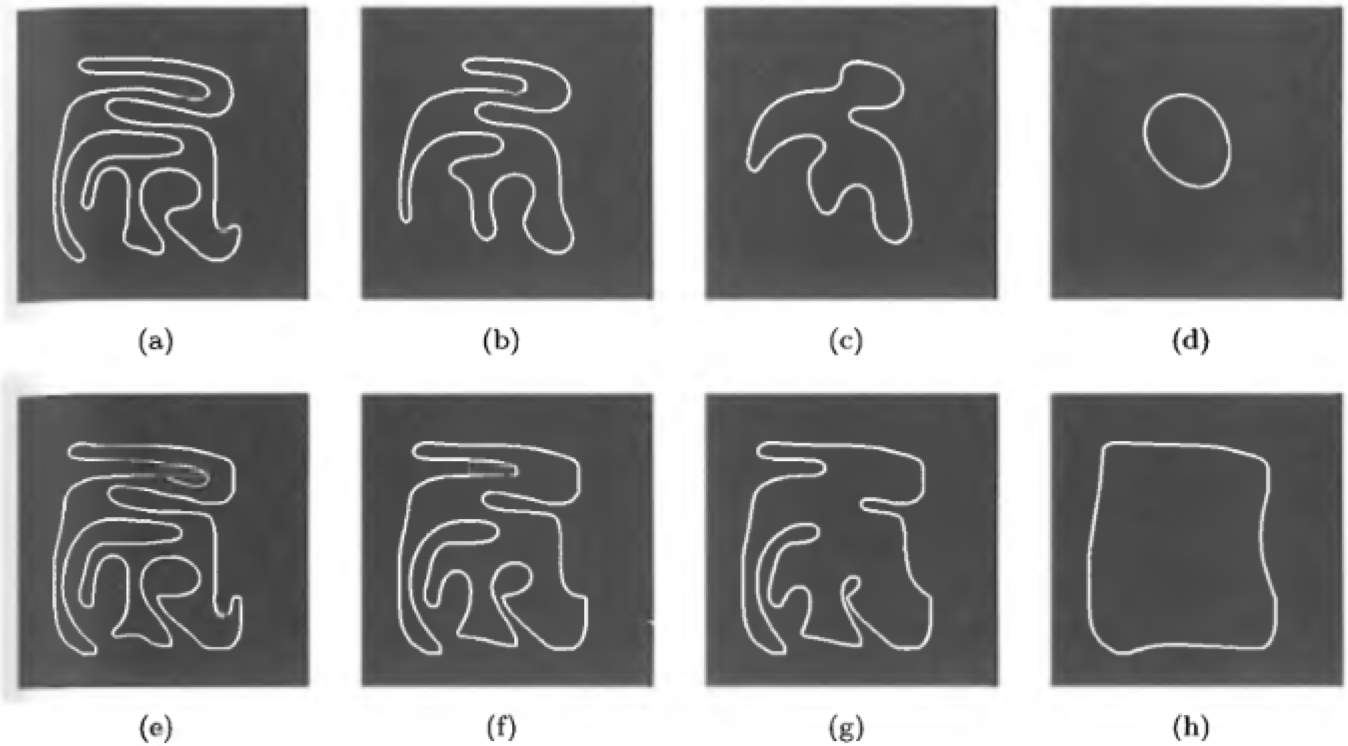


Figure 7.19: Evolution of a closed 2D curve using curvature deformation. (a–d) Using positive curvature, iterations 100, 2,000, 4,000, 17,000. (e–h) Using negative curvature, iterations 100, 2,000, 4,000, 17,000.

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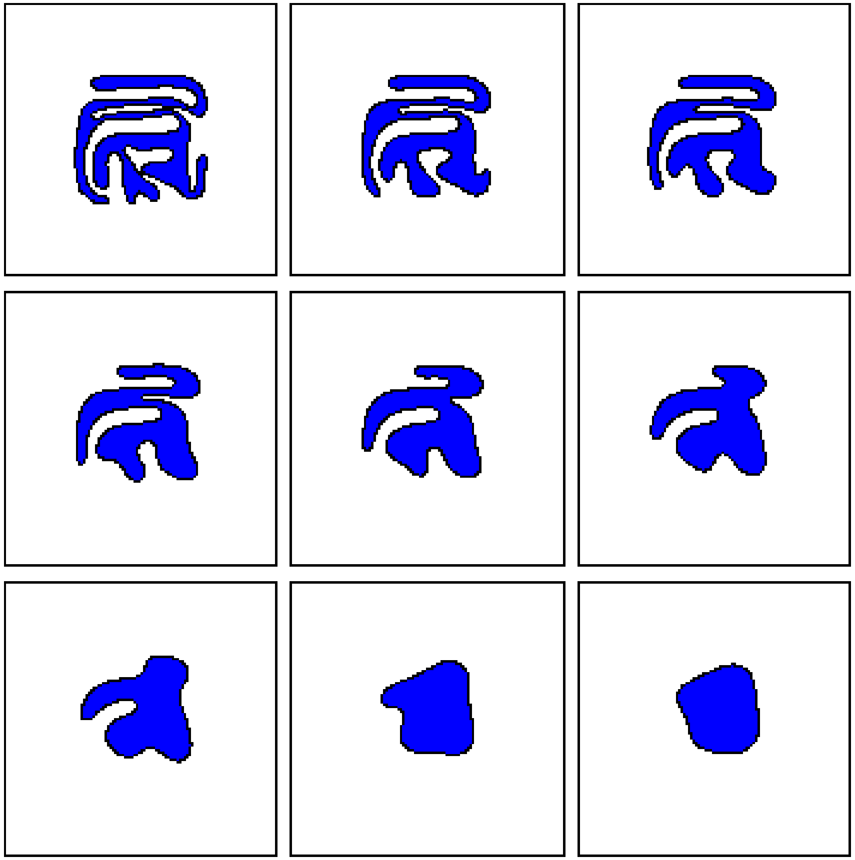
Example: Shape Simplification

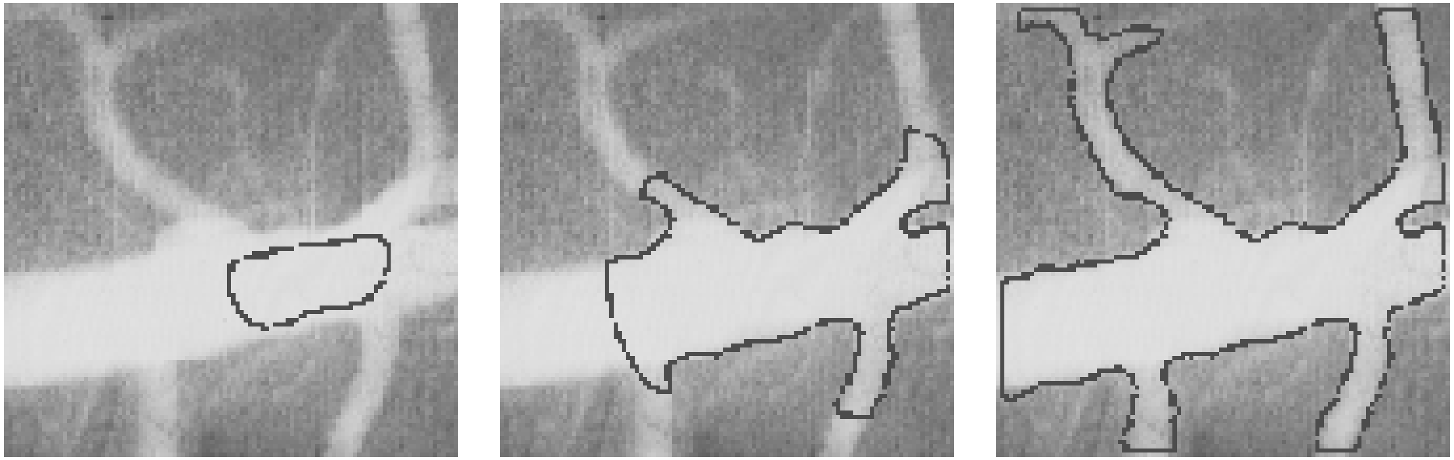
- **F** = 1 – 0.1κ where κ is the curvature at each contour point



Motion under Curvature: Collapse of a Curve to a Single Point.

Example: Segmentation

- Digital Subtraction Angiogram
- **F** based on image gradient and contour curvature



Evolving Front Driven by Function of Image Gradient.

4.2.2 Geometric deformable models-level sets and geodesic active contours

(a) (b) (c)

Figure 7.20: An example of embedding a curve as a level set. (a) A single curve. (b) The level set function where the curve is embedded as the zero level set $\phi[\mathbf{X}(s,t),t] = 0$ (in black). (c) The height map of the level set function with its zero level set depicted in black. *Courtesy of J. Prince, Johns Hopkins University.*

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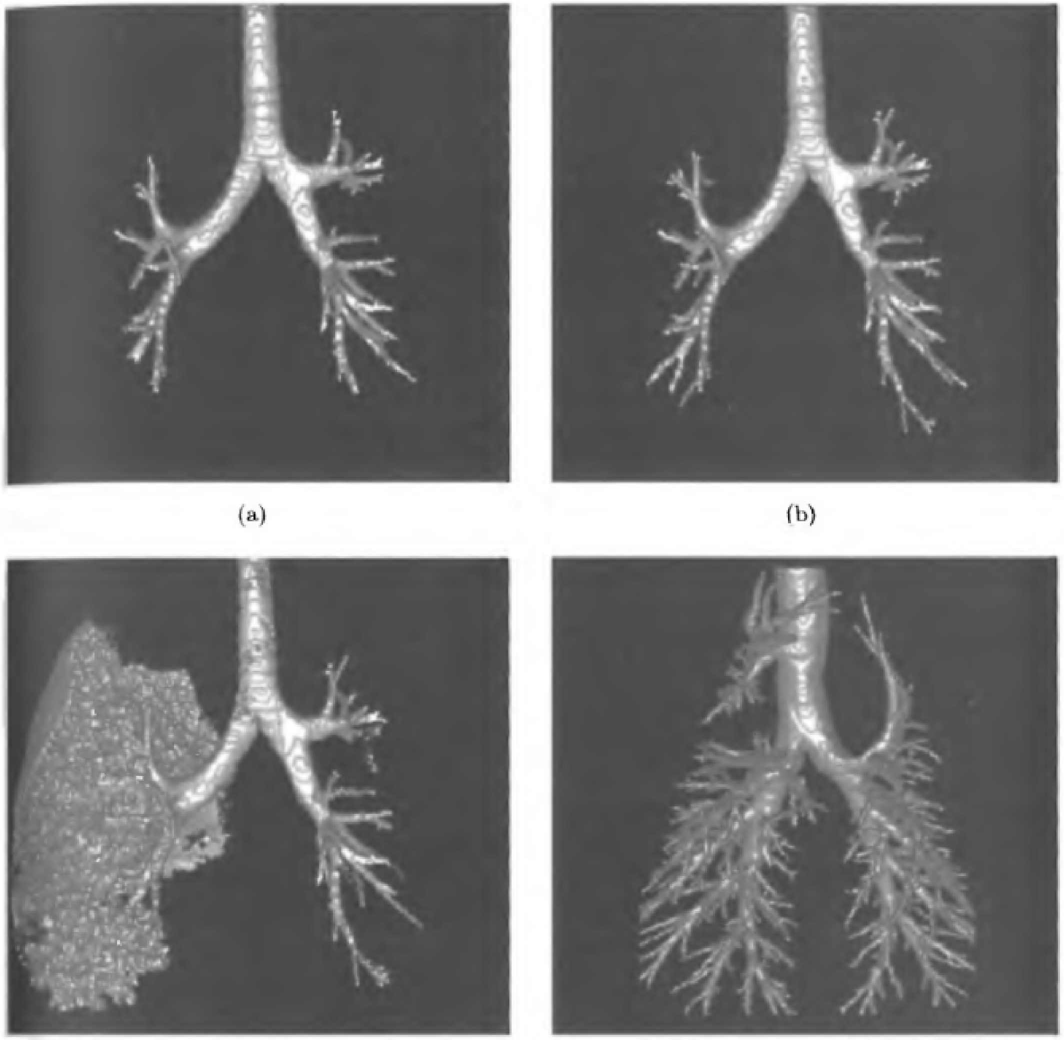
(a) (b) (c) (d)

Figure 7.21: Pulmonary airway tree segmentation using a 3D fast marching level set approach applied to X-ray computed tomography data. The speed function was defined as $V = 1/intensity$; the segmentation front moves faster in dark regions corresponding to air in CT images and slower in bright regions corresponding to airway walls. The stopping criterion uses a combination of gradient threshold value T_g and local image intensity threshold T_i—increasing gradient and image intensity slow down and stop the front propagation. (a) Human airway tree segmentation result employing T_i=6. (b) T_i=11. (c) T_i=13—a segmentation leak occurs. (d) Sheep airway tree segmentation—obtaining a larger number of airways is due to a higher X-ray dose yielding better quality image data.

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4.2.2 Geometric deformable models-level sets and geodesic active contours

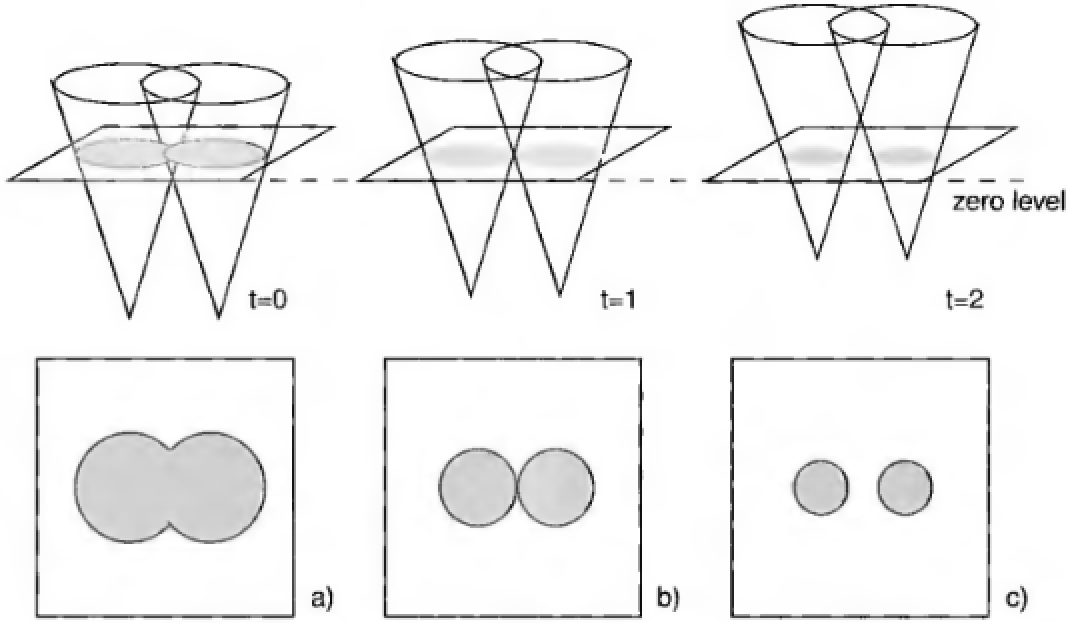

Figure 7.22: Topology change using level sets. As the level set function is updated for $t = 1, 2, 3$, the zero-level set changes topology, eventually providing a 2-object boundary.

4.2.2 Geometric deformable models-level sets and geodesic active contours

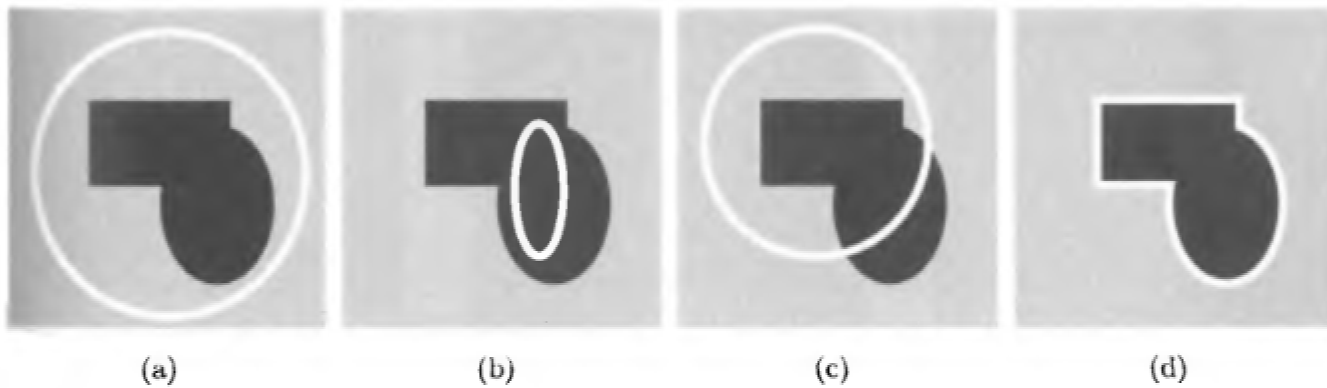

Figure 7.23: Chan–Vese energy functional. (a) $C_1(\phi) > 0$, $C_2(\phi) \approx 0$. (b) $C_1(\phi) \approx 0$, $C_2(\phi) > 0$. (c) $C_1(\phi) > 0$, $C_2(\phi) > 0$. (d) $C_1(\phi) \approx 0$, $C_2(\phi) \approx 0$.

Reference: Tony F. Chan, and Luminita A. Vese. “Active Contours Without Edges”. IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 2, FEBRUARY 2001

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4.2.2 Geometric deformable models-level sets and geodesic active contours

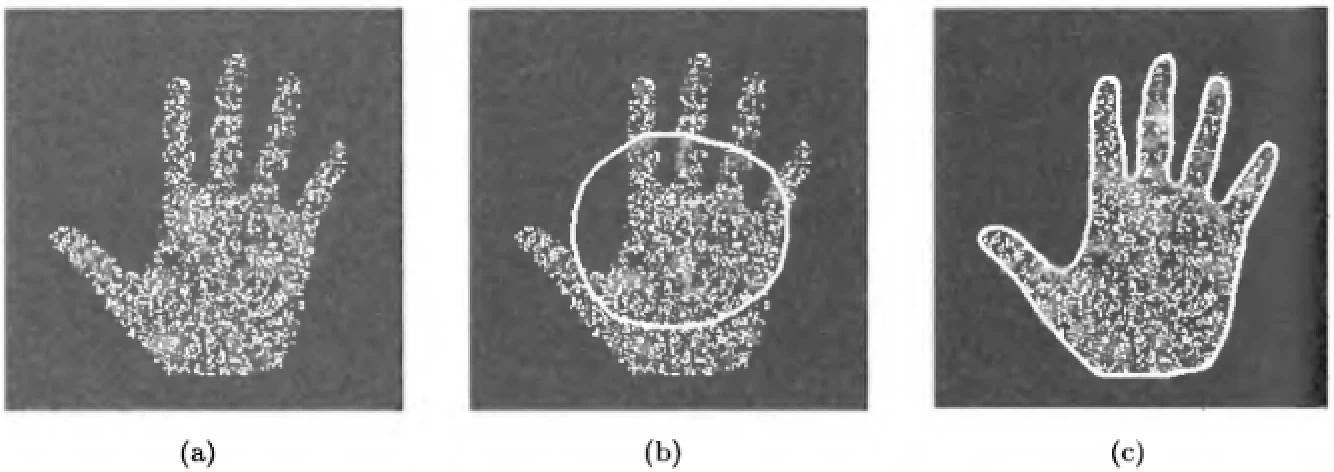

Figure 7.24: Chan-Vese level set segmentation. (a) Original image. (b) Initial contour. (c) Segmentation result.

4.2.2 Geometric deformable models-level sets and geodesic active contours

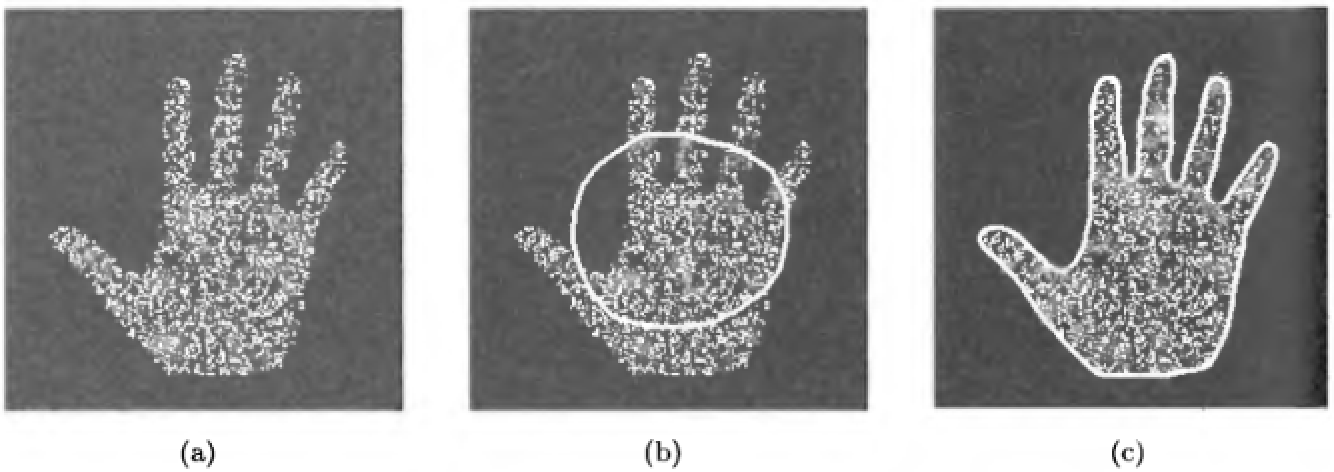

Figure 7.24: Chan-Vese level set segmentation. (a) Original image. (b) Initial contour. (c) Segmentation result.

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4.2.3 Fuzzy Connectivity

- Teach yourself. P283

Algorithm 7.4: Absolute fuzzy connectivity segmentation

1. Define properties of fuzzy adjacency and fuzzy affinity.
2. Determine the affinity values for all pairs of fuzzy adjacent spels.
3. Determine the segmentation seed element c.
4. Determine all possible paths between the seed c and all other image elements d_i in the image domain C (not forming loops) considering the fuzzy adjacency relationship.
5. For each path, determine its strength according as the minimum affinity along the path (equation 7.52).
6. For each image element d_j, determine its fuzzy connectedness $\mu_\psi(c, d_j)$ to the seed point c as the maximum strength of all possible paths $\langle c, \ldots, d_j \rangle$ (equation 7.53) and form an image connectedness map.
7. Threshold the connectedness map with an appropriate threshold t to segment the image into an object containing the seed c and the background.

4.2.3 Fuzzy Connectivity

Algorithm 7.5: Fuzzy object extraction

1. Define a seed-point c in the input image.
2. Form a temporary queue Q and a real-valued array f_c with one element $f_c(d)$ for each spel d.
3. For all spels $d \in C$, initialize array $f_c(d) := 0$ if $d \neq c$; $f_c(d) := 1$ if $d = c$.
4. For all spels $d \in C$ for which fuzzy spel adjacency $\mu_\psi(c, d) > 0$, add spel d to queue Q.
5. While the queue Q is not empty, remove spel d from queue Q and perform the following operations:

   $f_{\max} := \max_{e \in C} \min\big(f_c(e), \psi(d, e)\big)$
   if $f_{\max} > f_c(d)$ then
     $f_c(d) := f_{\max}$
     for all spels g for which $\psi(d, g) > 0$, add g to queue Q
   endif
   endwhile

6. Once the queue Q is empty, the connectedness map (C, f_c) is obtained.

4.2.3 Fuzzy Connectivity

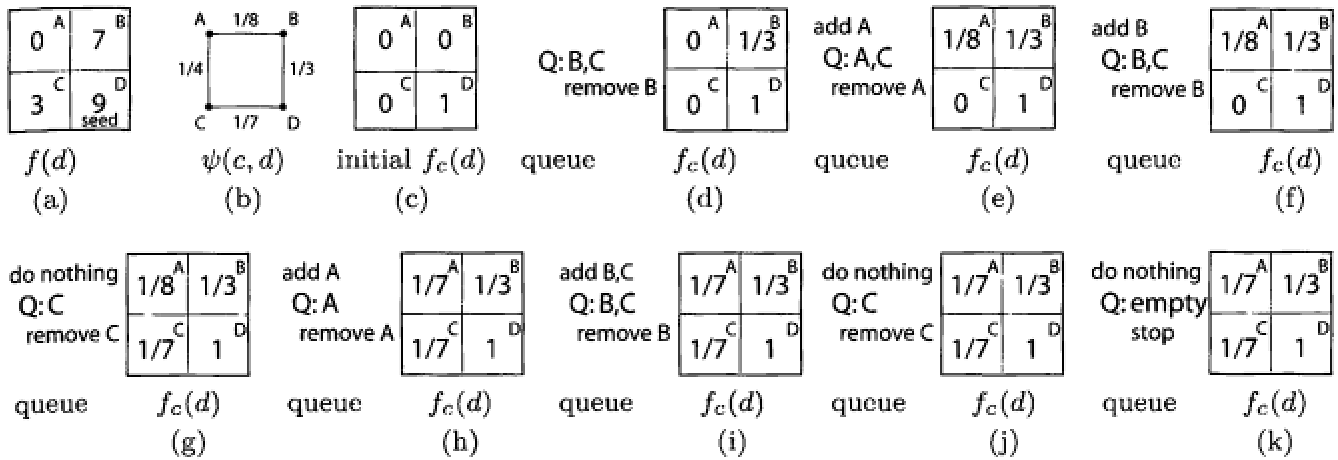


Figure 7.25: Fuzzy object extraction using Algorithm 7.5. (a) Image properties, which can be represented as image intensities. (b) Fuzzy affinity $\psi(c,d)$ calculated according to equation (7.54), $k_2 = 1$. (c) Initialized array $f_c(d)$. (d) Initial queue Q, temporary values of $f_c(d)$ after removal of spel B from queue Q. (e-j) Intermediate algorithm steps. (k) The queue Q is empty, stop. Values of array $f_c(d)$ represent the connectedness map.

4.2.3 Fuzzy Connectivity

Algorithm 7.6: Fuzzy object extraction with preset connectedness

1. Define a seed-point c in the input image.
2. Form a temporary queue Q and a real-valued array f_c with one element $f_c(d)$ for each spel d.
3. For all spels $d \in C$, initialize array $f_c(d) := 0$ if $d \neq c$; $f_c(d) := 1$ if $d = c$.
4. For all spels $d \in C$ for which fuzzy spel adjacency $\mu_\psi(c, d) > t$, add spel d to queue Q.
5. While the queue Q is not empty, remove spel d from queue Q and perform the following operations:

   $f_{\max} := \max_{e \in C} \min \big(f_c(e), \psi(d, e) \big)$
   if $f_{\max} > f_c(d)$ then
   $\quad f_c(d) := f_{\max}$
   $\quad$for all spels g for which $\psi(d, g) > 0$, add g to queue Q
   endif
   endwhile
6. Once the queue Q is empty, the connectedness map (C, f_c) is obtained.

4.2.3 Fuzzy Connectivity

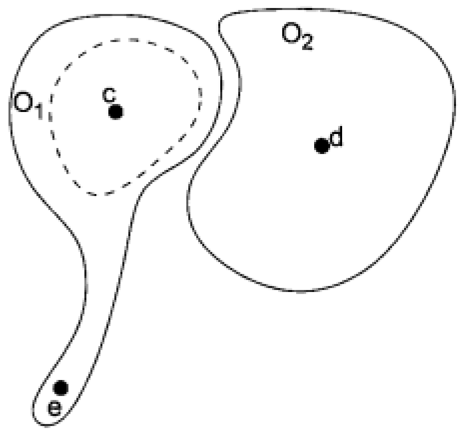
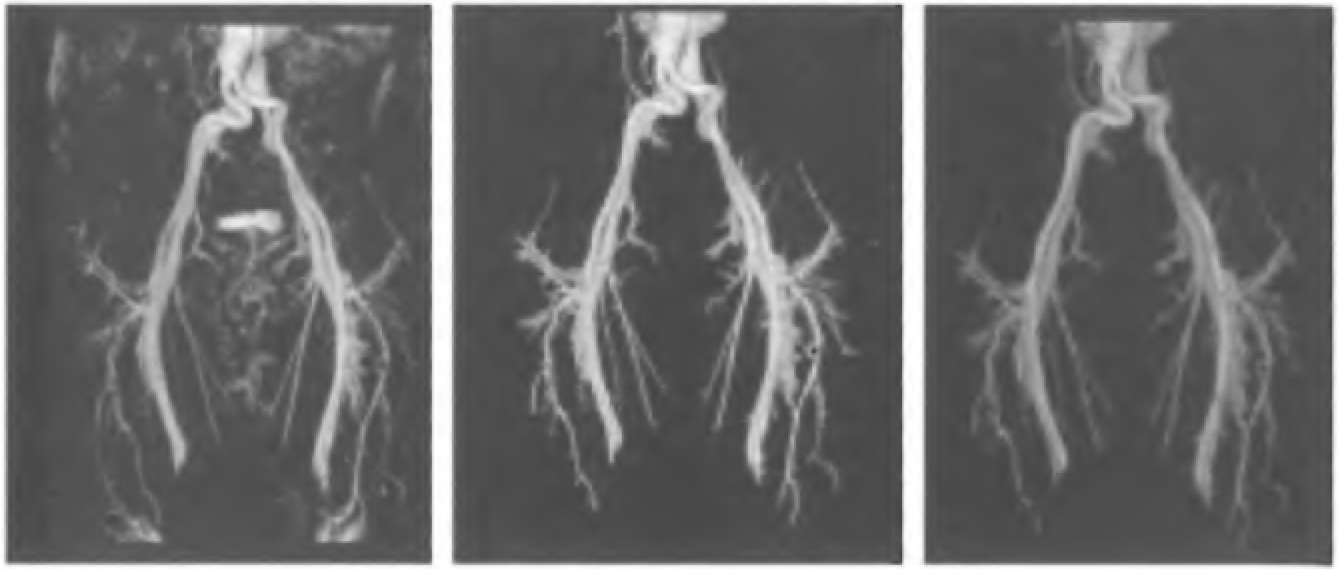


Figure 7.26: Segmentation task that can be solved by iterative fuzzy connectivity.

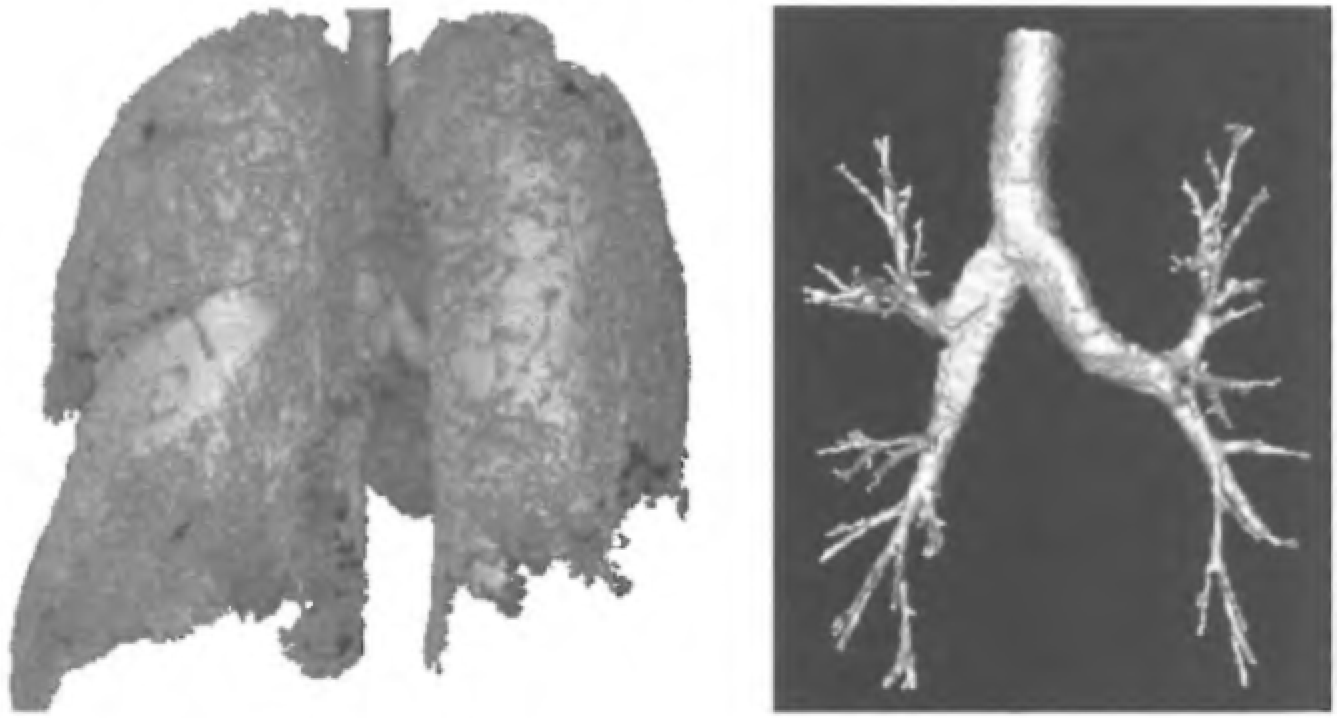
4.2.3 Fuzzy Connectivity



(a) (b) (c)

Figure 7.27: Segmentation and separation of vascular trees using fuzzy connectivity segmentation. (a) Maximum intensity projection image of the original magnetic resonance angiography data used for artery-vein segmentation in lower extremities. (b) Segmentation of the entire vessel tree using absolute fuzzy connectivity. (c) Artery–vein separation using relative fuzzy connectivity. *Courtesy of J. K. Udupa, University of Pennsylvania. A color version of this figure may be seen in the color inset—Plate 11.*

4.2.3 Fuzzy Connectivity



(a) (b)

Figure 7.28: Segmentation result using multi-seeded fuzzy connectivity approach. (a) Region growing segmentation results in a severe segmentation leak. (Emphysema patient, segmented with standard 3D region growing algorithm–the leak was unavoidable). (b) Multi-seeded fuzzy connectivity succeeded with the image segmentation using a standard setting of the method.

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Knowledge points

- Active contour models—snakes

  — A snake is an energy minimizing spline—the snake's energy depends on its shape and location within the image. Local minima of this energy then correspond to desired image properties.

  — Snakes are parametric deformable models.

  — The energy functional which is minimized is a weighted combination of internal and external forces.

  — Gradient vector flow field increases the effective area of snake attraction decreasing the snake's sensitivity to initialization and allowing to segment concave boundaries.

- Geometric deformable models

  — Geometric deformable models represent the developing surfaces by partial differential equations.

  — The movements of the propagating fronts are described by speed functions.

  — The evolving curves and/or surfaces are represented as level sets of higher dimensional functions yielding seamless treatment of topologic changes.

Project 1

- See "Project1 Segmentation of Objects"