Chapter 2 Digital Image Fundamentals

Why we need to know “digital image fundamentals” ?

Video of “cow tracking ”

Image capture

Preprocessing

Segmentation

Model fitting

Model prediction

Conclusion (understanding)

Fig. Flowchart of “cow tracking”

A Video = … + … Many frames

track.mpg

A frame = an image!

Image processing + Image understanding !

Outline

- 2.1 The image, its representations and properties

- 2.2 The image, its mathematical and physical background !

In greater theoretical depth!

Think about “images are intrinsically signals”?

2.1 The image, its representations and properties

- 2.1.1 Image representations, a few concepts
- 2.1.2 Image digitization
- 2.1.3 Digital image properties

2.1.1 Image representations, a few concepts

- -- Image functions
- -- Images as a stochastic process

2.1.1 -Image functions

$f(x, y, t)$

- The **image** can be modeled by a continuous function of two or three variables;

- arguments are co-ordinates *x, y* in a plane, while if images change in time a third variable *t* might be added.

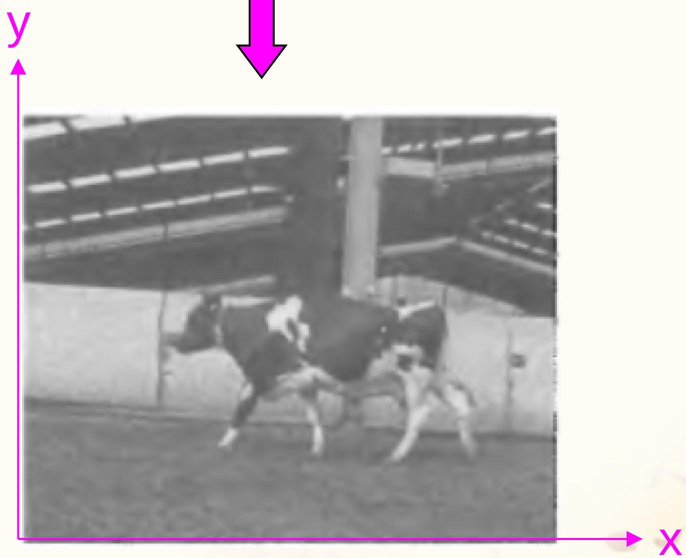
2.1.1 -Image functions

$f(x, y, t)$

A Video = ... + ... Many frames

track.mpg

while if images change in time a third variable *t* might be added.



A frame from a video

$f(x, y)$

arguments are co-ordinates *x*, *y* in a plane

2.1.1 -Image functions

$f(x, y, t)$

- The **image** can be modeled by a continuous function of two or three variables;
- arguments are co-ordinates x, y in a plane, while if images change in time a third variable t might be added.
- The image function values correspond to the brightness at image points.
- The function value can express other physical quantities as well (temperature, pressure distribution, distance from the observer, etc.).

2.1.1 -Image functions

$$f(x, y, t)$$

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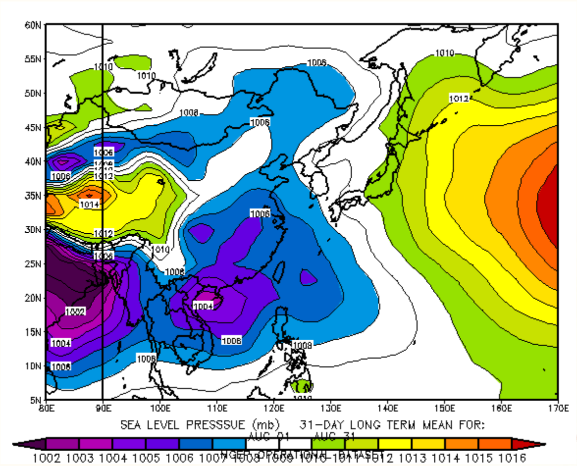
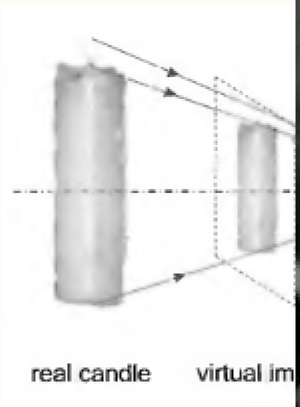


2.1.1 -Image functions

- The image on the human eye retina or on a TV camera sensor is intrinsically 2D. We shall call such a 2D image bearing information about brightness points an **intensity image**.
- The real world v cally 3D.
- The 2D intensit spective projection of the



2.1.1 -Image functions

- The First problem of computer vision:
- When 3D objects are mapped into the camera plane by perspective projection a lot of information disappears as such a transformation is not one-to-one.
- Recognizing or reconstructing objects in a 3D scene from one image is an ill-posed problem.
- Recovering information lost by perspective projection is only one, mainly geometric, problem of computer vision

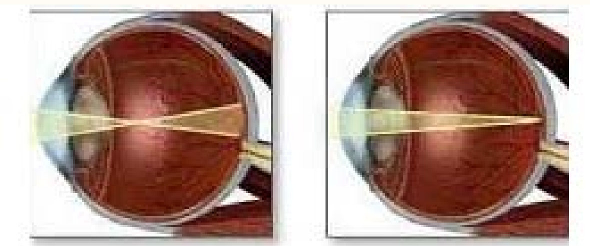
2.1.1 -Image functions

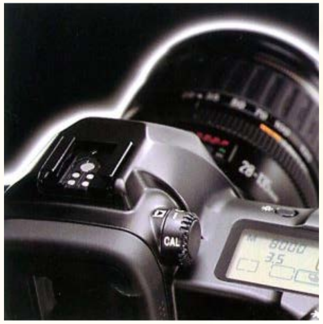
- The second problem of computer vision is how to understand image brightness.
- The only information available in an intensity image is brightness of the appropriate pixel, which is dependent on a number of independent factors such as

  --object surface reflectance properties (given by the surface material, microstructure and marking),

  --illumination properties,

  --and object surface orientation with respect to a viewer and light source.





2.1.1 -Image functions

- Image processing in 2D

  Some scientific and technical disciplines work with 2D images directly;

  Many basic and useful methods used in digital image analysis do not depend on whether the object was originally 2D or 3D.

- The problem of 3D understanding will be addressed explicitly later.

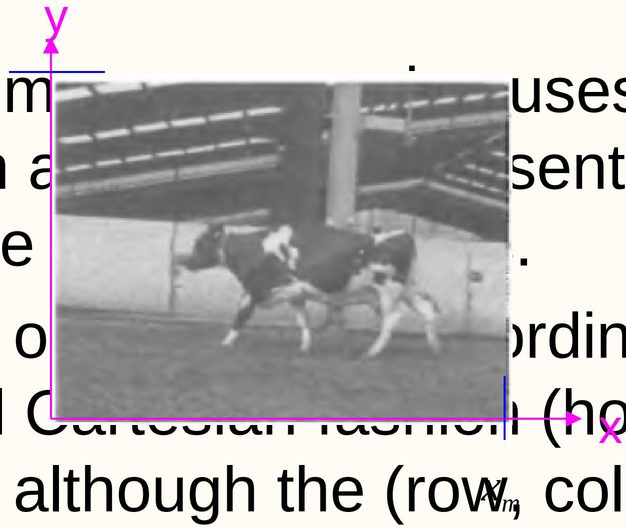
2.1.1 -Image functions

- Image processing often deals with static images, in which time *t* is constant.
- A monochromatic static image is represented by a continuous image function f(x,y) whose arguments are two co-ordinates in the plane.

$$f(x, y)$$

2.1.1 -Image functions

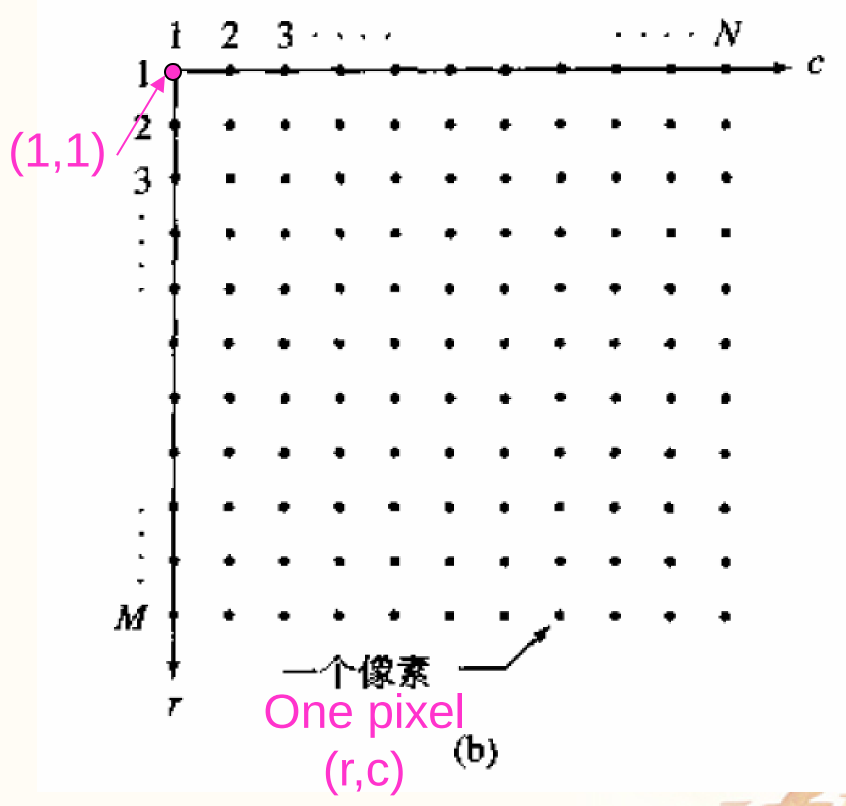
$$R = \{(x, y),\ 1 \le x \le x_m,\ 1 \le y \le y_n\}\ , \qquad (2.2)$$



- Computerized im uses digital image functions which a sented by matrices, so co-ordinates are .
- The customary o rdinates in an image is in the normal (horizontal x axis, vertical y axis), although the (row, column) orientation used in matrices is also quite often used in digital image processing.

Questions and Practices

- 1. Coordinate conventions used in Matlab?



2.1.1 -Image functions

$$R = \{(x,y),\ 1 \le x \le x_m,\ 1 \le y \le y_n\}\ , \qquad (2.2)$$

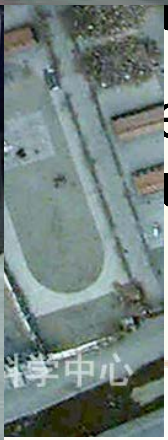
- The range of image function values is also limited; by convention, in monochromatic images the lowest value corresponds to black and the highest to white.
- Brightness values bounded by these limits are **gray levels**.

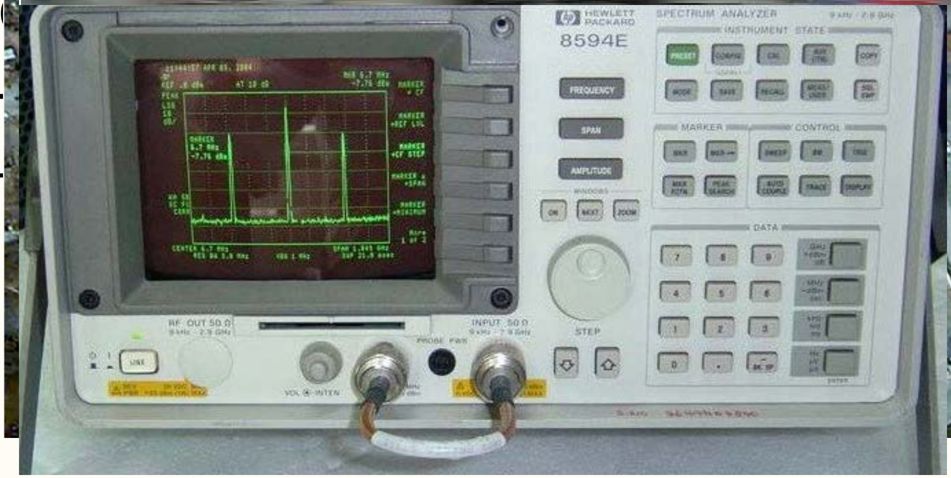
White, 255

black, 0

Do you remember "practice 1"?

2.1.1 -Image fun



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2.1.1 -Images as a stochastic process

- Images f(x,y) can be treated as deterministic functions or as realizations of stochastic processes.
- Mathematical tools used in image description have roots in linear system theory, integral transformations, discrete mathematics and the theory of stochastic processes.
- It will be discussed later (in section 2.2).

2.1.2 Image digitization

- -- Sampling
- -- Quantization

Think about “images are intrinsically signals”?

2.1.2 Image digitization

- Image captured by a sensor is expressed as a continuous function f(x,y) of two co-ordinates in the plane.
- Image digitization means that the function f(x,y) is **sampled** into a matrix with M rows and N columns.

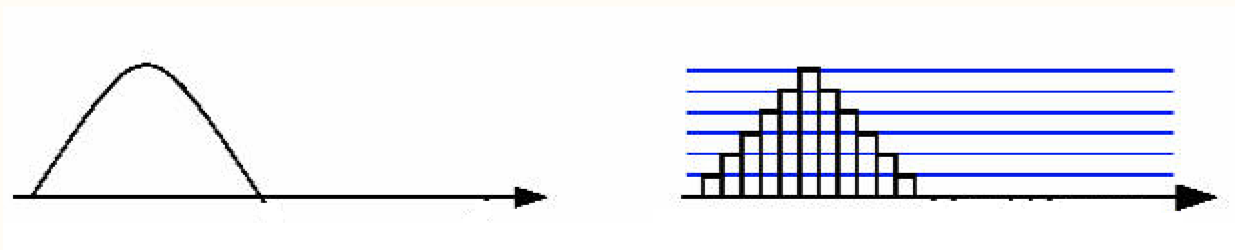
$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} \mathtt{f(1,1)} & \mathtt{f(1,2)} & \cdots & \mathtt{f(1,N)} \\ \mathtt{f(2,1)} & \mathtt{f(2,2)} & \cdots & \mathtt{f(2,N)} \\ \vdots & \vdots & & \vdots \\ \mathtt{f(M,1)} & \mathtt{f(M,2)} & \cdots & \mathtt{f(M,N)} \end{bmatrix}$$

In Matlab!

2.1.2 Image digitization

- The continuous range of the image function f(x,y) is split into K intervals.



Do you remember “discussion of Practice 1”?

K = ?

2.1.2 Image digitization

- The finer the sampling (i.e., the larger M and N) and quantization (the larger K) the better the approximation of the continuous image function f(x,y).

2.1.2 Image digitization

- Two questions should be answered in connection with image function sampling:
- First, the sampling period should be determined -- the distance between two neighboring sampling points in the image
- Second, the geometric arrangement of sampling points (sampling grid) should be set.
- It will be discussed later. (in section 2.2 “The image, its mathematical and physical background”)

2.1.2 -Sampling

- A continuous image is digitized at sampling points.
- These sampling points are ordered in the plane and their geometric relation is called the **grid**.
- Grids used in practice are mainly square or hexagonal.

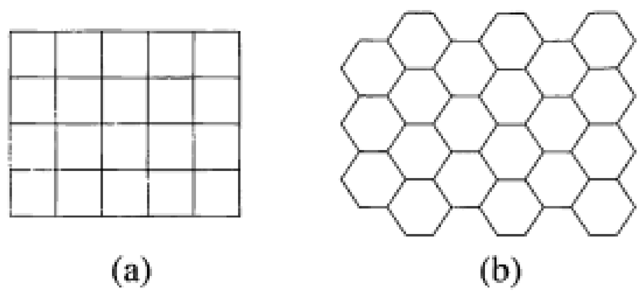


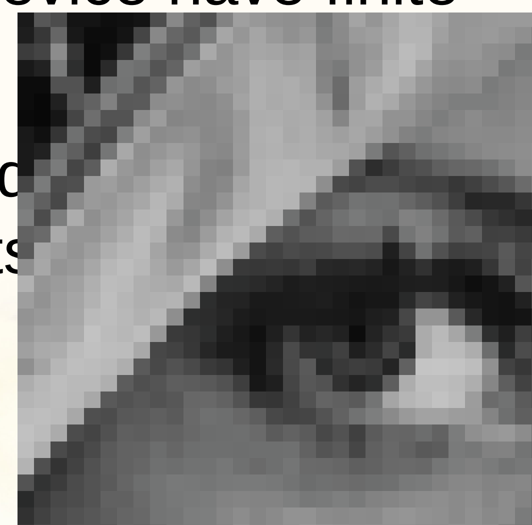
Figure 2.2: (a) Square grid. (b) Hexagonal grid.

2.1.2 -Sampling

- A continuous image function f(x,y) can be sampled using a discrete grid of sampling points in the plane.
- The image is sampled at points x = j (Delta_x), y = k (Delta_y)
- Two neighboring sampling points are separated by distance Delta_x along the x axis and Delta_y along the y axis.
- Distances Delta_x and Delta_y are called the **sampling interval** and the matrix of samples constitutes the discrete image.

2.1.2 -Sampling

- One infinitely small sampling point in the grid corresponds to one picture element (**pixel**) in the digital image.
- The set of pixels together covers the entire image
- Pixels captured by a real digitization device have finite



2.1.2 -quantization

- The image **quantization** assigns to each continuous sample an integer value.
- The continuous range of the image function f(x,y) is split into *K* intervals.

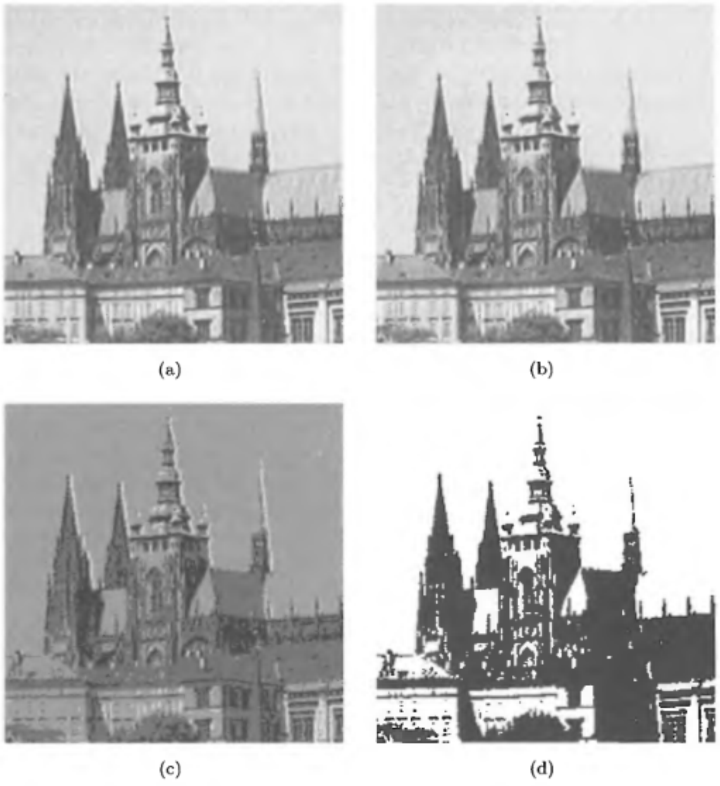


Figure 2.3: Brightness levels. (a) 64. (b) 16. (c) 4. (d) 2.

Review!
Discussion of “practice 1”

Practices

- 3. The maximum and minimum of the intensity value of the image “Lena.bmp” is?

Review!

2.1.1 -Image functions

$$R = \{(x, y),\ 1 \le x \le x_m,\ 1 \le y \le y_n\}\ , \tag{2.2}$$

- The range of image function values is also limited; by convention, in monochromatic images the lowest value corresponds to black and the highest to white.
- Brightness values bounded by these limits are **gray levels**.

White, 255

black, 0

2.1.2 -Quantization

- A magnitude of the sampled image is expressed as a digital value in image processing.
- The transition between continuous values of the image function (brightness) and its digital equivalent is called **quantization**.
- The number of quantization levels should be high enough for human perception of fine shading details in the image.

2.1.2 -Quantization

- Most digital image processing devices use quantization into *k* equal intervals.
- If b bits are used ... the number of brightness levels is $k=2^b$.
- Eight bits per pixel are commonly used, specialized measuring devices use twelve and more bits per pixel.

White, 255

black, 0

12bits
Or even
16 bits

2.1.3 Digital image properties

- --Metric and topological properties of digital images
- --Histograms
- --Entropy
- --Noise in images

2.1.3 -Metric properties of digital images

- **Distance** is an important example
- The distance between two pixels in a digital image is a significant quantitative measure.

2.1.3 -Metric properites of digital images

- The distance between points with co-ordinates (i,j) and (h,k) may be defined in several different ways;
- the **Euclidean** distance is defined by Eq.

$$D_E((i,j),(h,k)) = \sqrt{(i-h)^2 + (j-k)^2}$$

- **city block** distance ...

$$D_4((i,j),(h,k)) = | i-h | + | j-k |$$

- **chessboard** distance

$$D_8((i,j),(h,k)) = max\{| i-h |, | j-k |\}$$

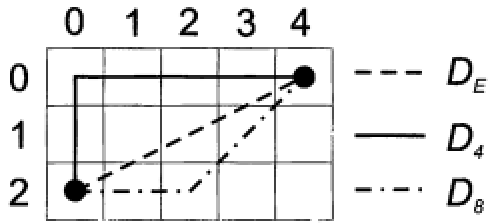
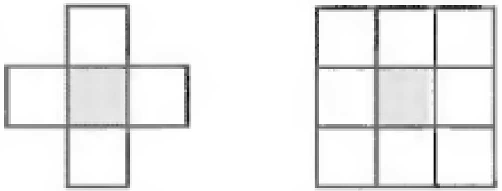


Figure 2.4: Distance metrics D_e, D_4, and D_8.

2.1.3 -Metric properites of digital images

- **Pixel adjacency** is another important concept in digital images.
- **4-neighborhood**
- **8-neighborhood** (Fig. 2.5)



(a) 4-neighborhood (b) 8-neighborhood

Figure 2.5: Neighborhood of the representative pixel (gray filled pixel in the middle).

2.1.3 -Metric properites of digital images

- It will become necessary to consider important sets consisting of several adjacent pixels -- **regions**.
- Region is a contiguous set.
- Figure 2.6 illustrates a binary image decomposed by the relation 'contiguous' into three regions.

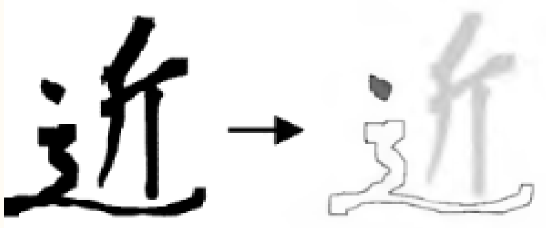


Figure 2.6: The relation 'to be contiguous' decomposes an image into individual regions. The Chinese "Jin" character meaning 'near from here' decomposes into 3 regions.

2.1.3 -Metric properites of digital images

- **Border** R is the set of pixels within the region that have one or more neighbors outside R ... **inner** borders, **outer** borders exist.
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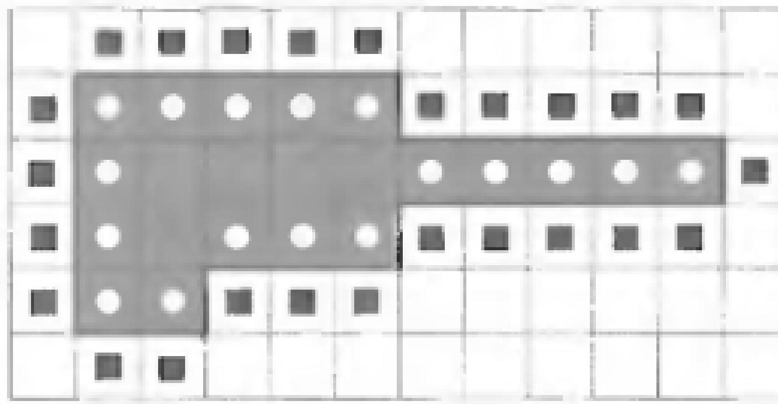
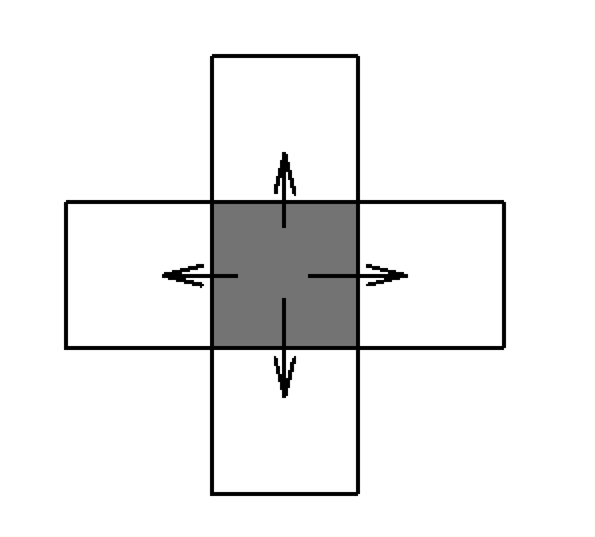


Figure 2.13: Inner borders of a region shown as white circles and outer borders shown as black squares. 4-neighborhood was considered.

2.1.3 -Metric properites of digital images

- **Crack edges** ... four crack edges are attached to each pixel, which are defined by its relation to its 4-neighbors. The direction of the crack edge is that of increasing brightness, and is a multiple of 90 degrees, while its magnitude is the absolute difference between the brightness of the relevant pair of pixels. (Fig. 2.12)



2.1.3 -Topological properties of digital images

- Topological properties of images are invariant to **rubber sheet transformations**.
- Stretching does not change contiguity of the object parts and does not change the number of holes in regions.

2.1.3 -Topological properties of digital images

- Convex hull is used to describe topological properties of objects.
- The convex hull is the smallest region which contains the object, such that any two points of the region can be connected by a straight line, all points of which belong to the region.

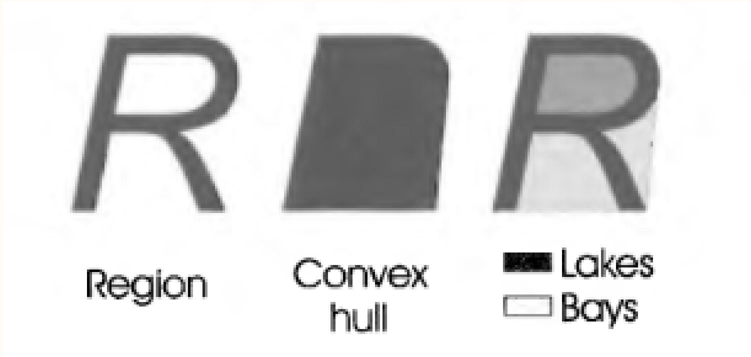


Figure 2.15: Description using topological components: An 'R' object, its convex hull, and the associated lakes and bays.

Paper "Segmentation of pulmonary nodules with morphological approaches and convexity models"

2.1.3 -Histograms

- **Brightness** **histogram** provides the frequency of the brightness value z in the image.

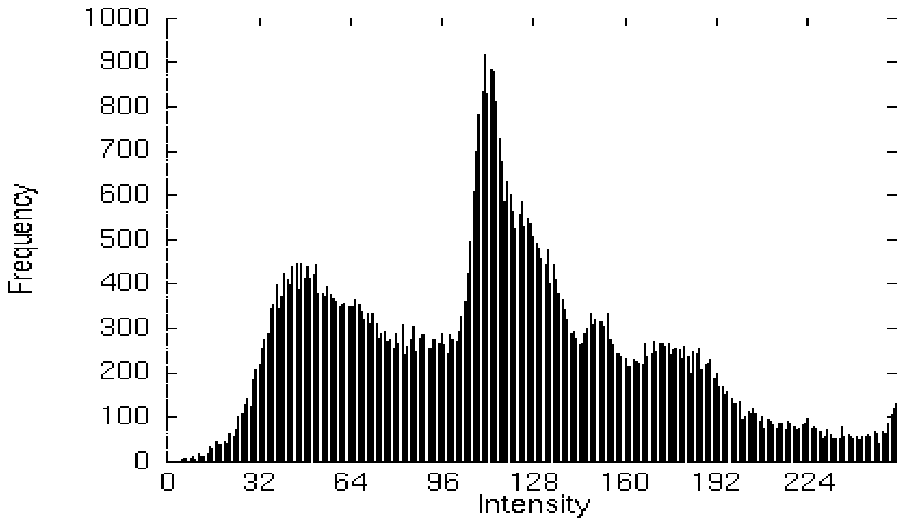


Figure 2.11 *A brightness histogram.*

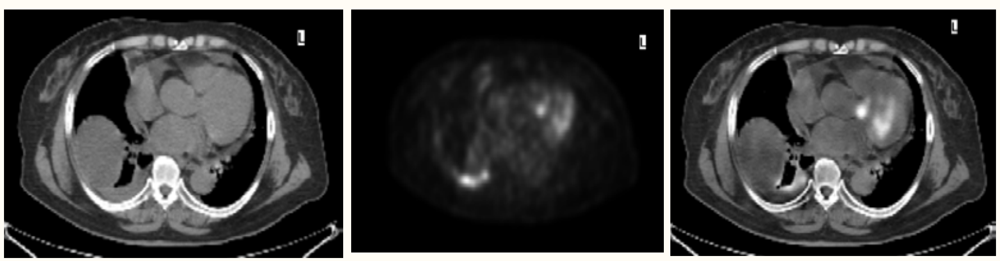
- Algorithm 2.1

How to draw Histograms?

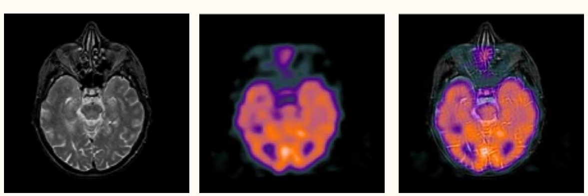
1. Assign zero values to all elements of the array h.
2. For all pixels (x, y) of the image f, increment h(f(x, y)) by one.

2.1.3 -Entropy

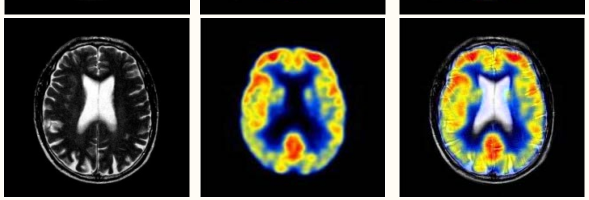
- If a probability density p is known then image information content can be estimated regardless of its interpretation using **entropy** H.
- The information-theoretic formulation of entropy comes from Shannon [Shannon, 1948] and is often called **information entropy**.



PET-CT image



MRI-SPECT image



MRI-PET image

It is often used in image registration and fusion.

2.1.3 -Noise in images

- Images are often degraded by random noise.
- Noise can occur during image capture, transmission or processing, and may be dependent on or independent of image content.

2.1.3 -Noise in images

- Noise is usually described by its probabilistic characteristics.
- **White noise -** constant power spectrum (its intensity does not decrease with increasing frequency); very crude approximation of image noise
- **Gaussian noise** is a very good approximation of noise that occurs in many practical cases
- probability density of the random variable is given by the Gaussian curve;
- 1D Gaussian noise - is the mean and is the standard deviation of the random variable.

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} ,$$

2.1.3 -Noise in images

- The signal-to-noise ratio is

$$\mathrm{SNR} = \frac{F}{E}$$

the total square value of the noise contribution:

$$E = \sum_{(x,y)} \nu^2(x,y)$$

the total square value of the observed signal

$$F = \sum_{(x,y)} f^2(x,y)\,.$$

2.1.3 -Noise in images

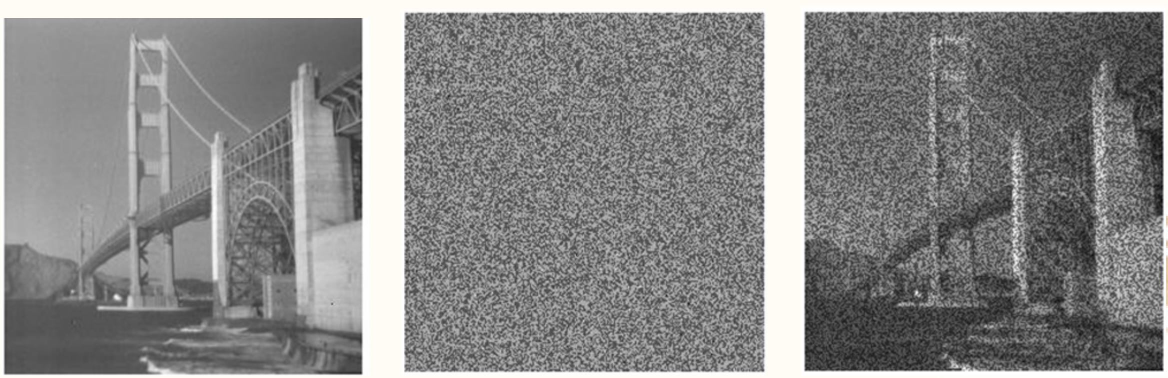
- Noise may be
- **additive**, noise and image signal g are independent

$$f(x,y) = g(x,y) + \nu(x,y)\,,$$

- **multiplicative**, noise is a function of signal magnitude

$$f = g + \nu g = g(1+\nu) \approx g\nu$$

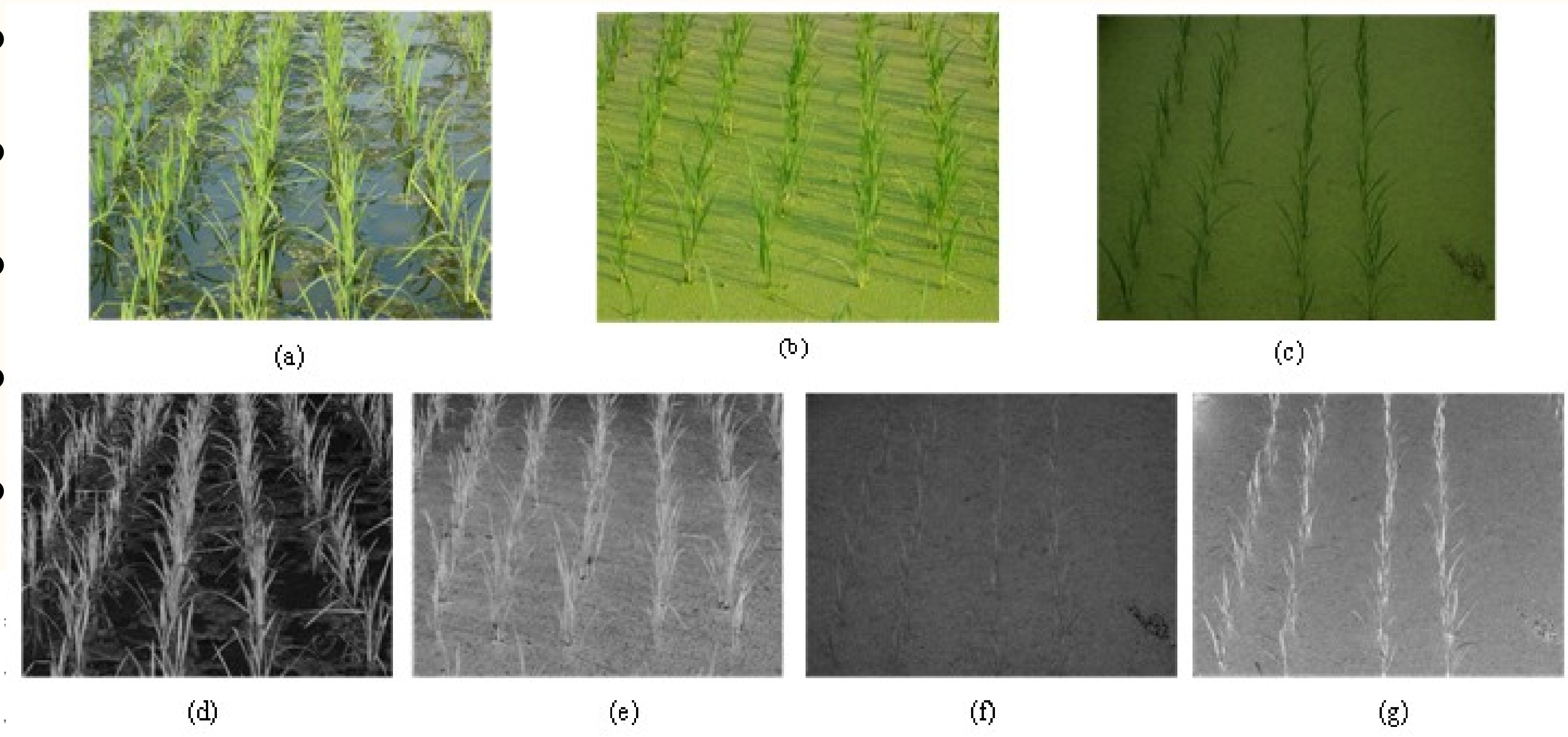
- **impulse** noise (saturated = salt and pepper noise)



2.1.4 Color images

- Teach yourself. P31-41.

In south China the paddy fields often have duckweeds and cyanobacterias



Device oriented, Uniform spaces

User oriented

Fig Segmentation of rice seedlings

2.2 The image, its mathematical and physical background

- 2.2.1 Overview
- 2.2.2 Linear integral transforms
- 2.2.3 Images as stochastic processes
- 2.2.4 Image formation physics

In greater theoretical depth!

2.2.1 Overview

- --Linearity
- --The Dirac distribution and convolution

Review!

2.2.1 -Linearity

- The notion of linearity will occur frequently in this book: this relates to vector (linear) spaces where commonly matrix algebra is used.
- Linearity also concerns more general elements of vector spaces, for instance, functions.
- The linear combination is a key concept in linear mathematics, permitting the expression of a new element of a vector space as a sum of known elements multiplied by coefficients (scalars, usually real numbers).
- A general linear combination of two vectors x. y can be written as ax + by, where a, b are scalars.

Review!

2.2.1 -Linearity

- Consider a mapping L between two linear spaces.
- It is called additive if

$$L(x+y) = Lx + Ly$$

- homogeneous if

$$L(ax) = aLx$$

for any scalar a.

Review!

2.2.1 -Linearity

- From a practical point of view, this means that the sum of inputs (respectively, multiple) results in the sum of the respective outputs (respectively, multiple).
- This property is also called a superposition principle. We call the mapping *L linear* if it is additive and homogeneous (i.e., satisfies the superposition principle).

Review!

2.2.1 -Linearity

- A linear mapping satisfies

$$L(ax + by) = aLx + bLy$$

for all vectors x, y and scalars a, b, i.e., it preserves linear combinations.

- The concept has been addressed in linearity control theory classes.

2.2.1 -The Dirac distribution and convolution

- The knowledge has been addressed in courses “Digital signal processing” or “Signals and systems”.
- These are fundamental motivators for appreciating the use of linear mathematical theory in considering image functions.

2.2.1 -The Dirac distribution and convolution

- An ideal impulse is an important input signal: the ideal impulse in the image plane is defined using the Dirac distribution $\delta(x, y)$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x, y)\, \mathrm{d}x\, \mathrm{d}y = 1\,, \tag{3.1}$$

and $\delta(x, y) = 0$ for all $(x, y) \neq 0$.

2.2.1 -The Dirac distribution and convolution

- The following equation is called the 'sifting property' of the Dirac distribution; it provides the value of the function f(x,y) at the point (λ, μ)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x-\lambda, y-\mu)\,\mathrm{d}x\,\mathrm{d}y = f(\lambda,\mu)\,. \tag{3.2}$$

- The 'sifting equation' (3.2) can be used to describe the sampling process of a continuous image function f(x,y).

2.2.1 -The Dirac distribution and convolution

- We may express the image function as a linear combination of Dirac pulses located at the points a, b that cover the whole image plane; samples are weighted by the image function f(x,y).

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\,\delta(a-x,b-y)\,\mathrm{d}a\,\mathrm{d}b = f(x,y)\,. \qquad (3.3)$$

2.2.1 -The Dirac distribution and convolution

- Convolution is an important operation in the linear approach to image analysis.
- The convolution is an integral which expresses the amount of overlap of one function *f(t)* as it is shifted over another function *h(t)*.

$$(f * h)(t) \equiv \int_0^t f(t)\, h(t-\tau)\, \mathrm{d}\tau \,. \tag{3.4}$$

$$(f * h)(t) \equiv \int_{-\infty}^{\infty} f(t)\, h(t-\tau)\, \mathrm{d}\tau = \int_{-\infty}^{\infty} f(t-\tau)\, h(t)\, \mathrm{d}\tau \,. \tag{3.5}$$

2.2.1 -The Dirac distribution and convolution

- Let *f, g, h* be functions and *a* a scalar constant. Convolution satisfies the following properties

$$f * h = h * f\,, \tag{3.6}$$

$$f * (g * h) = (f * g) * h\,, \tag{3.7}$$

$$f * (g + h) = (f * g) + (f * h)\,, \tag{3.8}$$

$$a\,(f * g) = (a\,f) * g = f * (a\,g)\,. \tag{3.9}$$

2.2.1 -The Dirac distribution and convolution

- Taking the derivative of *a* convolution gives

$$\frac{\mathrm{d}}{\mathrm{d}x}(f * h) = \frac{\mathrm{d}f}{\mathrm{d}x} * h = f * \frac{\mathrm{d}h}{\mathrm{d}x}\,. \qquad (3.10)$$

- We will see later that the above derivative of the convolution proved useful, e.g., in edge detection of images.

Think about “processing window/mask in image segmentation”!

2.2.1 -The Dirac distribution and convolution

- Convolution can be generalized to higher dimensions. Convolution of 2D functions *f* and *h* is denoted by *f *h*, and is defined by the integral

$$\begin{aligned}(f * h)(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(a, b)\, h(x - a, y - b)\, \mathrm{d}a\, \mathrm{d}b \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x - a, y - b)\, h(a, b)\, \mathrm{d}a\, \mathrm{d}b \qquad (3.11) \\ &= (h * f)(x, y)\end{aligned}$$

2.2.1 -The Dirac distribution and convolution

- In digital image analysis, the discrete convolution is expressed using sums instead of integrals.
- A digital image has a limited domain on the image plane.
- However, the limited domain does not prevent us from the use of convolutions as their results outside the image domain are zero.
- The convolution expresses a linear filtering process using the filter *h*;
- linear filtering is often used in local image pre-processing and image restoration.

2.2.1 -The Dirac distribution and convolution

- Linear operations calculate the resulting value in the output image pixel *g(i,j)* as a linear combination of image intensities in a local neighborhood *O* of the pixel *f{i,j)* in the input image.
- The contribution of the pixels in the neighborhood *O* is weighted by coefficients *h*

$$f(i,j) = \sum_{(m,n)\in\mathcal{O}} h(i-m,\, j-n)\, g(m,n)\,. \tag{3.12}$$

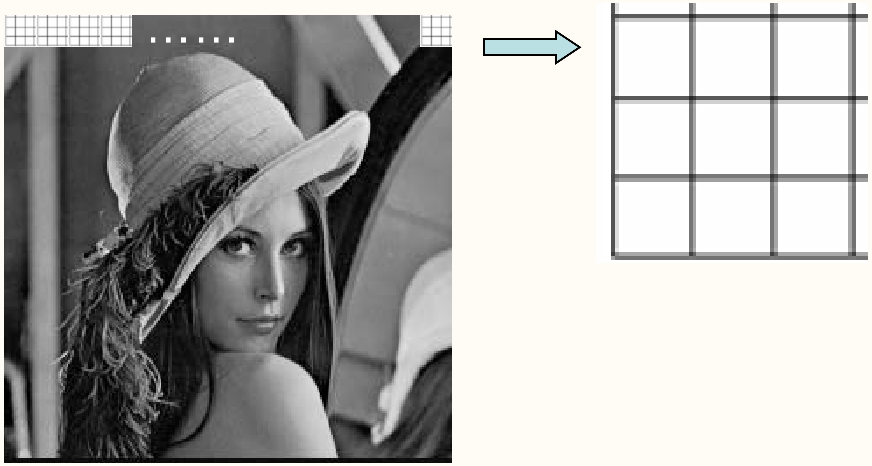
- Equation(3.12) is equivalent to discrete convolution with the kernel *h*, which is called a convolution mask.

Think about "processing window in image processing"!

2.2.1 -The Dirac distribution and convolution

- Rectangular neighborhoods O are often used with an odd number of pixels in rows and columns, enabling specification of the central pixel of the neighborhood.

Think about “processing window in image processing”!



2.2.2 Linear integral transforms

- Linear integral transforms are frequently employed in image processing. Using such transforms, images are treated as linear (vector) spaces.

- There are two basic and commonly used representations of image functions: the spatial domain (pixels) and the frequency domain (frequency spectra).

Think about “images are intrinsically signals”?

2.2.2 Linear integral transforms

- The image is expressed as a linear combination of some basis functions of some linear integral transform.
- For instance, the Fourier transform uses sines and cosines as basis functions. If linear operations are involved in the spatial domain (an important example of such linear operation is convolution) then there is a one-to-one mapping between the spatial and frequency representations of the image.
- Advanced signal/image processing goes beyond linear operations, and these non-linear image processing techniques are mainly used in the spatial domain.

Think about "Image processing in spatial domain and frequency domain"?

2.2.2 Linear integral transforms

- --Image as linear systems
- --Introduction to linear integral transforms
- --Fourier transform
- --Sampling
- --Eigen-analysis
- --Principal component analysis

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2.2.2 -Image as linear systems

- Images and their processing can be modeled as superpositions of point spread functions which are represented by Dirac pulses δ (equation 3.1).

- If this image representation is used, well-developed linear system theory can be employed.

2.2.2 -Image as linear systems

- Assume that the input image *f* is given by equation (3.3). The response *g* of the linear system $\mathcal{L}$ to the input image

$$
\begin{aligned}
g(x,y) = \mathcal{L}\{f(x,y)\} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\,\mathcal{L}\{\delta(x-a,y-b)\}\,\mathrm{d}a\,\mathrm{d}b \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a,b)\,h(x-a,y-b)\,\mathrm{d}a\,\mathrm{d}b = (f*h)(x,y)\,.
\end{aligned}
\tag{3.14}
$$

  where *h* is the impulse response of the linear system $\mathcal{L}$. In other words the output of the linear system is expressed as the convolution of the input image *f* with an impulse response *h* of the linear system $\mathcal{L}$.

2.2.2 -Image as linear systems

- If the Fourier transform is applied to equation (3.14) and the Fourier images are denoted by the respective capital letters then the following equation is obtained.

$$G(u, v) = F(u, v)\, H(u, v)\,. \tag{3.15}$$

- Equation (3.15) is often used in image pre-processing to express the behavior of smoothing or sharpening operations, and is considered further in Chapter 5.

Think about "the convolution theorem"?

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v)H(u, v).$$

2.2.2 -Image as linear systems

- Image processing can be approximated by linear systems in many cases.

2.2.2 -Introduction to linear integral transforms

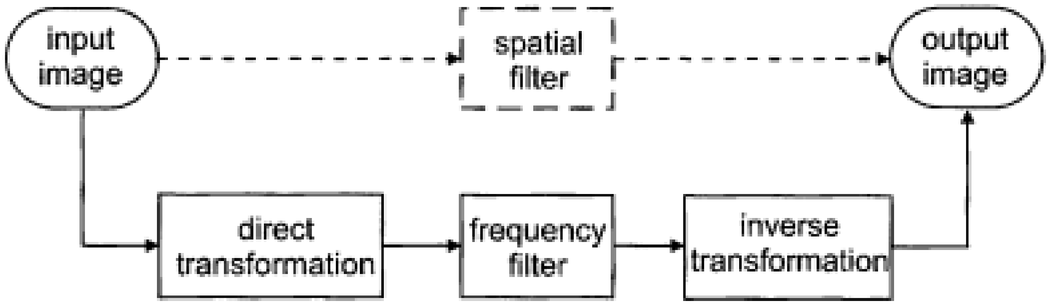


Figure 3.1: The image can be processed in either spatial or frequency domains. For linear operations, these two ways should provide equivalent results.

Review!

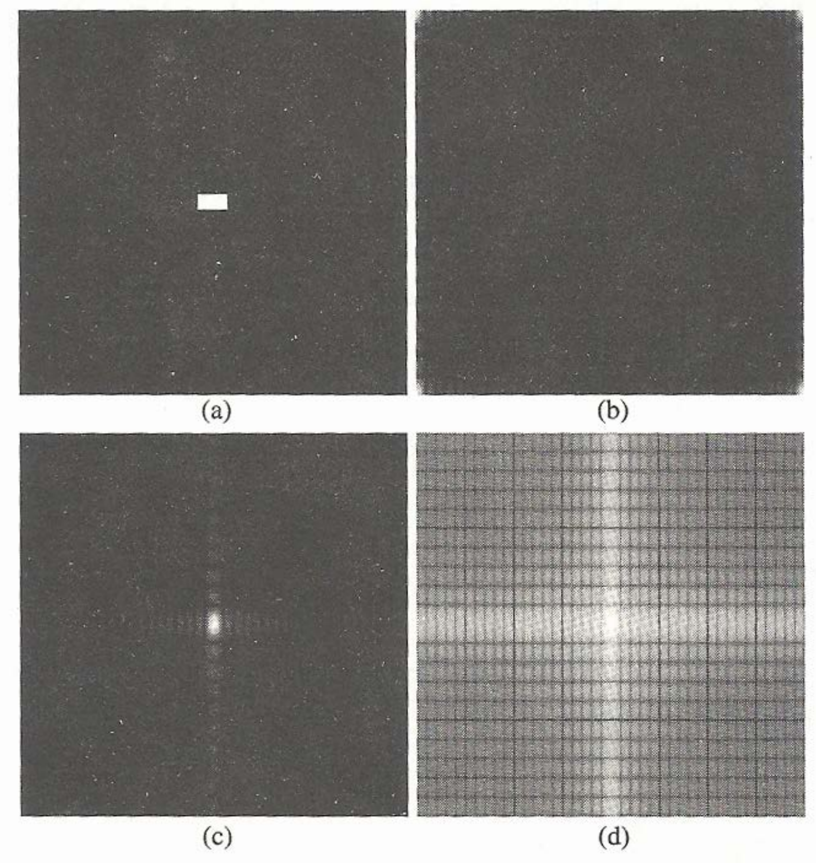
2.2.2 -1D Fourier transform

- The contents have been addressed in "Digital signal processing" or "Signals and systems" classes.
- Teach yourself. P53-58.

Review!

2.2.2 -2D Fourier transform

- The contents have been addressed in “Digital signal processing” or “Signals and systems” classes.
- Teach yourself. P58-61.



(a)A simple image.
(b)Fourier spectrum
(c)Centered spectrum
(d)Spectrum visually enhanced by a log transformation

Review!

2.2.2 - Sampling and the Shannon constraint

- Review the related contents.
- Teach yourself. P14-16.

Review!

2.1.2 Image digitization

- Image captured by a sensor is expressed as a continuous function f(x,y) of two co-ordinates in the plane.
- Image digitization means that the function f(x,y) is **sampled** into a matrix with M rows and N columns.
- The image **quantization** assigns to each continuous sample an integer value.
- The continuous range of the image function f(x,y) is split into K intervals.
- The finer the sampling (i.e., the larger M and N) and quantitation (the larger K) the better the approximation of the continuous image function f(x,y).

Review!

2.1.2 Image digitization

- Two questions should be answered in connection with image function sampling:
- First, the sampling period should be determined -- the distance between two neighboring sampling points in the image
- Second, the geometric arrangement of sampling points (sampling grid) should be set.

Review!

2.1.2 -Sampling

- A continuous image function f(x,y) can be sampled using a discrete grid of sampling points in the plane.
- The image is sampled at points x = j (Delta_x), y = k (Delta_y)
- Two neighboring sampling points are separated by distance Delta_x along the x axis and Delta_y along the y axis. Distances Delta_x and Delta_y are called the **sampling interval** and the matrix of samples constitutes the discrete image

2.2.2 -Sampling

- The ideal sampling s(x,y) in the regular grid can be represented using a collection of Dirac distributions (Eq.)

$$s(x,y) = \sum_{j=1}^{M} \sum_{k=1}^{N} \delta(x - j\Delta x, y - k\Delta y)$$

2.2.2 -Sampling

- The sampled image is the product of the continuous image f(x,y) and the sampling function s(x,y)

$$
\begin{aligned}
f_s(x,y) &= (f\,s)(x,y) \\
&= \sum_{j=1}^{M}\sum_{k=1}^{N} f(x,y)\,\delta(x-j\Delta x, y-k\Delta y) \\
&= f(x,y)\sum_{j=1}^{M}\sum_{k=1}^{N} \delta(x-j\Delta x, y-k\Delta y)
\end{aligned}
\qquad (3.40)
$$

P62

2.2.2 -Sampling

- The collection of Dirac distributions can be regarded as periodic with period x, y and expanded into a Fourier series (assuming that the sampling grid covers the whole plane (infinite limits)).

$$\sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(x - j\Delta x, y - k\Delta y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{mn}\, e^{2\pi i\left(\frac{mx}{\Delta x} + \frac{ny}{\Delta y}\right)}$$

where the coefficients of the Fourier expansion can be calculated as given in Eq.

$$a_{mn} = \frac{1}{\Delta x\, \Delta y} \int_{\frac{-\Delta x}{2}}^{\frac{\Delta x}{2}} \int_{\frac{-\Delta y}{2}}^{\frac{\Delta y}{2}} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(x - j\Delta x, y - k\Delta y)\, e^{-2\pi i\left(\frac{mx}{\Delta x} + \frac{ny}{\Delta y}\right)} dx\, dy$$

2.2.2 -Sampling

- The Fourier transform of the sampled image is the sum of periodically repeated Fourier transforms F(u,v) of the image.

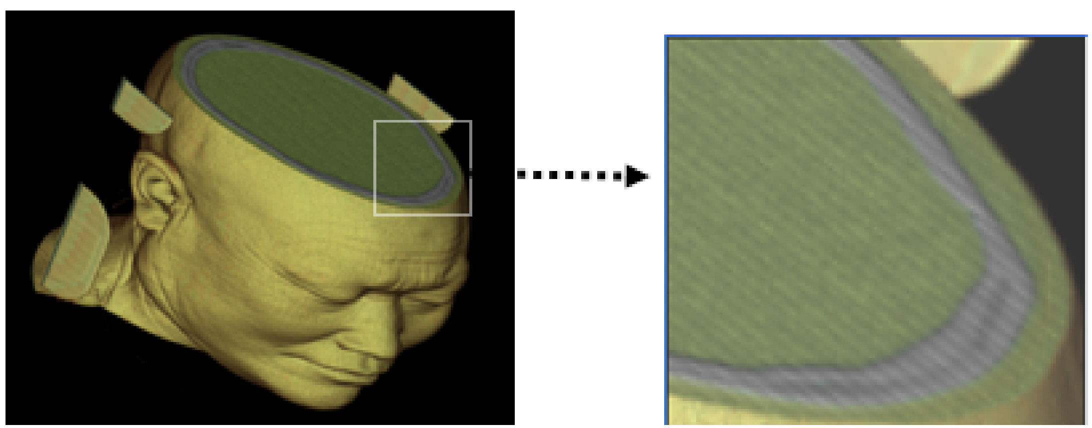
$$a_{mn} = \frac{1}{\Delta x \, \Delta y}$$

$$f_s(x, y) = f(x, y) \frac{1}{\Delta x \, \Delta y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{2\pi i \left(\frac{mx}{\Delta x} + \frac{ny}{\Delta y}\right)}$$

$$F_s(u, v) = \frac{1}{\Delta x \, \Delta y} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} F\left(u - \frac{j}{\Delta x}, v - \frac{k}{\Delta y}\right)$$

2.2.2 -Sampling

- Periodic repetition of the Fourier transform result F(u,v) may under certain conditions cause distortion of the image which is called **aliasing**; this happens when individual digitized components F(u,v) overlap.

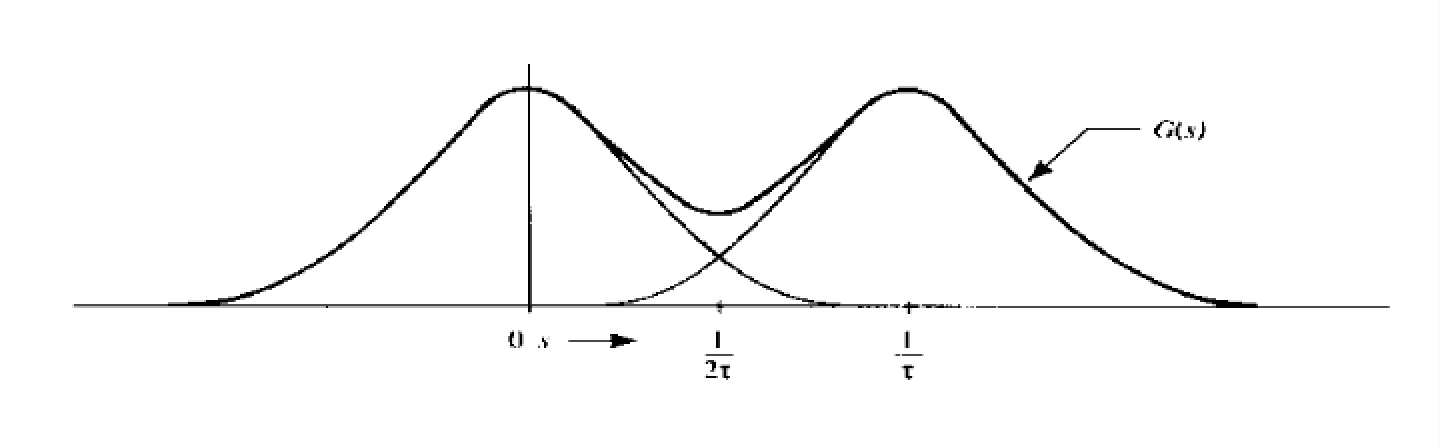


(a)整体图
(a) Whole image

(b)局部放大图
(b)Local zooming of Fig.4-9a

Streak
Aliasing
artifact

2.2.2 -Sampling



Overlapped!

PLS review the knowledge points
In the course “digital signal processing”!

2.2.2 -Sampling

- There is no aliasing if the image function f(x,y) has a band limited spectrum ... its Fourier transform F(u,v)=0 outside a certain interval of frequencies |u| > U; |v| > V.

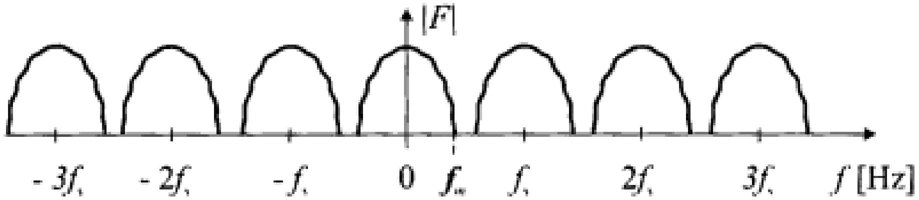


Figure 3.10: Repeated spectra of the 1D signal due to sampling. Non-overlapped case when $f_s \geq 2f_m$.

PLS review the knowledge points
In the course “digital signal processing”!

2.2.2 -Sampling

- As you know from general sampling theory, overlapping of the periodically repeated results of the Fourier transform F(u,v) of an image with band limited spectrum can be prevented if the sampling interval is chosen according to Eq. 3.43, pp63

$$\Delta x < \frac{1}{2U}, \Delta y < \frac{1}{2V}$$

2.2.2 -Sampling

- This is the **Shannon sampling theorem** that has a simple physical interpretation in image analysis: The sampling interval should be chosen in size such that it is less than or equal to half of the smallest interesting detail in the image.

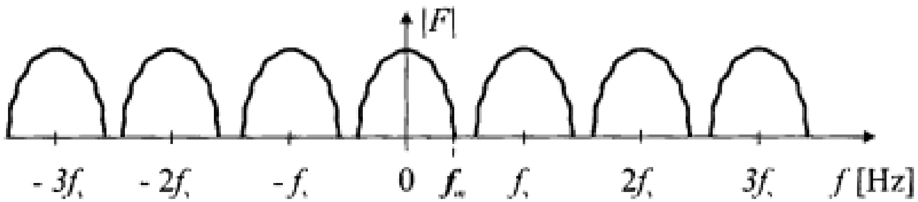
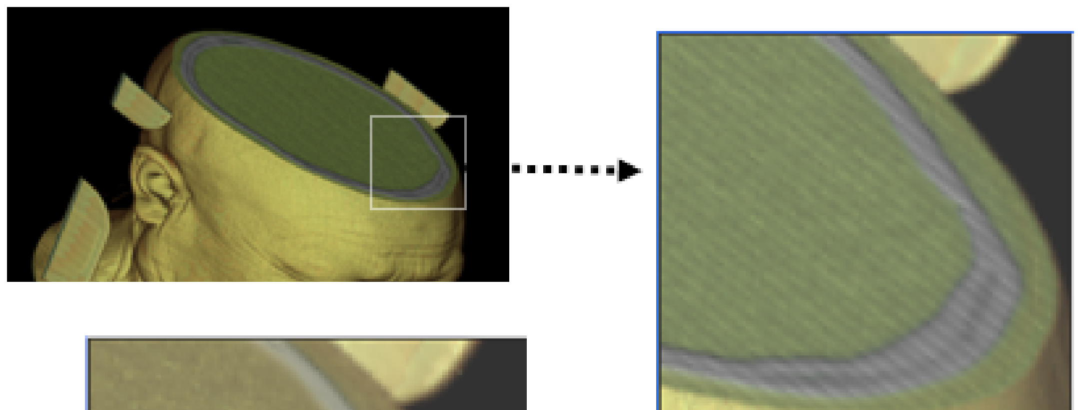


Figure 3.10: Repeated spectra of the 1D signal due to sampling. Non-overlapped case when $f_s \geq 2f_m$.

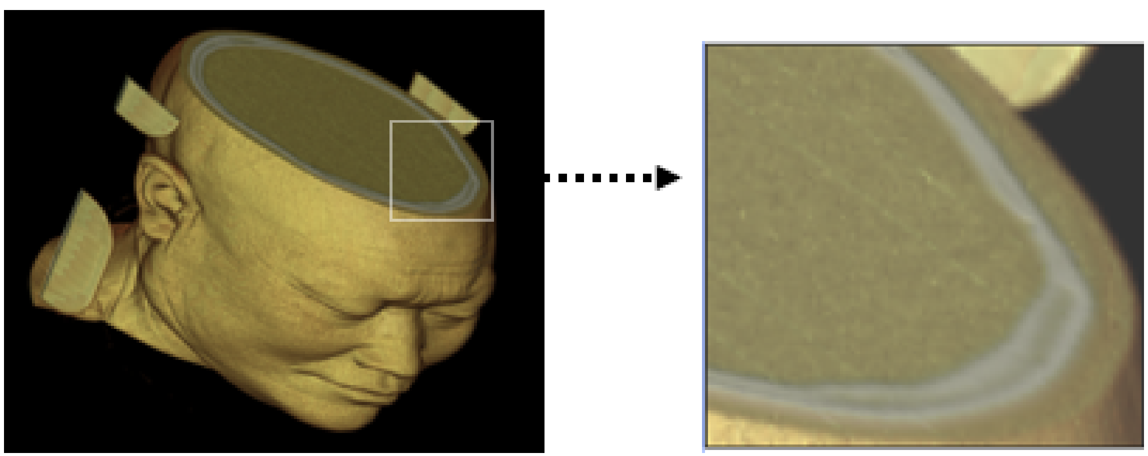
2.2.2 -Sampling

- Practical examples of digitization help to understand the reality of sampling.



(b)局部放大图
(b)Local zooming of Fig.4-9a

Streak
Aliasing
artifact



(a)整体图
(a)Whole image

(b)局部放大图
(b)Local zooming of Fig.4-10a

2.2.2 - Wavelet transform

- Teach yourself. P66-72.

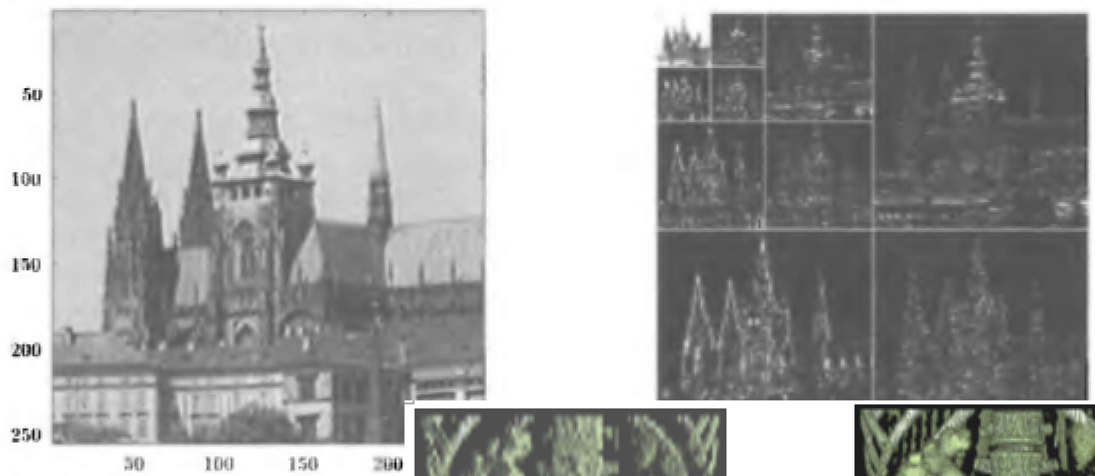


Figure 3.21: Decomposition by the
and decomposition to three levels per
and on the right four quadrants. T
quadrants correspond to detailed coe
and horizontal directions, respective
for level 2 at resolution 64 × 64 pix
str
cor

平滑逼近结果 (c) 0 级平滑逼近结果

(a)Level 2 smooth approximation (b)Level 1 smooth approximation (c)Level 0 smooth approximation

Think about “shearlet transform”?

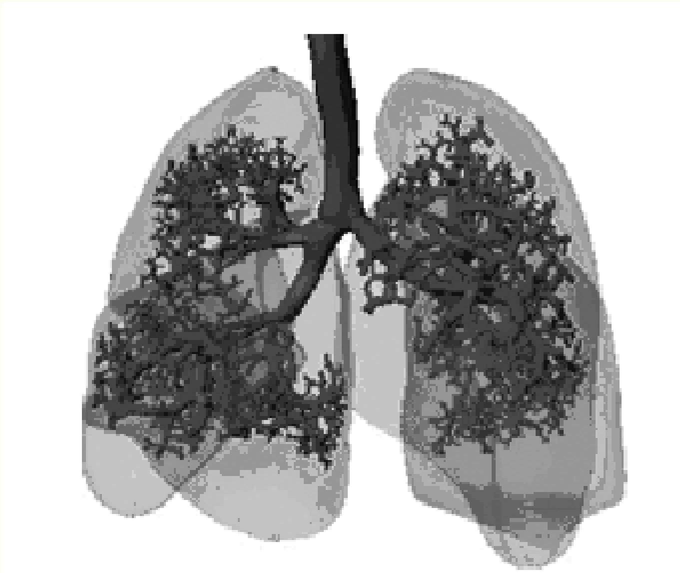
2.2.2 - Eigen-analysis

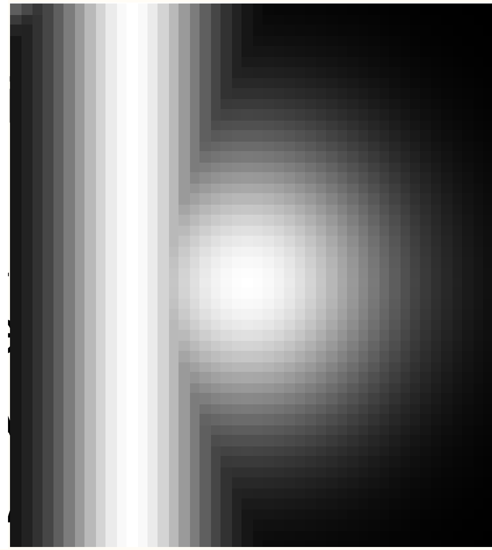
- Teach yourself. P72-73.
- Review the related knowledge.
- The contents have been addressed in “Linear algebra” classes.
- For an *n x n* square regular matrix A, eigen-vectors are solutions of the equation

$$A\mathbf{x} = \lambda\mathbf{x}, \tag{3.55}$$

where λ is called an **eigen-value** (which may be complex).

2.2.2 - Eigen-analysis





- s, seek to
  and general
  ndependence
- ecognition,
  ules”.

Think about “*Shape Index (A 3-D Geometric Feature)*”?

Think about “how to discriminate the cylinder-like vessel and sphere-like nodule in lung CT image”?

2.2.2 - Principal component analysis

- Teach yourself. P74-77.

PCA will be introduced further in chapter “Feature Extraction and Object Recognition”?

Figure 3.23: 32 original images of a boy

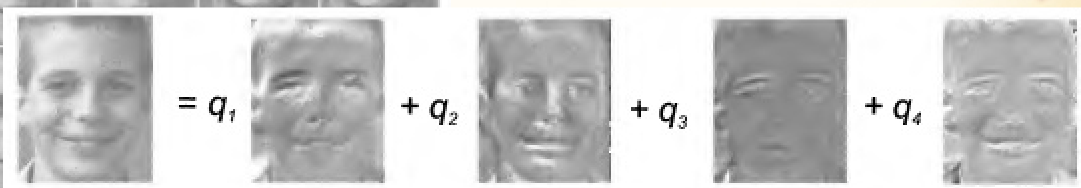


Figure 3.24: Reconstruction of the image from four basis vectors $\mathbf{b}_i$, $i = 1, \ldots, 4$ which can be displayed as images. The linear combination was computed as $q_1\mathbf{b}_1 + q_2\mathbf{b}_2 + q_3\mathbf{b}_3 + q_4\mathbf{b}_4 = 0.078\,\mathbf{b}_1 + 0.062\,\mathbf{b}_2 - 0.182\,\mathbf{b}_3 + 0.179\,\mathbf{b}_4$.

2.2.3 Images as stochastic processes

- Images are statistical in nature due to random changes and noise, and it is sometimes of advantage to treat image functions as realizations of a stochastic process.
- In such an approach, questions regarding image information content and redundancy can be answered using probability distributions, and simplifying probabilistic characterizations as the mean, dispersion, correlation functions, etc.

2.2.3 Images as stochastic processes

- A stochastic process (random process, random field) is a generalization of the random variable concept.
- We will constrain ourselves to stochastic processes of with two independent variables x. y which are the coordinates in the image.

2.2.3 Images as stochastic processes

A stochastic process f is entirely described by a collection of k-dimensional **distribution functions** P_k, $k = 1, 2, \ldots$. The distribution function of k arguments $z_1, \ldots, z_k$ is

$$P_k(z_1, \ldots, z_k; x_1, y_1, \ldots, x_k, y_k) = \mathcal{P}\left\{\phi(x_1, y_1) < z_1, \phi(x_2, y_2) < z_2, \ldots, \phi(x_k, y_k) < z_k\right\}, \tag{3.60}$$

where $\mathcal{P}$ denotes the probability of the conjunction of events listed in the brackets. The above equation expresses the dependence of k pixels $(x_1, y_1), \ldots, (x_k, y_k)$. For a complete probabilistic description, we would need these joint distribution functions for k equal to the number of pixels in the image.

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2.2.3 Images as stochastic processes

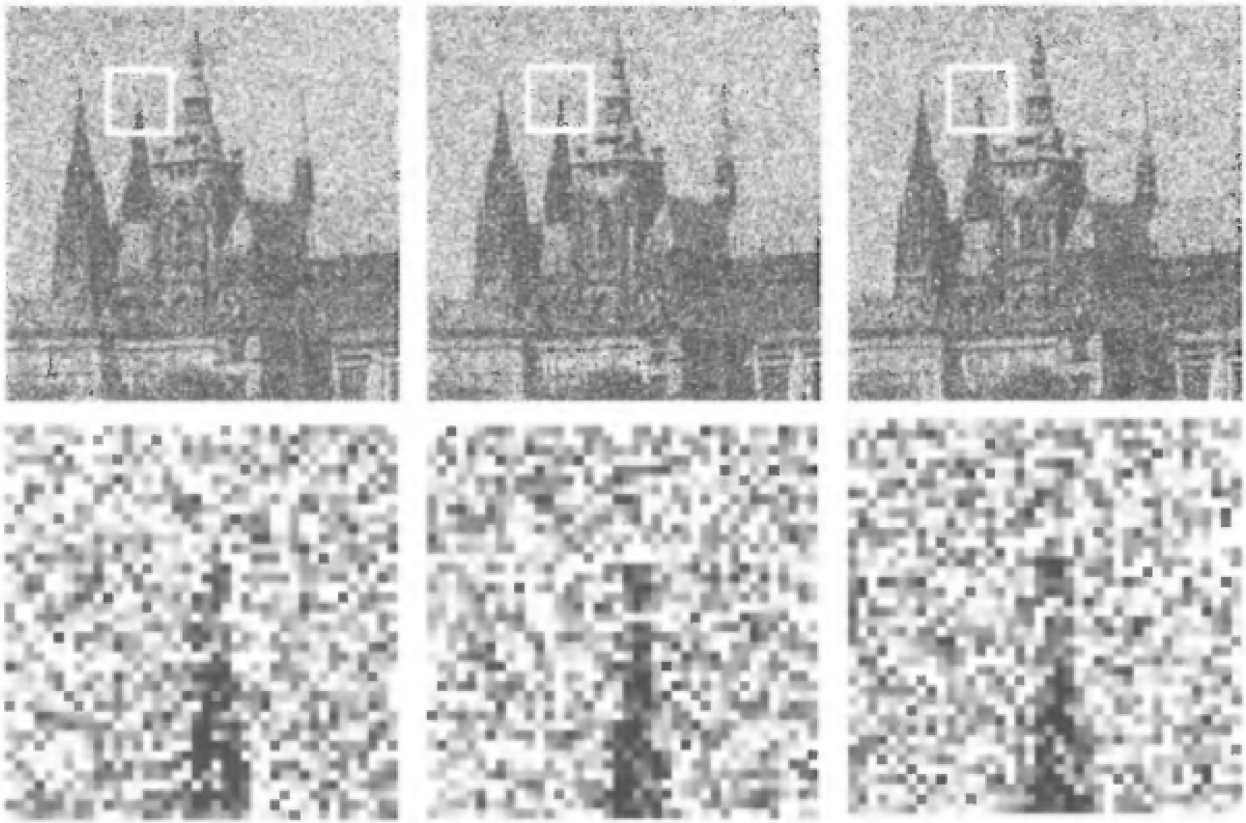

Figure 3.25: Three 256 × 256 images are shown as realizations of a stochastic process ϕ in the upper row. The crop windows is marked by the white squares in the three images. The content of these crop windows is enlarged in three images below. Notice that pixels really differ in the same locations in three realizations.

2.2.4 Image formation physics

- Teach yourself. P80-94.
- Image as radiometric measurements
- Image capture and geometric optics
- Lens abberations and radial distortion
- Image capture from a radiometric point of view
- Surface reflectance

Knowledge points

- Fundamental concepts and mathematical tools are introduced in this chapter which will be used throughout the course.

- Signals can be

  two-dimensional (e.g., images dependent on two co-ordinates in a plane),

  three-dimensional (e.g., describing an object in space), or higher-dimensional.

Questions and Practices

- See “Practice 2 Digital image processing”