PHYS639, Spring16, Problem 3

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A motion of a realistic pendulum can be described by the second order ODE

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin(\theta) - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t), \tag{1}$$

where the second term on the right side describes dissipation and the third term describes a periodic driving force. The energy of such a pendulum is given by

$$E = mgl[1 - \cos(\theta)] + \frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2. \tag{2}$$

You can use $l = 9.8$m for simplicity.

1. For a small initial angle $\theta_o = 0.1$ plot the angle and the energy as functions of time for
   (a) A non-damped ($q = 0$) unforced ($F_D = 0$) pendulum
   (b) Unforced pendulum with large ($q = 5$) and small ($q = 1$) friction
   (c) A forced pendulum ($F_D = 0.2$, $\Omega_D = 2$).
2. Check what happens when Ω_D approaches $\sqrt{g/l}$.
3. Make similar plots for a large initial angle $\theta_o = 0.5$ and a small friction $q = 0.5$, and two values for the driving force $F_D = 0.5$ and $F_D = 1.4$.
4. This is an example of a *chaotic* system where very small differences in the initial conditions will result in exponentially large differences with time. To observe this chaotic behavior make similar plots but with an initial angle of $\theta_o = 0.5001$.
5. *Poincaré sections* are useful for studying the dynamics of chaotic systems. Draw the Poincaré section for times $t \approx 2\pi n/\Omega_D$, where n is a sequence of integers, for the two values of the driving force.

Hint: A simple Euler's method of solving ODE may not be accurate enough for this problem. You will have to use something better e.g. Euler-Cromer or Runge-Kutta algorithms.