PHYS639, Spring16, Problem 4

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In units of $G = 1$ the magnitude of the force of gravitational interaction between any two objects is

$$F = \frac{m_1 m_2}{|\mathbf{r}_1 - \mathbf{r}_2|^2}, \tag{1}$$

the energy of each object is

$$E_i = \frac{m v_i^2}{2} - \frac{m}{|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{2}$$

and the angular momentum is

$$\mathbf{L}_i = m \mathbf{r}_i \times \boldsymbol{v}_i. \tag{3}$$

1. Compute and plot the orbit of a planet around a star. You can assume that the star is much heavier compared to the planet and is at rest. Compute kinetic and potential energies, and angular momentum as a function of time for various initial conditions.
   (a) Choose initial conditions so that the orbit is circular (Hint: $v_\oplus^2/r = m_\star/r$).
   (b) Use initial conditions such that the orbit is elliptic.
   (c) Use initial conditions such that the planet is not gravitationally bound.
2. Compute the period for three different elliptic orbits and verify the Kepler's law – $T^2/a^3 \sim$ const – where T is the period and a is the length of the semi-major axis.
3. Compute and plot the orbit of two planets around a star. You should account for interplanetary interactions. Increase the mass of the larger planet so that it has a visible effect on the trajectory of the smaller planet.
4. Simulate a motion of the earth and the moon around the sun.
5. Simulate a binary system of two massive stars of equal mass orbiting each other. Choose initial conditions so that the *center of mass* is not moving.
6. Simulate a system made of a star (solar mass), a large planet (1000 times Jupiter's mass), and a small planet (earth's mass). Account for all interactions. Choose initial conditions that resemble the distances between the sun, the Jupiter, and the earth.

Deliverables

You need to plot orbits, kinetic energies, potential energies, and angular momenta, for all relevant bodies in the system. If there is more than one body in motion also plot the total energy and angular momentum. Do this for the following systems.

1. A planet orbiting a star for a circular, elliptic, and unbound cases.
2. Two planets orbiting a star.
3. The earth and the moon around the sun.
4. Binary system of two stars of equal mass.
5. The sun, the Jupiter, and the earth.

Also generate following numbers

7. T^2/a^3 for three sets of initial conditions.