PHYS639, Spring16, Problem 1

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1. Using *Eulerś method* solve the ordinary differential equation describing radioactive decay,

$$\frac{dN}{dt} = -\frac{N}{\tau}, \tag{1}$$

where N is the number of radioactive atoms and τ is a characteristic timescale for decay. Assume some initial value $N(t = 0) = N_o$ and a numerical value for τ and plot N vs t. Do this for few different values of τ.

2. The population growth to some approximation can be modelled by the equation

$$\frac{dN}{dt} = aN - bN^2. \tag{2}$$

The birth rate (first term) is proportional to the number of specimens, and the death rate (second term) is proportional to the second power of N. For large values of N (assuming limited resources) the second term dominates. Solve this equation and plot N vs t for various values of N_o, a and b.

Bonus Problems

1. Radioactive substance A decays into another substance B at a rate τ_A. B then decays at a rate τ_B. This process can be modeled with a a set of differential equations,

$$\frac{dN_A}{dt} = -\frac{N_A}{\tau_A}, \tag{3}$$

$$\frac{dN_B}{dt} = \frac{N_A}{\tau_A} - \frac{N_B}{\tau_B}. \tag{4}$$

Solve this equation and plot N_A and N_B vs time for different values of τ_A/τ_B.

2. If B decays back into A, the set of differential equations above will become symmetric (assuming the same decay rate)

$$\frac{dN}{dt} = \frac{N_B}{\tau} - \frac{N_A}{\tau} \tag{5}$$

$$\frac{dN}{dt} = \frac{N_A}{\tau} - \frac{N_B}{\tau}. \tag{6}$$

Plot N_A and N_B vs time for different initial conditions and τ.