

Recursion Problems

Question: Print all subsequence of an array

arr = ~~[3, 2, 1]~~ [3, 1, 2]

Subsequences →

- $\{\emptyset\}$
- 3
- 1
- 2
- 3, 1
- 1, 2
- 3, 2
- 3, 1, 2

→ 8 subsequences

How can we take elements from array to create the subsequences?

For $\rightarrow \{\emptyset\}$ $\begin{matrix} \times & \times & \times \\ [3, 1, 2] \end{matrix}$

For $\rightarrow \{3\}$ $\begin{matrix} & \times & \times \\ \checkmark & & \\ [3, 1, 2] \end{matrix}$

For $\rightarrow \{1\}$ $\begin{matrix} \times & & \times \\ & \checkmark & \\ [3, 1, 2] \end{matrix}$

For $\rightarrow \{2\}$ $\begin{matrix} \times & \times & \checkmark \\ & & \\ [3, 1, 2] \end{matrix}$

For $\rightarrow \{3, 1\}$ $\begin{matrix} & \checkmark & \checkmark & \times \\ \checkmark & & & \\ [3, 1, 2] \end{matrix}$

For $\{1, 2\} \rightarrow \begin{matrix} \times & \checkmark & \checkmark \\ \times & & \\ [3, 1, 2] \end{matrix}$

For $\{3, 2\} \rightarrow \begin{matrix} \checkmark & \times & \checkmark \\ & \checkmark & \\ [3, 1, 2] \end{matrix}$

For $\{3, 1, 2\} \rightarrow \begin{matrix} \checkmark & \checkmark & \checkmark \\ & & \\ [3, 1, 2] \end{matrix}$

$\checkmark \rightarrow$ take
 $\times \rightarrow$ Not take

Pseudocode:

input = [3, 1, 2]

Subsequence (input, index, ans) {

if (index \geq len(input))

Print(ans)

return

ans.append(input[index]) \rightarrow Adding element (Take)

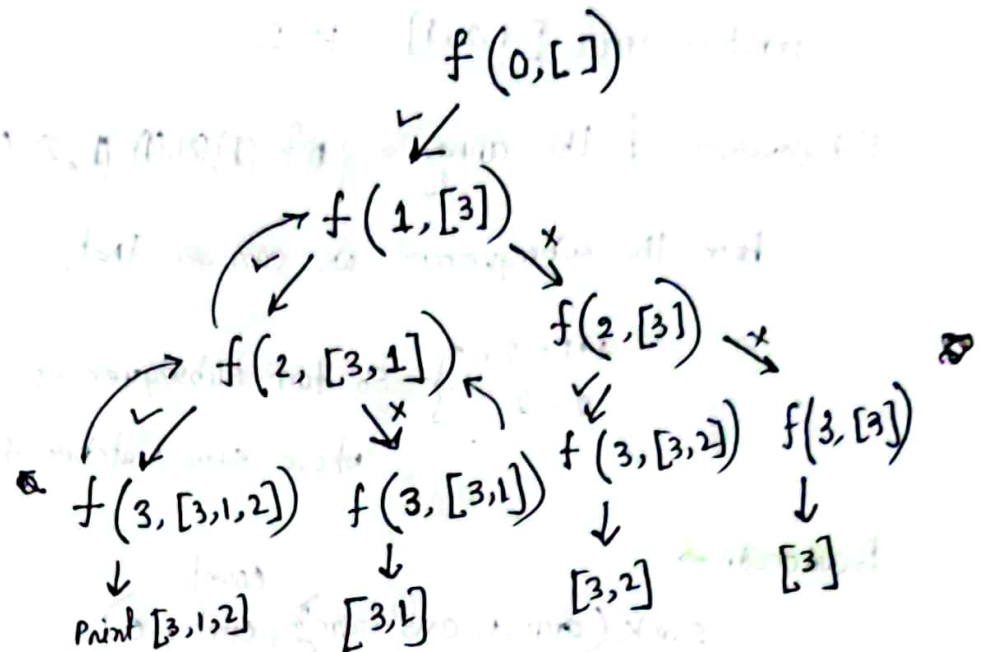
Subsequences(input, index+1, ans)

ans.remove(input[index]) \rightarrow Removing element (Not take)

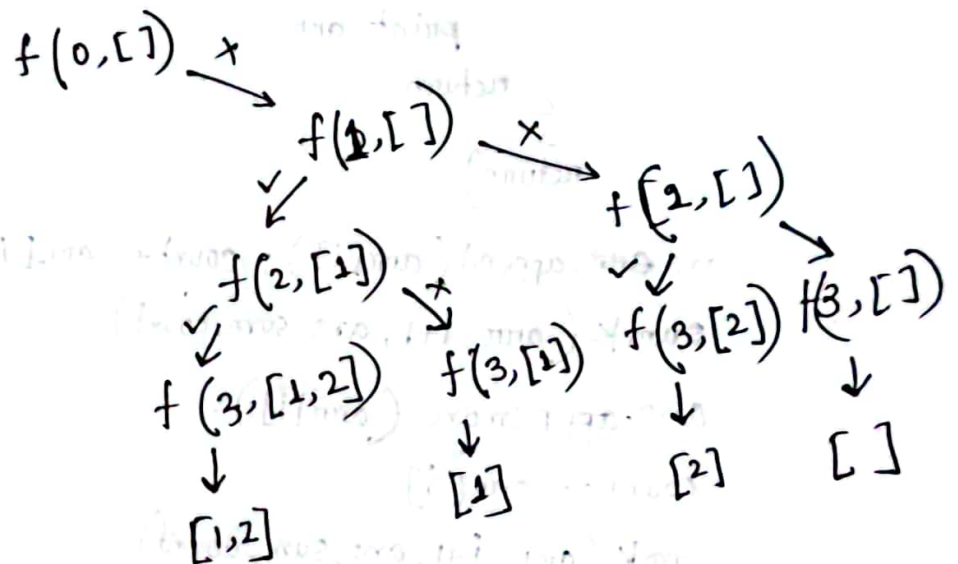
Subsequences(input, index+1, ans)

Recursion Tree of Subsequences of an array

Left Recursion



Right Recursion



Question: Print all the subsequence of an array whose sum is k.

input \rightarrow arr = [1, 2, 1], K = 2

Subsequences of this array = $\{\emptyset, (1), (2), (1), (1, 2), (2, 1), (1, 2, 1)\}$ ^(1,1)

From the subsequences, we can see that,

$\left. \begin{array}{l} 1+1=2, \\ 2=2 \end{array} \right\}$ So two subsequences are available that whose sum matches to K.

Pseudocode \rightarrow

sumK(arr, i, ans, sum, ^{count}count) ~~count = 0~~

{ if (i == len(arr)):

{ if sum == ^{count}ans:

print arr

} return

} return

ans.append(arr[i]); count += arr[i] [take]

sumK(arr, i+1, ans, sum, count)

ans.remove(arr[i]);

count -= arr[i]

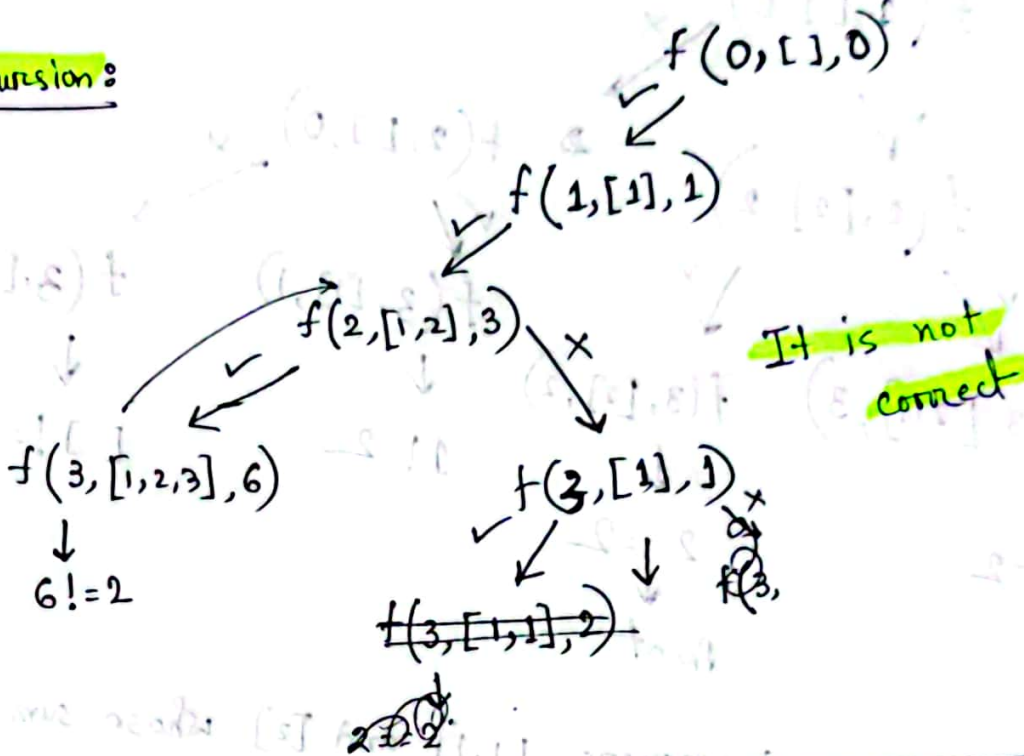
sumK(arr, i+1, ans, sum, count)

Recursion Tree of Sum K from subsequence array

Array $\rightarrow [1, 2, 1]$, Sum $\rightarrow K = 2$

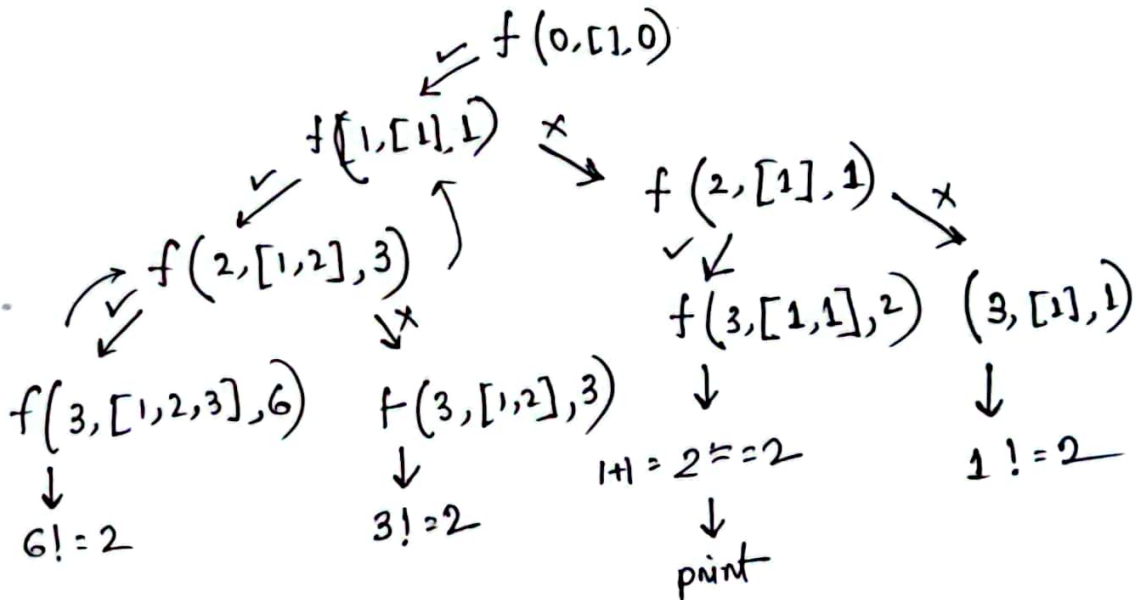
f(index, ans, count)

Left Recursion:



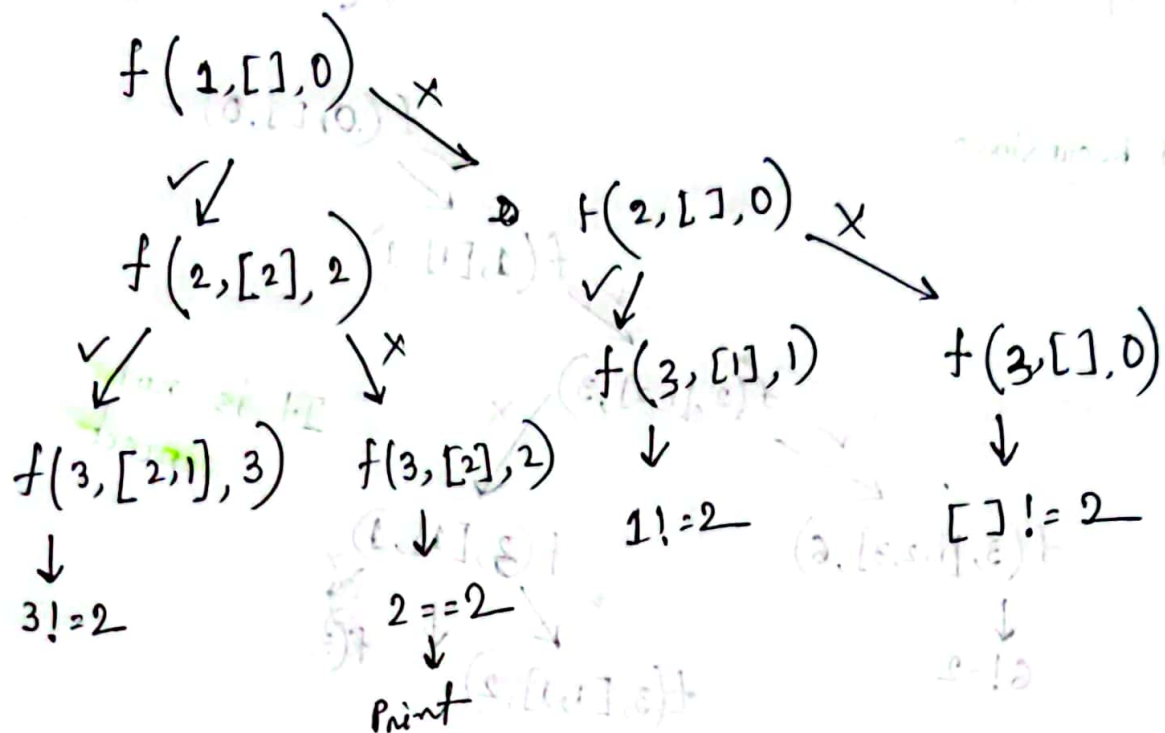
Left Recursion:

f(index, ans, count)



Right Recursion:

$f(\text{index, ans, count})$



So, we got our subsequences $[1, 1]$ and $[2]$ whose sum is $K=2$