دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

دانشکده مهندسی مکانیک
دانشگاه صنعتی امیرکبیر

Lecture 3_1:
Manipulator Kinematics
Forward Kinematics

Advanced Robotics

Hamed Ghafarirad

Outlines

- Introduction
- Link Description
- Link-Connection Description
- Convention for Affixing Frames to Links
- Manipulator Kinematic
- Frames with Standard Names

Introduction

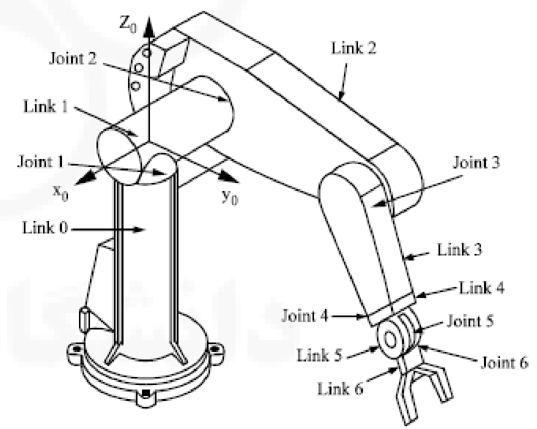
- **Kinematics:**

➢ The science of **motion** without regard to the **forces** that cause it.

- Position, velocity, acceleration & all higher order derivatives with respect to *time* or any *other variables*.

- **Forward Kinematic:**

➢ Compute the **position** and **orientation** of the manipulator linkages and end-effector (EE) *relative to the base frame* as a function of joint variables.

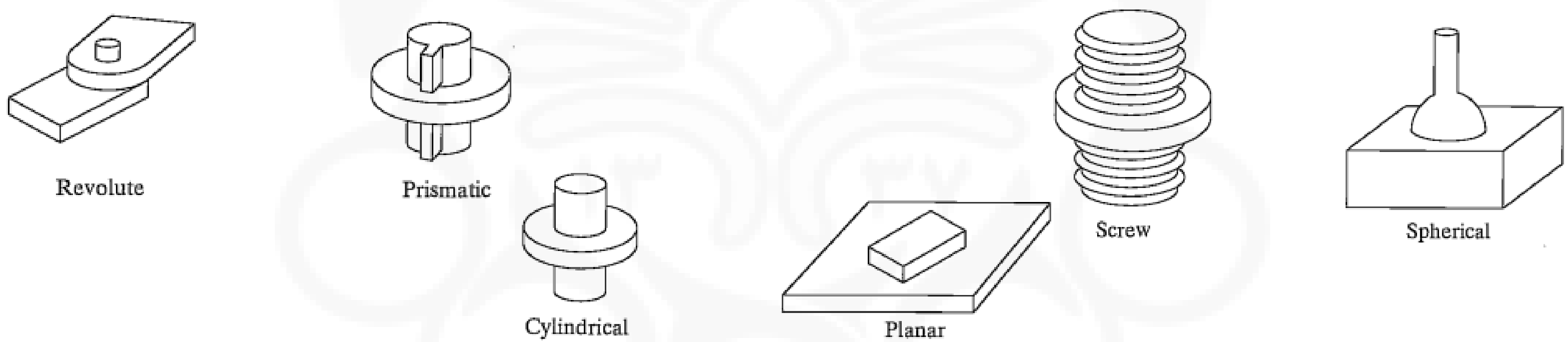


- $\boldsymbol{X} = \boldsymbol{f}(\Theta)$

- To perform:
  - ➢ Affix frames to the various parts of the robot mechanism.
  - ➢ Describe the relationship between frames (Frames Transformation).

Link Description

- **Manipulator:** A set of bodies connected in a chain by joints.
- Bodies are called **Links.**
- **Joints:** A connection between a neighboring pair of links.



- **Convention:** Joints exhibit just **one** degree of freedom.
- Most manipulators have **Revolute joints** or have Sliding joints (**Prismatic joints**).
- **Note:** A n DOF joint can be modeled as n joints of one DOF connected with $n-1$ links of zero length.

Link Description

- A ***n* DOF** robot has $n+1$ links numbered from *0 to n*.

- *Link 0* is the immobile base of the manipulator and *Link n* is the last moving link.

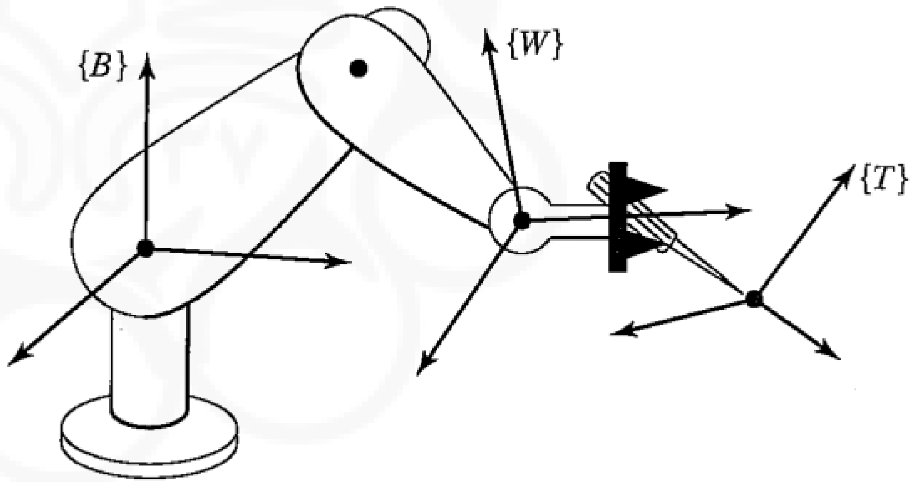
- Immobile Base: Link 0
- 1st moving link: Link 1

.   .

.   .

.   .

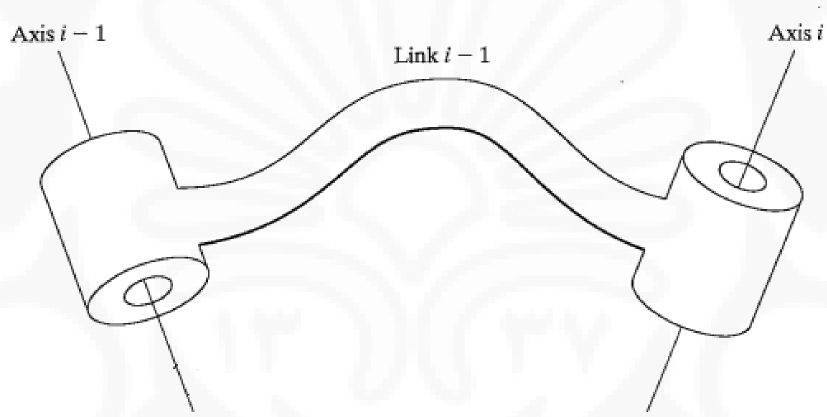
- Last moving link: Link *n*



- In order to position an end-effector generally in 3-space, a minimum of six joints is required (<u>Necessary Condition</u> but <u>not sufficient</u>).

Link Description

- **Link:** A rigid body that defines the relationship between two neighboring joint axes of a manipulator.

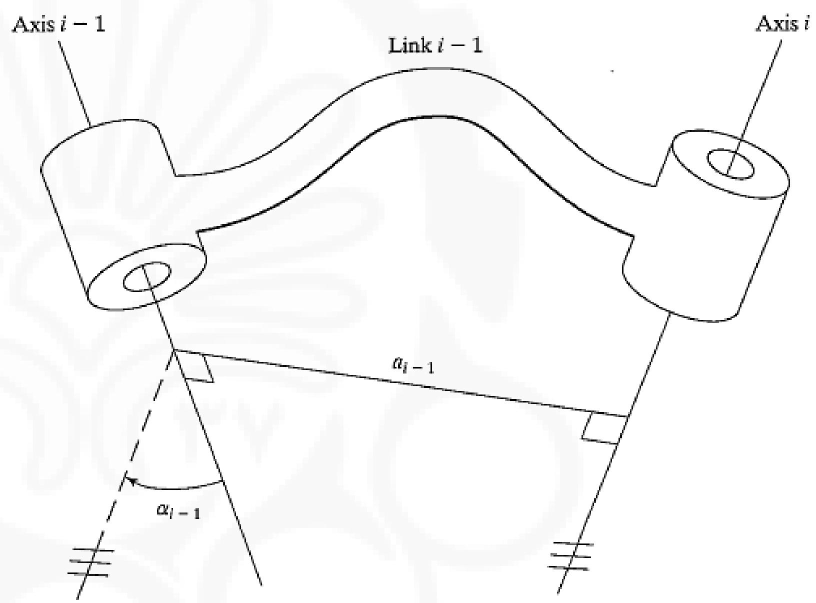


- ***Joint axis i*** is defined by a line in space, or a vector direction, about which *link* i Rotates (Or Slides) relative to *link* $i-1$.

- Each link (i) will be described by 4 parameters:
  - ➢ 2 parameters describe the **Link itself.**
  - ➢ 2 parameters describe the **Link's connection.**

Link Description

- 2 parameters which describe the **Link itself** are as follows:
  - **Link length a_{i-1}**
  - **Link twist α_{i-1}**



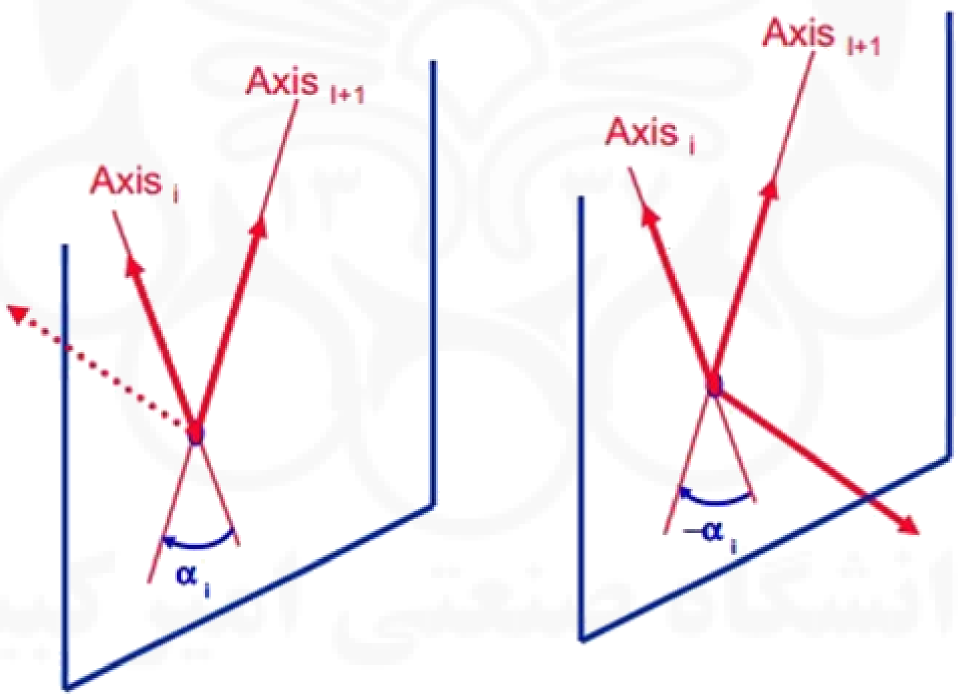
- **Link length a_{i-1} :** The distance between two joint axes measured along mutual perpendicular to both axes.

- **Note:** Mutual perpendicular always exists & is unique except for intersected axes & parallel axes.

- **Link twist α_{i-1}:** The angle between the two joint axes measured about a_{i-1} in the right hand sense.

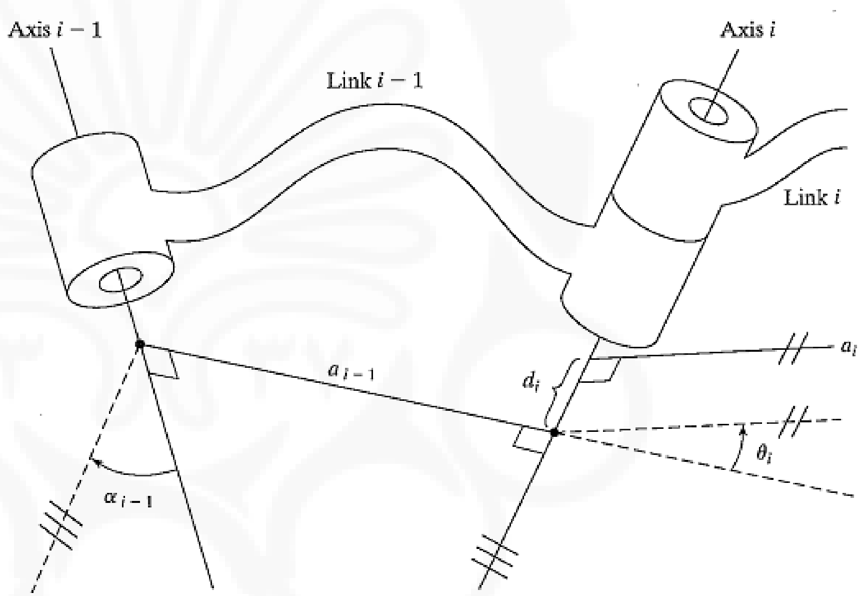
Link Description

- In the case of **intersecting axes** :
  - ➢ a_{i-1} would be zero (Zero link length).
  - ➢ α_{i-1} is measured in the plane containing both axes, but one is free to assign the sign of α_{i-1} arbitrarily.



Link-Connection Description

- 2 parameters which describe the **Link's connection** are as follows:
  - **Link offset d_i**
  - **Joint angle θ_i**



Intermediate Links in the Chain

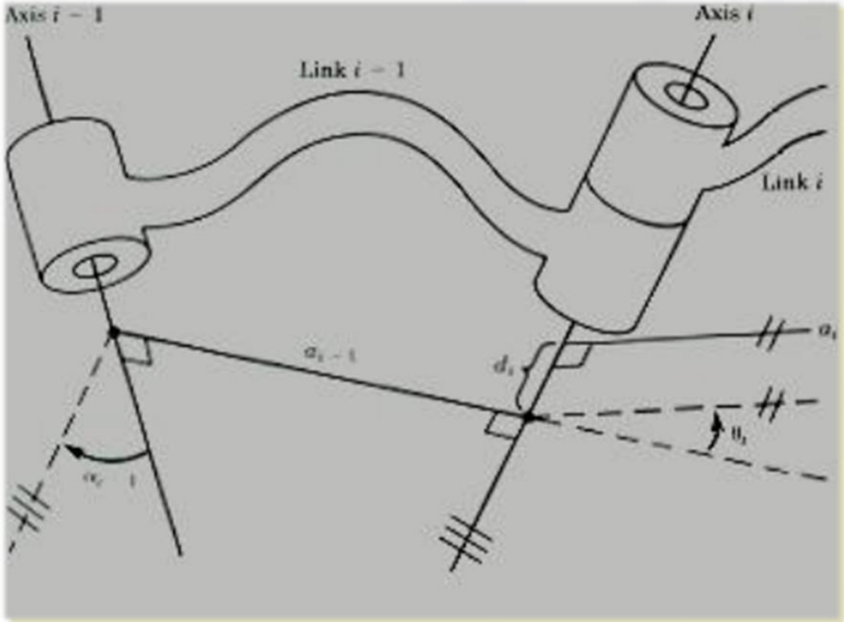
- **Link offset d_i :** The *signed* **distance** along common axis from one link to next.

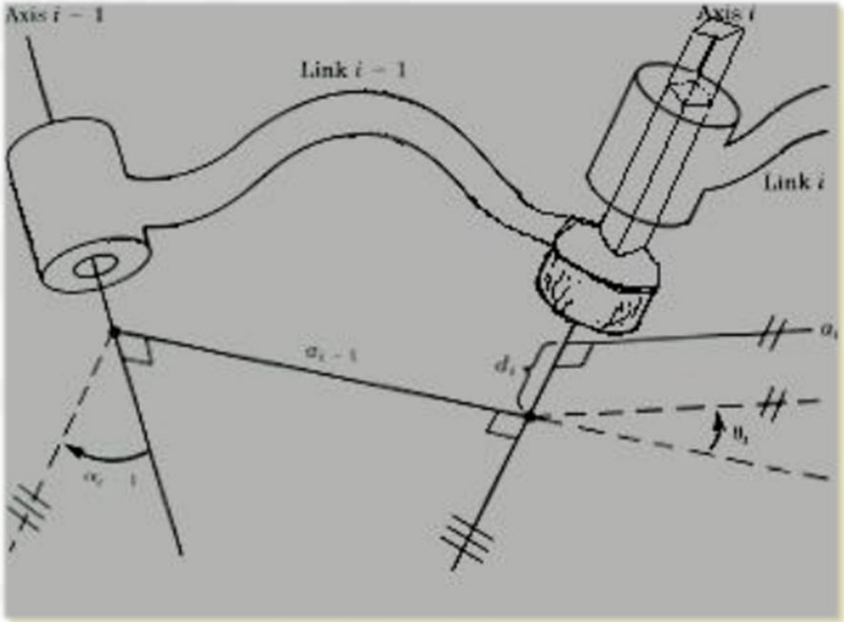
- **Joint angle θ_i :** The **rotation** about this common axis between one link and its neighbor.

← Outline

Link-Connection Description

❑ Intermediate Links in the Chain





θ_i is variable if the joint is revolute

d_i is variable if the joint is prismatic

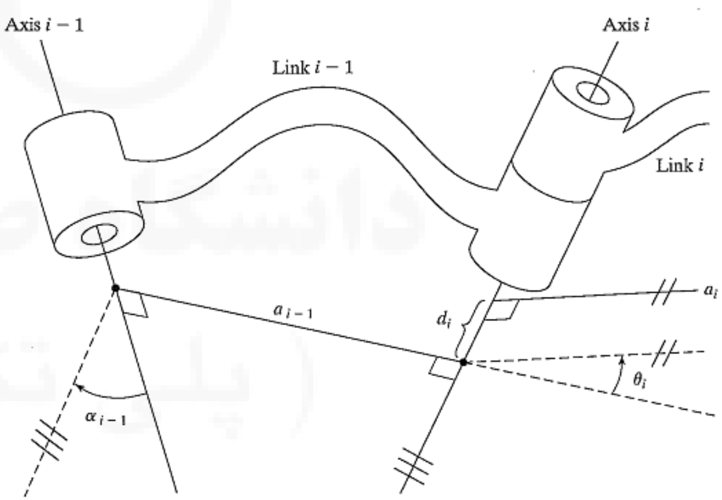
Link-Connection Description

❑ Link Parameters

- Any robot can be described kinematically by giving the values of four quantities for each link i .
  - ➢ a_i & α_i describe the Link itself (relative to the next link)
  - ➢ d_i & θ_i describe the Link's connection (relative to the previous link)

- The definition of mechanisms by means of these quantities is a convention usually called the **Denavit-Hartenberg** notation.

- Four quantities are called DH Parameters.

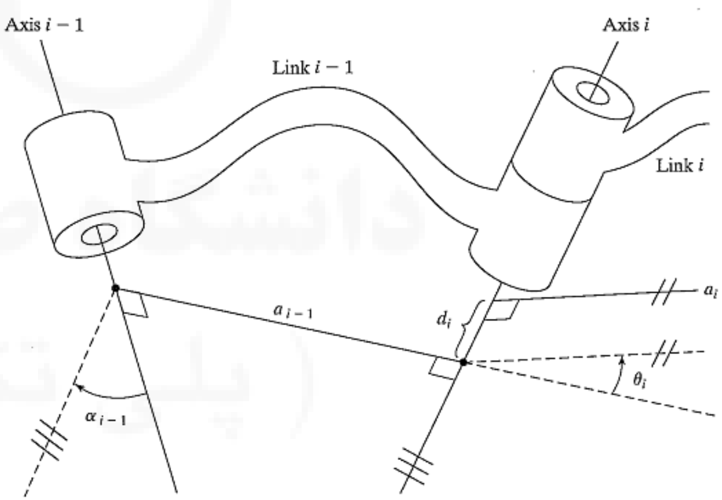


Link-Connection Description

Link Parameters

- Revolute joint: θ_i is the joint variable, and the other three quantities would be fixed link parameters.
- Prismatic joints: d_i is the joint variable, and the other three quantities are fixed link parameters.

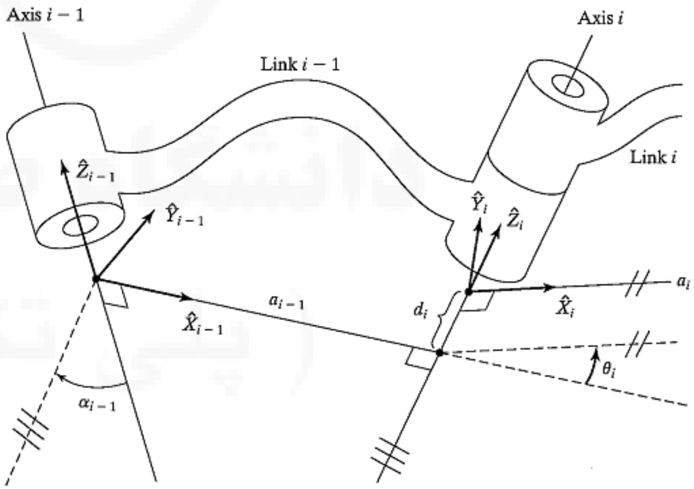
- To locate a link relative to its neighbor, a frame is attached to each link.
- Frame $\{i\}$ is attached rigidly to link i .



Convention for Affixing Frames to Links

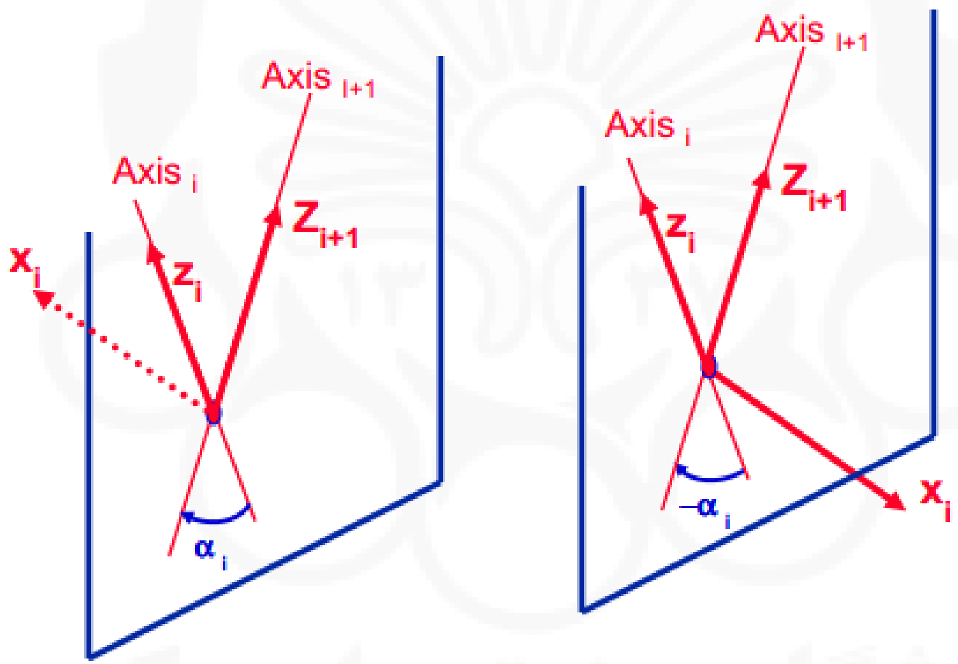
Intermediate link in the chain

- The $\hat{Z}_i$ axis of frame $\{i\}$, called $\hat{Z}_i$, are coincident with the joint axis i.
- The origin of the frame $\hat{Z}_i$ is located where the a_i perpendicular intersects the joint axis i
- $\hat{X}_i$ points along a_i in the direction from joint i to joint $i+1$.
- $\hat{Y}_i$ is formed by the right-hand rule to complete the i^{th} frame.



Convention for Affixing Frames to Links

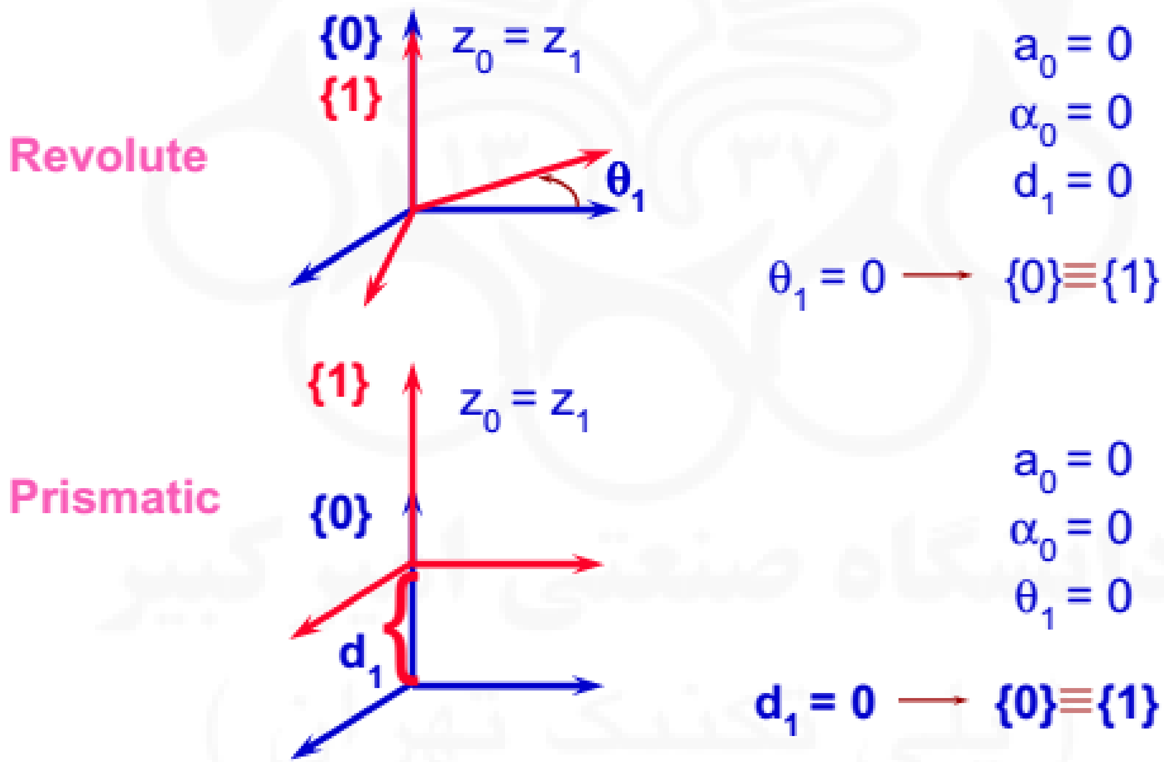
- **Intermediate link in the chain**
- **Intersecting Joint Axes**
  - $\hat{X}_i$ is normal to the plane of $\hat{Z}_i$ and $\hat{Z}_{i+1}$.



- Since α_i is measured in the right hand sense about $\hat{X}_i$, two choices of $\hat{X}_i$ lead to two sign of α_i .

Convention for Affixing Frames to Links

- **First & Last links in the chain**
- **Link 0 (Base Frame)**
  - It is arbitrary, so convenient to choose $\hat{Z}_0$ along axis 1 and locate frame {1} such that it coincides with frame {0} when joint variable 1 is zero.

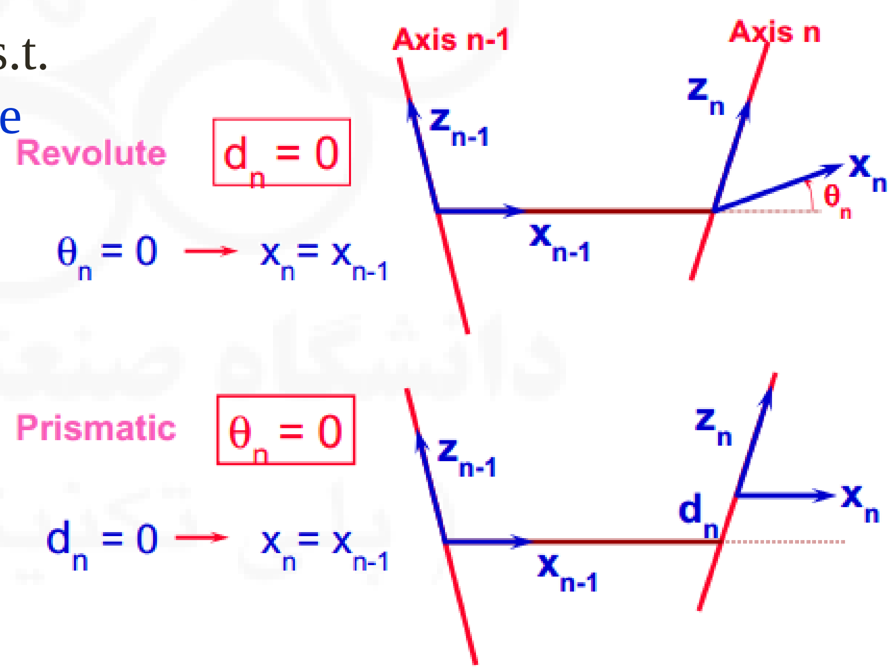


Convention for Affixing Frames to Links

- **First & Last links in the chain**
- **Link n**
  - For joint n **revolute**, $\hat{X}_N$ aligns with $\hat{X}_{N-1}$ when $\theta_n = 0$.
  - The origin of frame {N} is chosen so that $d_n = 0$.

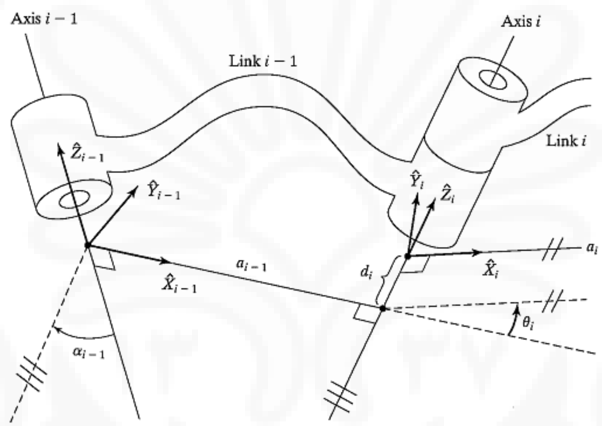
  - For joint n **prismatic**, the direction of $\hat{X}_N$ is chosen so that $\theta_n = 0$.
  - The origin of frame {N} is chosen at the intersection of $\hat{X}_{N-1}$ and joint axis n when $d_n = 0$.

- **Note:** It is arbitrary, but selected s.t. as many link parameter as possible to be zero.



Convention for Affixing Frames to Links

❑ Summary of the Link Parameters in Terms of the Link Frames



a_i = the distance from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured along $\hat{X}_i$

α_i = the angle from $\hat{Z}_i$ to $\hat{Z}_{i+1}$ measured about $\hat{X}_i$

d_i= the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$

θ_i= the angle from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured about $\hat{Z}_i$

Convention for Affixing Frames to Links

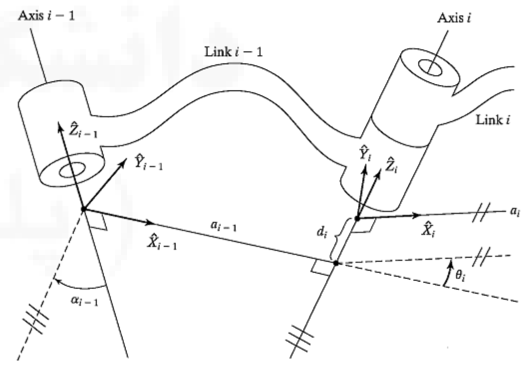
- **Note:** We usually choose $a_i > 0$, because it corresponds to a distance; however α_i, d_i and θ_i are signed quantities.

- **Non-uniqueness of frame attachment:**
  - To align $\hat{Z}_i$ axis with joint axis i, there are two choices of direction in which to point $\hat{Z}_i$.
  - In the case of intersecting joint axes (i.e., $a_i = 0$), there are two choices for the direction of $\hat{X}_i$.
  - When axes i and $i+1$ are parallel, the choice of origin location for $\{i\}$ is arbitrary (Generally chosen in order to cause d_i to be zero).

Convention for Affixing Frames to Links

❑ Summary of Link-Frame Attachment Procedure

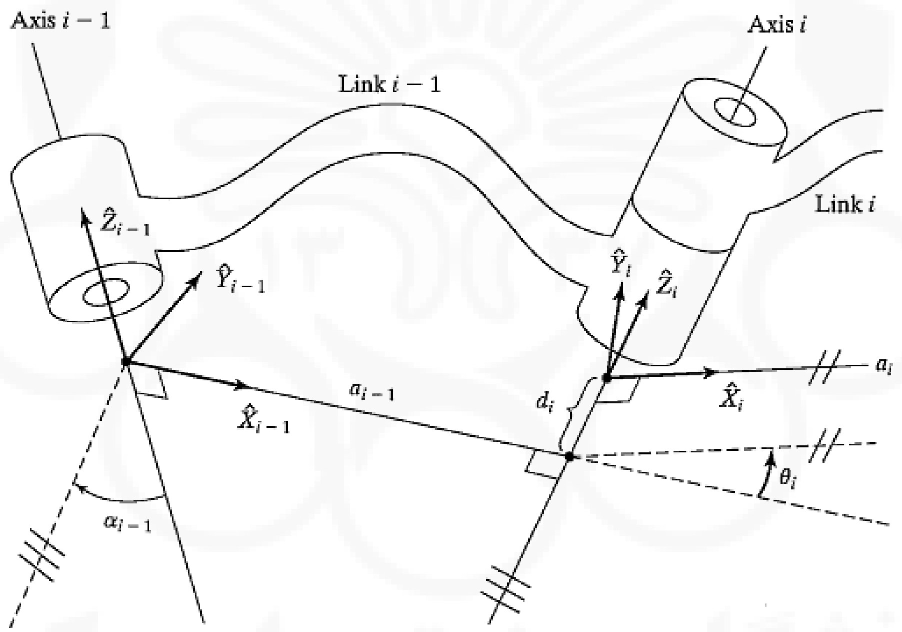
1) Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and $i+1$).
2) Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link-frame origin.
3) Assign the $\hat{Z}_i$ axis pointing along the ith joint axis.
4) Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axes intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axes.
5) Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6) Assign $\{0\}$ to match $\{1\}$ when the first joint variable is zero. For $\{N\}$, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.



Convention for Affixing Frames to Links

- **Note:** Final objective is expressing frame $\{i\}$ relative to frame $\{i-1\}$.

- ***Q:*** Which parameters are important between two frames $\{i-1\}\&\{i\}$?



Convention for Affixing Frames to Links

- **Note:** Final objective is expressing frame $\{i\}$ relative to frame $\{i-1\}$.

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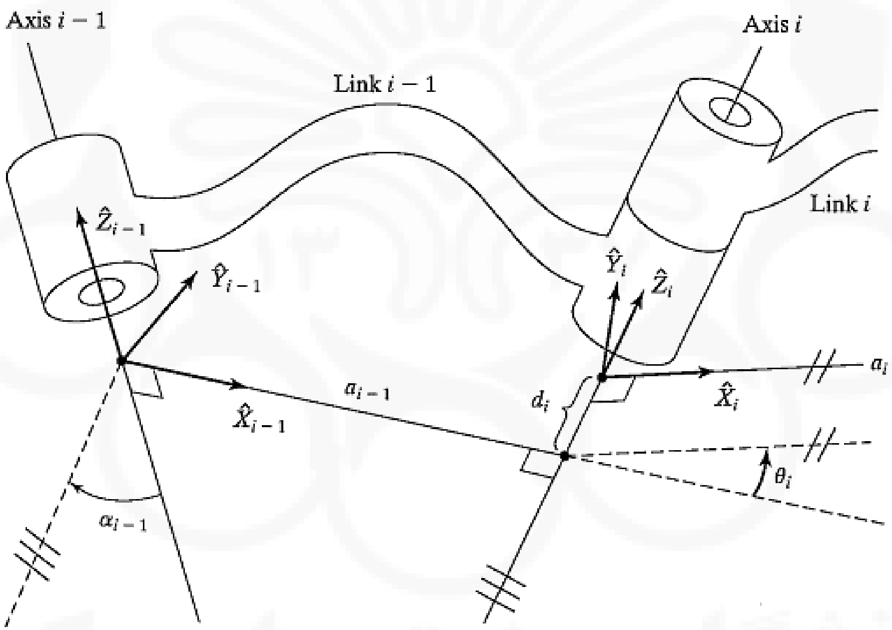
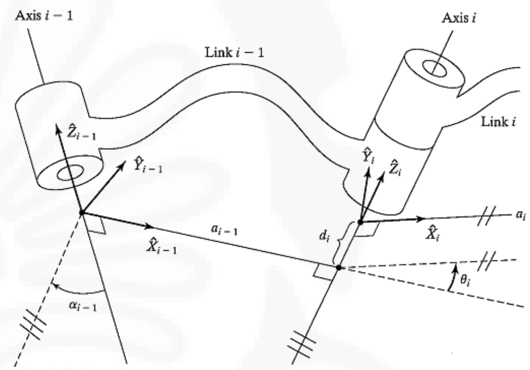


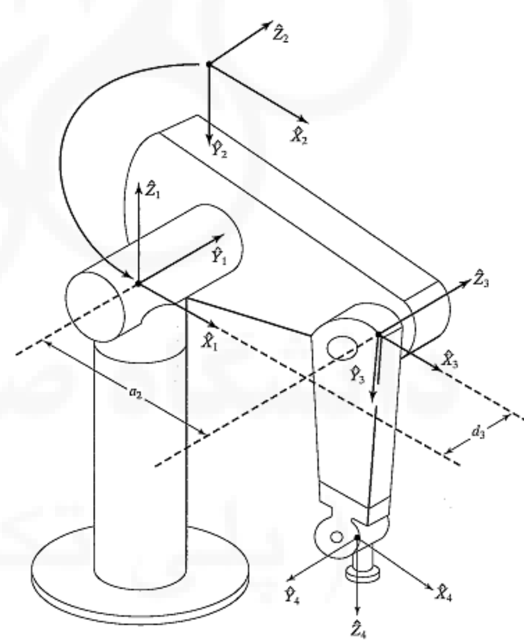
$\{i-1\}\&\{i\}$	α_{i-1}	a_{i-1}	d_i	θ_i

Convention for Affixing Frames to Links

- After frames assignment, the following table should be completed for any given manipulator.

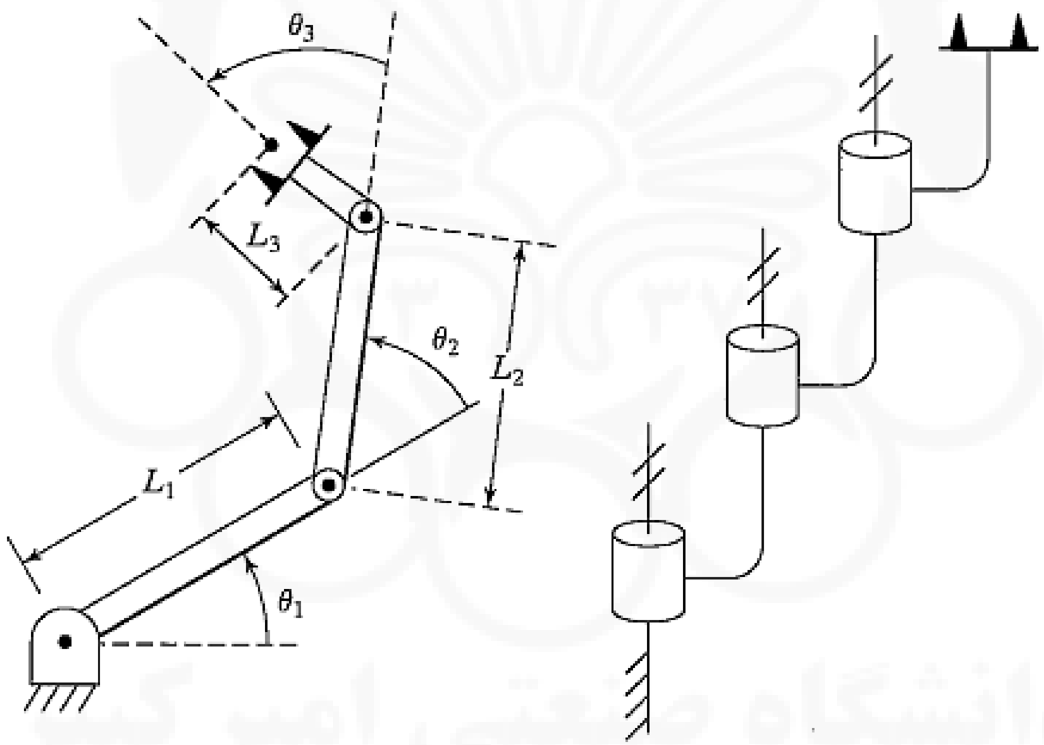
i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1				
2				
...				
...				
n-1				
n				





Convention for Affixing Frames to Links

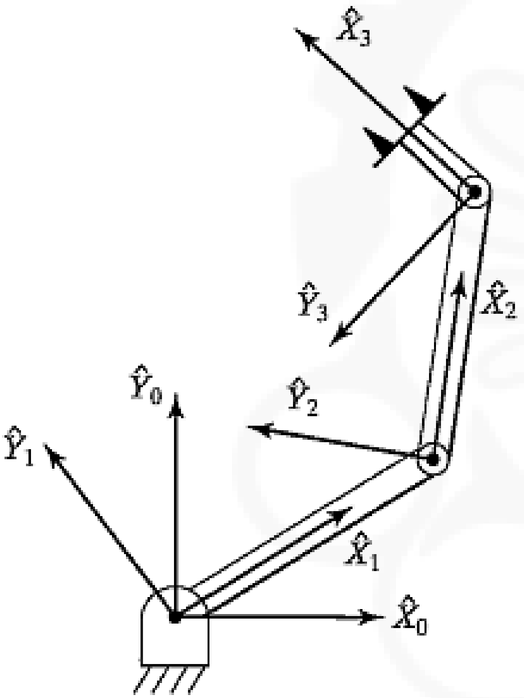
❖ **Example 1: RRR (3R) Manipulator**



- Hash marks on the axes indicate that they are mutually parallel.

Convention for Affixing Frames to Links

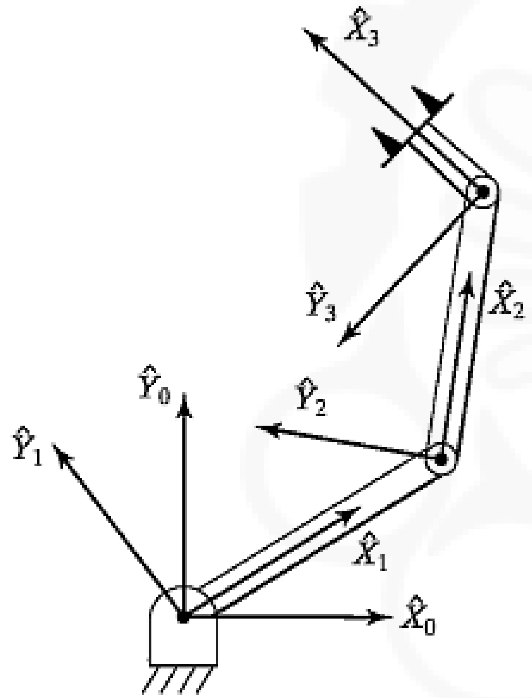
❖ **Example 1: RRR (3R) Manipulator**



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

Convention for Affixing Frames to Links

❖ **Example 1: RRR (3R) Manipulator**

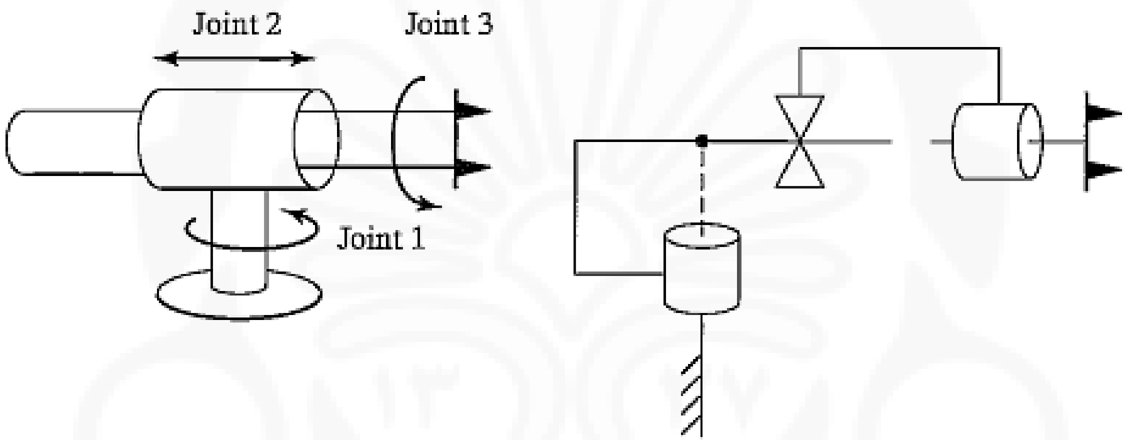


i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

- The arm lies in a plane with all $\hat{Z}$ axes parallel, there are no link offsets, all d_i are zero.
- Joint axes are all parallel and all the $\hat{Z}$ axes are taken as pointing out of the paper, all α_i are zero.
- l_3 does not appear in the link parameters.

Convention for Affixing Frames to Links

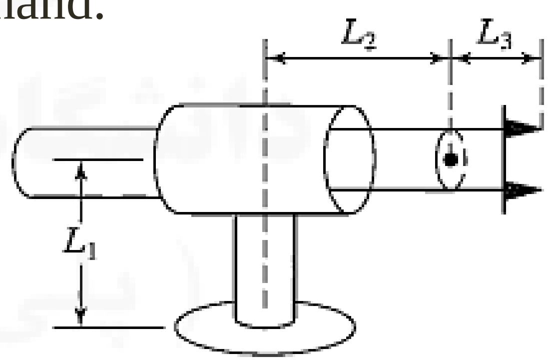
❖ Example 2: RPR Manipulator



- **Polar robot** whose first two joints are analogous to **polar coordinates** when viewed from above.

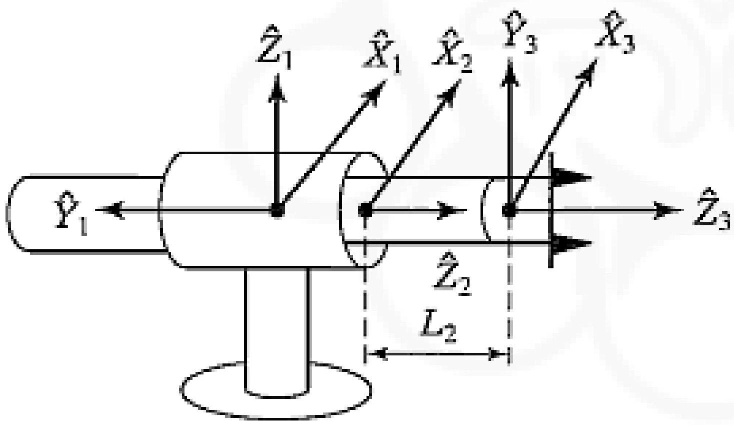
- The last joint (joint 3) provides "roll" for the hand.

- Figure shows the prismatic joint at the minimum extension.



Convention for Affixing Frames to Links

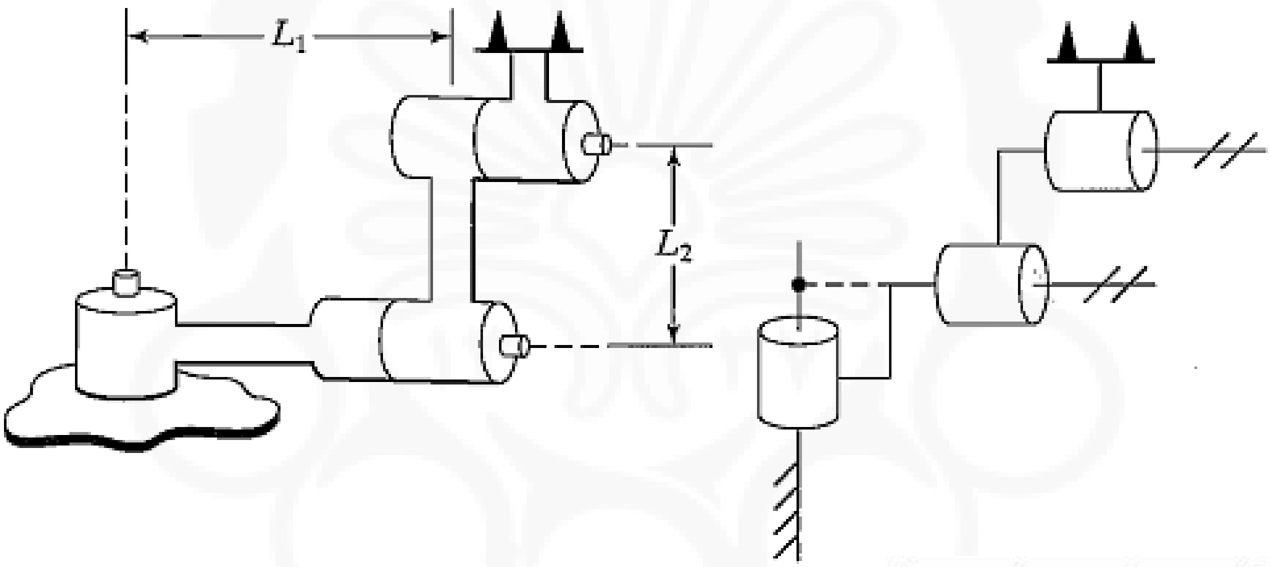
❖ **Example 2: RPR Manipulator**



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

Convention for Affixing Frames to Links

❖ **Example 3: RRR Non-Planar Manipulator**



❖ ***Q:*** Assign the appropriate frames and fill the table !

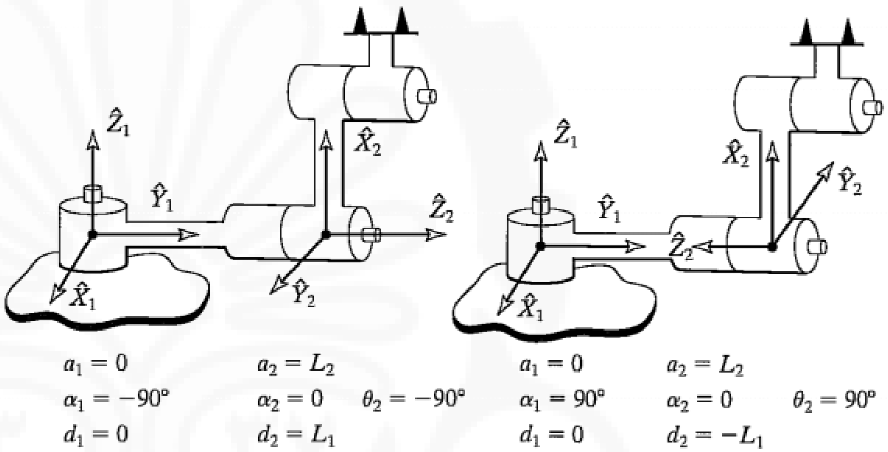
▪ **Note:** Non-uniqueness of frame assignments.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				

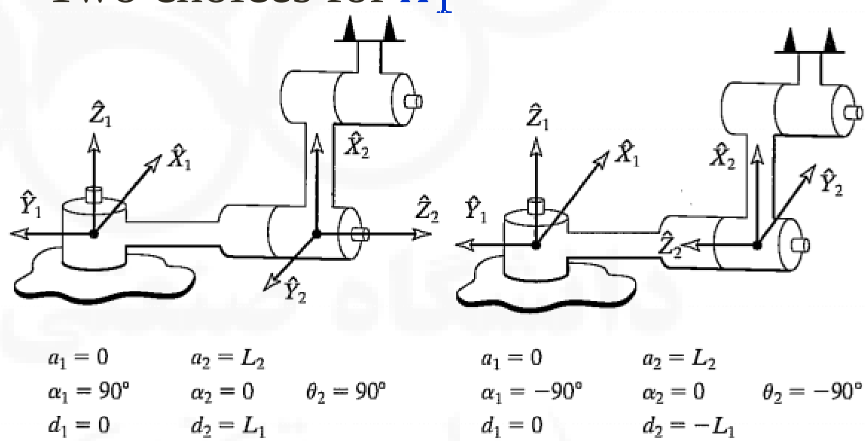
Convention for Affixing Frames to Links

❖ **Example 3: RRR Non-Planar Manipulator**

- Two Possible choices of $\hat{Z}_2$



- Joint 1 and 2 axes intersect → Two choices for $\hat{X}_1$



- Four more possibilities for $\hat{Z}_1$ pointing downward

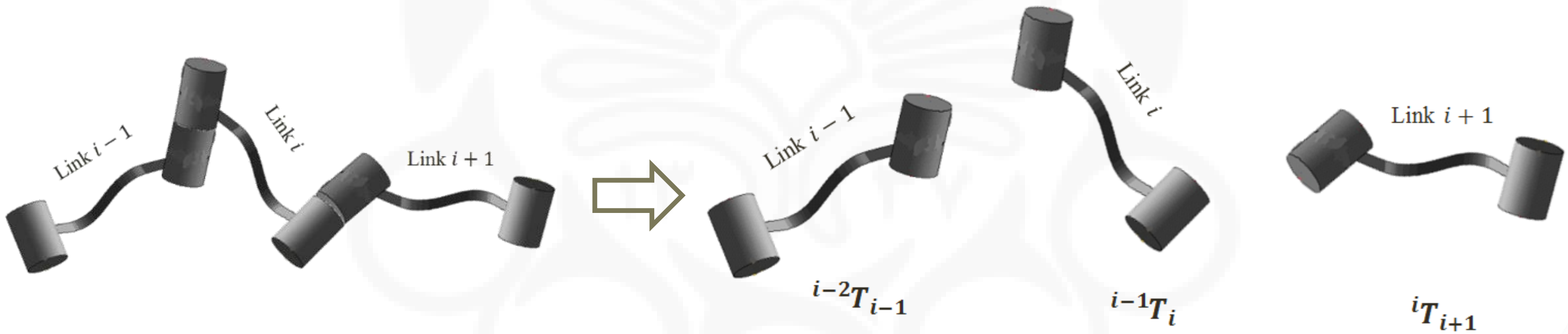
Manipulator Kinematic

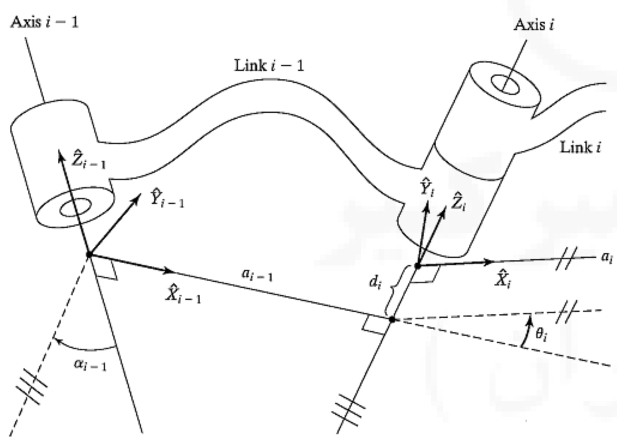
- **Derivation of Link Transformation**
- **Objective:** Transform frame $\{n\}$ relative to frame $\{0\}$ $({}^{0}\boldsymbol{T}_{\boldsymbol{n}})$.

Manipulator Kinematic

❑ Derivation of Link Transformation

- **Objective:** Transform frame $\{n\}$ relative to frame $\{0\}$ $({}^{0}T_{n})$.
- By defining a frame for each link, the kinematic problem is broken to n subproblem, i.e. getting ${}^{i-1}T_{i}$.





$$ {}^{0}T_{n} = {}^{0}T_{1}\ {}^{1}T_{2} \ldots {}^{i-1}T_{i} \ldots {}^{n-1}T_{n} $$

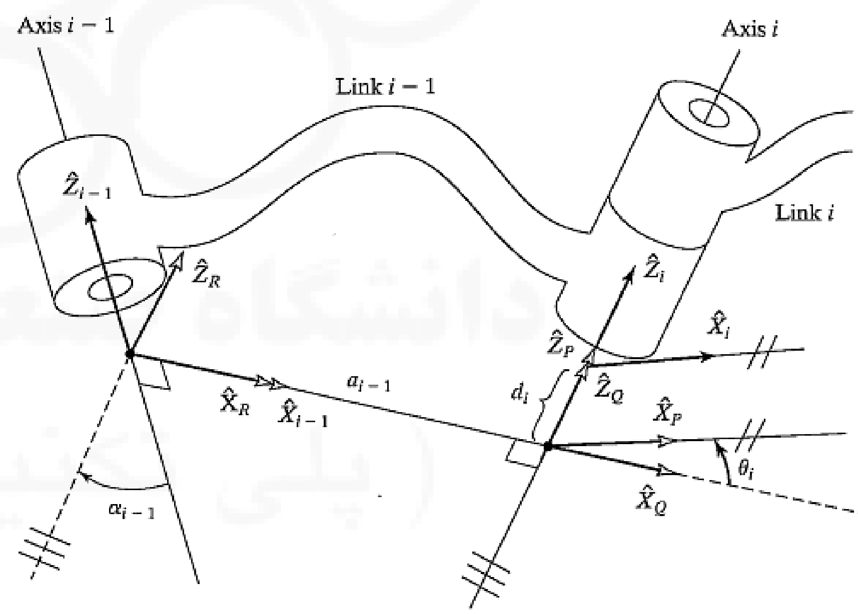
Manipulator Kinematic

Derivation of Link Transformation

- **Objective:** Transform frame $\{i\}$ relative to frame $\{i-1\}$ $\left({}^{i-1}T_i\right)$.

- Define **3 Intermediate Frames** for each link, $\{P\}$, $\{Q\}$ and $\{R\}$: (**New 4 subproblems**)

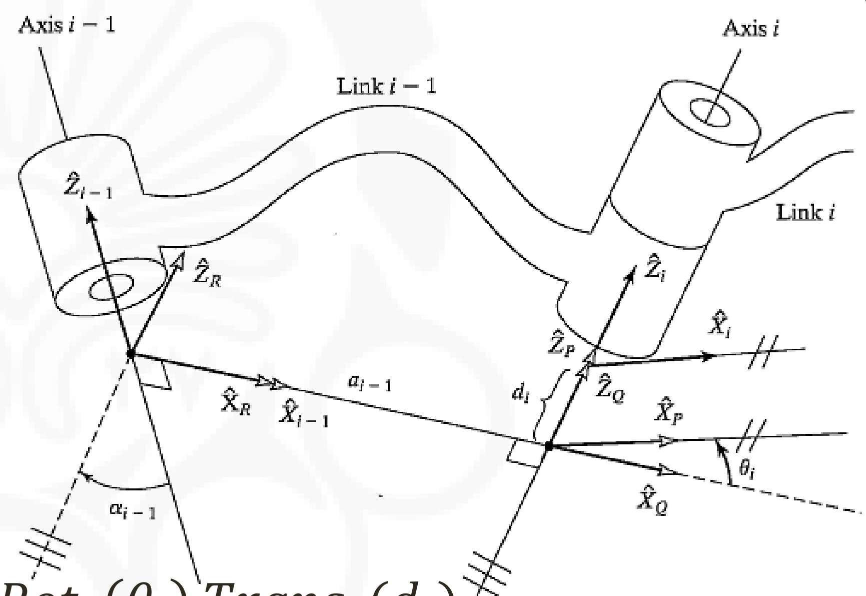
- Frame $\{\boldsymbol{R}\}$ differs from $\{i-1\}$ only by a rotation α_{i-1}.
- Frame $\{\boldsymbol{Q}\}$ differs from $\{R\}$ only by a translation a_{i-1} .
- Frame $\{\boldsymbol{P}\}$ differs from $\{Q\}$ only by a rotation θ_i .
- Frame $\{\boldsymbol{i}\}$ differs from $\{P\}$ only by a translation d_i .



Manipulator Kinematic

❑ Derivation of Link Transformation

- **Objective:** Transform frame $\{i\}$ relative to frame $\{i-1\}$ $\left({}^{i-1}\boldsymbol{T}_{\boldsymbol{i}}\right)$.



$${}^{i-1}T_i = {}^{i-1}T_R \; {}^{R}T_Q \; {}^{Q}T_P \; {}^{P}T_i$$

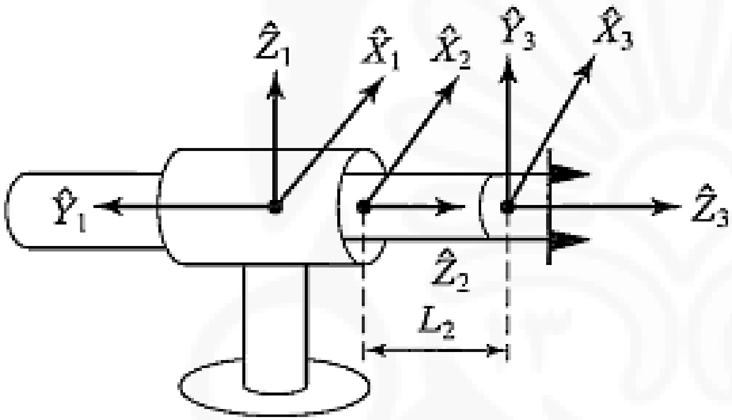
$${}^{i-1}T_i = Rot_x(\alpha_{i-1})\, Trans_x\,(a_{i-1}\,)\, Rot_z(\theta_i)\, Trans_z(d_i)$$

$${}^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Manipulator Kinematic

❑ Derivation of Link Transformation

❖ Example 1: Cylindrical Manipulator



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

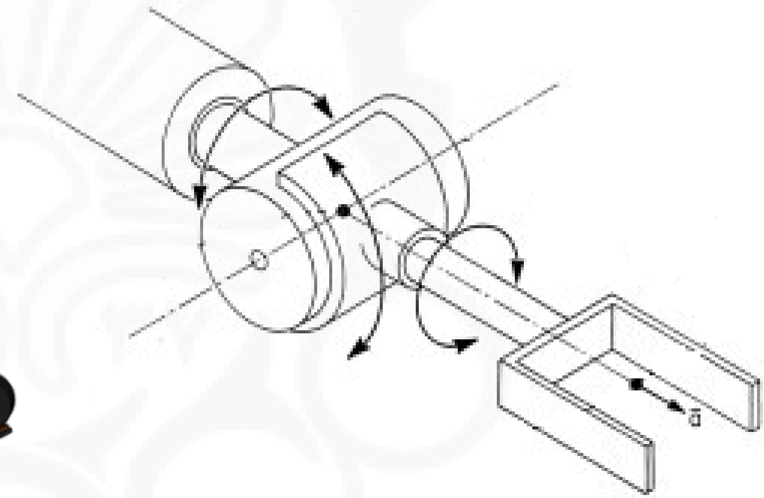
$$
{}^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- To check them against common sense:
  - ➢ The elements of the fourth column of each transform should give the coordinates of the origin of the next higher frame.

Manipulator Kinematic

❑ Derivation of Link Transformation

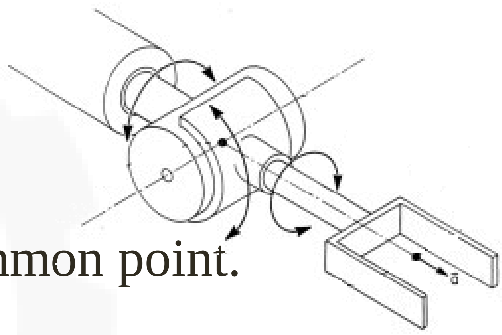
❖ Example 2: PUMA 560 (6R Manipulator)



- A 6DOF rotary-joint manipulator.
- First joint rotates the whole body in vertical plane.
- Joints 2 axis is perpendicular to that of joint 1.
- Joint 2 and 3 are parallel.
- **Three DOF wrist** is installed at the end of link 3.

Manipulator Kinematic

- **Derivation of Link Transformation**
- **Example 2: PUMA 560 (6R Manipulator)**
- **Spherical Wrist**

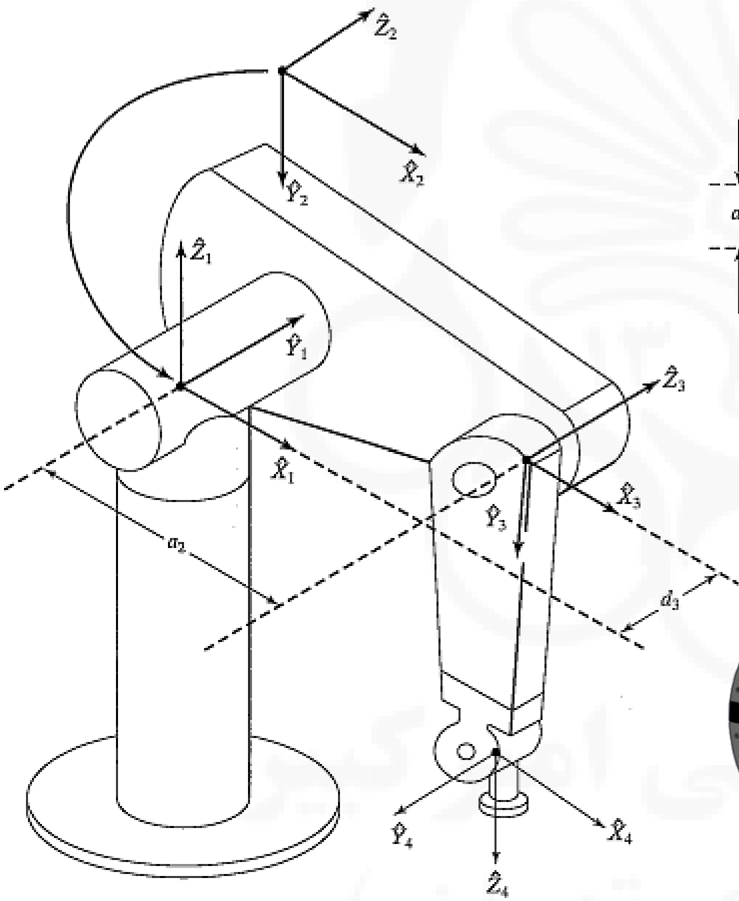


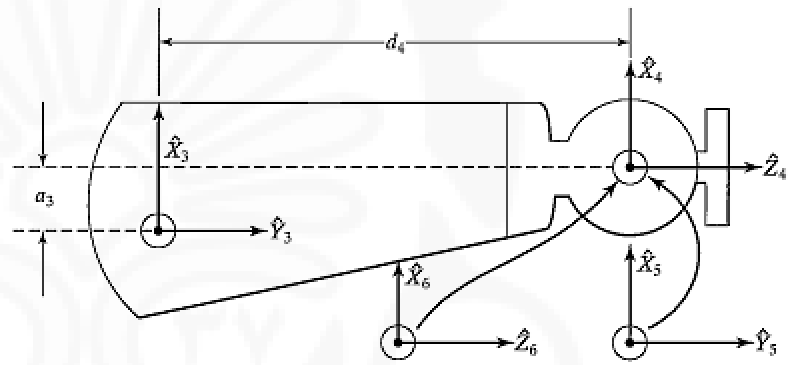
  - Joint axes 4, 5, 6 (the wrist) intersects at a common point.
  - They are also mutually orthogonal.
  - These joints constitute the wrist of the manipulator.
  - Called **Spherical Wrist** because when the point of intersection of the three wrist axes, i.e. C fixed, all points of the wrist move on spheres centered at C.
  - Results in decoupling the position and orientation.
  - A property that greatly simplifies the Inverse Kinematic solution.
  - Manipulator whose last three joint have intersecting axes called **Decoupled Manipulators**.

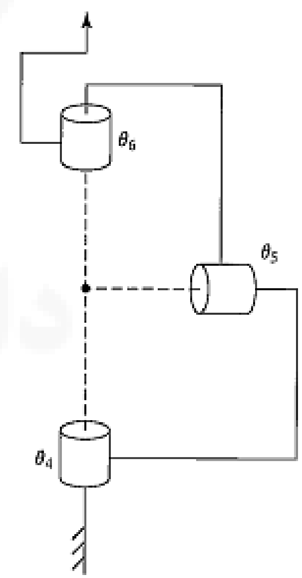
Manipulator Kinematic

❑ Derivation of Link Transformation

❖ Example 2: PUMA 560 (6R Manipulator)





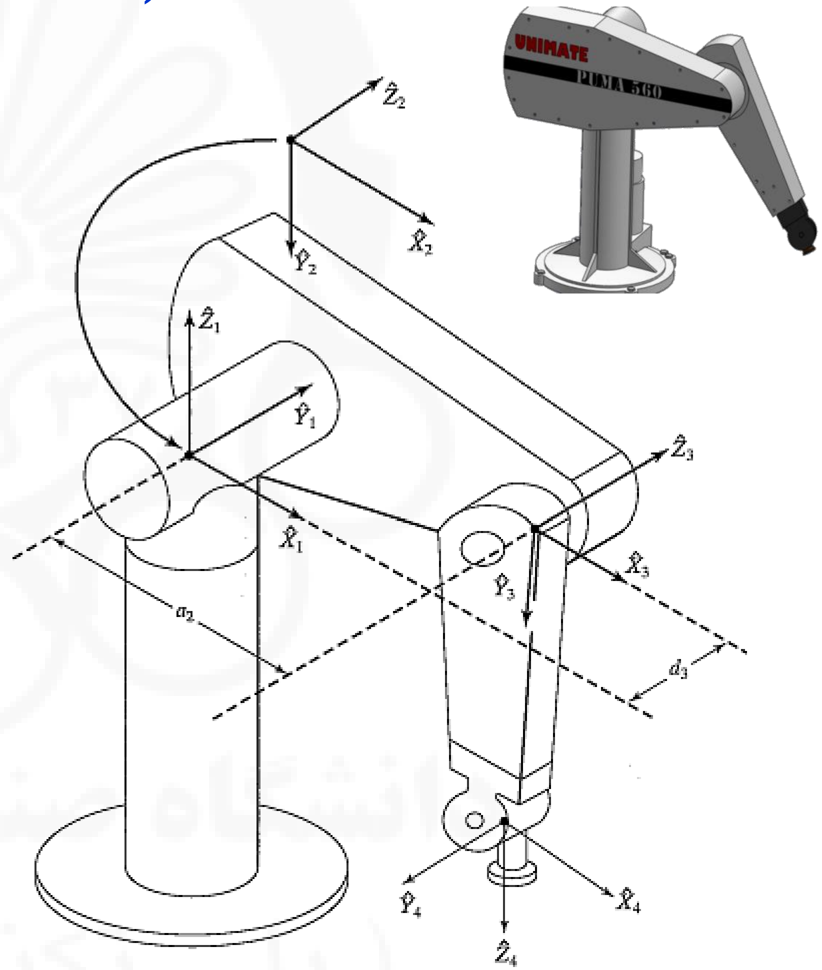


Manipulator Kinematic

❑ Derivation of Link Transformation

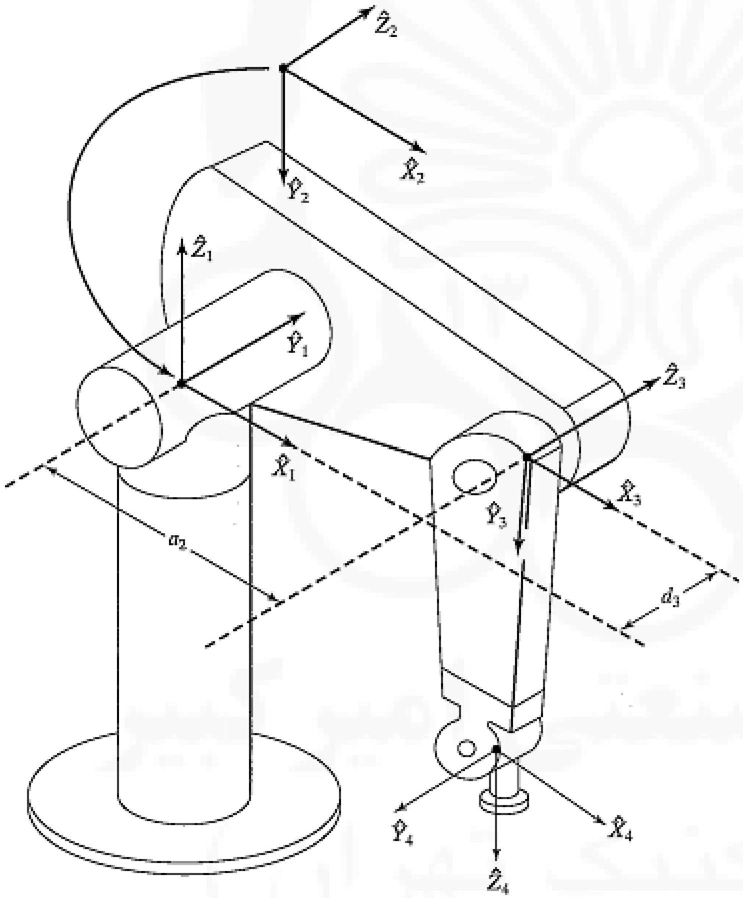
❖ Example 2: PUMA 560 (6R Manipulator)

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6



Manipulator Kinematic

- **Derivation of Link Transformation**
- **Example 2: PUMA 560 (6R Manipulator)**



$$
{}^{0}T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}T_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}T_5 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}T_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Manipulator Kinematic

- **Derivation of Link Transformation**
- **Example 2: PUMA 560 (6R Manipulator)**

$$ {}^4T_6 = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3T_6 = {}^3T_4\,{}^4T_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

- **Joints 2 and 3 are parallel**, easier to use sum of angles.

$$ {}^1T_3 = {}^1T_2\,{}^2T_3 = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Manipulator Kinematic

- **Derivation of Link Transformation**
- **Example 2: PUMA 560 (6R Manipulator)**

$$
{}^{1}T_6 = {}^{1}T_3\,{}^{3}T_6 = \begin{bmatrix} {}^{1}r_{11} & {}^{1}r_{12} & {}^{1}r_{13} & {}^{1}p_x \\ {}^{1}r_{21} & {}^{1}r_{22} & {}^{1}r_{23} & {}^{1}p_y \\ {}^{1}r_{31} & {}^{1}r_{32} & {}^{1}r_{33} & {}^{1}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\boxed{\begin{aligned}
{}^{1}r_{11} &= c_{23}\left(c_4c_5c_6 - s_4s_6\right) - s_{23}s_5s_6 \\
{}^{1}r_{21} &= -s_4c_5c_6 - c_4s_6 \\
{}^{1}r_{31} &= -s_{23}\left(c_4c_5c_6 - s_4s_6\right) - c_{23}s_5c_6 \\
{}^{1}r_{12} &= -c_{23}\left(c_4c_5s_6 + s_4c_6\right) + s_{23}s_5s_6 \\
{}^{1}r_{22} &= s_4c_5s_6 - c_4c_6 \\
{}^{1}r_{32} &= s_{23}\left(c_4c_5c_6 + s_4c_6\right) + c_{23}s_5s_6 \\
{}^{1}r_{13} &= -c_{23}c_4s_5 - s_{23}c_5 \\
{}^{1}r_{23} &= s_4s_5 \\
{}^{1}r_{33} &= s_{23}c_4s_5 - c_{23}c_5
\end{aligned}}
$$

$$
\begin{aligned}
{}^{1}p_x &= a_2c_2 + a_3c_{23} - d_4s_{23} \\
{}^{1}p_y &= d_3 \\
{}^{1}p_z &= -a_3s_{23} - a_2s_2 - d_4c_{23}
\end{aligned}
$$

Manipulator Kinematic

❑ Derivation of Link Transformation

❖ Example 2: PUMA 560 (6R Manipulator)

$$ {}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

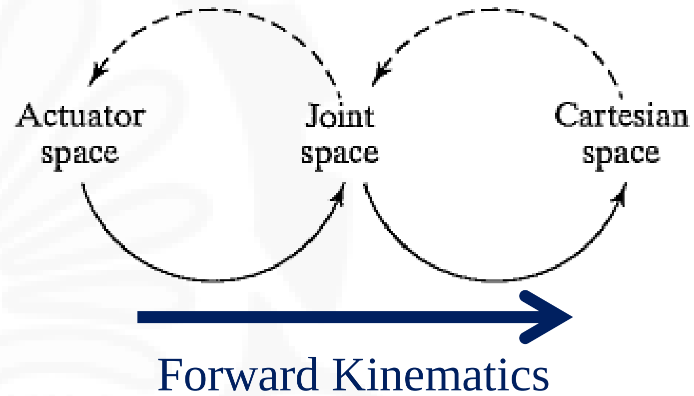
$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$
$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$
$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1 $$
$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1 $$
$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}. $$

Manipulator Kinematic

❑ Joint Space, Cartesian Space & Actuator Space



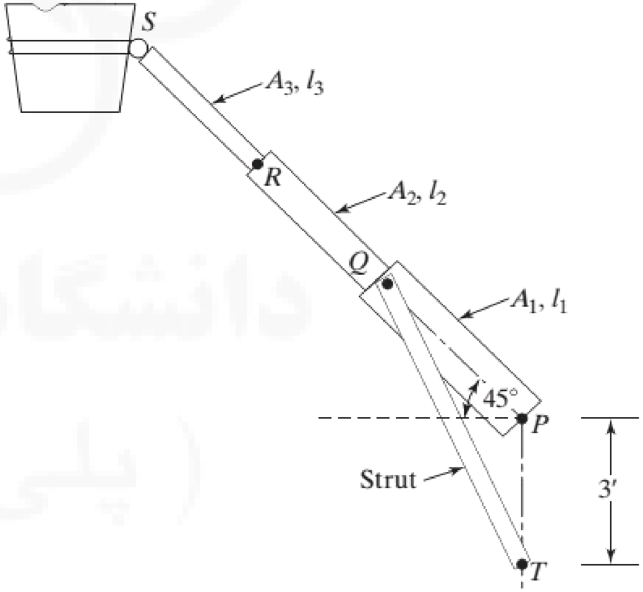
- **Joint Space**
  - ➢ The position of all the links of a n DOF manipulator can be specified with a set of n joint variables (d_i & θ_i).
  - ➢ It is often referred to as $n \times 1$ joint vector.
  - ➢ The space of all such joint vectors is referred to as ***joint space***.

- **Cartesian Space**
  - ➢ Description of position and orientation of the manipulator along orthogonal axes.
  - ➢ It is expressed based on the knowledge of the joint-space description.
  - ➢ Sometimes, called ***task space*** or ***operational space***.

Manipulator Kinematic

- **Joint Space, Cartesian Space & Actuator Space**
- **Actuator Space**

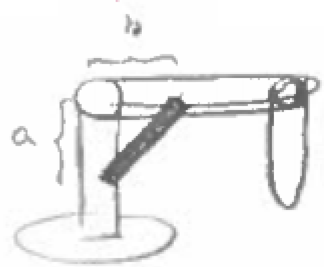
- Sometimes one or more linear actuators are used to rotate a revolute joint.
- Consider the actuator position since sensors measure actuator vector.
- Computations needed to get joint vector from actuator vector.
- The space of all such actuator vector is called ***actuator space***.

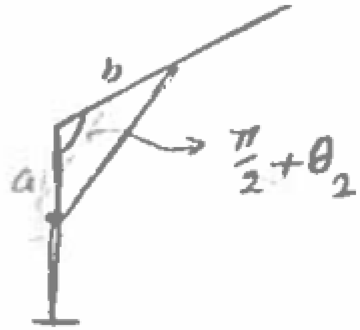


Manipulator Kinematic

❑ Joint Space, Cartesian Space & Actuator Space

❖ Example: Linear Actuator





a, b are fixed

Actuator Variable : A_2 → تغیر طول عضو محرک

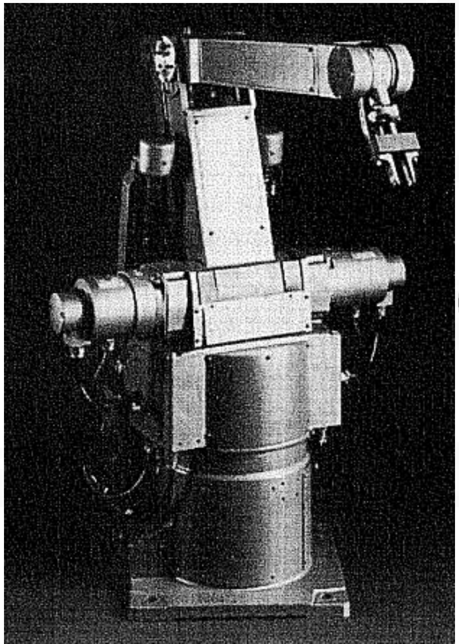
$$A_2 = \left[a^2+b^2-2ab\cos\left(\frac{\pi}{2}+\theta_2\right)\right]^{\frac{1}{2}} - \sqrt{a^2+b^2} \Longrightarrow \theta_2 = f_1(A_2)$$

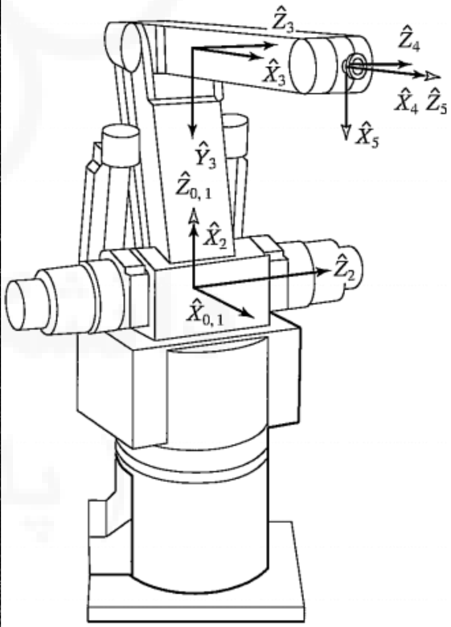
$\theta_1, \theta_2, \theta_3 \longrightarrow$ forward kinematics

$$\Rightarrow X = f_2(\theta_1, \theta_2, \theta_3) = f_2(\theta_1, f_1(A_2), \theta_3) = f_3(\theta_1, A_2, \theta_3)$$

Manipulator Kinematic

- **Joint Space, Cartesian Space & Actuator Space**
- **Example: The Yasukawa Motoman L-3**
- A popular 5-DOF industrial manipulator.
- It is not a simple open kinematic chain.
- It uses two linear actuators coupled to links 2 and 3 with four-bar linkages.
- One purpose is to increase the structural rigidity of the main linkages of the robot.

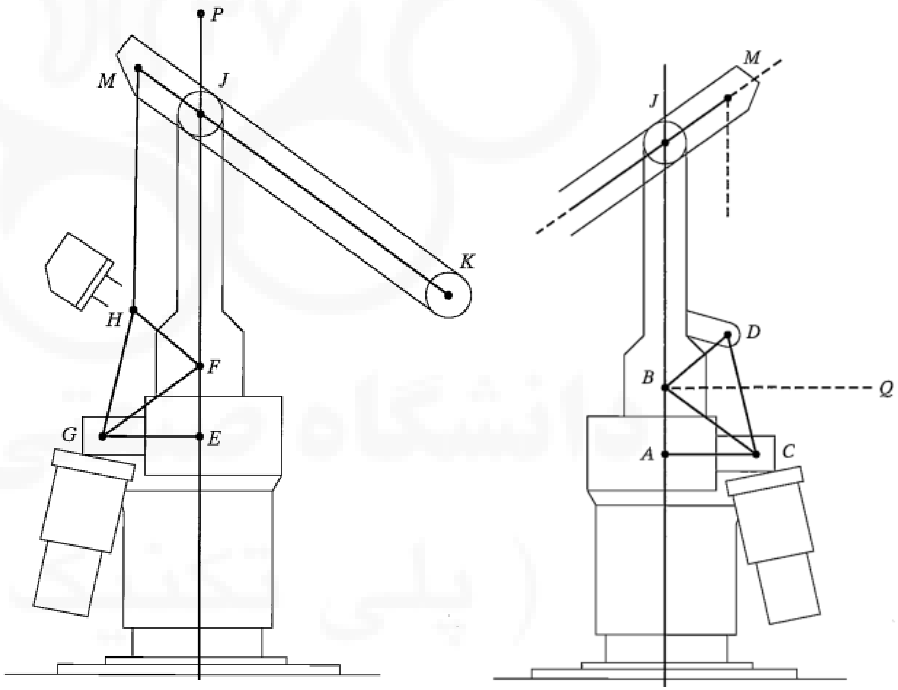




Manipulator Kinematic

- **Joint Space, Cartesian Space & Actuator Space**
- **Example: The Yasukawa Motoman L-3**
- Actuator 2 is used to position joint 2; while it is doing so, link 3 remains in the same orientation relative to the base of the robot.

- Actuator 3 is used to adjust the orientation of link 3 relative to the base of the robot (rather than relative to the preceding link as in a serial-kinematic-chain robot).



Manipulator Kinematic

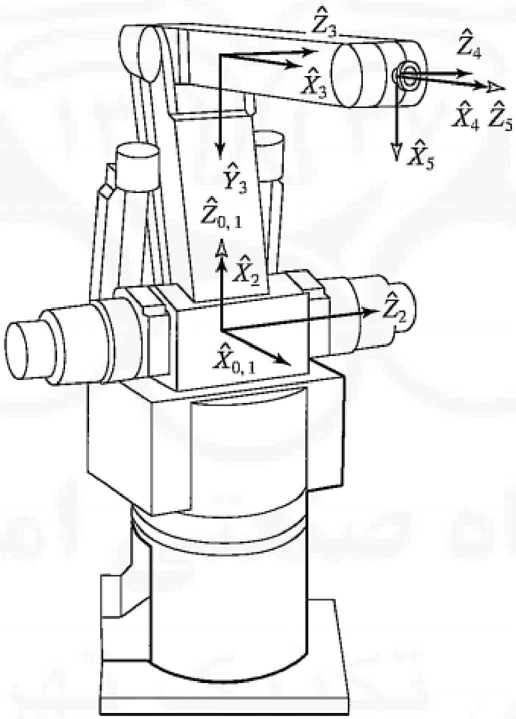
- **Joint Space, Cartesian Space & Actuator Space**
- **Example: The Yasukawa Motoman L-3**
- Solving the kinematics in two stages:
  - **First**, solving for joint angles from actuator positions;

$$
\begin{aligned}
\theta_1 &= k_1 A_1 + \lambda_1, \\
\theta_2 &= \cos^{-1}\left(\frac{(k_2 A_2 + \lambda_2)^2 - \alpha_2^2 - \beta_2^2}{-2\alpha_2\beta_2}\right) + \tan^{-1}\left(\frac{\phi_2}{\gamma_2}\right) + \Omega_2 - 270°, \\
\theta_3 &= \cos^{-1}\left(\frac{(k_3 A_3 + \lambda_3)^2 - \alpha_3^2 - \beta_3^2}{-2\alpha_3\beta_3}\right) - \theta_2 + \tan^{-1}\left(\frac{\phi_3}{\gamma_3}\right) - 90°, \\
\theta_4 &= -k_4 A_4 - \theta_2 - \theta_3 + \lambda_4 + 180°, \\
\theta_5 &= -k_5 A_5 + \lambda_5.
\end{aligned}
$$

  - Mapping a set of actuator values (A_i) to the equivalent set of joint values (θ_i).

Manipulator Kinematic

- **Joint Space, Cartesian Space & Actuator Space**
- **Example: The Yasukawa Motoman L-3**
- Solving the kinematics in two stages:
  - **Second**, solving for Cartesian position and orientation of the last link from joint angles. (treat the system as a simple open-kinematic-chain SR device).



Manipulator Kinematic

- **Joint Space, Cartesian Space & Actuator Space**
- **Example: The Yasukawa Motoman L-3**
- Solving the kinematics in two stages:
  - **Second**, solving for Cartesian position and orientation of the last link from joint angles. (treat the system as a simple open-kinematic-chain SR device)

i	$\alpha_i - 1$	$a_i - 1$	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	θ_4
5	$90°$	0	0	θ_5

$$
{}^0_5T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
r_{11} &= c_1 c_{234} c_5 - s_1 s_5, \\
r_{21} &= s_1 c_{234} c_5 + c_1 s_5, \\
r_{31} &= -s_{234} c_5, \\
r_{12} &= -c_1 c_{234} s_5 - s_1 c_5, \\
r_{22} &= -s_1 c_{234} s_5 + c_1 c_5, \\
r_{32} &= s_{234} s_5, \\
r_{13} &= c_1 s_{234}, \\
r_{23} &= s_1 s_{234}, \\
r_{33} &= c_{234},
\end{aligned}
$$

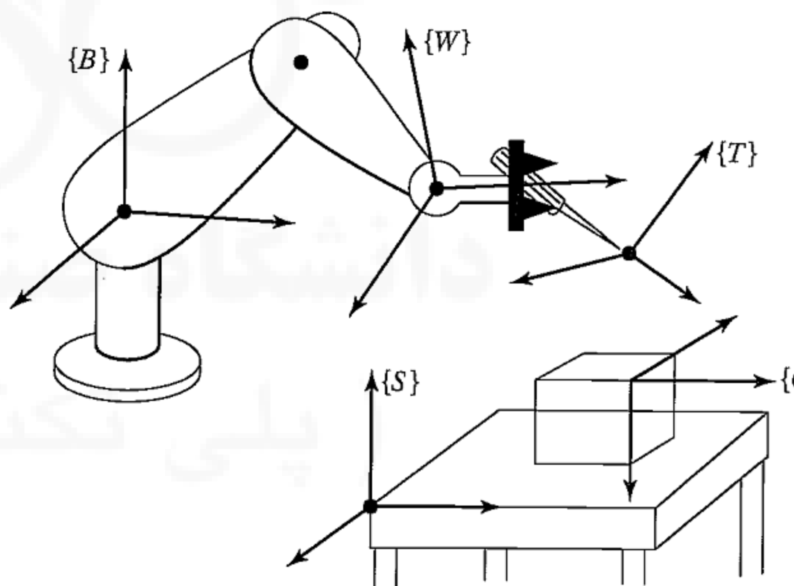
$$
\begin{aligned}
p_x &= c_1(l_2 c_2 + l_3 c_{23}), \\
p_y &= s_1(l_2 c_2 + l_3 c_{23}), \\
p_z &= -l_2 s_2 - l_3 s_{23}.
\end{aligned}
$$

Frames with Standard Names

- All robot motions will be described in terms of these frames:

- **Base frame {B}**
  - Located at the base of the manipulator (Another name for frame {0}).
  - Affixed to a nonmoving part of the robot.

- **Station frame {S}**
  - Located in a task-relevant location (e.g. at the corner of the table on which the robot is to work).
  - It is sometimes called world frame or universe frame.
  - It is specified wrt the Base frame.



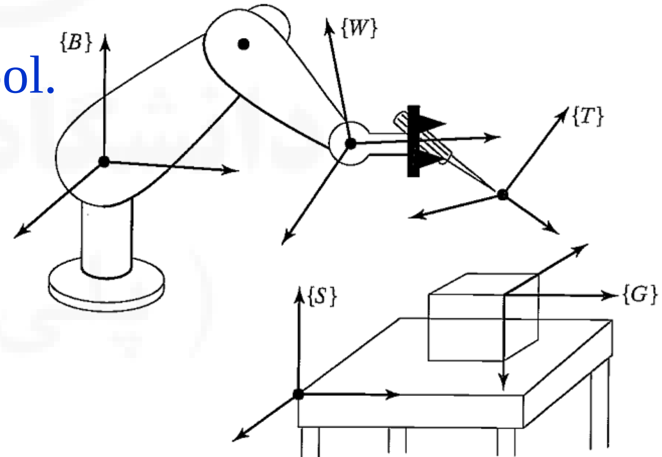
Frames with Standard Names

- All robot motions will be described in terms of these frames:

- **Wrist frame {W}**
  - Affixed to the last link of the manipulator (another name for frame $\{N\}$).
  - It is defined wrt the base of the manipulator.

- **Tool frame {T}**
  - Affixed to the end of the tool the robot is holding.
  - If the hand is empty, $\{T\}$ is located with the origin between the fingertip of the robot.

- **Goal frame {G}**
- Location to which the robot is to move the tool.
- At the end of motion, tool frame brings coincident to the goal frame.



The END

- **References:**

1) .