دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

دانشکده مهندسی مکانیک
دانشگاه صنعتی امیرکبیر

Lecture 1_3:
Introduction

Descriptions, Kinematics & Dynamics

Robotics

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Outlines

Definition

- The initiation was from 1949 by MIT Servo Lab (Numerical Control Machines).

- Machines which are limited to one class of task: **Fixed Automation**

- **Robot**: A **reprogrammable**, **multi-functional** manipulator designed to move material parts, or tools or other specified devices through variable programmed motions for the performance of a **variety of tasks.**

- **Industrial robot:**
- A mechanical device which can be **programmed** to perform a **wide variety** of applications.

❖ What is difference between Robot & Manipulator?

Description of the Objects Location

- **Concern:** *Location* of **objects** in three-dimensional space.

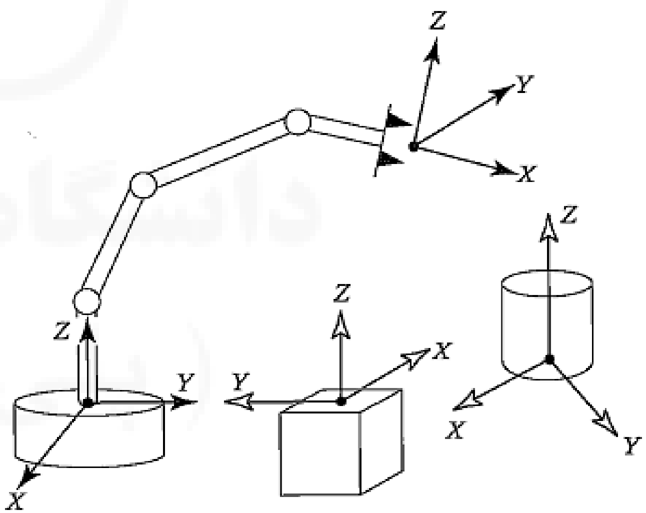
- **Objects:** Links of the manipulator, parts and tools and other objects in environment.

- Two attributes: **Position** and **Orientation**

- **Description method:**
  - Attach a coordinate system (**Frame**), rigidly to the object.
  - Describe its position and orientation with respect to some **Reference** coordinate system.

- *Any frame* can serve as a **reference** system.

- Challenge: **Transforming** or changing the description from one frame to another.



Kinematics

- The science that treats motion without regard to the forces which cause it.
- Position, velocity, acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable ???)
- It expresses the geometrical and time-based properties of the motion.
- It includes:
  - Forward Kinematics
  - Inverse Kinematics

Kinematics

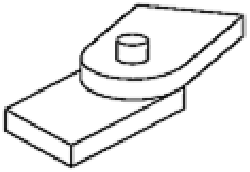
Manipulator Joints

- Manipulators consist of nearly rigid **links**, connected by **joints** that allow **relative motion** of neighboring links.

- **Note:** *Position sensors* measure the **relative position** of neighboring links.

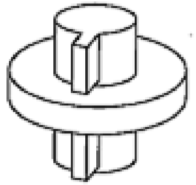
- **Two Joint Types:**

➢ Rotary or **Revolute** joints (**R**):
The relative displacement is called **joint angle.**



Revolute

➢ Sliding or **Prismatic** joints (**P**):
The relative displacement is called **joint offset.**



Prismatic

Kinematics

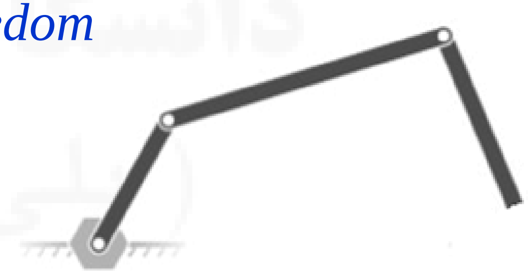
❑ Degrees of Freedom

- The number of **independent position variables** that would have to be specified in order to locate all parts of the mechanism.

❖ Example: A four-bar linkage

- A serial manipulator is an **open kinematic chain**, and because each joint position is usually defined with a single variable, so …

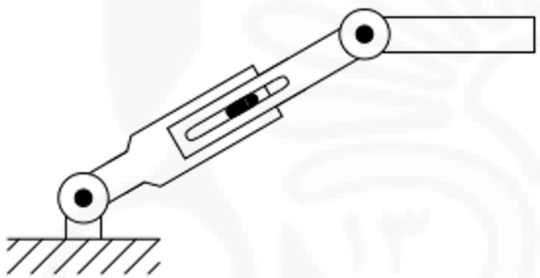
  Number of joints = Number of degrees of freedom

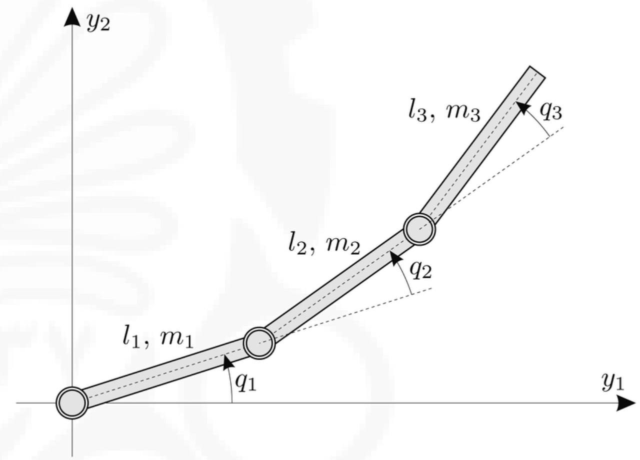


Kinematics

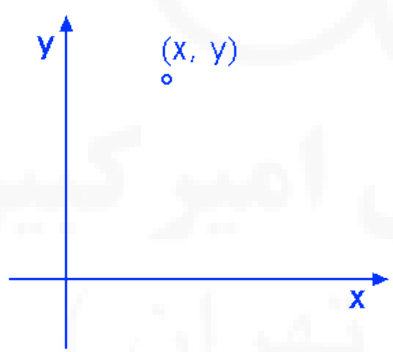
❑ Degrees of Freedom

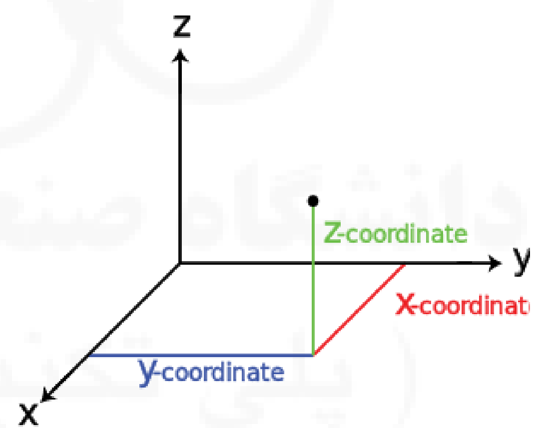
- Two types of DoF :
  - ➢ Manipulator DoF





  - ➢ Motion DoF





Kinematics

- **Degrees of Freedom**
- In 2-D plane:
- In 3-D Cartesian space:
- A 6-DOF Robot:

- A 8-DOF Robot:

← Outline

Kinematics

- **Degrees of Freedom**
- In 2-D plane:
  A Robot Hand can have maximum 3 DOF: 2 translations and 1 rotation.

- In 3-D Cartesian space:
  A Robot Hand can have maximum 6 DOF: 3 translations and 3 rotations.

- A 6 DOF Robot:
  Robot Hand can *generally* move freely in operational space, along 3 translational axes and around 3 rotational axes.

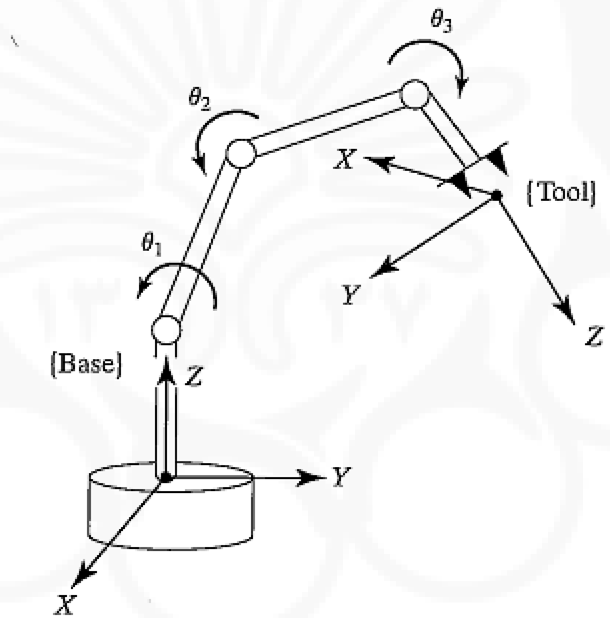
- A 8-DOF Robot:
  This Robot has 8 DOF in *actuator joint space* and can have 6 motions in Robot *operational space*.
- This is a **redundant** Robot. (Why Redundancy?)

Kinematics

❑ Forward Kinematics

- Given a set of <u>joint variables</u>, compute the position and orientation of the tool frame relative to the base frame.

- $X = f(\theta)$



Forward

$\theta \longrightarrow X$

$\dot{\theta} \longrightarrow \dot{X}$

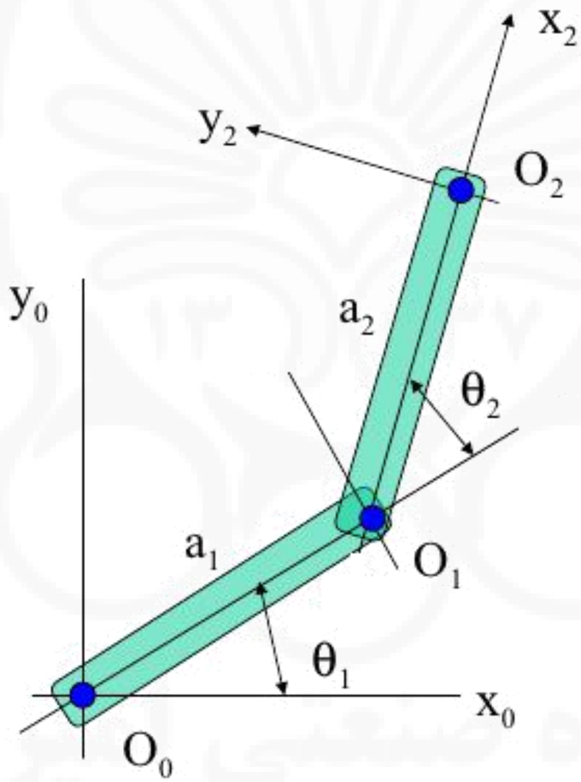
$\ddot{\theta} \longrightarrow \ddot{X}$

- **FK** is a **Mapping** of manipulator position from a **joint space** description into a **Cartesian space** description.

- **Cartesian space:** Task space or Operational space.

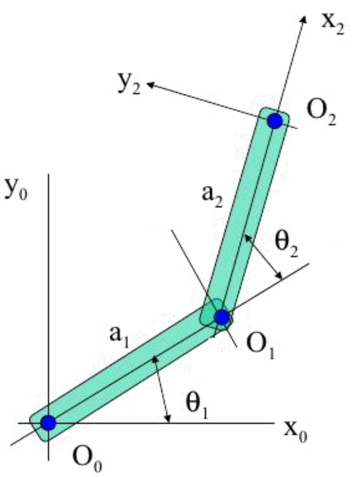
Kinematics

- **Forward Kinematics**
- Example: Two Links Manipulator



Kinematics

- **Forward Kinematics**
- Example: Two Links Manipulator



$$\begin{cases} \text{Tool Frame } (x_2, y_2) \\ \text{Base Frame } (x_0, y_0) \end{cases}$$

- Position:

$$\begin{cases} x_t = a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) \\ y_t = a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2) \end{cases}$$

- Orientation (Direction Cosines):

$x_2 \cdot x_0 = \cos(\theta_1+\theta_2)$ $\quad$ $y_2 \cdot x_0 = -\sin(\theta_1+\theta_2)$

$x_2 \cdot y_0 = \sin(\theta_1+\theta_2)$ $\quad$ $y_2 \cdot y_0 = \cos(\theta_1+\theta_2)$

$$\Rightarrow R = \begin{bmatrix} x_2 \cdot x_0 & y_2 \cdot x_0 \\ x_2 \cdot y_0 & y_2 \cdot y_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix}$$

Kinematics

❑ Inverse kinematics

- Given the position and orientation of the end-effector, calculate **all possible** sets of joint angles

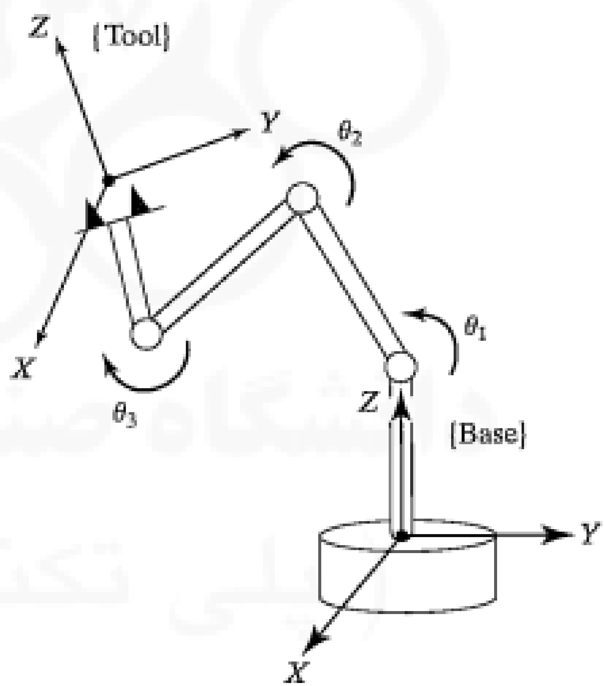
- $\theta = f^{-1}(\mathrm{X}) = g(X)$

- **Mapping** of "locations": from 3-D Cartesian space into the joint space.

❖ ***Q:*** Why inverse kinematics?

Inverse

$$\begin{aligned} X &\longrightarrow \theta \\ \dot{X} &\longrightarrow \dot{\theta} \\ \ddot{X} &\longrightarrow \ddot{\theta} \end{aligned}$$



Kinematics

- **Inverse kinematics**
- **IK** is a Complicated geometrical problem, routinely solved thousands of times daily in *human biological system*.

- Solution of this problem is the most important element in a manipulator system.

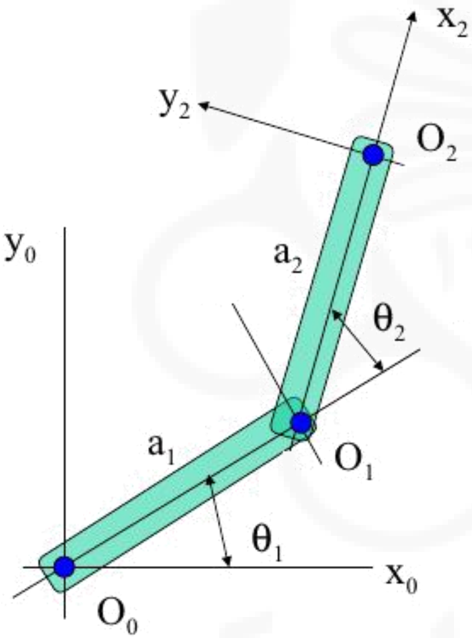
- **Early robots** were simply moved (sometimes by hand) to desired locations, which were then recorded as a set of joint values.

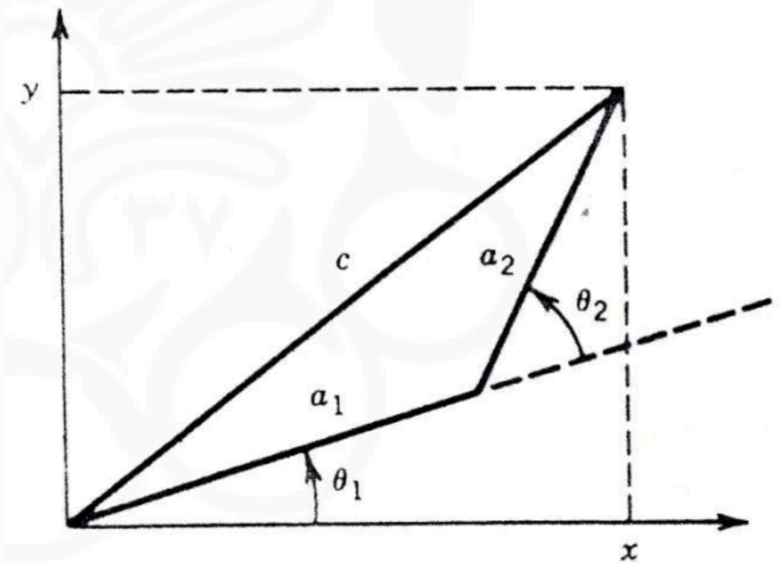
- **Solution** is so complicated:
  - Nonlinear equations
  - Not always easy (or even possible) in a closed form (Existence)
  - Multiple solutions

- The existence or nonexistence of a kinematic solution determines the manipulator **workspace.**

Kinematics

- **Inverse kinematics**
- Example: Two Links Manipulator



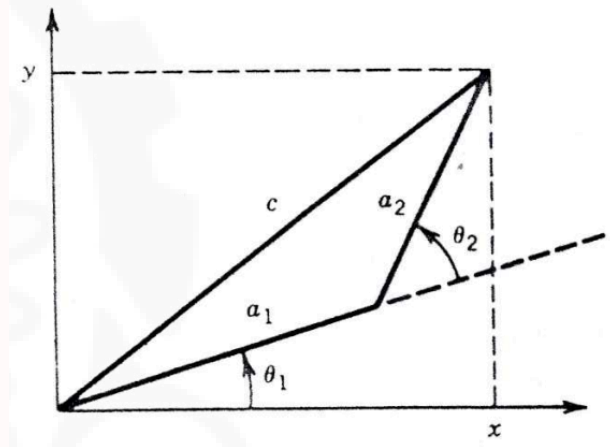


Kinematics

- **Inverse kinematics**
- Example: Two Links Manipulator
- Law of cosines:

$$c^2 = a_1^2 + a_2^2 - 2a_1a_2 \underbrace{\cos(\pi-\theta_2)}_{-\cos\theta_2}$$

$$c^2 = x^2 + y^2$$



- Finding θ_2:

$$\cos\theta_2 = \frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2} = M$$

$$\sin\theta_2 = \pm\sqrt{1-M^2} \implies \theta_2 = \tan^{-1}\frac{\pm\sqrt{1-M^2}}{M}$$

- Finding θ_1:

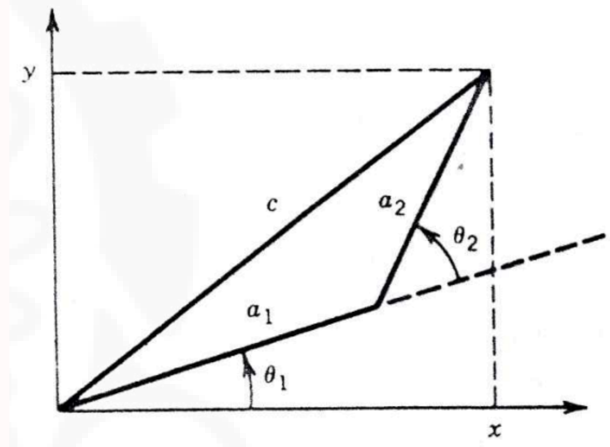
$$x = a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \longrightarrow \theta_1 \checkmark \rightarrow \text{for each } \theta_2$$

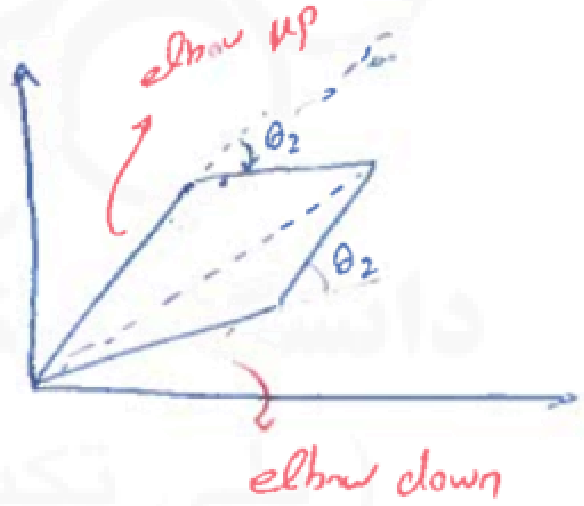
Kinematics

❑ **Inverse kinematics**

❖ Example: Two Links Manipulator

▪ Two configurations:





▪ IK Problem:

➢ Nonlinear Equations

➢ Closed Form Solutions

➢ Multiple Solutions

Velocities, Static Forces, Singularities

- **Velocities:**
- In addition to static positioning problems, we may analyze manipulators in **motion**.

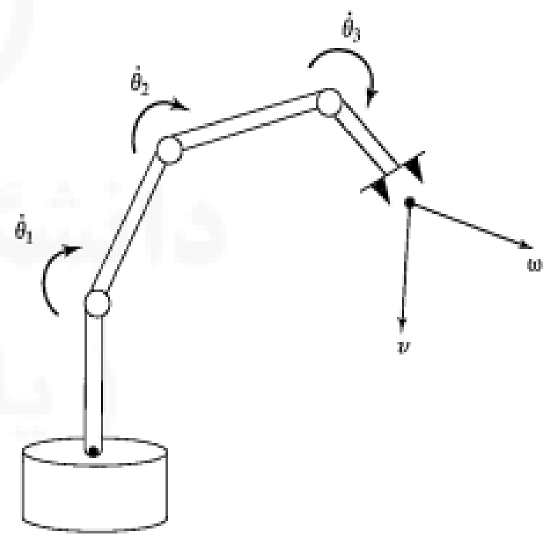
- kinematic differentiation expresses a mapping from velocities in joint space to velocities in Cartesian space.

$$If \quad X = f(\theta) \qquad So \qquad \dot{X} = \frac{\partial f}{\partial \theta}\,\dot{\theta} = J(\theta)\,\dot{\theta}$$

- $J(\theta)$ is called the Jacobian matrix of the manipulator.

- The Jacobian matrix is configuration dependent.

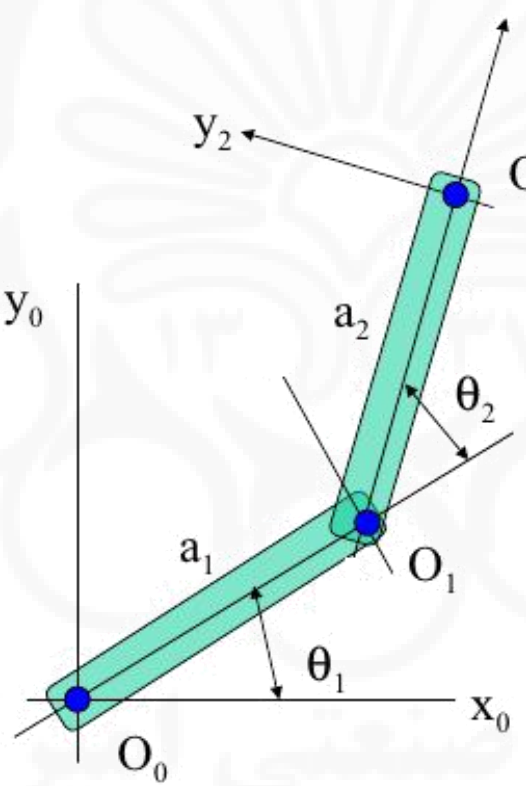
- At certain points, called singularities, this mapping, $J(\theta)$, is not invertible. ($|J(\theta)| = 0$).



- ***Q:*** So what?

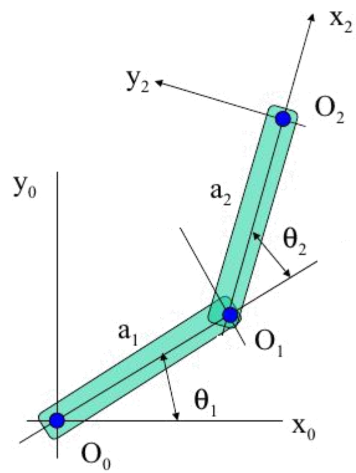
Velocities, Static Forces, Singularities

- **Velocities:**
- Example 1: Two Links Manipulator



❑ **Velocities:**

❖ Example 1: Two Links Manipulator



- FK:

$$\begin{cases} x = a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) \\ y = a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2) \end{cases}$$

- Differentiation:

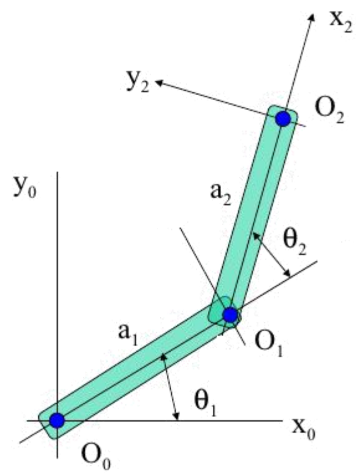
$$\dot{x} = -a_1 \dot{\theta}_1 \sin\theta_1 - a_2 \dot{\theta}_1 \sin(\theta_1+\theta_2) - a_2 \dot{\theta}_2 \sin(\theta_1+\theta_2)$$

$$\dot{y} = a_1 \dot{\theta}_1 \cos\theta_1 + a_2 \dot{\theta}_1 \cos(\theta_1+\theta_2) + a_2 \dot{\theta}_2 \cos(\theta_1+\theta_2)$$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) & -a_2 \sin(\theta_1+\theta_2) \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & a_2 \cos(\theta_1+\theta_2) \end{bmatrix}}_{J(\theta)} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Velocities, Static Forces, Singularities

- **Velocities:**
- Example 1: Two Links Manipulator
- Inverse velocity problem:



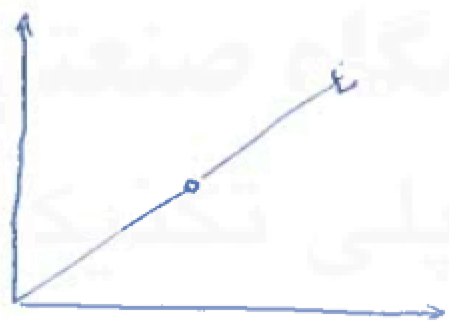
$$\begin{cases} \sin\theta_1 = S_1 \\ \sin(\theta_1+\theta_2) = S_{12} \end{cases} \qquad \cos\theta_1 = C_1 \qquad \cos(\theta_1+\theta_2) = C_{12}$$

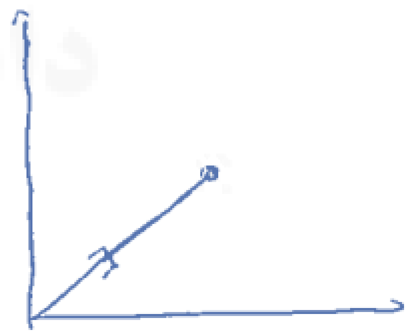
- $\dot{\theta} = J^{-1}\dot{X} = \dfrac{J^*}{|J|}\dot{X}$

$$\dot{\theta} = \frac{1}{a_1 a_2 \sin\theta_2}\begin{bmatrix} a_2 C_{12} & a_2 S_{12} \\ -a_1 C_1 - a_2 C_{12} & -a_1 S_1 - a_2 S_{12} \end{bmatrix}\dot{X}$$

$|J| = 0$ if $\begin{cases} \theta_2 = 0 \\ \theta_2 = \pi \end{cases}$

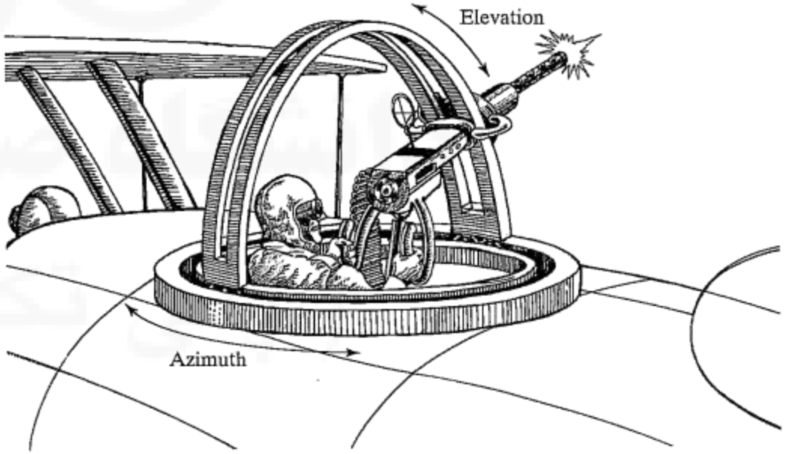
Singularities





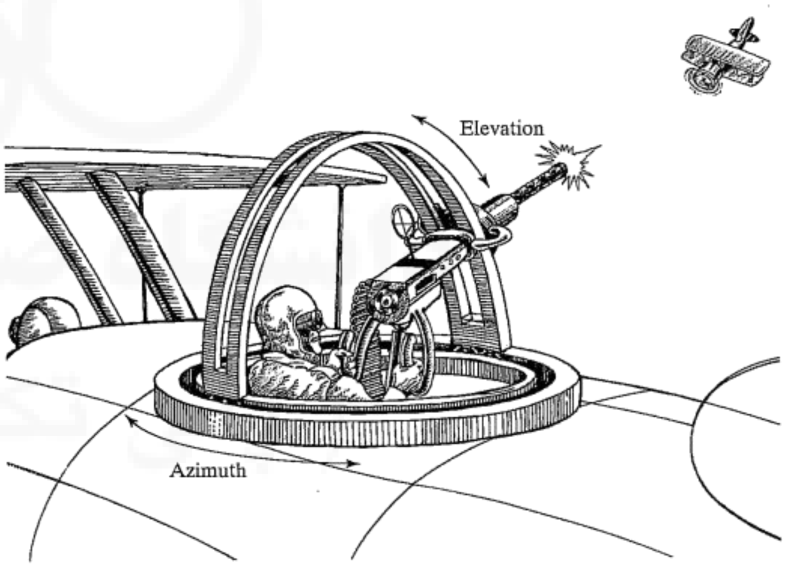
Velocities, Static Forces, Singularities

- **Velocities:**
- Example 2: World War I
- A mechanism that rotates about two axes, azimuth and elevation.
- It can direct the stream of bullets in any direction in the upper semisphere.
- With it directed **straight up**, their direction aligns with the axis of rotation of the azimuth rotation.
- At exactly this point, the azimuth rotation does not cause a change in the direction.
- Our mechanism has become locally degenerate at this location.



Velocities, Static Forces, Singularities

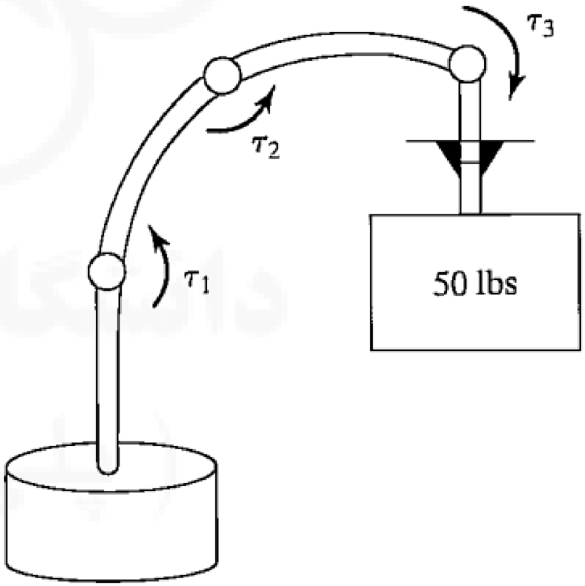
- **Velocities:**
- Example 2: World War I
- This phenomenon is called a **singularity** of the mechanism.
- These singularity conditions do not prevent a robot arm from positioning anywhere within its workspace.
- However, they can cause problems with motions of the arm in their neighborhood.



Velocities, Static Forces, Singularities

Static Forces

- Manipulators do not always move through space.
- They sometimes **touch** a workpiece or work surface and apply a static force.
- Statics: Study of forces and moments apart from motion.
- **Goal:** Given a desired contact force and moment, what set of joint torques is required to generate them?

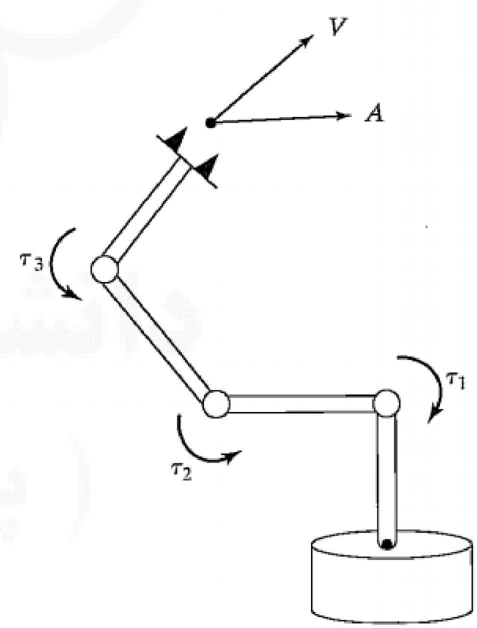


Dynamics

- A field of study devoted to studying the **required forces** to cause motion.

- Motion caused by a complex set of joint actuator torques:
  - Accelerate a manipulator from rest.
  - Glide at a constant end effector velocity.
  - Finally decelerate to a stop.

- It depends on the spatial and temporal attributes of the path, the mass properties of the links and payload, friction in the joints, and …

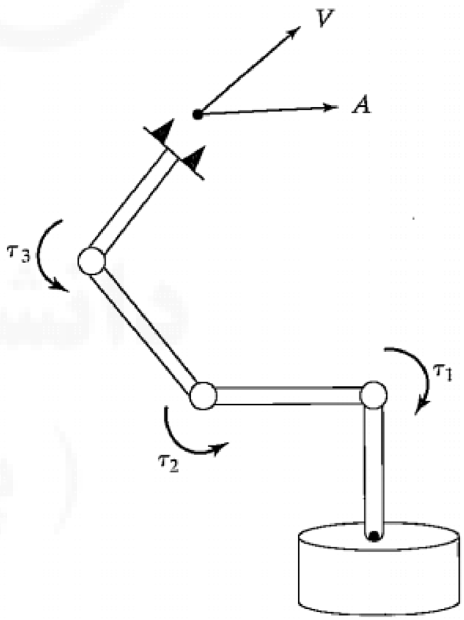


Dynamics

- **Two applications:**

1) **Simulation** (Forward)
Simulate how a manipulator would move under application of a set of actuator torques (F , $\tau \rightarrow X$, θ).

2) **Manipulator Control** (Inverse)
Calculating the required actuator torque functions to follow a desired path ($X_d, \theta_d \rightarrow F$, τ).



The END

- **References:**

1) .