Nicholas Nhat Tran 1002027150

CSE 4309 Assignment 2

Task 1 (20 pts)

p(S) = 0.95
p(M) = 0.05

p(O | S) = 0.9
p(U | S) = 0.1

p(O | M) = 0.2
p(U | M) = 0.8

Part a:

Part a: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the sensor is placed in Maine?

$$p(M \mid U) = \frac{p(U \mid M)p(M)}{p(U)} = \frac{(0.8)(0.05)}{p(U)}$$

$$p(U) = p(U \mid M) * p(M) + p(U \mid S) * p(S)$$

$$= 0.8 * 0.05 + 0.1 * 0.95 = 0.04 + 0.095 = 0.135$$

$$p(M \mid U) = \frac{(0.8)(0.05)}{0.135} = 0.296296296$$

Part b:

Part b: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the second e-mail also indicates a daily high under 80 degrees?

$p(U_2 \mid U_1) = p(U_2 \mid U_1, M)*p(M \mid U_1) + p(U_2 \mid U_1, S)*p(S \mid U_1)$

$p(U_2 \mid U_1, M) = p(U_2 \mid M) = 0.8$

$p(U_2 \mid U_1, M) = p(U_2 \mid S) = 0.1$

$p(S \mid U) = 1 - p(M \mid U) = 1 - 0.296296296 = 0.703703704$

$p(U_2 \mid U_1) = 0.8*0.296296296 + 0.1*0.703703704 = 0.307407407$

Part c:

Part c: What is the probability that the first three e-mails all indicate daily highs under 80 degrees?

$p(U_1 \cap U_2 \cap U_3) = p(U_1 \cap U_2 \cap U_3 \mid M)*p(M) + p(U_1 \cap U_2 \cap U_3 \mid S)*p(S)$

$p(U_1 \cap U_2 \cap U_3 \mid M) = p(U \mid M)^3 = 0.8^3 = 0.512$

$p(U_1 \cap U_2 \cap U_3 \mid S) = p(U \mid S)^3 = 0.1^3 = 0.001$

$p(U_1 \cap U_2 \cap U_3) = 0.512*0.05 + 0.001*0.95 = 0.02655$

Task 2 (10 pts)

Function P is a function defined on a set of samples S = {A, B, C, D}. We do not know the value of P for all samples, but we know that P(A) = 0.3 and P(B) = 0.6. What can you say about whether P is a valid probability function? Is P definitely a probability function, possibly a probability function, or definitely not a probability function? Justify your answer.

S = {A, B, C, D}
p(A) = 0.3
p(B) = 0.6

P is possibly a probability function because the space S contains known atomic events that fit the rule of non-negativity (the probability of any event cannot be less than 0) and the remaining unknown atomic events could possibly have a sum fitting the rule of normalization (the sum of probabilities of all possible atomic events is 1). Example: The sum of p(A) = 0.3 + p(B) = 0.6 + p(C) = 0.05 + p(D) = 0.05 would equal 1, which points to the possibility of fitting the description of a probability function.

Task 3 (10 pts)

Function P is a function defined on the set of real numbers. We do not know the value of P for all cases, but we know that P(x) = 0.3 when 0 <= x <= 10. What can you say about whether P is a valid probability density function? Is P definitely a probability density function, possibly a probability density function, or definitely not a probability density function? Justify your answer.

P = {R}
P(x) = 0.3 when 0 <= x <= 10

P is definitely not a probability density function because equation $\int_0^{10} 0.3dx = 0.3(10) - 0.3(0) = 3.0$ breaks the rule of normalization (the total area under the curve must equal one).

Task 4 (10 pts)

This problem refers to the Bayes Classifier as applied to the problem of "boxes and fruits" that we covered in class. A description of that example is in slides 13-19 from the Bayes Classifier presentation. Suppose that the training stage computed the correct probabilities:

- $p(B = r) = 0.4$
- $p(B = b) = 0.6$
- $p(F = a \mid B = r) = 0.25$
- $p(F = o \mid B = r) = 0.75$
- $p(F = a \mid B = b) = 0.75$
- $p(F = o \mid B = b) = 0.25$

Suppose that we get a test object x by following the experiment protocol from slide 13 (we randomly pick a box, with the given priors $p(B = r) = 0.4$ and $p(B = b) = 0.6$, and then we randomly pick a fruit from the box). What is the probability that the classifier will give the correct output for x? Justify your answer.

$p(B = r) = 0.4$
$p(B = b) = 0.6$

$p(F = o) = p(F = o \mid B = r)*p(B = r) + p(F = o \mid B = b)*p(B = b)$
$= 0.75*0.4 + 0.25*0.6 = 0.45$
$p(F = a) = p(F = a \mid B = r)*p(B = r) + p(F = a \mid B = b)*p(B = b)$
$= 0.25*0.4 + 0.75*0.6 = 0.55$

$p(F = a \mid B = r) = 0.25$
$p(F = o \mid B = r) = 0.75$

$p(F = a \mid B = b) = 0.75$
$p(F = o \mid B = b) = 0.25$

$p(B = b \mid F = o) = p(F = o \mid B = b)p(B = b)/p(F = o) = 0.25*0.6/0.45 =$
0.333333333
$p(B = r \mid F = o) = p(F = o \mid B = r)p(B = r)/p(F = o) = 0.75*0.4/0.45 =$
0.666666667
Since $p(B = r \mid F = o) > p(B = b \mid F = o)$, the classifier picks the **red** box for every orange.

$p(B = b \mid F = a) = p(F = a \mid B = b)p(B = b)/p(F = a) = 0.75*0.6/0.55 = 0.818181818$

$p(B = r \mid F = a) = p(F = a \mid B = r)p(B = r)/p(F = a) = 0.25*0.4/0.55 = 0.181818182$

Since $p(B = b \mid F = a) > p(B = r \mid F = a)$, the classifier picks the **blue** box for every apple.

$p(correct) = p(F = a \mid B = b)*p(b) + p(F = o \mid B = r)*p(r)$
$= 0.75*0.6 + 0.75*0.4 = 0.75$

Task 5 (50 pts)

Training

```
PS C:\Users\malwa\GitHub Repos\cse4309\Assignment 2> python .\naive_bayes_main.py
Class 1, attribute 1, mean =  0.52, std =  0.10
Class 1, attribute 2, mean =  0.54, std =  0.10
Class 1, attribute 3, mean =  0.52, std =  0.07
Class 1, attribute 4, mean =  0.41, std =  0.17
Class 1, attribute 5, mean =  0.50, std =  0.01
Class 1, attribute 6, mean =  0.00, std =  0.01
Class 1, attribute 7, mean =  0.50, std =  0.05
Class 1, attribute 8, mean =  0.24, std =  0.05
Class 2, attribute 1, mean =  0.45, std =  0.11
Class 2, attribute 2, mean =  0.45, std =  0.10
Class 2, attribute 3, mean =  0.53, std =  0.06
Class 2, attribute 4, mean =  0.23, std =  0.11
Class 2, attribute 5, mean =  0.50, std =  0.04
Class 2, attribute 6, mean =  0.00, std =  0.01
Class 2, attribute 7, mean =  0.49, std =  0.06
Class 2, attribute 8, mean =  0.33, std =  0.14
Class 3, attribute 1, mean =  0.43, std =  0.10
Class 3, attribute 2, mean =  0.48, std =  0.11
Class 3, attribute 3, mean =  0.36, std =  0.06
Class 3, attribute 4, mean =  0.22, std =  0.08
Class 3, attribute 5, mean =  0.51, std =  0.05
Class 3, attribute 6, mean =  0.00, std =  0.01
Class 3, attribute 7, mean =  0.51, std =  0.04
Class 3, attribute 8, mean =  0.27, std =  0.09
Class 4, attribute 1, mean =  0.79, std =  0.07
Class 4, attribute 2, mean =  0.76, std =  0.07
Class 4, attribute 3, mean =  0.38, std =  0.06
Class 4, attribute 4, mean =  0.32, std =  0.11
Class 4, attribute 5, mean =  0.50, std =  0.01
Class 4, attribute 6, mean =  0.00, std =  0.01
Class 4, attribute 7, mean =  0.51, std =  0.07
Class 4, attribute 8, mean =  0.27, std =  0.09
Class 5, attribute 1, mean =  0.74, std =  0.16
Class 5, attribute 2, mean =  0.62, std =  0.13
Class 5, attribute 3, mean =  0.42, std =  0.08
Class 5, attribute 4, mean =  0.30, std =  0.12
Class 5, attribute 5, mean =  0.50, std =  0.01
Class 5, attribute 6, mean =  0.00, std =  0.01
Class 5, attribute 7, mean =  0.51, std =  0.06
Class 5, attribute 8, mean =  0.24, std =  0.04
Class 6, attribute 1, mean =  0.54, std =  0.14
Class 6, attribute 2, mean =  0.50, std =  0.12
Class 6, attribute 3, mean =  0.51, std =  0.05
Class 6, attribute 4, mean =  0.24, std =  0.10
Class 6, attribute 5, mean =  0.50, std =  0.01
Class 6, attribute 6, mean =  0.49, std =  0.39
Class 6, attribute 7, mean =  0.51, std =  0.03
Class 6, attribute 8, mean =  0.24, std =  0.05
Class 7, attribute 1, mean =  0.48, std =  0.11
Class 7, attribute 2, mean =  0.47, std =  0.09
Class 7, attribute 3, mean =  0.54, std =  0.06
Class 7, attribute 4, mean =  0.22, std =  0.12
Class 7, attribute 5, mean =  0.50, std =  0.04
Class 7, attribute 6, mean =  0.00, std =  0.03
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```
Class 7, attribute 7, mean =  0.50, std =  0.06
Class 7, attribute 8, mean =  0.26, std =  0.09
Class 8, attribute 1, mean =  0.74, std =  0.11
Class 8, attribute 2, mean =  0.73, std =  0.11
Class 8, attribute 3, mean =  0.49, std =  0.05
Class 8, attribute 4, mean =  0.29, std =  0.07
Class 8, attribute 5, mean =  0.50, std =  0.01
Class 8, attribute 6, mean =  0.00, std =  0.01
Class 8, attribute 7, mean =  0.46, std =  0.08
Class 8, attribute 8, mean =  0.23, std =  0.02
Class 9, attribute 1, mean =  0.55, std =  0.14
Class 9, attribute 2, mean =  0.56, std =  0.16
Class 9, attribute 3, mean =  0.51, std =  0.07
Class 9, attribute 4, mean =  0.20, std =  0.07
Class 9, attribute 5, mean =  0.50, std =  0.01
Class 9, attribute 6, mean =  0.00, std =  0.01
Class 9, attribute 7, mean =  0.53, std =  0.05
Class 9, attribute 8, mean =  0.24, std =  0.05
Class 10, attribute 1, mean =  0.78, std =  0.06
Class 10, attribute 2, mean =  0.73, std =  0.12
Class 10, attribute 3, mean =  0.48, std =  0.11
Class 10, attribute 4, mean =  0.33, std =  0.07
Class 10, attribute 5, mean =  1.00, std =  0.01
Class 10, attribute 6, mean =  0.00, std =  0.01
Class 10, attribute 7, mean =  0.55, std =  0.02
Class 10, attribute 8, mean =  0.23, std =  0.01
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Classification

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classification accuracy= 0.4483
PS C:\Users\malwa\GitHub Repos\cse4309\Assignment 2>
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