

CSE 4309 Assignment 1

Task 1 (5 pts)

```
def factorial(n):  
    result = 1  
    for i in range(2, (n+1)):  
        result = result * i;  
  
    return result
```

The time complexity is $\Theta(n)$.

Task 2 (10 pts)

```
def factorial_recursive(n):  
    if n == 0 or n == 1:  
        return 1  
    else:  
        return n * factorial_recursive(n-1)
```

Task 3 (10 pts)

```
def foo(n):  
    result = 0  
    for i in range(1, n+1):  
        for j in range(1, i+1):  
            result = result + 1  
    return result
```

Time time complexity is $\Theta(n^2)$.

Task 4 (5 pts)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad B = \begin{bmatrix} e \\ f \end{bmatrix}$$

What is the result of matrix multiplication $A \cdot B$?

$A = 2 \times 2$ matrix

$B = 2 \times 1$ matrix

The number of A 's rows is equal to the number of B 's columns, so this can be multiplied.

Multiplying A 's rows $(a \ b)$ and $(c \ d)$ and multiplying by B 's column $(e \ f)$, we get $(a \cdot e + b \cdot f)$ and $(c \cdot e + d \cdot f)$. This is the resulting matrix:

$$A \cdot B = \begin{bmatrix} ae+bf \\ ce+df \end{bmatrix}$$

Task 5 (5 pts)

Consider function $f(x) = 3x^2 + 5x - 7$.

Part a: What is the first derivative $f'(x)$? Provide a specific formula as a function of x .

$$f'(x) = 6x + 5$$

Part b: What is $f'(5)$? Your answer should be a real number.

$$f'(5) = 6(5) + 5 = 30 + 5 = 35$$

Part c: What is the second derivative $f''(x)$? Provide a specific formula as a function of x .

$$f''(x) = 6$$

Part d: What is $f''(5)$? Your answer should be a real number.

$$f''(5) = 6$$

Task 6 (5 pts)

Consider function $f(x, y) = 3x^2y + 5x - 7y$.

Part a: What is the partial derivative of f with respect to x ? Provide a specific formula as a function of x and y .

$$f_x(x, y) = 6xy + 5$$

Part b: What is the partial derivative of f with respect to x when $x=5$ and $y=2$? Your answer should be a real number.

$$f_x(x=5, y=2) = 6(5)(2) + 5 = 60 + 5 = 65$$

Task 7 (10 pts)

In this task, we denote by $P(x)$ the probability of event x . A and B are two events that are independent of each other. $P(A) = 0.3$ and $P(B) = 0.6$.

Compute the following quantities:

- $P(A \text{ and } B)$.
- $P(A \text{ or } B)$.
- $P(\text{not}(A))$.
- $P(A | B)$ (i.e., the conditional probability of A given B).

$$P(A \text{ and } B) = P(A) * P(B) = 0.3 * 0.6 = 0.18$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.3 + 0.6 - 0.18 = 0.72$$

$$P(\text{not}(A)) = 1 - P(A) = 0.7$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.18}{0.6} = 0.3$$

Task 8 (10 pts)

Color	Price \$20 to \$40	Price \$50 to \$70	Price \$80 to \$100
red	40	70	35
green	15	50	30
blue	60	20	80

Color	Price \$20 to \$40	Price \$50 to \$70	Price \$50 to \$70	Total
red	40	70	35	145
green	15	50	30	95
blue	60	20	80	160
Total	115	140	145	400

The above table shows, for a certain hat store, the number of hats in their inventory, for each combination of color and price. For example, the inventory contains 40 red hats at a price between \$20 and \$40. Using that table:

Part a: Determine $P(\text{price} < \$75)$, i.e., the probability that a hat costs less than \$75.

$$P(\text{price} < \$75) = \frac{(40+15+60)+(70+50+20)}{(40+15+60)+(70+50+20)+(35+30+80)} = 0.6375$$

Part b: Determine $P(\text{price} < \$75 \mid \text{color}=\text{green})$, i.e., the conditional probability that the price of a hat is under \$75, given that the color of that hat is green.

$$P(\text{price} < \$75 \mid \text{color}=\text{green}) = \frac{P(\text{price} < \$75 \text{ and } \text{color}=\text{green})}{P(\text{color}=\text{green})} = \frac{0.6375 \cdot 95/400}{95/400}$$

$$= \frac{0.15140625}{0.2375} = 0.6375$$

Part c: Determine $P(\text{price} < 75, \text{color}=\text{green})$, i.e., the joint probability that the price of a hat is under 75 and the color of that hat is green.

$$P(\text{price} < \$75, \text{color}=\text{green}) = P(\text{price} < \$75) * P(\text{color}=\text{green} \mid \text{price} < \$75)$$

$$= 0.6375 * \frac{P(\text{price} < \$75 \text{ and } \text{color}=\text{green})}{p(\text{price} < \$75)} = 0.6375 * \frac{0.6375 \cdot 95/400}{0.6375} = 0.15140625$$

Task 9 (10 pts)

Two hens lay a combined total of two eggs in two days. If this rate of egg production per hen per day continues, how many eggs do ten hens lay in ten days?

2 hens lay 2 eggs in 2 days

$$\text{Rate} = \frac{2 \text{ eggs}}{2 \text{ hens} * 2 \text{ days}} = 0.5 \text{ eggs/hen-day}$$

$$10 \text{ hens} * 10 \text{ days} * 0.5 \text{ eggs/hen-day} = 50 \text{ eggs}$$

Task 12 (5 pts)

Part a: Towards the end of October, when the deadline for assignment 5 approaches, a student sends the instructor the following email:

C: No, extensions are not provided except in case of an emergency documented in writing (too much other work, computer/network problems do not qualify). However, remember that you can resubmit (or make late submissions) until Friday December 9. For the purposes of making a B, C, or D semester grade, the resubmission score fully replaces the original grade. So, if you do not make the deadline and you get a 0, you can fully replace that score with a resubmission. The only caveat is that the resubmission score will not be considered for the purposes of giving an A grade for the semester.

Part b: Suppose that instead of "three midterms this week", the reason for the extension request was a computer crash or a network problem. In that case, which of the above three responses should the student expect?

C: No, extensions are not provided except in case of an emergency documented in writing (too much other work, computer/network problems do not qualify). However, remember that you can resubmit (or make late submissions) until Friday December 9. For the purposes of making a B, C, or D semester grade, the resubmission score fully replaces the original grade. So, if you do not make the deadline and you get a 0, you can fully replace that score with a resubmission. The

only caveat is that the resubmission score will not be considered for the purposes of giving an A grade for the semester.

Task 13 (5 pts)

C: Use at your own risk. The lectures have provided all the information that you need to implement everything from scratch. Your solution should produce the correct outputs for the test inputs that we will use during grading. If you get that done using an existing library, that is fine. If the library produces results that do not match my specifications, you bear the responsibility.