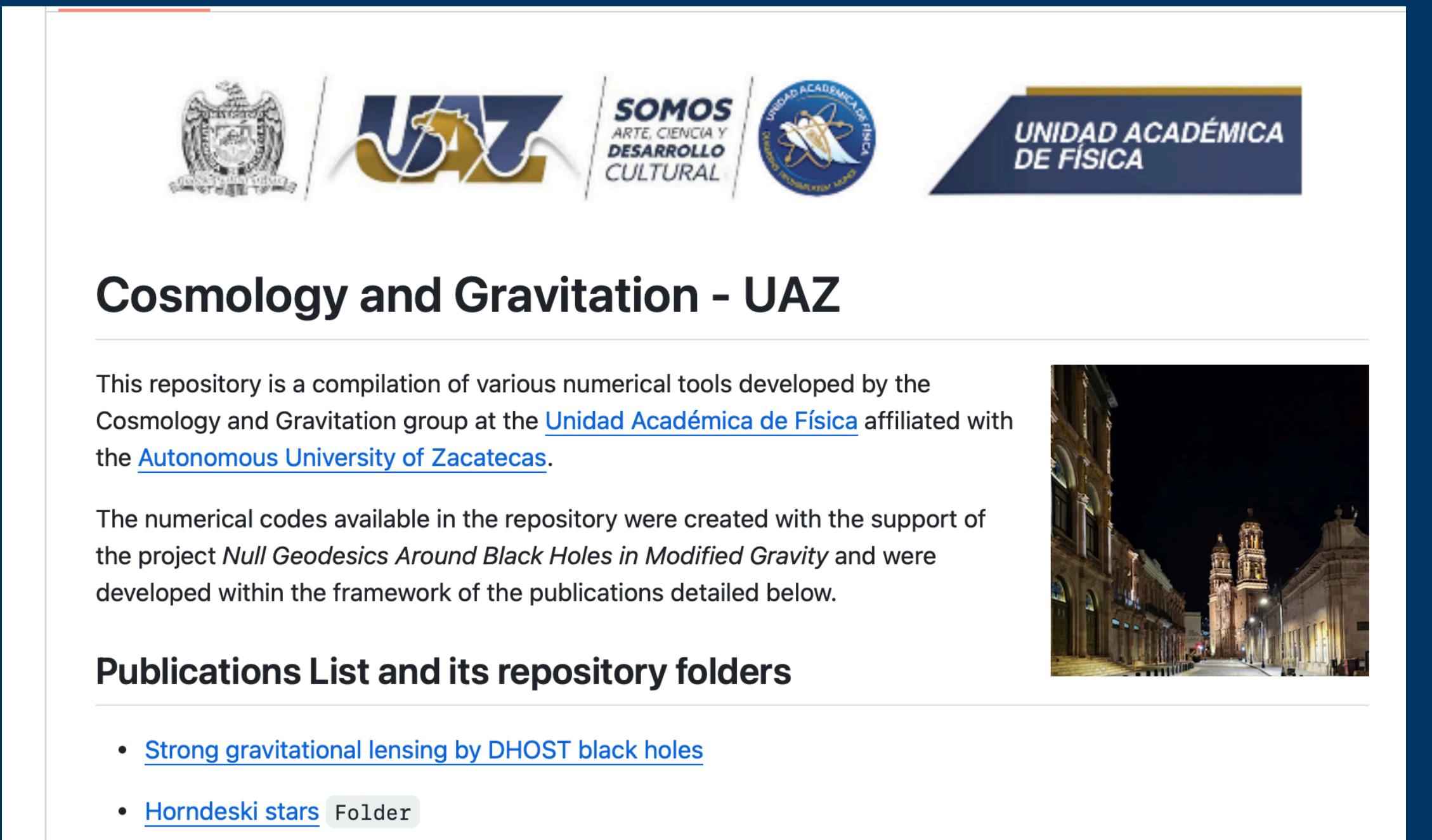


Taller de Cosmología y Gravitación

Herramientas de programación y estadística.



The screenshot shows the homepage of a GitHub repository. At the top, there are logos for the University of Guanajuato, UAZ, SOMOS (Arte, Ciencia y Desarrollo Cultural), and the Unidad Académica de Física. Below the header, the title "Cosmology and Gravitation - UAZ" is displayed. A paragraph of text explains that the repository contains numerical tools developed by the Cosmology and Gravitation group at the Unidad Académica de Física, affiliated with the Autonomous University of Zacatecas. It also mentions support from the project *Null Geodesics Around Black Holes in Modified Gravity*. A photograph of a city street at night is shown on the right side. The footer lists publications and repository folders, including links to "Strong gravitational lensing by DHOST black holes" and "Horndeski stars Folder".



Gravitación, Métodos numéricos y Python

Dr. Armando A. Roque Estrada



Estructura

- I. Relatividad General /Objetos compactos
- II. Estrellas barionicas
- III. Estrellas escalares
- IV. Estrellas hibridas

Relatividad General (RG) y sus modificaciones

Relatividad General+Materia

unidades $\hbar = c = 1$

$$S_{EH} = \int d^4x \sqrt{-g} \left(\underbrace{\frac{1}{16\pi G} R[g_{\mu\nu}]}_{\text{gravedad}} + \underbrace{\mathcal{L}_m}_{\text{materia}} \right)$$

ecuaciones de movimiento \longrightarrow curvatura $\equiv R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{materia}}$

Primeros triunfos (pruebas clásicas)

- Precesión del perihelio de Mercurio.
- Deflexión de la luz al pasar cerca del Sol.
- Corrimiento al rojo gravitacional de la luz.

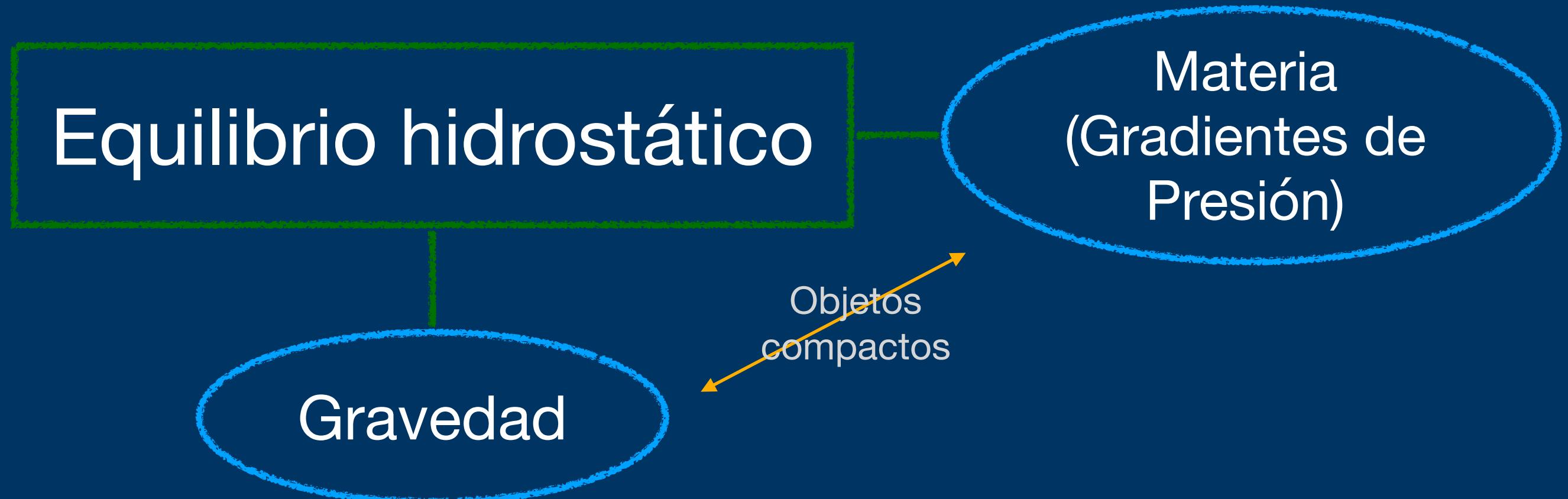
TEOREMA DE LOVELOCK:

La RG es la única teoría para un campo de spin dos no masivo, local, en 4-dimensiones, invariante de Lorentz, y cuyas ecuaciones son de 2do orden:

$$E^{\mu\nu} = \alpha \sqrt{-g} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) + \lambda \sqrt{-g} g^{\mu\nu}$$

- ▶ Considerar nuevos campos más allá (o en lugar) del tensor métrico, p. ej. escalar, vector, tensorial.
- ▶ Nuevas reglas: torsión, no-local, dimensiones superiores,
- ▶ Añadir derivadas de la métrica mayores de segundo orden (posible inconveniente la inestabilidad de Ostrogradsky, modos fantasma, etc.).

Objetos compactos como laboratorios.



+ TianQin (en el espacio)



Nuevos detectores capaces de mejorar las resoluciones actuales

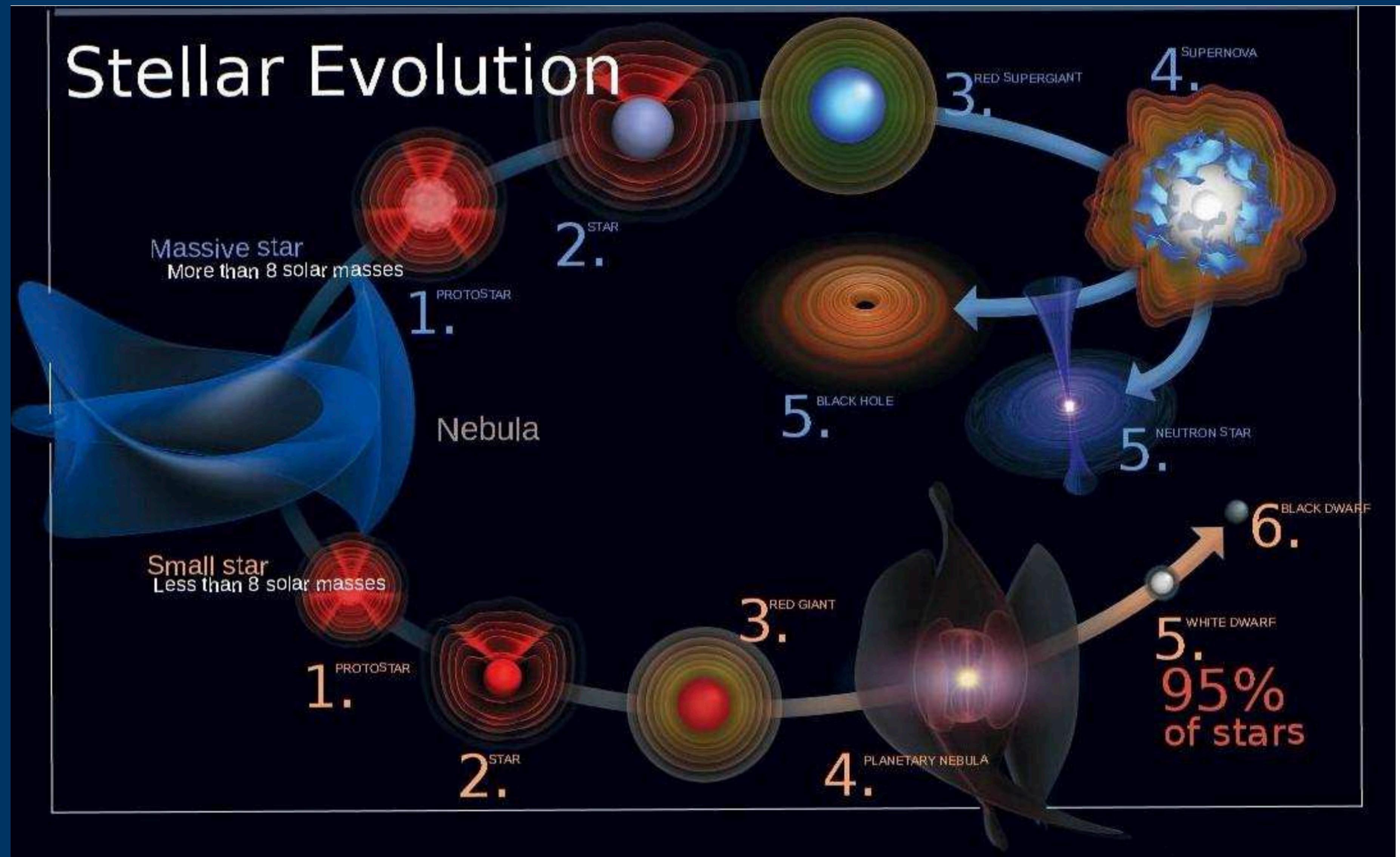
Primer paso o aproximación: considerar objetos estáticos, esféricamente simétricos y que pueden ser descritos mediante la métrica

$$ds^2 = -N^2(r)dt^2 + g^2(r)dr^2 + r^2d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

ansatz usado en todo el trabajo

Existen propuestas teóricas que intentan explicar mediante objetos compactos sumamente extendidos, y cuyos constituyentes son nuevos campos, el comportamiento *atípico* en las curvas de rotación y dispersión de velocidades en las galaxias. (no se aborda en este curso)

Estrellas y Estrellas de Neutrones



Estrellas Bariónicas

Gravedad $f(R)$

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} f(R[g_{\mu\nu}]) + \mathcal{L}_m \right)$$

formalismo métrico

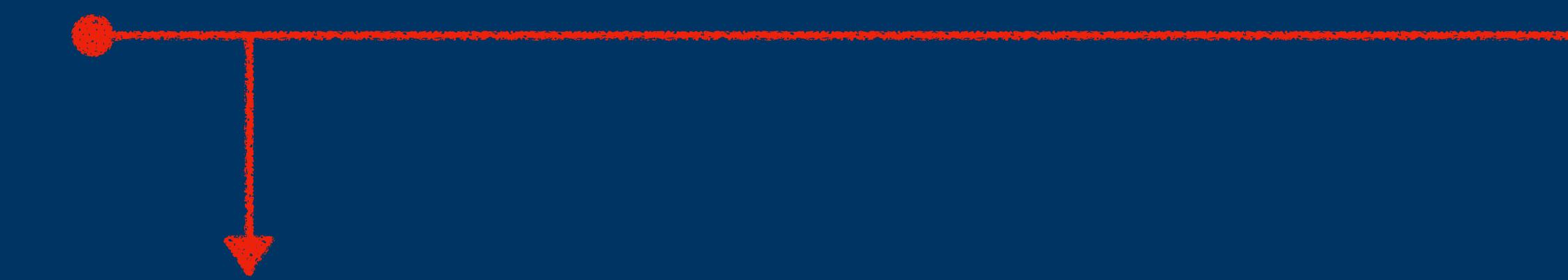
$$f_R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + \boxed{g_{\mu\nu} \square f_R - \nabla_\nu \nabla_\mu f_R} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{materia}}$$

curvatura (Modificación a la RG)

Casos

$$f(R) = R \rightarrow \text{RG}$$

$$f(R) = R + \alpha R^2 \rightarrow \text{Gravedad cuadrática}$$



Derivadas de Orden Superior

$$R_{\mu\nu} R^{\mu\nu},$$

$\square R, \dots$

$$f(G)$$

$$f(R)$$

$$(\Lambda)RG$$

$$\alpha \sqrt{-g} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) + \lambda \sqrt{-g} g^{\mu\nu}$$

Dimensiones Superiores
Cuerdas & Branas

Generalización de
 S_{EH}

No Locales
 $f(R/\square)$

Nuevos Campos

escalar vectorial tensorial
DHOST PROCA bigravity
EKG E. Aether EBI

Nuevas Consideraciones

$f(T)$
CDT

Objetos auto-gravitantes, estáticos y esféricamente simétricos

$$\frac{N'}{N} = \frac{1}{2kr(2f_R + rR'f_{RR})} \left\{ r^2 T^{11} + kg^2 \left[r^2 f + (2 - r^2 R) f_R \right] - 2k (f_R + 2rR'f_{RR}) \right\}$$

$$\frac{g'}{g} = \frac{1}{2kN^2r(2f_R + rR'f_{RR})} \left\{ g^2 r^2 T^{00} + kN^2 \left[g^2 \left(-r^2 f + [-2 + r^2 R] f_R \right) + 2 \left(f' + r [(2R' + rR'') f_{RR} + rR'^2 f_{RRR}] \right) \right] \right\}$$

$$R'' = \left(-\frac{2}{r} - \frac{N'}{N} + \frac{g'}{g} \right) R' + \frac{g^2 (T + 4kf - 2kRf_R)}{6kf_{RR}} - \frac{R'^2 f_{RRR}}{f_{RR}}$$

T^{00} y T^{11} son las componentes 0-0 y 1-1 (respectivamente) de un tensor de energía-momento correspondiente a un fluido perfecto oscuro



$$T^{\mu\nu} = (\rho + pm) u^\mu u^\nu + g^{\mu\nu} pm$$

ecuación dinámica del grado extra de libertad

$$\nabla_\mu T^{\mu\nu} = 0$$

Ecuación de estado de la MO

$$f(R) = R$$

Tolman-Oppenheimer-Volkoff (TOV)

$$f(R) \rightarrow \text{arbitraria}$$

Tolman-Oppenheimer-Volkoff modificadas (TOVm)

Reto, obtener Newton

$$\begin{aligned} \frac{dM(r)}{dr} &= 4\pi r^2 \rho_m(r) \\ \frac{dP(r)}{dr} &= -G \frac{M(r)\rho_m(r)}{r^2} \end{aligned}$$

Materia fermiónica auto-interactuante en el límite degenerado

$$T^{00} = -\rho u^0 u^0 = m^4 [\bar{\rho} + \bar{\rho}_{int}] u^0 u^0$$

$$T^{ij} = g^{ij} p_m = m^4 [\bar{p}_m + \bar{p}_{int}] g^{ij}$$

$$F(r, p, t) = \frac{1}{4\pi^3} \frac{1}{1 + e^{[E(p) - \mu(r)]/k_B T(r)}}, \quad T(r) = 0$$

$$-\rho = \int E F(r, p, t) d^3 p$$

$$p_m = \frac{1}{3} \int U \cdot p F(r, p, t) d^3 p$$

$$E_F(r) \equiv \mu(r) = m\sqrt{\Phi(r) + 1} = \sqrt{p_F(r)^2 + m^2}$$

$$\bar{\rho}(x_F) = \frac{1}{8\pi^2} \left[(2x_F^3 + x_F)(1 + x_F^2)^{1/2} - \ln \left(x_F + \sqrt{x_F^2 + 1} \right) \right]$$

$$\bar{p}_m(x_F) = \frac{1}{24\pi^2} \left[(2x_F^3 - 3x_F)(1 + x_F^2)^{1/2} + 3 \ln \left(x_F + \sqrt{x_F^2 + 1} \right) \right]$$

► $\mathcal{L}^{\text{int}} = -g\bar{\chi}\chi\psi \rightarrow \rho_{int} \sim n^2/m_I^2$

$$\bar{p}_{int} = \frac{1}{9\pi^4} y^2 x_F^6$$

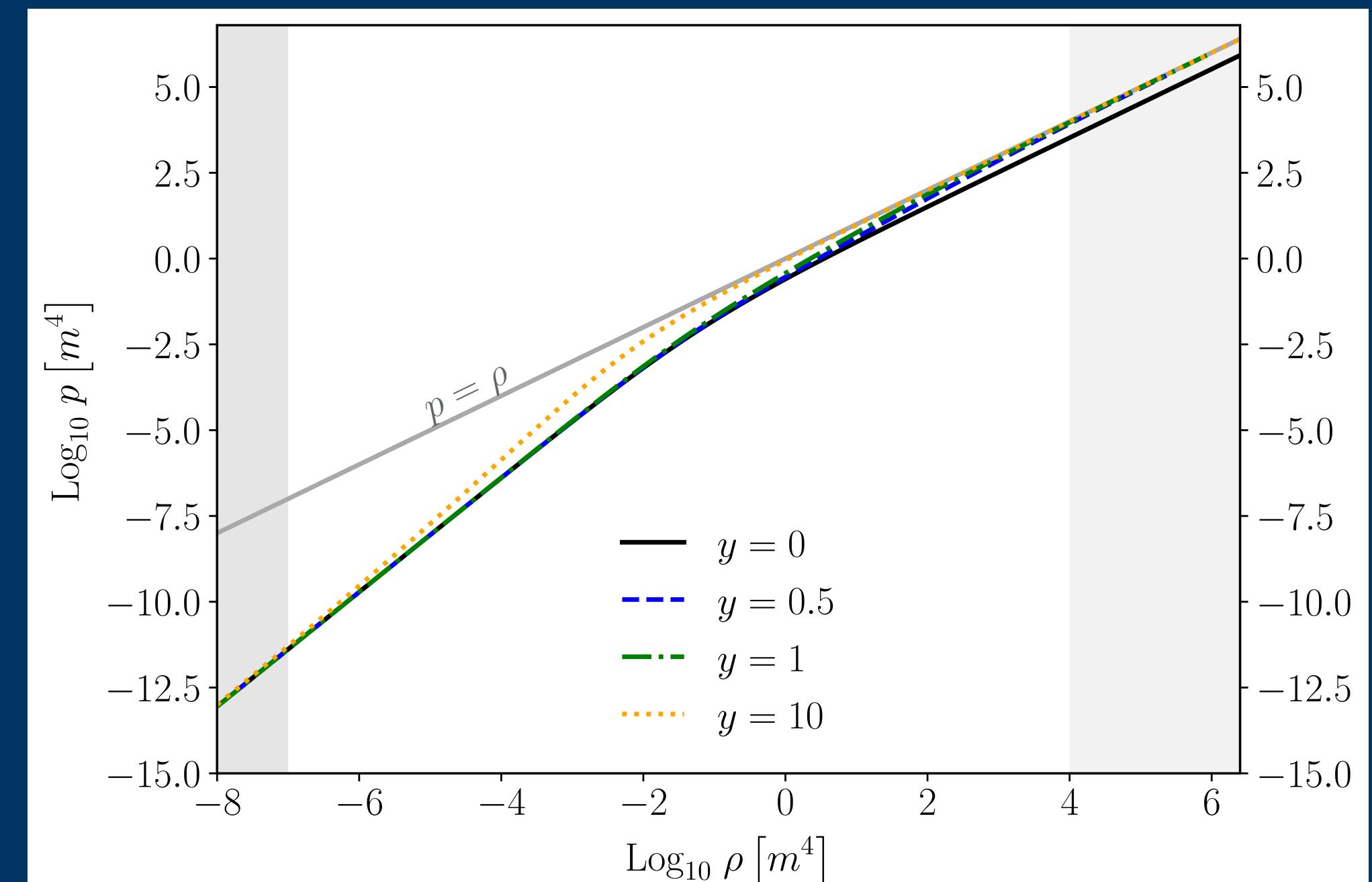
$$\bar{\rho}_{int} = \frac{1}{9\pi^4} y^2 x_F^6$$

G. Narain, et al.

$$m_I = m_\psi/g$$

$$y = m/m_I = g m/m_\psi$$

$n \rightarrow$ densidad de partículas



$$x_F = p_F/m$$

$$0 < x_F < 20$$

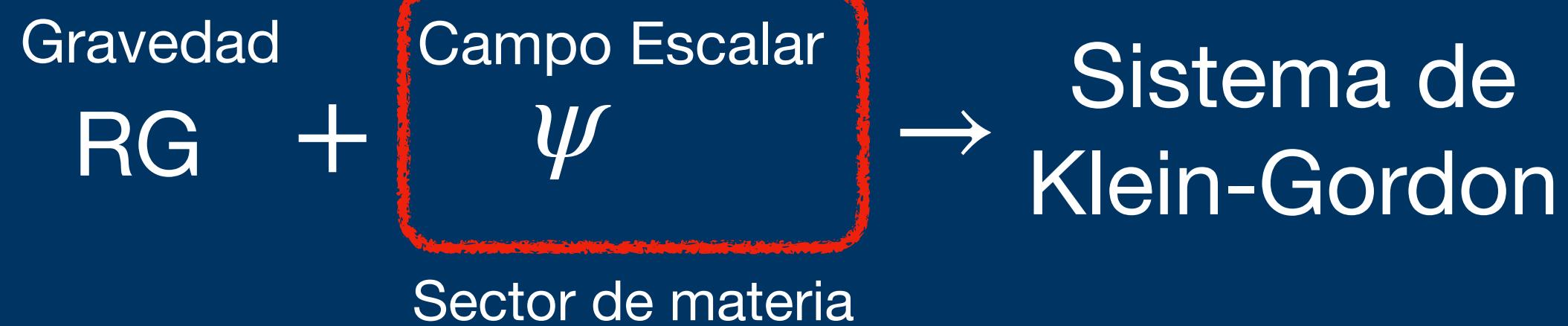
$$0 \leq y \leq 10$$

¡Empecemos a Divertirnos!

Estrellas Escalares

¿Qué es una estrella de bosones?

Ingredientes



La acción

$$S[g_{\mu\nu}, \psi] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \nabla_\mu \psi \nabla^\mu \psi - V(\psi) \right]$$

La física

“Presión” repulsiva

Equilibrio hidrostático

Gravedad

Objetos compactos localizados

PHYSICAL REVIEW

VOLUME 172, NUMBER 5

25 AUGUST 1968

Klein-Gordon Geon*

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(Received 4 March 1968)

A study of the spherically symmetric eigenstates of the Klein-Gordon Einstein equations (Klein-Gordon geons) reveals that these geons have properties that are uniquely different from other gravitating systems that have been studied. The equilibrium states of these geons seem analogous to other gravitating systems; but when the question of stability is considered from a thermodynamical viewpoint, it is shown that, in contrast with other systems, adiabatic perturbations are forbidden. The reason is that the equations of state for the thermodynamical variables are not algebraic equations, but instead are differential equations. Consequently, the usual concept of an equation of state breaks down when Klein-Gordon geons are considered. When the question of stability is reconsidered in terms of infinitesimal perturbations of the basic

PHYSICAL REVIEW

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Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

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AND

SILVANO BONAZZOLA‡

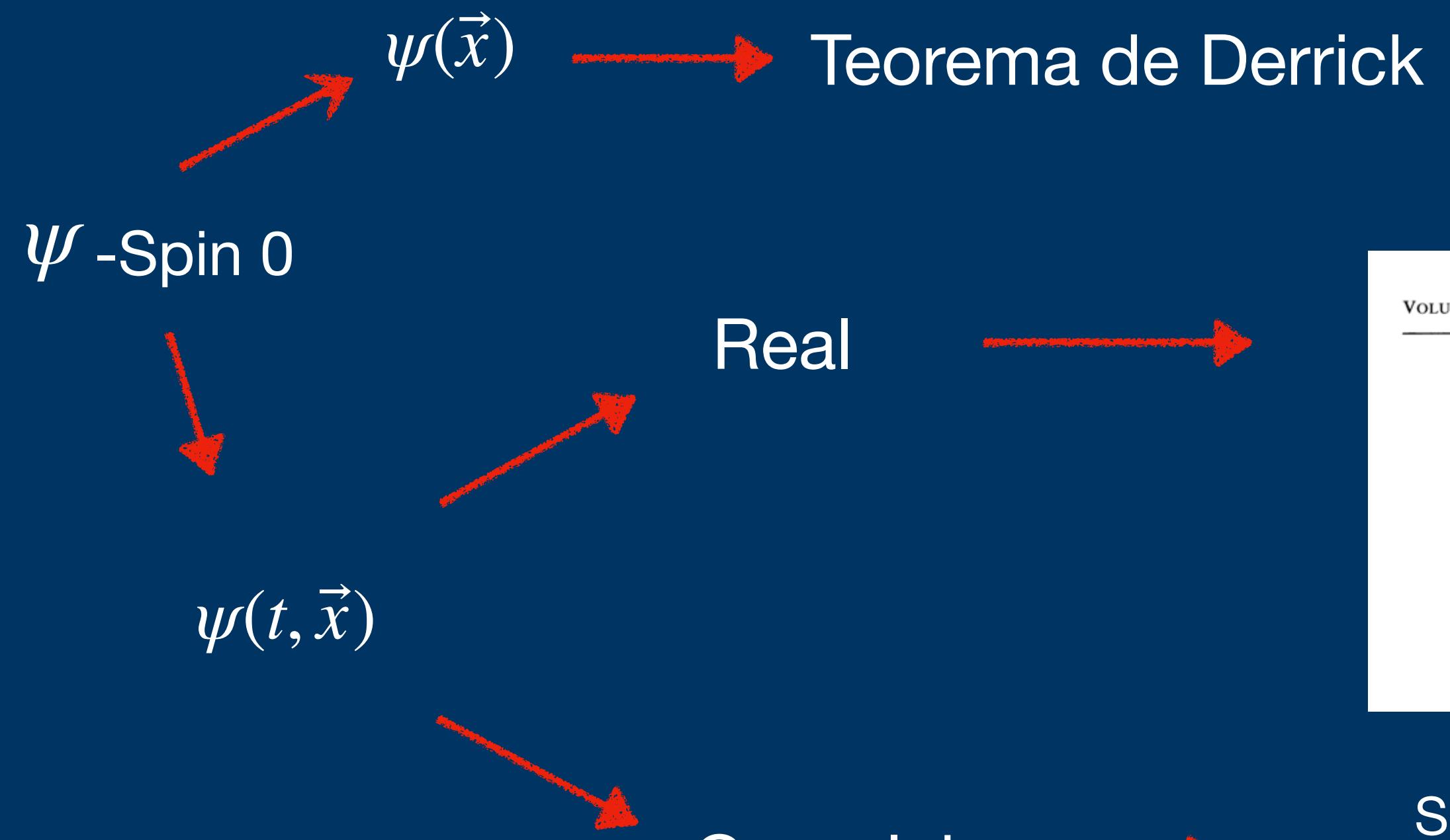
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(Received 4 February 1969)

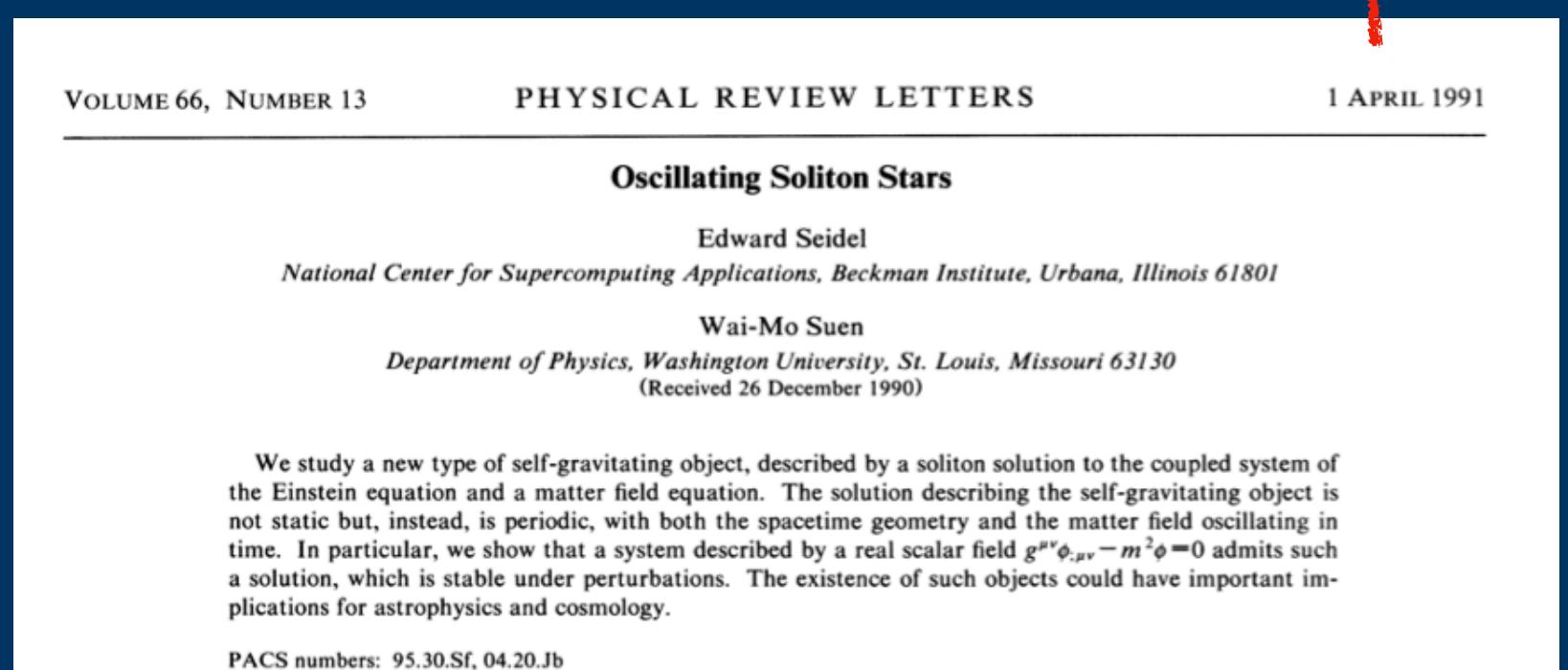
A method of self-consistent fields is used to study the equilibrium configurations of a system of self-gravitating scalar bosons or spin-½ fermions in the ground state without using the traditional perfect-fluid approximation or equation of state. The many-particle system is described by a second-quantized free field, which in the boson case satisfies the Klein-Gordon equation in general relativity, $\nabla_\alpha \nabla^\alpha \phi = \mu^2 \phi$, and in the fermion case the Dirac equation in general relativity $\gamma^\alpha \nabla_\alpha \psi = \mu \psi$ (where $\mu = mc/\hbar$). The coefficients of the metric $g_{\alpha\beta}$ are determined by the Einstein equations with a source term given by the mean value $\langle \phi | T_{\mu\nu} | \phi \rangle$ of the energy-momentum tensor operator constructed from the scalar or the spinor field. The state vector $|\phi\rangle$ corresponds to the ground state of the system of many particles. In both cases, for completeness, a nonrelativistic Newtonian approximation is developed, and the corrections due to special and general relativity explicitly are pointed out. For N bosons, both in the region of validity of the Newtonian treatment (density from 10^{-40} to 10^{44} g cm $^{-3}$, and number of particles from 10 to 10^{40}) as well as in the relativistic region (density $\sim 10^{44}$ g cm $^{-3}$, number of particles $\sim 10^{40}$), we obtain results completely different from those of a traditional fluid analysis. The energy-momentum tensor is anisotropic. A critical mass is found for a system of $N \sim [(\text{Planck mass})/m]^2 \sim 10^{40}$ (for $m \sim 10^{-25}$ g) self-gravitating bosons in the ground state, above which mass gravitational collapse occurs. For N fermions, the binding energy of typical particles is $G^2 m^6 N^{4/3} \hbar^{-2}$ and reaches a value $\sim mc^2$ for $N \sim N_{\text{crit}} \sim [(\text{Planck mass})/m]^3 \sim 10^{87}$ (for $m \sim 10^{-24}$ g, implying mass $\sim 10^{33}$ g, radius $\sim 10^6$ cm, density $\sim 10^{15}$ g/cm 3). For densities of this order of

Nota: $c = \hbar = 1$

El campo escalar:



Soluciones localizadas y estacionarias de Eq. KG en 3D o dimensiones más altas son inestables



$$N^2(t, r) = 1 + \sum_{j=0}^{\infty} N_{2j}(r) \cos(2j\omega_0 t),$$

$$g^2(t, r) = 1 + \sum_{j=0}^{\infty} g_{2j}(r) \cos(2j\omega_0 t),$$

$$\phi(t, r) = \sum_{j=1}^{\infty} \phi_{2j-1}(r) \cos[(2j-1)\omega_0 t].$$

oscilones

El potencial

$V[|\psi|] = m^2 |\psi|^2 + M^4 \sum_{n=2}^{\infty} \frac{v_{2n}}{(2n)!} \left| \frac{\psi}{M} \right|^{2n}$

término de masa

términos de auto-interacción

$\sim |\psi|^4$

$\sim c_1 |\psi|^4 + c_2 |\psi|^6$

- Campos con Spin 1: Estrellas de Proca

$$\mathcal{L}_M = -\frac{1}{2} F_{\mu\nu}^* F^{\mu\nu} - m_0^2 A_\mu^* A^\mu - \lambda (A_\mu^* A^\mu)^2,$$

$$A_\mu(t, \vec{x}) = e^{-i\omega t} (f(r), ig(r), 0, 0).$$

- Campos con Spin 2: teorías Bimétricas

Resumen:

