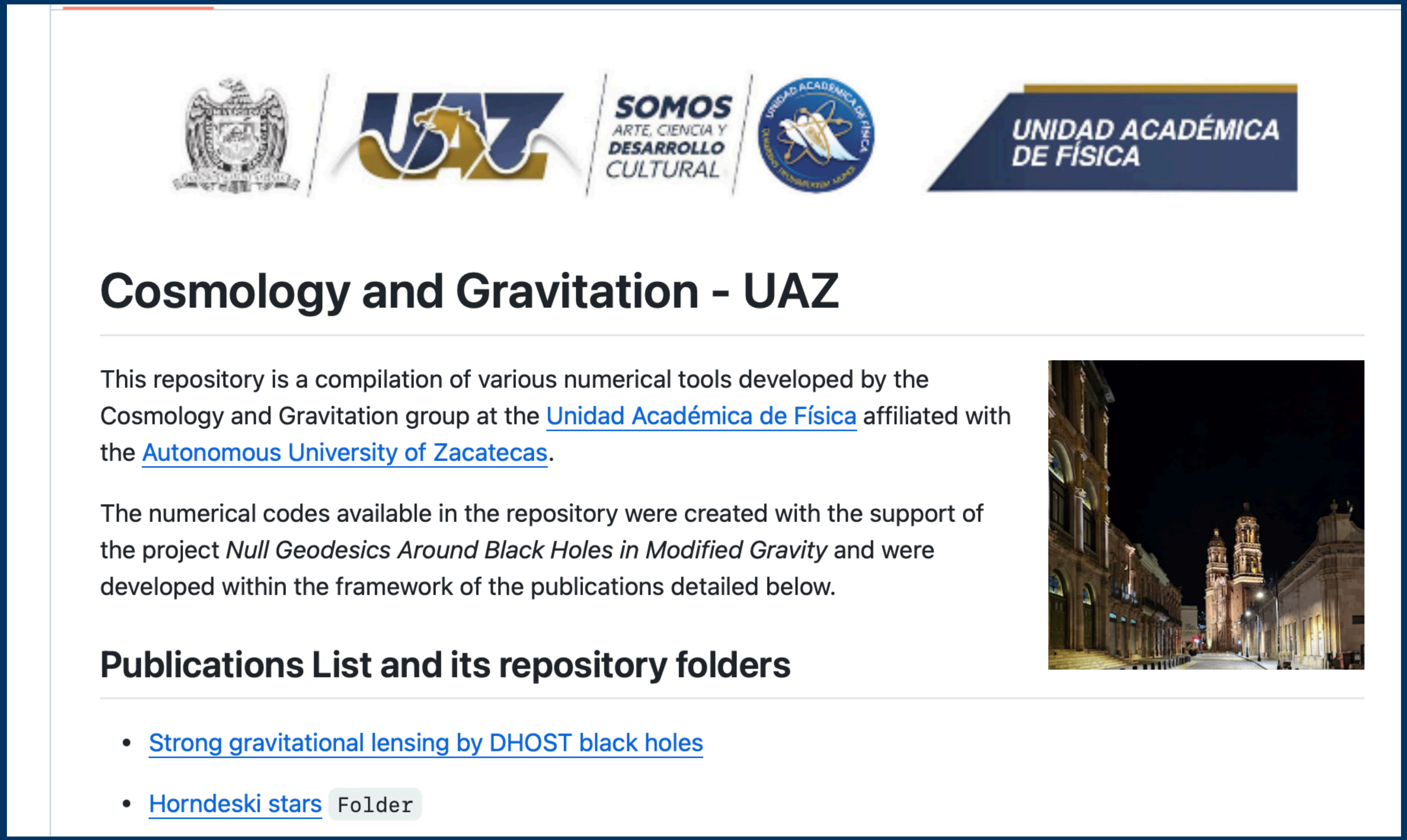
Taller de Cosmología y Gravitación

Herramientas de programación y estadística.



Cosmology and Gravitation - UAZ

This repository is a compilation of various numerical tools developed by the Cosmology and Gravitation group at the Unidad Académica de Física affiliated with the Autonomous University of Zacatecas.

The numerical codes available in the repository were created with the support of the project *Null Geodesics Around Black Holes in Modified Gravity* and were developed within the framework of the publications detailed below.

Publications List and its repository folders

- Strong gravitational lensing by DHOST black holes
- Horndeski stars `Folder`

Gravitación, Métodos numéricos y Python

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CONACYT
Consejo Nacional de Ciencia y Tecnología

Estructura

I. Relatividad General /Objetos compactos
II. Estrellas barionicas
III. Estrellas escalares
IV. Estrellas hibridas

Relatividad General (RG) y sus modificaciones

Relatividad General+Materia

unidades $\hbar = c = 1$

$$S_{\text{EH}} = \int d^4x\sqrt{-g}\left(\underbrace{\frac{1}{16\pi G}R[g_{\mu\nu}]}_{\text{gravedad}} + \underbrace{\mathscr{L}_m}_{\text{materia}}\right) \xrightarrow[\text{de movimiento}]{\text{ecuaciones}} \underbrace{G_{\mu\nu}}_{\text{curvatura}} \equiv R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G\,\underbrace{T_{\mu\nu}}_{\text{materia}}$$

Primeros triunfos (pruebas clásicas)

- Precesión del perihelio de Mercurio.
- Deflexión de la luz al pasar cerca del Sol.
- Corrimiento al rojo gravitacional de la luz.

TEOREMA DE LOVELOCK:

La RG es la única teoría para un campo de spin dos no masivo, local, en 4-dimensiones, invariante de Lorentz, y cuyas ecuaciones son de 2do orden:

$$E^{\mu\nu} = \alpha\sqrt{-g}\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) + \lambda\sqrt{-g}\,g^{\mu\nu}$$

- Considerar nuevos campos más allá (o en lugar) del tensor métrico, p. ej. escalar, vector, tensorial.
- Nuevas reglas: torsión, no-local, dimensiones superiores, … .
- Añadir derivadas de la métrica mayores de segundo orden (posible inconveniente la inestabilidad de Ostrogradsky, modos fantasmas, etc.).

Objetos compactos como laboratorios.

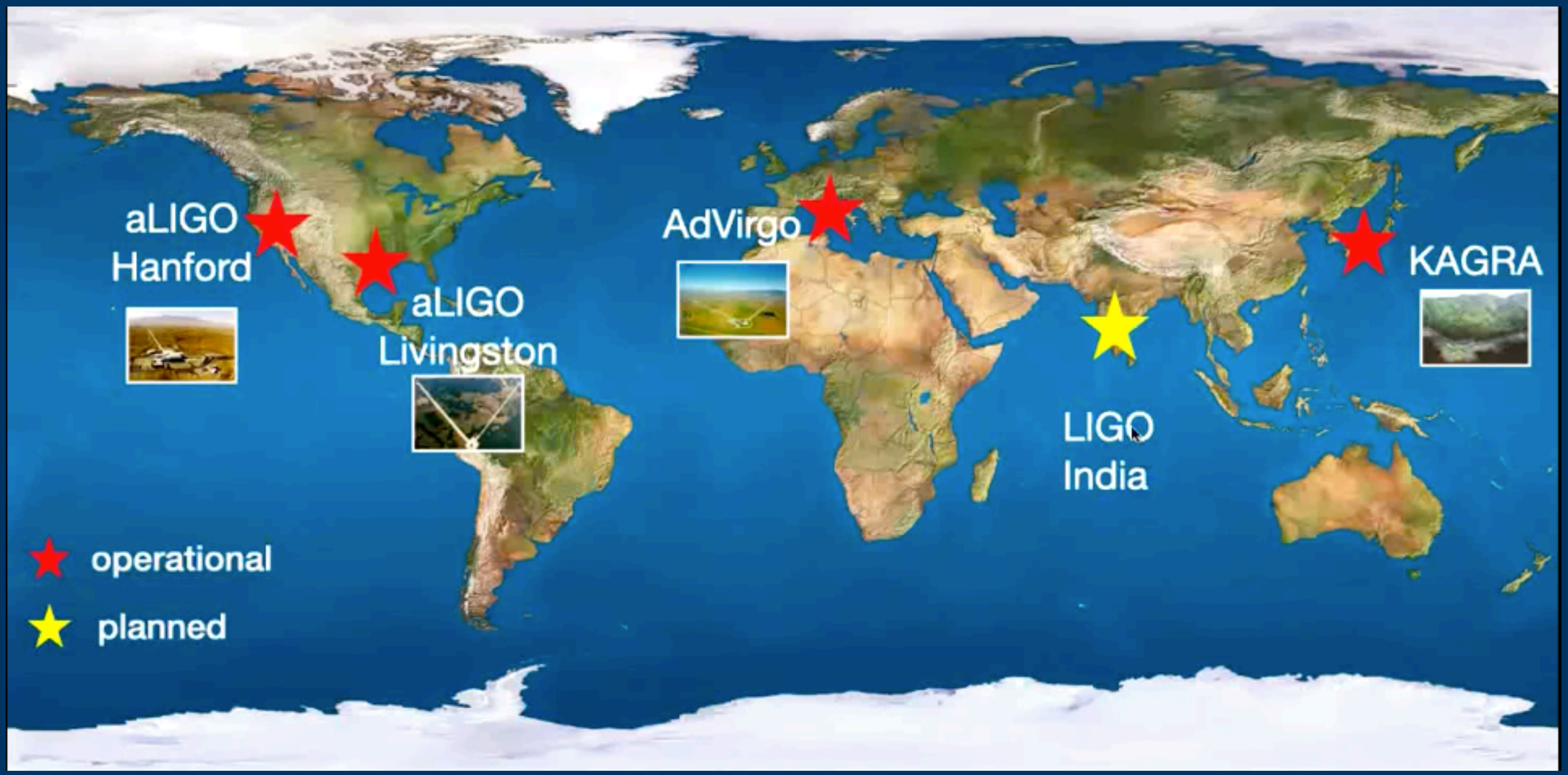
Equilibrio hidrostático

Materia (Gradientes de Presión)

Objetos compactos

Gravedad

+ TianQin (en el espacio)



Nuevos detectores capaces de mejorar las resoluciones actuales

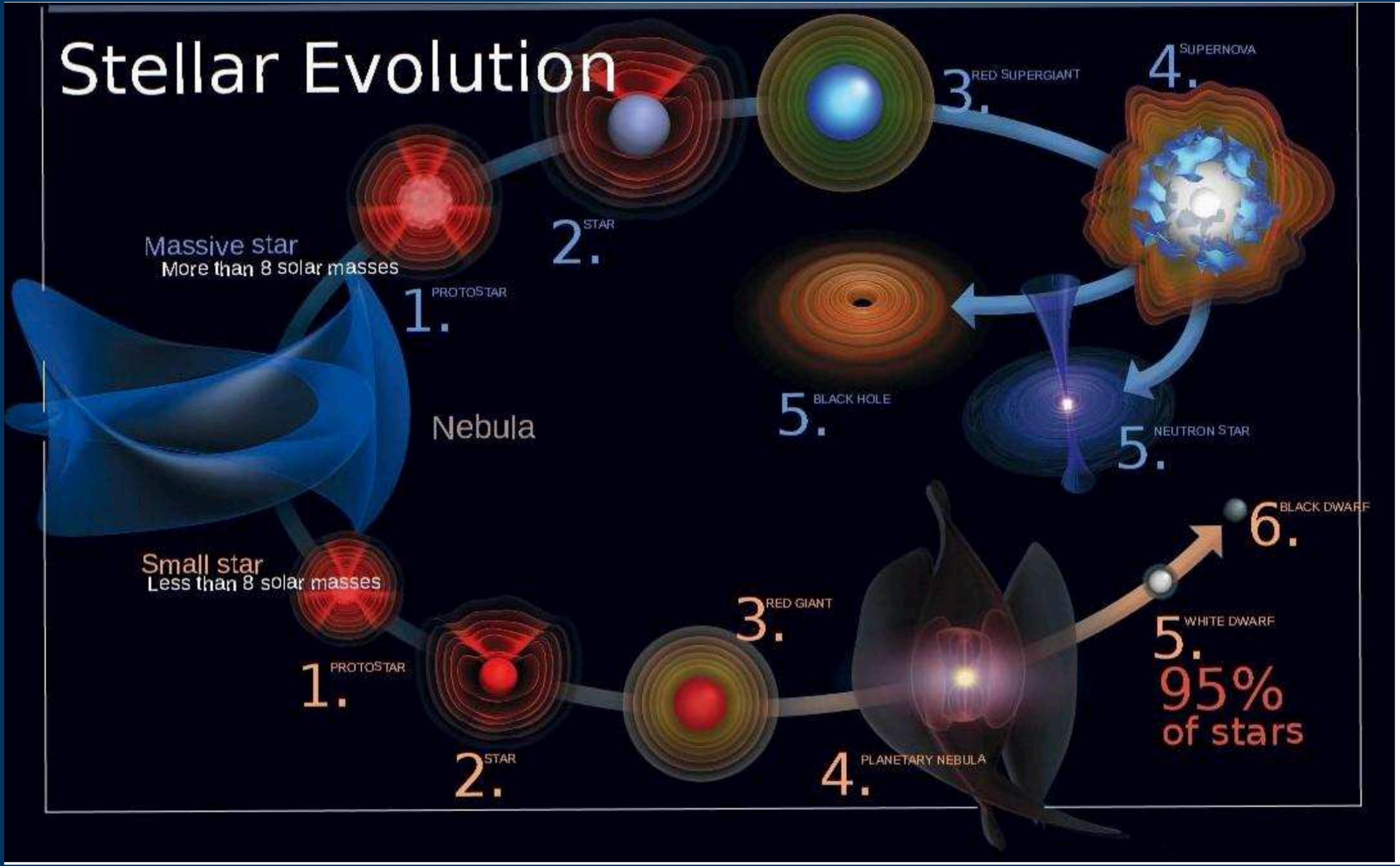
Primer paso o aproximación: considerar objetos estáticos, esféricamente simétricos y que pueden ser descritos mediante la métrica

$$ds^2 = -N^2(r)dt^2 + g^2(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

ansatz usado en todo el trabajo

Existen propuestas teóricas que intentan explicar mediante objetos compactos sumamente extendidos, y cuyos constituyentes son nuevos campos, el comportamiento *atípico* en las curvas de rotación y dispersión de velocidades en las galaxias. (no se aborda en este curso)

Estrellas y Estrellas de Neutrones



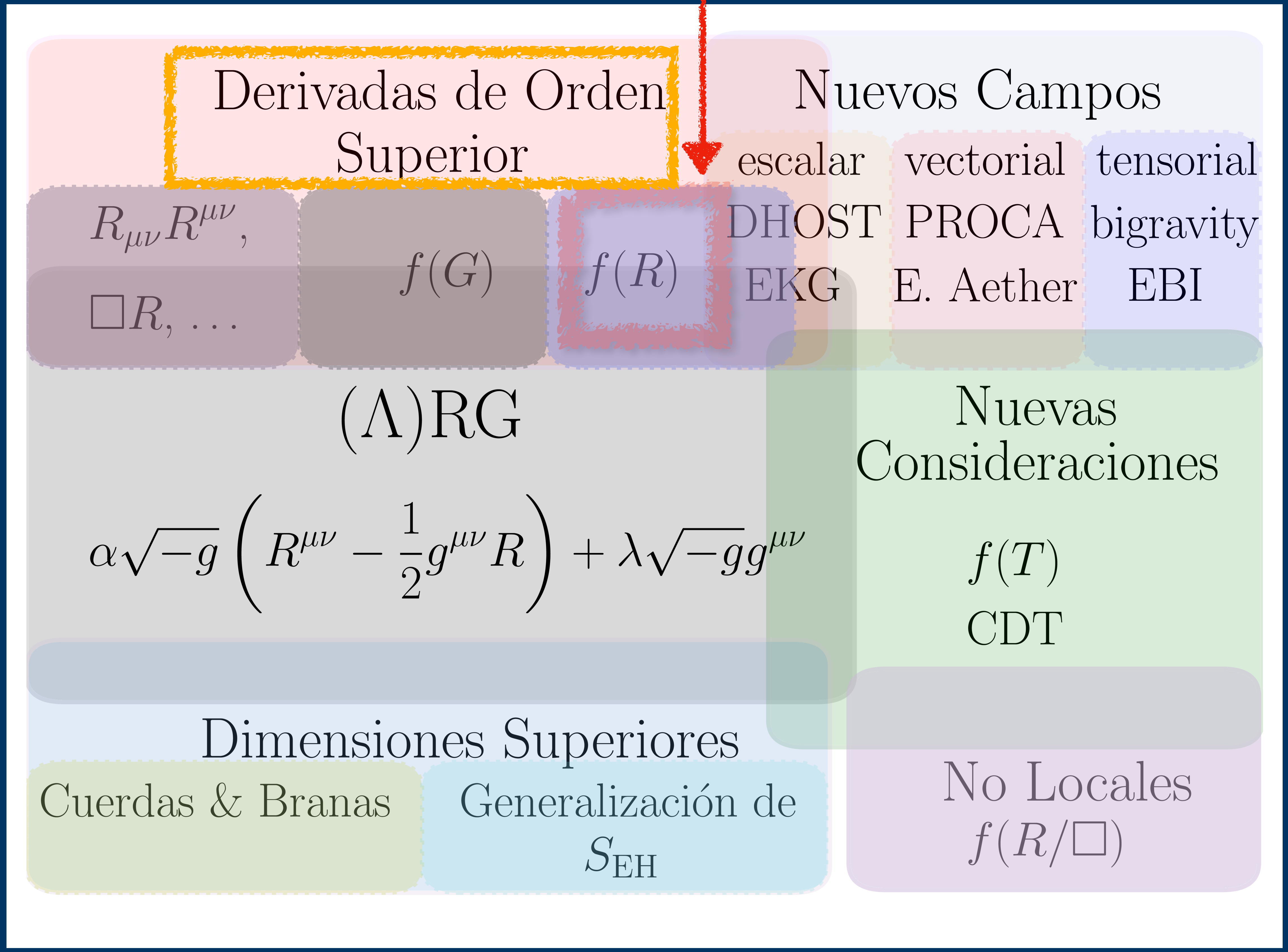
Estrellas Bariónicas

Gravedad $f(R)$

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} f(R[g_{\mu\nu}]) + \mathscr{L}_{\mathrm{m}} \right)$$

formalismo métrico

$$\underbrace{f_R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + \boxed{g_{\mu\nu} \Box f_R - \nabla_\nu \nabla_\mu f_R}}_{\text{curvatura (Modificación a la RG)}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{materia}}$$



Casos

$f(R) = R \rightarrow$ RG

$f(R) = R + \alpha R^2 \rightarrow$ Gravedad cuadrática

Objetos auto-gravitantes, estáticos y esféricamente simétricos

$f(R) = R$

$\downarrow$

Tolman-Oppenheimer-Volkoff (TOV)

$$\frac{N'}{N} = \frac{1}{2\,kr\left(2f_R + rR'f_{RR}\right)}\left\{r^2 T^{11} + kg^2\left[r^2 f + \left(2 - r^2 R\right)f_R\right] - 2k\left(f_R + 2rR'f_{RR}\right)\right\}$$

¿?

$$\frac{g'}{g} = \frac{1}{2\,kN^2 r\left(2f_R + rR'f_{RR}\right)}\left\{g^2 r^2 T^{00} + kN^2\left[g^2\left(-r^2 f + \left[-2 + r^2 R\right]f_R\right) + 2\left(f' + r\left[(2R' + rR'')f_{RR} + rR'^2 f_{RRR}\right]\right)\right]\right\}$$

$$\boxed{R'' = \left(-\frac{2}{r} - \frac{N'}{N} + \frac{g'}{g}\right)R' + \frac{g^2\left(T + 4kf - 2kRf_R\right)}{6kf_{RR}} - \frac{R'^2 f_{RRR}}{f_{RR}}}$$

ecuación dinámica del grado extra de libertad

$f(R) \to$ arbitraria

$\downarrow$

Tolman-Oppenheimer-Volkoff modificadas (TOVm)

T^{00} y T^{11} son las componentes 0-0 y 1-1 (respectivamente) de un tensor de energía-momento correspondiente a un fluido perfecto oscuro

¿?

$$T^{\mu\nu} = \left(\rho + \mathrm{p_m}\right)u^\mu u^\nu + g^{\mu\nu}\mathrm{p_m}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Ecuación de estado de la MO

Reto, obtener Newton

$$\begin{aligned}\frac{dM(r)}{dr} &= 4\pi r^2 \rho_m(r)\\ \frac{dP(r)}{dr} &= -G\frac{M(r)\rho_m(r)}{r^2}\end{aligned}$$

Materia fermiónica auto-interactuarte en el límite degenerado

$$T^{00} = -\rho\, u^0 u^0 = m^4 \left[\bar{\rho} + \bar{\rho}_{int}\right] u^0 u^0$$

$$T^{ij} = g^{ij}\, \mathrm{p}_m = m^4 \left[\bar{\mathrm{p}}_m + \bar{\mathrm{p}}_{int}\right] g^{ij}$$

$$\mathscr{L}^{\mathrm{int}} = -\, g\bar{\chi}\chi\psi \quad \rightarrow \quad \rho_{int} \sim n^2/m_I^2$$

$$\boxed{\begin{aligned} \bar{\rho}_{int} &= \frac{1}{9\pi^4}\, y^2 x_F^6 \\ \bar{p}_{int} &= \frac{1}{9\pi^4}\, y^2 x_F^6 \end{aligned}}$$

G. Narain, et al.

$m_I = m_\psi / g$

$y = m/m_I = g\, m/m_\psi$

$n \rightarrow$ densidad de partículas

$$F(r,p,t) = \frac{1}{4\pi^3}\, \frac{1}{1 + e^{[E(p)-\mu(r)]/k_B T(r)}}, \quad T(r) = 0$$

$$-\rho = \int E\, F(r,p,t)\, d^3p$$

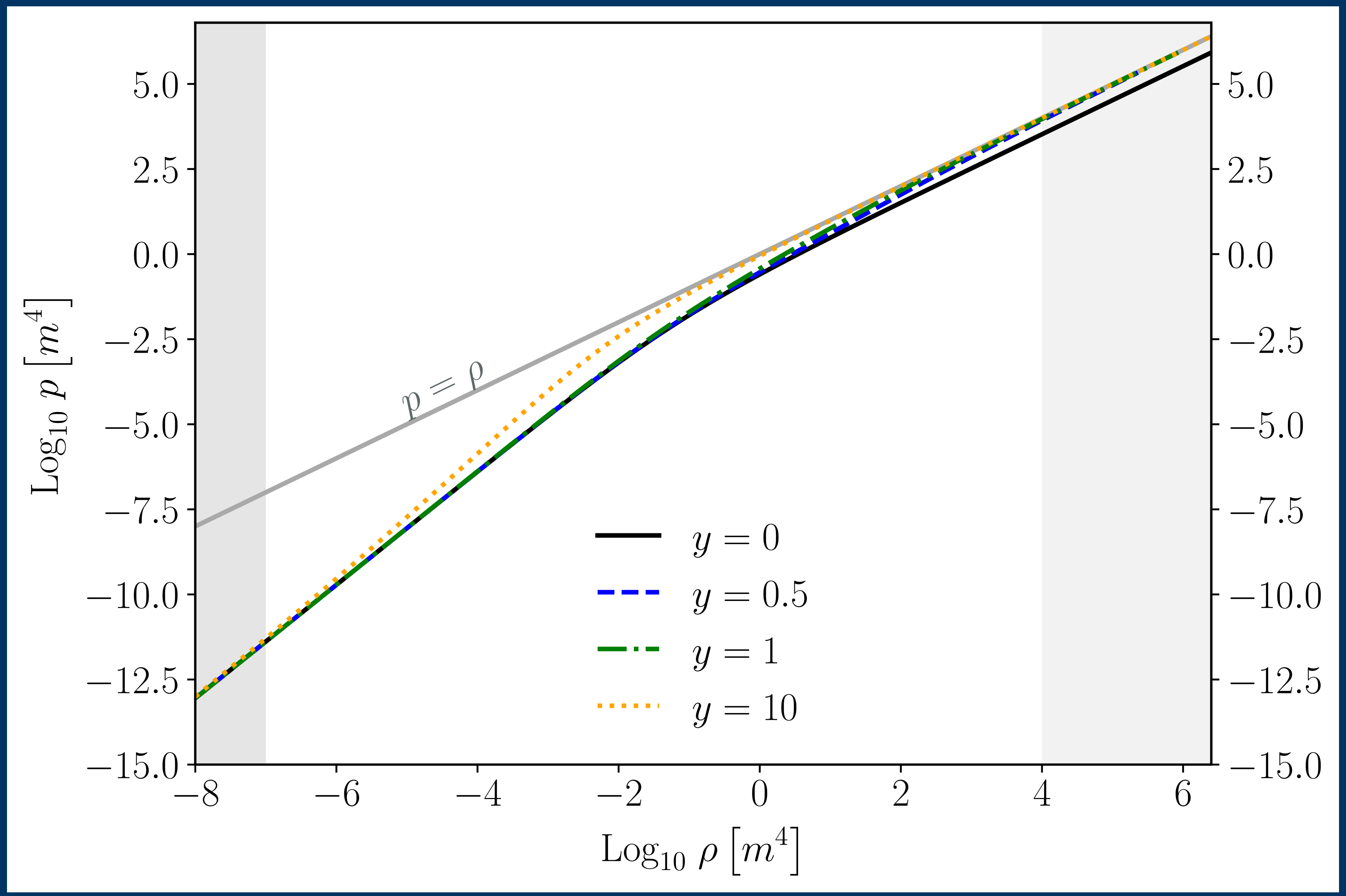
$$\mathrm{p}_m = \frac{1}{3}\int U \cdot p\, F(r,p,t)\, d^3p$$

$$\boxed{f(E) = \begin{cases} 1 & \text{si} \quad \mathrm{E}(p) \leq \mathrm{E_F}(r), \\ 0 & \text{si} \quad \mathrm{E}(p) > \mathrm{E_F}(r)\,. \end{cases}}$$

$$E_F(r) \equiv \mu(r) = m\sqrt{\Phi(r) + 1} = \sqrt{p_F(r)^2 + m^2}$$

$$\bar{\rho}(x_F) = \frac{1}{8\pi^2}\left[(2x_F^3 + x_F)(1 + x_F^2)^{1/2} - \ln\left(x_F + \sqrt{x_F^2 + 1}\right)\right]$$

$$\bar{\mathrm{p}}_m(x_F) = \frac{1}{24\pi^2}\left[\left(2x_F^3 - 3x_F\right)(1 + x_F^2)^{1/2} + 3\ln\left(x_F + \sqrt{x_F^2 + 1}\right)\right]$$



$x_F = p_F/m \qquad 0 < x_F < 20 \qquad 0 \leq y \leq 10$

¡Empecemos a Divertirnos!

Estrellas Escalares

¿Qué es una estrella de bosones?

Ingredientes

Gravedad

RG + $\boxed{\text{Campo Escalar } \psi}$ $\rightarrow$ Sistema de Klein-Gordon

Sector de materia

La acción

$$S[g_{\mu\nu}, \psi] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \boxed{\nabla_\mu \psi \nabla^\mu \psi - V(\psi)} \right]$$

La física

"Presión" repulsiva

Gravedad

Equilibrio hidrostático $\rightarrow$ Objetos compactos localizados

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Klein-Gordon Geon*

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(Received 4 March 1968)

A study of the spherically symmetric eigenstates of the Klein-Gordon Einstein equations (Klein-Gordon geons) reveals that these geons have properties that are uniquely different from other gravitating systems that have been studied. The equilibrium states of these geons seem analogous to other gravitating systems; but when the question of stability is considered from a thermodynamical viewpoint, it is shown that, in contrast with other systems, adiabatic perturbations are forbidden. The reason is that the equations of state for the thermodynamical variables are not algebraic equations, but instead are differential equations. Consequently, the usual concept of an equation of state breaks down when Klein-Gordon geons are considered. When the question of stability is reconsidered in terms of infinitesimal perturbations of the basic

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Systems of Self-Gravitating Particles in General Relativity and the Concept of an Equation of State*

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(Received 4 February 1969)

A method of self-consistent fields is used to study the equilibrium configurations of a system of self-gravitating scalar bosons or spin-$\frac{1}{2}$ fermions in the ground state without using the traditional perfect-fluid approximation or equation of state. The many-particle system is described by a second-quantized free field, which in the boson case satisfies the Klein-Gordon equation in general relativity, $\nabla_\alpha \nabla^\alpha \phi = \mu^2 \phi$, and in the fermion case the Dirac equation in general relativity $\gamma^\alpha \nabla_\alpha \psi = \mu \psi$ (where $\mu = mc/\hbar$). The coefficients of the metric $g_{\alpha\beta}$ are determined by the Einstein equations with a source term given by the mean value $\langle \phi | T_{\mu\nu} | \phi \rangle$ of the energy-momentum tensor operator constructed from the scalar or the spinor field. The state vector $\langle \phi |$ corresponds to the ground state of the system of many particles. In both cases, for completeness, a nonrelativistic Newtonian approximation is developed, and the corrections due to special and general relativity explicitly are pointed out. For N bosons, both in the region of validity of the Newtonian treatment (density from 10^{-80} to 10^{54} g cm^{-3}, and number of particles from 10 to 10^{40}) as well as in the relativistic region (density $\sim 10^{54}$ g cm^{-3}, number of particles $\sim 10^{40}$), we obtain results completely different from those of a traditional fluid analysis. The energy-momentum tensor is anisotropic. A critical mass is found for a system of $N \sim [(\text{Planck mass})/m]^2 \sim 10^{40}$ (for $m \sim 10^{-25}$ g) self-gravitating bosons in the ground state, above which mass gravitational collapse occurs. For N fermions, the binding energy of typical particles is $G^2 m^5 N^{4/3} \hbar^{-2}$ and reaches a value $\sim mc^2$ for $N \sim N_{crit} \sim [(\text{Planck mass})/m]^3 \sim 10^{57}$ (for $m \sim 10^{-24}$ g, implying mass $\sim 10^{33}$ g, radius $\sim 10^6$ cm, density $\sim 10^{15}$ g/cm^3). For densities of this order of

Nota: $c = \hbar = 1$

El campo escalar:

ψ -Spin 0

$\psi(\vec{x})$ → Teorema de Derrick

$\boxed{\text{Soluciones localizadas y estacionarias de Eq. KG en 3D o dimensiones más altas son inestables}}$

$\psi(t,\vec{x})$

Real →

Volume 66, Number 13 PHYSICAL REVIEW LETTERS 1 April 1991

Oscillating Soliton Stars

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(Received 26 December 1990)

We study a new type of self-gravitating object, described by a soliton solution to the coupled system of the Einstein equation and a matter field equation. The solution describing the self-gravitating object is not static but, instead, is periodic, with both the spacetime geometry and the matter field oscillating in time. In particular, we show that a system described by a real scalar field $g^{\mu\nu}\phi_{;\mu\nu}-m^2\phi=0$ admits such a solution, which is stable under perturbations. The existence of such objects could have important implications for astrophysics and cosmology.

PACS numbers: 95.30.Sf, 04.20.Jb

$$N^2(t,r)=1+\sum_{j=0}^{\infty}N_{2j}(r)\cos(2j\omega_0 t),$$
$$g^2(t,r)=1+\sum_{j=0}^{\infty}g_{2j}(r)\cos(2j\omega_0 t),$$
$$\phi(t,r)=\sum_{j=1}^{\infty}\phi_{2j-1}(r)\cos[(2j-1)\omega_0 t].$$

oscilones

Complejo →

Simetría $U(1)$

$$\psi(t,\vec{x})=\sigma(r)e^{-i\omega t}$$

→

- Q-balls
- Boson stars

El potencial

$$V[|\psi|]=\boxed{m^2|\psi|^2}+M^4\sum_{n=2}^{\infty}\frac{v_{2n}}{(2n)!}\boxed{\left|\frac{\psi}{M}\right|^{2n}}$$

término de masa

términos de auto-interacción

$\sim|\psi|^4$

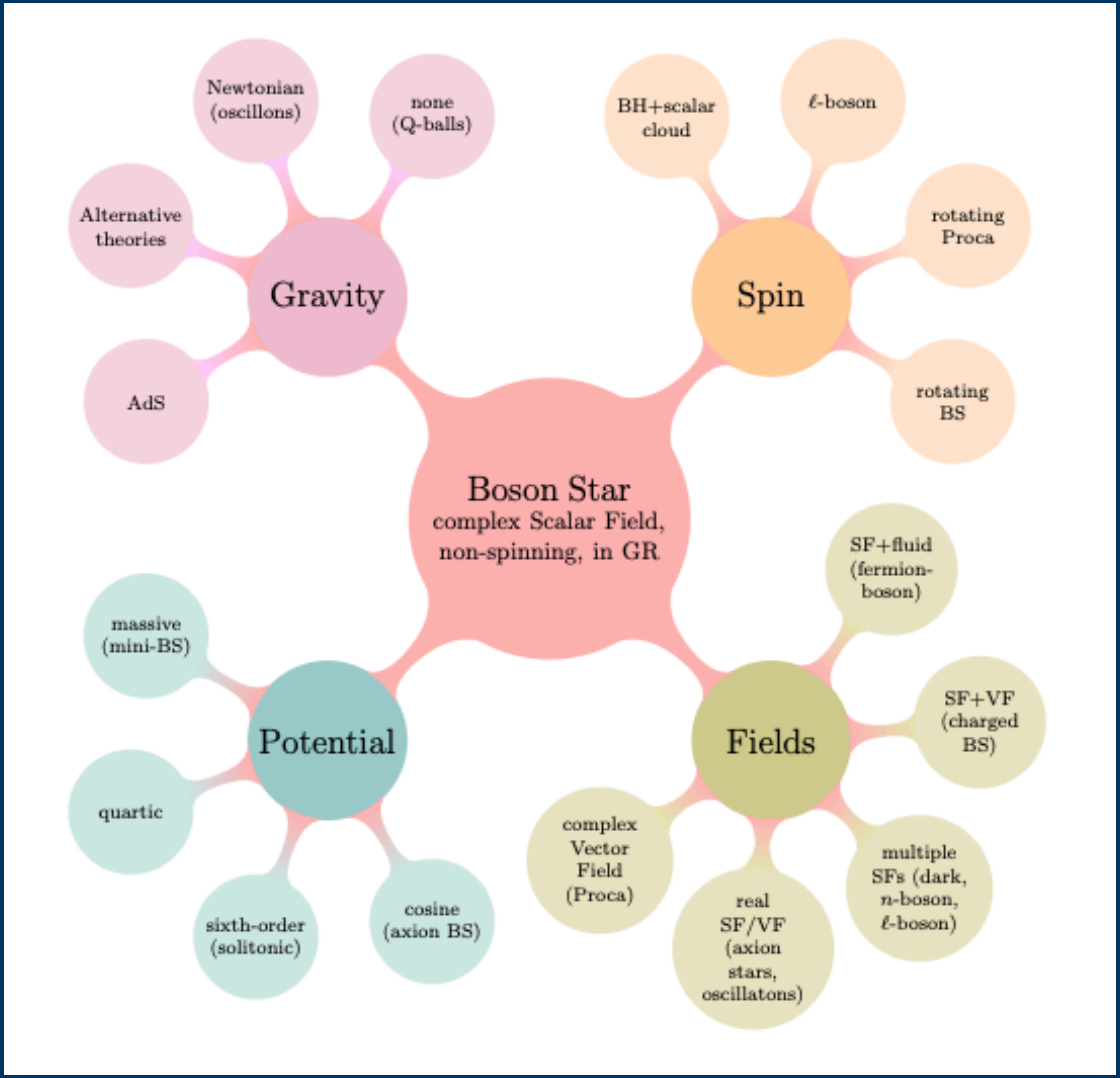
$\sim c_1|\psi|^4+c_2|\psi|^6$

- Campos con Spin 1: Estrellas de Proca

$$\mathcal{L}_M=-\frac{1}{2}F^*_{\mu\nu}F^{\mu\nu}-m_0^2A^*_\mu A^\mu-\lambda(A^*_\mu A^\mu)^2,$$
$$A_\mu(t,\vec{x}) = e^{-i\omega t}\left(f(r),ig(r),0,0\right).$$

- Campos con Spin 2: teorías Bimetric

Resumen:



arXiv:1202.5809v5