Solution to Exercise 6 Artificial Intelligence

12/6

Problem 6.1: The man in the painting

- A sibling is another child of one's parents.

$$\forall x, y \quad Sibling(x, y) \Leftrightarrow x \neq y \wedge \exists p \quad Parent(p, x) \wedge Parent(p, y)$$ 1, 2

- Parent and child are inverse relations.

$$\forall p, c \quad Parent(p, c) \Leftrightarrow Child(c, p)$$ 3, 4

Problem 6.1.1:

- Every son is a male child, and every male child is a son.

$$\forall s, p \quad Son(s, p) \Leftrightarrow Child(s, p) \wedge Male(s)$$ 5, 6

- Every father is a male parent, and every male parent is a father.

$$\forall p, c \quad Father(p, c) \Leftrightarrow Parent(p, c) \wedge Male(p)$$ 7, 8

Problem 6.1.2:

From the problem definition, we know that the constant Me is male. Therefore, we add the following sentence to our knowledge base.

$$Male(Me)$$ 12

From the problem definition, we also know that the person in the painting is male.

$$Male(That)$$ 11

Problem 6.1.3:

- Brothers and sisters have I none.

$$\forall x \quad \neg Sibling(x, Me) \wedge \neg Sibling(Me, x)$$ 9

- That man's father is my father's son.

$$\exists f_1, f_2 \quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2)$$ 10

Problem 6.1.4:

We analyse the sentence

> ... but that man's father is my father's son.

The phrase "my father's son" could be either "Me" or "My Sibling". However, from previous constraint

> Brothers and sisters have I none, ...

we know that it could not be "My Sibling" because it does not exist. Therefore, the phrase "my father's son" is "Me". Using this equality, we know from the first quote above that "That man's father" is "Me". Therefore, "That man" is the son of "Me".

Problem 6.1.5:

We want to prove that $\alpha : Son(That, Me)$. We, first, transform the rule into Conjunctive Normal Form (CNF).

1. A sibling is another child of one's parents ($\Leftarrow$).

$$\begin{aligned}
& \forall x, y \quad Sibling(x,y) \;\Leftarrow\; (x \neq y) \wedge \exists p \quad Parent(p,x) \wedge Parent(p,y) \\
\equiv \quad & \langle \text{by removing implication} \rangle \\
& \forall x, y \quad Sibling(x,y) \;\vee\; \neg((x \neq y) \wedge \exists p \quad Parent(p,x) \wedge Parent(p,y)) \\
\equiv \quad & \langle \text{by pushing } \neg \text{ inwards} \rangle \\
& \forall x, y \quad Sibling(x,y) \;\vee\; (x = y) \vee \forall p \quad \neg Parent(p,x) \vee \neg Parent(p,y) \\
\equiv \quad & \langle \text{by dropping universal quantifier} \rangle \\
& Sibling(x,y) \;\vee\; (x = y) \vee \neg Parent(p,x) \vee \neg Parent(p,y)
\end{aligned}$$

$$\boxed{Sibling(x,y) \;\vee\; (x = y) \vee \neg Parent(p,x) \vee \neg Parent(p,y)} \tag{1}$$

2. A sibling is another child of one's parents ($\Rightarrow$).

$$\begin{aligned}
& \forall x, y \quad \neg Sibling(x,y) \vee ((x \neq y) \wedge \exists p \quad Parent(p,x) \wedge Parent(p,y)) \\
\equiv \quad & \langle \text{by skolemisation} \rangle \\
& \forall x, y \quad \neg Sibling(x,y) \vee ((x \neq y) \wedge Parent(F(x,y),x) \wedge Parent(F(x,y),y)) \\
\equiv \quad & \langle \text{by dropping universal quantifier} \rangle \\
& \neg Sibling(x,y) \vee ((x \neq y) \wedge Parent(F(x,y),x) \wedge Parent(F(x,y),y)) \\
\equiv \quad & \langle \text{by distributivity of } \vee \text{ over } \wedge \rangle \\
& (\neg Sibling(x,y) \vee (x \neq y)) \quad \wedge (\neg Sibling(x,y) \vee Parent(F(x,y),x)) \\
& \qquad\qquad\qquad\qquad\qquad \wedge (\neg Sibling(x,y) \vee Parent(F(x,y),y))
\end{aligned}$$

$$\boxed{\neg Sibling(x,y) \vee (x \neq y)} \tag{2}$$

$$\boxed{\neg Sibling(x,y) \vee Parent(F(x,y),x)} \tag{3}$$

$$\boxed{\neg Sibling(x,y) \vee Parent(F(x,y),y)} \tag{4}$$

3. Parent and child are inverse relations ($\Rightarrow$).

$$\begin{aligned}
& \forall p,c \quad Parent(p,c) \ \Rightarrow \ Child(c,p) \\
\equiv \quad & \langle \text{by removing implication} \rangle \\
& \forall p,c \quad \neg Parent(p,c) \ \vee \ Child(c,p) \\
\equiv \quad & \langle \text{by dropping universal quantifier} \rangle \\
& \neg Parent(p,c) \vee Child(c,p)
\end{aligned}$$

$$\boxed{\neg Parent(p,c) \vee Child(c,p)} \tag{5}$$

4. Parent and child are inverse relations ($\Leftarrow$).

$$\begin{aligned}
& \forall p,c \quad Child(c,p) \ \Rightarrow \ Parent(p,c) \\
\equiv \quad & \langle \text{by removing implication} \rangle \\
& \forall p,c \quad \neg Child(c,p) \ \vee \ Parent(p,c) \\
\equiv \quad & \langle \text{by dropping universal quantifier} \rangle \\
& \neg Child(c,p) \vee Parent(p,c)
\end{aligned}$$

$$\boxed{\neg Child(c,p) \vee Parent(p,c)} \tag{6}$$

5. Every son is a male child ($\Rightarrow$).

$$\begin{aligned}
& \forall s,p \quad Son(s,p) \ \Rightarrow \ Child(s,p) \wedge Male(s) \\
\equiv \quad & \langle \text{by removing implication } \rangle \\
& \forall s,p \quad \neg Son(s,p) \ \vee \ (Child(s,p) \wedge Male(s)) \\
\equiv \quad & \langle \text{by dropping universal quantifier } \rangle \\
& \neg Son(s,p) \ \vee \ (Child(s,p) \wedge Male(s)) \\
\equiv \quad & \langle \text{by distributivity of } \vee \text{ over } \wedge \rangle \\
& (\neg Son(s,p) \ \vee \ Child(s,p)) \wedge (\neg Son(s,p) \ \vee \ Male(s))
\end{aligned}$$

$$\boxed{\neg Son(s,p) \ \vee \ Child(s,p)} \tag{7}$$

$$\boxed{\neg Son(s,p) \ \vee \ Male(s)} \tag{8}$$

6. Every son is a male child ($\Leftarrow$).

$$\begin{aligned}
& \forall s,p \quad Son(s,p) \ \Leftarrow \ Child(s,p) \wedge Male(s) \\
\equiv \quad & \langle \text{by removing implication } \rangle \\
& \forall s,p \quad Son(s,p) \ \vee \ \neg(Child(s,p) \wedge Male(s)) \\
\equiv \quad & \langle \text{by dropping universal quantifier } \rangle \\
& Son(s,p) \ \vee \ \neg(Child(s,p) \wedge Male(s)) \\
\equiv \quad & \langle \text{by de Morgan's rule} \rangle \\
& Son(s,p) \ \vee \ \neg Child(s,p) \vee \neg Male(s)
\end{aligned}$$

$$\boxed{Son(s,p) \ \vee \ \neg Child(s,p) \vee \neg Male(s)} \tag{9}$$

7. Every father is a male parent ($\Rightarrow$).

$$\begin{aligned}
& \forall p, c \quad Father(p,c) \;\Rightarrow\; Parent(p,c) \wedge Male(p) \\
\equiv & \quad \langle \text{by removing implication} \rangle \\
& \forall p, c \quad \neg Father(p,c) \;\vee\; (Parent(p,c) \wedge Male(p)) \\
\equiv & \quad \langle \text{by dropping universal quantifier} \rangle \\
& \neg Father(p,c) \;\vee\; (Parent(p,c) \wedge Male(p)) \\
\equiv & \quad \langle \text{by distributivity of } \vee \text{ over } \wedge \rangle \\
& (\neg Father(p,c) \;\vee\; Parent(p,c) \;\wedge\; (\neg Father(p,c) \;\vee\; Male(p))
\end{aligned}$$

$$\boxed{\neg Father(p,c) \;\vee\; Parent(p,c)} \tag{10}$$

$$\boxed{\neg Father(p,c) \;\vee\; Male(p)} \tag{11}$$

8. Every father is a male parent ($\Leftarrow$).

$$\begin{aligned}
& \forall p, c \quad Father(p,c) \;\Leftarrow\; Parent(p,c) \wedge Male(p) \\
\equiv & \quad \langle \text{by removing implication} \rangle \\
& \forall p, c \quad Father(p,c) \;\vee\; \neg(Parent(p,c) \wedge Male(p)) \\
\equiv & \quad \langle \text{by de Morgan's rule} \rangle \\
& \forall p, c \quad Father(p,c) \;\vee\; \neg Parent(p,c) \vee \neg Male(p) \\
\equiv & \quad \langle \text{by dropping universal quantifier} \rangle \\
& Father(p,c) \;\vee\; \neg Parent(p,c) \vee \neg Male(p)
\end{aligned}$$

$$\boxed{Father(p,c) \;\vee\; \neg Parent(p,c) \vee \neg Male(p)} \tag{12}$$

9. Brothers and sisters have I none.

$$\begin{aligned}
& \forall x \quad \neg Sibling(x, Me) \wedge \neg Sibling(Me, x) \\
\equiv & \quad \langle \text{by dropping quantifier} \rangle \\
& \neg Sibling(x, Me) \wedge \neg Sibling(Me, x)
\end{aligned}$$

$$\boxed{\neg Sibling(x, Me)} \tag{13}$$

$$\boxed{\neg Sibling(Me, x)} \tag{14}$$

10. That man's father is my father's son.

$$\begin{aligned}
& \exists f_1, f_2 \quad Father(f_1, That) \wedge Father(f_2, Me) \wedge Son(f_1, f_2) \\
\equiv & \quad \langle \text{by skolemisation} \rangle \\
& Father(F_1, That) \wedge Father(F_2, Me) \wedge Son(F_1, F_2)
\end{aligned}$$

$$\boxed{Father(F_1, That)} \tag{15}$$

$$\boxed{Father(F_2, Me)} \tag{16}$$

$$\boxed{Son(F_1, F_2)} \tag{17}$$

11. Sex of the person in the painting.

$$\boxed{Male(That)} \tag{18}$$

12. Sex of the person standing in front of the painting.

$$\boxed{Male(Me)} \tag{19}$$

13. negation of the goal

$$\boxed{\neg Son(That, Me)} \tag{20}$$

Formal proof. We start with the negation of the goal

KB $\vee \neg \alpha$ is unsatisfiable $\Leftrightarrow$ KB $\models \alpha$

$$\begin{aligned}
& \neg Son(That, Me) \\
\Rightarrow \quad & \langle \text{by Rule 9 with } \{s/That, p/Me\} \rangle \\
& \neg Child(That, Me) \vee \neg Male(That) \\
\Rightarrow \quad & \langle \text{by Rule 18} \rangle \\
& \neg Child(That, Me) \\
\Rightarrow \quad & \langle \text{by Rule 5, with } \{c/That, p/Me\} \rangle \\
& \neg Parent(Me, That)
\end{aligned}$$

$$\begin{tikzcd}
\boxed{\mathbf{Son(s,p)} \ \vee \ \neg Child(s,p) \vee \neg Male(s)} \arrow[dr] & \boxed{\neg Son(That, Me)} \arrow[d] \\
\boxed{\mathbf{Male(That)}} \arrow[dr] & \boxed{\neg Child(That, Me) \vee \neg Male(That)} \arrow[d] \\
\boxed{\neg Parent(p,c) \vee \mathbf{Child(c,p)}} \arrow[dr] & \boxed{\neg Child(That, Me)} \arrow[d] \\
& \boxed{\neg Parent(Me, That)}
\end{tikzcd}$$

Figure 1: Resolution proof for $\neg Parent(Me, That)$

We add this to resolution clause.

$$\boxed{\neg Parent(Me, That)} \tag{21}$$

We, then, start with the clause 17.

$$\begin{aligned}
& Son(F_1, F_2) \\
\Rightarrow \quad & \langle \text{by Rule 7 with } \{s/F_1, p/F_2\} \rangle \\
& Child(F_1, F_2) \\
\Rightarrow \quad & \langle \text{by Rule 6 with } \{c/F_1, p/F_2\} \rangle \\
& Parent(F_2, F_1) \\
\Rightarrow \quad & \langle \text{by Rule 1 with } \{x/F_1, p/F_2\} \rangle \\
& Sibling(F_1, y) \vee (F_1 = y) \vee \neg Parent(F_2, y) \\
\Rightarrow \quad & \langle \text{by Rule 10 with } \{p/F_2, c/y\} \rangle \\
& Sibling(F_1, y) \vee (F_1 = y) \vee \neg Father(F_2, y)
\end{aligned}$$

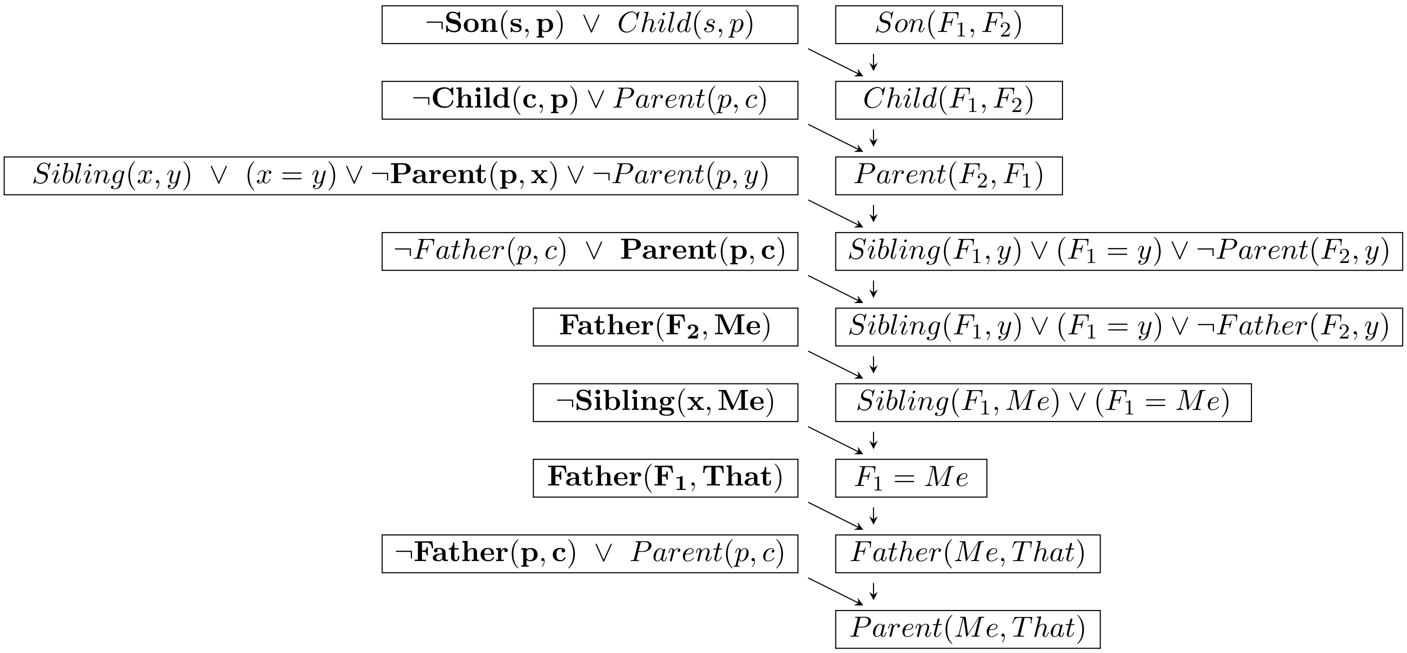

Figure 2: Resolution proof for $Parent(Me, That)$

$\Rightarrow$ $\langle$by Rule 16 with $\{y/Me\}\rangle$
$Sibling(F_1, Me) \vee (F_1 = Me)$
$\Rightarrow$ $\langle$by Rule 13 with $\{x/F_1\}$ $\rangle$
$(F_1 = Me)$
$\Rightarrow$ $\langle$by demodulation rule with rule 15$\rangle$
$Father(Me, That)$
$\Rightarrow$ $\langle$by rule 10 with $\{p/Me, c/That\}\rangle$
$Parent(Me, That)$
$\Rightarrow$ $\langle$by the last clause added 21$\rangle$
$\{\}$

*

Problem 6.2: Backward chaining

Proof sketch.

$$
\begin{array}{ll}
 & 7 \leq 3+9 \\
\Leftarrow & \langle \text{by Rule 8 with } \{x/7, z/3+9\} \rangle \\
 & 7 \leq y \quad \wedge \quad y \leq 3+9 \\
\Leftarrow & \langle \text{by Rule 4 with } \{x/7, y/7+0\} \rangle \\
 & \text{true} \quad \wedge \quad 7+0 \leq 3+9 \\
\Leftarrow & \langle \text{by true rule} \rangle \\
 & 7+0 \leq 3+9 \\
\Leftarrow & \langle \text{by Rule 8 with } \{x/7+0, z/3+9\} \rangle \\
 & 7+0 \leq y \quad \wedge \quad y \leq 3+9 \\
\Leftarrow & \langle \text{by Rule 6 with } \{y/0+7, x_6/7, y_6/0\} \rangle \\
 & \text{true} \quad \wedge \quad 0+7 \leq 3+9 \\
\Leftarrow & \langle \text{by true rule} \rangle \\
 & 0+7 \leq 3+9 \\
\Leftarrow & \langle \text{by Rule 8 with } \{w/0, y/3, x/7, z/9\} \rangle \\
 & 0 \leq 3 \quad \wedge \quad \leq 7 \leq 9 \\
\Leftarrow & \langle \text{by Rule 1 and Rule 2} \rangle \\
 & \text{true} \quad \wedge \quad \text{true} \\
\Leftarrow & \langle \text{by true rule} \rangle \\
 & \text{true}
\end{array}
$$

We apply the algorithm provided in the slide.

1. goals : $\{7 \leq 3+9\}$
   $q' \leftarrow \textsc{Subst}(\emptyset, 7 \leq 3+9)$
   $\theta' \leftarrow \{x_8/7,\ z_8/3+9\}$
   new $\leftarrow \{x_8 \leq y_8,\ y_8 \leq z_8\}$

   Using rule $\boxed{\forall x_8, y_8, z_8 \quad x_8 \leq y_8 \wedge y_8 \leq z_8 \ \Rightarrow \ x_8 \leq z_8}$.

2. goals : $\{x_8 \leq y_8,\ y_8 \leq z_8\}$
   $q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9\}, x_8 \leq y_8)$
   $\theta' \leftarrow \{x_4/7, y_8/7+0\}$
   new $\leftarrow \{y_8 \leq z_8\}$

   Using rule $\boxed{\forall x_4, \quad x_4 \leq x_4 + 0}$.

3. goals : $\{y_8 \leq z_8\}$
   $q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0\}, y_8 \leq z_8)$
   $\theta' \leftarrow \{x'_8/7+0,\ z'_8/3+9\}$
   new $\leftarrow \{x'_8 \leq y'_8,\ y'_8 \leq z'_8\}$

   Using rule $\boxed{\forall x'_8, y'_8, z'_8 \quad x'_8 \leq y'_8 \wedge y'_8 \leq z'_8 \ \Rightarrow \ x'_8 \leq z'_8}$.

4. goals : $\{x'_8 \leq y'_8,\ y'_8 \leq z'_8\}$
   $q' \leftarrow \textsc{Subst}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x'_8/7+0,\ z'_8/3+9\}, x'_8 \leq y'_8)$

$\theta' \leftarrow \{y_8'/0+7,\ x_6/7,\ y_6/0\}$
new $\leftarrow \{y_8' \leq z_8'\}$

$\boxed{\forall x_6, y_6 \quad x_6 + y_6 \leq y_6 + x_6}$.

5. goals : $\{y_8' \leq z_8'\}$
   $q' \leftarrow \text{SUBST}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x_8'/7+0,\ z_8'/3+9, y_8'/0+7,\ x_6/7,\ y_6/0\}, y_8' \leq z_8')$
   $\theta' \leftarrow \{w_7/0,\ y_7/3,\ x_7/7,\ z_7/9\}$
   new $\leftarrow \{w_7 \leq y_7,\ x_7 \leq z_7\}$

   $\boxed{\forall w_7, x_7, y_7, z_7 \quad w_7 \leq y_7 \wedge x_7 \leq z_7 \ \Rightarrow\ w_7 + x_7 \leq y_7 + z_7}$

6. new : $\{w_7 \leq y_7,\ x_7 \leq z_7\}$
   $q' \leftarrow \text{SUBST}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x_8'/7+0,\ z_8'/3+9, y_8'/0+7,\ x_6/7,\ y_6/0,\ w_7/0,\ y_7/3,\ x_7/7,\ z_7/9\},$
   $w_7 \leq y_7)$
   $\theta' \leftarrow \emptyset$
   new $\leftarrow \{x_7 \leq z_7\}$

   $\boxed{0 \leq 3}$

7. goals : $\{x_7 \leq z_7\}$
   $q' \leftarrow \text{SUBST}(\{x_8/7,\ z_8/3+9,\ x_4/7,\ y_8/7+0,\ x_8'/7+0,\ z_8'/3+9, y_8'/0+7,\ x_6/7,\ y_6/0,\ w_7/0,\ y_7/3,\ x_7/7,\ z_7/9\},$
   $w_7 \leq y_7)$
   $\theta' \leftarrow \emptyset$
   new $\leftarrow \emptyset$

   $\boxed{7 \leq 9}$