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Solution to Exercise 5 Artificial Intelligence

Problem 5.1: Syntax of first-order logic

Problem 5.1.1:

1. This is **not** a term because g needs three arguments.
2. This is **not** a term because f needs two arguments.
3. This is a term because
   - g is a function;
   - x is a variable, hence a term;
   - $f(y, z)$ is a term because
     - f is a function;
     - y is a variable, hence a term;
     - z is a variable, hence a term;
   - d is a constant, hence a term;
4. This is **not** a term because symbol h is not in $\mathcal{F}$

Problem 5.1.2:

1. This is a sentence because
   - S is a predicate;
   - m is a constant, hence a term;
   - x is a variable, hence a term.
2. This is a sentence because
   - B is a predicate;
   - m is a constant, hence a term;
   - $f(m)$ is a term because
     - f is a function;
     - m is a constant, hence a term.
3. This is **not** a sentence because the first argument of B, that is $B(m, x)$, is not a term but predicate because

- B is a predicate;
- m is a constant, hence a term;
- x is a variable, hence a term.

4. This is a sentence because
   - $B(x, y)$ is a sentence because
     - B is a predicate;
     - x, y are variables, hence terms.
   - $\exists z\ S(z, y)$ is a sentence because
     - z, y are variables, hence terms;
     - S is predicate.
5. This is a sentence because
   - $S(x, y)$ is a sentence because
     - S is a predicate;
     - x, y are variables, hence terms.
   - $S(y, f(f(x)))$ is a sentence because
     - S is a predicate;
     - y is a variable, hence a term;
     - $f(f(x))$ is a term because
       * f is a function;
       * $f(x)$ is a term because
         · f is a function;
         · x is a variable hence a term.

Problem 5.2: Universal and existential quantifier

Problem 5.2.1: Yes. The sentence evaluates to true according to these condition. The fact that Chris does not go to heaven has nothing to do with this sentence. This sentence only applies to good people only — which, in this case, only Anna is a good person and Chris not.

Problem 5.2.2: This formalisation evaluates to false. To see this, apply the extended interpretation of universal quantifier. This sentence becomes $G(\text{Anna}) \wedge H(\text{Anna}) \wedge G(\text{Ben}) \wedge H(\text{Ben}) \wedge G(\text{Chris}) \wedge H(\text{Chris})$. Since $G(Ben)$ and $G(Chris)$ are false, then the whole extended interpretation is also false.

Problem 5.2.3: False. No one is a good person and goes to heaven.

Problem 5.2.4: No. This formalisation evaluates to true. This is due to the implication form of the body of the existential quantifier. Since none of these people are good, the implication evaluates to true since it says nothing about bad people. Hence, the existential quantifier also evaluates to true because it has extended interpretation of $G(\text{Anna}) \Rightarrow H(\text{Anna}) \ \vee\ G(\text{Ben}) \Rightarrow H(\text{Ben}) \ \vee\ G(\text{Chris}) \Rightarrow H(\text{Chris})$. Then, it clearly is not a faithful formalisation of the sentence “Some good people go to heaven.”

Problem 5.3: Narcissist Mary

NB $\forall$, $\Rightarrow$

$\exists$, $\wedge$

1. Mary admires every professor.

$$\forall x \quad P(x) \Rightarrow A(m, x)$$

2. Some professor admires Mary.

$$\exists x \quad P(x) \wedge A(x, m)$$

3. Mary admires herself.

$$A(m, m)$$

4. No student attended every lecture

$$\neg \exists x \forall y \quad S(x) \wedge (L(y) \Rightarrow B(x, y))$$

or equivalently, "Every student has a lecture he or she does not attend"

$$\forall x \exists y \quad S(x) \Rightarrow (L(y) \wedge \neg B(x, y))$$

5. No lecture was attended by every student

$$\neg \exists x \forall y \quad L(x) \wedge (S(y) \Rightarrow B(y, x))$$

or equivalently, "Every lecture must have some student misses the lecture"

$$\forall x \exists y \quad L(x) \Rightarrow (S(y) \wedge \neg B(y, x))$$

6. No lecture was attended by any student

$$\neg \exists x \exists y \quad L(x) \wedge S(y) \wedge B(y, x)$$

or equivalently, "Every lecture is missed by every student"

$$\forall x \forall y \quad L(x) \Rightarrow (S(y) \Rightarrow \neg B(y, x))$$

or equivalently

$$\forall x \forall y \quad (L(x) \wedge S(y)) \Rightarrow \neg B(y, x)$$

Problem 5.4: Sherlock Holmes and Professor Moriarty

1. Holmes can trap anyone who can trap Moriarty.

$$\forall x \quad T(x,m) \Rightarrow T(h,x)$$

2. Holmes can trap anyone whom Moriarty can trap.

$$\forall x \quad T(m,x) \Rightarrow T(h,x)$$

3. Holmes can trap anyone who can be trapped by Moriarty.

$$\forall x \quad T(m,x) \Rightarrow T(h,x)$$

4. If anyone can trap Moriarty, then Holmes can.

$$(\exists x \quad T(x,m)) \Rightarrow T(h,m)$$

5. If everyone can trap Moriarty, then Holmes can.

$$(\forall x \quad T(x,m)) \Rightarrow T(h,m)$$

6. Anyone who can trap Holmes can trap Moriarty.

$$\forall x \quad T(x,h) \Rightarrow T(x,m)$$

7. No one can trap Holmes unless he can trap Moriarty.

$$\forall x \quad \neg T(x,m) \Rightarrow \neg T(x,h)$$

or equivalently

$$\neg \exists x \quad \neg T(x,m) \wedge T(x,h)$$

8. Everyone can trap someone who cannot trap Moriarty.

$$\forall x \, \exists y \quad \neg T(y,m) \wedge T(x,y)$$

9. Anyone who can trap Holmes can trap anyone whom Holmes can trap.

$$\forall x \quad T(x,h) \Rightarrow (\forall y \quad T(h,y) \Rightarrow T(x,y))$$

or equivalently

$$\forall x,y \quad T(x,h) \Rightarrow (T(h,y) \Rightarrow T(x,y))$$

or equivalently

$$\forall x,y \quad T(x,h) \wedge T(h,y) \Rightarrow T(x,y)$$