Lecture 7 Recap

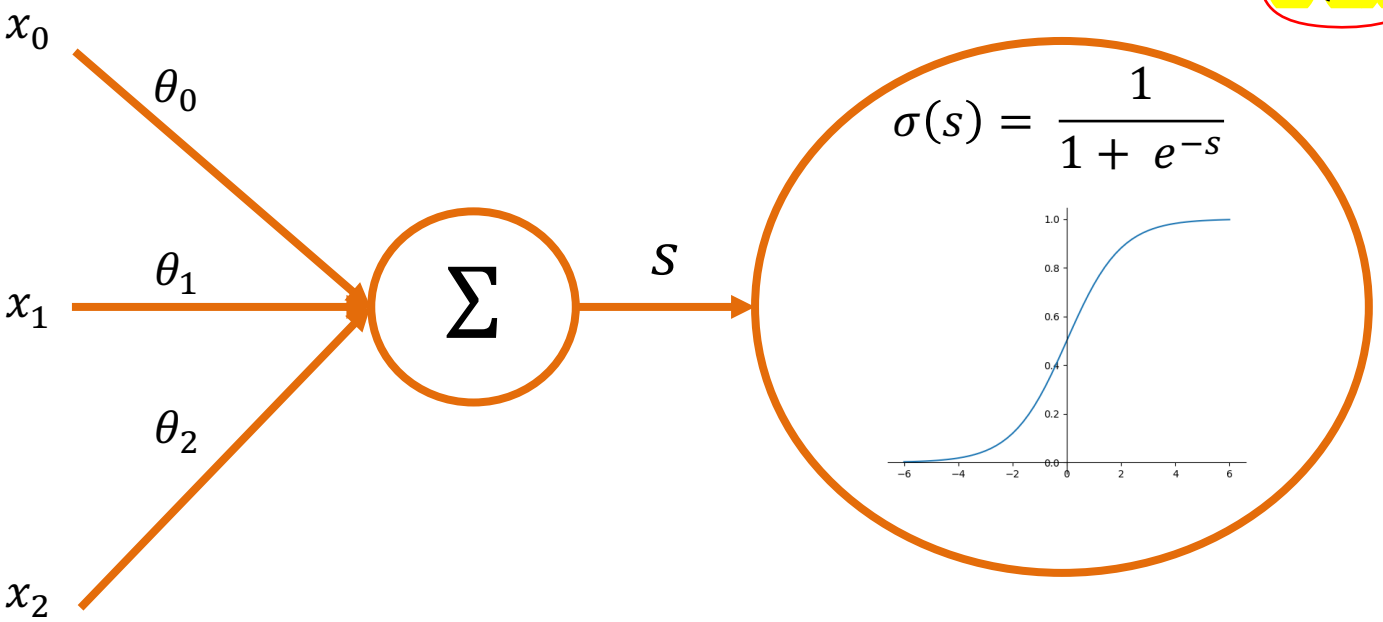
Naïve Losses: L2 vs L1

- L2 Loss:
  - $L^2 = \sum_{i=1}^{n} (y_i - f(x_i))^2$
  - Sum of squared differences (SSD)
  - Prone to outliers
  - Compute-efficient (optimization)
  - Optimum is the mean

- L1 Loss:
  - $L^1 = \sum_{i=1}^{n} |y_i - f(x_i)|$
  - Sum of absolute differences
  - Robust
  - Costly to compute
  - Optimum is the median

Binary Classification: Sigmoid

$$\sigma(\boldsymbol{x}, \boldsymbol{\theta}) = \frac{1}{1 + e^{-\Sigma \theta_i x_i}}$$



1

Can be interpreted as a probability

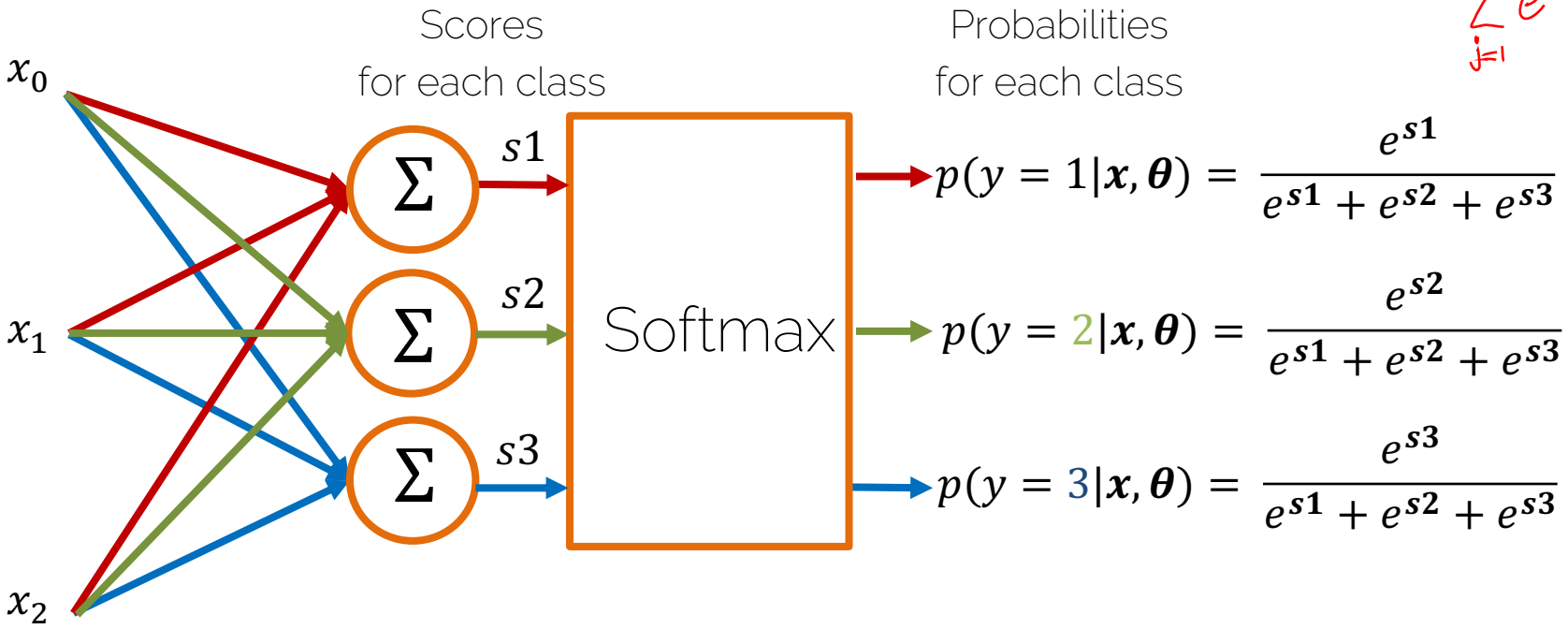
$p(y = 1|x, \boldsymbol{\theta})$

0

Softmax Formulation

- What if we have multiple classes?

$$\sigma(z) = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_i}}$$



Example: Hinge vs Cross-Entropy

Hinge Loss: $L_i = \sum_{k \neq y_i} \max(0, s_k - s_{y_i} + 1)$

Cross Entropy: $L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_k e^{s_k}}\right)$

Given the following scores for $\boldsymbol{x_i}$:		Hinge loss:	Cross Entropy loss:
Model 1	$s = [5, -3, 2]$	$\max(0, -3 - 5 + 1) + \max(0, 2 - 5 + 1) = 0$	$-\ln\left(\frac{e^5}{e^5 + e^{-3} + e^2}\right) = 0.05$
Model 2	$s = [5, 10, 10]$	$\max(0, 10 - 5 + 1) + \max(0, 10 - 5 + 1) = 12$	$-\ln\left(\frac{e^5}{e^5 + e^{10} + e^{10}}\right) = 5.70$
Model 3	$s = [5, -20, -20]$ $y_i = 0$	$\max(0, -20 - 5 + 1) + \max(0, -20 - 5 + 1) = 0$	$-\ln\left(\frac{e^5}{e^5 + e^{-20} + e^{-20}}\right) = 2 * 10^{-11}$

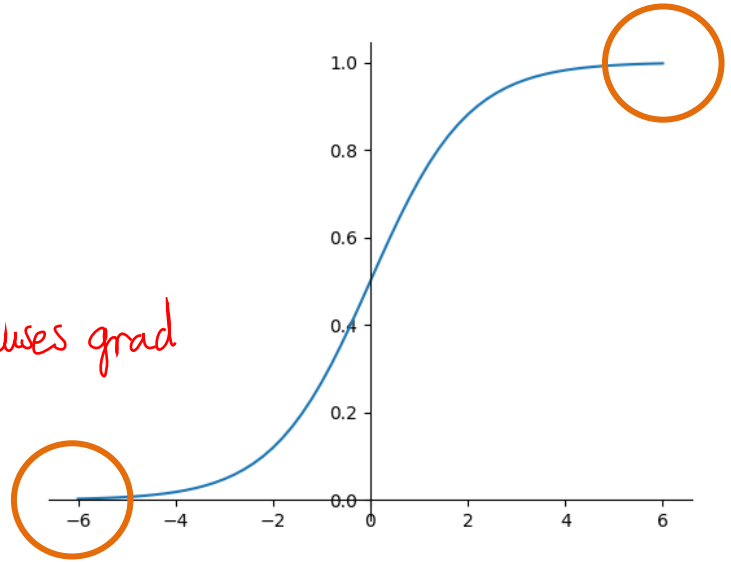
- Cross Entropy *always* wants to improve! (loss never 0)
- Hinge Loss saturates.

Sigmoid Activation

Forward

$$\sigma(s) = \frac{1}{1 + e^{-s}}$$

$$\frac{\partial L}{\partial w} = \frac{\partial s}{\partial w}\frac{\partial L}{\partial s}$$



✗ Saturated neurons kill the gradient flow

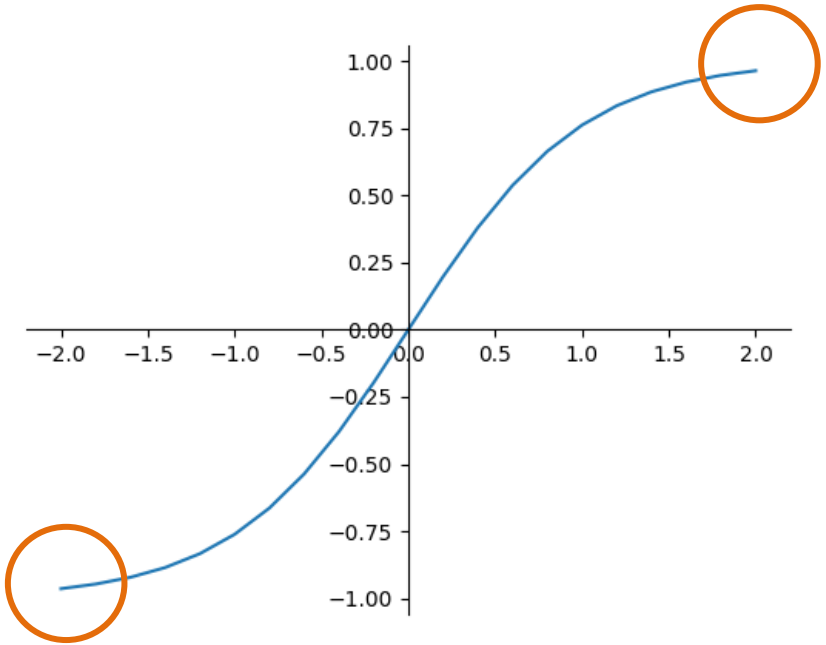
NB Non-Zero centered causes grad to be either +ve / -ve at the same time

$$\frac{\partial L}{\partial s} = \frac{\partial \sigma}{\partial s}\frac{\partial L}{\partial \sigma}$$

$$\frac{\partial \sigma}{\partial s}$$

$$\frac{\partial L}{\partial \sigma}$$

TanH Activation



✘ Still saturates

✓ Zero-centered

[LeCun et al. 1991] Improving Generalization Performance in Character Recognition

Rectified Linear Units (ReLU)

mostly with CNNs

✗ Dead ReLU

What happens if a ReLU outputs zero?

Large and consistent gradients ✓

✓ Fast convergence

✓ Does not saturate

[Krizhevsky et al. NeurIPS 2012] ImageNet Classification with Deep Convolutional Neural Networks

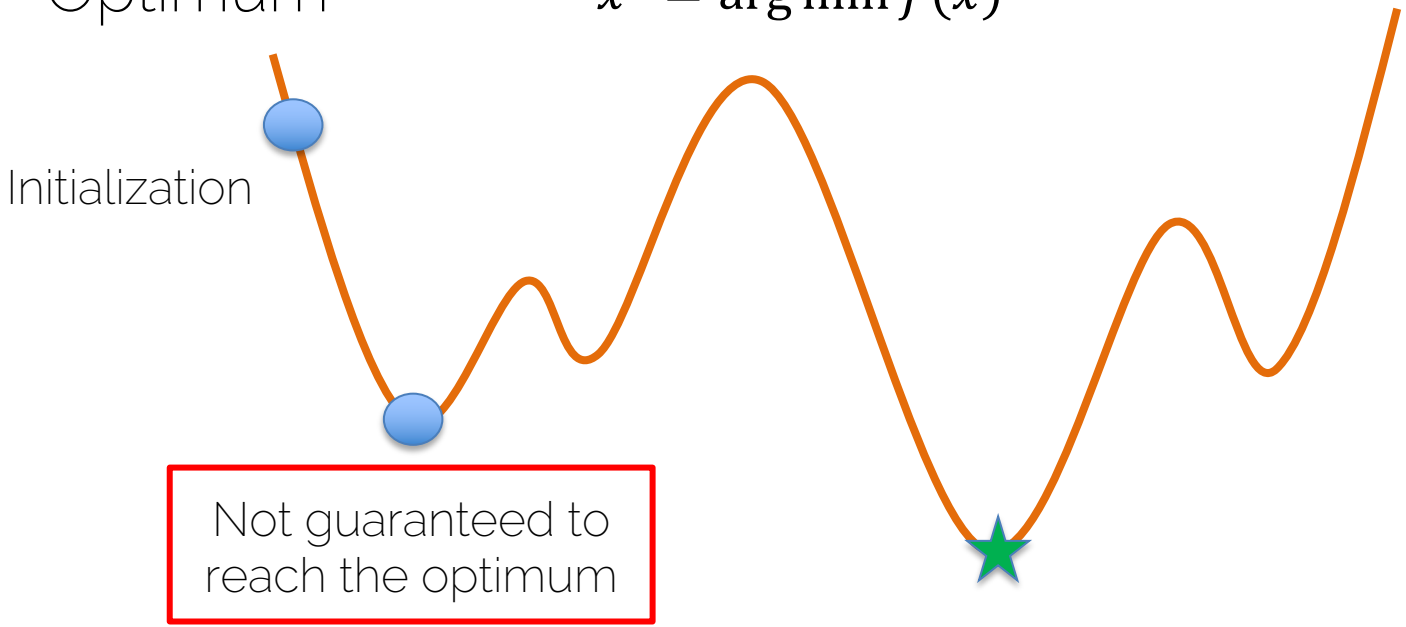
Quick Guide

- Sigmoid is not really used.
- ReLU is the standard choice.
- Second choice are the variants of ReLU or Maxout.
- Recurrent nets will require TanH or similar.

↳ avoiding the vanishing gradients

Initialization is Extremely Important

- Optimum $x^* = \arg\min f(x)$



Xavier Initialization

- How to ensure the variance of the output is the same as the input?

$$(nVar(w)Var(x))$$

$$= 1$$

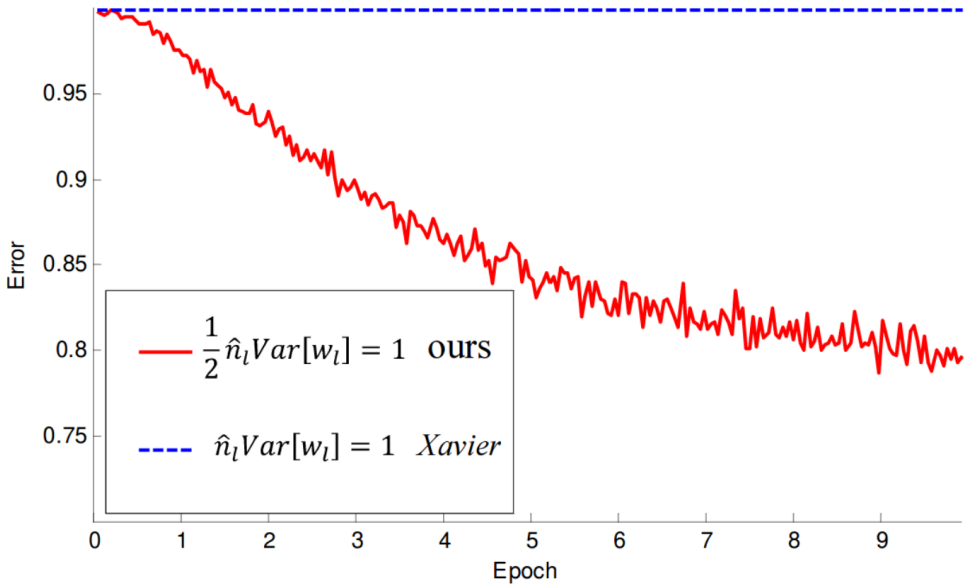
$$Var(w) = \frac{1}{n}$$

with tanh activation fn

ReLU Kills Half of the Data

$$Var(w) = \frac{2}{n}$$

It makes a huge difference!



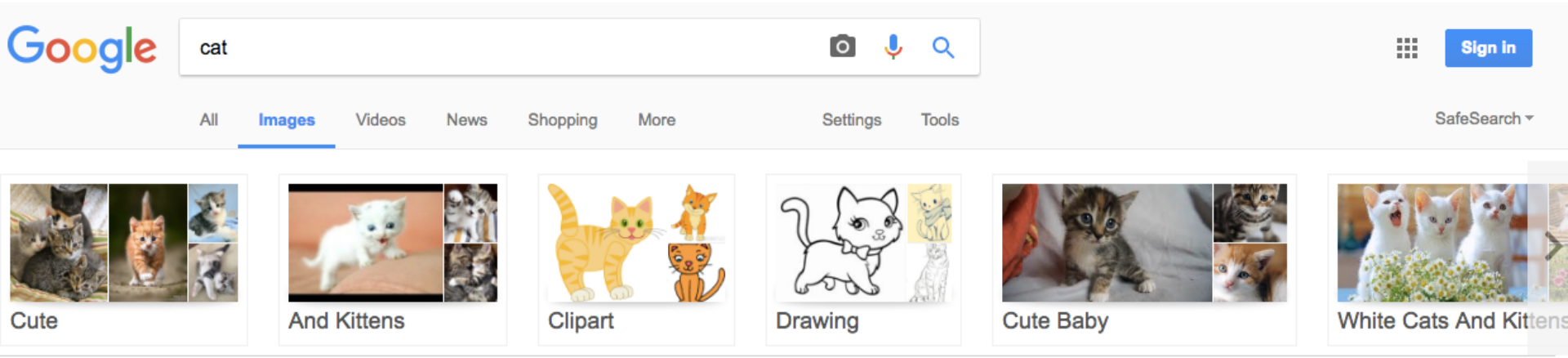
[He et al., ICCV'15] He Initialization

Lecture 8

Data Augmentation

Data Augmentation

- A classifier has to be invariant to a wide variety of transformations



Pose Appearance Illumination

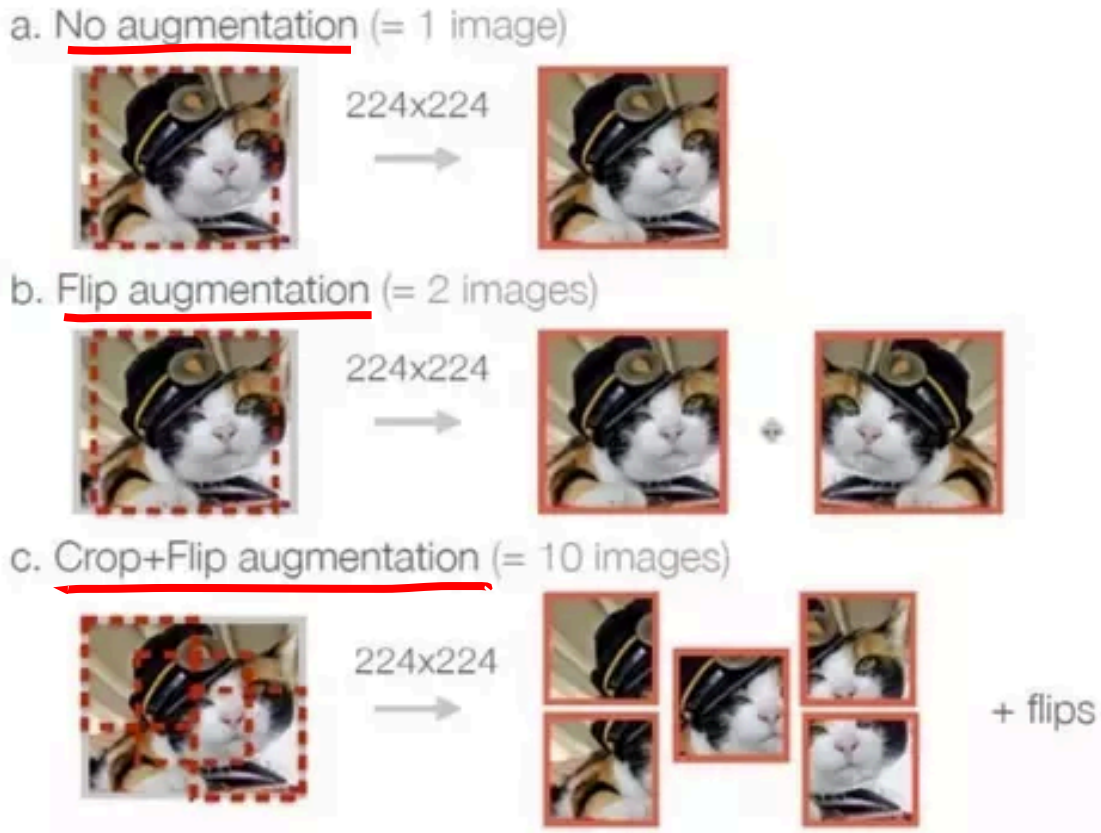
Data Augmentation

GOAL

- A classifier has to be invariant to a wide variety of transformations

- Helping the classifier: synthesize data simulating plausible transformations

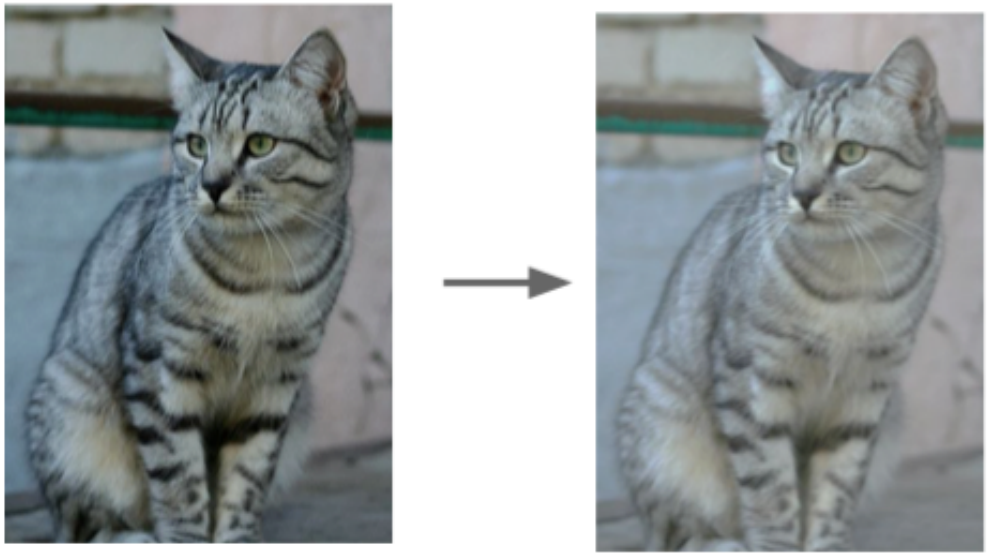
Data Augmentation



[Krizhevsky et al., NIPS'12] ImageNet

Data Augmentation: Brightness

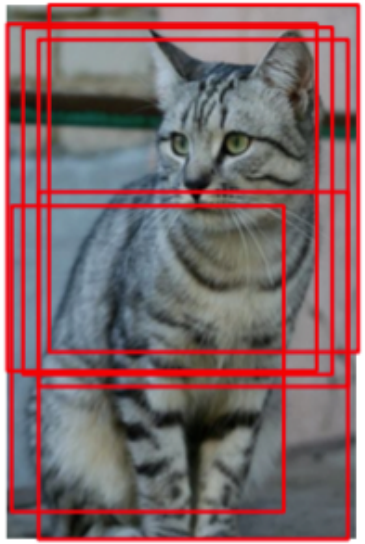
- Random brightness and contrast changes



[Krizhevsky et al., NIPS'12] ImageNet

Data Augmentation: Random Crops

- Training: random crops
  - Pick a random L in [256,480]
  - Resize training image, short side L
  - Randomly sample crops of 224x224



- Testing: fixed set of crops
  - Resize image at N scales
  - 10 fixed crops of 224x224: (4 corners + 1 center) × 2 flips

[Krizhevsky et al., NIPS'12] ImageNet

When comparing two networks make sure to use the same data augmentation!

- Consider data augmentation a part of your network design

regard it as a hyperparameter

Data augmentation is considered as a form of regularization

Advanced Regularization

Weight Decay

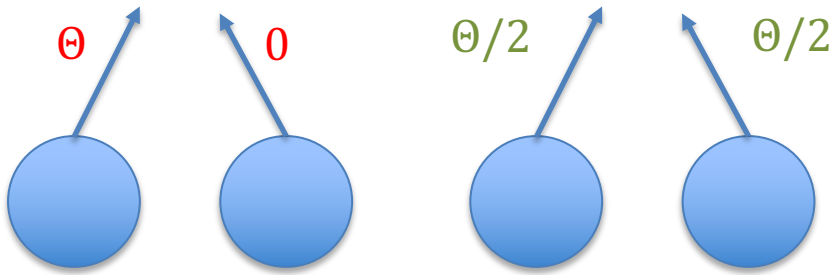
is essentially

- L2 regularization

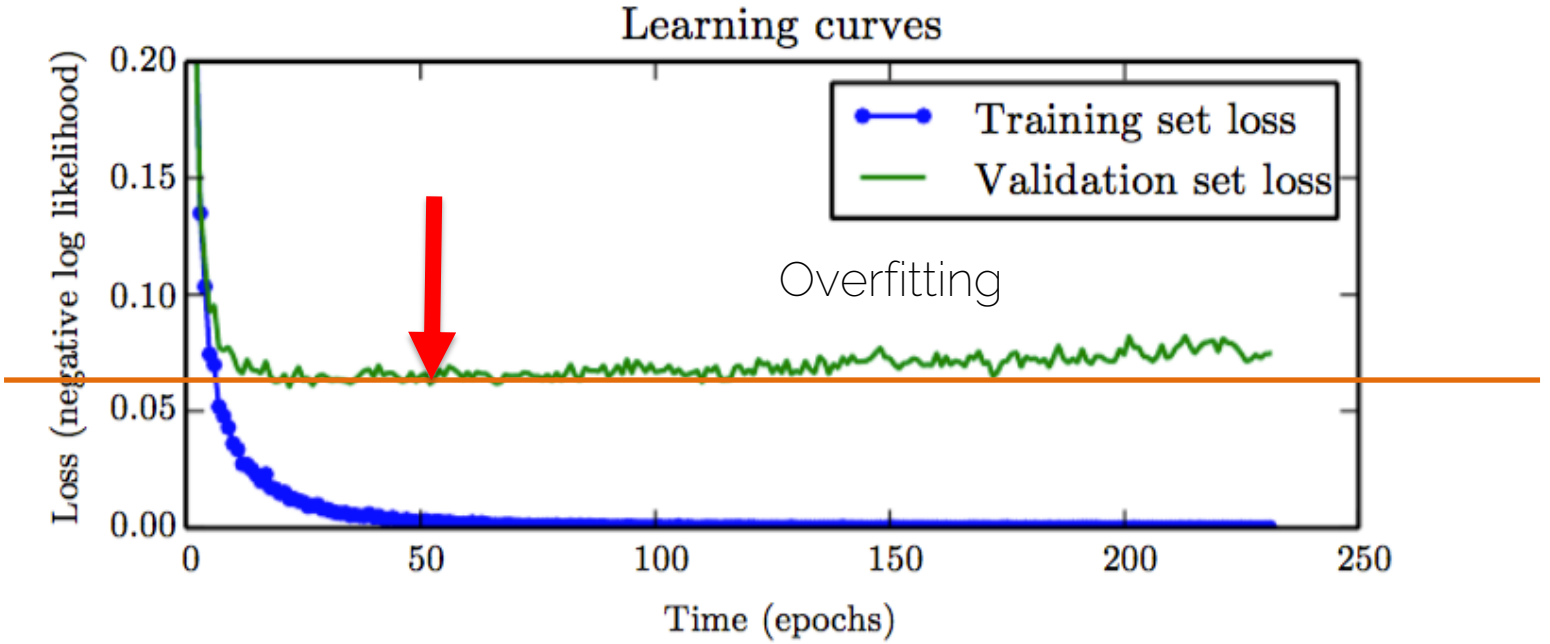
$$\Theta_{k+1} = \Theta_k - \epsilon \nabla_\Theta (\Theta_k, x, y) - \lambda \theta_k$$

Learning rate | Gradient | Gradient of L2-regularization

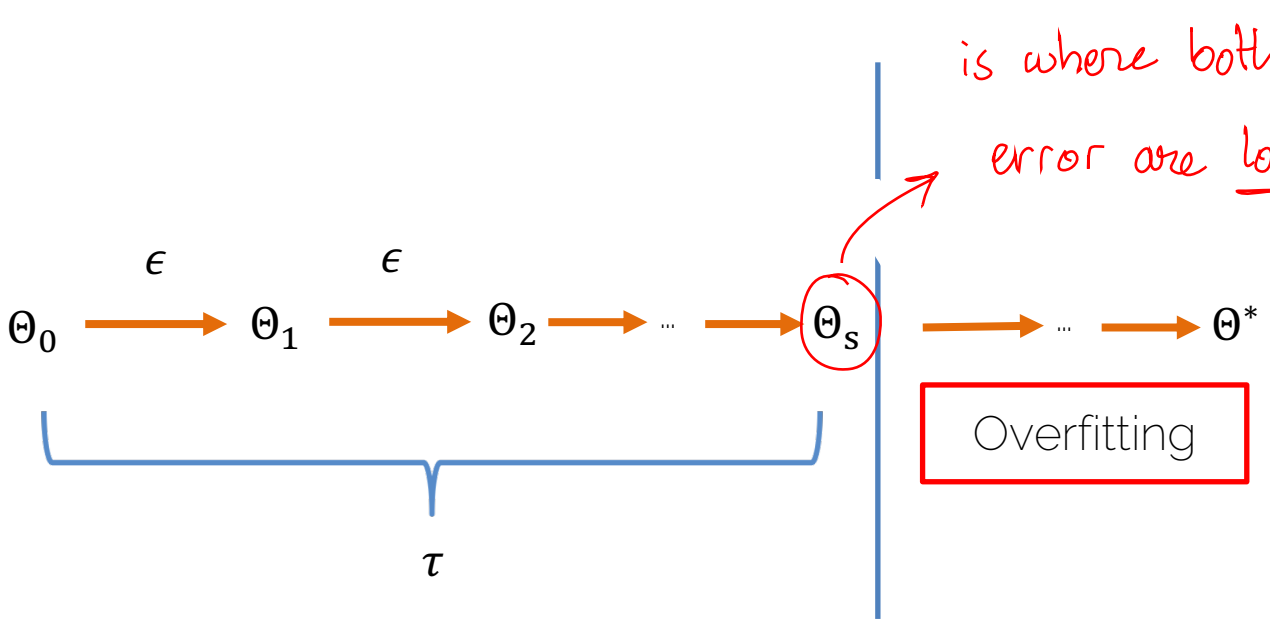
- Penalizes large weights
- Improves generalization



2 Early Stopping



Easy form of regularization



3 Bagging and Ensemble Methods

- Train multiple models and average their results

- E.g., use a different algorithm for optimization or change the objective function / loss function.

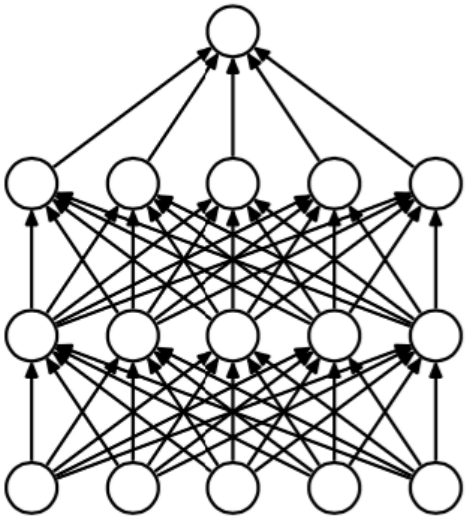
* If errors are uncorrelated, the expected combined error will decrease linearly with the ensemble size

the more models we train, the less the error

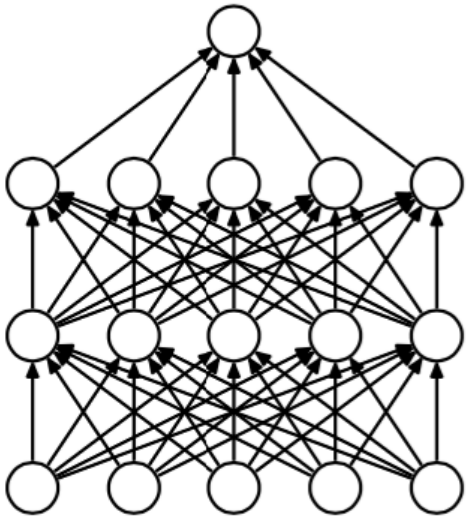
* Each model shall be specialized in specific features, making the combined prediction better.

- Bagging: uses k different datasets

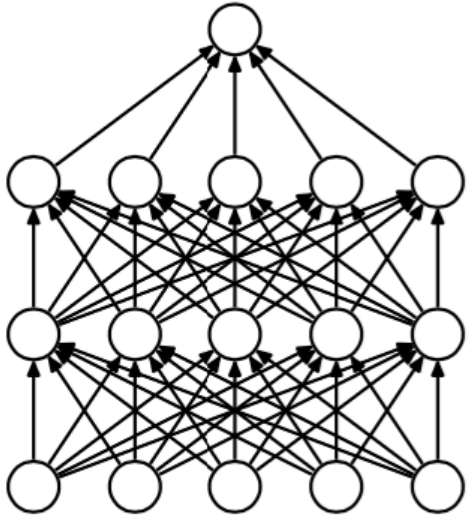
→ We divide training sets into diff subset, which might overlap



Training Set 1



Training Set 2



Training Set 3

Image Source: [Srivastava et al., JMLR'14] Dropout

Dropout

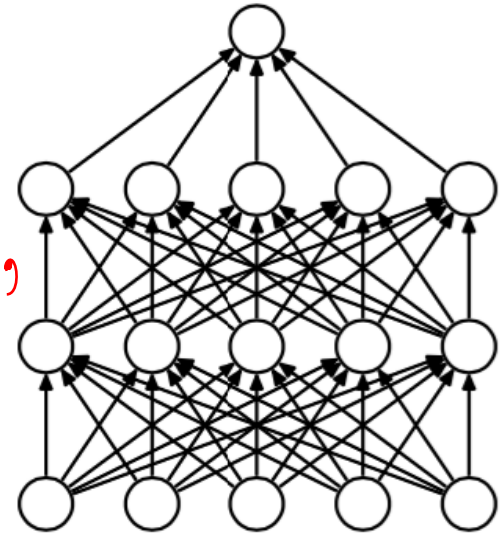
Very Important

Dropout

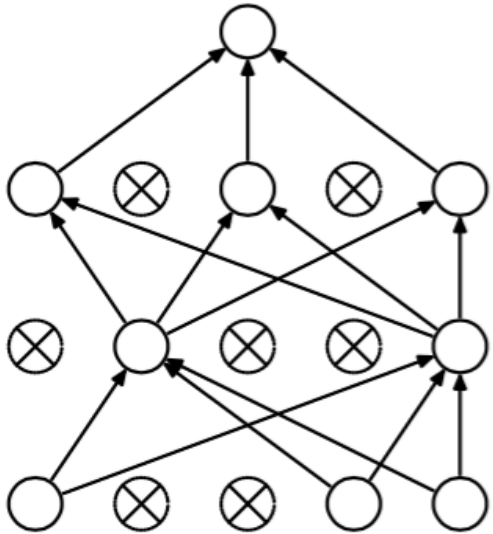
- Disable a random set of neurons (typically 50%)

NB

During backprop, neurons are pushed to learn more features



(a) Standard Neural Net



(b) After applying dropout.

Forward

[Srivastava et al., JMLR'14] Dropout

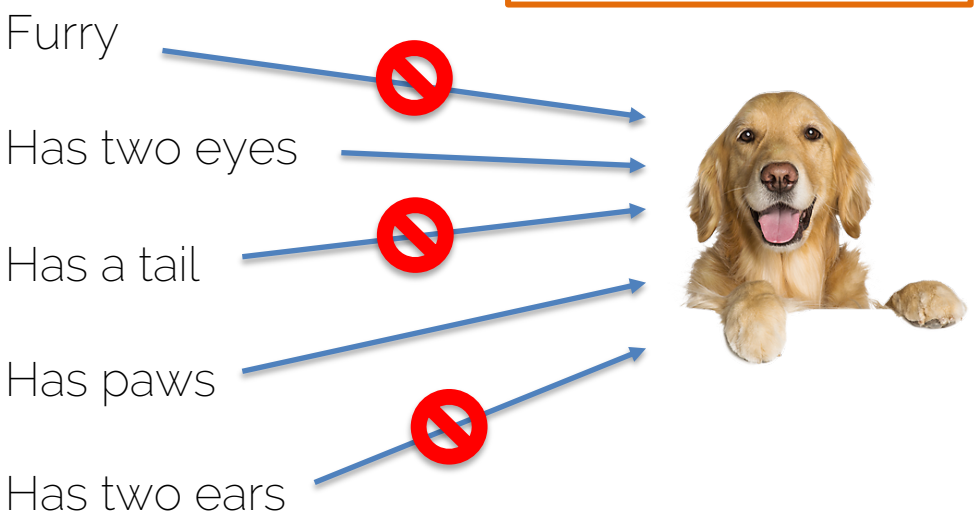
Dropout: Intuition

NB Instead of having more specialized neurons, less neurons shall learn more.

diff rep

"Contra Specialization Technique"

Redundant representations



[Srivastava et al., JMLR'14] Dropout

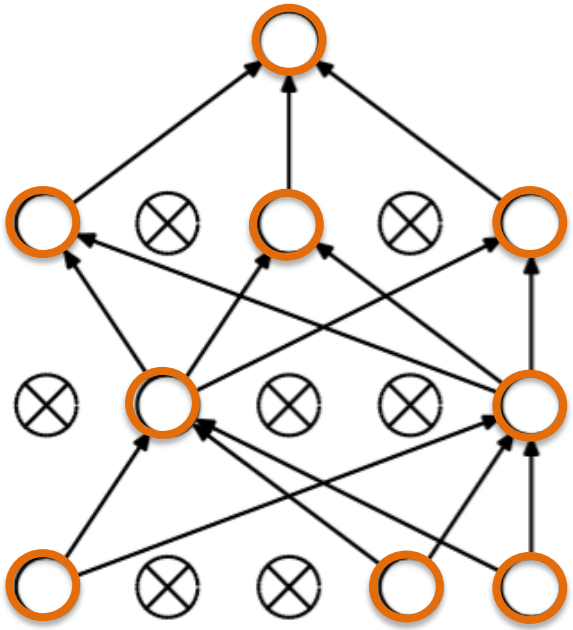
- Using half the network = half capacity
  - Redundant representations
  - Base your scores on more features

- Consider it as a model ensemble

Such features shall be learnt from more neurons, instead just few specialized ones.

→ Such neurons usually overfit

- Two models in one



(b) After applying dropout.

Model 1

Model 2

[Srivastava et al., JMLR'14] Dropout

- Using half the network = half capacity
  - Redundant representations
  - Base your scores on more features

- Consider it as two models in one
  - Training a large ensemble of models, each on different set of data (mini-batch) and with SHARED parameters

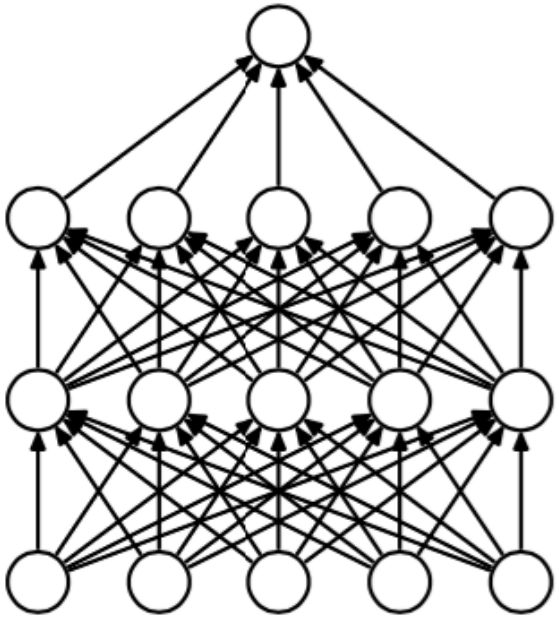
Reducing co-adaptation between neurons

[Srivastava et al., JMLR'14] Dropout

Dropout: Test Time

Most important conclusion

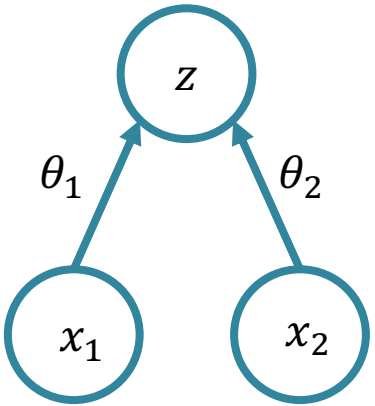
- All neurons are "turned on" – no dropout



Conditions at train and test time are not the same

[Srivastava et al., JMLR'14] Dropout

- Test:
- Train:



Dropout probability $p = 0.5$

$$z = (\theta_1 x_1 + \theta_2 x_2) \cdot p$$

$$E[z] = \frac{1}{4}(\theta_1 0 + \theta_2 0 + \theta_1 x_1 + \theta_2 0 + \theta_1 0 + \theta_2 x_2 + \theta_1 x_1 + \theta_2 x_2) = \frac{1}{2}(\theta_1 x_1 + \theta_2 x_2)$$

to get the same output intesting as the output in training

Weight scaling inference rule

scaling the weights of each neuron during testing by P

Dropout: Verdict

- Efficient bagging method with parameter sharing

- Try it!

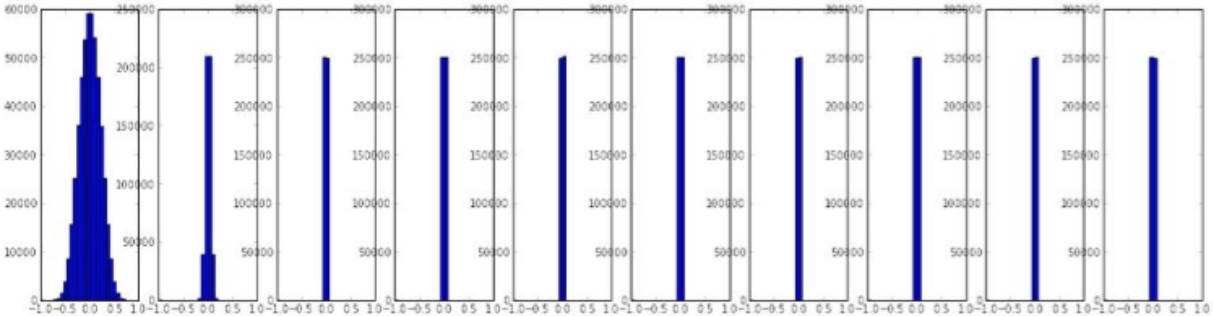
- Dropout reduces the effective capacity of a model → larger models, more training time

Batch Normalization

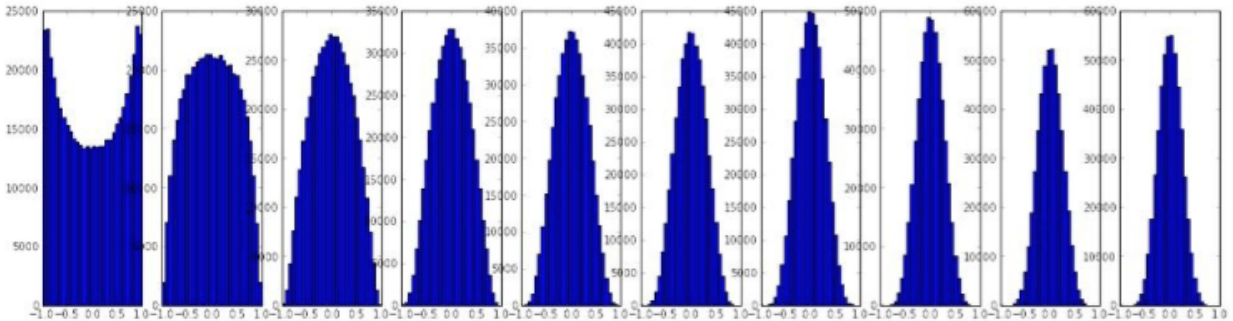
Our Goal

- All we want is that our activations do not die out

Vanishing gradients



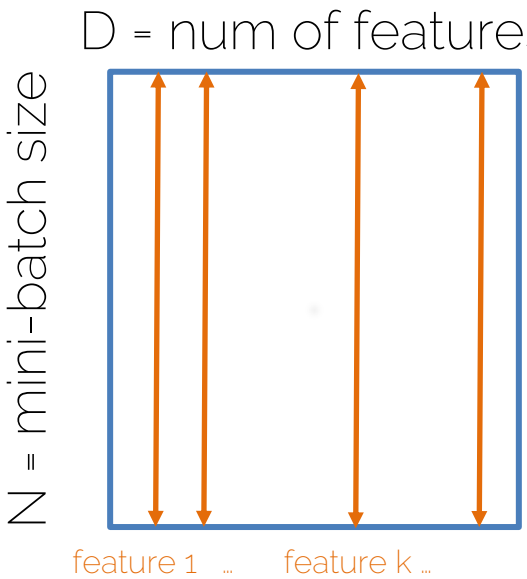
with proper initialization



Batch Normalization

- Wish: Unit Gaussian activations (in our example)
- Solution: let's do it

even after processing multiple layers



NB

No distr shall be converted into Unit Gaussian!

Unit Gaussian is already in the input, we are just normalizing it for the output.

Mean of your mini-batch examples over feature k

$$\hat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{Var[x^{(k)}]}}$$

Unit gaussian

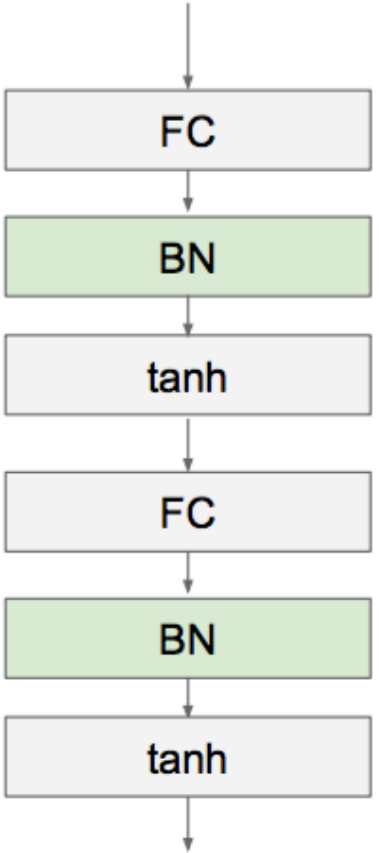
- In each dimension of the features, you have a unit gaussian (in our example)

- For NN in general → BN normalizes the mean and variance of the inputs to your activation functions

[Ioffe and Szegedy, PMLR'15] Batch Normalization

BN Layer

A layer to be applied after Fully Connected (or Convolutional) layers and before non-linear activation functions



[Ioffe and Szegedy, PMLR'15] Batch Normalization

Batch Normalization

Important

- 1. Normalize

$$\hat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{Var[x^{(k)}]}}$$

Differentiable function so we can backprop through it….

- 2. Allow the network to change the range

$$y^{(k)} = \gamma^{(k)} \hat{x}^{(k)} + \beta^{(k)}$$

These parameters will be optimized during backprop

- 1. Normalize

$$\hat{\boldsymbol{x}}^{(k)} = \frac{\boldsymbol{x}^{(k)} - E[\boldsymbol{x}^{(k)}]}{\sqrt{Var[\boldsymbol{x}^{(k)}]}}$$

- 2. Allow the network to change the range

$$\boldsymbol{y}^{(k)} = \gamma^{(k)}\hat{\boldsymbol{x}}^{(k)} + \beta^{(k)}$$

backprop

The network *can* learn to undo the normalization

$$\gamma^{(k)} = \sqrt{Var[\boldsymbol{x}^{(k)}]}$$

$$\beta^{(k)} = E[\boldsymbol{x}^{(k)}]$$

- Ok to treat dimensions separately?
  Shown empirically that even if features are not correlated, convergence is still faster with this method

- You can set all biases of the layers before BN to zero, because they will be cancelled out by BN anyway

BN: Train vs Test

- Train time: mean and variance is taken over the mini-batch

$$\hat{\boldsymbol{x}}^{(k)} = \frac{\boldsymbol{x}^{(k)} - E[\boldsymbol{x}^{(k)}]}{\sqrt{Var[\boldsymbol{x}^{(k)}]}}$$

- Test-time: what happens if we can just process one image at a time?
  - No chance to compute a meaningful mean and variance

Very Important

Training: Compute mean and variance from mini-batch 1,2,3 …

fixed

Testing: Compute mean and variance by running an exponentially weighted averaged across training mini-batches. Use them as σ^2_{test} and μ_{test} .

$$Var_{running} = \beta_m * Var_{running} + (1 - \beta_m) * Var_{minibatch}$$

$$\mu_{running} = \beta_m * \mu_{running} + (1 - \beta_m) * \mu_{minibatch}$$

β_m: momentum (hyperparameter)

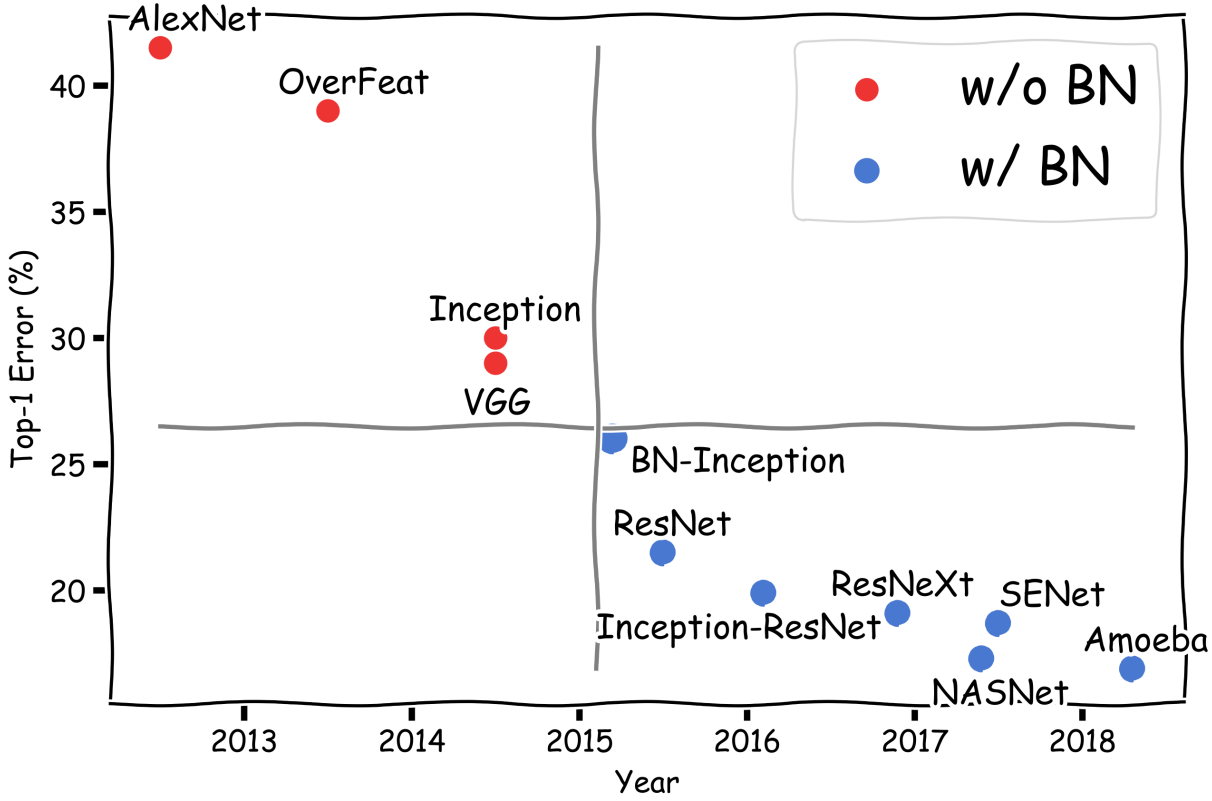
BN: What do you get?

- Very deep nets are much easier to train → more stable gradients

- A much larger range of hyperparameters works similarly when using BN

[Ioffe and Szegedy, PMLR'15] Batch Normalization

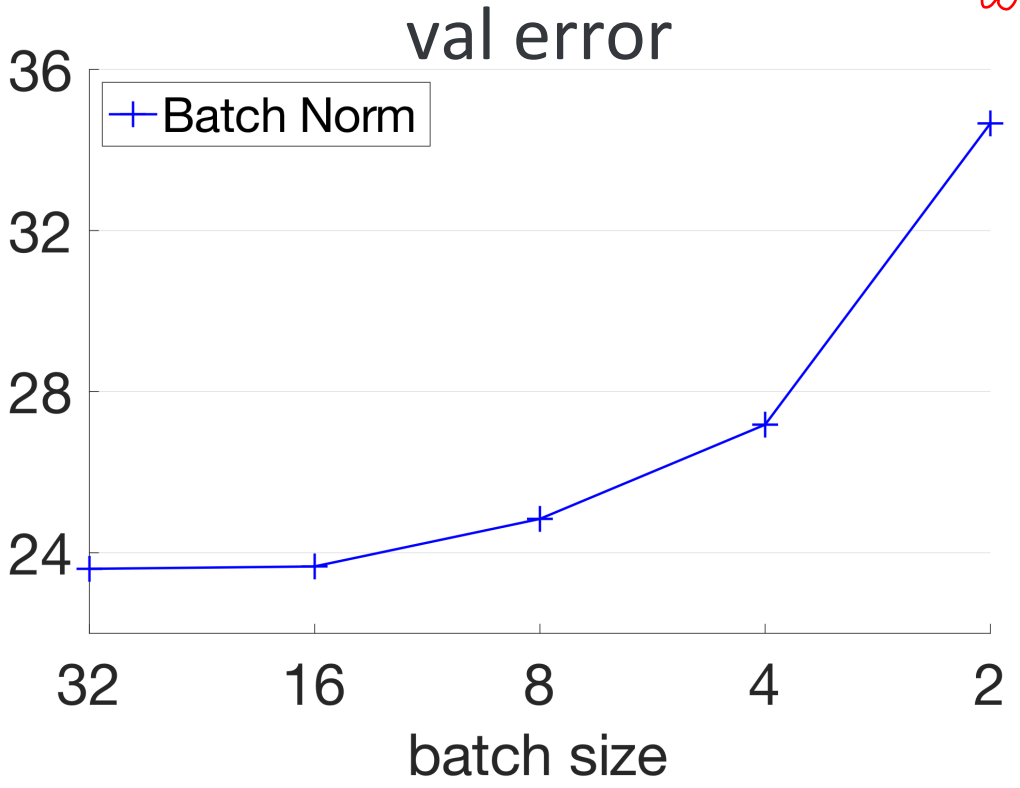
BN: A Milestone



[Wu and He, ECCV'18] Group Normalization

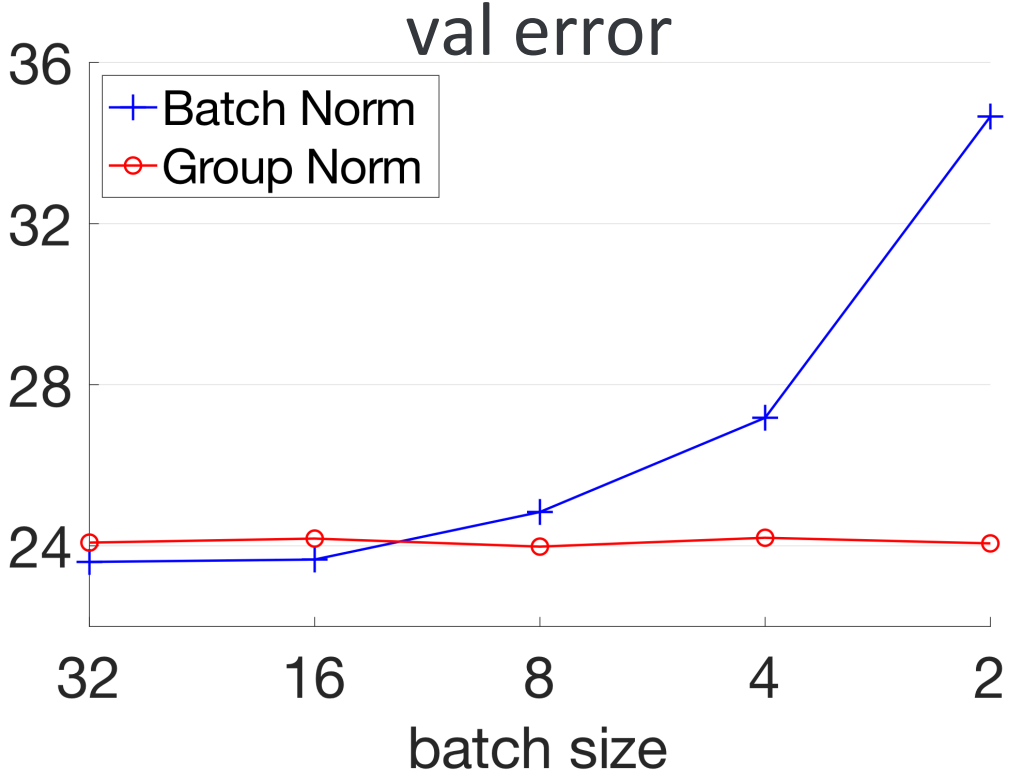
BN: Drawbacks

→ does not work well with small batches



[Wu and He, ECCV'18] Group Normalization

Other Normalizations

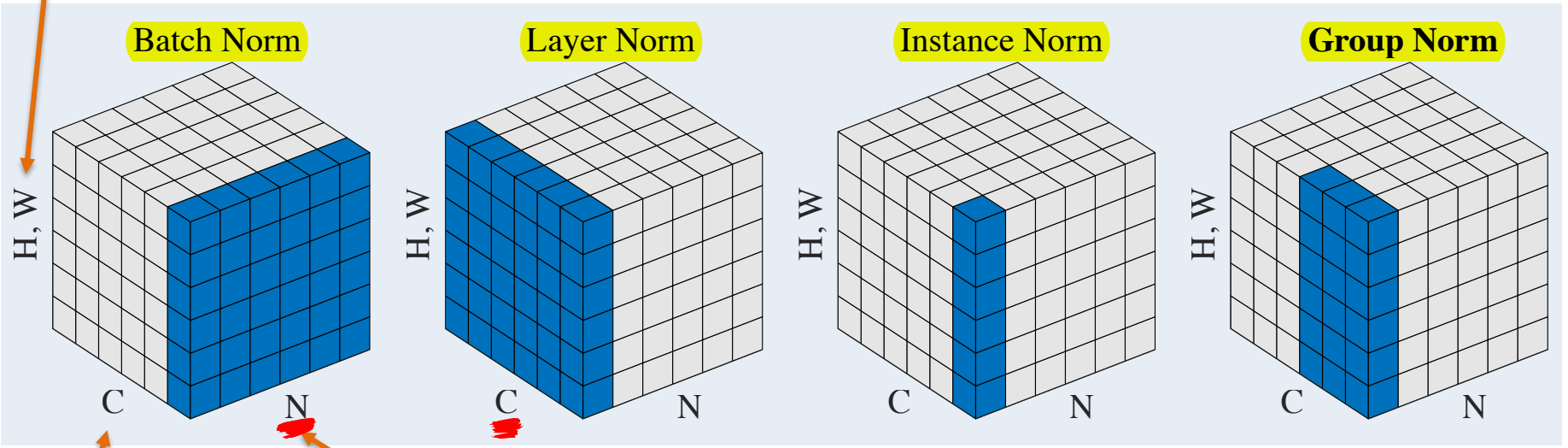


[Wu and He, ECCV'18] Group Normalization

Other Normalizations

Understand

Image size



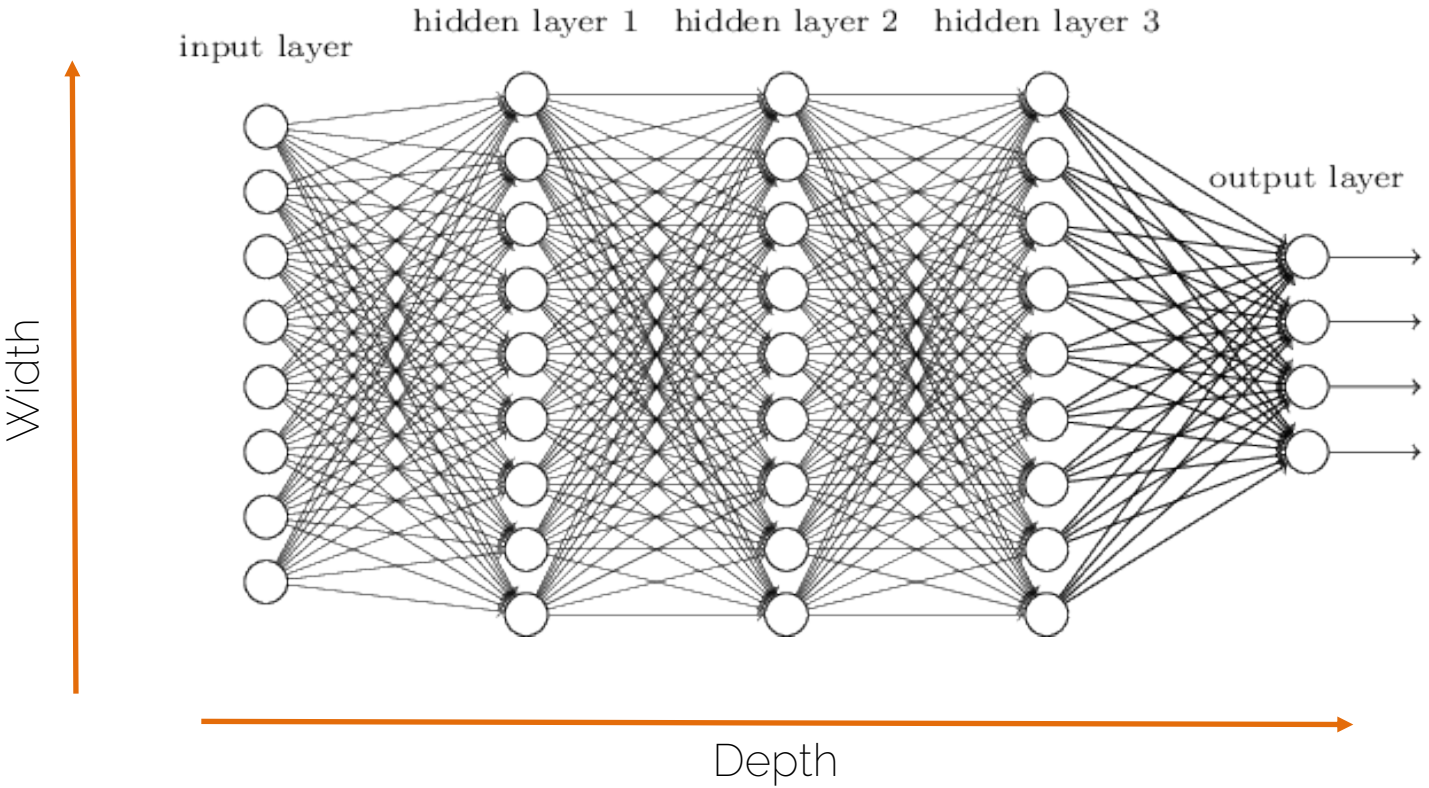
Number of elements in the batch

Number of channels

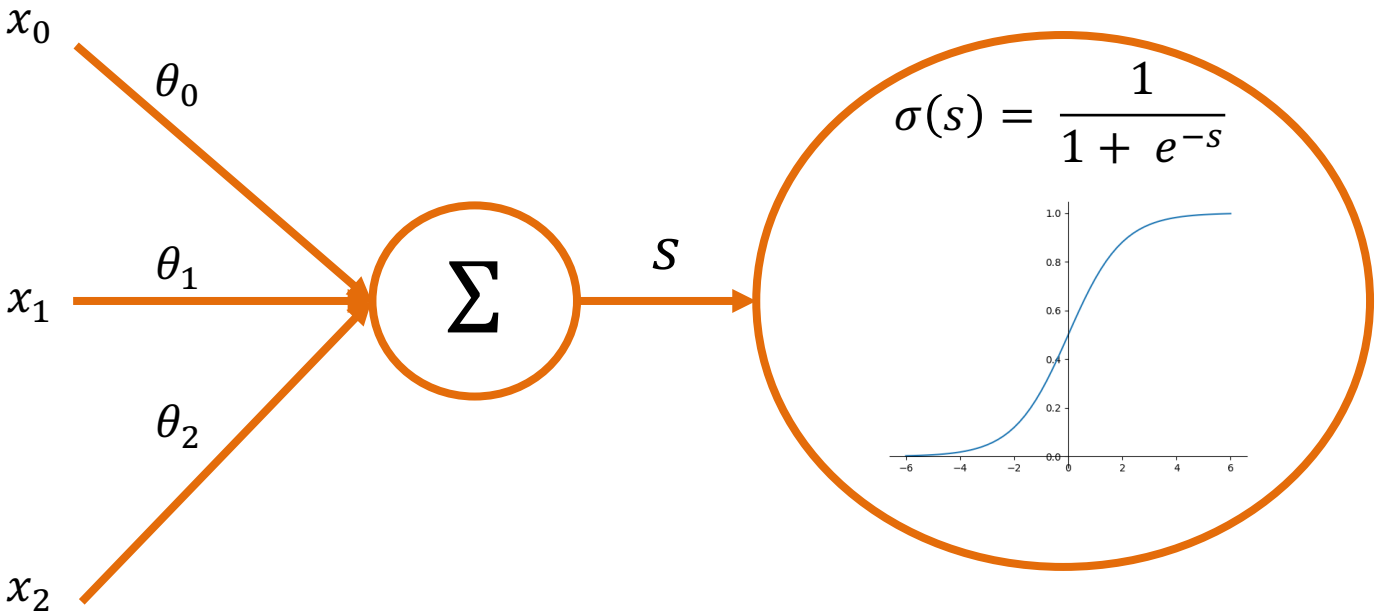
[Wu and He, ECCV'18] Group Normalization

What We Know

What do we know so far?

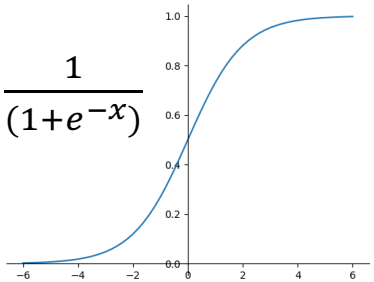


Concept of a 'Neuron'

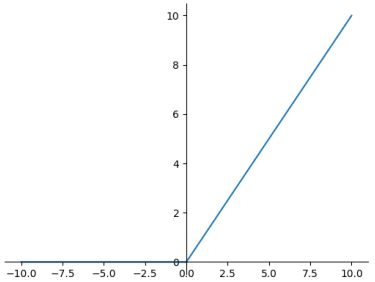


Activation Functions (non-linearities)

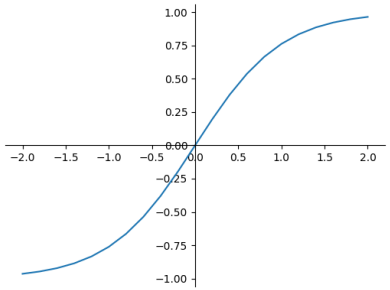
- Sigmoid: $\sigma(x) = \frac{1}{(1+e^{-x})}$



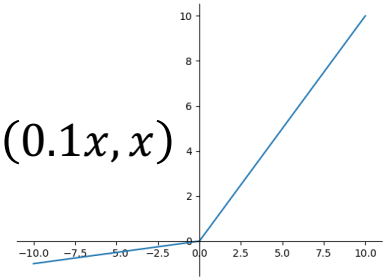
- ReLU: $\max(0, x)$



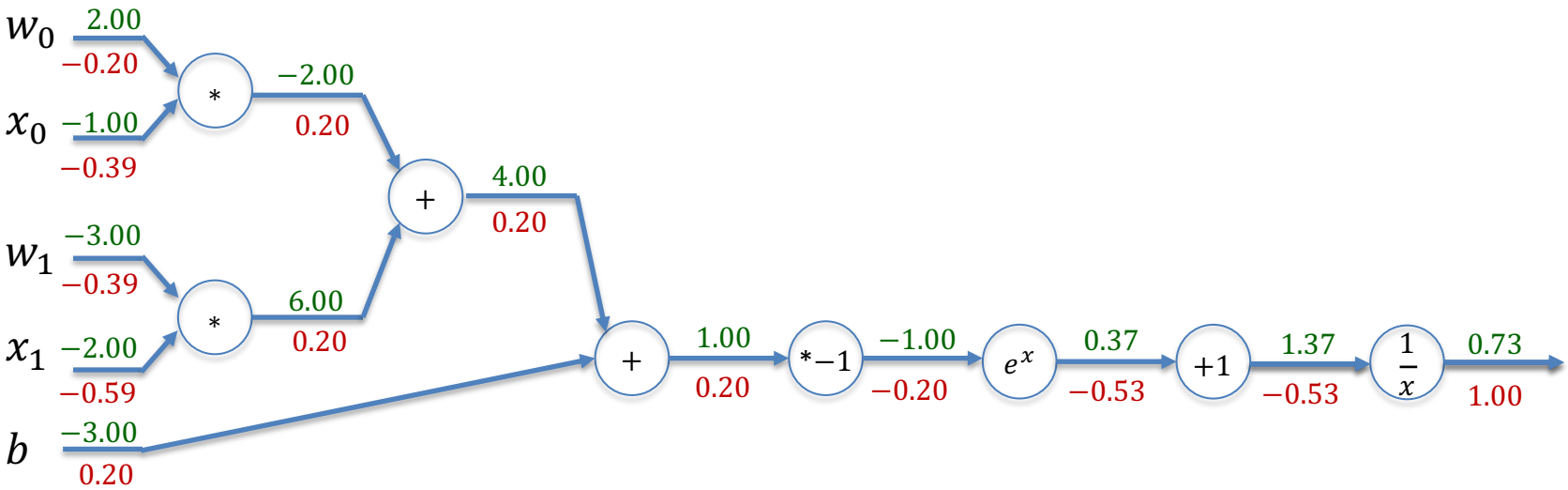
- TanH: $\tanh(x)$



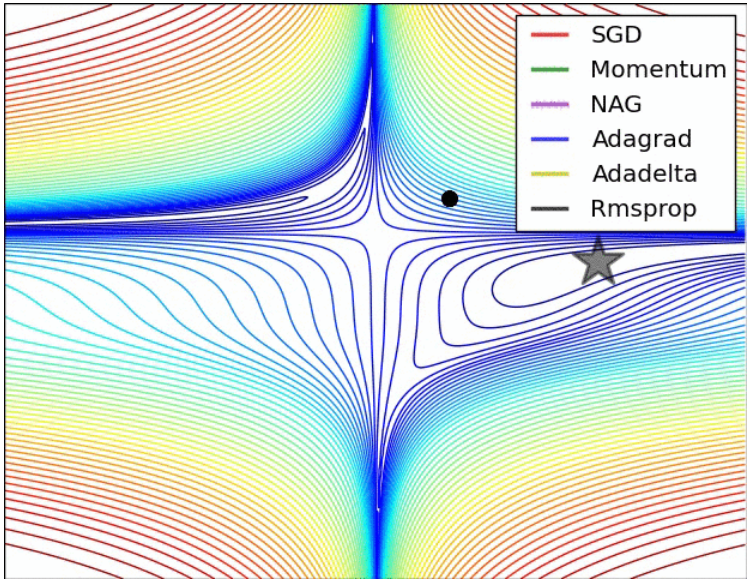
- Leaky ReLU: $\max(0.1x, x)$



Backpropagation



SGD Variations (Momentum, etc.)

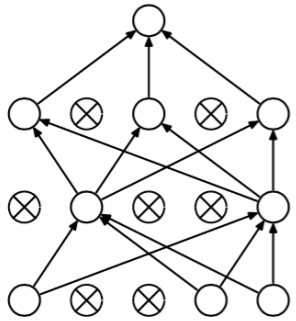


Data Augmentation

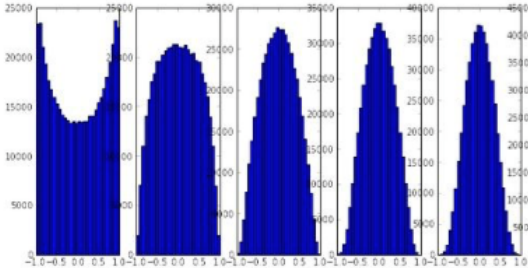
Batch-Norm

$$\hat{\boldsymbol{x}}^{(k)} = \frac{\boldsymbol{x}^{(k)} - E\left[\boldsymbol{x}^{(k)}\right]}{\sqrt{Var\left[\boldsymbol{x}^{(k)}\right]}}$$

Dropout


(b) After applying dropout.

Weight Initialization (e.g., Xavier/2)



Weight Regularization

e.g., L^2-reg: $R^2(\boldsymbol{W}) = \sum_{i=1}^{N} w_i^2$

Why not simply more Layers?

- We cannot make networks arbitrarily complex

- Why not just go deeper and get better?
  - No structure!!
  - It is just brute force!
  - Optimization becomes hard
  - Performance plateaus / drops!

FC drawbacks

See you next week!

References

- Goodfellow et al. “Deep Learning” (2016),
  - Chapter 6: Deep Feedforward Networks
- Bishop “Pattern Recognition and Machine Learning” (2006),
  - Chapter 5.5: Regularization in Network Nets
- http://cs231n.github.io/neural-networks-1/
- http://cs231n.github.io/neural-networks-2/
- http://cs231n.github.io/neural-networks-3/