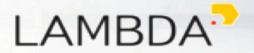


Model-free reinforcement learning





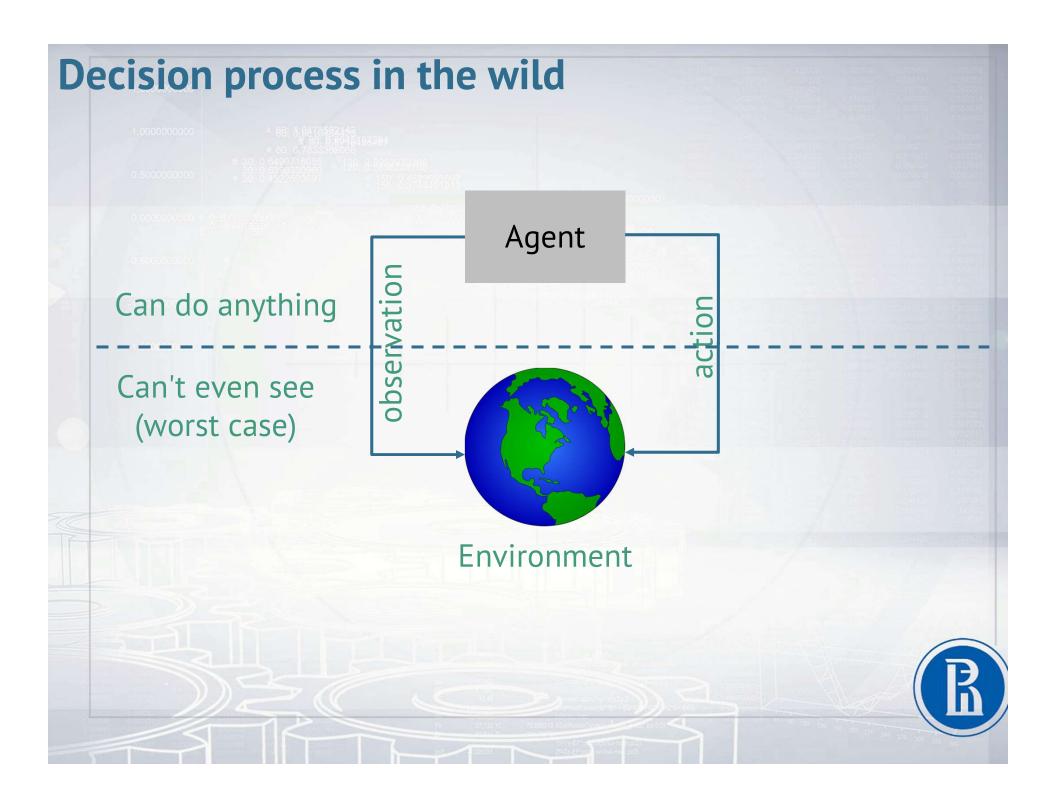


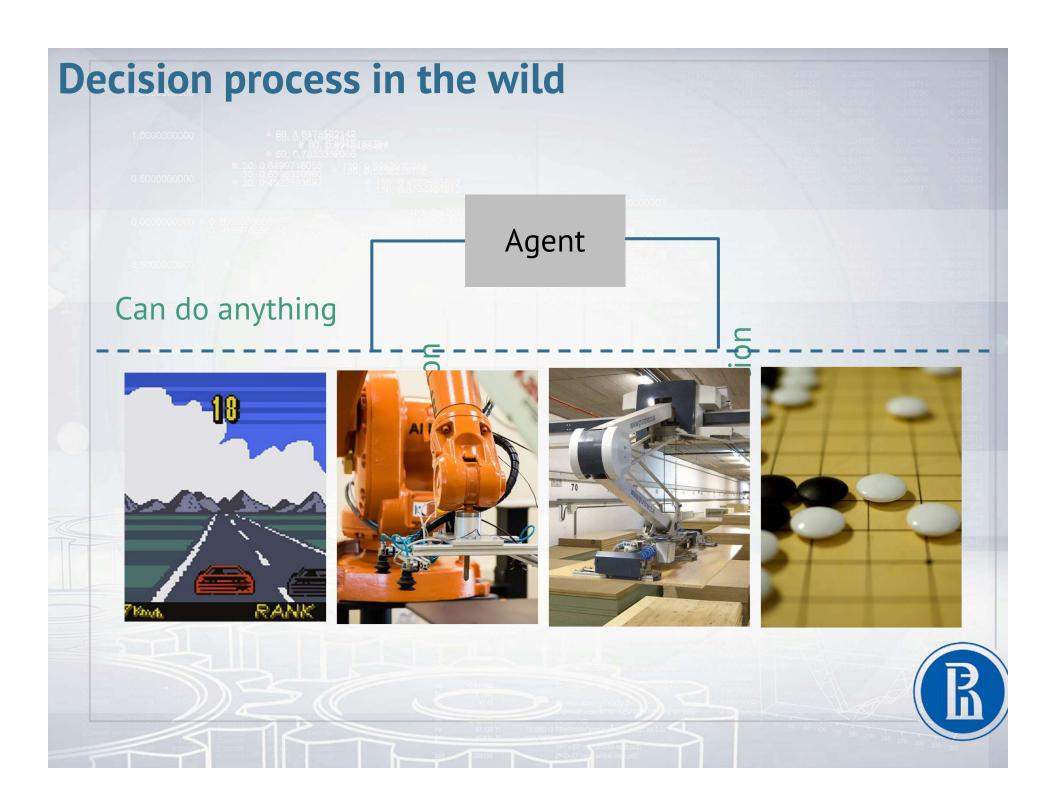
## What we learned

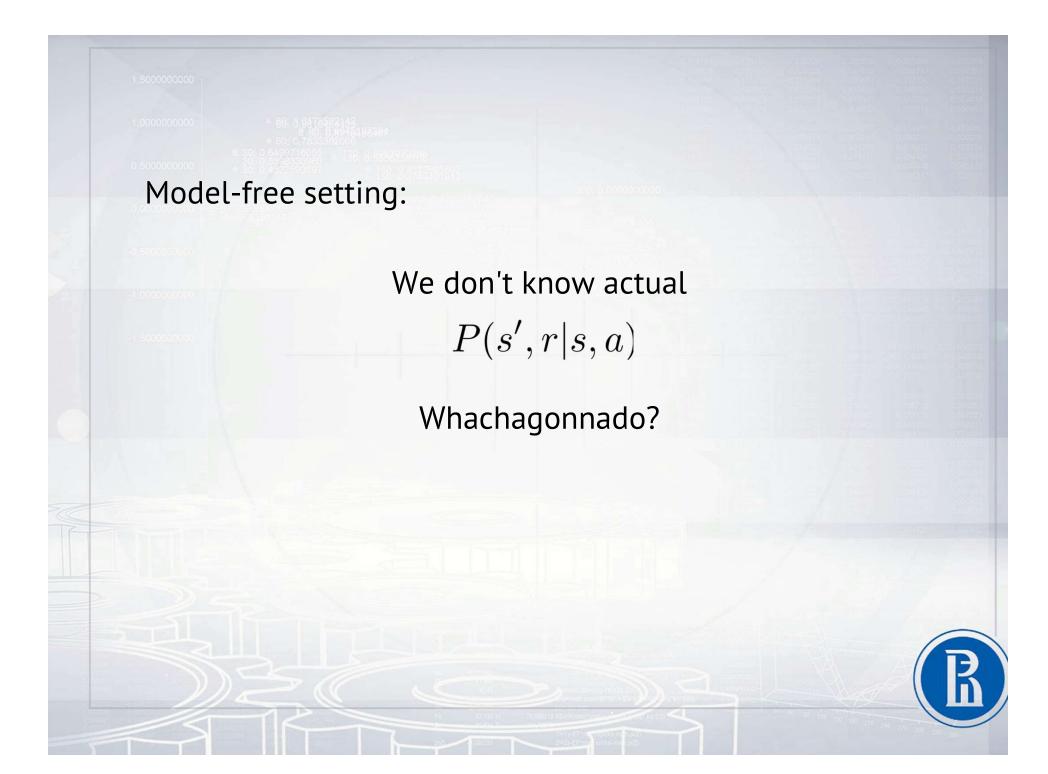
- V(s) and Q(s,a)
  - If we know V or Q → we have optimal policy
  - · We can learn them with dynamic programming

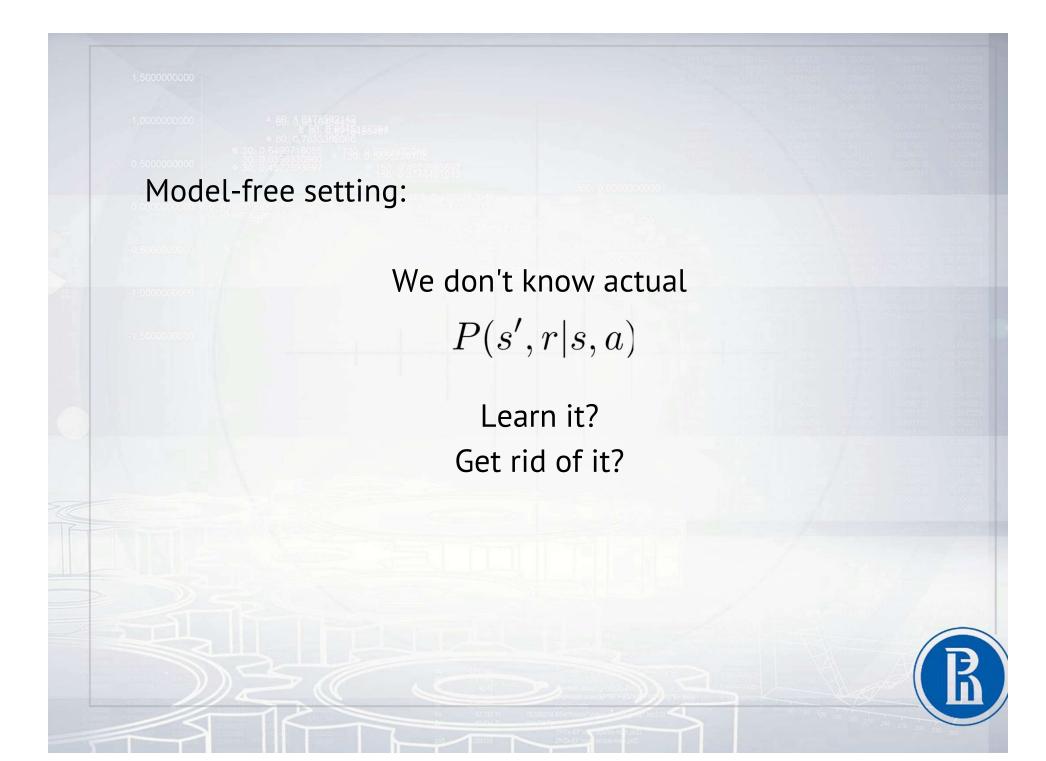
$$V_{i+1}(s) := \max_{a} [r(s,a) + \gamma \cdot \underset{s' \sim P(s'|s,a)}{E} V_{t}(s')]$$











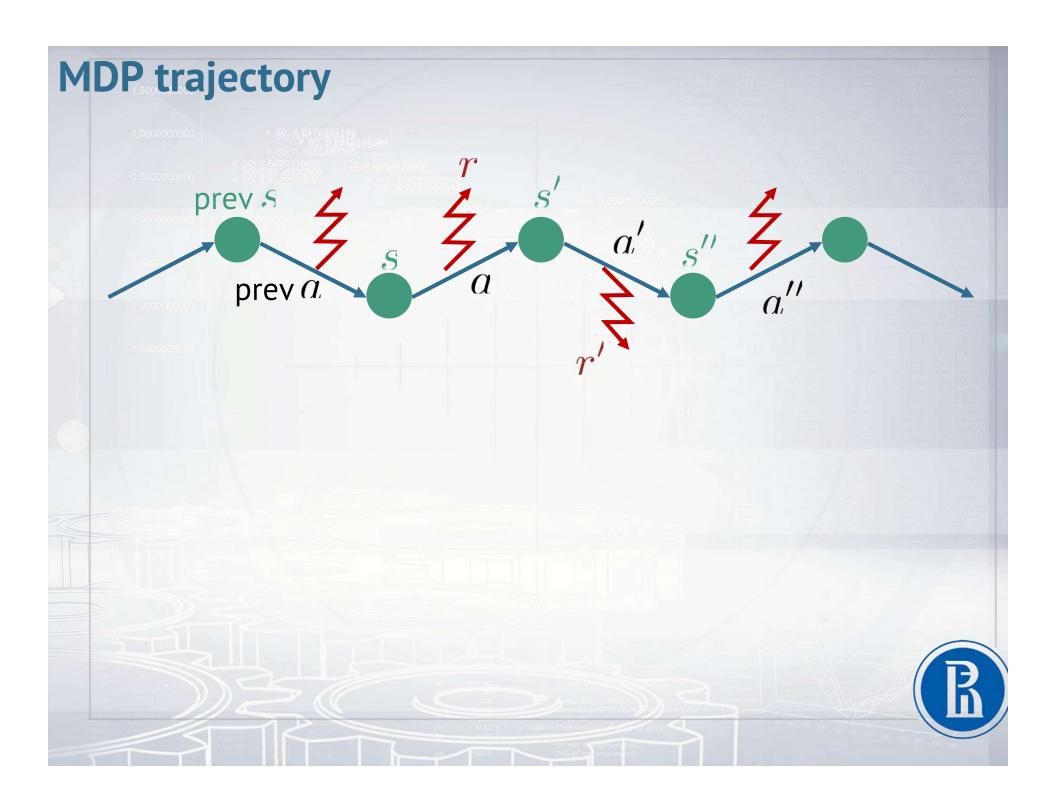
### What we learned

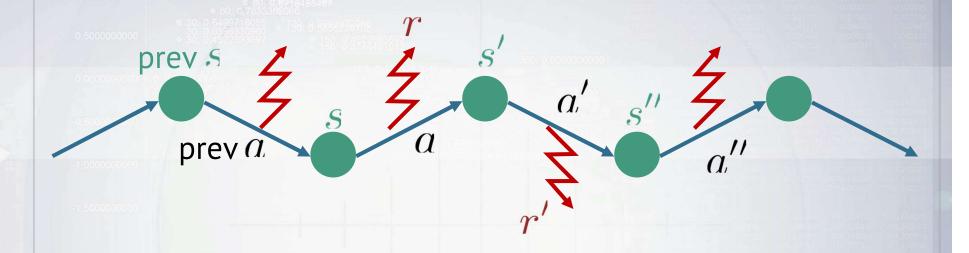
- V(s) and Q(s,a)
  - If we know V or  $Q \rightarrow$  we have optimal policy
  - We can learn them with dynamic programming

$$V_{i+1}(s) := \max_{a} [r(s,a) + \gamma \cdot \underbrace{E}_{s' \sim P(s'|s,a)} V_t(s')]$$

over both terms Can't enumerate all

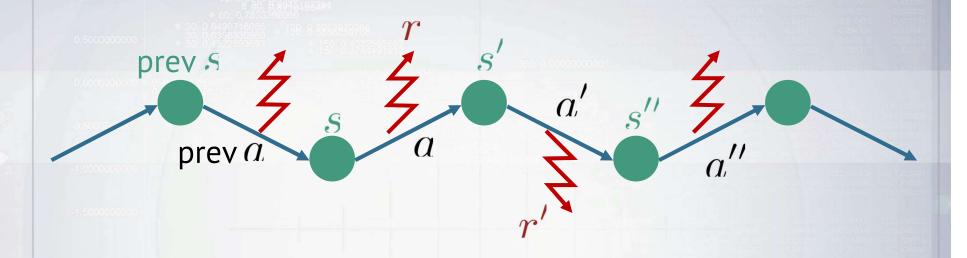






- Trajectory is a sequence of
  - states (s)
  - actions (a)
  - rewards (r)
- We can only sample trajectories

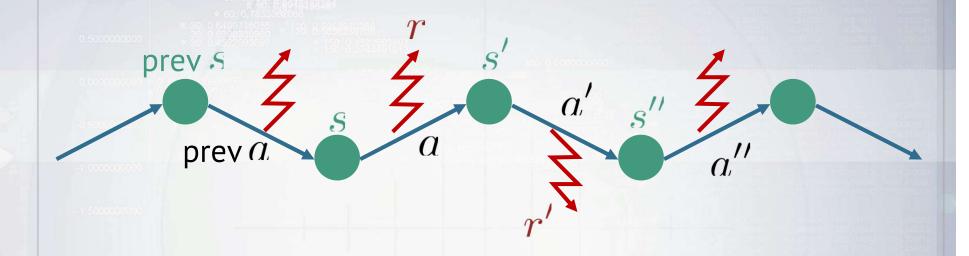




- Trajectory is a sequence of
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  - rewards (r)
- We can only sample trajectories

Q: What to learn? V(s) or Q(s,a)





- Trajectory is a sequence of
  - states (s)
  - actions (a)
  - rewards (r)
- We can only sample trajectories

Q: What to learn? V(s) or Q(s,a)

V(s) is useless without P(s'|s,a)



# From V to Q

One approach: action Q- values

$$Q(s,a) = r(s,a) + \gamma \cdot V(s')$$

Action value Q(s,a) is the expected total reward G agent gets from state s by taking action a and following policy  $\pi$  from next state.

 $\pi(s) : argmax_a Q(s, a)$ 



### Idea 1: monte-carlo

- Get all trajectories containing particular (s, a)
- Estimate G(s,a) for each trajectory
- Average them to get expectation

not always practical to implement

takes a lot of sessions



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• Remember we can improve Q(s,a) iteratively!

$$Q(s_t, a_t) \leftarrow \underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$



• Remember we can improve Q(s,a) iteratively!

$$Q(s_t, a_t) \leftarrow \underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

The 
$$Q^st(s,a)$$

That's value for  $\pi*$  aka optimal policy



• Remember we can improve Q(s,a) iteratively!

$$Q(s_t, a_t) \leftarrow \underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

Tha
$$\mathbf{Q}^*(s,a)$$

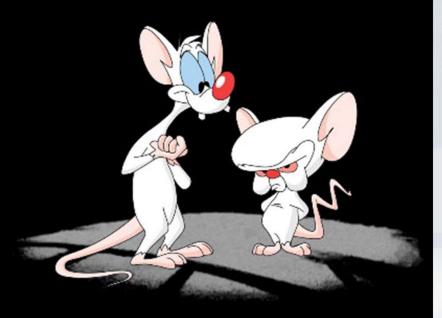
That's value for  $\pi*$  aka optimal policy

That's something we don't have

What do we do?



# THE SAME THING WE DO EVERY NIGHT, PINKY



APPROXIMATE!

imgflip.com

Авторство



Replace expectation with sampling

$$E_{r_t,s_{t+1}} + \gamma \cdot \max_{a'} Q(s_{t+1}, a') \approx \frac{1}{N} \sum_{i} r_i + \gamma \cdot \max_{a'} Q(s_i^{next}, a')$$

however, since it will be very noisy, we can not directly use it as Q-value.



Replace expectation with sampling

$$E_{r_t,s_{t+1}} + \gamma \cdot \max_{a'} Q(s_{t+1}, a') \approx \frac{1}{N} \sum_{i} r_i + \gamma \cdot \max_{a'} Q(s_i^{next}, a')$$

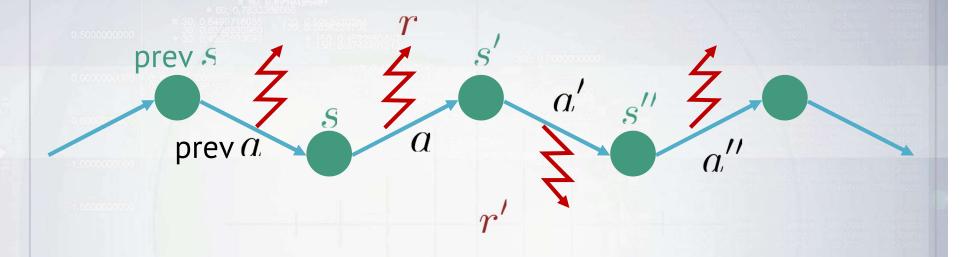
Use moving average with just one sample!

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot max_{a'}Q(s_{t+1}, a')) + (1 - \alpha)Q(s_t, a_t)$$

Exponentially weighted moving average

Is a smoother version of the prev noisy version



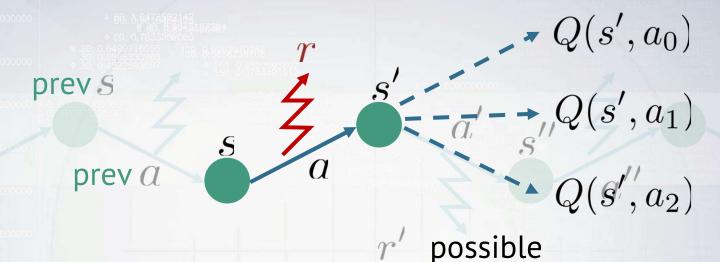


- Trajectory is a sequence of
  - states (s)
  - actions (a)
  - rewards (r)



# **Q-learning** prev S prev a Initialize Q(s,a) with zeros . Loop: - Sample $\langle s, a, r, s' \rangle$ from env

# **Q-learning**



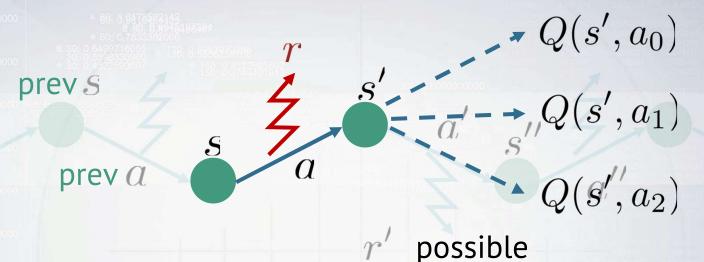
Initialize Q(s,a) with zeros

- . Loop:
  - Sample  $\langle s, a, r, s' \rangle$  from env
  - Compute  $\hat{Q}(s,a) = r(s,a) + \gamma maxQ(s',a_i)$

actions



# **Q-learning**



Initialize Q(s,a) with zeros

- . Loop:
  - Sample  $\langle s, a, r, s' \rangle$  from env
  - Compute  $\hat{Q}(s,a) = r(s,a) + \gamma maxQ(s',a_i)$
  - Update  $\hat{Q}(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

actions

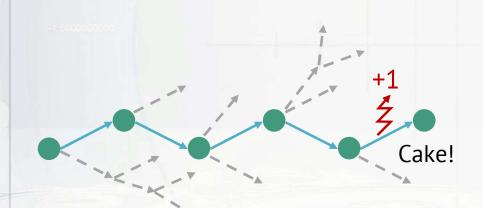
## Recap

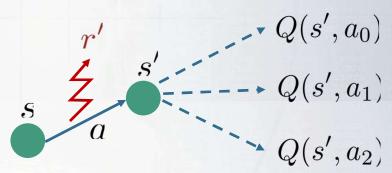
#### **Monte-carlo**

**Temporal Difference** 

Averages Q over sampled paths

Uses recurrent formula for Q





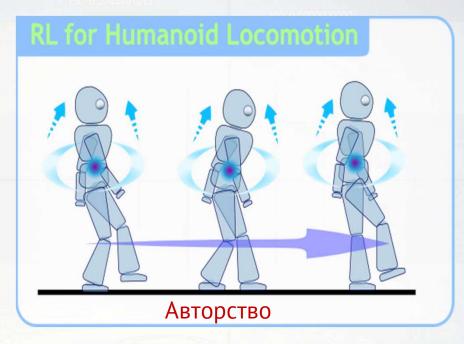


# Nuts and bolts: MC vs TD

Monte-carlo	Temporal Difference
<u>Averages Q</u> over sampled paths	Uses r <u>ecurrent formu</u> la for Q
N <u>eeds full tra</u> jectory to learn	Le <u>arns from partial</u> trajectory Works with <u>infinite MDP</u>
L <u>ess reliant on markov</u> p <u>roper</u> ty	N <u>eeds less experie</u> nce to learn

# What could possibly go wrong?

Our mobile robot learns to walk.



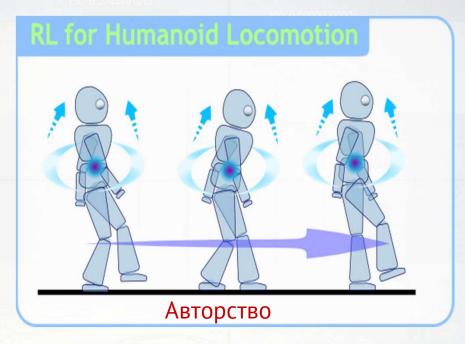
 $\label{eq:local_equation} \begin{aligned} & \operatorname{Initia} Q(s,a) & \text{are zeros} \\ & \operatorname{robot} \operatorname{uses} \operatorname{argmax} Q(s,a) \end{aligned}$ 

He has just learned to crawl with positive reward!



# What could possibly go wrong?

Our mobile robot learns to walk.



 $\label{eq:local_equation} \begin{aligned} & \operatorname{Initia} Q(s,a) & \text{are zeros} \\ & \operatorname{robot} \operatorname{uses} \operatorname{argmax} Q(s,a) \end{aligned}$ 

Too bad, now he will never learn to walk upright =(



# What could possibly go wrong?

New problem:

If our agent al<u>ways takes "best" actions</u> from his current point of view,

How will he ever learn that other actions may be better than his current best one?

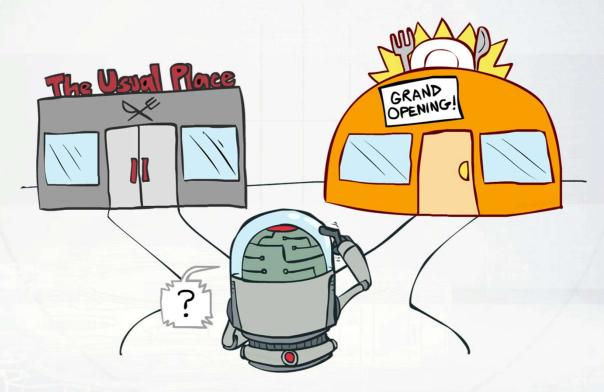
Ideas?

Stuck in local optima



# **Exploration Vs Exploitation**

Balance between using what you learned and trying to find something even better



Авторство



# **Exploration Vs Exploitation**

#### Strategies:

- $\varepsilon$  greedy
  - With probability take random action; otherwise take optimal action.

## One drawback:

All other actions, except the optimal, will be explored with the same prob i.e equally attractive



# **Exploration Vs Exploitation**

#### Strategies:

- $\varepsilon$  greedy
  - With probability  $\varepsilon$  take random action; otherwise take optimal action.
- Softmax
  - Pick action proportional to softmax of shifted normalized Q-values.

$$P(a) = software(\frac{Q(a)}{\tau})$$

More cool stuff coming later

all Qualues will be approx equal

# **Exploration over time**

#### Idea:



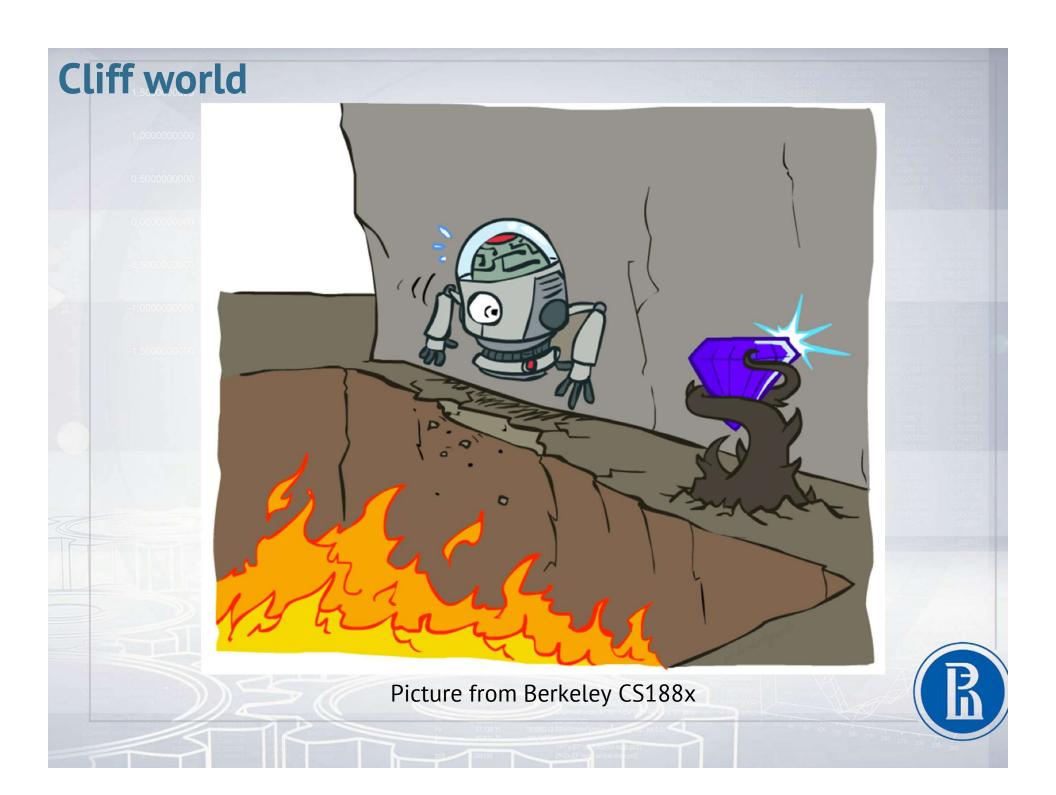
If you want to converge to optimal policy, you need to gradually reduce exploration

#### **Example:**

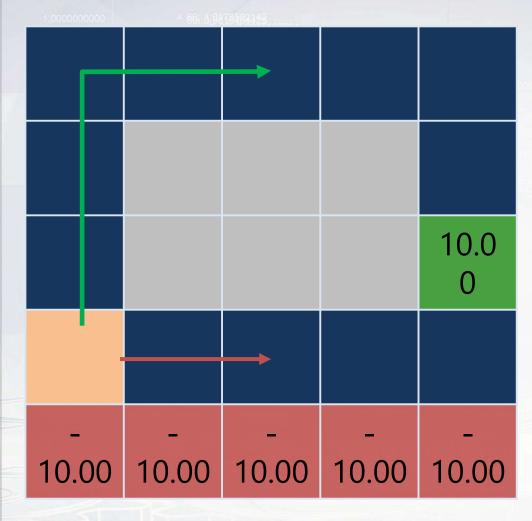
Initialize  $\varepsilon$  - greedy  $\varepsilon=0.5$  , then gradually reduce it

- If  $\varepsilon \to 0$  , it's greedy in the limit
- Be careful with non-stationary environments





## **Cliff world**



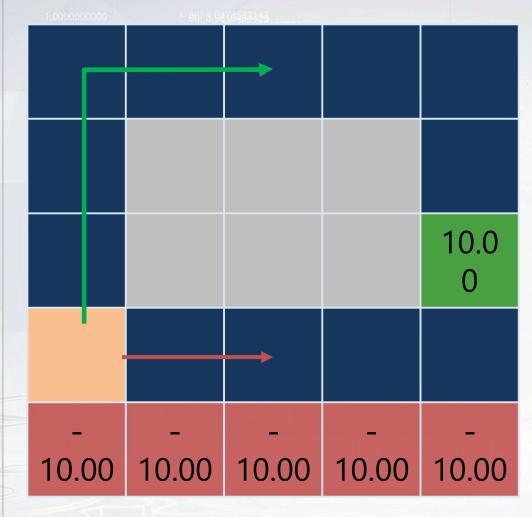
#### **Conditions**

- Q-learning
- $\gamma = 0.99 \ \epsilon = 0.1$
- · no slipping

Trivia: What will q-learning learn?



## Cliff world



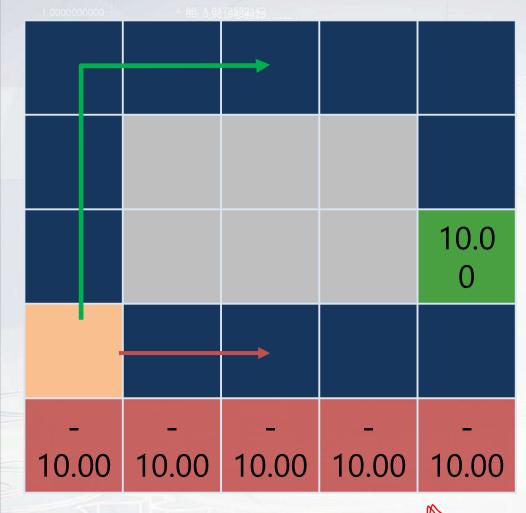
#### Conditions

- Q-learning
- $\gamma = 0.99 \ \epsilon = 0.1$
- · no slipping

Trivia: What will qlearning learn?
follow the short
Will it mathize reward?



### **Cliff world**



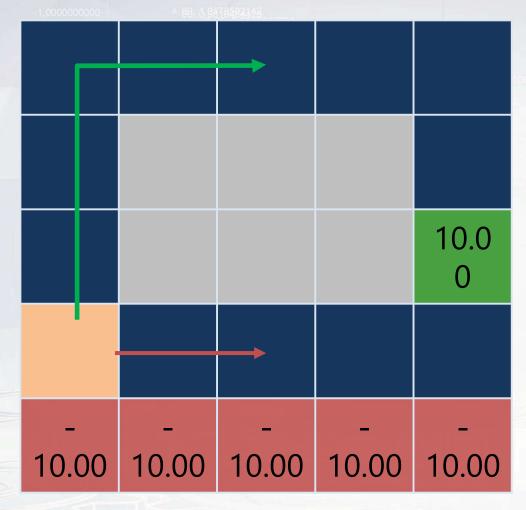
#### Conditions

- Q-learning
- $\gamma = 0.99 \ \epsilon = 0.1$
- no slipping

Trivia: What will qlearning learn?
follow the short
Will it maximize reward?
no, robot will fall due
to epsilon-greedy
"exploration"

no matter how long you train

#### **Cliff world**



#### **Conditions**

- Q-learning
- $\gamma = 0.99 \ \epsilon = 0.1$
- no slipping

Decisions must account for actual policy!

e.g.  $\varepsilon$  - greedy policy

\* longer path in this case will be better, using E-greedy



# Generalized update rule

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

better Q(s,a)



# **Q-learning VS SARSA**

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

**Q-learning** 

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

won't fet in

with exploration



## **Exploration over time**

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

better (2(5,0)

**Q-learning** 

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

SARSA

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \underbrace{E}_{a' \sim \pi(a'|s')} Q(s',a')$$



## **Exploration over time**

Update rule (from Bellman eq.)

$$Q(s_t, a_t) \leftarrow \alpha \cdot \hat{Q}(s_t, a_t) + (1 - \alpha)Q(s_t, a_t)$$

**Q-learning** 

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \max_{a'} Q(s',a')$$

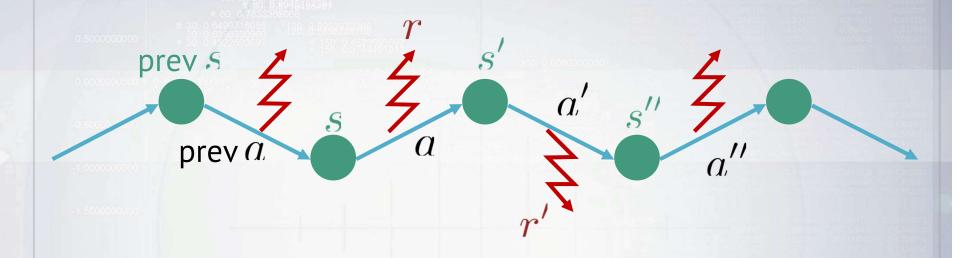
SARSA

Expected from 
$$S' \sim P(s'|s,a)$$

$$\hat{Q}(s,a) = r(s,a) + \gamma \cdot \underset{a' \sim \pi(a'|s')}{E} Q(s',a')$$



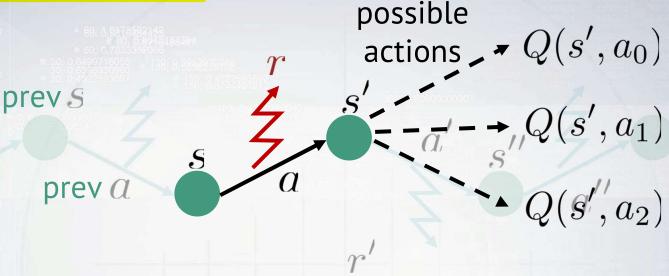
# **MDP** trajectory



- Trajectory is a sequence of
  - states (s)
  - actions (a)
  - rewards (r)
- Can be infinite, we can't wait that long



# Recap: Q-learning



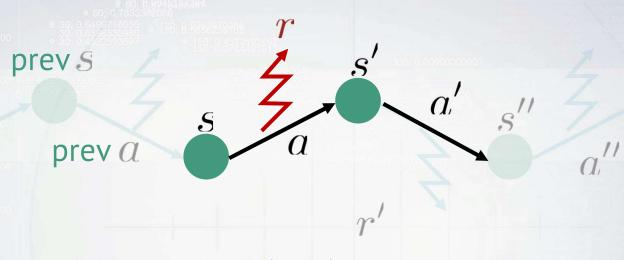
$$\forall s \in S, \forall a \in A, Q(s, a) \leftarrow 0$$

Initialize Q(s,a) with zeros

Loop:

- Sample  $\langle s, a, r, s' \rangle$  from env
- Compute  $\hat{Q}(s,a) = r(s,a) + \gamma \max_{a_i} Q(s',a_i)$
- Update  $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

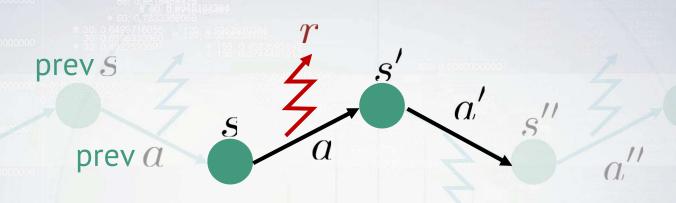
# SARSA



 $\forall s \in S, \forall a \in A, Q(s,a) \leftarrow 0$ Initialize Q(s,a) with zeros Loop:

- Sample  $\langle s, a, r, s', a \rangle$  from env
- Compute  $\hat{Q}(s,a) = r(s,a) + \gamma Q(s',a')$
- Update  $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + \stackrel{a_i}{(1-\alpha)} Q(s,a)$

#### **SARSA**



 $\forall s \in S, \forall a \in A, Q(s, a) \leftarrow 0^{\text{actions}}$ 

Initialize Q(s,a) with zeros

Loop:

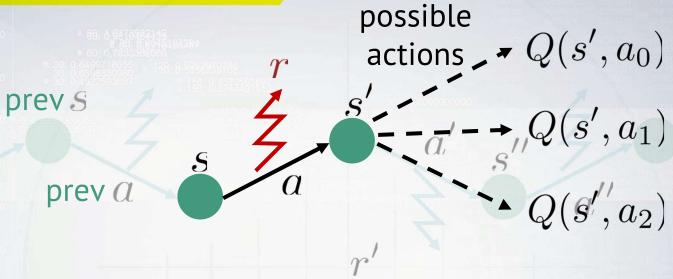
• Sample  $\langle s, a, r, s', a \rangle$  from env

hence "SARSA"

from env <u>next action</u> (not max)

- Compute  $\hat{Q}(s,a) = r(s,a) + \gamma Q(s',a')$
- Update  $Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha)Q(s,a)$

# **Expected value SARSA**



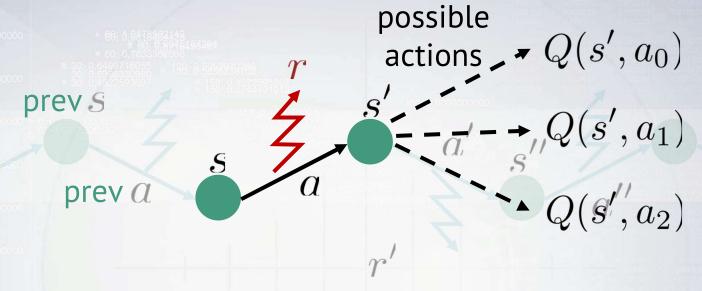
$$\forall s \in S, \forall a \in A, Q(s, a) \leftarrow 0$$

Initialize Q(s,a) with zeros

Loop:

- Sample  $\langle s, a, r, s' \rangle$ from env
- Compute  $\hat{Q}(s,a)=r(s,a)+\gamma$  E  $Q(s',a_i)$  Update  $Q(s,a)\leftarrow\alpha\cdot\hat{Q}(s,a)+(1-\alpha)Q(s,a)$

## **Expected value SARSA**



 $\forall s \in S, \forall a \in A, Q(s, a) \leftarrow 0$ 

Initialize Q(s,a) with zeros

Loop:

• Sample  $\langle s, a, r, s' \rangle$ from env

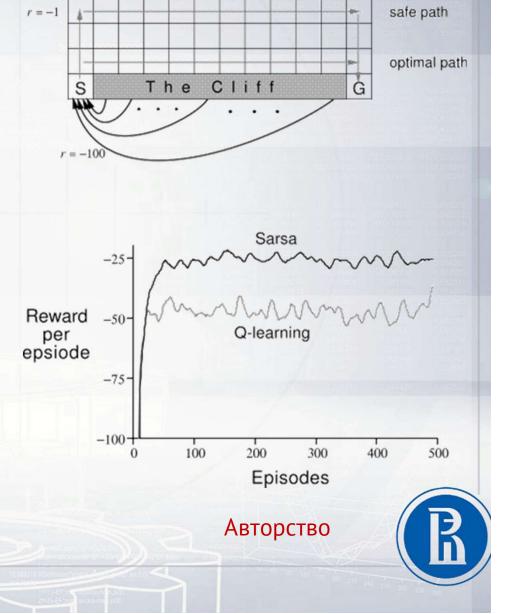
• Compute  $\hat{Q}(s,a)=r(s,a)+\gamma \mathop{E}_{a_i\sim\pi(a|s')}Q(s',a_i)$  • Update  $Q(s,a)\leftarrow\alpha\cdot\hat{Q}(s,a)+(1-\alpha)Q(s,a)$ 

**Expected value** 

## **Difference**

 SARSA gets optimal rewards under current exploration strategy

 O-learning policy would be optimal without exploration



Two problem setups

0,5000000000	off-policy
Agent <u>can</u> p <u>ick action</u> s	Agent <u>can't</u> pick actions
Most <u>obvious setu</u> p :)	Learning with exploration, playing without exploration
Agent al <u>ways follow</u> s his <b>own</b> policy	Learning from expert (expert is imperfect)
	Learning from sessions (recorded data)

X agent is trained here on diff policy than the current one

learn off-policy

On-policy algorithms can't

on-policy off-policy Agent can pick actions Agent **can't** pick actions

Off-policy algorithms can

learn on-policy learn optimal policy even if (but they be faster/better) agent takes random actions

Two problem setups

Trivia: which of Q-learning, SARSA and exp. val. SARSA will only work on-policy?



Two problem setups on-policy off-policy Agent **can** pick actions Agent **can't** pick actions On-policy algorithms can't Off-policy algorithms can learn off-policy learn on-policy SARSA **Q**-learning more coming soon Expected Value SARSA

Trivia: will Crossentropy Method converge if it learns off-policyfrom agent that takes random actions?





Two problem setups

on-policy	off-policy
Agent <b>can</b> pick actions	Agent <b>can't</b> pick actions
On-policy algorithms can't learn off-policy	Off-policy algorithms <b>can</b> learn on-policy
SARSA	Q-learning
more coming soon	Expected Value SARSA

Trivia: will Crossentropy Method converge if it learns off-policyfrom agent that takes random Wellipmo:)



Two problem setups

on-policy	off-policy
Agent <b>can</b> pick actions	Agent <b>can't</b> pick actions
On-policy algorithms can't learn off-policy	Off-policy algorithms <b>can</b> learn on-policy
SARSA	<u>Q-learning</u>
more coming soon	Expected Value SARSA



# **Experience replay**

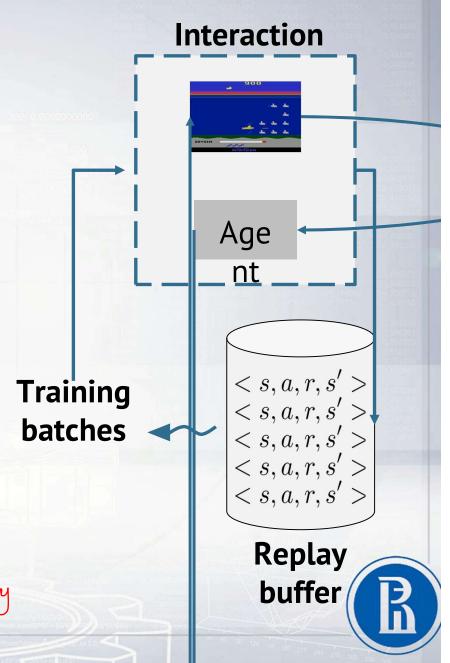
Idea: store several past interactions < s, a, r, s' > Train on random subsamples

\*\* Agent is trained not only on immediate State actual rewards, but it can actually store its prev interactions

Hence, sampling can be done from this

large pool.

Obviously, however, such samples are probably worse than your current policy



# **Experience replay**

Idea: store several past interactions < s, a, r, s' > Train on random subsamples

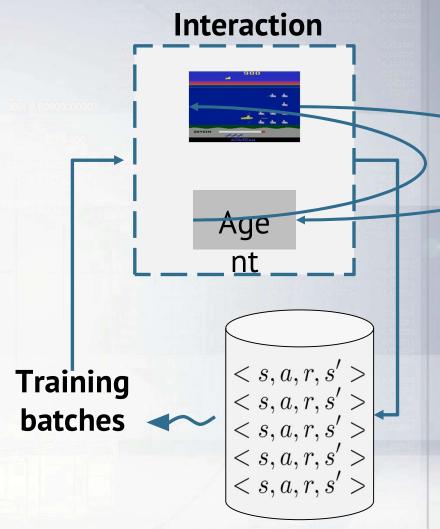
#### **Training curriculum:**

- play 1 step and record it
- pick N random transitions to train

Profit: you don't need to re-visit same (s,a) many times to learn it.

Only works with off-policy algorithms!

Btw, why only them?





# **Experience replay**

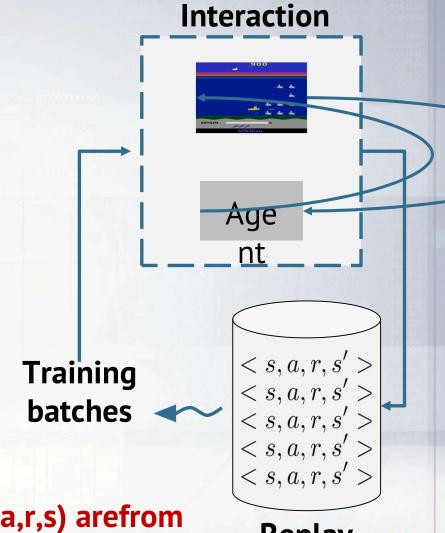
Idea: store several past interactions < s, a, r, s' > Train on random subsamples

#### **Training curriculum:**

- play 1 step and record it
- pick N random transitions to train

Profit: you don't need to re-visit same (s,a) many times to learn it.

Y Only works with off-policy algorithms!



Old (s,a,r,s) arefrom older/weakerversion of policy!





