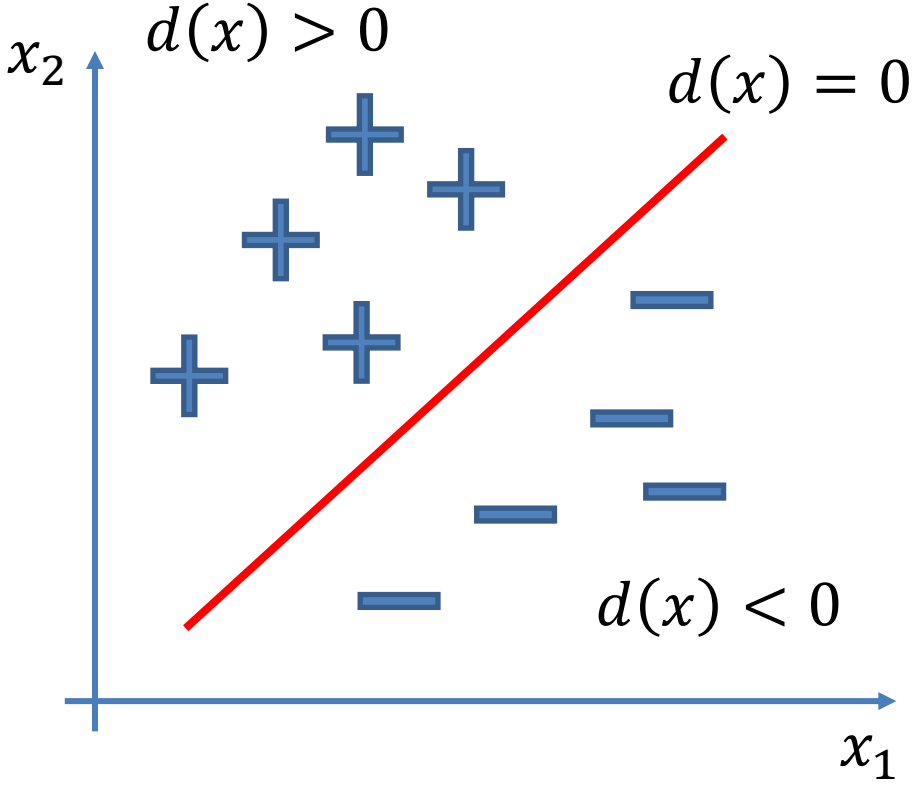
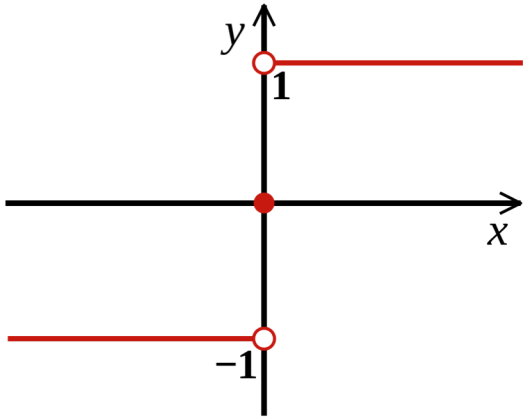
Intro

- In this video we will talk about the simplest neural network – multi-layer perceptron (MLP)

Let's recall linear binary classification

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$
- Decision function: $d(x) = \boldsymbol{w_0} + \boldsymbol{w_1} x_1 + \boldsymbol{w_2} x_2$
- Algorithm: $a(x) = \text{sign}(d(x))$

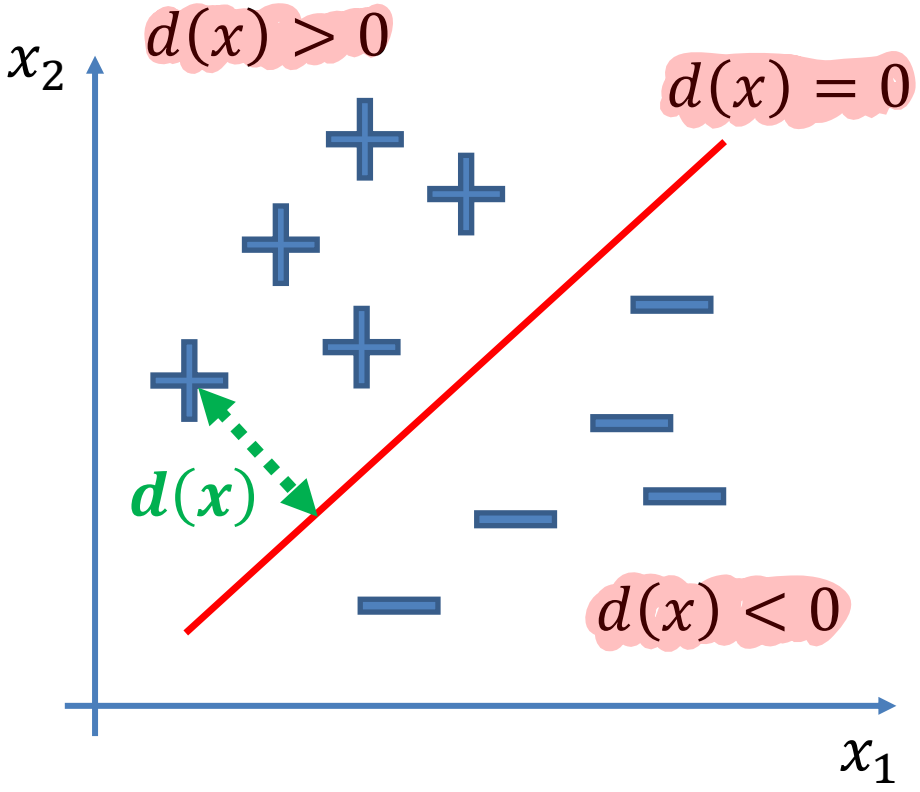


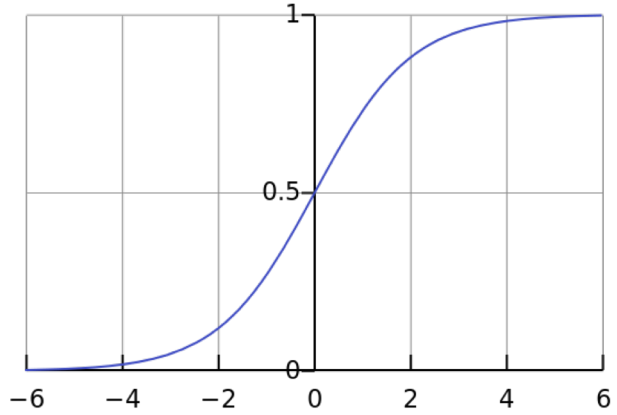


$\text{sign}(x)$

Logistic regression

- Predicts probability of the positive class (+1)
- Decision function: $d(x) = \boldsymbol{w_0} + \boldsymbol{w_1} x_1 + \boldsymbol{w_2} x_2$ reflecting confidence
- Algorithm: $a(x) = \sigma\big(d(x)\big)$

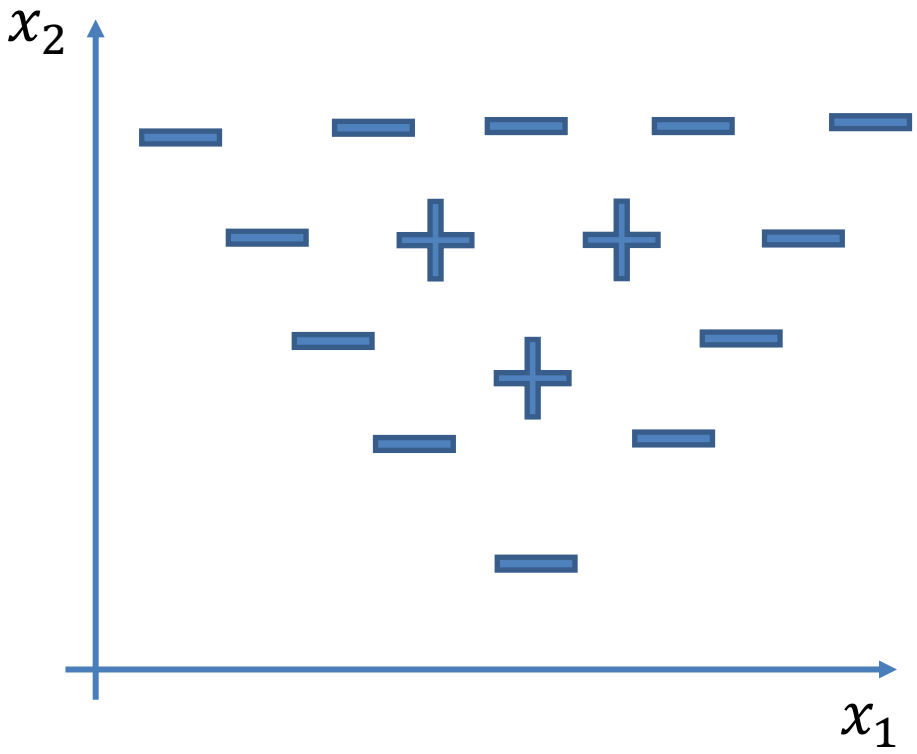




$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

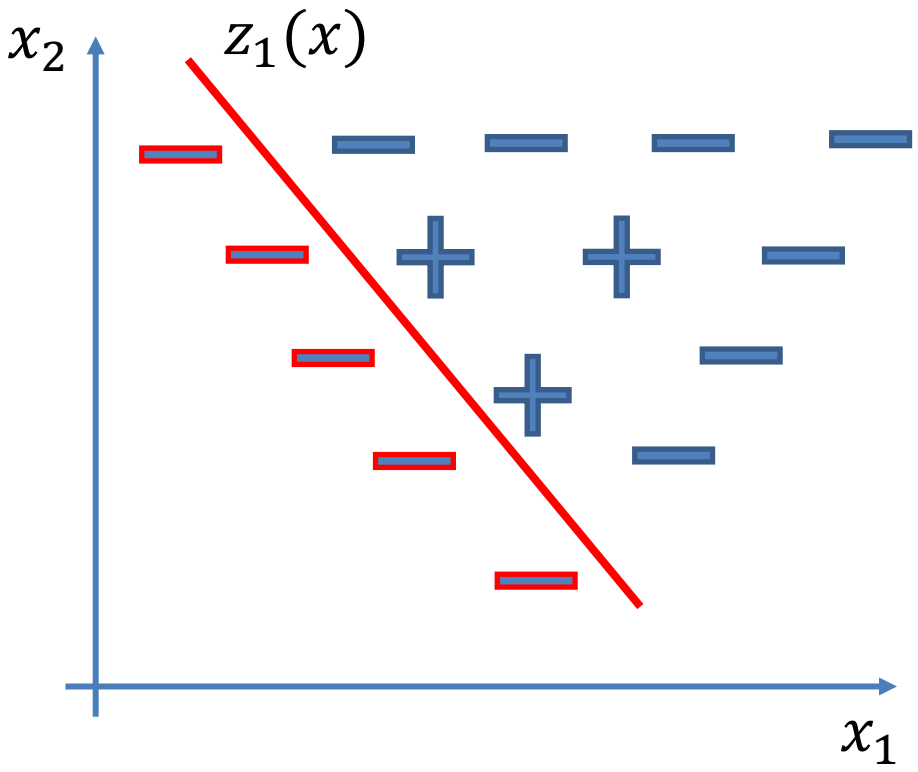
Triangle problem

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$



Triangle problem: solving a subproblem

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$

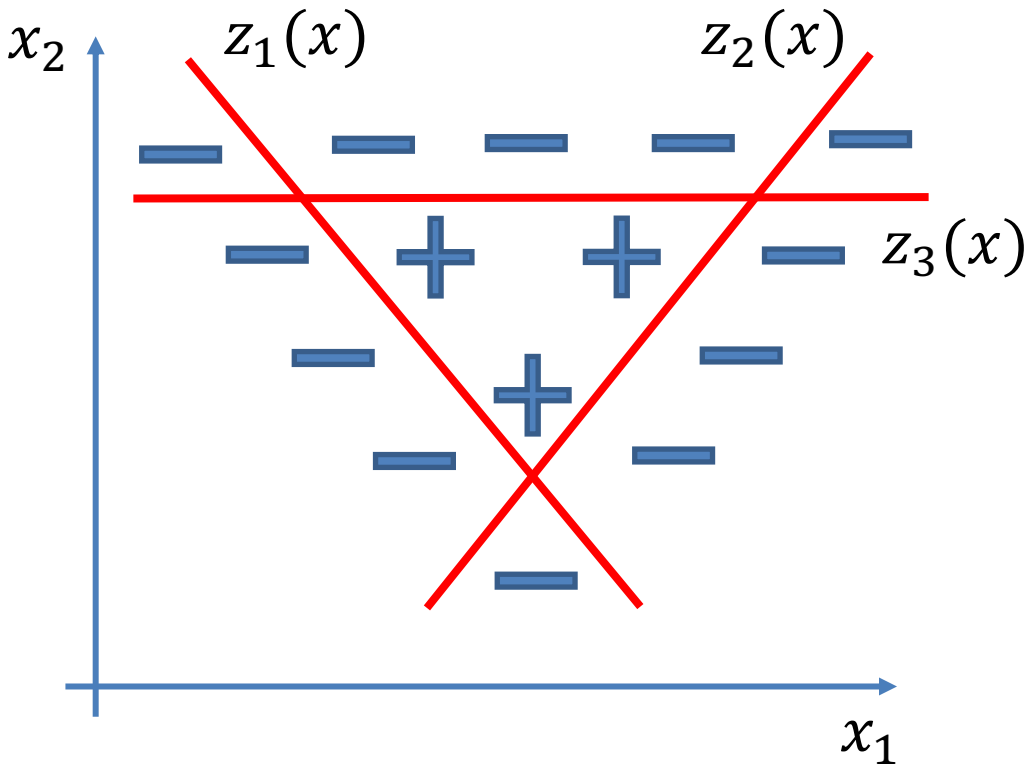


This line helps us to separate minuses on the left

$$z_1 = \sigma(\boldsymbol{w_{0,1}} + \boldsymbol{w_{1,1}} x_1 + \boldsymbol{w_{2,1}} x_2)$$

A logistic regression per a subproblem

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$



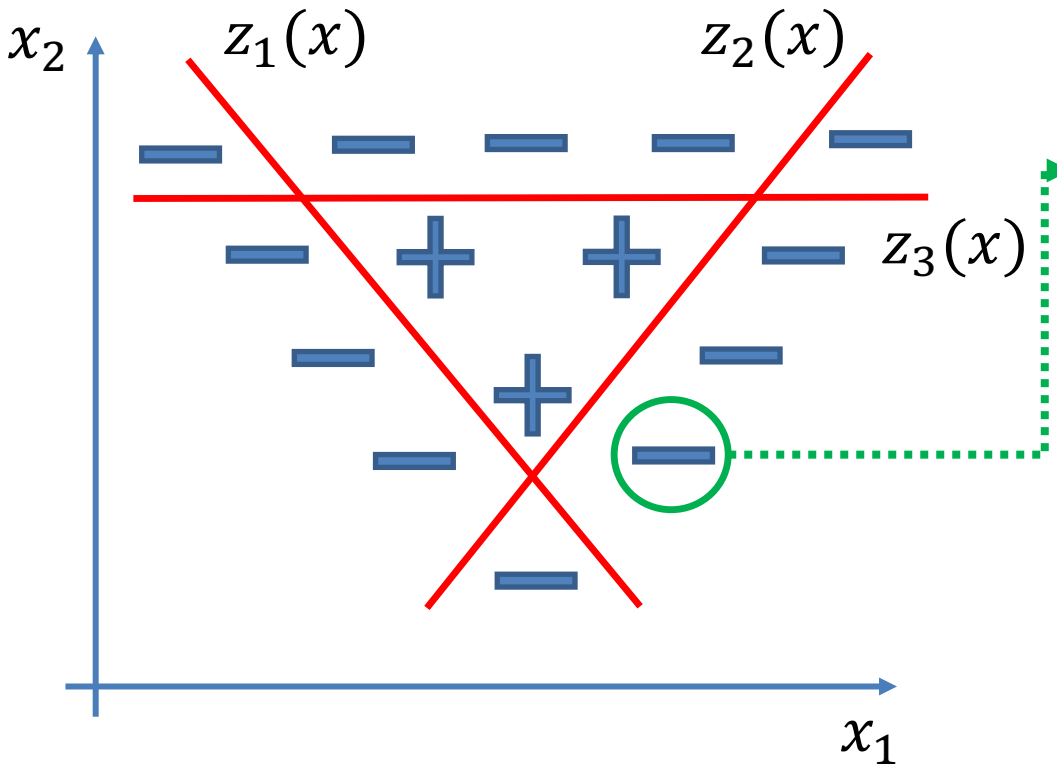
Imagine that we've somehow found these 3 lines

$$z_i = \sigma(\boldsymbol{w_{0,i}} + \boldsymbol{w_{1,i}} x_1 + \boldsymbol{w_{2,i}} x_2)$$

These lines give us new features

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$

transformation



New features:

$z_1(x)$	$z_2(x)$	$z_3(x)$	y
0.6	0.3	0.8	-1
0.7	0.7	0.7	+1

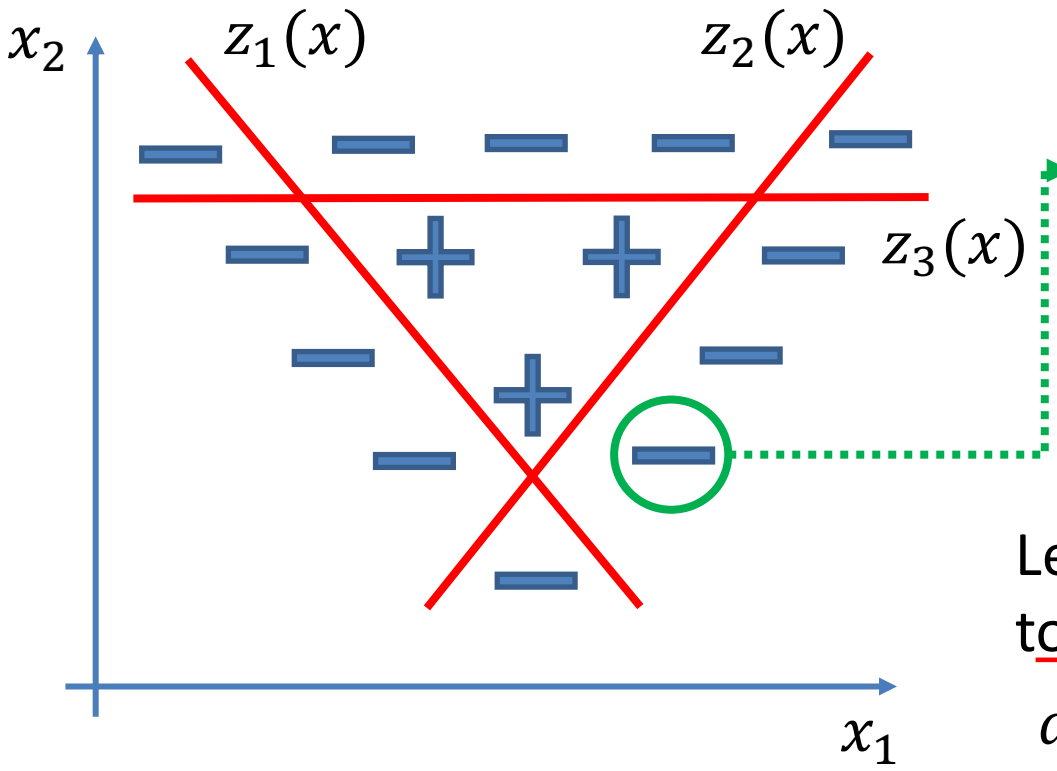
$(x_1, x_2) \rightarrow (z_1, z_2, z_3)$

What to do next?

$$z_i = \sigma(\boldsymbol{w_{0,i}} + \boldsymbol{w_{1,i}} x_1 + \boldsymbol{w_{2,i}} x_2)$$

Final algorithm

- Features: $x = (x_1, x_2)$
- Target: $y \in \{+1, -1\}$



$$z_i = \sigma(\boldsymbol{w_{0,i}} + \boldsymbol{w_{1,i}} x_1 + \boldsymbol{w_{2,i}} x_2)$$

New features:

$z_1(x)$	$z_2(x)$	$z_3(x)$	y
0.6	0.3	0.8	-1
0.7	0.7	0.7	+1

$$(x_1, x_2) \rightarrow (z_1, z_2, z_3)$$

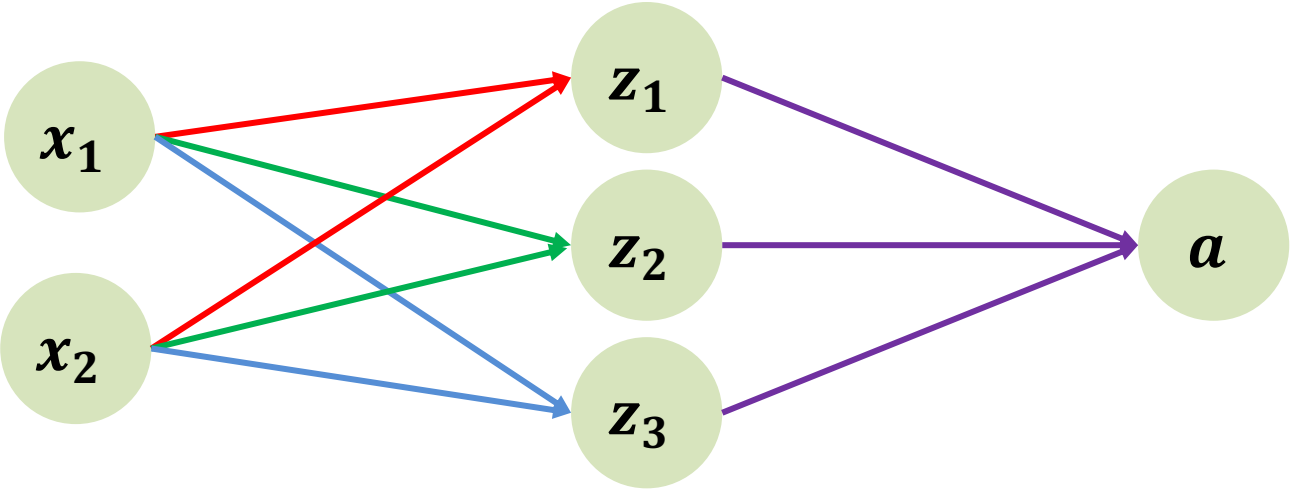
Let's build a linear model on top of these new features:

$$a(x) = \sigma(\boldsymbol{w_0} + \boldsymbol{w_1} z_1(x) + \boldsymbol{w_2} z_2(x) + \boldsymbol{w_3} z_3(x))$$

We still don't know how to find these 4 lines

- But we know how our algorithm will predict once we find them:
  - $z_i = \sigma(\boldsymbol{w_{0,i}} + \boldsymbol{w_{1,i}} x_1 + \boldsymbol{w_{2,i}} x_2)$
  - $a(x) = \sigma(\boldsymbol{w_0} + \boldsymbol{w_1} z_1(x) + \boldsymbol{w_2} z_2(x) + \boldsymbol{w_3} z_3(x))$

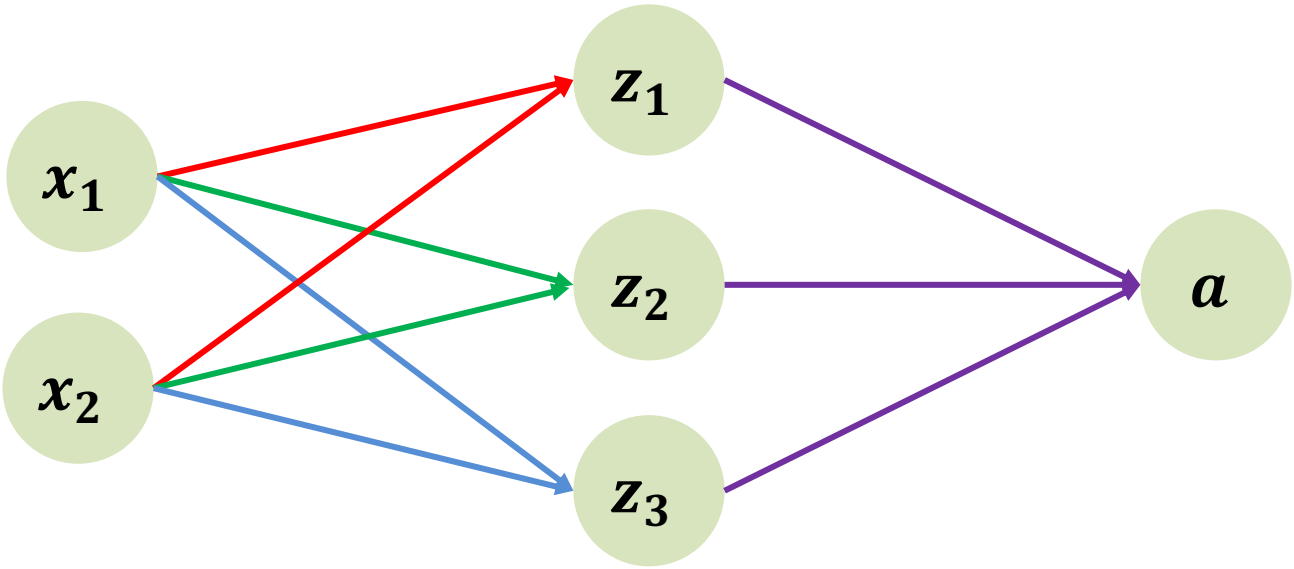
Let's rewrite our algorithm in terms of a **computation graph**:



Nodes: computed variables ($\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{z_1}, \boldsymbol{z_2}, \boldsymbol{z_3}, \boldsymbol{a}$)
Edges: dependencies (we need $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ to compute $\boldsymbol{z_1}$)

Our computation graph has a name

- Multi-layer perceptron (MLP):



Input layer **Hidden layer** **Output layer**

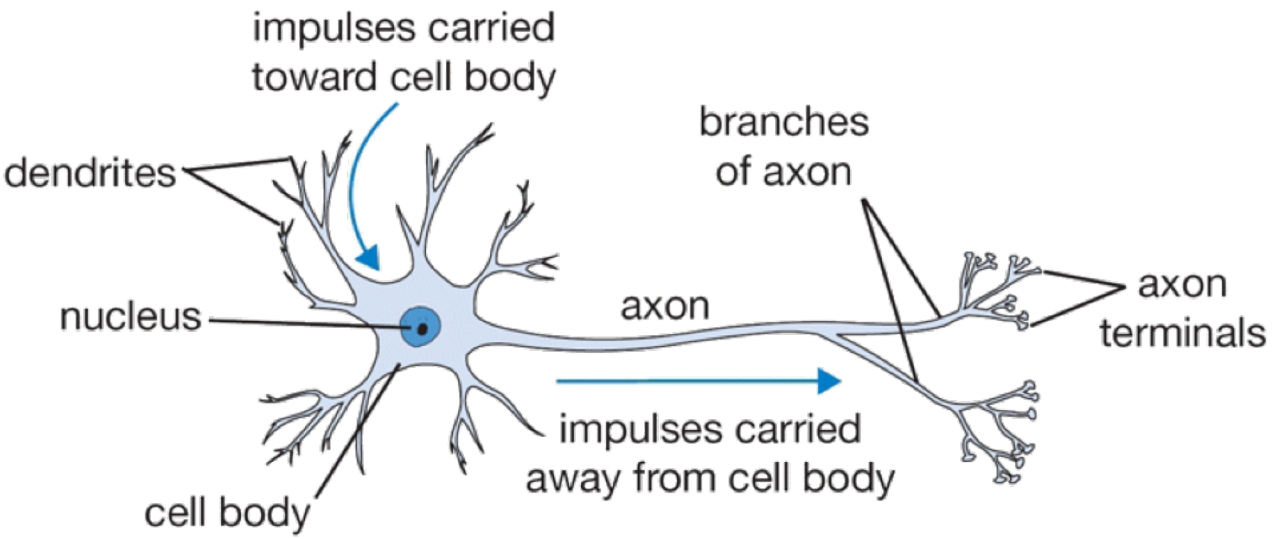
Features

Here each node is a **neuron**:

1. Take a linear combination of inputs
2. Apply **activation** function (e.g. $\sigma(x)$)

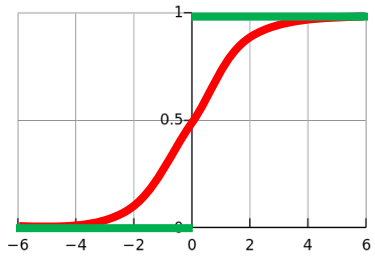
Why is it called a neuron?

- Neuron in a human brain:

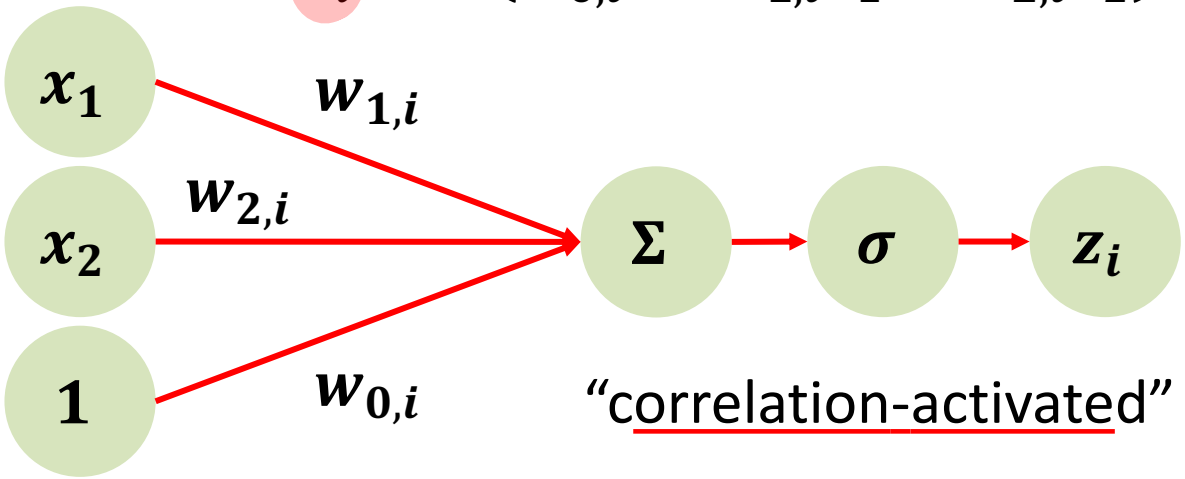


- Artificial neuron:

$$z_i = \sigma(w_{0,i} + w_{1,i}x_1 + w_{2,i}x_2)$$



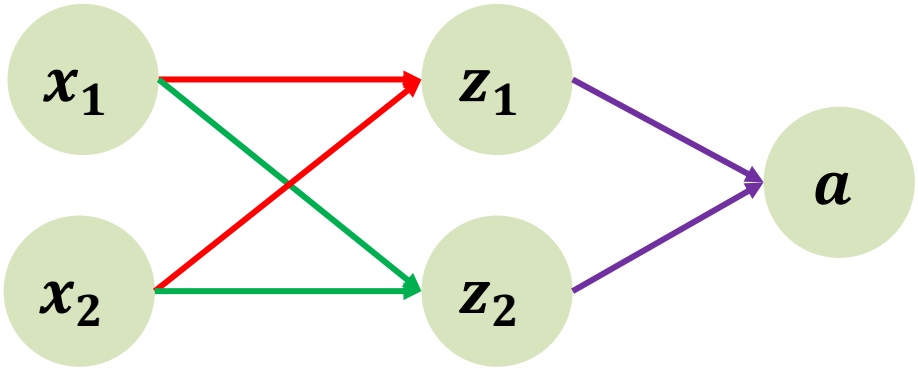
"smooth indicator"



"correlation-activated"

We need a non-linear activation function!

- Let's see what happens if we throw away $\sigma(x)$:



$$z_1 = w_{1,1}x_1 + w_{2,1}x_2$$

$$z_2 = w_{1,2}x_1 + w_{2,2}x_2$$

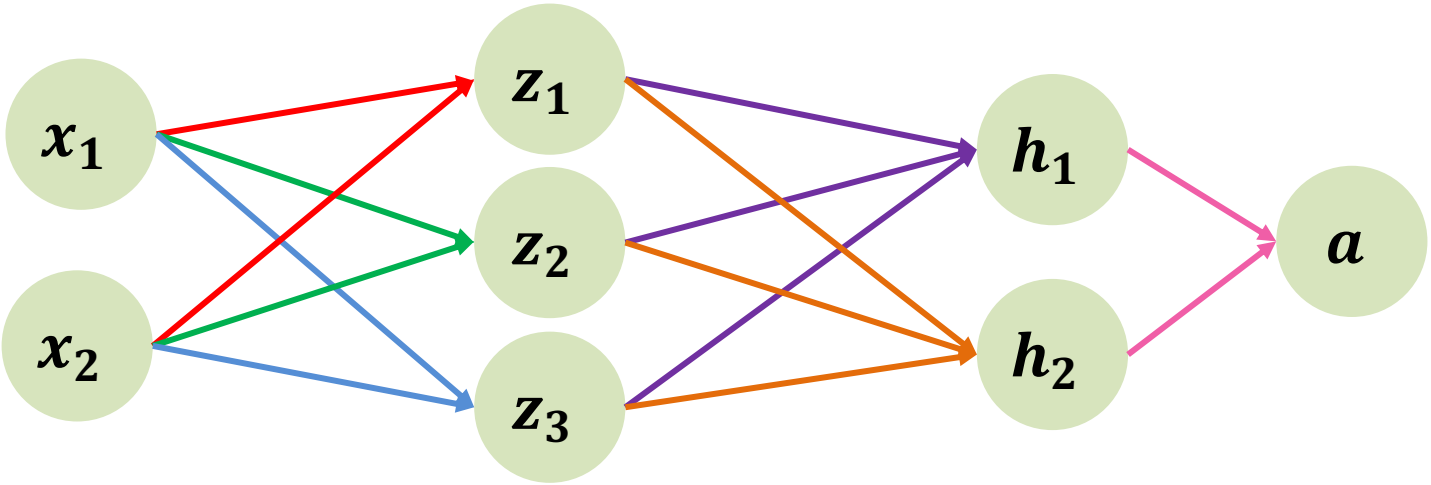
$$a = w_1 z_1 + w_2 z_2$$

- Our algorithm turns into a fancy **linear function**!
  - $a = (w_1 w_{1,1} + w_2 w_{1,2})x_1 + (w_1 w_{2,1} + w_2 w_{2,2})x_2$

MLP overview

- MLP is an example of **artificial neural network**

- MLP can have many hidden layers:



- **Architecture** of an MLP:
  - Number of layers
  - Number of neurons in each layer
  - Activation function

- Hidden layer in MLP:
  - Dense layer
  - Fully-connected layer

or conv

or literally anything

How to train your MLP?

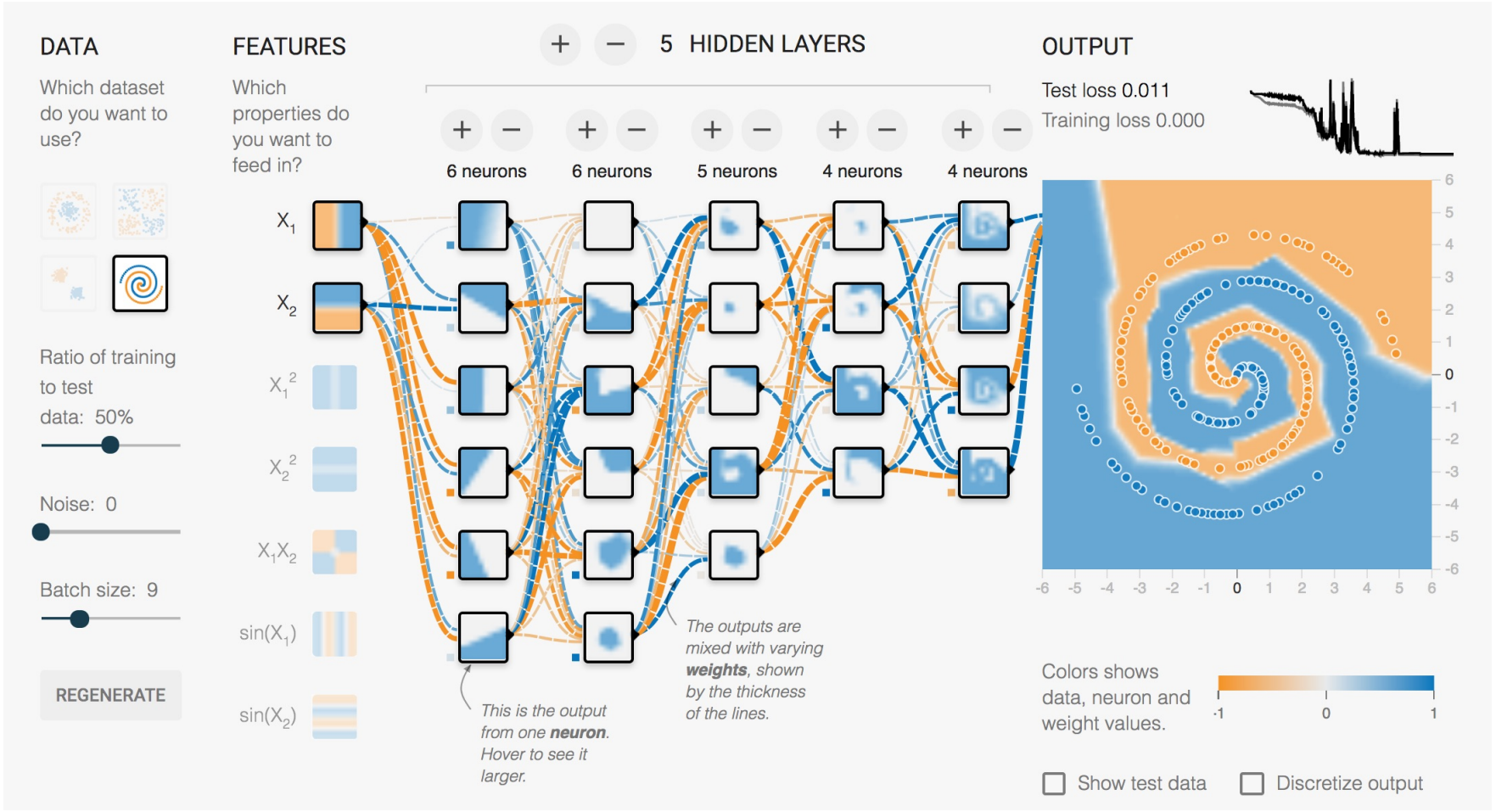
- We know how to train one neuron (e.g. logistic regression): SGD
- Let's do the same for the whole MLP!

Bob chilling at a local optima

https://hackernoon.com/life-is-gradient-descent-880c60ac1be8

Check out MLP training demo

- http://playground.tensorflow.org (change activation to ReLU)



How to train your MLP?

- Other problems:

  We can have many hidden layers (hyper-parameter) → we need to calculate gradients **automatically**!

  - We can have many neurons → we need to calculate gradients **fast**!

- In the next video we will solve these problems!

Intro

- In this video we will learn how to compute the gradients for MLP **automatically**.

Chain rule

- We know derivatives for simple functions:

$$\frac{dx^2}{dx} = 2x \qquad \frac{de^x}{dx} = e^x \qquad \frac{d\ln(x)}{dx} = \frac{1}{x}$$

- Let's take a composite function:

$$z_1 = z_1(x_1, x_2)$$
$$z_2 = z_2(x_1, x_2)$$
$$p = p(z_1, z_2)$$

where z_1, z_2, p are differentiable

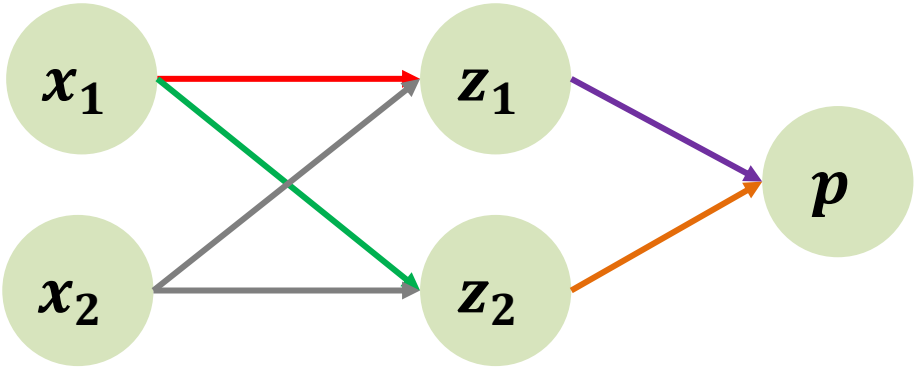
Chain rule: $$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

Example for $h(x) = f(x)g(x)$:

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial f}\frac{\partial f}{\partial x} + \frac{\partial h}{\partial g}\frac{\partial g}{\partial x} = g\frac{\partial f}{\partial x} + f\frac{\partial g}{\partial x}$$

Derivatives computation graph

- Let's take our simple computation graph:

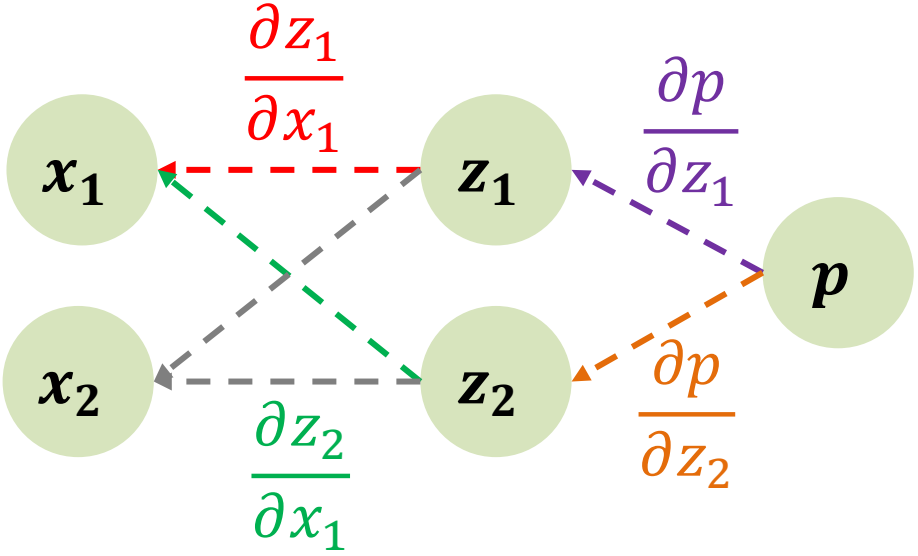


$$z_1 = z_1(x_1, x_2)$$

$$z_2 = z_2(x_1, x_2)$$

$$p = p(z_1, z_2)$$

- And construct a new graph of derivatives:

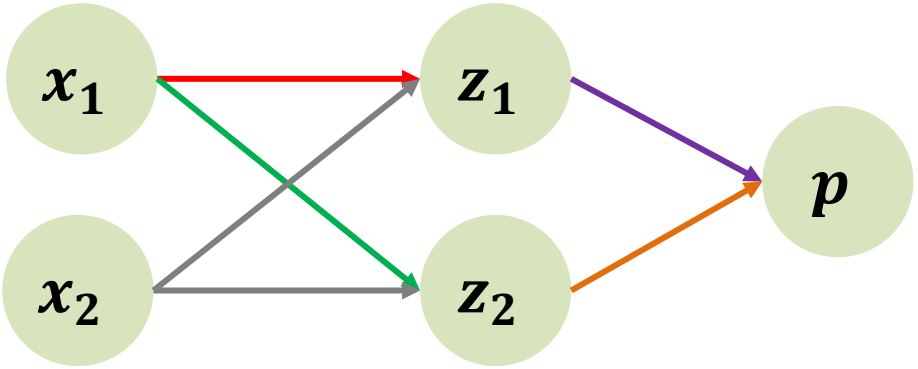


going backward

Each edge is assigned to derivative of origin w.r.t. destination

Derivatives computation graph

- Let's take our simple computation graph:

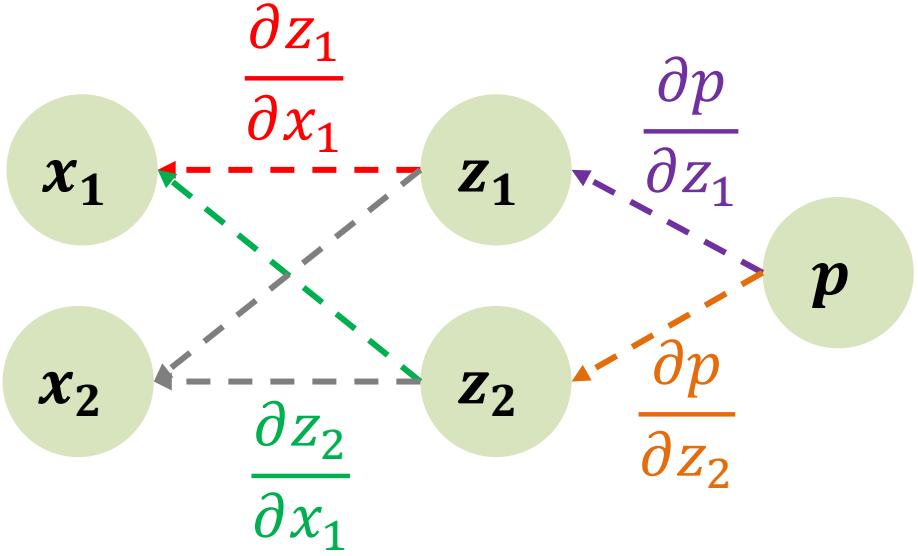


$$z_1 = z_1(x_1, x_2)$$

$$z_2 = z_2(x_1, x_2)$$

$$p = p(z_1, z_2)$$

- And construct a new graph of derivatives:

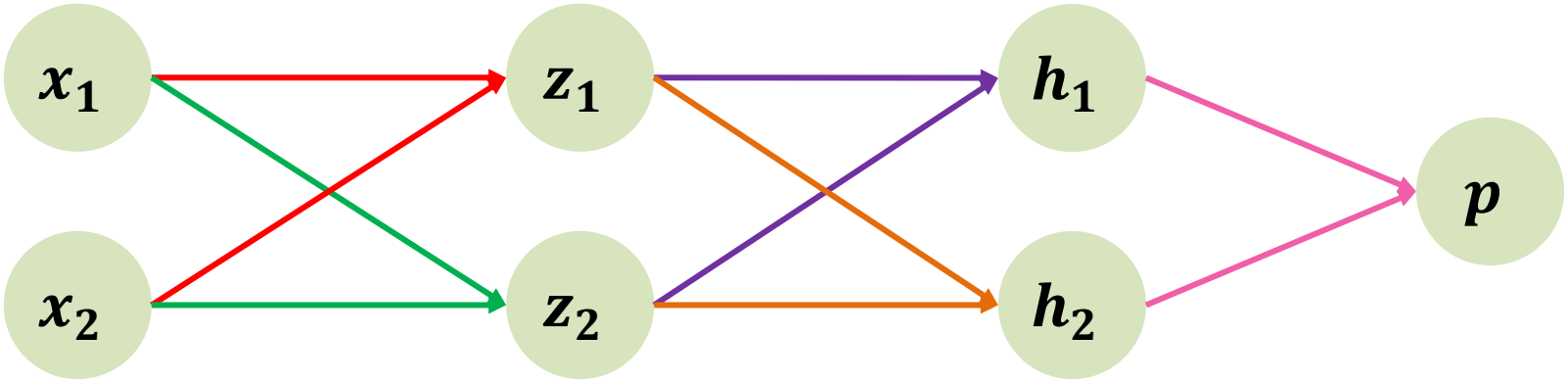


$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

You can see how
a **chain rule** works

Let's go deeper

- A little bit more composite function:



$$z_1 = z_1(x_1, x_2) \qquad h_1 = h_1(z_1, z_2)$$

$$z_2 = z_2(x_1, x_2) \qquad h_2 = h_2(z_1, z_2)$$

$$p = p(h_1, h_2)$$

Let's go deeper

Chain rule: $$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial x_1}$$

Let's go deeper

Chain rule: $$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial x_1}$$

$$\frac{\partial h_1}{\partial x_1} = \frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

$$\frac{\partial h_2}{\partial x_1} = \frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

Let's go deeper

Chain rule: $$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial x_1}$$

$$\frac{\partial h_1}{\partial x_1} = \frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

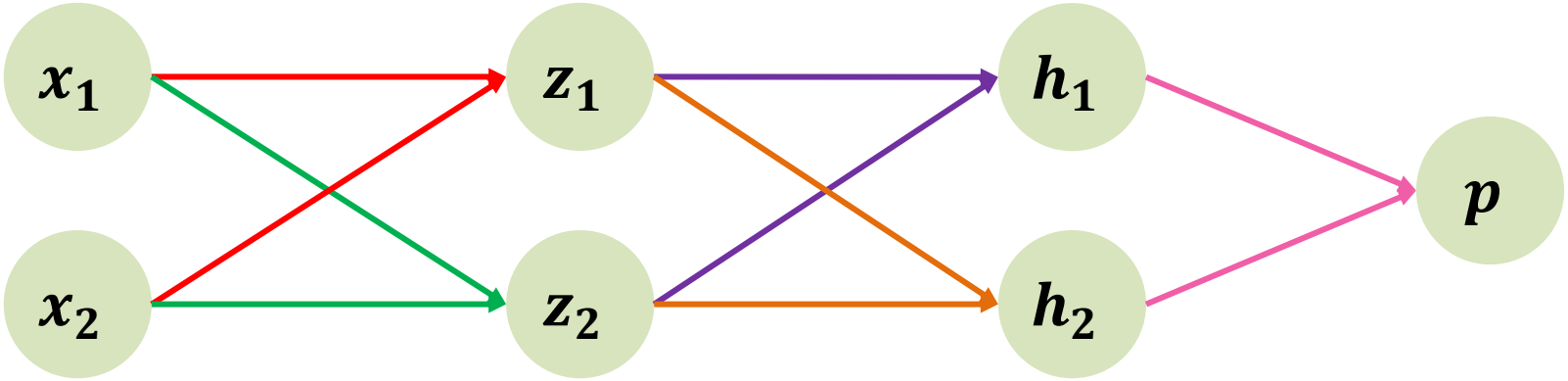
$$\frac{\partial h_2}{\partial x_1} = \frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\left(\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1}\right) + \frac{\partial p}{\partial h_2}\left(\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}\right)$$

$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

Let's check out the derivatives graph!

Let's go deeper



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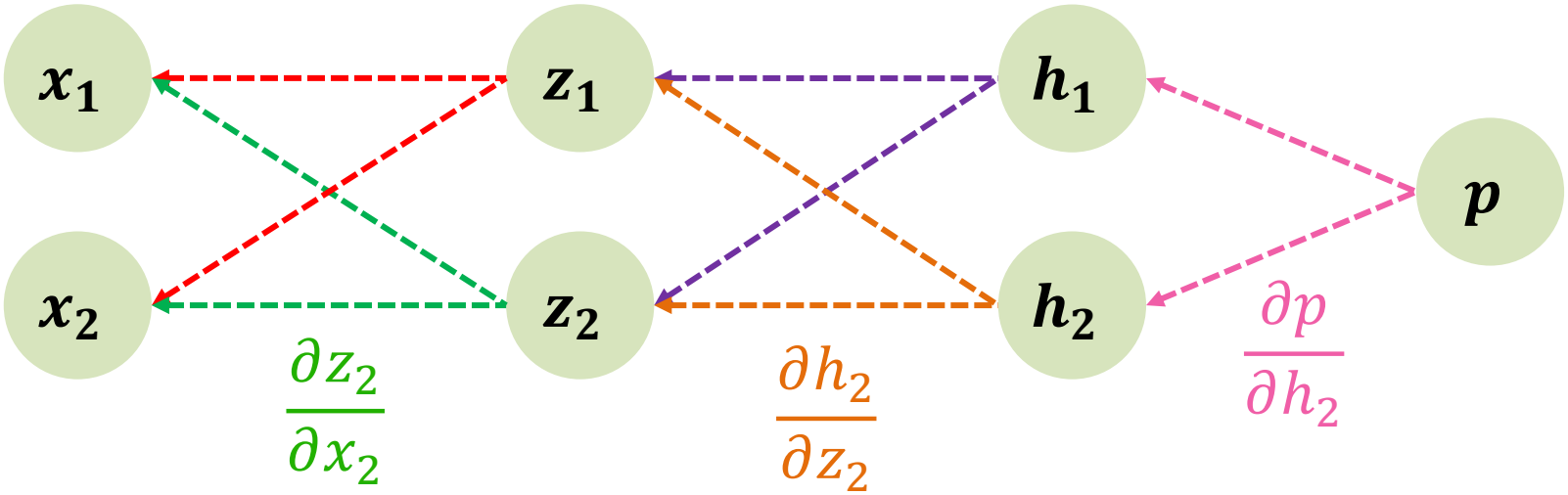
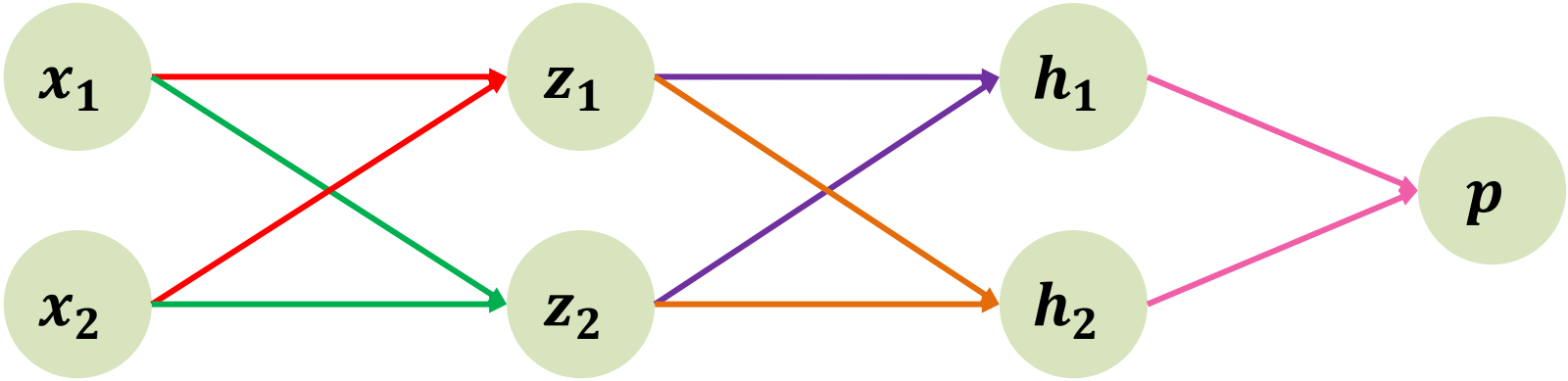
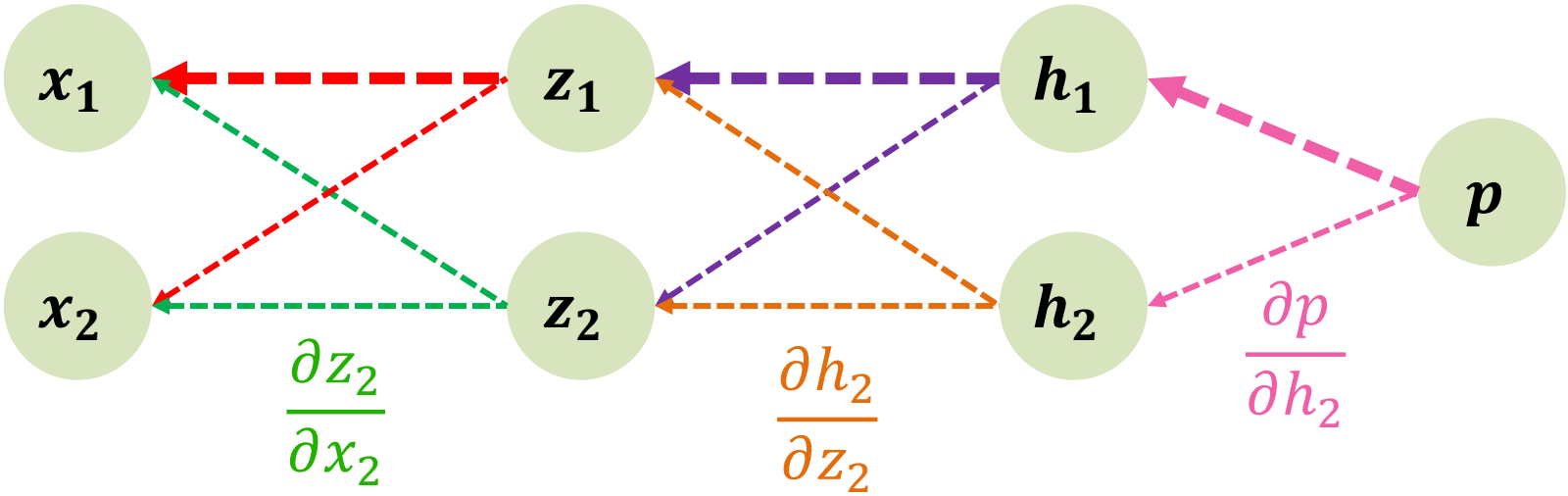


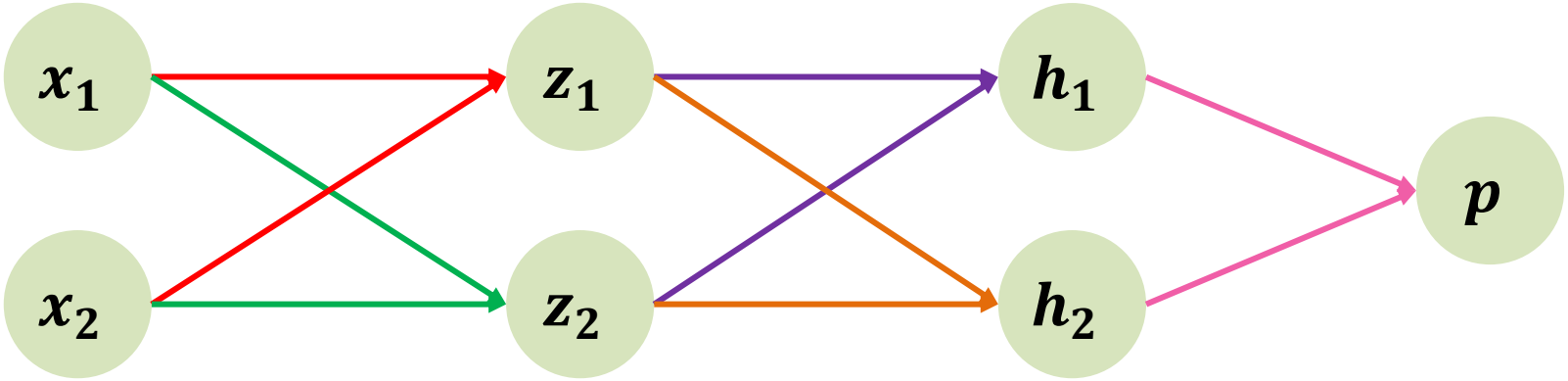
Let's go deeper



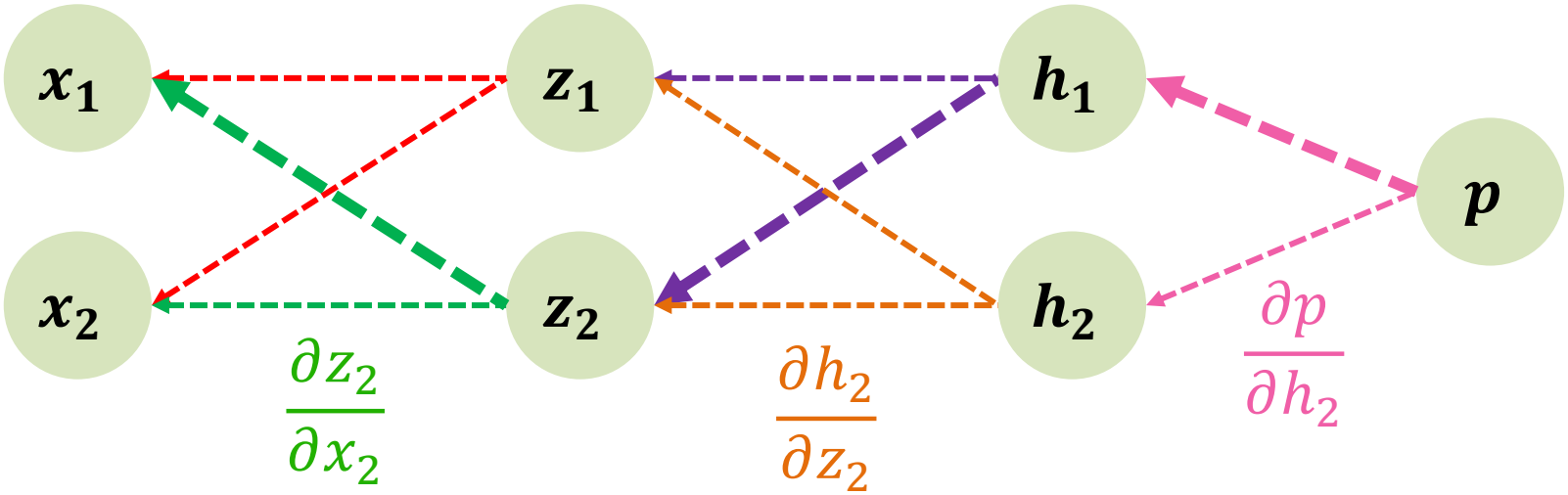
$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$



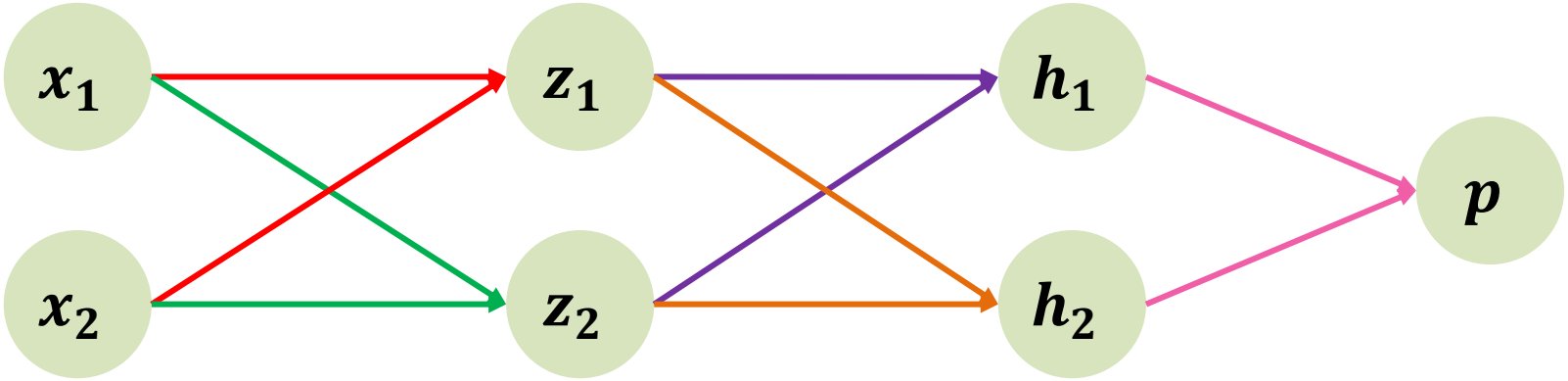
Let's go deeper



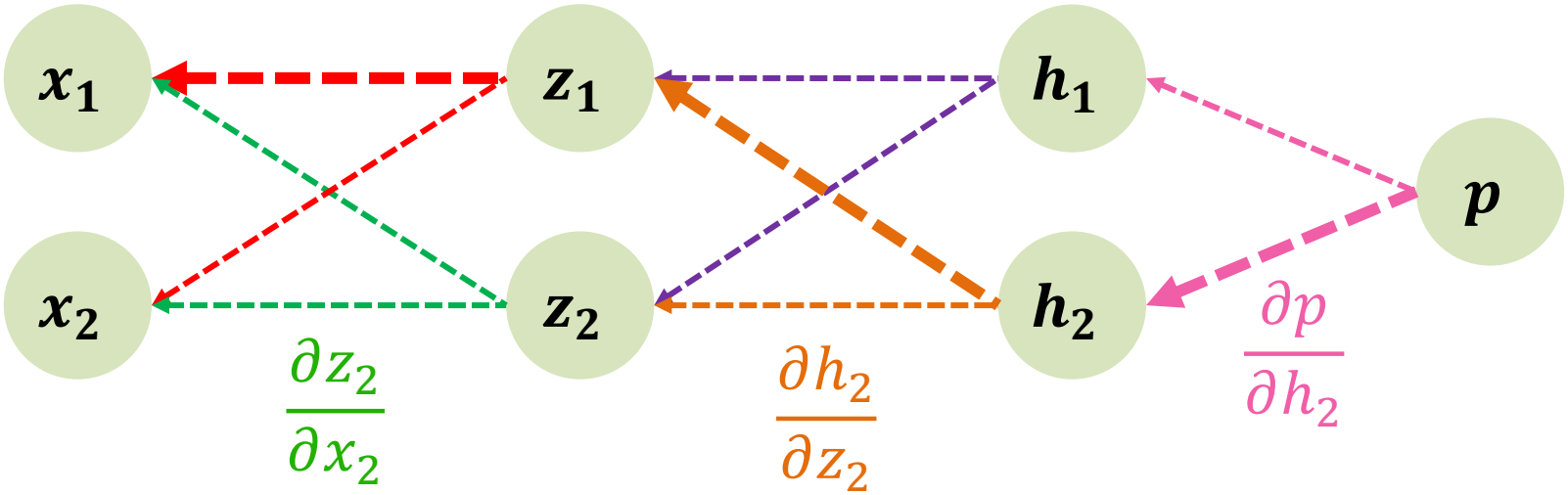
$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \boxed{\frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1}} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$



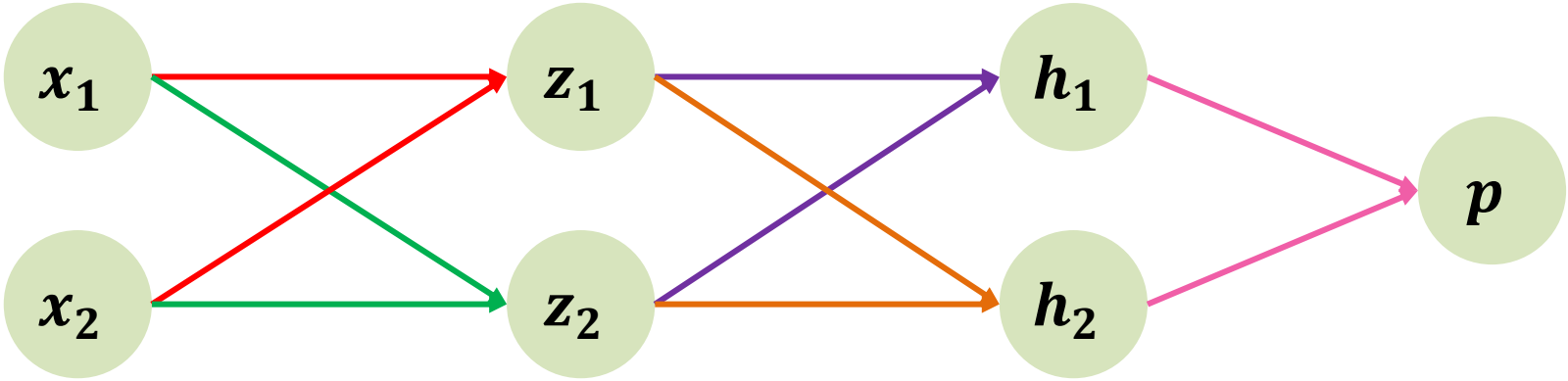
Let's go deeper



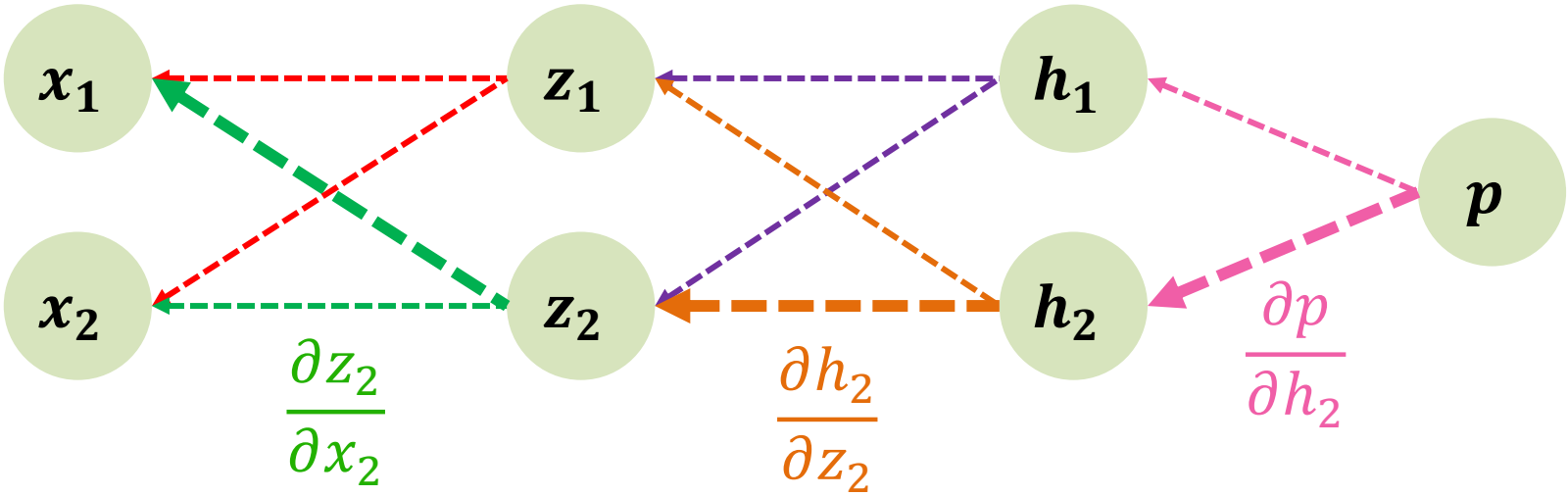
$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \boxed{\frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1}} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$



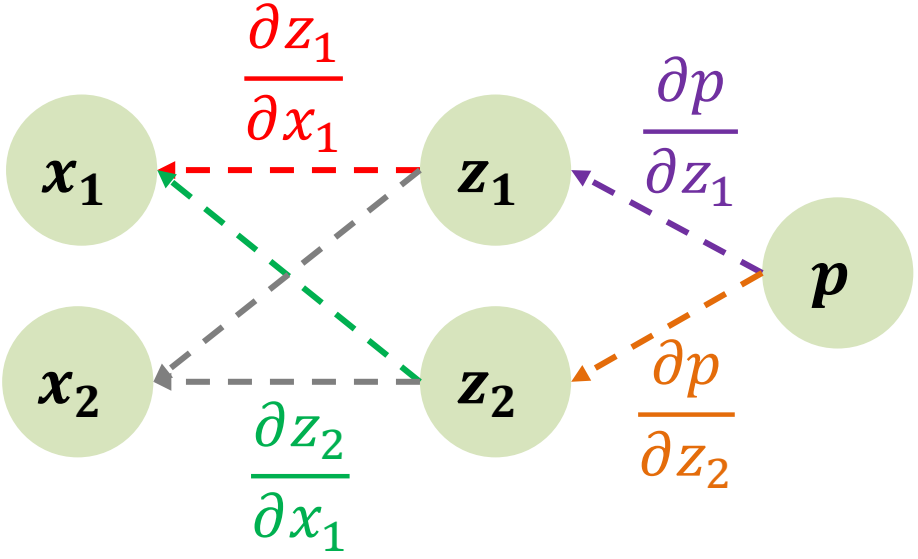
Let's go deeper



$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \boxed{\frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}}$$



How this graph of derivatives helps

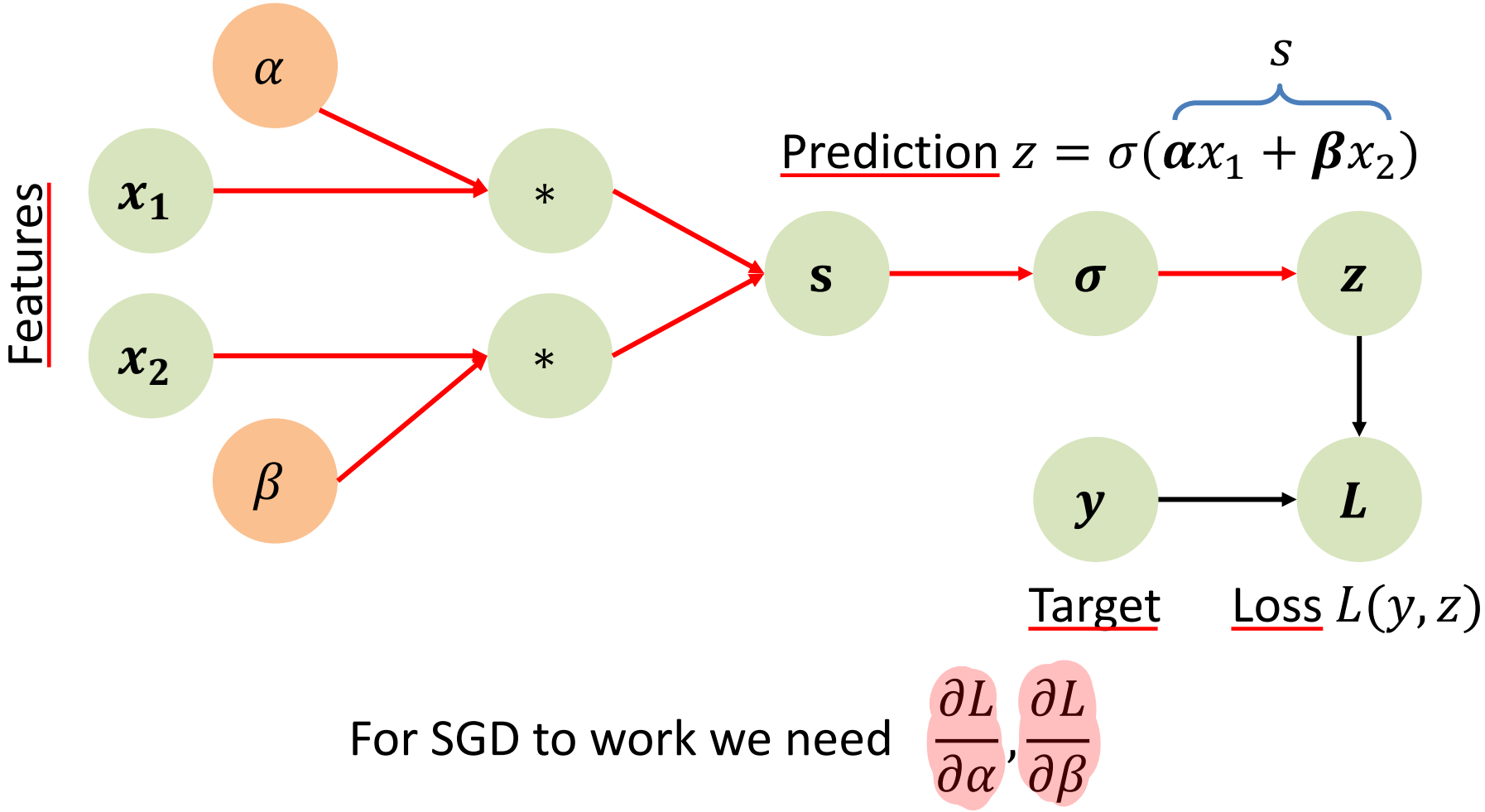


$$\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial z_2}\frac{\partial z_2}{\partial x_1}$$

How to calculate a derivative of node a w.r.t. node b:

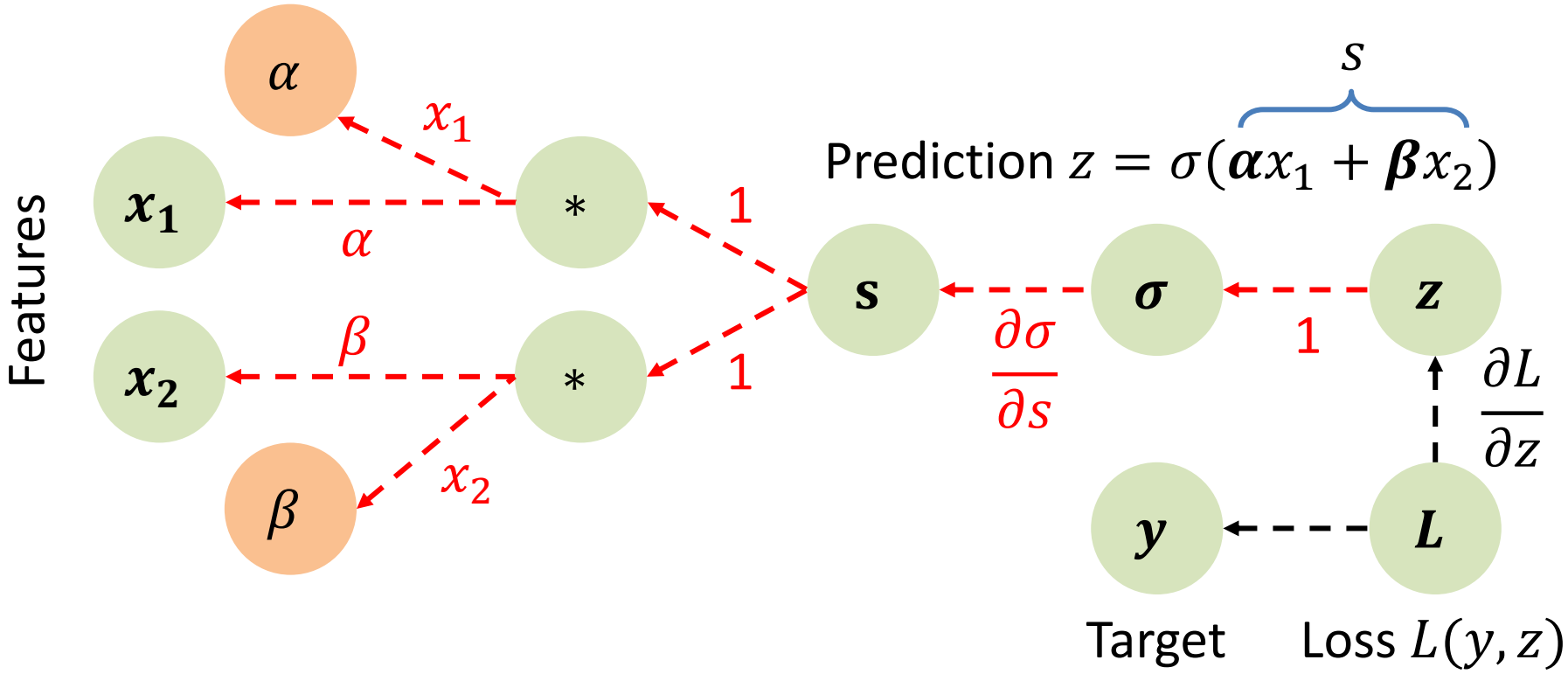
- Find an unvisited path from a to b
- Multiply all edge values along this path
- Add to the resulting derivative

How chain rule helps to train a neuron



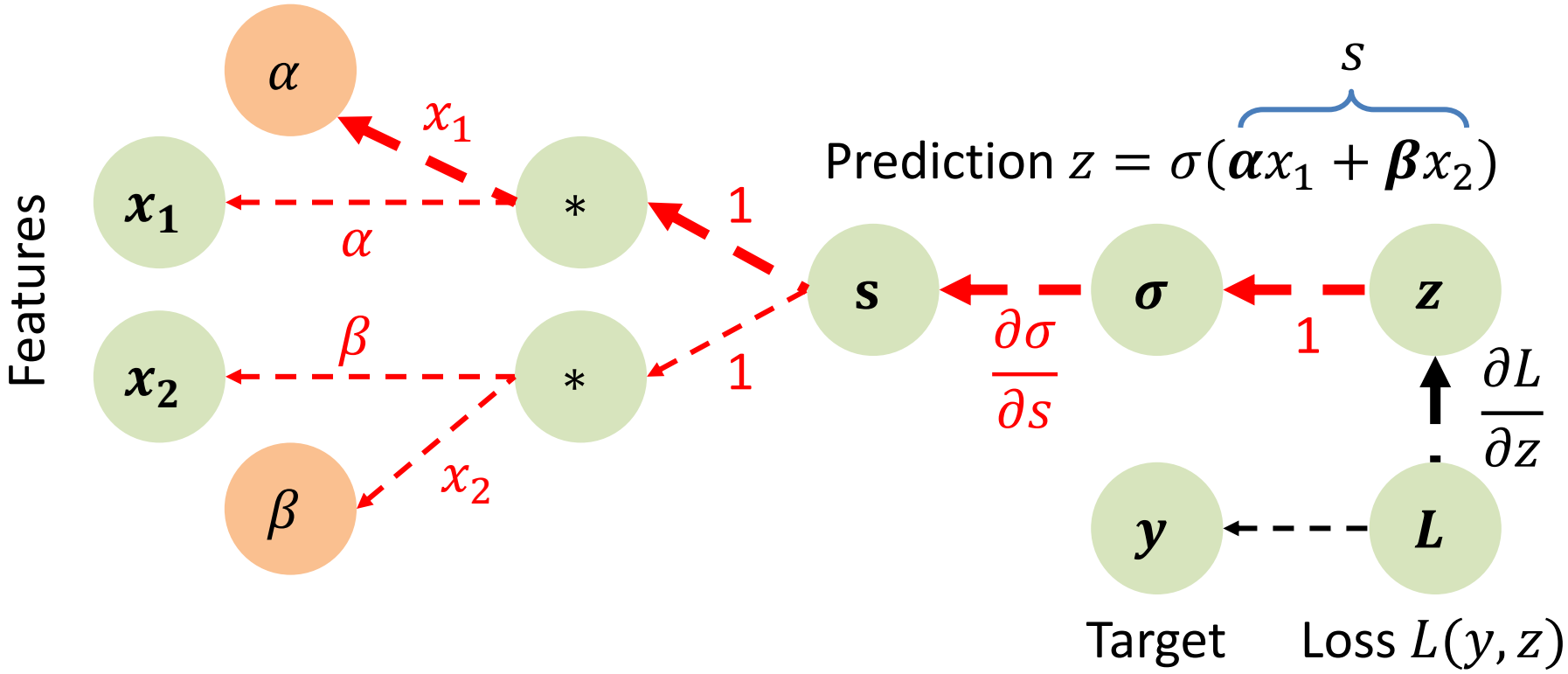
For SGD to work we need $\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}$

Derivatives computation graph



For SGD to work we need $\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}$

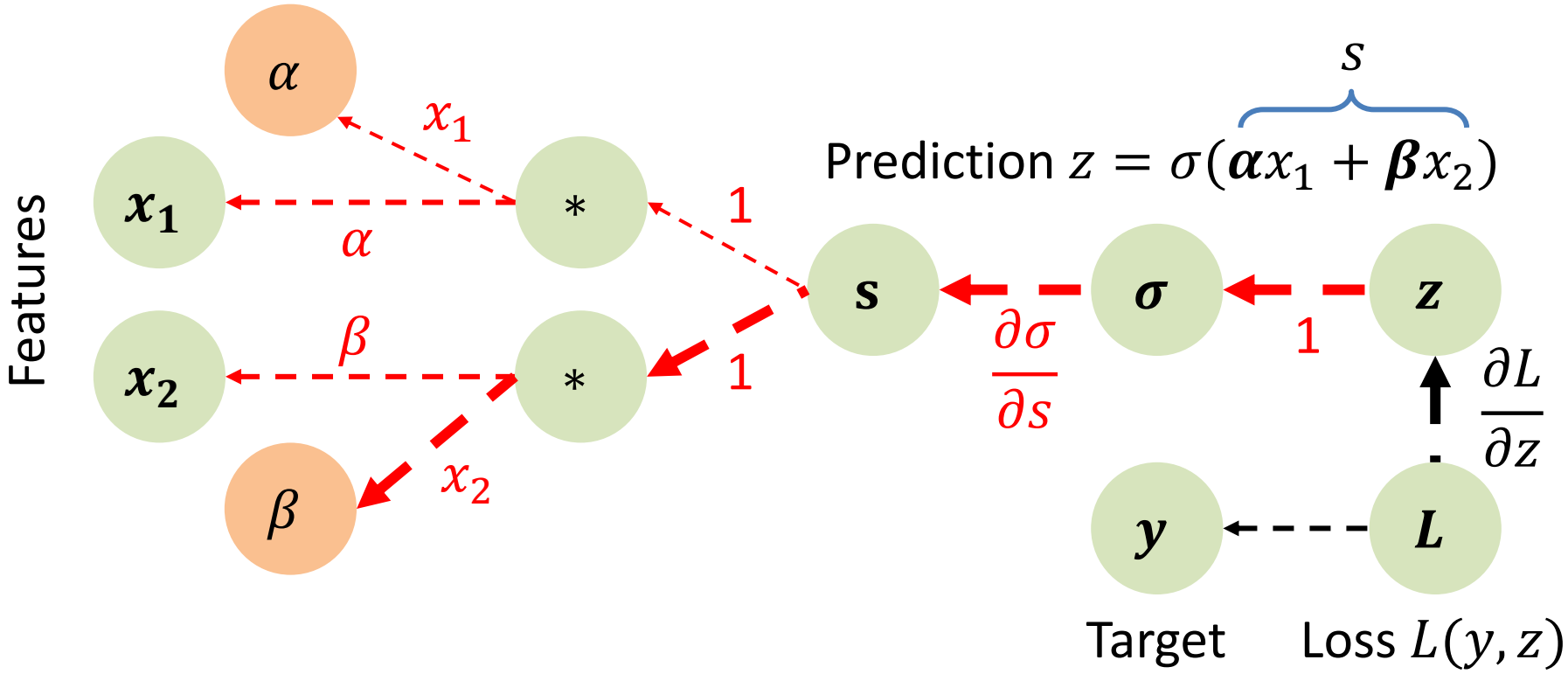
Derivatives computation graph



For SGD to work we need $\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}$

$$\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial z} \frac{\partial \sigma}{\partial s} x_1$$

Derivatives computation graph



For SGD to work we need $\frac{\partial L}{\partial \alpha}, \frac{\partial L}{\partial \beta}$

$$\frac{\partial L}{\partial \alpha} = \frac{\partial L}{\partial z}\frac{\partial \sigma}{\partial s} x_1 \qquad \frac{\partial L}{\partial \beta} = \frac{\partial L}{\partial z}\frac{\partial \sigma}{\partial s} x_2$$

Summary

- We can use chain rule to compute derivatives of composite functions

- We can use a computation graph of derivatives to compute them **automatically**

- In the next video we will find out how to do this **fast!**

Intro

- In this video we will find out how to apply a chain rule **efficiently**!

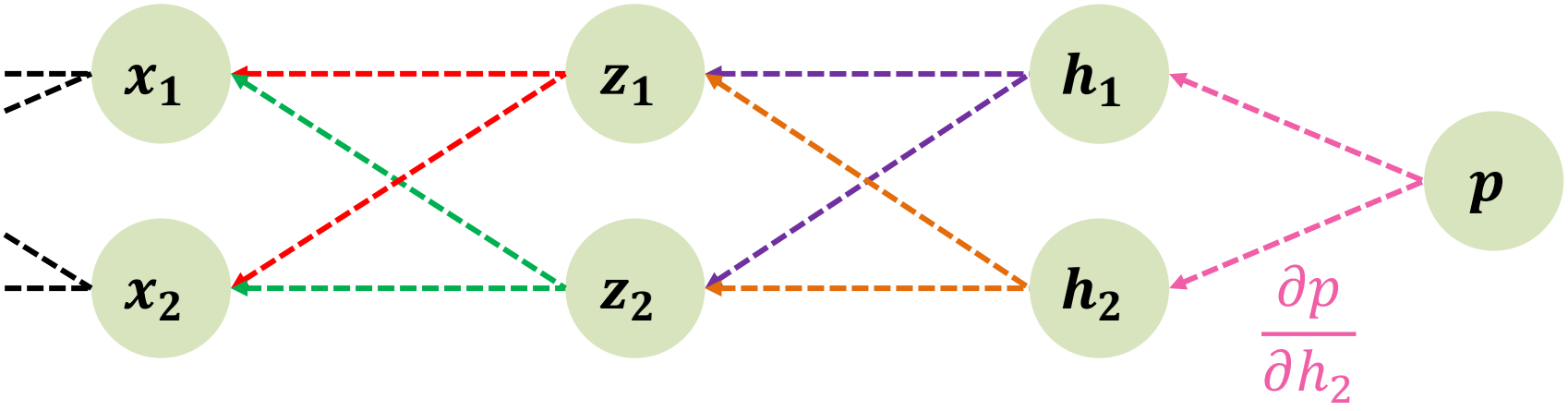
Let's look at MLP with 3 hidden layers

Let's drill down to an actual parameter of MLP:

$$h_2 = \sigma(\boldsymbol{w_0} + \boldsymbol{w_1} z_1 + \boldsymbol{w_2} z_2)$$

Gradient Descent:

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial p}\frac{\partial p}{\partial w_1} = \frac{\partial L}{\partial p}\frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial w_1}$$



We need to do this efficiently!

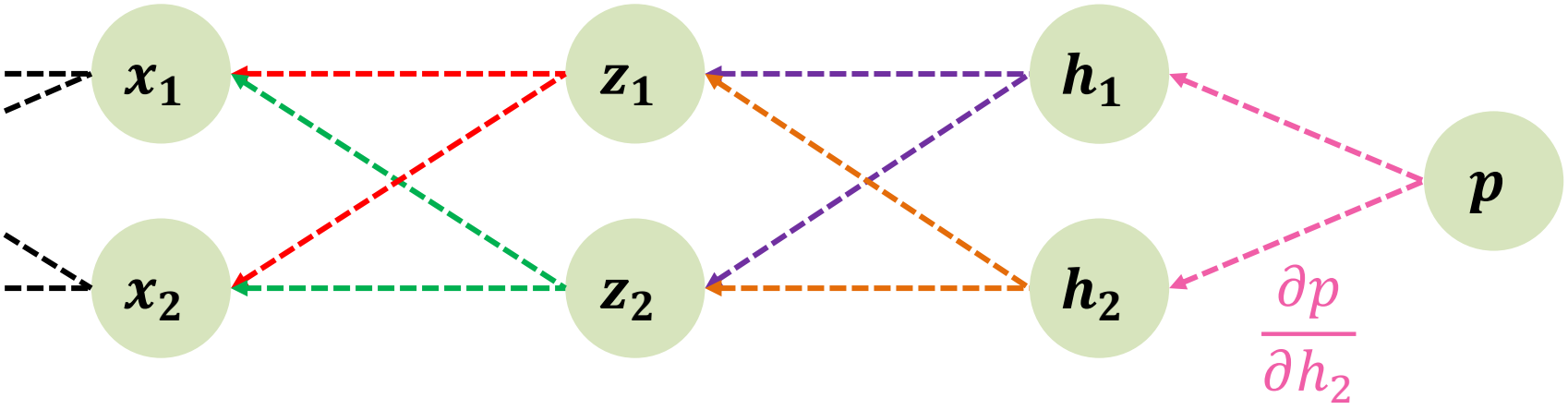
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

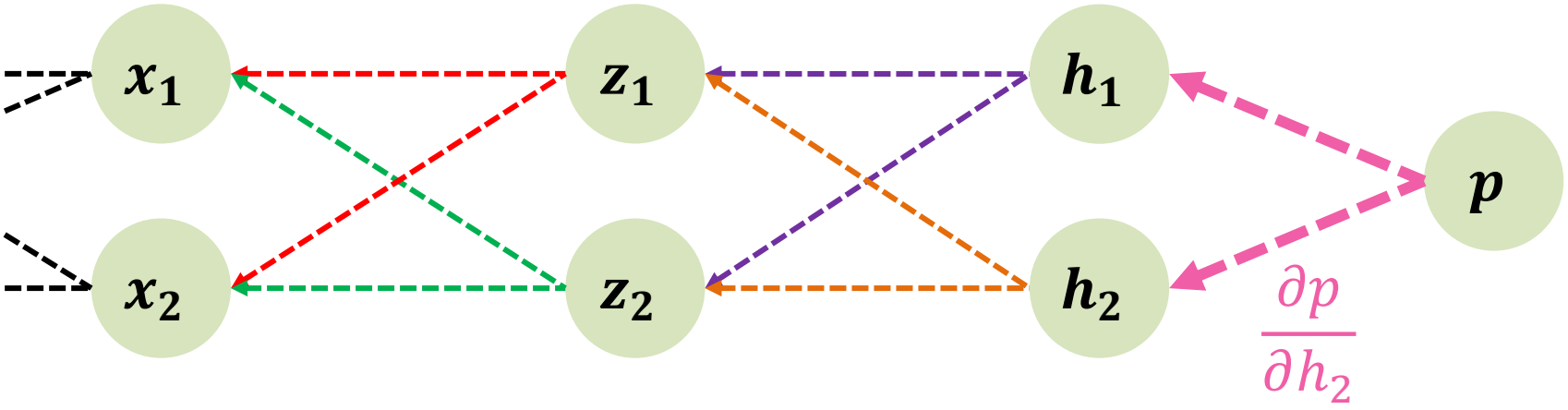
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

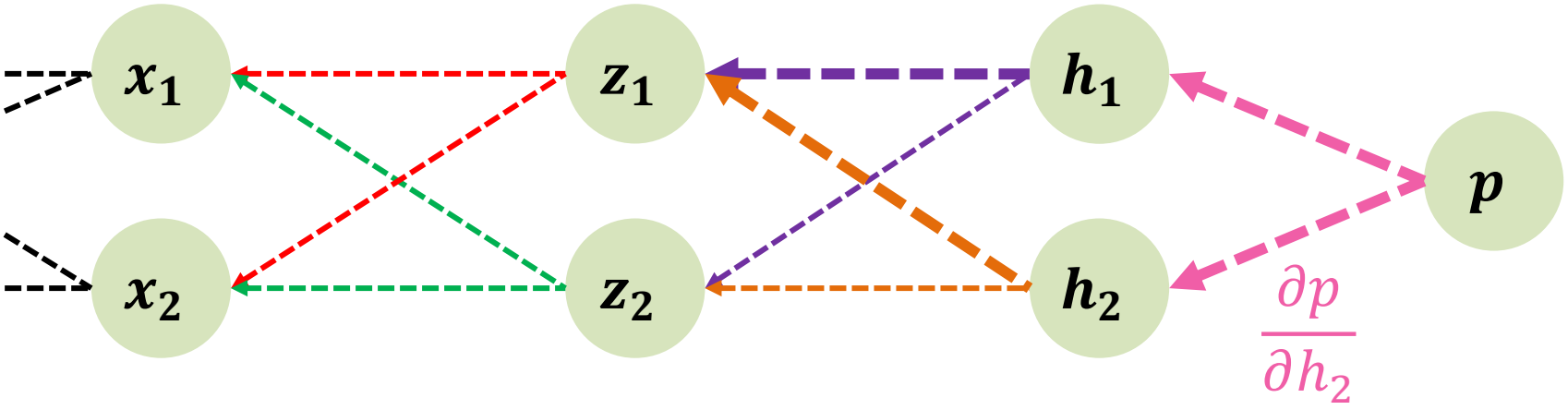
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\boxed{\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

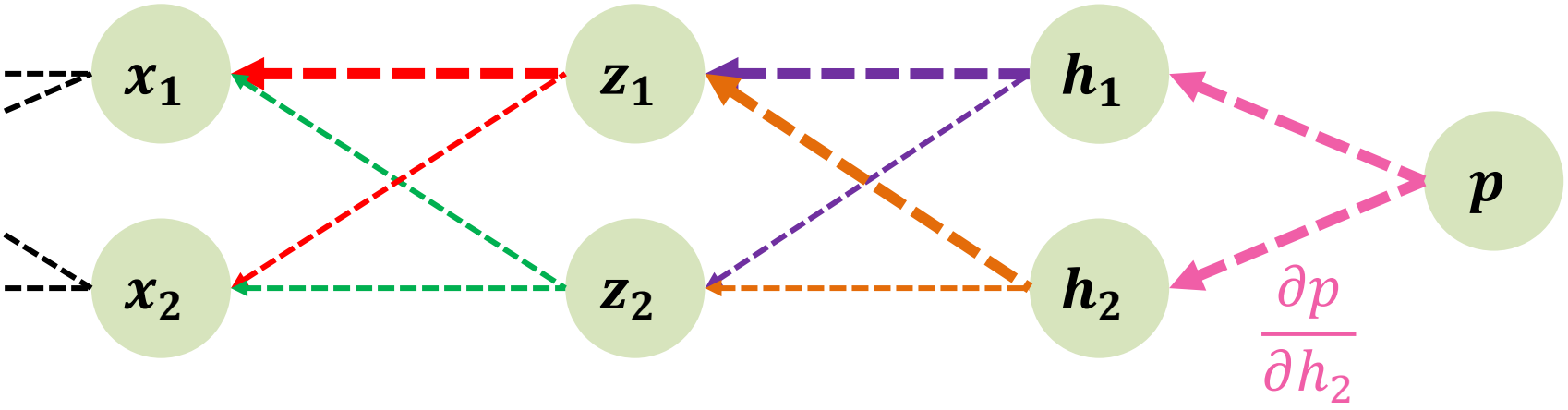
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \boxed{\frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_1}} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

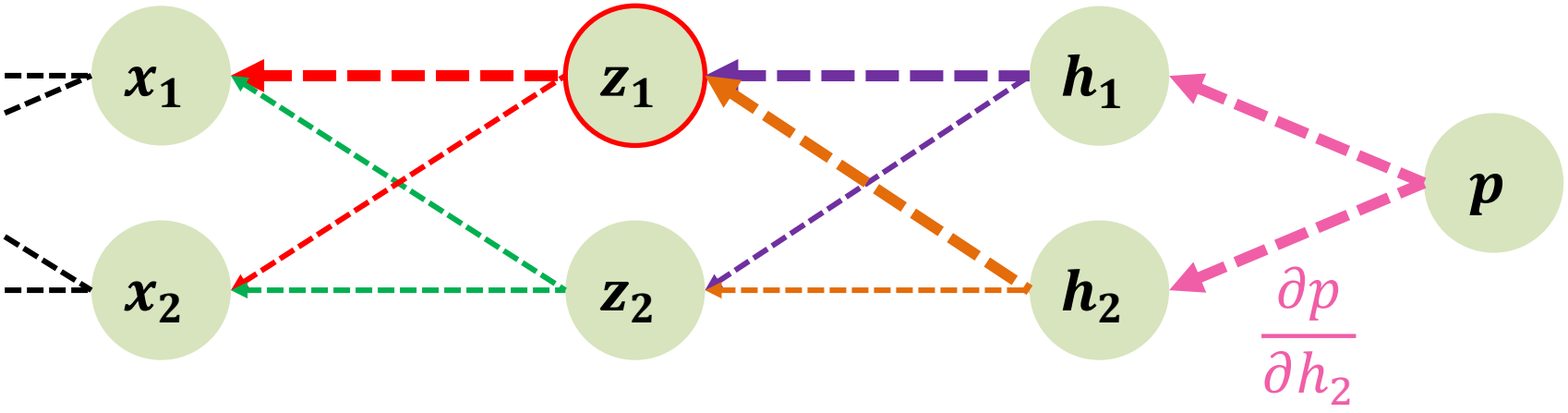
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \left(\frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\right)\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

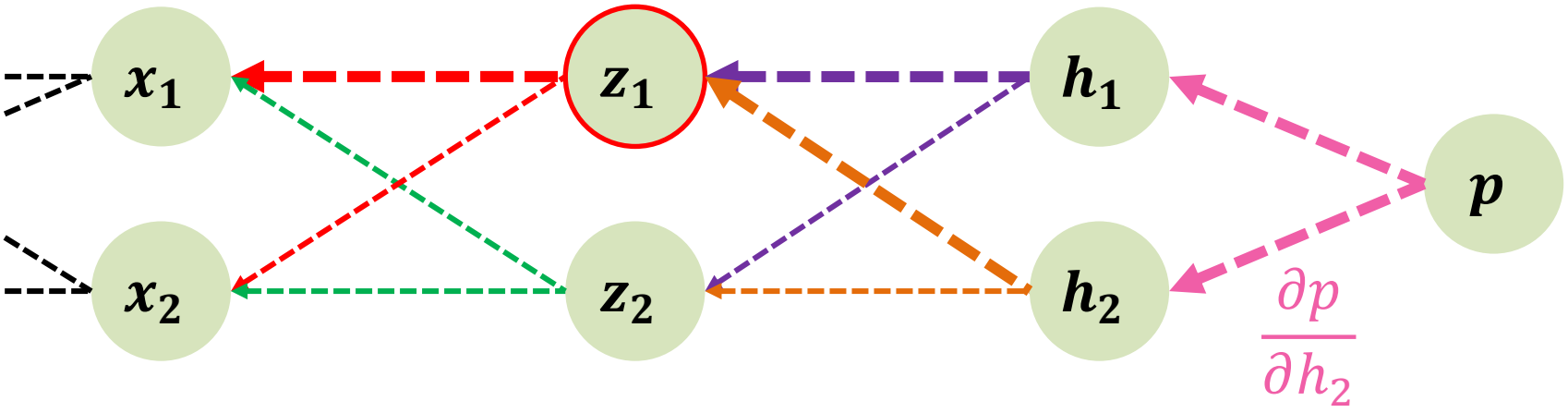
3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

2: $\frac{\partial p}{\partial z_1} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}$ $\frac{\partial p}{\partial z_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}$

1: $\frac{\partial p}{\partial x_1} = \left(\frac{\partial p}{\partial z_1}\right)\frac{\partial z_1}{\partial x_1} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_1} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_1}$

$\frac{\partial p}{\partial x_2} = \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_1}\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



We can reuse previous computations

3: $\frac{\partial p}{\partial h_1}$ $\frac{\partial p}{\partial h_2}$

We will need these for GD

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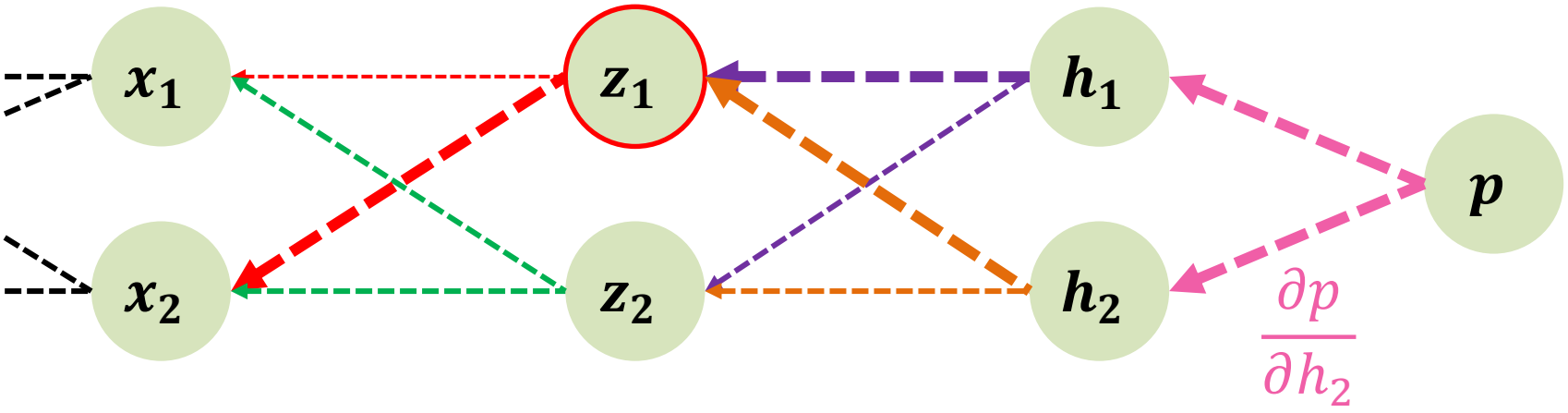
$\frac{\partial p}{\partial x_2} = \left(\frac{\partial p}{\partial z_1}\right)\frac{\partial z_1}{\partial x_2} + \frac{\partial p}{\partial h_1}\frac{\partial h_1}{\partial z_2}\frac{\partial z_2}{\partial x_2} + \frac{\partial p}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial x_2}$



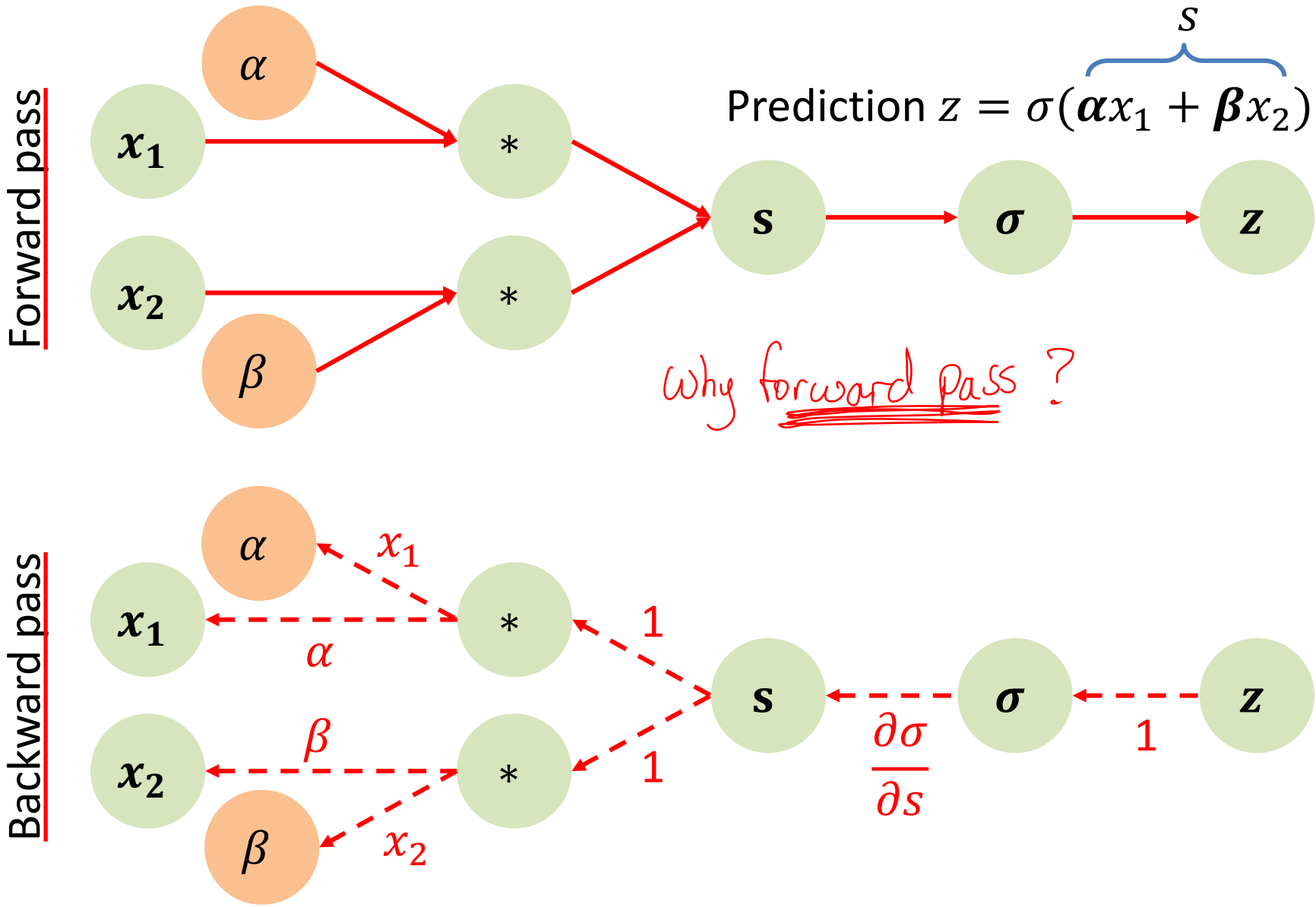
This is called reverse-mode differentiation

- In application to neural networks it has one more name: **back-propagation**.

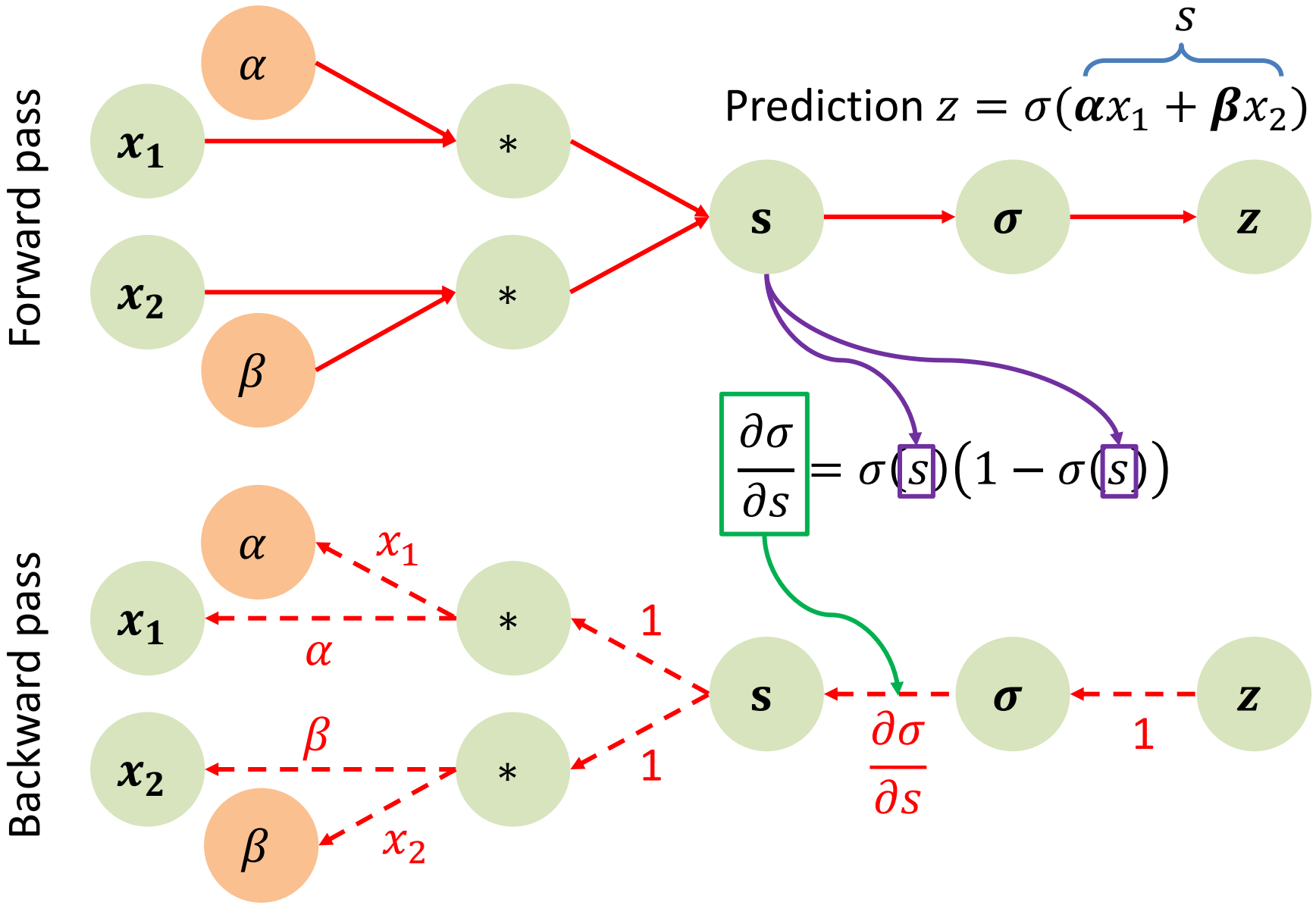
- It works **fast**, because we reuse computations from previous steps.

- In fact, for each edge we compute its value only once. And multiply by its value exactly once.

Back-propagation (Back-prop)

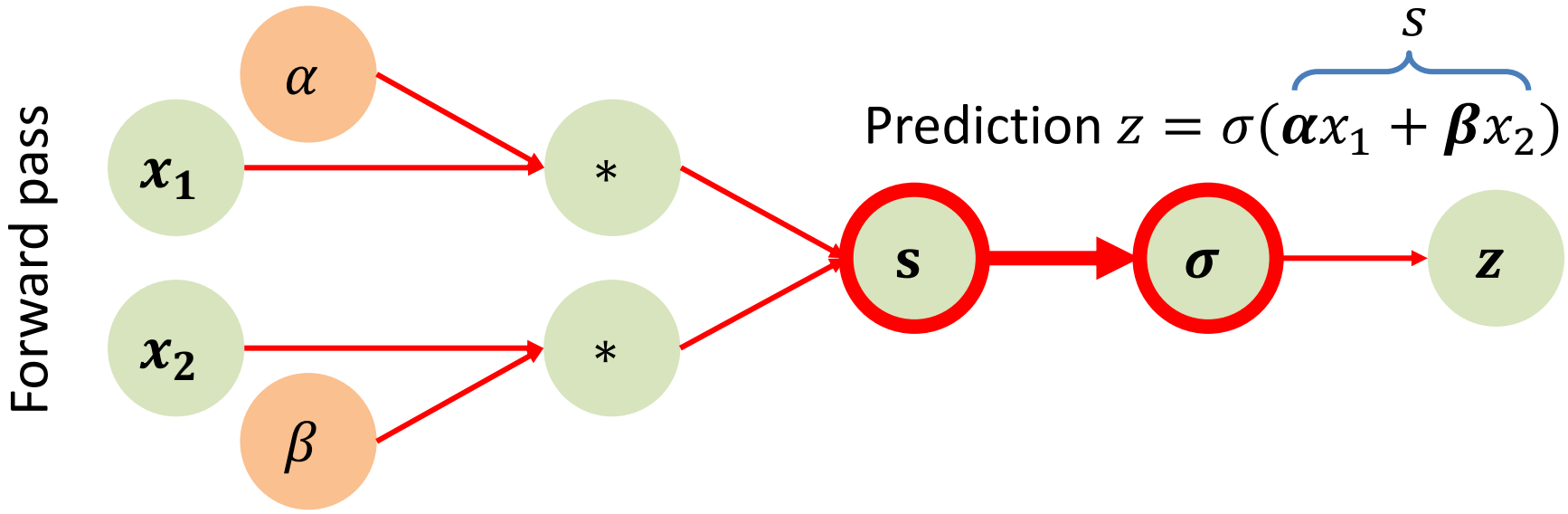


Back-propagation (Back-prop)



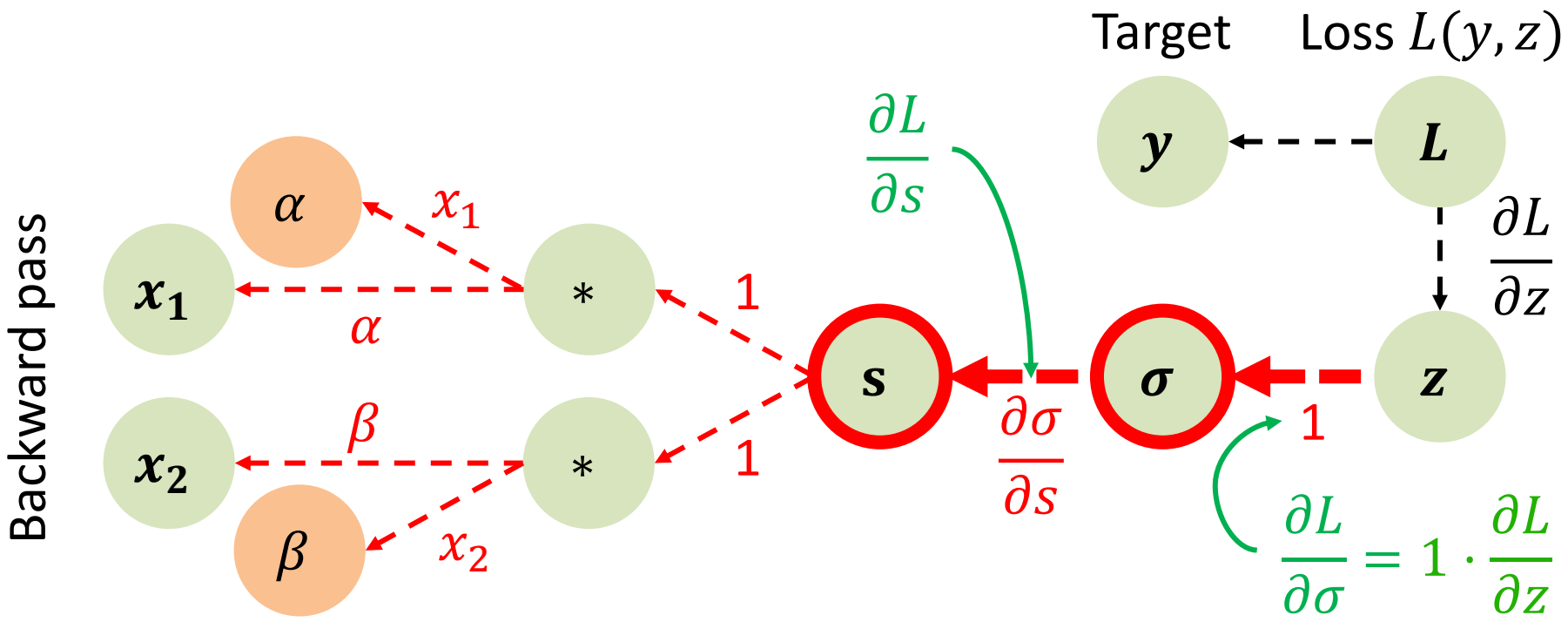
Forward pass interface

Let's implement a sigmoid activation node!

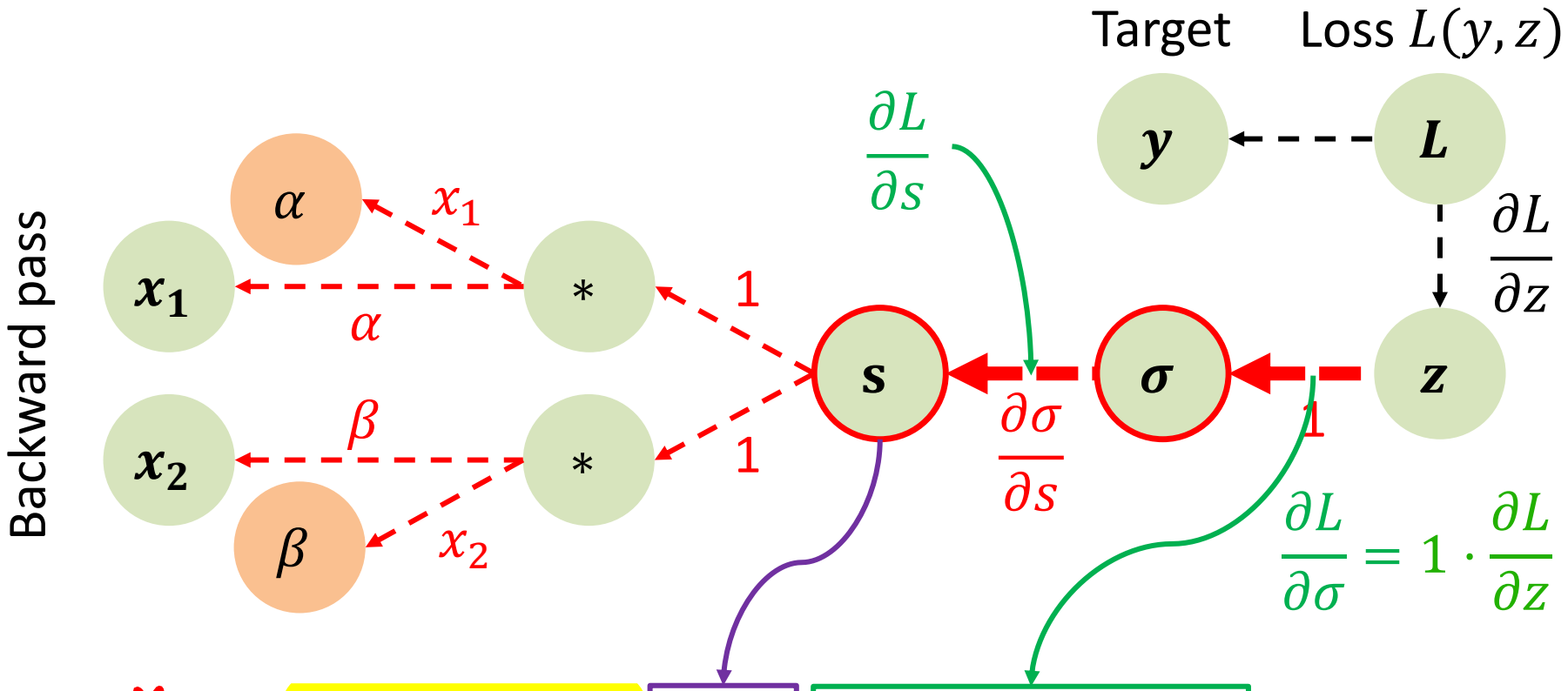



```
def forward_pass(inputs):
    return 1. / (1 + np.exp(-inputs))
```

Backward pass interface



Backward pass interface




```
def backward_pass(inputs, incoming_gradient):
    sigmoid = 1. / (1 + np.exp(-inputs))
    return sigmoid * (1 - sigmoid) * incoming_gradient
```

$$\frac{\partial L}{\partial s} = \frac{\partial \sigma}{\partial s} \cdot \frac{\partial L}{\partial \sigma}$$

Summary

- Back-propagation is a name for an **automatic** reverse-mode differentiation
- Back-propagation is **fast**
- And this is how neural nets are trained today
- In the next video we will look at MLP implementation details

Intro

- In this video we will talk about MLP implementation details

Dense layer as a matrix multiplication

- We can compute 2 neurons with linear activation, 3 inputs and no bias term as a matrix multiplication:

$$(x_1 \quad x_2 \quad x_3) \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = (z_1 \quad z_2)$$

which is equivalent to:

$$z_1 = x_1 w_{1,1} + x_2 w_{2,1} + x_3 w_{3,1}$$
$$z_2 = x_1 w_{1,2} + x_2 w_{2,2} + x_3 w_{3,2}$$

and is written as: $xW = z$

- Matrix multiplication can be done faster on both CPU (e.g. **BLAS**) and GPU (e.g. **cuBLAS**).

- Matrix multiplication with **numpy** is much faster than **Python** loops.

Backward pass for a dense layer

- Forward pass:

$$xW = z \qquad (x_1 \quad x_2 \quad x_3) \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = (z_1 \quad z_2)$$

- For backward pass we need $\frac{\partial L}{\partial W}$, where $L(z_1, z_2)$ is our scalar loss.

$$\frac{\partial L}{\partial W} = \begin{bmatrix} \frac{\partial L}{\partial w_{1,1}} & \frac{\partial L}{\partial w_{1,2}} \\ \frac{\partial L}{\partial w_{2,1}} & \frac{\partial L}{\partial w_{2,2}} \\ \frac{\partial L}{\partial w_{3,1}} & \frac{\partial L}{\partial w_{3,2}} \end{bmatrix}$$

which is convenient for SGD:

$$W_{new} = W - \gamma \frac{\partial L}{\partial W}$$

Backward pass for a dense layer

- Forward pass:

$$xW = z \qquad (x_1 \quad x_2 \quad x_3) \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = (z_1 \quad z_2)$$

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let's apply a chain rule

$$\frac{\partial L}{\partial w_{i,j}} = \sum_k \frac{\partial L}{\partial z_k} \frac{\partial z_k}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} \frac{\partial z_j}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} x_i$$

$$z_j = x_1 w_{1,j} + x_2 w_{2,j} + x_3 w_{3,j}$$

Backward pass for a dense layer

- Forward pass:

$$xW = z \qquad (x_1 \quad x_2 \quad x_3) \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = (z_1 \quad z_2)$$

- For backward pass we need $\frac{\partial L}{\partial W}$, where $L(z_1, z_2)$ is our scalar loss.

$$\frac{\partial L}{\partial W} = \begin{bmatrix} \frac{\partial L}{\partial w_{1,1}} & \frac{\partial L}{\partial w_{1,2}} \\ \frac{\partial L}{\partial w_{2,1}} & \frac{\partial L}{\partial w_{2,2}} \\ \frac{\partial L}{\partial w_{3,1}} & \frac{\partial L}{\partial w_{3,2}} \end{bmatrix}$$

$$\frac{\partial L}{\partial w_{i,j}} = \sum_k \frac{\partial L}{\partial z_k} \frac{\partial z_k}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} \frac{\partial z_j}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} x_i$$

$$\frac{\partial L}{\partial z} = \left(\frac{\partial L}{\partial z_1} \quad \frac{\partial L}{\partial z_2} \right)$$

gradient vector

$$\frac{\partial L}{\partial W} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \left(\frac{\partial L}{\partial z_1} \quad \frac{\partial L}{\partial z_2} \right) = x^T \frac{\partial L}{\partial z}$$

Forward pass for mini-batches

- Usually we do forward pass in mini-batches.

Batch of 2: $\begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix} \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{pmatrix}$

Matrix notation: $XW = Z$

1st neuron for 2nd sample: $z_{\mathbf{2},1} = x_{\mathbf{2},1}w_{1,1} + x_{\mathbf{2},2}w_{2,1} + x_{\mathbf{2},3}w_{3,1}$

2nd row of X

1st col of W

Backward pass for mini-batches

loss wrt W

- SGD with mini-batches takes a sum of losses for each sample.

Batch of 2: $\begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix} \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{pmatrix}$

SGD step: $\frac{\partial L_b}{\partial W} = \frac{\partial L(z_{\mathbf{1},1}, z_{\mathbf{1},2})}{\partial W} + \frac{\partial L(z_{\mathbf{2},1}, z_{\mathbf{2},2})}{\partial W}$

For one sample: $\frac{\partial L}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} \frac{\partial z_j}{\partial w_{i,j}} = \frac{\partial L}{\partial z_j} x_i$ *we know this*

For 2 samples: $\frac{\partial L_b}{\partial w_{i,j}} = \frac{\partial L}{\partial z_{\mathbf{1},j}} x_{\mathbf{1},i} + \frac{\partial L}{\partial z_{\mathbf{2},j}} x_{\mathbf{2},i}$

batch loss $L_b = L(z_{1,1}, z_{1,2}) + L(z_{2,1}, z_{2,2})$

Backward pass for mini-batches

- SGD with mini-batches takes a sum of losses for each sample.

Batch of 2: $$\begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix} \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{pmatrix}$$

SGD step: $$\frac{\partial L_b}{\partial W} = \frac{\partial L(z_{\mathbf{1},1}, z_{\mathbf{1},2})}{\partial W} + \frac{\partial L(z_{\mathbf{2},1}, z_{\mathbf{2},2})}{\partial W}$$

For 2 samples: $$\frac{\partial L_b}{\partial w_{i,j}} = \frac{\partial L}{\partial z_{1,j}} x_{1,i} + \frac{\partial L}{\partial z_{2,j}} x_{2,i}$$

$$\frac{\partial L_b}{\partial W} = X^T \frac{\partial L}{\partial Z} \quad \Big| \quad X^T = \begin{pmatrix} x_{1,1} & x_{2,1} \\ x_{1,2} & x_{2,2} \\ x_{1,3} & x_{2,3} \end{pmatrix} \quad \Big| \quad \frac{\partial L}{\partial Z} = \begin{pmatrix} \frac{\partial L}{\partial z_{1,1}} & \frac{\partial L}{\partial z_{1,2}} \\ \frac{\partial L}{\partial z_{2,1}} & \frac{\partial L}{\partial z_{2,2}} \end{pmatrix}$$

Backward pass for mini-batches

- SGD with mini-batches takes a sum of losses for each sample.

Batch of 2: $\begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix} \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{pmatrix}$

SGD step: $\frac{\partial L_b}{\partial W} = \frac{\partial L(z_{\mathbf{1},1}, z_{\mathbf{1},2})}{\partial W} + \frac{\partial L(z_{\mathbf{2},1}, z_{\mathbf{2},2})}{\partial W}$

Example

For 2 samples: $\frac{\partial L_b}{\partial \boldsymbol{w_{3,2}}} = \frac{\partial L}{\partial z_{1,2}} x_{1,3} + \frac{\partial L}{\partial z_{2,2}} x_{2,3}$ *let's check*

$\frac{\partial L_b}{\partial W} = X^T \frac{\partial L}{\partial Z}$ | $X^T = \begin{pmatrix} x_{1,1} & x_{2,1} \\ x_{1,2} & x_{2,2} \\ \boldsymbol{x_{1,3}} & \boldsymbol{x_{2,3}} \end{pmatrix}$ | $\frac{\partial L}{\partial Z} = \begin{pmatrix} \frac{\partial L}{\partial z_{1,1}} & \boldsymbol{\frac{\partial L}{\partial z_{1,2}}} \\ \frac{\partial L}{\partial z_{2,1}} & \boldsymbol{\frac{\partial L}{\partial z_{2,2}}} \end{pmatrix}$

Backward pass for X (used in MLP)

loss wrt X

- $\boldsymbol{X}$ could contain outputs of a previous hidden layer:

Why update inputs!!

Batch of 2: $\begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \end{pmatrix} \cdot \begin{bmatrix} w_{1,1} & w_{1,2} \\ w_{2,1} & w_{2,2} \\ w_{3,1} & w_{3,2} \end{bmatrix} = \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{2,1} & z_{2,2} \end{pmatrix}$

SGD step: $\frac{\partial L_b}{\partial X} = \frac{\partial L(z_{\mathbf{1},1}, z_{\mathbf{1},2})}{\partial X} + \frac{\partial L(z_{\mathbf{2},1}, z_{\mathbf{2},2})}{\partial X}$

For 1 sample: *let's apply a chain rule*

$$\frac{\partial L(z_{\boldsymbol{i},1}, z_{\boldsymbol{i},2})}{\partial x_{\boldsymbol{i},j}} = \sum_k \frac{\partial L}{\partial z_{i,k}} \frac{\partial z_{i,k}}{\partial x_{i,j}} = \sum_k \frac{\partial L}{\partial z_{i,k}} w_{j,k} = \sum_k \frac{\partial L}{\partial z_{i,k}} w_{k,j}^T$$

For 2 samples: $\frac{\partial L_b}{\partial X} = \frac{\partial L}{\partial Z} W^T$

contributes 1 non-zero row

Fast Python implementation

- Forward pass for a **dense layer** with **numpy**:

```
def forward_pass(X, W):
    return X.dot(W)
```

$$XW = Z$$

- Backward pass with **numpy**:

incoming grad

```
def backward_pass(X, W, dZ):
    dX = dZ.dot(W.T)
    dW = X.T.dot(dZ)
    return dX, dW
```

$$\frac{\partial L_b}{\partial X} = \frac{\partial L}{\partial Z} W^T$$

$$\frac{\partial L_b}{\partial W} = X^T \frac{\partial L}{\partial Z}$$

- One more reason to have $\frac{\partial L}{\partial Z}$ in backward pass **interface**:
  - Otherwise, we would need $\frac{\partial Z}{\partial X}$ and $\frac{\partial Z}{\partial W}$, which is scary!

Summary

- Forward pass for a **dense layer** is a matrix multiplication
- Backward pass is a matrix multiplication as well
- Efficient for mini-batches on both CPU and GPU
- Easy to code it with **numpy**
- In the next video we will take a quick look at other matrix derivatives

Intro

- In this video we will take a look at other cases of matrix derivatives

Jacobian

- Let's take a composition of two vector functions:

$$g(x): (x_1, \dots, x_n) \to (g_1, \dots, g_m)$$

$$f(g): (g_1, \dots, g_m) \to (f_1, \dots, f_k)$$ *this will be useful for RNN*

$$h(x) = f(g(x)): (x_1, \dots, x_n) \to (h_1, \dots, h_k) = (f_1, \dots, f_k)$$

- The matrix of partial derivatives $\frac{\partial h_i}{\partial x_j}$ is called the Jacobian:

$$J^h = \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_k}{\partial x_1} & \cdots & \frac{\partial h_k}{\partial x_n} \end{pmatrix}$$

Jacobian

- Let's take a composition of two vector functions:

$$g(x): (x_1, \dots, x_n) \to (g_1, \dots, g_m)$$

$$f(g): (g_1, \dots, g_m) \to (f_1, \dots, f_k)$$ *this will be useful for RNN*

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$$J^h = \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_k}{\partial x_1} & \cdots & \frac{\partial h_k}{\partial x_n} \end{pmatrix}$$

$$J^h_{i,j} = \frac{\partial h_i}{\partial x_j} = \sum_l \frac{\partial f_i}{\partial g_l}\frac{\partial g_l}{\partial x_j} = \sum_l J^f_{i,l} \cdot J^g_{l,j}$$

Chain rule: $J^h = J^f \cdot J^g$

Matrix by matrix derivative

- Matrix product:

$$C = AB = \begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} \cdot \begin{pmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{pmatrix} =$$
$$\begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$

- How to calculate the derivative $\frac{\partial C}{\partial A}$?

3D x 4D → 7D

$$\frac{\partial C}{\partial A} = \left\{ \frac{\partial c_{i,j}}{\partial a_{k,l}} \right\}_{i,j,k,l}$$

This is a **tensor** (high-dimensional array)!

Chain rule

$$C = AB = \begin{pmatrix} a_{1,1}b_{1,1} + a_{1,2}b_{2,1} & a_{1,1}b_{1,2} + a_{1,2}b_{2,2} \\ a_{2,1}b_{1,1} + a_{2,2}b_{2,1} & a_{2,1}b_{1,2} + a_{2,2}b_{2,2} \end{pmatrix}$$

$$\frac{\partial C}{\partial A} = \begin{pmatrix} \frac{\partial c_{1,1}}{\partial A} & \frac{\partial c_{1,2}}{\partial A} \\ \frac{\partial c_{2,1}}{\partial A} & \frac{\partial c_{2,2}}{\partial A} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} b_{1,1} & b_{2,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} b_{1,2} & b_{2,2} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ b_{1,1} & b_{2,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ b_{1,2} & b_{2,2} \end{bmatrix} \end{pmatrix}$$

Possible notation

Let's say we have $\frac{\partial L}{\partial C} = \begin{bmatrix} \alpha & \beta \\ \gamma & \eta \end{bmatrix}$

$$\frac{\partial L}{\partial A} = \sum_{i,j} \frac{\partial L}{\partial c_{i,j}} \frac{\partial c_{i,j}}{\partial A} = \alpha \frac{\partial c_{1,1}}{\partial A} + \beta \frac{\partial c_{1,2}}{\partial A} + \gamma \frac{\partial c_{2,1}}{\partial A} + \eta \frac{\partial c_{2,2}}{\partial A} =$$

$$\begin{bmatrix} \alpha b_{1,1} + \beta b_{1,2} & \alpha b_{2,1} + \beta b_{2,2} \\ \gamma b_{1,1} + \eta b_{1,2} & \gamma b_{2,1} + \eta b_{2,2} \end{bmatrix}$$

What's bad about it

- Our chain rule is a **linear combination** of matrices:

$$\frac{\partial L}{\partial A} = \sum_{i,j} \frac{\partial L}{\partial c_{i,j}} \frac{\partial c_{i,j}}{\partial A} = \alpha \frac{\partial c_{1,1}}{\partial A} + \beta \frac{\partial c_{1,2}}{\partial A} + \gamma \frac{\partial c_{2,1}}{\partial A} + \eta \frac{\partial c_{2,2}}{\partial A} =$$

$$\begin{bmatrix} \alpha b_{1,1} + \beta b_{1,2} & \alpha b_{2,1} + \beta b_{2,2} \\ \gamma b_{1,1} + \eta b_{1,2} & \gamma b_{2,1} + \eta b_{2,2} \end{bmatrix}$$

Problems:

- **Crunching a lot of zeros:**

$$\frac{\partial C}{\partial A} = \begin{pmatrix} \frac{\partial c_{1,1}}{\partial A} & \frac{\partial c_{1,2}}{\partial A} \\ \frac{\partial c_{2,1}}{\partial A} & \frac{\partial c_{2,2}}{\partial A} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} b_{1,1} & b_{2,1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} b_{1,2} & b_{2,2} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ b_{1,1} & b_{2,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ b_{1,2} & b_{2,2} \end{bmatrix} \end{pmatrix}$$

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- Our chain rule is a **linear combination** of matrices:

$$\frac{\partial L}{\partial A} = \sum_{i,j} \frac{\partial L}{\partial c_{i,j}} \frac{\partial c_{i,j}}{\partial A} = \alpha \frac{\partial c_{1,1}}{\partial A} + \beta \frac{\partial c_{1,2}}{\partial A} + \gamma \frac{\partial c_{2,1}}{\partial A} + \eta \frac{\partial c_{2,2}}{\partial A} =$$

$$\begin{bmatrix} \alpha b_{1,1} + \beta b_{1,2} & \alpha b_{2,1} + \beta b_{2,2} \\ \gamma b_{1,1} + \eta b_{1,2} & \gamma b_{2,1} + \eta b_{2,2} \end{bmatrix}$$

- **Performance issues**
  - **e.g. no such operation in BLAS**

What's bad about it

- Our chain rule is a **linear combination** of matrices:

$$\frac{\partial L}{\partial A} = \sum_{i,j} \frac{\partial L}{\partial c_{i,j}} \frac{\partial c_{i,j}}{\partial A} = \alpha \frac{\partial c_{1,1}}{\partial A} + \beta \frac{\partial c_{1,2}}{\partial A} + \gamma \frac{\partial c_{2,1}}{\partial A} + \eta \frac{\partial c_{2,2}}{\partial A} =$$

$$\begin{bmatrix} \alpha b_{1,1} + \beta b_{1,2} & \alpha b_{2,1} + \beta b_{2,2} \\ \gamma b_{1,1} + \eta b_{1,2} & \gamma b_{2,1} + \eta b_{2,2} \end{bmatrix}$$

We know how to compute $\frac{\partial L}{\partial A}$ skipping $\frac{\partial C}{\partial A}$ tensor computation:

$$\frac{\partial L}{\partial C} = \begin{bmatrix} \alpha & \beta \\ \gamma & \eta \end{bmatrix} \qquad B^T = \begin{pmatrix} b_{1,1} & b_{2,1} \\ b_{1,2} & b_{2,2} \end{pmatrix}$$

From MLP lecture: $\frac{\partial L}{\partial A} = \frac{\partial L}{\partial C} B^T = \begin{bmatrix} \alpha & \beta \\ \gamma & \eta \end{bmatrix} \begin{pmatrix} b_{1,1} & b_{2,1} \\ b_{1,2} & b_{2,2} \end{pmatrix}$

We're still fine

- Thankfully, in deep learning frameworks we need to calculate gradients of a **scalar loss** with respect to another **scalar, vector, matrix or tensor**.

- This's how tf.gradients() in TensorFlow works.

- Deep learning frameworks have **optimized** versions of backward pass for standard layers.

Summary

- For vector functions chain rule says that you need to multiply respective **Jacobians**.

- Matrix by matrix derivative is a **tensor**

- Chain rule for such tensors is **not very useful** in practice

- Thankfully, in deep learning frameworks we usually need to track gradients of a **scalar loss** with respect to all other parameters.

Intro

- In this video we will overview basic principles of TensorFlow

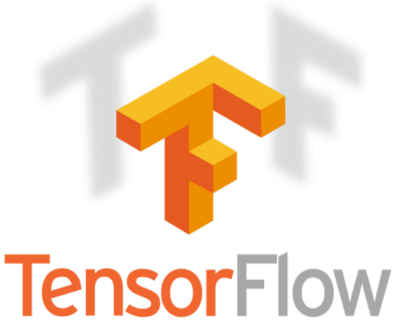
TensorFlow DL framework

- We will use it in Jupyter Notebook with Python 3 kernel

```
import numpy as np
import tensorflow as tf
```

- We will overview Python API for TensorFlow 1.2+

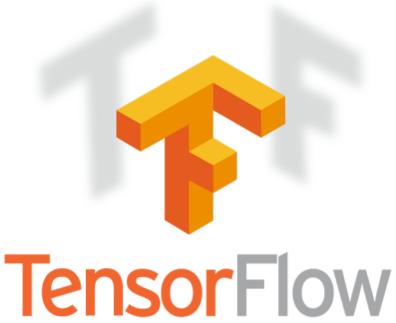
- APIs in other languages exist: Java, C++, Go
  - Python API is at present the most complete and the easiest to use
  - https://www.tensorflow.org/api_docs/



What is TensorFlow?

1. A tool to describe computational graphs
   - The foundation of computation in TensorFlow is the **Graph** object. This holds a network of nodes, each representing one **operation**, connected to each other as inputs and outputs.

2. A runtime for execution of these graphs
   - On CPU, GPU, TPU, …
   - On one node or in distributed mode



Why this name

- **Input** to any operation will be a collection of **tensors** (multi-dimensional arrays)

- **Output** will be a collection of tensors as well.

- We will have a graph of operations, each of which transforms tensors into another tensors, so it’s a kind of a flow of tensors

Why this name

- **Input** to any operation will be a collection of **tensors** (multi-dimensional arrays)

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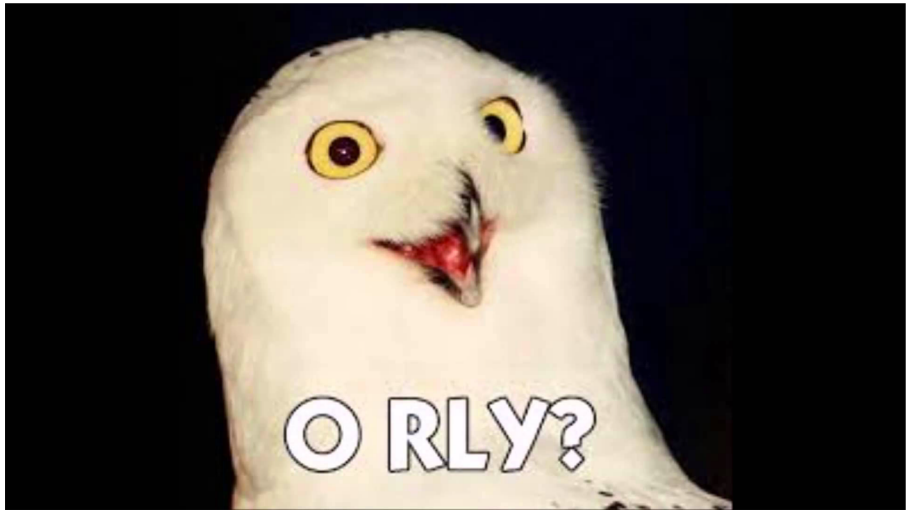


How the input looks like

- **Placeholder**
  - This is placeholder for a tensor, which will be fed during graph execution (e.g. input features)
  - x = tf.placeholder(tf.float32, (None, 10))
- **Variable**
  - This is a tensor with some value that is updated during execution (e.g. weights matrix in MLP)
  - w = tf.get_variable("w", shape=(10, 20), dtype=tf.float32)
  - w = tf.Variable(tf.random_uniform((10, 20)), name="w")
- **Constant**
  - This is a tensor with constant value, that cannot be changed
  - c = tf.constant(np.ones((4, 4)))

Operation example

- Matrix product:

```
x = tf.placeholder(tf.float32, (None, 10))
w = tf.Variable(tf.random_uniform((10, 20)), name="w")
z = x @ w
# z = tf.matmul(x, w)
print(z)
```

- Output:

```
Tensor("matmul:0", shape=(?, 20), dtype=float32)
```

- We don’t do any computations here, we just **define** the graph!

Computational graph

- TensorFlow creates a **default graph** after importing
  - All the operations will go there by default
  - You can get it with tf.get_default_graph(), which returns an instance of tf.Graph.

- You can **create your own** graph variable and define operations there:

```
g = tf.Graph()
with g.as_default():
    pass
```

- You can **clear** the default graph like this:

```
tf.reset_default_graph()
```

Jupyter Notebook cells

- If you run this cell **3 times**:

```
x = tf.placeholder(tf.float32, (None, 10))
```

- This is what you get in your **default graph**:
  - Using tf.get_default_graph().get_operations()

```
[<tf.Operation 'Placeholder' type=Placeholder>,
 <tf.Operation 'Placeholder_1' type=Placeholder>,
 <tf.Operation 'Placeholder_2' type=Placeholder>]
```

- **Your graph is cluttered**!
  - Clear your graph with tf.reset_default_graph()

Operations and tensors

- Every **node** in our graph is an **operation**:

```
x = tf.placeholder(tf.float32, (None, 10), name="x")
```

- Listing nodes with tf.get_default_graph().get_operations():

```
[<tf.Operation 'x' type=Placeholder>]
```

- How to get **output tensors** of operation:
  - tf.get_default_graph().get_operations()[0].outputs
  - Output: [<tf.Tensor **'x:0'** shape=(?, 10) dtype=float32>]

Running a graph

- A **tf.Session** object encapsulates the environment in which tf.Operation objects are executed, and tf.Tensor objects are evaluated.
- Create a session:

```
s = tf.InteractiveSession()
```

- Defining a graph:

```
a = tf.constant(5.0)
b = tf.constant(6.0)
c = a * b
```

- Running a graph:

```
print(c)  # here just looking at the type
print(s.run(c))  # that's how you run the graph
```

- Output:

```
Tensor("mul:0", shape=(), dtype=float32)
30.0
```

Running a graph

- Operations are written in C++ and executed on CPU or GPU.

- tf.Session owns necessary resources to execute your graph, such as tf.Variable, that occupy RAM.

- It is important to release these resources when they are no longer required with tf.Session.close()

Initialization of variables

- A variable has an initial value:
  - Tensor: tf.Variable(tf.random_uniform((10, 20)), name="w")
  - Initializer: tf.get_variable("w", shape=(10, 20), dtype=tf.float32)

- You need to run some code to **compute that initial value** in graph execution environment

- This is done with a call in your session s:

```
s.run(tf.global_variables_initializer())
```

- Without it you will get “Attempting to use uninitialized value” errors

Example

- Definition:

```
tf.reset_default_graph()
a = tf.constant(np.ones((2, 2), dtype=np.float32))
b = tf.Variable(tf.ones((2, 2)))
c = a @ b
```

- Running attempt:

```
s = tf.InteractiveSession()
s.run(c)
```

Output: **“Attempting to use uninitialized value”** error

- Running properly:

```
s.run(tf.global_variables_initializer())
s.run(c)
```

Output: array([[2.,2.],[2.,2.]], dtype=float32)

Feeding placeholder values

- Definition:

```
tf.reset_default_graph()
a = tf.placeholder(np.float32, (2, 2))
b = tf.Variable(tf.ones((2, 2)))
c = a @ b
```

- Running attempt:

```
s = tf.InteractiveSession()
s.run(tf.global_variables_initializer())
s.run(c)
```

Output: "**You must feed a value for placeholder tensor**" error

- Running properly:

```
s.run(tf.global_variables_initializer())
s.run(c, feed_dict={a: np.ones((2, 2))})
```

Output: array([[2.,2.],[2.,2.]], dtype=float32)

Summary

- TensorFlow: defining and running computational graphs
- Nodes of a graph are operations, that convert a collection of tensors into another collection of tensors
- In Python API you **define** the graph, you **don't execute** it along the way
  - In 1.5+ the latter mode is supported: eager execution
- You create a **session** to execute your graph (fast C++ code on CPU or GPU)
- Session **owns** all the resources (tensors eat RAM)

Intro

- In this video we will train our first model in TensorFlow

Optimizers in TensorFlow

- Let's define **f** as a square of variable **x**:

```
import numpy as np
import tensorflow as tf

tf.reset_default_graph()
x = tf.get_variable("x", shape=(), dtype=tf.float32)
f = x ** 2
```

- Let's say we want to *minimize* the value of **f** w.r.t **x**:

```
optimizer = tf.train.GradientDescentOptimizer(0.1)
step = optimizer.minimize(f, var_list=[x])
```

Trainable variables

- You don't have to specify all the optimized variables:

```
step = optimizer.minimize(f, var_list=[x])
step = optimizer.minimize(f)
```

- Because all variables are **trainable** by default**:**

```
x = tf.get_variable("x", shape=(), dtype=tf.float32)
x = tf.get_variable("x", shape=(), dtype=tf.float32, trainable=True)
```

- You can get all of them:

```
tf.trainable_variables()
```

- Output:

```
[<tf.Variable 'x:0' shape=() dtype=float32_ref>]
```

Making gradient descent steps

- Now we need to create a **session** and **initialize** variables:

```
s = tf.InteractiveSession()
s.run(tf.global_variables_initializer())
```

- We are ready to make 10 **gradient descent** steps:

```
for i in range(10):
    _, curr_x, curr_f = s.run([step, x, f])
    print(curr_x, curr_f)
```

- Output:

```
0.448929 0.314901
0.359143 0.201537
…
0.0753177 0.00886368
0.0602542 0.00567276
```

GD step is already applied to x

Logging with tf.Print

- We can **evaluate** tensors and print them like this:

```
for i in range(10):
    _, curr_x, curr_f = s.run([step, x, f])
    print(curr_x, curr_f)
```

- Or we can pass our tensor of interest through **tf.Print**:

```
...
f = x ** 2
f = tf.Print(f, [x, f], "x, f:")
...
for i in range(10):
    s.run([step, f])
```

```
x, f:[1.5879565][2.521606]
x, f:[1.2703652][1.6138278]
...
x, f:[0.26641488][0.070976891]
x, f:[0.2131319][0.04542521]
```

This is Jupyter Server stdout.
Not visible in the Notebook!

Logging with TensorBoard

- We can add so-called **summaries**:

```
tf.summary.scalar('curr_x', x)
tf.summary.scalar('curr_f', f)
summaries = tf.summary.merge_all()
```

- This is how we log these summaries:

Run number

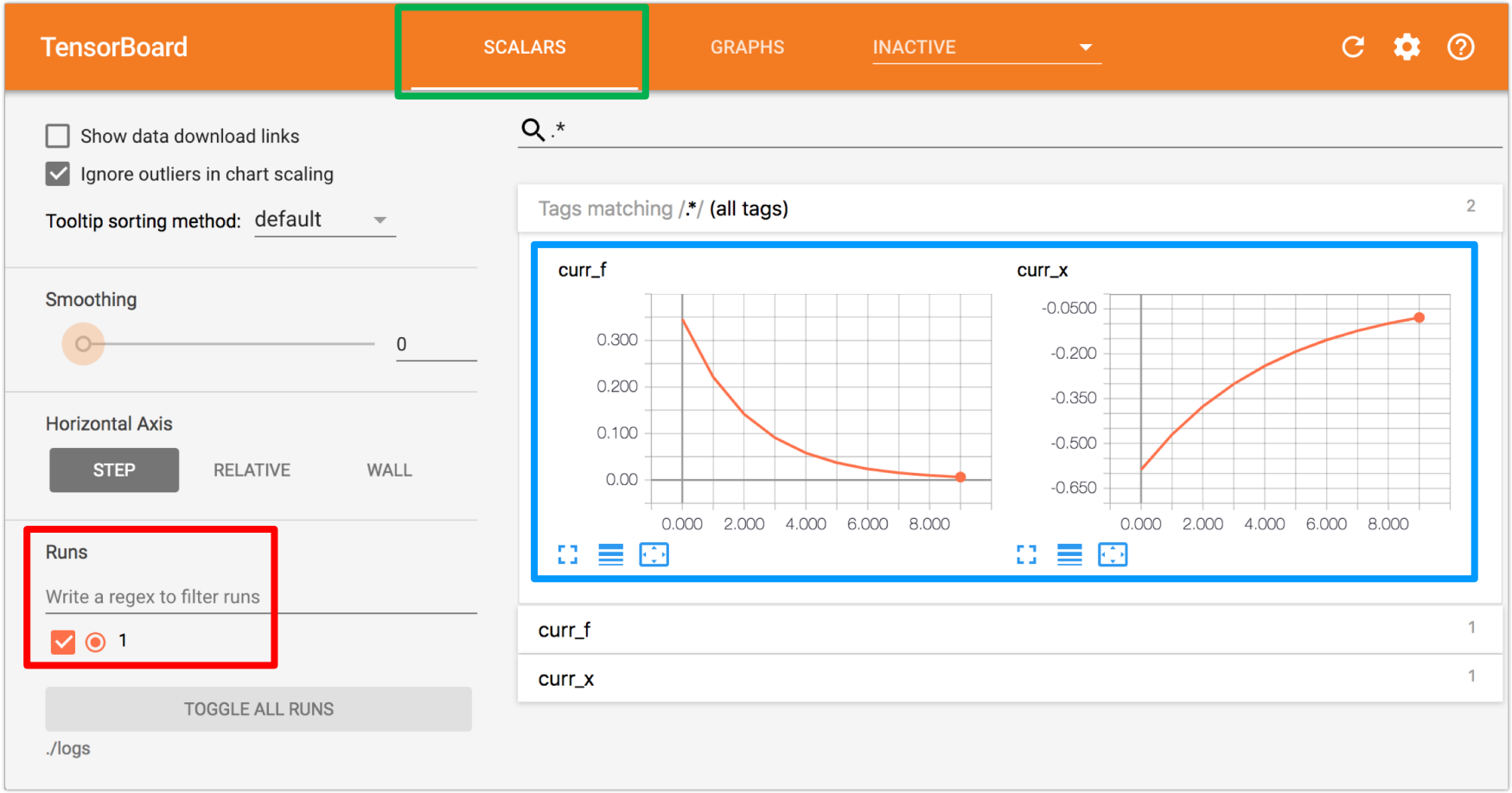
```
s = tf.InteractiveSession()
summary_writer = tf.summary.FileWriter("logs/1", s.graph)
s.run(tf.global_variables_initializer())
for i in range(10):
    _, curr_summaries = s.run([step, summaries])
    summary_writer.add_summary(curr_summaries, i)
    summary_writer.flush()
```

Launching TensorBoard

- Now you can launch TensorBoard via bash:

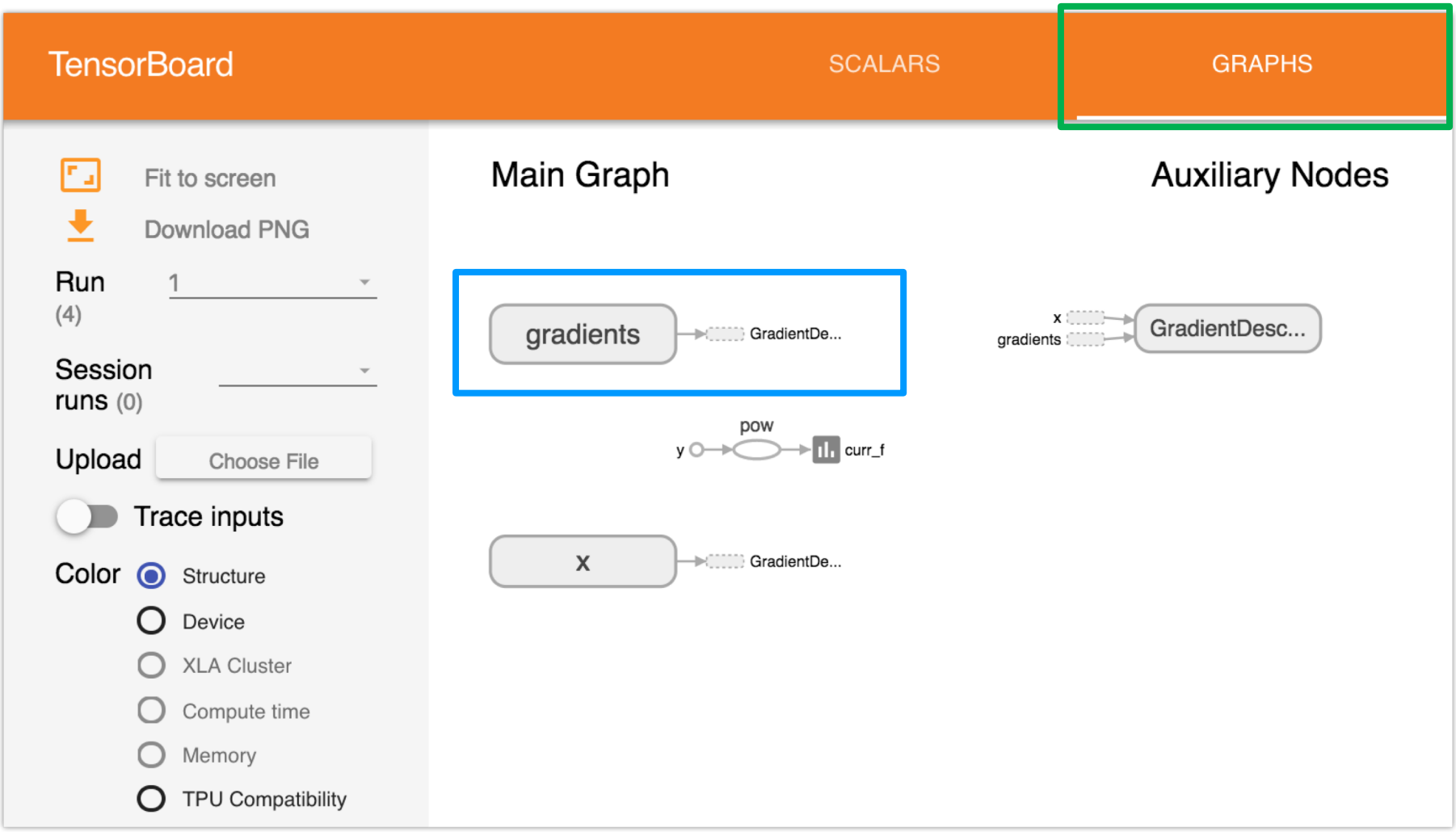
  tensorboard --logdir=./logs

- And open **http://127.0.0.1:6006** in your browser.



Visualizing graph in TensorBoard

- You can see that **gradients computation** is a part of our graph:



Solving a linear regression

- Let's generate a model dataset:

```
N = 1000
D = 3
x = np.random.random((N, D))
w = np.random.random((D, 1))
y = x @ w + np.random.randn(N, 1) * 0.20
```

Solving a linear regression

- We will need **placeholders** for input data:

```
tf.reset_default_graph()
features = tf.placeholder(tf.float32, shape=(None, D))
target = tf.placeholder(tf.float32, shape=(None, 1))
```

- This is how we make **predictions**:

```
weights = tf.get_variable("w", shape=(D, 1), dtype=tf.float32)
predictions = features @ weights
```

- And define our **loss**:

```
loss = tf.reduce_mean((target - predictions) ** 2)
```

- And **optimizer**:

```
optimizer = tf.train.GradientDescentOptimizer(0.1)
step = optimizer.minimize(loss)
```

Solving a linear regression

- Gradient descent:

```
s = tf.InteractiveSession()
s.run(tf.global_variables_initializer())
for i in range(300):
    _, curr_loss, curr_weights = s.run(
        [step, loss, weights], feed_dict={features: x, target: y})
    if i % 50 == 0:
        print(curr_loss)
```

Filling placeholders

- Ground truth weights:

[0.11649134,0.82753164,0.46924019]

- Found weights:

[0.13715988,0.79555332,0.47024861]

Model checkpoints

- We can save variables' state with **tf.train.Saver**:

```
s = tf.InteractiveSession()
saver = tf.train.Saver(tf.trainable_variables())
s.run(tf.global_variables_initializer())
for i in range(300):
    _, curr_loss, curr_weights = s.run(
        [step, loss, weights], feed_dict={features: x, target: y})
    if i % 50 == 0:
        saver.save(s, "logs/2/model.ckpt", global_step=i)
        print(curr_loss)
```

Model checkpoints

- We can **list** last checkpoints:

```
saver.last_checkpoints
```

```
['logs/2/model.ckpt-50', 'logs/2/model.ckpt-100',
'logs/2/model.ckpt-150', 'logs/2/model.ckpt-200',
'logs/2/model.ckpt-250']
```

- We can **restore** a previous checkpoint like this:

```
saver.restore(s, "logs/2/model.ckpt-50")
```

- Only variables' values are restored, which means that you need to define a graph in *the same* way **before restoring** a checkpoint.

Summary

- TensorFlow has **built-in optimizers** that do back-propagation automatically.

- TensorBoard provides **tools for visualizing** your training progress.

- TensorFlow allows you to **checkpoint** your graph to restore its state later *(you need to define it in exactly the same way though)*

Deep Learning@Coursera

week2 outro

Deep Learning is...

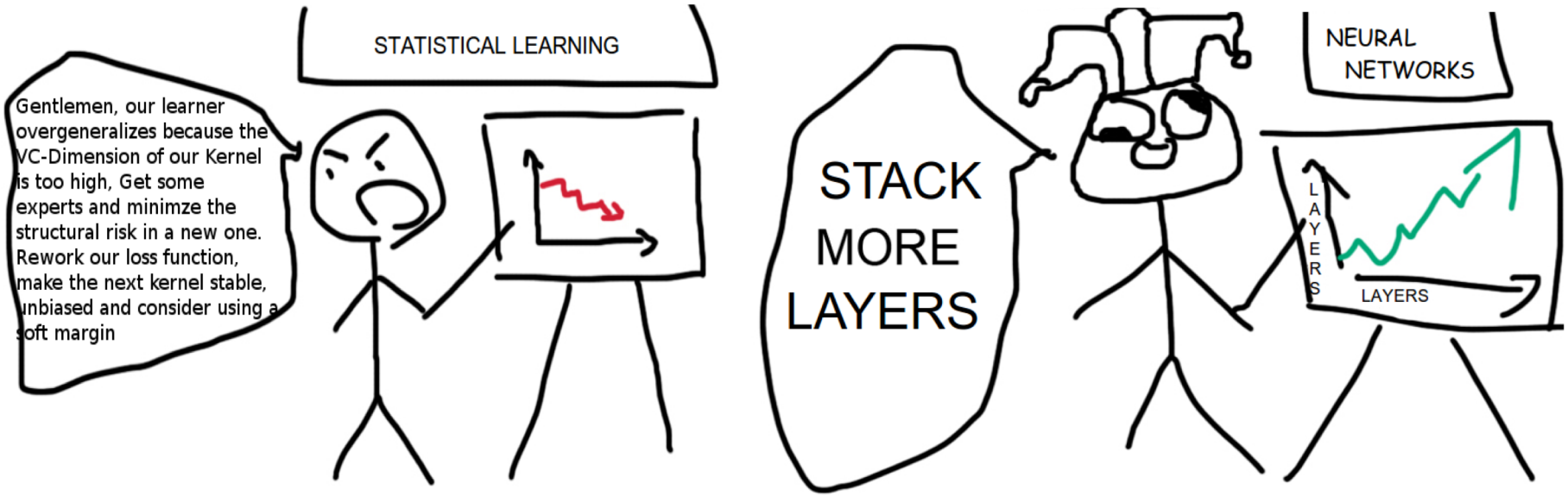
Yandex
Data Factory

LAMBDA

British Hedgehog
Preservation Society

Not magic!

Don't expect deep learning to solve all your problems for free.
For it won't.



https://i.warosu.org/data/sci/img/0073/62/1435656449422.png

Not magic

Book of grudges

- No core theory
  - Relies on intuitive reasoning

Not magic

Book of grudges

- No core theory
  - Relies on intuitive reasoning
- Needs tons of data
  - You need either large dataset or heavy wizardry

Not magic

Book of grudges

- No core theory
  - Relies on intuitive reasoning
- Needs tons of data
  - You need either large dataset or heavy wizardry
- Computationally heavy
  - Running on mobiles/embedded is a challenge

Not magic

Book of grudges

- No core theory
  - Relies on intuitive reasoning
- Needs tons of data
  - You need either large dataset or heavy wizardry
- Computationally heavy
  - Running on mobiles/embedded is a challenge
- Pathologically overhyped
  - People expect of it to make wonders

Deep learning is a language

Deep learning is a language

in which you can hint your model
on what you want it to learn

Deep learning is a language

Example

Say, you train classifier on two sets of features

Raw features

High-level features

Target

Deep learning is a language

Say, you train classifier on two sets of features

Raw features

Car photo (image pixels)

High-level features

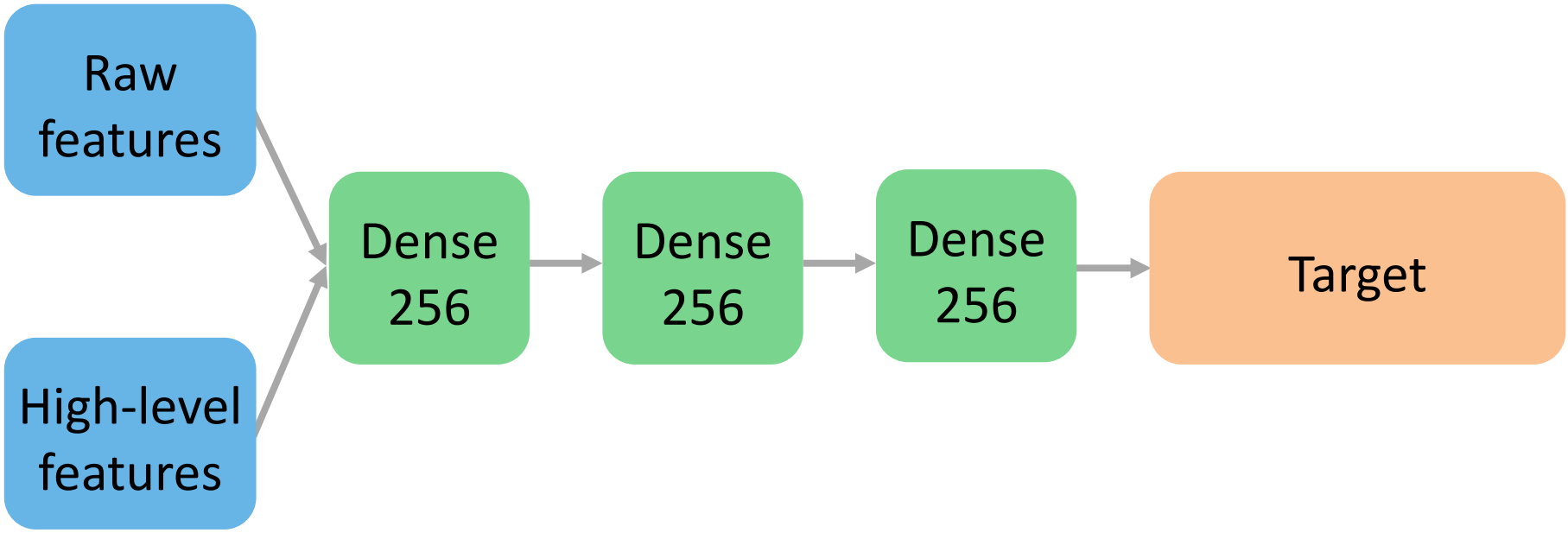
Car brand, model, age, blemishes

Car price

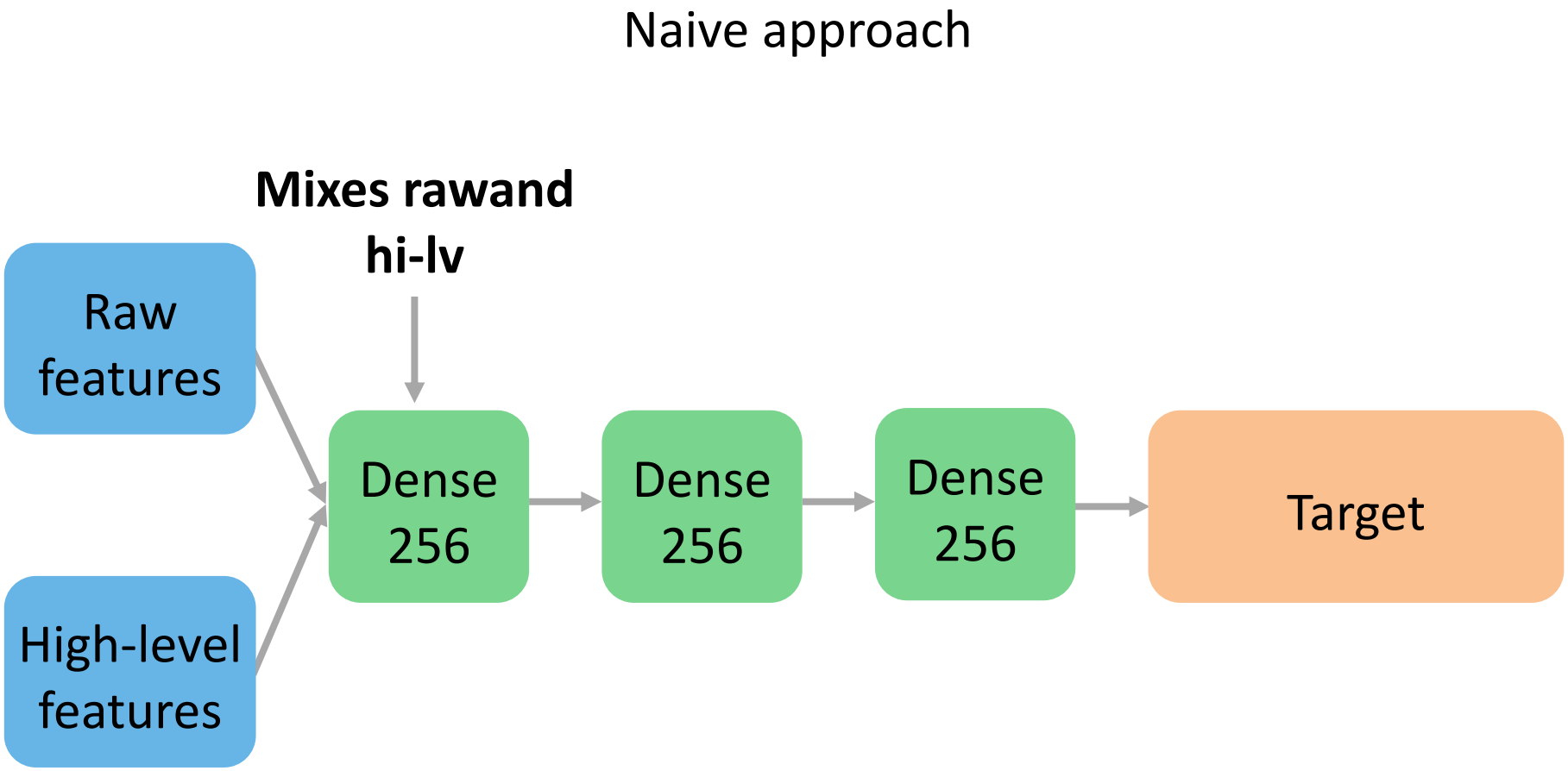
Target

Deep learning is a language

Naive approach

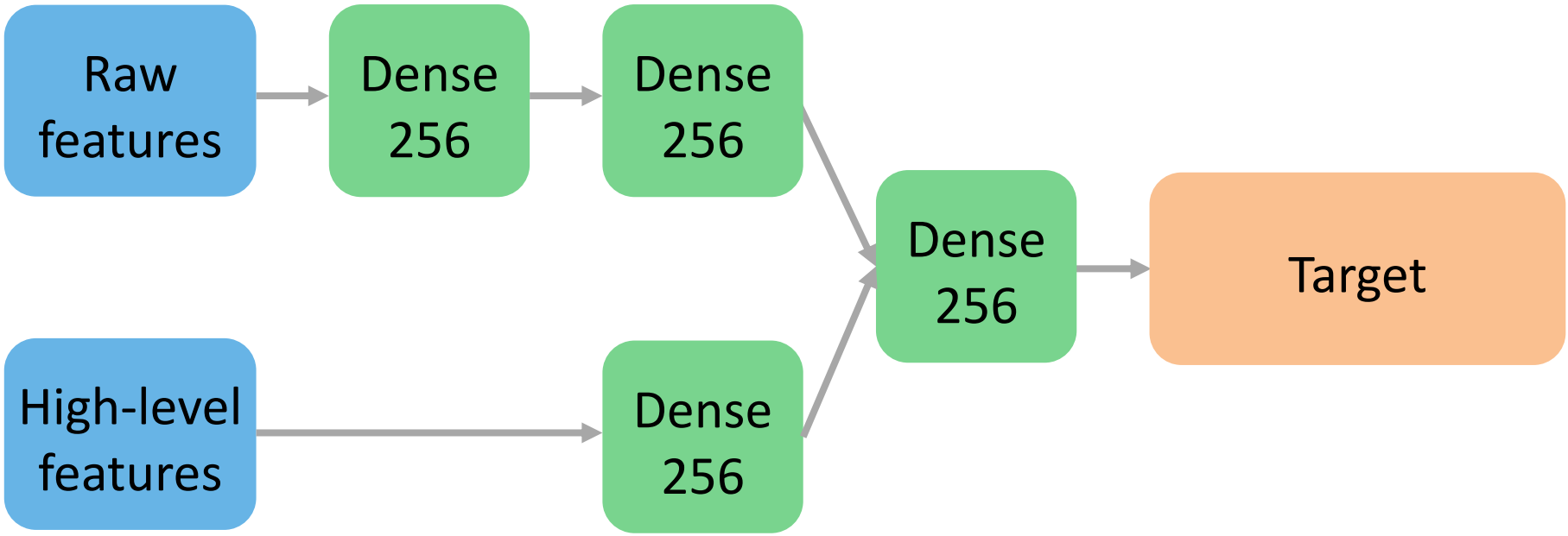


Deep learning is a language

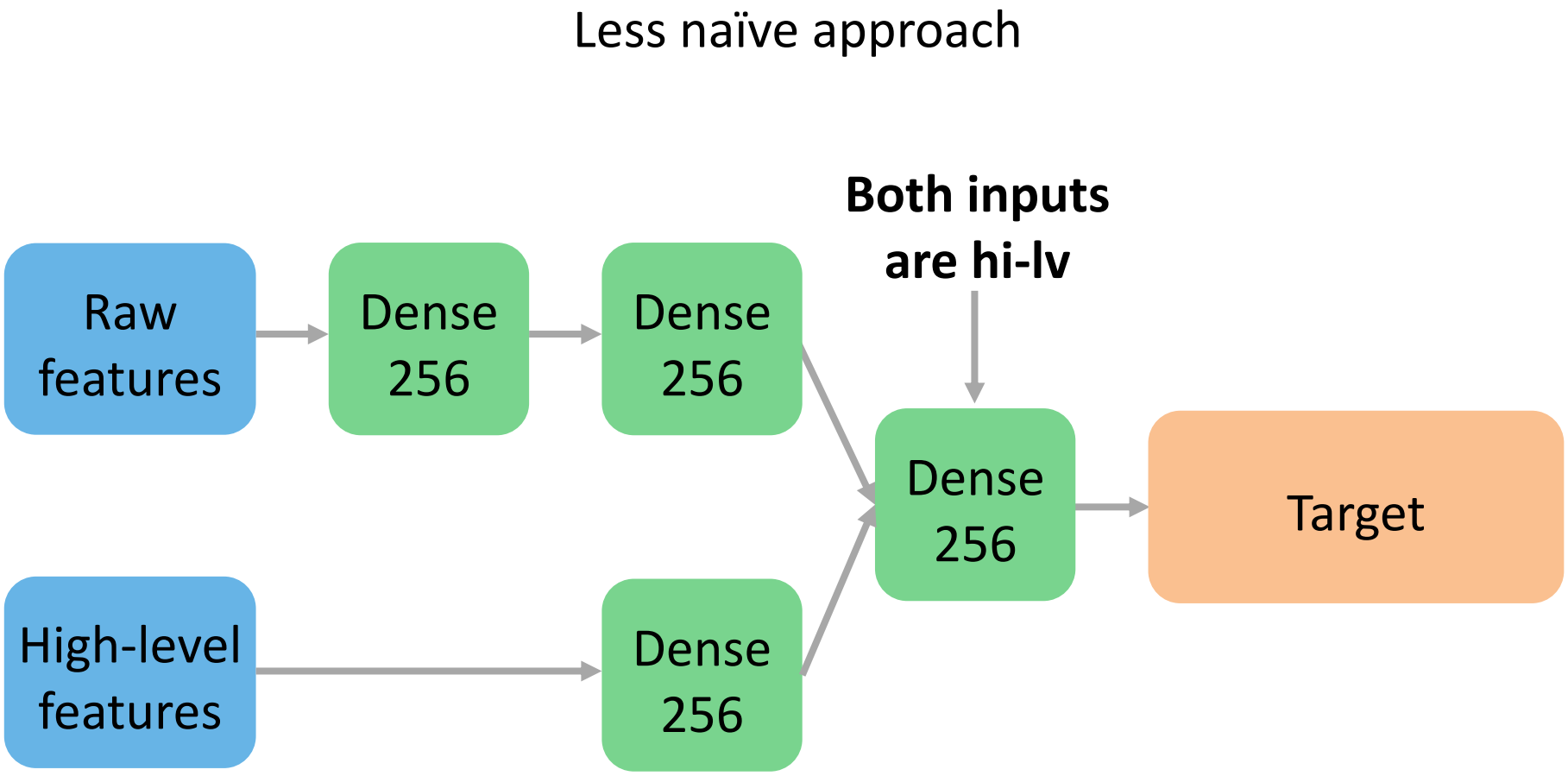


Deep learning is a language

Less naïve approach

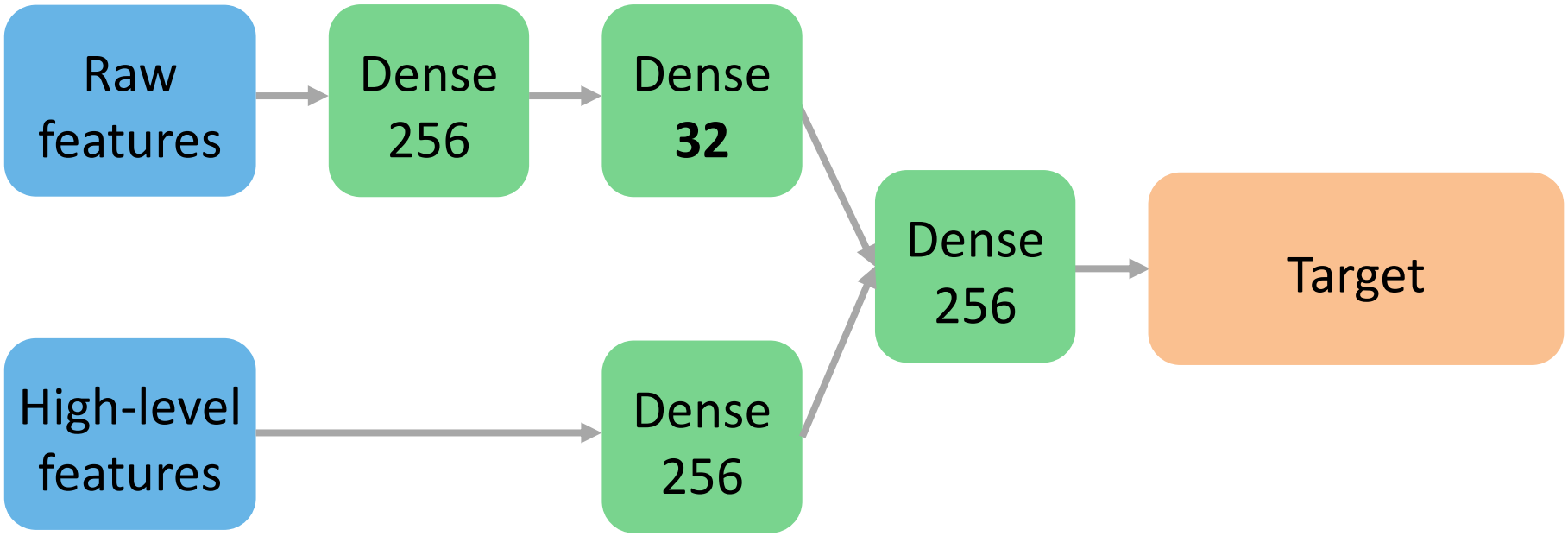


Deep learning is a language



Deep learning is a language

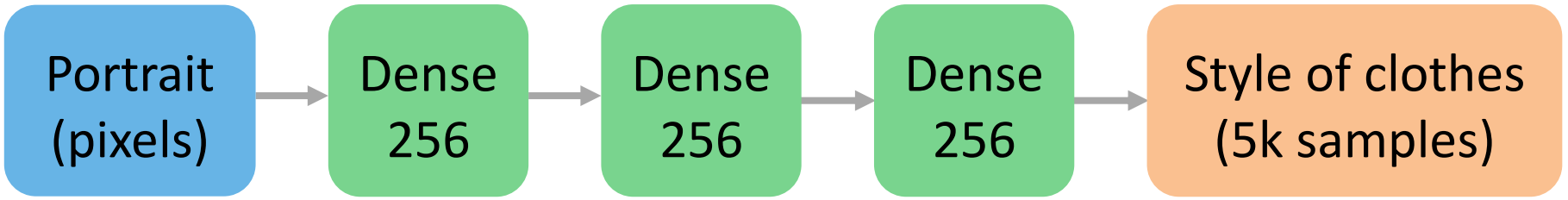
"Image features should be less important"
if that's what you want to say



Deep learning is a language

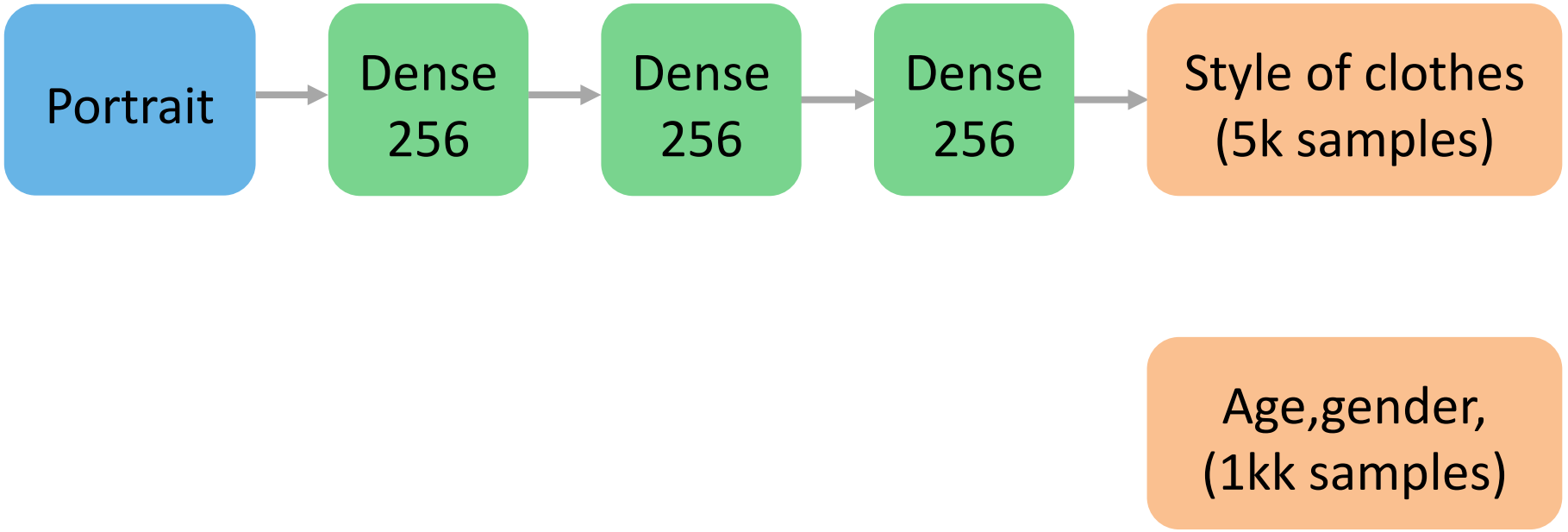
Example

You have a small dataset



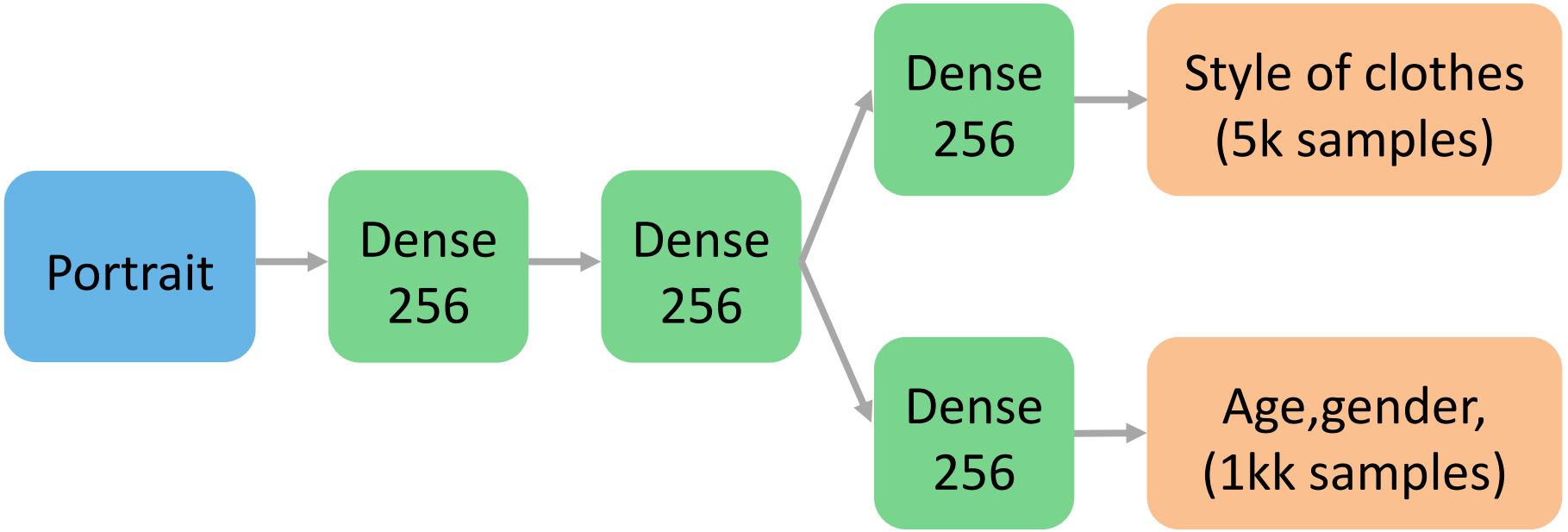
Deep learning is a language

You have a small dataset
and a larger dataset with similar task



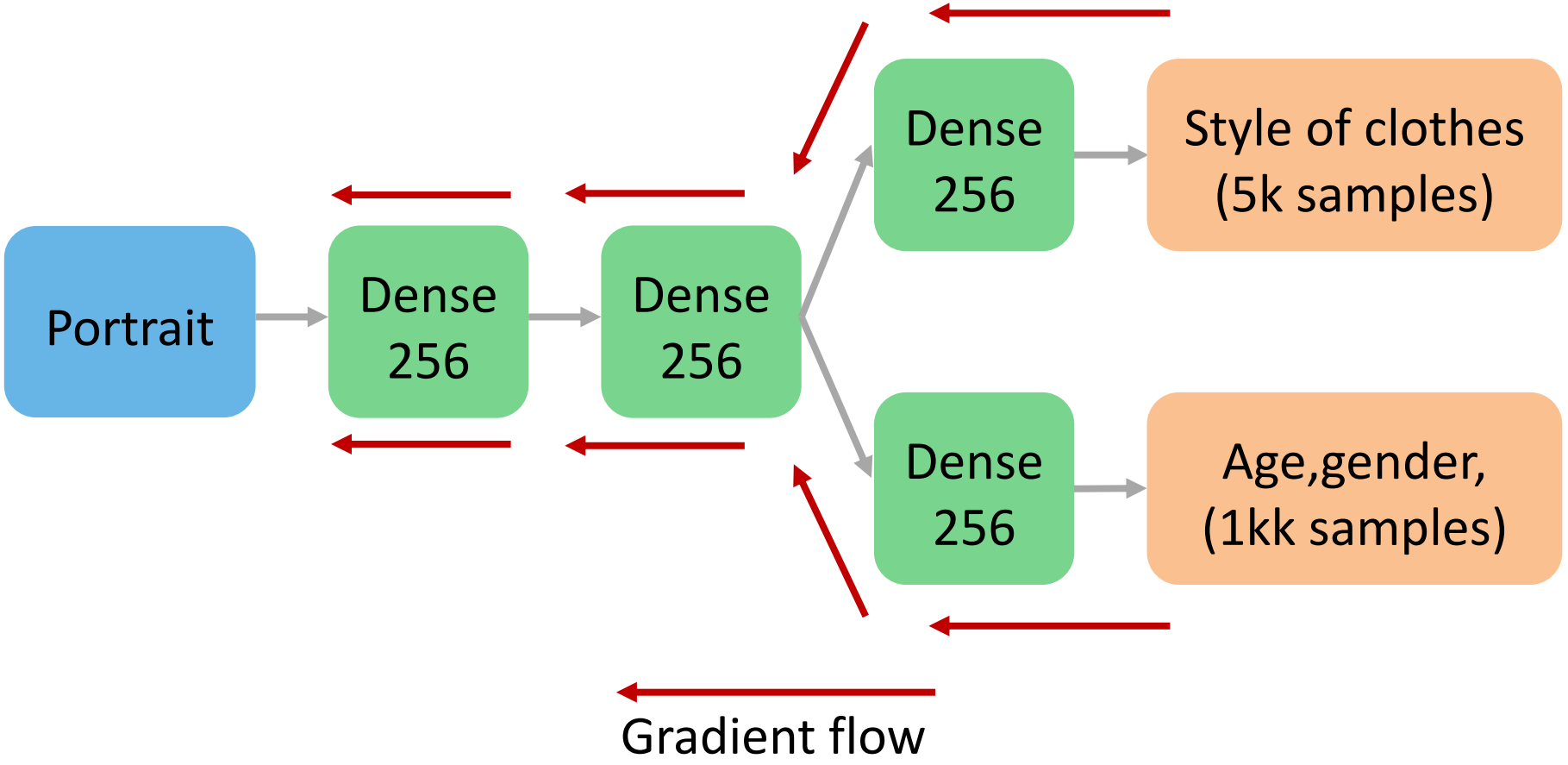
Deep learning is a language

You have a small dataset
and a larger dataset with similar task



Deep learning is a language

I want to learn features for style classification
that also help determine age & gender



Deep learning is a language

For images:

- “I want to classify cats regardless where they are”
- “I don't want model to be indifferent to small shifts”

For texts:

- “Model should reconstruct the underlying process”

In general:

- “I don't want model to trust single feature too much”
- “I want my features to be sparse”