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COLLEGE OF SOFTWARE BEIHANG UNIVERSITY 软件学院

Chapter 6
Parallel Program Development

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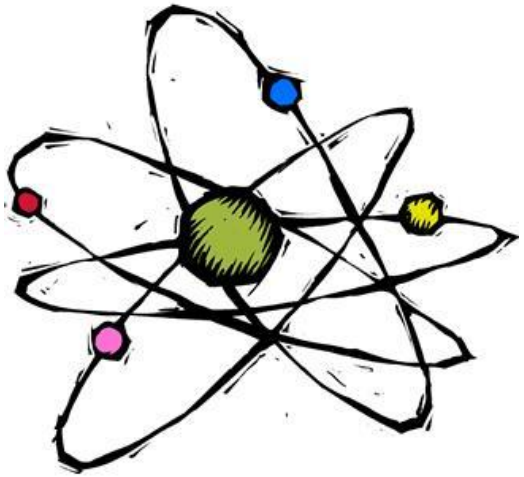
Contents

- The n-body problem

  —Two n-body solvers.

- The traveling salesman problem

  —Tree search.

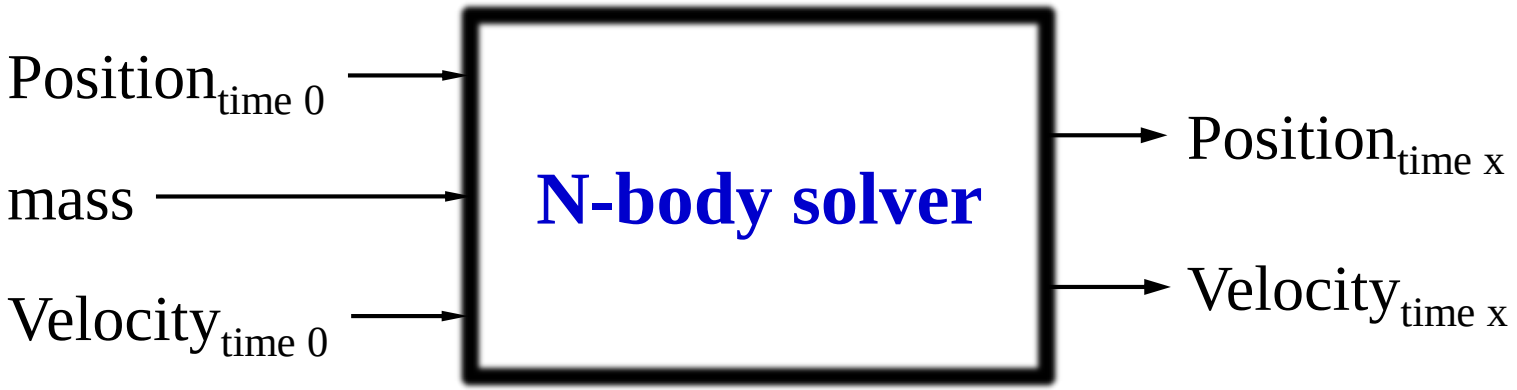


THE N-BODY PROBLEM

The N-Body Problem

- Find the positions and velocities of a collection of interacting particles over a period of time.

- An n-body solver is a program that finds the solution to an n-body problem by simulating the behavior of the particles.



Simulating Motion of Planets

- Determine the positions and velocities:
  - Newton's second law of motion. $\mathbf{F} = m\boldsymbol{a} = m\mathbf{s}''$
  - Newton's law of universal gravitation. $\mathbf{F} = \frac{Gm_1m_2}{r^2}$

(6.1)

$$\mathbf{f}_{qk}(t) = -\frac{Gm_q m_k}{\left|\mathbf{s}_q(t) - \mathbf{s}_k(t)\right|^3}\left[\mathbf{s}_q(t) - \mathbf{s}_k(t)\right]$$

(6.2)

$$\mathbf{F}_q(t) = \sum_{\substack{k=0 \\ k\neq q}}^{n-1} \mathbf{f}_{qk} = -Gm_q \sum_{\substack{k=0 \\ k\neq q}}^{n-1} \frac{m_k}{\left|\mathbf{s}_q(t) - \mathbf{s}_k(t)\right|^3}\left[\mathbf{s}_q(t) - \mathbf{s}_k(t)\right]$$

(6.3)

$$\mathbf{s}_q''(t) = -G \sum_{\substack{j=0 \\ j \neq q}}^{n-1} \frac{m_j}{\left|\mathbf{s}_q(t) - \mathbf{s}_j(t)\right|^3} \left[\mathbf{s}_q(t) - \mathbf{s}_j(t)\right]$$

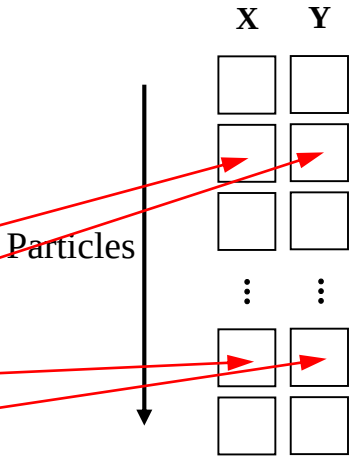
$$t = 0, \Delta t, 2\Delta t, \cdots, T\Delta t$$

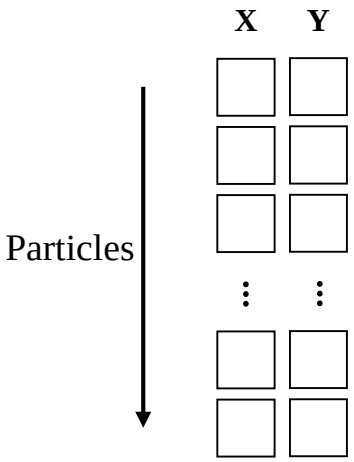
Serial Pseudo-Code

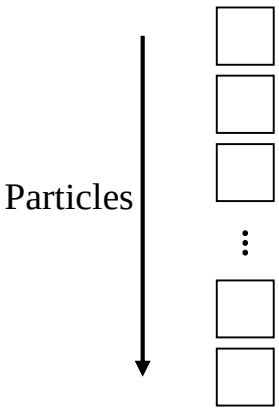
```
Get input data;
Print positions and velocities of particles;    /*输出初始值*/

for each timestep {
    for each particle q
        Compute total force on q;
    for each particle q
        Compute position and velocity of q;

    if (timestep output)
        Print positions and velocities of particles;
}
```

Computation of the Forces

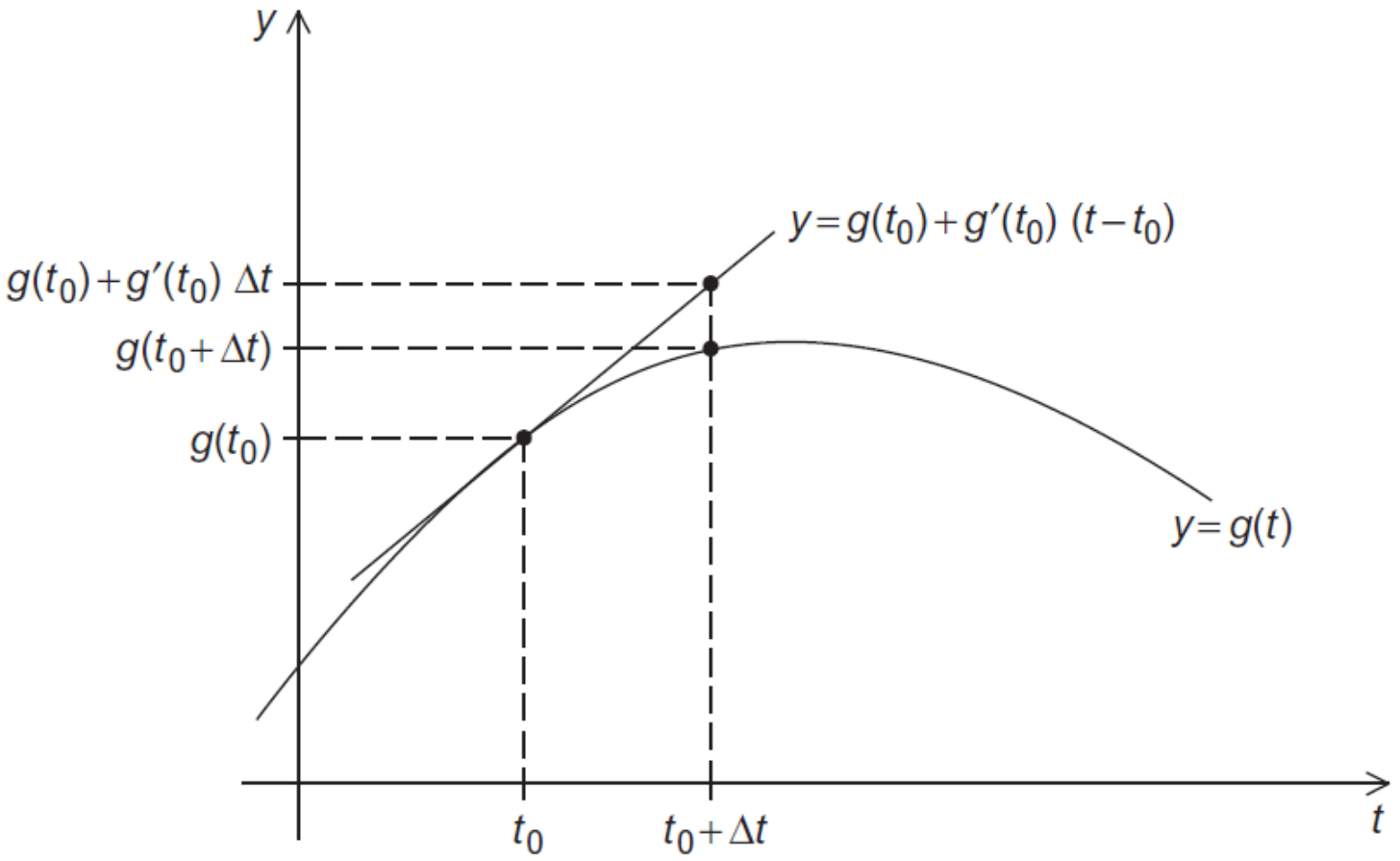
```
for each particle q {
    for each particle k != q {
        x_diff = pos[q][X] - pos[k][X];
        y_diff = pos[q][Y] - pos[k][Y];
        dist = sqrt(x_diff*x_diff + y_diff*y_diff);
        dist_cubed = dist*dist*dist;
        forces[q][X] -= G*masses[q]*masses[k]/dist_cubed*x_diff;
        forces[q][Y] -= G*masses[q]*masses[k]/dist_cubed*y_diff;
    }
}
```







Using the Tangent Line to Approximate a Function



$\mathbf{s}_q(\Delta t)$ and $\mathbf{v}_q(\Delta t)$

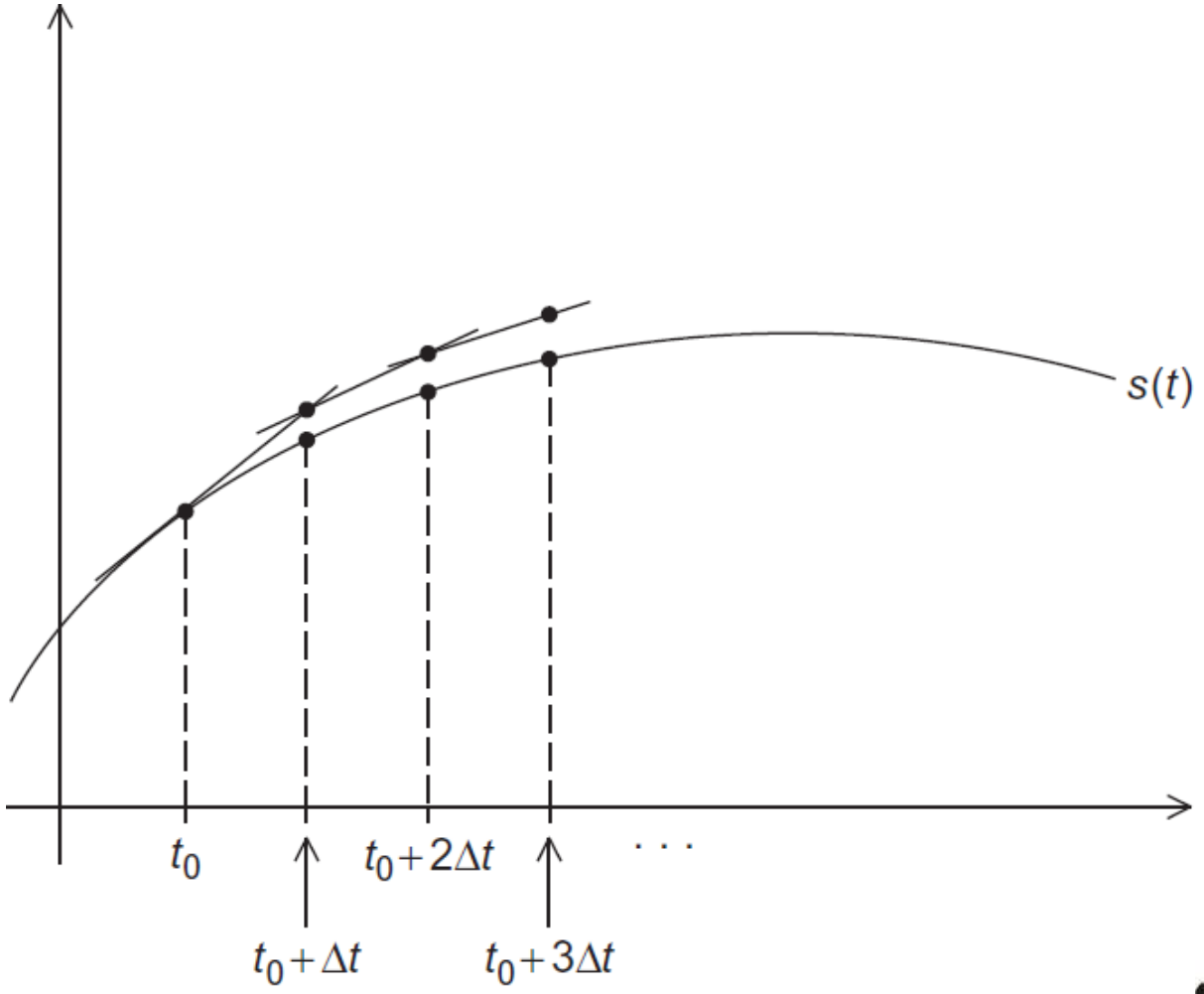
(6.4)

$$\mathbf{s}_q(\Delta t) \approx \mathbf{s}_q(0) + \Delta t \mathbf{s}_q'(0) = \mathbf{s}_q(0) + \Delta t \mathbf{v}_q(0)$$

(6.5)

$$\mathbf{v}_q(\Delta t) \approx \mathbf{v}_q(0) + \Delta t \mathbf{v}_q'(0) = \mathbf{v}_q(0) + \Delta t \boldsymbol{\alpha}_q(0)$$
$$= \mathbf{v}_q(0) + \Delta t \frac{1}{m_q} \mathbf{F}_q(0)$$

Euler's Method



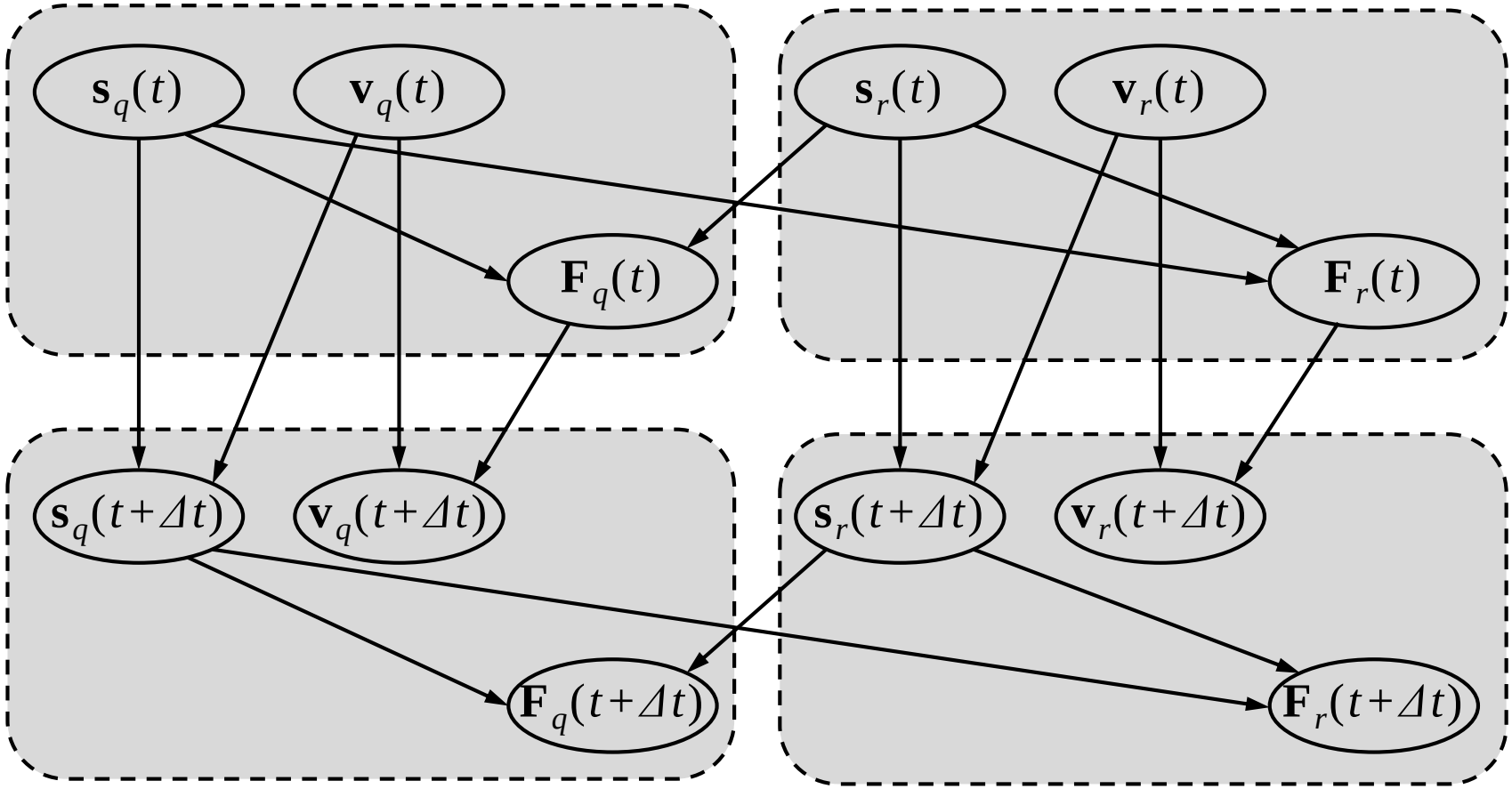
Code for Computing Position and Velocity

```
pos[q][X] += delta_t * vel[q][X];
pos[q][Y] += delta_t * vel[q][Y];
vel[q][X] += delta_t/masses[q] * forces[q][X];
vel[q][Y] += delta_t/masses[q] * forces[q][Y];
```

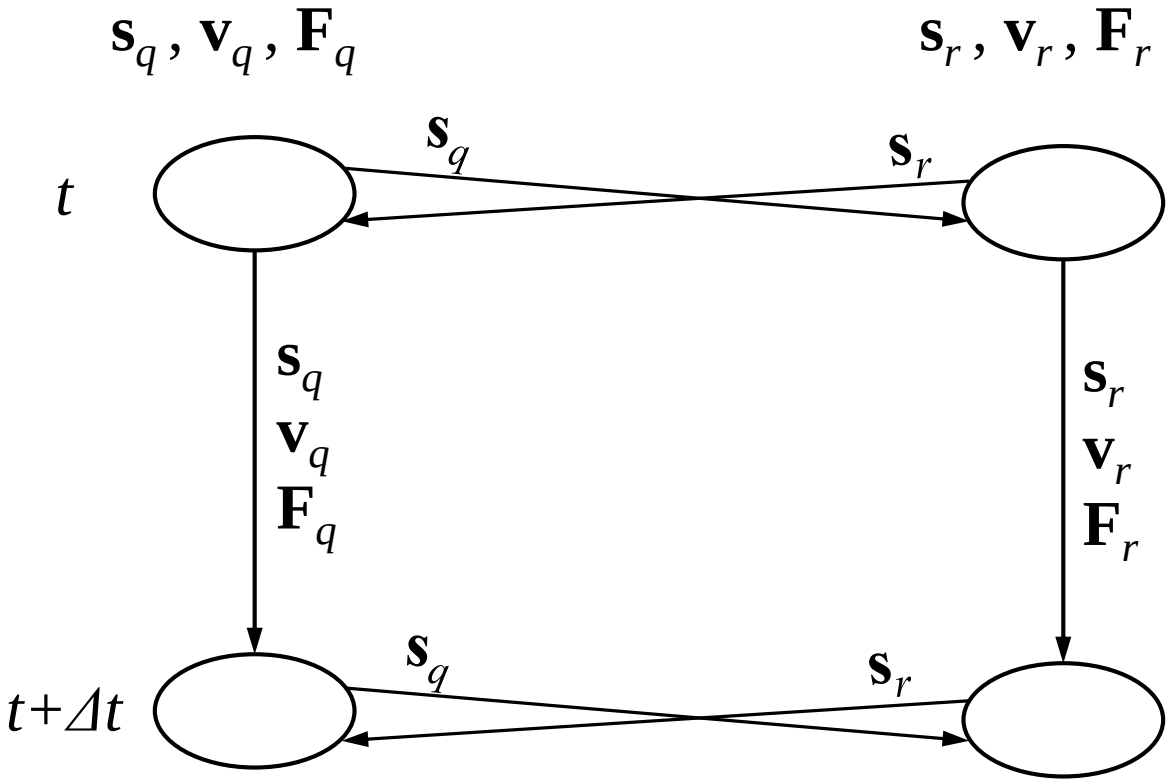
Parallelizing the N-Body Solvers

- Apply Foster's methodology.
- Initially, we want a lot of tasks.
- Start by making our tasks the computations of the positions, the velocities, and the total forces at each timestep.

Communications among Tasks in the Basic N-Body Solver



Communications among Agglomerated Tasks in the Basic N-Body Solver



Serial Pseudo-Code

```
for each timestep {
    for each particle q
        Compute total force on q;
    for each particle q
        Compute position and velocity of q;

    if (timestep output)
        Print positions and velocities of particles;
}
```

In principle, parallelizing the two inner for loops will map tasks/particles to cores.

First Attempt

```
for each timestep {
#  pragma omp parallel for
   for each particle q
      Compute total force on q;
#  pragma omp parallel for
   for each particle q
      Compute position and velocity of q;

   if (timestep output)
      Print positions and velocities of particles;
}
```

Let's check for race conditions caused by loop-carried dependences.

First Loop—Calculate Forces

```
# pragma omp parallel for
  for each particle q {
    forces[q][X] = forces[q][Y] = 0;
    for each particle k != q {
      x_diff = pos[q][X] - pos[k][X];
      y_diff = pos[q][Y] - pos[k][Y];
      dist = sqrt(x_diff*x_diff + y_diff*y_diff);
      dist_cubed = dist*dist*dist;
      forces[q][X] -= G*masses[q]*masses[k] / dist_cubed*x_diff;
      forces[q][Y] -= G*masses[q]*masses[k] / dist_cubed*y_diff;
    }
}
```

no race conditions

Second Loop—Calculate Positions and Velocities

```
# pragma omp parallel for
  for each particle q {
     pos[q][X] += delta_t * vel[q][X];
     pos[q][Y] += delta_t * vel[q][Y];
     vel[q][X] += delta_t / masses[q] * forces[q][X];
     vel[q][Y] += delta_t / masses[q] * forces[q][Y];
}
```

no race conditions

But Repeated Forking and Joining of Threads

```
# pragma omp parallel
  for each timestep {
#    pragma omp for
     for each particle q
        Compute total force on q;
#    pragma omp for
     for each particle q
        Compute position and velocity of q;

     if (timestep output)
        Print positions and velocities of particles;
}
```

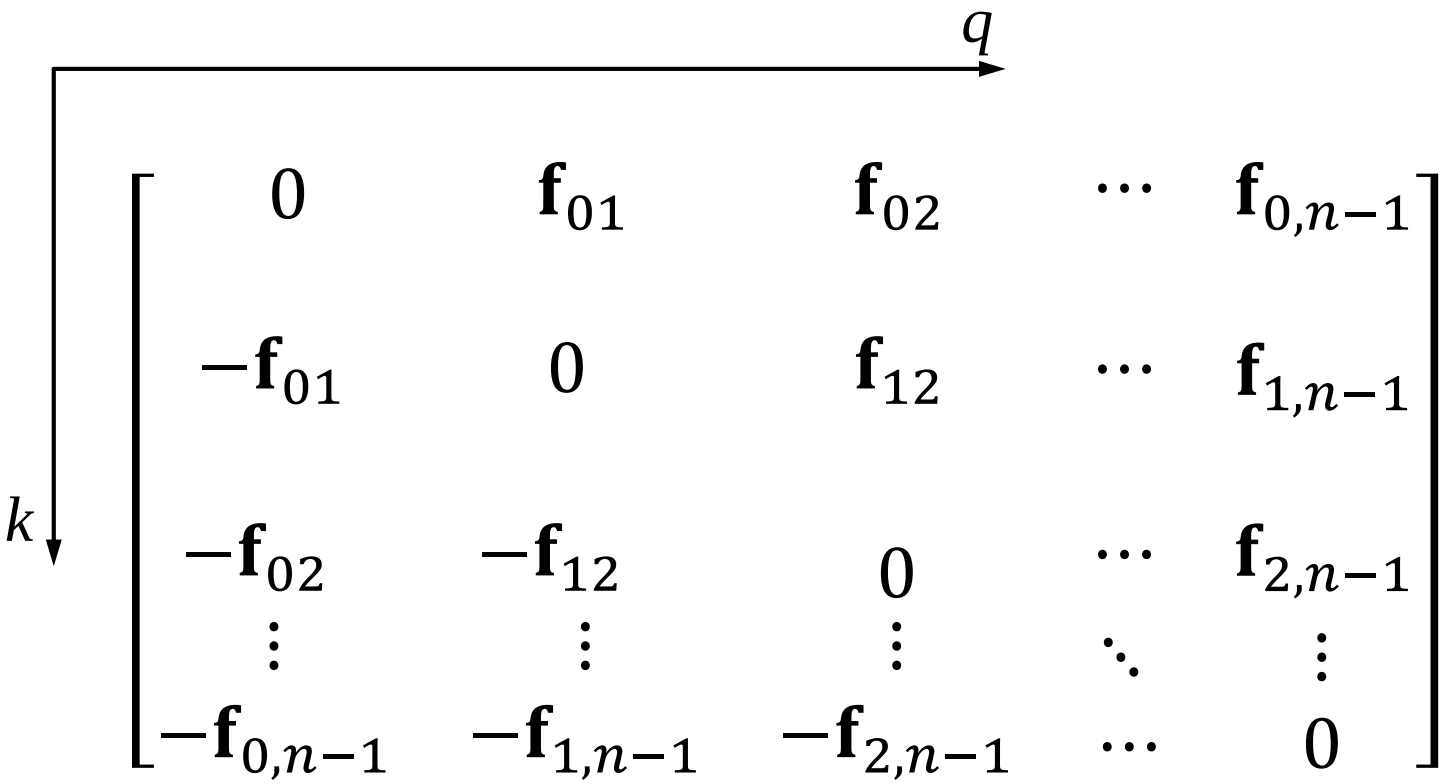
The same team of threads will be used in both loops and for every iteration of the outer loop.

But every thread will print all the positions and velocities.

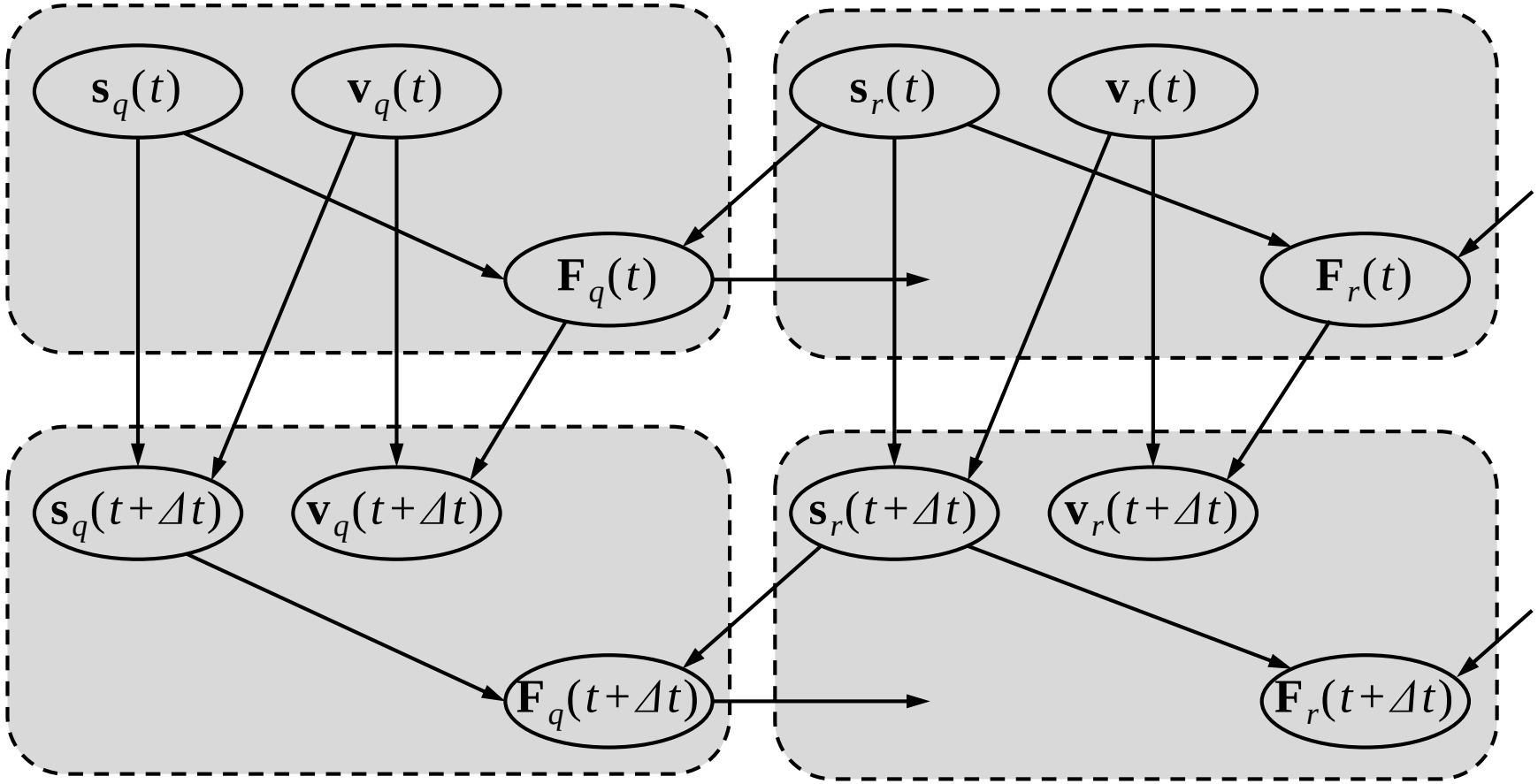
Adding the `Single` Directive

```
# pragma omp parallel
  for each timestep {
#    pragma omp for
     for each particle q
        Compute total force on q;
#    pragma omp for
     for each particle q
        Compute position and velocity of q;

     if (timestep output) {
#       pragma omp single
        Print positions and velocities of particles;
     }
}
```

But, There is a Reduced Algorithm



Communications among Tasks in the Reduced N-Body Solver



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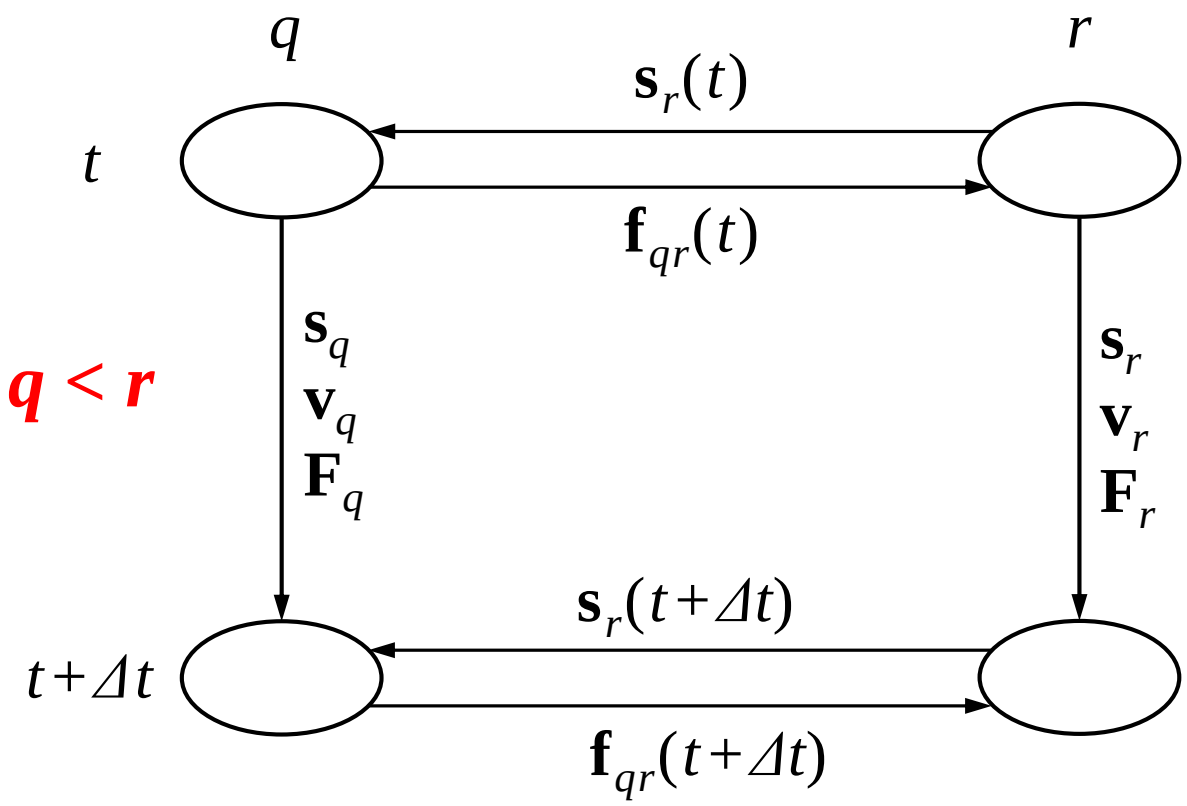
Communications among Agglomerated Tasks in the Reduced N-Body Solver



A Reduced Algorithm for Computing N-Body Forces

```
for each particle q
   forces[q] = 0.0;
for each particle q {
   for each particle k > q {
      x_diff = pos[q][X] - pos[k][X];
      y_diff = pos[q][Y] - pos[k][Y];
      dist = sqrt(x_diff*x_diff + y_diff*y_diff);
      dist_cubed = dist*dist*dist;
      force_qk[X]= -G*masses[q]*masses[k]/dist_cubed * x_diff;
      force_qk[Y]= -G*masses[q]*masses[k]/dist_cubed * y_diff;

      forces[q][X] += force_qk[X];
      forces[q][Y] += force_qk[Y];
      forces[k][X] -= force_qk[X];
      forces[k][Y] -= force_qk[Y];
   }
}
```

Parallelizing the Reduced Solver Using OpenMP

```
# pragma omp parallel
  for each timestep {
#     pragma omp for
      for each particle q
         forces[q] = 0.0;
#     pragma omp for
      for each particle q
         Compute total force on q;
#     pragma omp for
      for each particle q
         Compute position and velocity of q;

      if (timestep output) {
#        pragma omp single
         Print positions and velocities of particles;
      }
}
```

Computing the Total Force on Particle Q in the Reduced Algorithm

```
# pragma omp for
  for each particle q
     force_qk[X] = force_qk[Y] = 0;
     for each particle k > q {
        x_diff = pos[q][X] - pos[k][X];
        y_diff = pos[q][Y] - pos[k][Y];
        dist = sqrt(x_diff*x_diff + y_diff*y_diff);
        dist_cubed = dist*dist*dist;
        force_qk[X] = -G*masses[q]*masses[k]/dist_cubed * x_diff;
        force_qk[Y] = -G*masses[q]*masses[k]/dist_cubed * y_diff;

        forces[q][X] += force_qk[X];
        forces[q][Y] += force_qk[Y];
        forces[k][X] -= force_qk[X];
        forces[k][Y] -= force_qk[Y];
}
```

Problems

Suppose we have 2 threads and 4 particles and we're using a block partition of the particles.

$$\mathbf{F}_3 = -\mathbf{f}_{03} - \mathbf{f}_{13} - \mathbf{f}_{23}$$

Thread 0		Thread 1	
$\mathbf{F}_0$	$\mathbf{f}_{01}$	$\mathbf{f}_{02}$	$\mathbf{f}_{03}$
$\mathbf{f}_{01}$	$\mathbf{F}_1$	$\mathbf{f}_{12}$	$\mathbf{f}_{13}$
$\mathbf{f}_{02}$	$\mathbf{f}_{12}$	$\mathbf{F}_2$	$\mathbf{f}_{23}$
$\mathbf{f}_{03}$	$\mathbf{f}_{13}$	$\mathbf{f}_{23}$	$\mathbf{F}_3$

- Thread 0 will compute $\mathbf{f}_{03}$ and $\mathbf{f}_{13}$, while thread 1 will compute $\mathbf{f}_{23}$. Thus, the updates to forces[3] create a race condition.
- In fact, this is the case in general. Updates to the elements of the forces array introduce race conditions into the code.

First Solution Attempt

Before all the updates to forces

```
#  pragma omp critical
   {
      forces[q][X] += force_qk[X];
      forces[q][Y] += force_qk[Y];
      forces[k][X] -= force_qk[X];
      forces[k][Y] -= force_qk[Y];
   }
```

Access to the forces array will be effectively serialized!!!

Second Solution Attempt

```
omp_set_lock(&locks[q]);
forces[q][X] += force_qk[X];
forces[q][Y] += force_qk[Y];
omp_unset_lock(&locks[q]);

omp_set_lock(&locks[k]);
forces[k][X] -= force_qk[X];
forces[k][Y] -= force_qk[Y];
omp_unset_lock(&locks[k]);
```

Use one lock for each particle.

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Third Solution Attempt

```
# pragma omp for
  for each particle q
     force_qk[X] = force_qk[Y] = 0;
     for each particle k > q {
        ……
        forces[q][X] += force_qk[X];
        forces[q][Y] += force_qk[Y];
        forces[k][X] -= force_qk[X];
        forces[k][Y] -= force_qk[Y];
}
```

Phase I

```
# pragma omp for
  for each particle q
     force_qk[X] = force_qk[Y] = 0;
     for each particle k > q {
        ……
        loc_forces[q][X] += force_qk[X];
        loc_forces[q][Y] += force_qk[Y];
        loc_forces[k][X] -= force_qk[X];
        loc_forces[k][Y] -= force_qk[Y];
     }
```

Phase II

```
# pragma omp for
  for (q = 0; q < n; q++) {
     forces[q][X] = forces[q][Y] = 0;
     for (thread = 0; thread < thread_count; thread++){
        forces[q][X] += loc_forces[thread][q][X];
        forces[q][Y] += loc_forces[thread][q][Y];
     }
  }
```

先算
再分

Revised Algorithm – Phase I

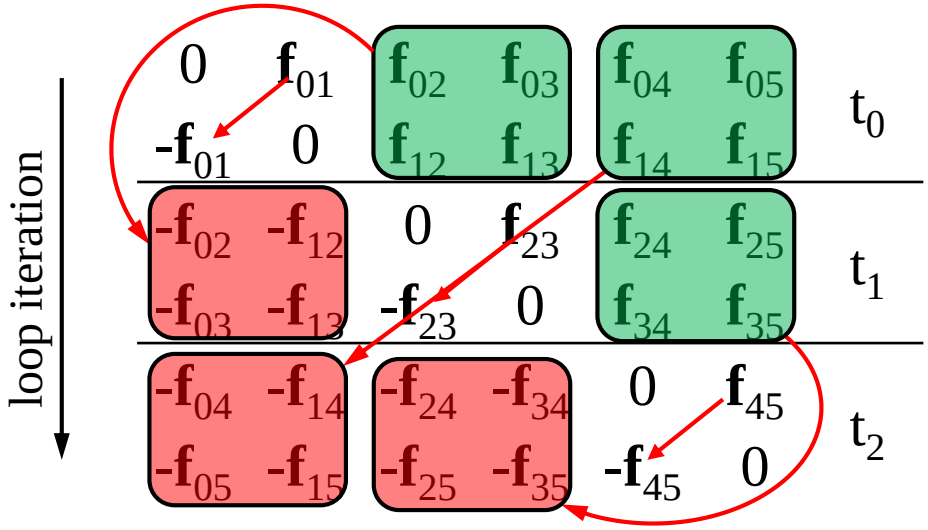
```
# pragma omp for
  for each particle q {
     force_qk[X] = force_qk[Y] = 0;
     for each particle k > q {
        x_diff = pos[q][X]-pos[k][X];
        y_diff = pos[q][Y]-pos[k][Y];
        dist = sqrt(x_diff * x_diff + y_diff * y_diff);
        dist_cubed = dist * dist * dist;
        force_qk[X] = -G * masses[q]* masses[k]/ dist_cubed * x_diff;
        force_qk[Y] = -G * masses[q]* masses[k]/ dist_cubed * y_diff;

        loc_forces[my_rank][q][X] += force_qk[X];
        loc_forces[my_rank][q][Y] += force_qk[Y];
        loc_forces[my_rank][k][X] -= force_qk[X];
        loc_forces[my_rank][k][Y] -= force_qk[Y];
     }
  }
```

Revised Algorithm – Phase II

```
# pragma omp for
  for (q = 0; q < n; q++) {
     forces[q][X] = forces[q][Y] = 0;
     for (thread = 0; thread < thread_count; thread++){
        forces[q][X] += loc_forces[thread][q][X];
        forces[q][Y] += loc_forces[thread][q][Y];
     }
  }
```

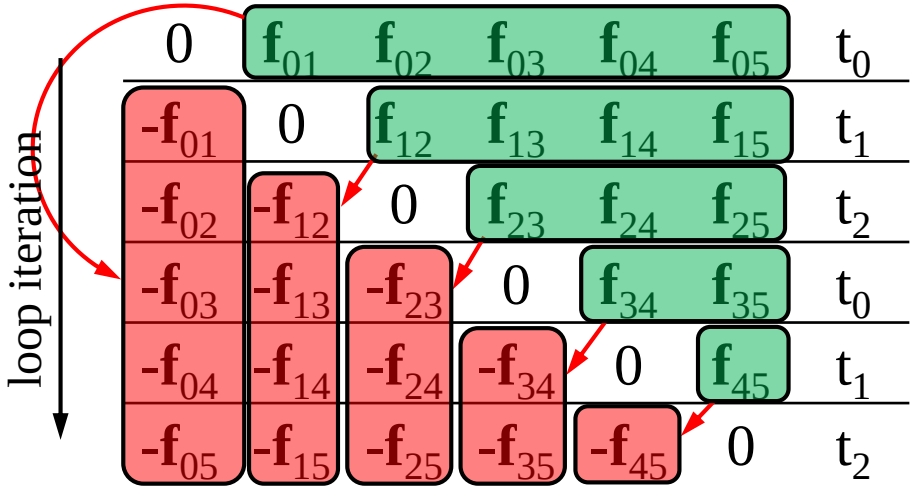
First Phase Computations for Reduced Algorithm with Block Partition

		Thread		
Thread	Particle	0	1	2
0	0	$\mathbf{f}_{01} + \mathbf{f}_{02} + \mathbf{f}_{03} + \mathbf{f}_{04} + \mathbf{f}_{05}$	0	0
	1	$-\mathbf{f}_{01} + \mathbf{f}_{12} + \mathbf{f}_{13} + \mathbf{f}_{14} + \mathbf{f}_{15}$	0	0
1	2	$-\mathbf{f}_{02} - \mathbf{f}_{12}$	$\mathbf{f}_{23} + \mathbf{f}_{24} + \mathbf{f}_{25}$	0
	3	$-\mathbf{f}_{03} - \mathbf{f}_{13}$	$-\mathbf{f}_{23} + \mathbf{f}_{34} + \mathbf{f}_{35}$	0
2	4	$-\mathbf{f}_{04} - \mathbf{f}_{14}$	$-\mathbf{f}_{24} - \mathbf{f}_{34}$	$\mathbf{f}_{45}$
	5	$-\mathbf{f}_{05} - \mathbf{f}_{15}$	$-\mathbf{f}_{25} - \mathbf{f}_{35}$	$-\mathbf{f}_{45}$



First Phase Computations for Reduced Algorithm with Cyclic Partition

		Thread		
Thread	Particle	0	1	2
0	0	$\mathbf{f}_{01}+\mathbf{f}_{02}+\mathbf{f}_{03}+\mathbf{f}_{04}+\mathbf{f}_{05}$	0	0
1	1	$-\mathbf{f}_{01}$	$\mathbf{f}_{12}+\mathbf{f}_{13}+\mathbf{f}_{14}+\mathbf{f}_{15}$	0
2	2	$-\mathbf{f}_{02}$	$-\mathbf{f}_{12}$	$\mathbf{f}_{23}+\mathbf{f}_{24}+\mathbf{f}_{25}$
0	3	$-\mathbf{f}_{03}+\mathbf{f}_{34}+\mathbf{f}_{35}$	$-\mathbf{f}_{13}$	$-\mathbf{f}_{23}$
1	4	$-\mathbf{f}_{04}-\mathbf{f}_{34}$	$-\mathbf{f}_{14}+\mathbf{f}_{45}$	$-\mathbf{f}_{24}$
2	5	$-\mathbf{f}_{05}-\mathbf{f}_{35}$	$-\mathbf{f}_{15}-\mathbf{f}_{45}$	$-\mathbf{f}_{25}$



Parallelizing the Solvers Using Pthreads

- By default local variables in Pthreads are private. So all shared variables are global in the Pthreads version.
- The principle data structures in the Pthreads version are identical to those in the OpenMP version: vectors are two-dimensional arrays of doubles, and the mass, position, and velocity of a single particle are stored in a struct.
- The forces are stored in an array of vectors.

Parallelizing the Solvers Using Pthreads

- Startup for Pthreads is basically the same as the startup for OpenMP: the main thread gets the command line arguments, and allocates and initializes the principle data structures.
- The main difference between the Pthreads and the OpenMP implementations is in the details of parallelizing the inner loops.
- Since Pthreads has nothing analogous to a `parallel for` directive, we must explicitly determine which values of the loop variables correspond to each thread's calculations.

Parallelizing the Solvers Using Pthreads

- Another difference between the Pthreads and the OpenMP versions has to do with barriers.
- At the end of a parallel for OpenMP has an implied barrier.
- We need to add explicit barriers after the inner loops when a race condition can arise.
- The Pthreads standard includes a barrier. However, some systems don’t implement it.
- If a barrier isn't defined we must define a function that uses a Pthreads condition variable to implement a barrier.

Parallelizing the Basic Solver Using MPI

- Choices with respect to the data structures:
  - Each process stores the entire global array of particle masses.
  - Each process only uses a single n-element array for the positions.
  - Each process uses a pointer loc_pos that refers to the start of its block of pos.
  - So on process 0 local_pos = pos;

    on process 1 local_pos = pos + loc_n;

    etc.

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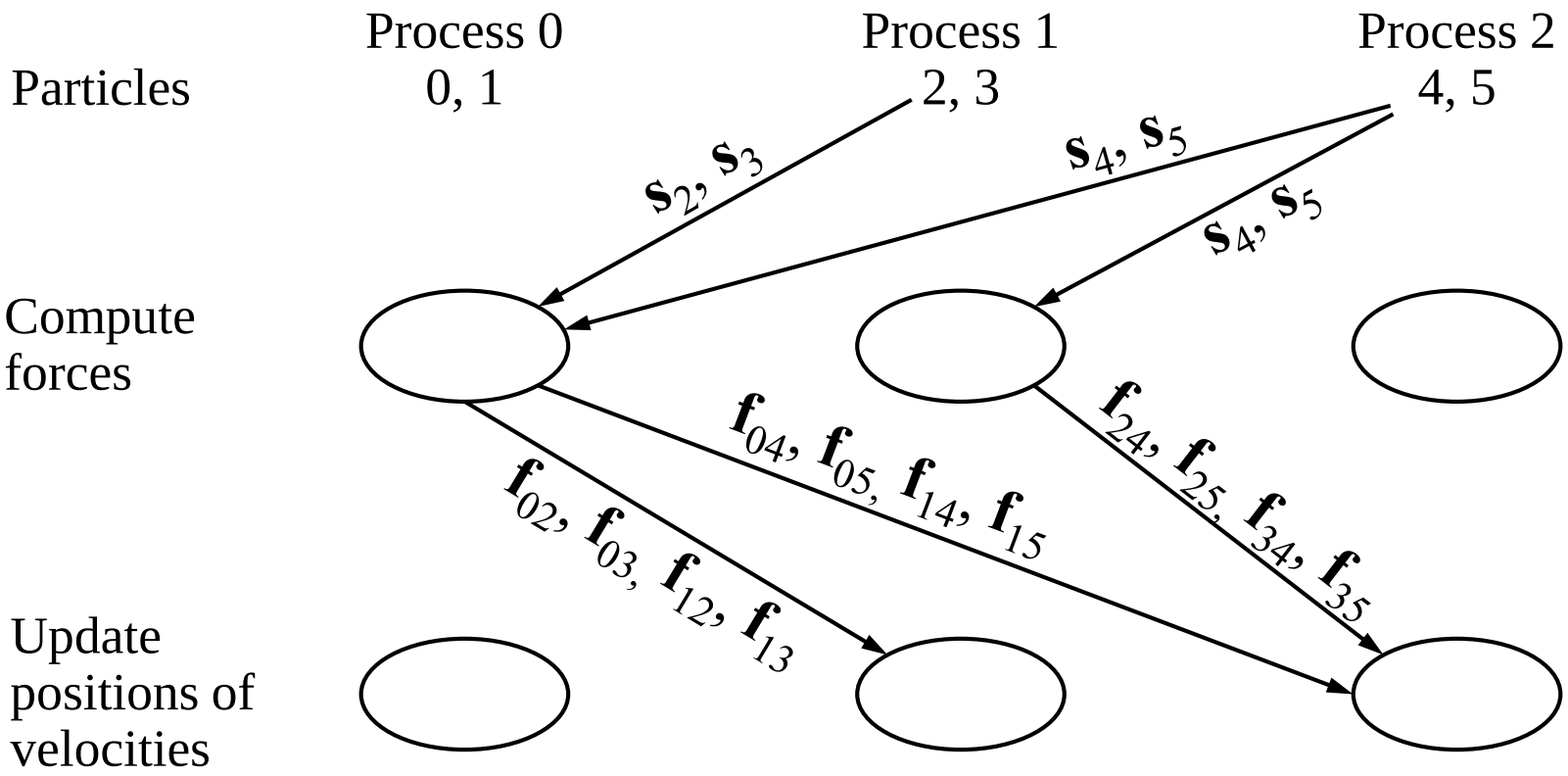
Pseudo-Code for the MPI Version of the Basic N-Body Solver

```
Get input data;
for each timestep {
   for each local particle loc_q
      Compute total force on loc_q;
   for each local particle loc_q
      Compute position and velocity of loc_q;
   Allgather local positions into global pos array;
   if (timestep output)
      Print positions and velocities of particles;
}
```

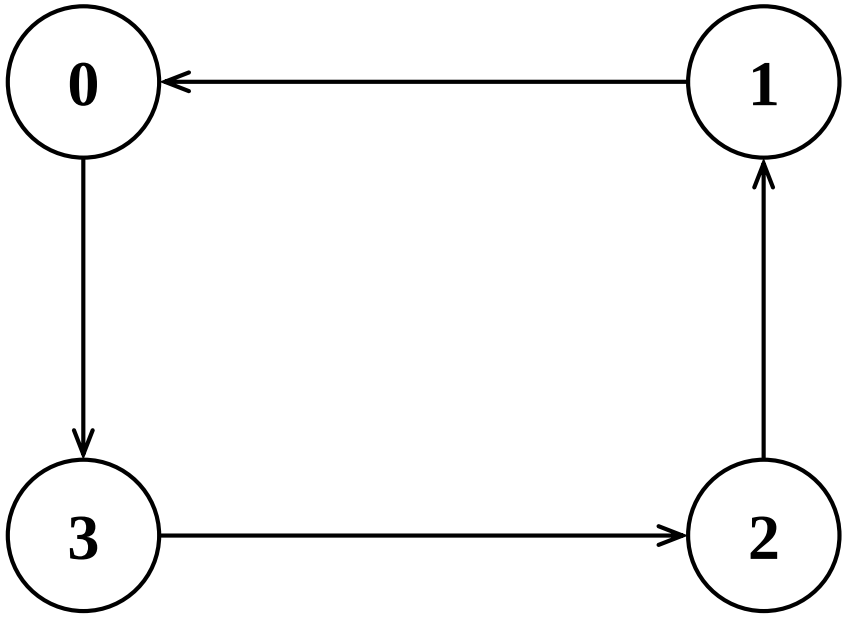
Pseudo-code for Output

```
Gather velocities onto process 0;
if (my_rank == 0) {
    Print timestep;
    for each particle
        Print pos[particle] and vel[particle]
}
```

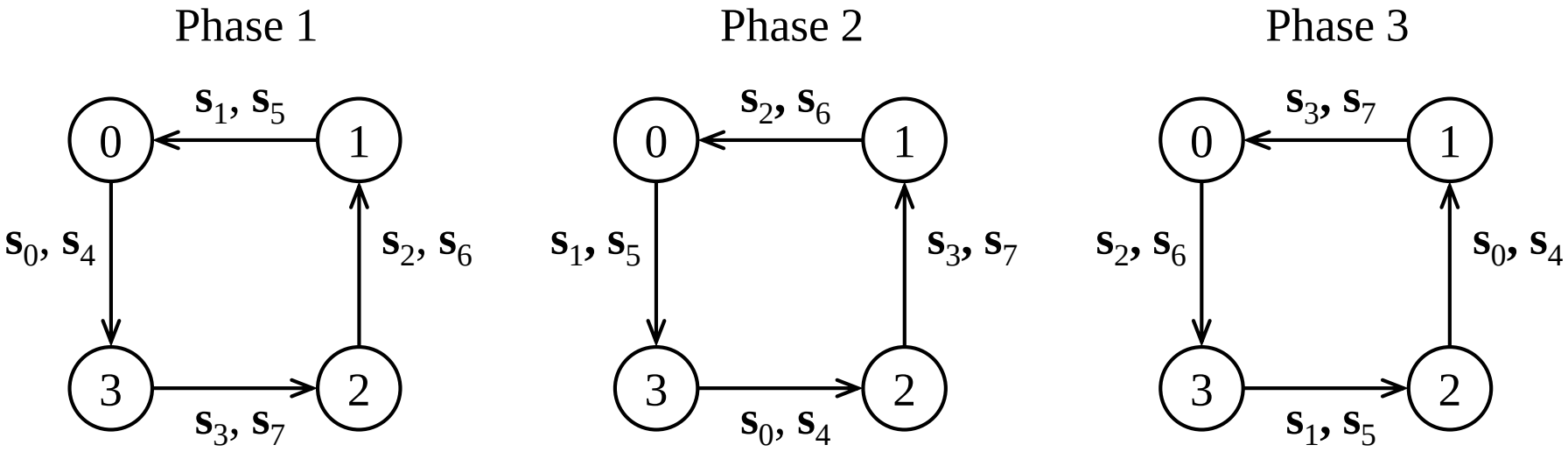
Communication in a Possible MPI Implementation of the N-Body Solver (for a Reduced Solver)



A Ring of Processes



Ring Pass of Positions



Computation of Forces in Ring Pass (1)

Time	Variable	Process 0	Process 1
Start	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $0, 0$ $\mathbf{s}_0, \mathbf{s}_2$ $0, 0$	$\mathbf{s}_1, \mathbf{s}_3$ $0, 0$ $\mathbf{s}_1, \mathbf{s}_3$ $0, 0$
After Comp of Forces	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $\mathbf{f}_{02}, 0$ $\mathbf{s}_0, \mathbf{s}_2$ $0, -\mathbf{f}_{02}$	$\mathbf{s}_1, \mathbf{s}_3$ $\mathbf{f}_{13}, 0$ $\mathbf{s}_1, \mathbf{s}_3$ $0, -\mathbf{f}_{13}$
After First Comm	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $\mathbf{f}_{02}, 0$ $\mathbf{s}_1, \mathbf{s}_3$ $0, -\mathbf{f}_{13}$	$\mathbf{s}_1, \mathbf{s}_3$ $\mathbf{f}_{13}, 0$ $\mathbf{s}_0, \mathbf{s}_2$ $0, -\mathrm{f}_{02}$
After Comp of Forces	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $\mathbf{f}_{01}+\mathbf{f}_{02}+\mathbf{f}_{03}, \mathbf{f}_{23}$ $\mathbf{s}_1, \mathbf{s}_3$ $-\mathbf{f}_{01}, -\mathbf{f}_{03}-\mathbf{f}_{13}-\mathbf{f}_{23}$	$\mathbf{s}_1, \mathbf{s}_3$ $\mathbf{f}_{12}+\mathbf{f}_{13}, 0$ $\mathbf{s}_0, \mathbf{s}_2$ $0, -\mathbf{f}_{02}-\mathbf{f}_{12}$

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Computation of Forces in Ring Pass (2)

Time	Variable	Process 0	Process 1
After Second Comm	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $\mathbf{f}_{01}+\mathbf{f}_{02}+\mathbf{f}_{03}, \mathbf{f}_{23}$ $\mathbf{s}_0, \mathbf{s}_2$ $0, -\mathbf{f}_{02} - \mathbf{f}_{12}$	$\mathbf{s}_1, \mathbf{s}_3$ $\mathbf{f}_{12}+\mathbf{f}_{13}, 0$ $\mathbf{s}_1, \mathbf{s}_3$ $-\mathbf{f}_{01}, -\mathbf{f}_{03}-\mathbf{f}_{13}-\mathbf{f}_{23}$
After Comp of Forces	loc_pos loc_forces tmp_pos tmp_forces	$\mathbf{s}_0, \mathbf{s}_2$ $\mathbf{f}_{01}+\mathbf{f}_{02}+\mathbf{f}_{03}, -\mathbf{f}_{02}-\mathbf{f}_{12}+\mathbf{f}_{23}$ $\mathbf{s}_0, \mathbf{s}_2$ $0, -\mathbf{f}_{02}-\mathbf{f}_{12}$	$\mathbf{s}_1, \mathbf{s}_3$ $-\mathbf{f}_{01}+\mathbf{f}_{12}+\mathbf{f}_{13}, -\mathbf{f}_{03}-\mathbf{f}_{13}-\mathbf{f}_{23}$ $\mathbf{s}_1, \mathbf{s}_3$ $-\mathbf{f}_{01}, -\mathbf{f}_{03}-\mathbf{f}_{13}-\mathbf{f}_{23}$

Pseudo-Code for the MPI Implementation of the Reduced N-body Solver

```
source = (my_rank + 1) % comm_sz;
dest = (my_rank - 1 + comm_sz) % comm_sz;
Copy loc_pos into tmp_pos;
loc_forces = tmp_forces = 0;

Compute forces due to interactions among local particles;
for (phase = 1; phase < comm_sz; phase++) {
   Send current tmp_pos and tmp_forces to dest;
   Receive new tmp_pos and tmp_forces from source;
   /* Owner of the positions and forces we're receiving */
   owner = (my_rank + phase) % comm_sz;
   Compute forces due to interactions among my particles
      and owner's particles;
}
Send current tmp_pos and tmp_forces to dest;
Receive new tmp_pos and tmp_forces from source;
```

Loops Iterating through Global Particle Indexes

```
for (loc_part1 = 0; loc_part1 < loc_n-1; loc_part1++)
  for (loc_part2 = loc_part1+1; loc_part2 < loc_n; loc_part2++)
    Compute_force(loc_pos[loc_part1], masses[loc_part1],
        tmp_pos[loc_part2], masses[loc_part2],
        loc_forces[loc_part1], tmp_forces[loc_part2]);
```

```
for (loc_part1 = 0, glb_part1 = my_rank;
     loc_part1 < loc_n-1;
     loc_part1++, glb_part1 += comm_sz)
   for (glb_part2 = First_index(glb_part1, my_rank, owner, comm_sz),
        loc_part2 = Global_to_local(glb_part2, owner, loc_n);
        loc_part2 < loc_n;
        loc_part2++, glb_part2 += comm_sz)
        Compute_force(loc_pos[loc_part1], masses[loc_part1],
              tmp_pos[loc_part2], masses[glb_part2],
              loc_forces[loc_part1], tmp_forces[loc_part2]);
```

Performance of the MPI N-Body Solvers

Processes	Basic	Reduced
1	17.30	8.68
2	8.65	4.45
4	4.35	2.30
8	2.20	1.26
16	1.13	0.78

(in seconds)

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Run-Times for OpenMP and MPI N-Body Solvers

Processes/ Threads	OpenMP		MPI	
	Basic	Reduced	Basic	Reduced
1	15.13	8.77	17.30	8.68
2	7.62	4.42	8.65	4.45
4	3.85	2.26	4.35	2.30

(in seconds)

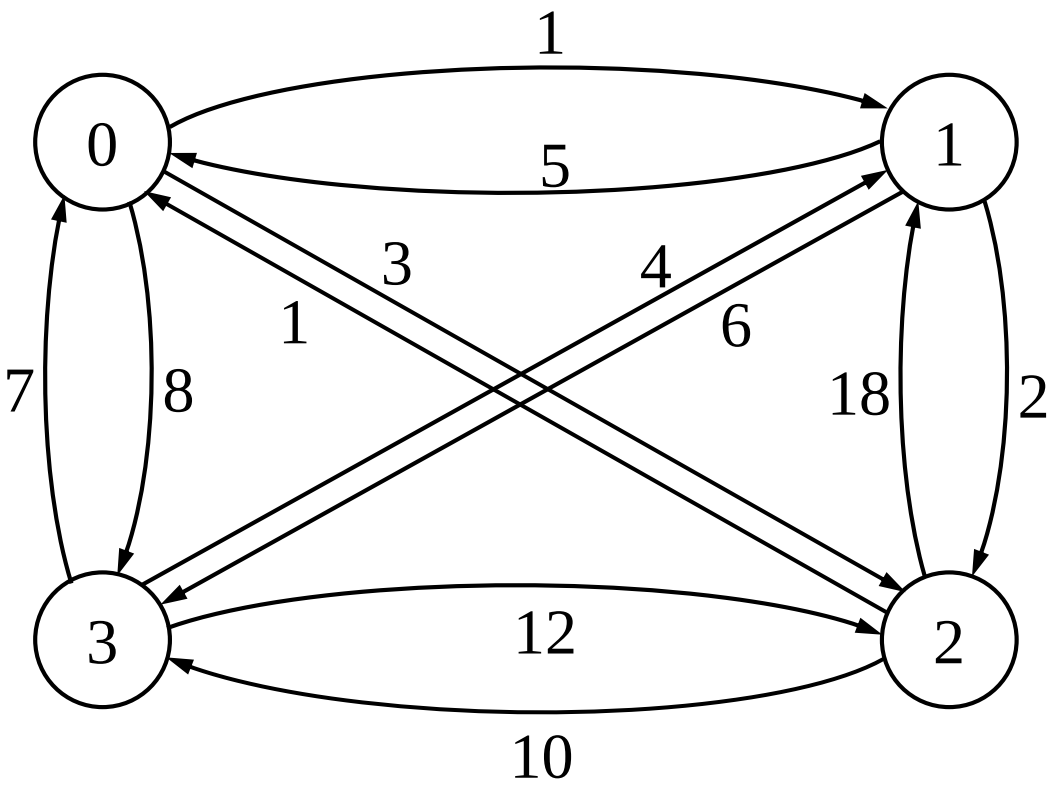
THE TRAVELING SALESMAN PROBLEM
--TREE SEARCH

Tree Search Problem (TSP)

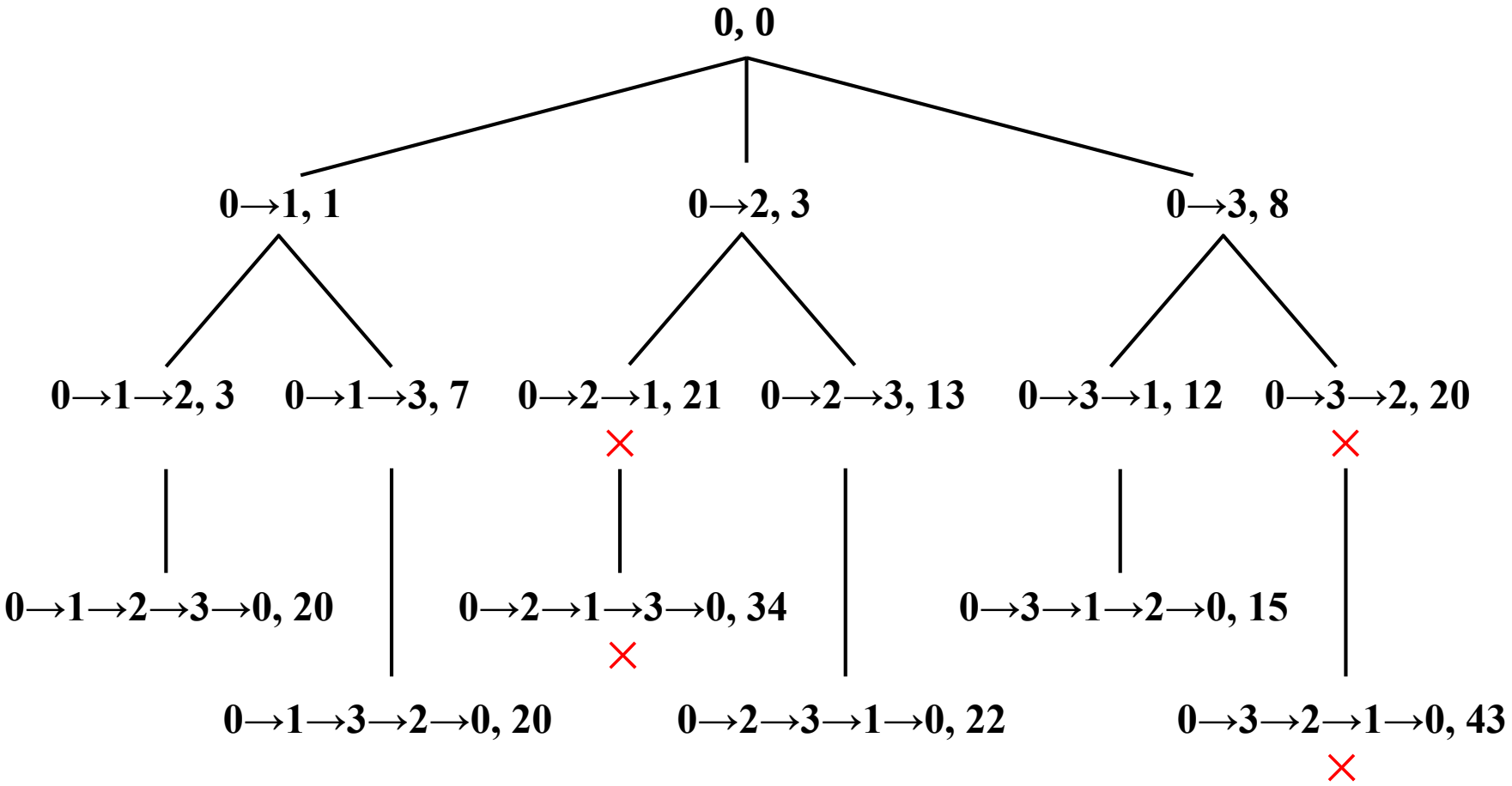
- An NP-complete problem.
- No known solution to TSP that is better in all cases than exhaustive search.
- Ex., the travelling salesperson problem, finding a minimum cost tour.

A Four-City TSP

$$\begin{bmatrix} 0 & 1 & 3 & 8 \\ 5 & 0 & 2 & 6 \\ 1 & 18 & 0 & 10 \\ 7 & 4 & 12 & 0 \end{bmatrix}$$



Search Tree for Four-City TSP



Pseudo-Code for a Recursive Solution to TSP Using Depth-First Search

```
void Depth_first_search(tour_t tour) { // 部分回路
   city_t city;

   if (City_count(tour) == n) { // 走过的城市为n，构成完全回路
      if (Best_tour(tour))        // 检测该完全回路代价是否更小
         Update_best_tour(tour);                  // 更新最佳回路
   } else {
      for each neighboring city  // 看是否还有没被访问过的城市
         if (Feasible(tour, city)) {    // 检测city是否被访问过
            Add_city(tour, city);     // 把city添加到已访问城市
            Depth_first_search(tour);              // 递归搜索
            Remove_last_city(tour);     // 删除之，因已被访问
         }
   }
} /* Depth_first_search */
```

Pseudo-Code for a First Depth-First Solution to TSP without Recursion

```
for (city = n-1; city >= 1; city--)
    Push(stack, city);          // 所有未去过的城市，一律逆序入栈
while (!Empty(stack)) {         // 循环终止条件是栈为空
    city = Pop(stack);
    if (city == NO_CITY)        // 栈尾，退回
        Remove_last_city(curr_tour);
    else {                                  // 尚有未去过的城市，继续前往
        Add_city(curr_tour, city);          // 将城市加入当前回路中
        if (City_count(curr_tour) == n) {   // 构成完全回路
            if (Best_tour(curr_tour))
                Update_best_tour(curr_tour);
            Remove_last_city(curr_tour);
        } else {
            Push(stack, NO_CITY);
            for (nbr = n-1; nbr >= 1; nbr--)   // 遍历剩余的城市
                if (Feasible(curr_tour, nbr))
                    Push(stack, nbr);
        }
    } /* if Feasible */
} /* while !Empty */
```

Pseudo-Code for a Second Depth-First Solution to TSP without Recursion

```
Push_copy(stack, tour); //Tour that visits only the hometown
while (!Empty(stack)) {
   curr_tour = Pop(stack);
   if (City_count(curr_tour) == n) {
      if (Best_tour(curr_tour))
         Update_best_tour(curr_tour);
   } else {
      for (nbr = n-1; nbr >= 1; nbr--)
         if (Feasible(curr_tour, nbr)) {
            Add_city(curr_tour, nbr);
            Push_copy(stack, curr_tour);
            Remove_last_city(curr_tour);
         }
   }
   Free_tour(curr_tour);
}
```

Run-Times of the Three Serial Implementations of Tree Search

Recursive	First Iterative	Second Iterative
30.5	29.2	32.9

(in seconds)

The digraph contains 15 cities.

All three versions visited approximately 95,000,000 tree nodes.

Making Sure We Have the "Best Tour" (1)

- When a process finishes a tour, it needs to check if it has a better solution than recorded so far.
- The global Best_tour function only reads the global best cost, so we don't need to tie it up by locking it. There's no contention with other readers.
- If the process does not have a better solution, then it does not attempt an update.

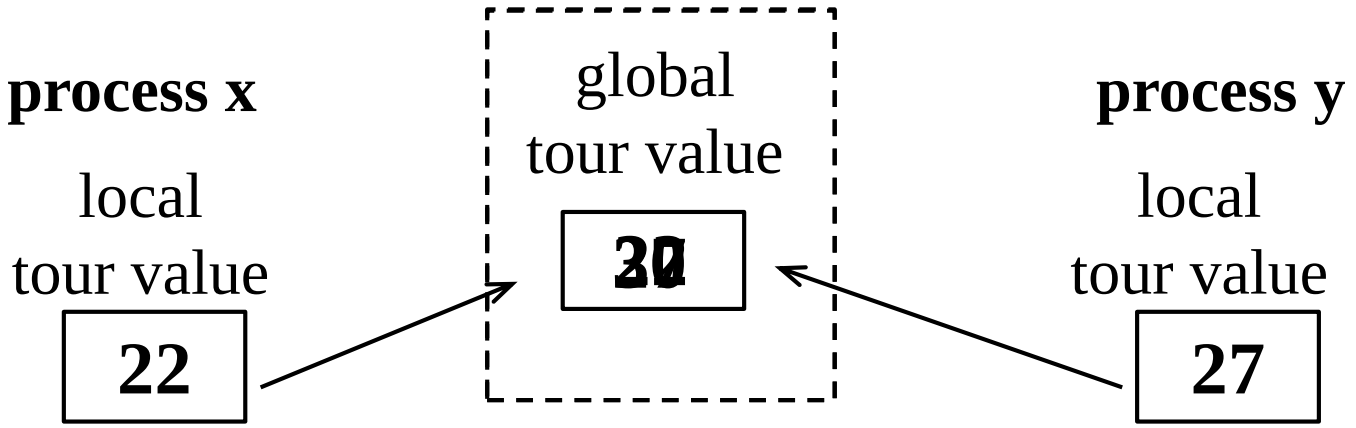
Making Sure We Have the "Best Tour" (2)

- If another thread is updating while we read, we may see the old value or the new value.
- The new value is preferable, but to ensure this would be more costly than it is worth.

Making Sure We Have the "Best Tour" (3)

- In the case where a thread tests and decides it has a better global solution, we need to ensure two things:
  1) That the process locks the value with a mutex, preventing a race condition.
  2) In the possible event that the first check was against an old value while another process was updating, we do not put a worse value than the new one that was being written.
- We handle this by locking, then testing again.

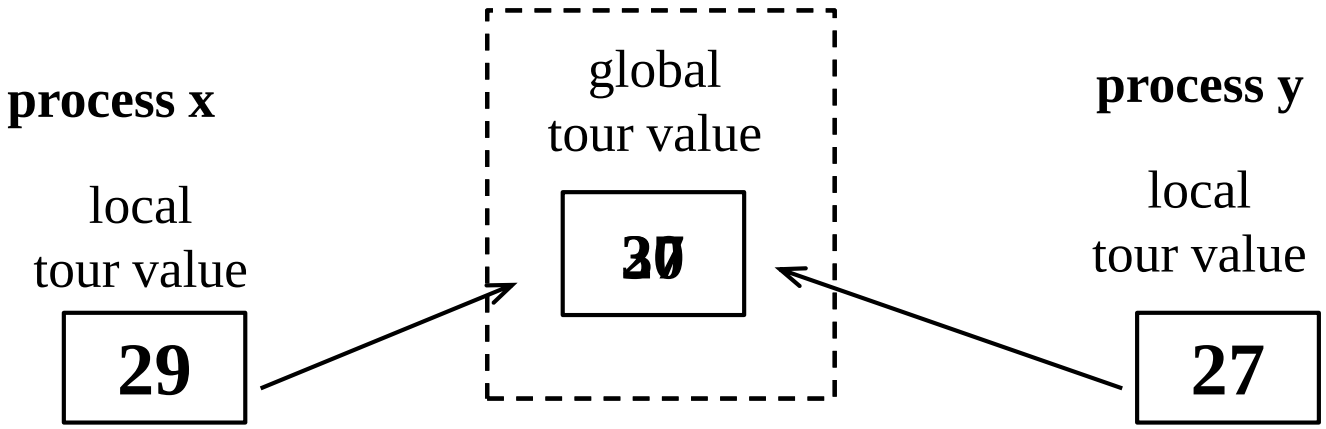
First Scenario



3. test
6. lock
7. test again
8. update
9. unlock

1. test
2. lock
4. update
5. unlock

Second Scenario



3. test
6. lock
7. test again
8. unlock

1. test
2. lock
4. update
5. unlock

Pseudo-Code for a Pthreads Implementation of a Statically Parallelized Solution to TSP

```
Partition tree(my_rank, my_stack);

while (!Empty(my_stack)) {
   curr_tour = Pop(my_stack);
   if (City_count(curr_tour) == n) {
      if (Best_tour(curr_tour))
         Update_best_tour(curr_tour);
   } else {
      for (city = n-1; city >= 1; city--)
         if (Feasible(curr_tour, city)) {
            Add_city(curr_tour, city);
            Push_copy(my_stack, curr_tour);
            Remove_last_city(curr_tour)
         }
   }
   Free_tour(curr_tour);
}
```

Dynamic Parallelization of Tree Search Using Pthreads

- Termination issues.
- Code executed by a thread before it splits:
  - It checks that it has at least two tours in its stack.
  - It checks that there are threads waiting.
  - It checks whether the new_stack variable is NULL.

Pseudo-Code for Pthreads Terminated Function (1)

```
if (my_stack_size >= 2 && threads_in_cond_wait > 0 &&
      new_stack == NULL) {
   lock term_mutex;
   if (threads_in_cond_wait > 0 && new_stack == NULL) {
      Split my_stack creating new_stack;
      pthread_cond_signal(&term_cond_var);
   }
   unlock term_mutex;
   return 0;     /* Terminated = false; don't quit */
} else if (!Empty(my_stack))      /* Keep working */
   return 0; /* Terminated = false; don't quit */
} else {  /* my_stack is empty */
   lock term_mutex;
   if (threads_in_cond_wait == thread_count-1){
                                 /* Last thread running */
      threads_in_cond_wait++;
      pthread_cond_broadcast(&term_cond_var);
      unlock term_mutex;
      return 1; /* Terminated = true; quit */
```

Pseudo-Code for Pthreads Terminated Function (2)

```
    } else { /* Other threads still working, wait for work */
        threads_in_cond_wait++;
        while (pthread_cond_wait(&term_cond_var, &term_mutex)!=0);
        /* We've been awakened */
        if (threads_in_cond_wait < thread_count) {
                                                /* We got work */
            my_stack = new_stack;
            new_stack = NULL;
            threads_in_cond_wait--;
            unlock term_mutex;
            return 0;    /* Terminated = false */
        } else {    /* All threads done */
            unlock term_mutex;
            return 1;    /* Terminated = true; quit */
        }
    } /* else wait for work */
} /* else my_stack is empty */
```

Grouping the Termination Variables

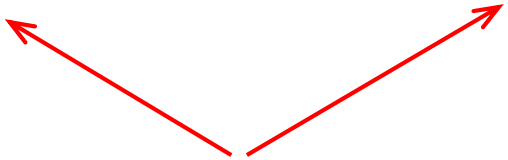
```
typedef struct {
    my_stack_t       new_stack;
    int              threads_in_cond_wait;
    pthread_cond_t   term_cond_var;
    pthread_mutex_t  term_mutex;
} term_struct;
typedef term_struct* term_t;

term_t term; // global variable
```

Run-times of Pthreads tree Search Programs

15-city problems

	First Problem			Second Problem		
Threads	Serial	Static	Dynamic	Serial	Static	Dynamic
1	32.9	32.7	34.7 (0)	26.0	25.8	27.5 (0)
2		27.9	28.9 (7)		25.8	19.2 (6)
4		25.7	25.9 (47)		25.8	9.3 (49)
8		23.8	22.4 (180)		24.0	5.7 (256)

(in seconds)



numbers of times stacks were split

Parallelizing the Tree Search Programs Using OpenMP

- Same basic issues implementing the static and dynamic parallel tree search programs as Pthreads.
- A few small changes can be noted.

Pthreads

```
if (my_rank == whatever)
```

OpenMP

```
#  pragma omp single
```

OpenMP Emulated Condition Wait

```
/* Global variables */
int awakened_thread = -1;
int work_remains = 1;    /* true */
. . .
omp_unset_lock(&term_lock);
while (awakened_thread != my_rank && work_remains);
omp_set_lock(&term_lock);
```

Performance of OpenMP and Pthreads Implementations of Tree Search

Th	First Problem				Second Problem			
	Static		Dynamic		Static		Dynamic	
	OMP	Pth	OMP	Pth	OMP	Pth	OMP	Pth
1	32.5	32.7	33.7 (0)	34.7 (0)	25.6	25.8	26.6 (0)	27.5 (0)
2	27.7	27.9	28.0 (6)	28.9 (7)	25.6	25.8	18.8 (9)	19.2 (6)
4	25.4	25.7	33.1 (75)	25.9 (47)	25.6	25.8	9.8 (52)	9.3 (49)
8	28.0	23.8	19.2 (134)	22.4 (180)	23.8	24.0	6.3 (163)	5.7 (256)

(in seconds)

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IMPLEMENTATION OF TREE SEARCH USING MPI AND STATIC PARTITIONING

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Sending a Different Number of Objects to Each Process in the Communicator

```
int MPI_Scatterv(
      void*          sendbuf          /* in  */,
      int*           sendcounts       /* in  */,
      int*           displacements    /* in  */,
      MPI_Datatype   sendtype         /* in  */,
      void*          recvbuf          /* out */,
      int            recvcount        /* in  */,
      MPI_Datatype   recvtype         /* in  */,
      int            root             /* in  */,
      MPI_Comm       comm             /* in  */);
```

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Gathering a Different Number of Objects from Each Process in the Communicator

```
int MPI_Gatherv(
      void*          sendbuf          /* in  */,
      int            sendcount        /* in  */,
      MPI_Datatype   sendtype         /* in  */,
      void*          recvbuf          /* out */,
      int*           recvcounts       /* in  */,
      int*           displacements    /* in  */,
      MPI_Datatype   recvtype         /* in  */,
      int            root             /* in  */,
      MPI_Comm       comm             /* in  */);
```

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Checking to See if a Message is Available

```
int MPI_Iprobe(
            int           source        /* in  */,
            int           tag           /* in  */,
            MPI_Comm      comm          /* in  */,
            int*          msg_avail_p   /* out */,
            MPI_Status*   status_p      /* out */);
```

```
if (My_avail_tour_count(my_stack) >= 2) {
   Fulfill_request(my_stack);
   return false;  /* Still more work */
} else { /* At most 1 available tour */
   Send_rejects(); /* Tell everyone who's requested */
                   /* work that I have none         */
   if (!Empty_stack(my_stack)) {
      return false;  /* Still more work */
   } else { /* Empty stack */
      if (comm_sz == 1) return true;
      Out_of_work();
      work_request_sent = false;
      while (1) {
         Clear_msgs(); /* Msgs unrelated to work, termination */
         if (No_work_left()) {
            return true; /* No work left. Quit */
         } else if (!work_request_sent) {
            Send_work_request();  /* Request work from someone */
            work_request_sent = true;
         } else {
            Check_for_work(&work_request_sent, &work_avail);
            if (work_avail) {
               Receive_work(my_stack);
               return false;
            }
         }
      } /* while */
   } /* Empty stack */
} /* At most 1 available tour */
```

Terminated Function for a Dynamically Partitioned TSP solver that Uses MPI.

Modes and Buffered Sends

- MPI provides four modes for sends.
  - Standard
  - Synchronous
  - Ready
  - Buffered

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Printing the Best Tour

```
struct {
   int cost;
   int rank;
} loc_data, global_data;

loc_data.cost = Tour_cost(loc_best_tour);
loc_data.rank = my_rank;

MPI_Allreduce(&loc_data, &global_data, 1, MPI_2INT, MPI_MINLOC, comm);
if (global_data.rank == 0) return;  /* 0 already has the best tour */
if (my_rank == 0)
   Receive best tour from process global_data.rank;
else if (my_rank == global_data.rank)
   Send best tour to process 0;
```

Terminated Function for a Dynamically Partitioned TSP Solver with MPI (1)

```
if (My_avail_tour_count(my_stack) >= 2) {
   Fulfill_request(my_stack);
   return false;  /* Still more work */
} else { /* At most 1 available tour */
   Send_rejects();   /* Tell everyone who's requested */
                     /* work that I have none          */
   if (!Empty_stack(my_stack)) {
      return false;  /* Still more work */
   } else {  /* Empty stack */
      if (comm_sz == 1) return true;
      Out_of_work();
      work_request_sent = false;
      while (1)
          Clear_msgs(); {/* Msgs unrelated to work, termination */
          if (No_work_left()) {
             return true; /* No work left. Quit */
```

Terminated Function for a Dynamically Partitioned TSP Solver with MPI (2)

```
            } else if (!work_request_sent) {
               Send_work_request(); /* Request work from someone */
               work_request_sent = true;
            } else {
               Check_for_work(&work_request_sent, &work_avail);
               if (work_avail) {
                  Receive_work(my_stack);
                  return false;
               }
            }
         } /* while */
      } /* Empty stack */
   } /* At most 1 available tour */
```

Packing Data into a Buffer of Contiguous Memory

```
int MPI_Pack(
      void*          data_to_be_packed   /* in     */,
      int            to_be_packed_count  /* in     */,
      MPI_Datatype   datatype            /* in     */,
      void*          contig_buf          /* out    */,
      int            contig_buf_size     /* in     */,
      int*           position_p          /* in/out */,
      MPI_Comm       comm                /* in     */);
```

Unpacking Data from a Buffer of Contiguous Memory

```
int MPI_Unpack(
        void*          contig_buf        /* in     */,
        int            contig_buf_size   /* in     */,
        int*           position_p        /* in/out */,
        void*          unpacked_data     /* out    */,
        int            unpack_count      /* in     */,
        MPI_Datatype   datatype          /* in     */,
        MPI_Comm       comm              /* in     */);
```

Table 6.10 Termination Events that Result in an Error

Time	Process 0	Process 1	Process 2
0	Out of Work Notify 1, 2 oow = 1	Out of Work Notify 0, 2 oow = 1	Working oow = 0
1	Send request to 1 oow = 1	Send Request to 2 oow = 1	Recv notify fr 1 oow = 1
2	oow = 1	Recv notify fr 0 oow = 2	Recv request fr 1 oow = 1
3	oow = 1	oow = 2	Send work to 1 oow = 0
4	oow = 1	Recv work fr 2 oow = 1	Recv notify fr 0 oow = 1
5	oow = 1	Notify 0 oow = 1	Working oow = 1
6	oow = 1	Recv request fr 0 oow = 1	Out of work Notify 0, 1 oow = 2
7	Recv notify fr 2 oow = 2	Send work to 0 oow = 0	Send request to 1 oow = 2
8	Recv 1st notify fr 1 oow = 3	Recv notify fr 2 oow = 1	oow = 2
9	Quit	Recv request fr 2 oow = 1	oow = 2

Performance of MPI and Pthreads Implementations of Tree Search

	First Problem				Second Problem			
	Static		Dynamic		Static		Dynamic	
Th/Pr	Pth	MPI	Pth	MPI	Pth	MPI	Pth	MPI
1	35.8	40.9	41.9 (0)	56.5 (0)	27.4	31.5	32.3 (0)	43.8 (0)
2	29.9	34.9	34.3 (9)	55.6 (5)	27.4	31.5	22.0 (8)	37.4 (9)
4	27.2	31.7	30.2 (55)	52.6 (85)	27.4	31.5	10.7 (44)	21.8 (76)
8		35.7		45.5 (165)		35.7		16.5 (161)
16		20.1		10.5 (441)		17.8		0.1 (173)

(in seconds)

Concluding Remarks (1)

- In developing the reduced MPI solution to the n-body problem, the “ring pass” algorithm proved to be much easier to implement and is probably more scalable.

- In a distributed memory environment in which processes send each other work, determining when to terminate is a nontrivial problem.

Concluding Remarks (2)

- When deciding which API to use, we should consider whether to use shared- or distributed-memory.

- We should look at the memory requirements of the application and the amount of communication among the processes/threads.

Concluding Remarks (3)

- If the memory requirements are great or the distributed memory version can work mainly with cache, then a distributed memory program is likely to be much faster.

- On the other hand if there is considerable communication, a shared memory program will probably be faster.

Concluding Remarks (4)

- In choosing between OpenMP and Pthreads, if there's an existing serial program and it can be parallelized by the insertion of OpenMP directives, then OpenMP is probably the clear choice.

- However, if complex thread synchronization is needed then Pthreads will be easier to use.