

## 并行程序设计

邵兵 2022年3月1日

## 课程目的

本课程的主要目的是让学生认识到并行计算的重要性,在硬件层面了解主要的并行计算体系结构,在软件层面掌握如何使用MPI、Pthreads和OpenMP编写并行程序,并对并行程序的性能进行评估,了解主要的并行算法。最后,还将介绍最先进的CUDA等并行计算架构。



## ■ 教材

- Peter S. Pacheco, An Introduction to Parallel Programming, 机械工业出版社,2011年1月
- Peter S. Pacheco著 邓倩妮等译《并行程序设计导论》, 机械工业出版社, 2010年5月

## ■ 参考书

- 陈国良编著,《并行计算——结构·算法编程》,机械工业出版社,2011年6月。
- 教学方法与考核方式(不及格无补考!)
  - 课堂考勤(10%)
  - 平时书面作业(10%)
  - 平时编程作业(40%)
  - 大作业天梯赛(40%)



## 主讲老师及助教

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# Chapter 1 Parallel Computing

软件学院 邵兵 2022年3月1日

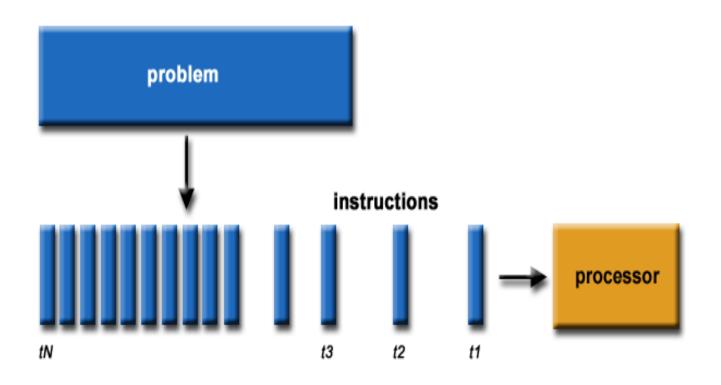
#### **Contents**

- **Overview**
- > Concepts
- > Parallel computer memory architectures
- > Parallel programming models
- > Designing parallel programs
- **≻** Parallel examples





## Serial Computing



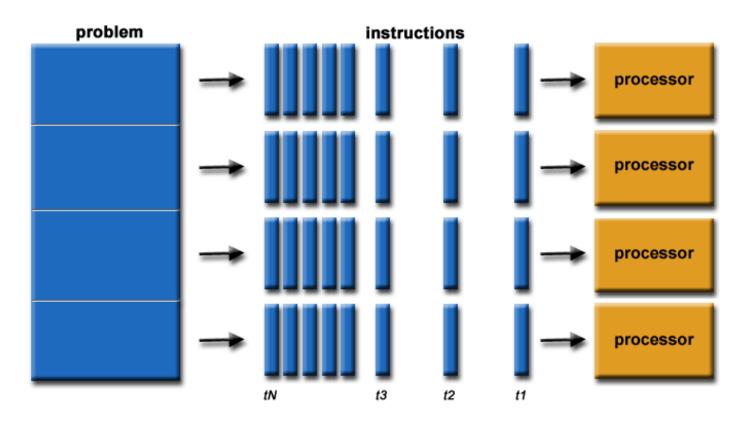


## Serial Computing:

- A problem is broken into a discrete series of instructions
- Instructions are executed sequentially one after another
- Executed on a single processor
- Only one instruction may execute at any moment in time



## Parallel Computing





## Parallel Computing:

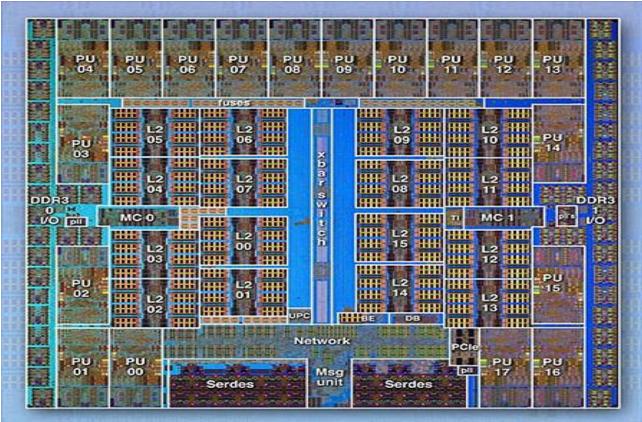
- A problem is broken into discrete parts that can be solved concurrently
- Each part is further broken down to a series of instructions
- Instructions from each part execute simultaneously on different processors
- An overall control/coordination mechanism is employed





## Parallel Computers:

Hardware perspective



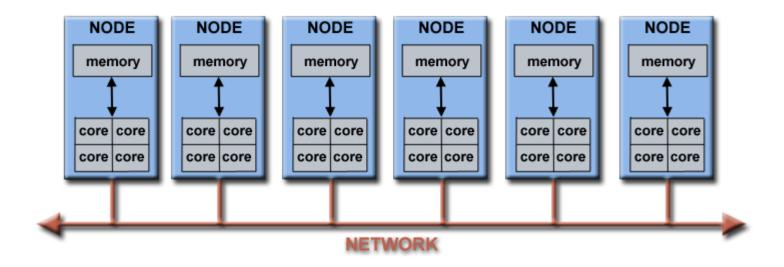
IBM BG/Q Compute Chip with 18 cores (PU) and 16 L2 Cache units (L2)





## Parallel Computers:

Networks connect perspective

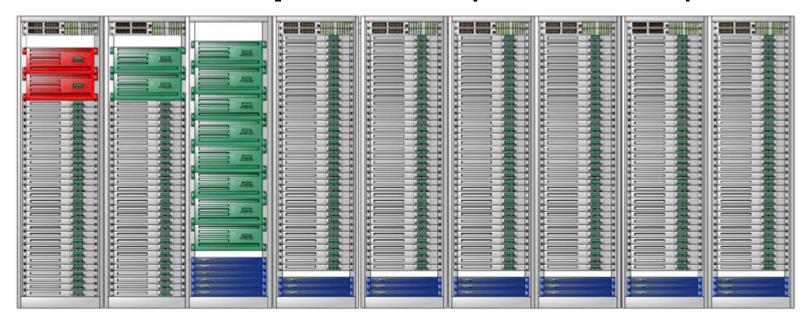


Networks connect multiple stand-alone computers (nodes) to make larger parallel computer clusters.



## Parallel Computers:

### for example: LLNL parallel computer



compute node

infiniband switch

management hardware



login / remote partition server node



gateway node



## Why Use Parallel Computing?

- SAVE TIME AND MONEY:
  - a. throwing more resources at a task will shorten its time to completion
  - b. can be built from cheap, commodity components





## Why Use Parallel Computing?

- SOLVE LARGER / MORE COMPLEX PROBLEMS:
  - Many problems are so large and/or complex that it is impractical or impossible to solve them on a single computer, especially given limited computer memory.

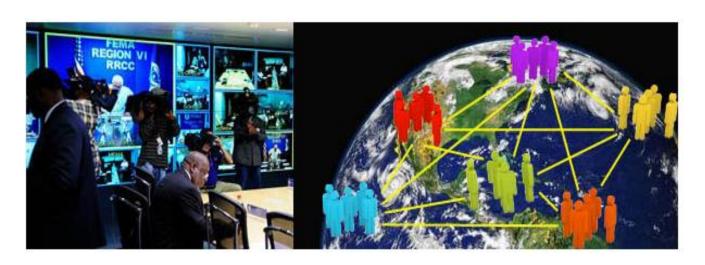




## Why Use Parallel Computing?

#### PROVIDE CONCURRENCY:

Multiple compute resources can do many things simultaneously.



Example: Web search engines/databases processing millions of transactions every second



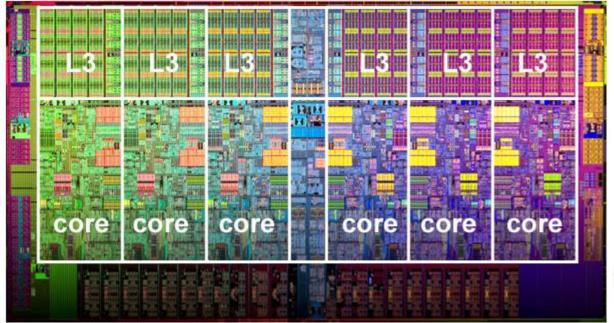
- Why Use Parallel Computing?
  - TAKE ADVANTAGE OF NON-LOCAL RESOURCES:
    - Using compute resources on a wide area network, or even the Internet when local compute resources are scarce or insufficient.





- Why Use Parallel Computing?
  - MAKE BETTER USE OF UNDERLYING PARALLEL HARDWARE:

Serial programs run on modern computers(are parallel in architecture with multiple processors/cores)"waste" potential computing power.



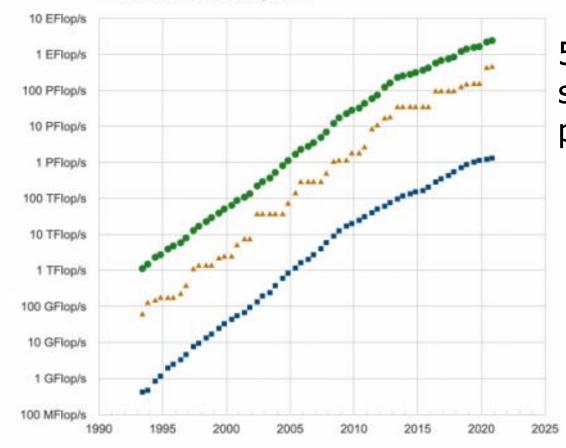
Intel Xeon processor with 6 cores and 6 L3 cache units



#### The future

• Exaflop =  $10^{18}$  calculations per second.

#### Performance Development



500,000x increase in supercomputer performance



## **TOP10 Supercomputers (November, 2021)**

### (https://www.top500.org/lists/top500/2021/11/)

Rank	System	Country	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	Supercomputer Fugaku	Japan	7,630,848	442,010.00	537,212.00	29,899
2	Summit	United States	2,414,592	148,600.0	200,794.9	10,096
3	Sierra	United States	1,572,480	94,640.0	125,712.0	7,438
4	Sunway TaihuLight	China	10,649,600	93,014.6	125,435.9	15,371
5	Perlmutter	United States	761,856	70,870.0	93,750.0	2,589
6	Selene	United States	555,520	63,460.0	79,215.0	2,646
7	Tianhe-2A	China	4,981,760	61,444.5	100,678.7	18,482
8	JUWELS Booster Module	Germany	449,280	44,120.0	70,980.0	1,764
9	HPC5	Italy	669,760	35,450.0	51,720.8	2,252
10	Voyager-EUS2	United States	253,440	30,050.0	39,531.2	

21

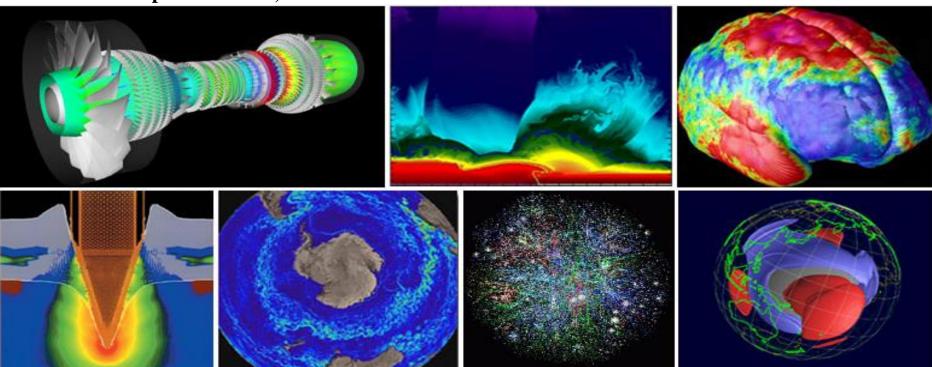
## Who is Using Parallel Computing?

### Science and Engineering:

- 1. Atmosphere, Earth, Environment
- 3. Bioscience, Biotechnology, Genetics
- 5. Geology, Seismology
- 7. Electrical Engineering, Circuit Design, Microelectronics
- 8. Computer Science, Mathematics



- 4. Chemistry, Molecular Sciences
- 6. Mechanical Engineering

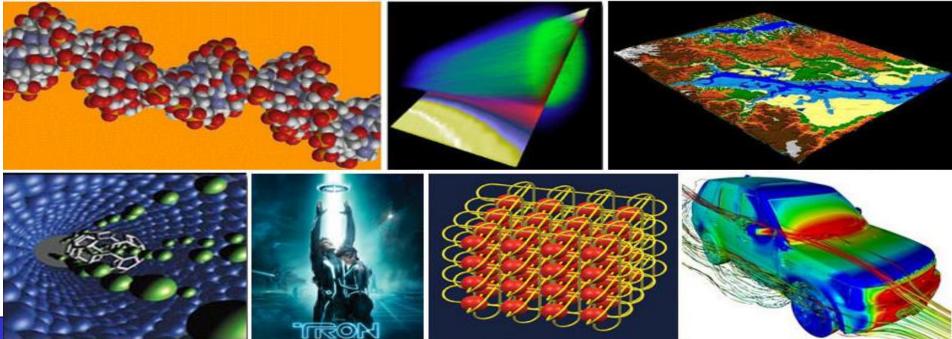


## Who is Using Parallel Computing?

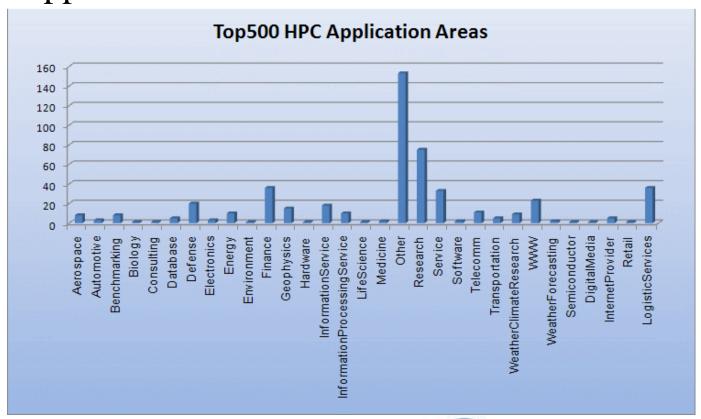
- Industrial and Commercial:
  - 1. "Big Data", databases, data mining
  - 3. Web search engines
  - 5. Pharmaceutical design

- 2. Oil exploration
- 4. Medical imaging and diagnosis
- 6. Financial and economic modeling
- 7. Management of national and multi-national corporations
- 8. Advanced graphics and virtual reality 9. Networked video and multi-media technologies

10. Collaborative work environments



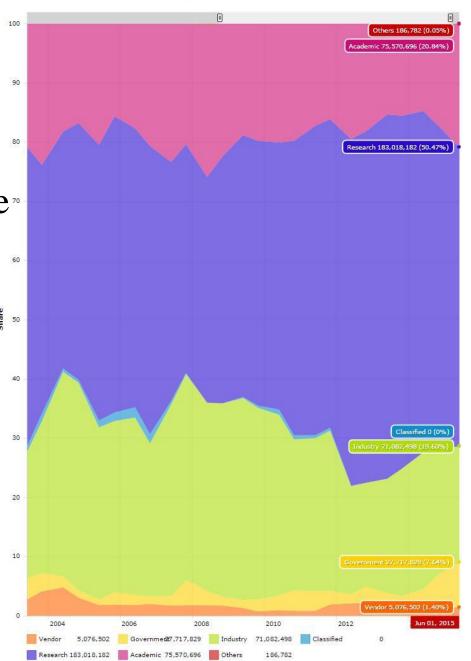
- Who is Using Parallel Computing?
  - Global Applications:





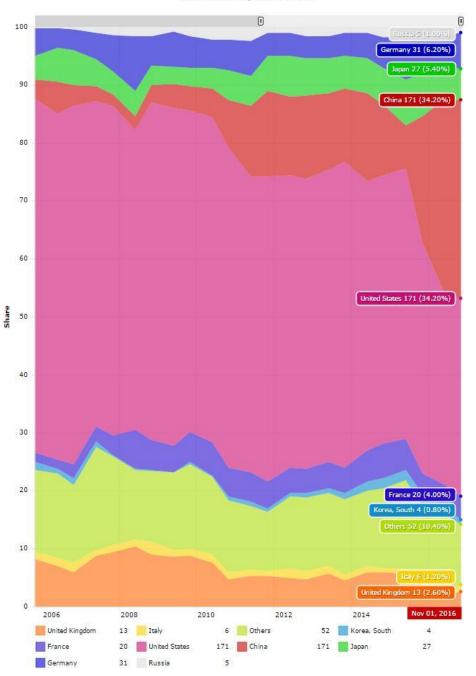
Who is Using Parallel Computing?

Segments – Performance Share<sup>70</sup>



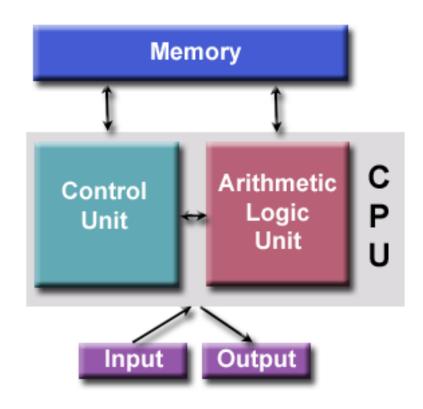
Who is Using Parallel Computing?

Countries – Systems share





#### Von Neumann Architecture



Basic computing architecture

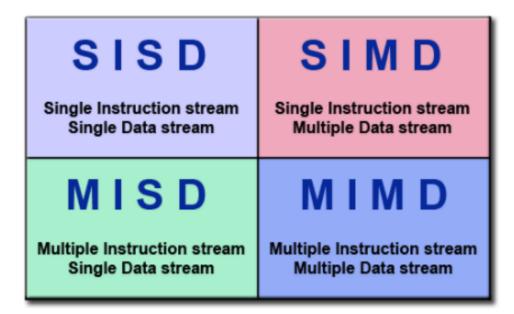


John von Neumann circa 1940s (Source: LANL archives)



## Flynn's Classical Taxonomy

 according to how they can be classified along the two independent dimensions of *Instruction* Stream and *Data Stream*





#### SISD

Single Instruction stream Single Data stream

## Flynn's Classical Taxonomy

Single Instruction, Single Data (SISD):



UNIVAC1



**CDC 7600** 



**IBM 360** 



PDP1

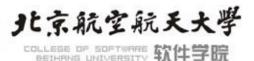


CRAY1



**Dell Laptop** 



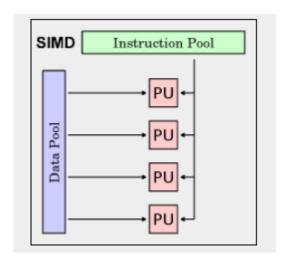


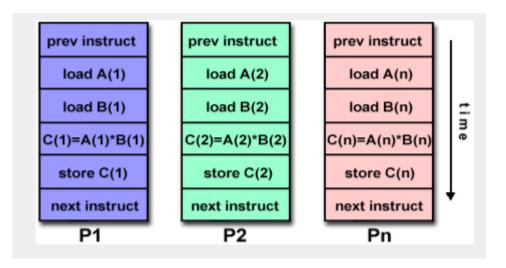
#### SIMD

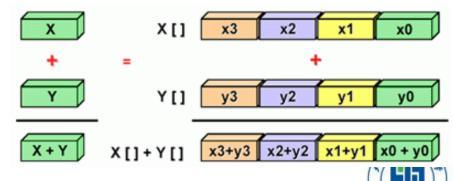
Single Instruction stream Multiple Data stream

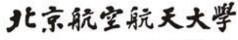
## Flynn's Classical Taxonomy

Single Instruction, Multiple Data (SIMD):









#### SIMD

Single Instruction stream Multiple Data stream

## Flynn's Classical Taxonomy

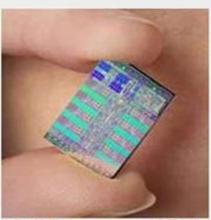
Single Instruction, Multiple Data (SIMD):



ILLIAC IV



MasPar



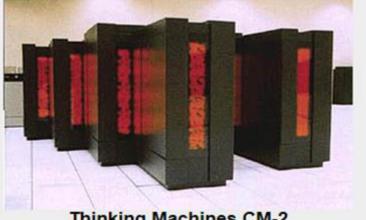
Cell Processor (GPU)



Cray X-MP



Cray Y-MP

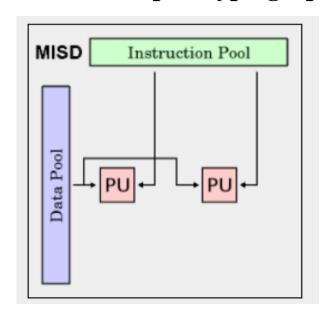


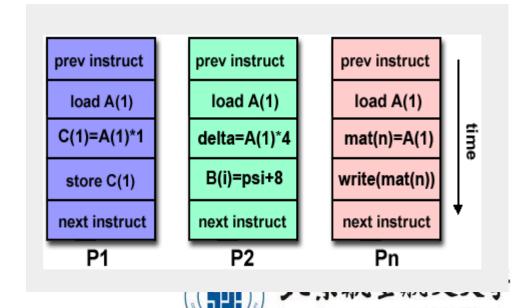
Thinking Machines CM-2

## MISD Multiple Instruction stream Single Data stream

## Flynn's Classical Taxonomy

- Multiple Instruction, Single Data (MISD):
   Few (if any) actual examples have ever existed. Some conceivable uses might be:
  - > multiple frequency filter
  - multiple cryptography algorithms





#### MIMD

Multiple Instruction stream Multiple Data stream

## Flynn's Classical Taxonomy

• Multiple Instruction, Multiple Data (MIMD):



**IBM POWER5** 



HP/Compaq Alphaserver





**AMD Opteron** 



Cray XT3

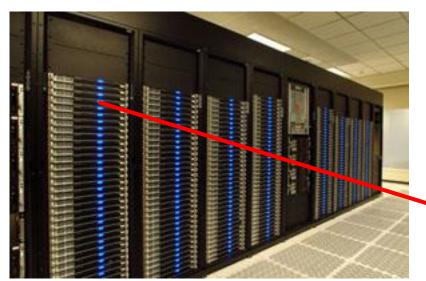


IBM BG/L BEIHANG UNIVERSITY キルトナゴエ

## Some General Parallel Terminology

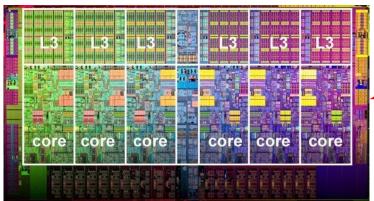
- Supercomputing / High Performance Computing (HPC)
- Node
- CPU / Socket / Processor / Core
- Task
- Pipelining
- Shared Memory
- Symmetric Multi-Processor (SMP)
- Distributed Memory



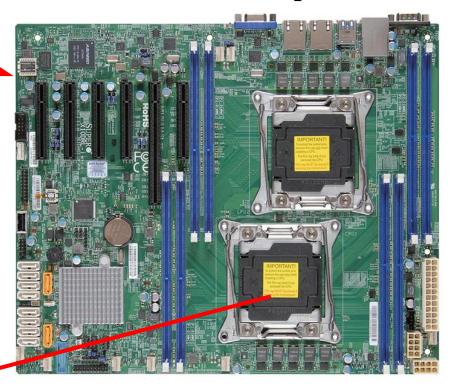


Supercomputer – each blue light is a node

## **CPU/Processer/Socket – each** has multiple cores/processers.



#### Node – standalone Von Neumann computer



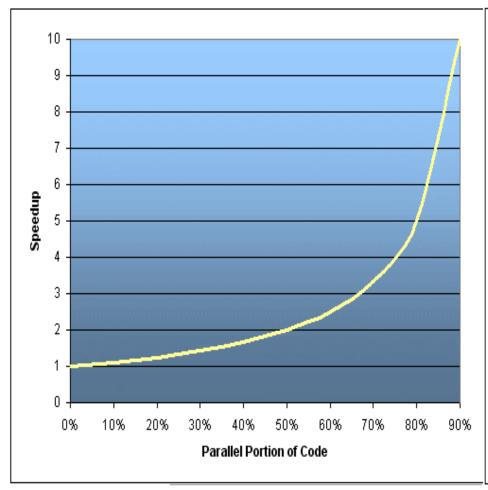


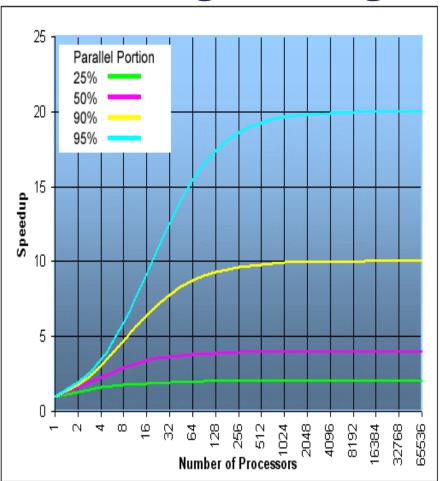
# Some General Parallel Terminology

- Communications
- Synchronization
- Computational Granularity
- Observed Speedup
- Parallel Overhead
- Massively Parallel
- Embarrassingly Parallel
- Scalability



# Limits and Costs of Parallel Programming







北京航空航天大學 COLLEGE OF SOFTWARE 软件学院

- Limits and Costs of Parallel Programming
  - Complexity:
    - Design
    - Coding
    - Debugging
    - Tuning
    - Maintenance



# Limits and Costs of Parallel Programming

- Portability:
  - All of the usual portability issues associated with serial programs apply to parallel programs.
  - sometimes to the point of requiring code modifications in order to effect portability.
  - Operating systems can play a key role in code portability issues.
  - Hardware architectures are characteristically highly variable and can affect portability.



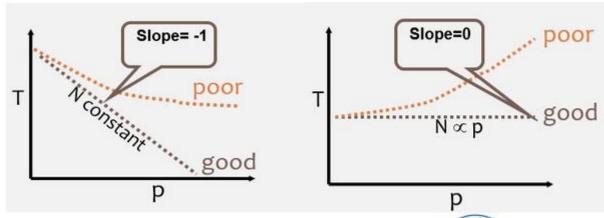
# Limits and Costs of Parallel Programming

- Resource Requirements:
  - The amount of memory required can be greater for parallel codes than serial codes,
  - For short running parallel programs, there can actually be a decrease in performance.



# Limits and Costs of Parallel Programming

- Scalability:
  - Strong scaling
    - to run the same problem size faster
  - Weak scaling
    - to run larger problem in same amount of time



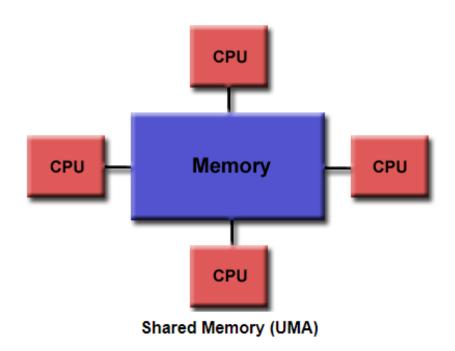


# Shared Memory

- General Characteristics:
  - shared memory machines have been classified as *UMA* and *NUMA*, based upon memory access times.

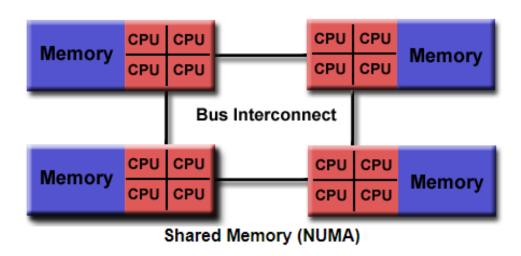


- Shared Memory
  - General Characteristics:
  - Uniform Memory Access: UMA





- Shared Memory
  - General Characteristics:
  - Non-Uniform Memory Access: NUMA





# Shared Memory

#### Advantages:

- provides a user-friendly programming perspective to memory.
- data sharing between tasks is both fast and uniform.

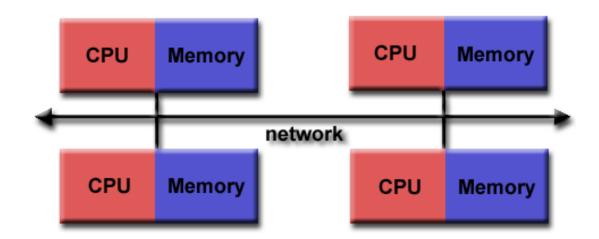
#### Disadvantages:

lack of scalability between memory and CPUs.



# Distributed Memory

 distributed memory systems vary widely but share a common characteristic.





# Distributed Memory

#### Advantage:

- Memory is scalable with the number of processors.
- Can use commodity, off-the-shelf processors and networking.

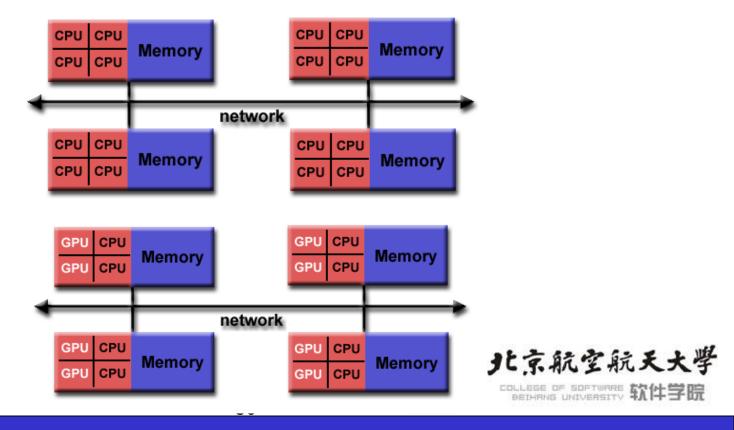
#### Disadvantage:

- It's difficult to map existing data structures, based on global memory, to memory organization.
- Non-uniform memory access times data residing on a remote node takes longer to access than node local data.



# Hybrid Distributed-Shared Memory

 The largest and fastest computers in the world today employ both shared and distributed memory architectures.



- Hybrid Distributed-Shared Memory
  - Advantages and Disadvantages
    - Whatever is common to both shared and distributed memory architectures.
    - Increased scalability is an important advantage
    - Increased programmer complexity is an important disadvantage



# Parallel Programming

# **Models**



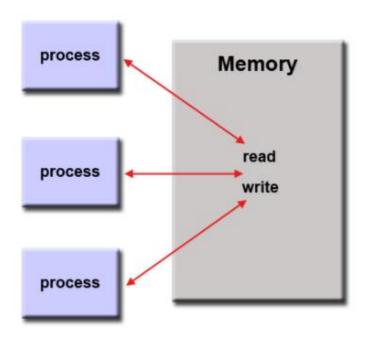
#### Models

- 1. Shared Memory (without threads)
- 2. Threads
- 3. Distributed Memory / Message Passing
- 4. Data Parallel
- 5. Hybrid
- 6. SPMD and MPMD



#### 1. Shared Memory Model (without threads)

Processes/tasks share a common address space, which they read and write to asynchronously.



Implementations: On standalone shared memory machines, native operating systems, compilers and/ or hardware provide support for shared memory programming

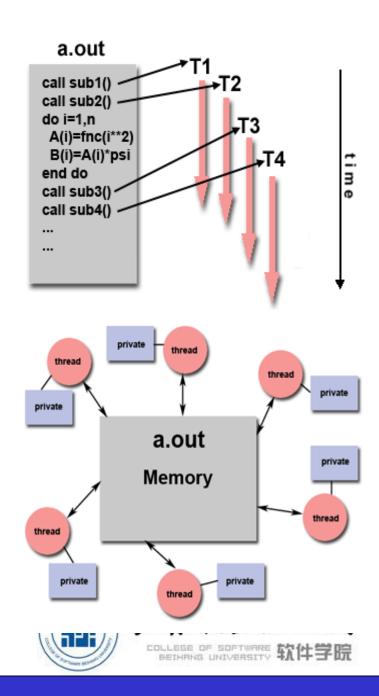


#### 2. Threads Model

shared memory programming.

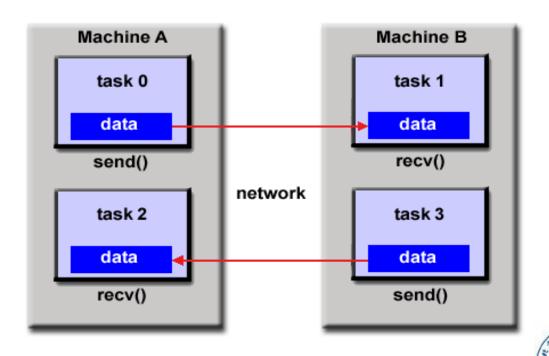
#### **Implementations:**

- A library of subroutines that are called from within parallel source code.
- A set of compiler directives imbedded in either serial or parallel source code.



#### 3. Distributed Memory / Message Passing Model

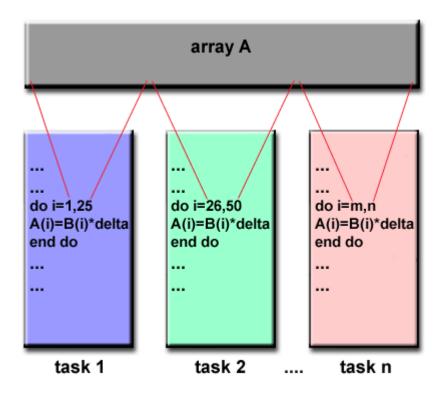
- Tasks that use their own local memory during computation.
- Data transfer usually requires cooperative operations.
- Data communications through sending and receiving messages.





#### 4. Data Parallel Model (Partitioned Global Address Space -PGAS)

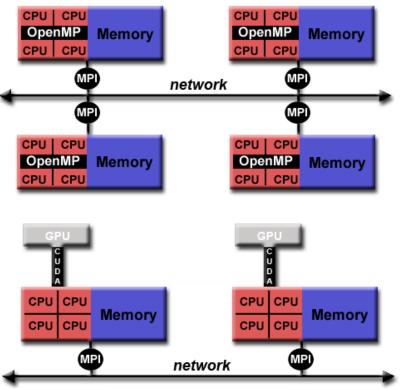
- Address space is treated globally
- Focuses on performing operations on a data set.
- Tasks perform the same operation on their partition of work.





#### 5. Hybrid Model

- Threads perform computationally intensive kernels using local, on-node data.
- Communications between processes on different nodes occurs over the network using MPI.

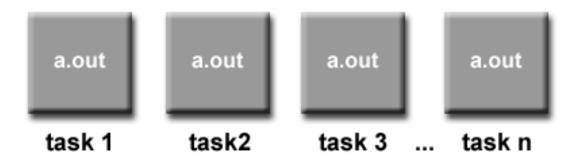


Lend to most popular (currently) hardware environment of clustered multi/many-core machines.

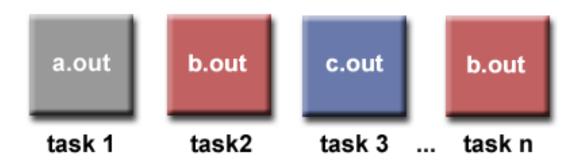


#### 6. SPMD and MPMD

Single Program Multiple Data (SPMD):



• Multiple Program Multiple Data (MPMD):







#### 1. Automatic vs. Manual Parallelization

- Manual parallelization: time consuming, complex, errorprone and *iterative* process.
- Automatic parallelization: ⇒ parallelizing compiler
  - Fully Automatic
    - compiler analyzes the source code and identifies opportunities for parallelism.
  - Programmer Directed
    - Using "compiler directives" or possibly compiler flags



# 2. Understand the Problem and the Program

1. First step in developing parallel software is to first understand the problem.

If you are starting with a serial program, this necessitates understanding the existing code also.

- 2. Determine whether or not the problem that can actually be parallelized.
- A parallelizable problem: The calculation of the minimum energy conformation.
- ➤ a little-to-no parallelism problem : Calculation of the Fibonacci series (0,1,1,2,3,5,8,13,21,...) by use of the formula: F(n) = F(n-1) + F(n-2)



# 2. Understand the Problem and the Program

- 3. Identify the program's *hotspots*.
- ➤ Know where most of the real work is being done.
- Focus on parallelizing the hotspots, ignore those sections of the program that account for little CPU usage.
- 4. Identify bottlenecks in the program.

> disproportionately slow areas, or cause parallelizable work to

halt or be deferred?

reduce or eliminate unnecessary slow areas.



# 2. Understand the Problem and the Program

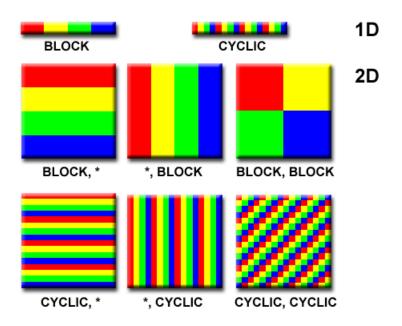
- 5. Identify inhibitors to parallelism.
- ➤ One common class of inhibitor is *data dependence*
- 6. Investigate other algorithms
- This may be the single most important consideration
- 7. Take advantage of optimized third party parallel software.
- ➤ IBM's ESSL, Intel's MKL, AMD's AMCL, etc.

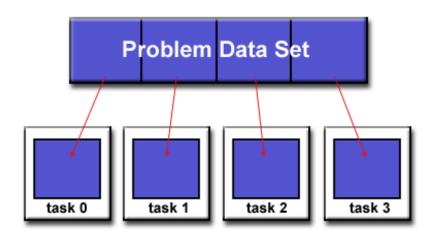


# 3. Partitioning

#### **Domain Decomposition:**

- the data associated with a problem is decomposed
- ways to partition data:



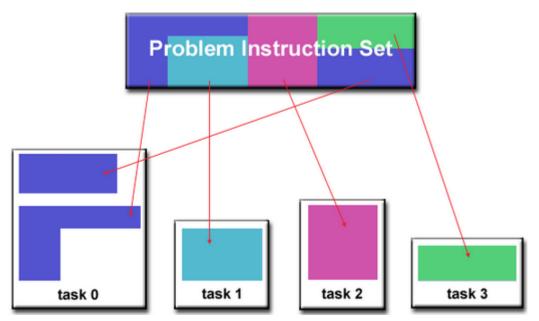




# 3. Partitioning

#### **Functional Decomposition:**

Problem is decomposed according to the work that must be done.

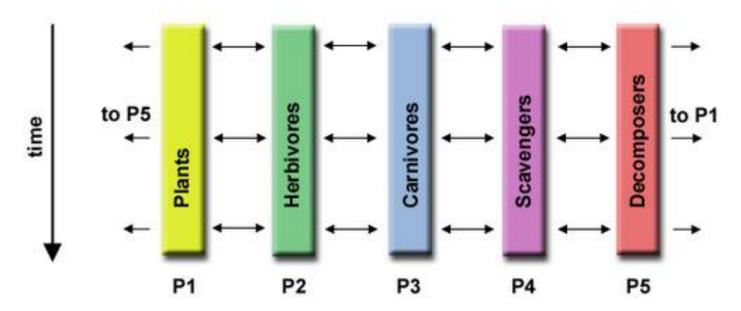




# 3. Partitioning

#### **Functional Decomposition:**

Ecosystem Modeling

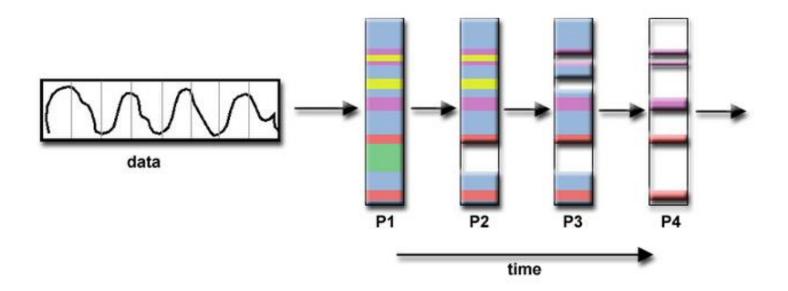




# 3. Partitioning

#### **Functional Decomposition:**

Signal Processing

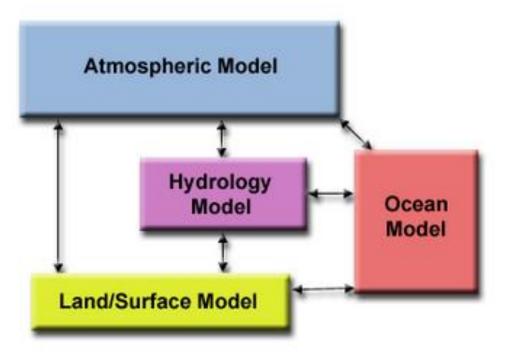




# 3. Partitioning

#### **Functional Decomposition:**

Climate Modeling





# How do we write parallel programs?

# Task parallelism

 Partition various tasks carried out solving the problem among the cores.

# Data parallelism

- Partition the data used in solving the problem among the cores.
- Each core carries out similar operations on it's part of the data.



# **Professor P**

15 questions300 exams



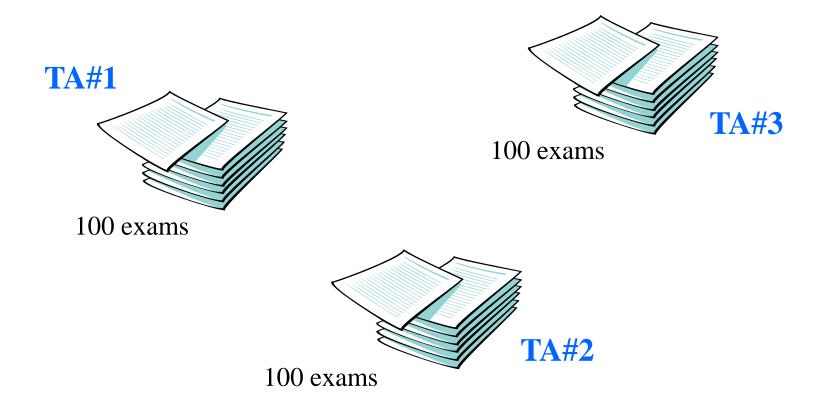


# **Professor P's grading assistants**





# Division of work – data parallelism





# **Division of work – task parallelism**

**TA#1** 



TA#3

Questions 11 - 15

Questions 1 - 5



Questions 6 - 10

**TA#2** 

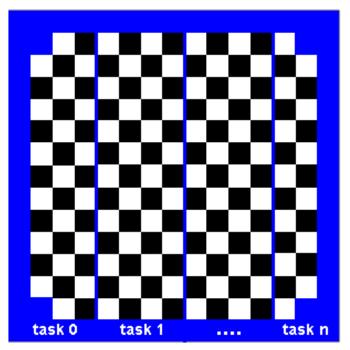


#### 4. Communications

#### Who Needs Communications?

You DON'T need communications:

• *embarrassingly parallel* - little or no communications are required, no need for tasks to share data.



Eg. every pixel in a black and white image needs to have its color reversed.

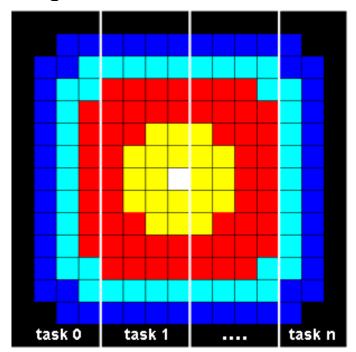


#### 4. Communications

#### Who Needs Communications?

You DO need communications:

 Most parallel applications are not quite so simple, and do require tasks to share data with each other.



Eg. 2-D heat diffusion problem



#### 4. Communications

#### **Factors to Consider:**

- Communication overhead
- Latency vs. Bandwidth
- Visibility of communications
- Synchronous vs. asynchronous communications
  - > Synchronous communications are often referred to as *blocking* communications
  - Asynchronous communications are often referred to as *non-blocking* communications

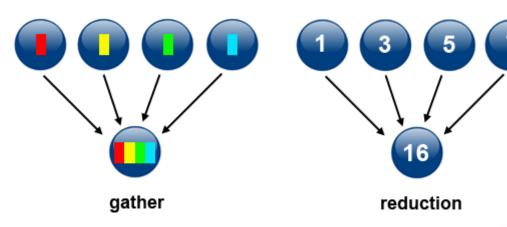


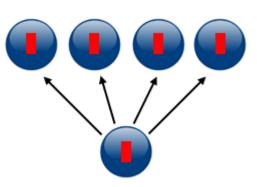
## 4. Communications

#### **Factors to Consider:**

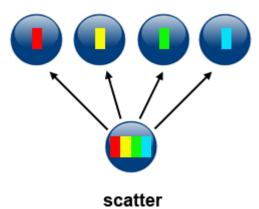
- Scope of communications
  - > *Point-to-point* involves two tasks.
  - > Collective involves more than two tasks

#### Eg.

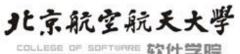


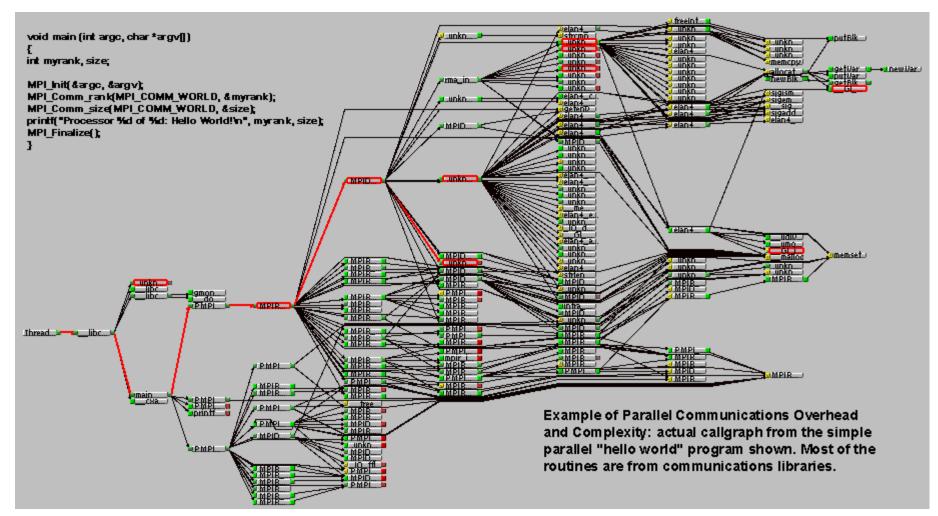


broadcast











# 5. Synchronization

#### **Types of Synchronization**

#### Barrier:

Each task performs its work until it reaches the barrier. It then stops, or "blocks".

#### Lock / semaphore:

Typically used to serialize (protect) access to global data or a section of code.

#### Synchronous communication operations:

Involves only those tasks executing a communication operation



# 6. Data Dependencies

#### Definition:

- > A *dependence* exists between program statements when the order of statement execution affects the results of the program.
- > A *data dependence* results from multiple use of the same location(s) in storage by different tasks.



# 6. Data Dependencies

Examples:

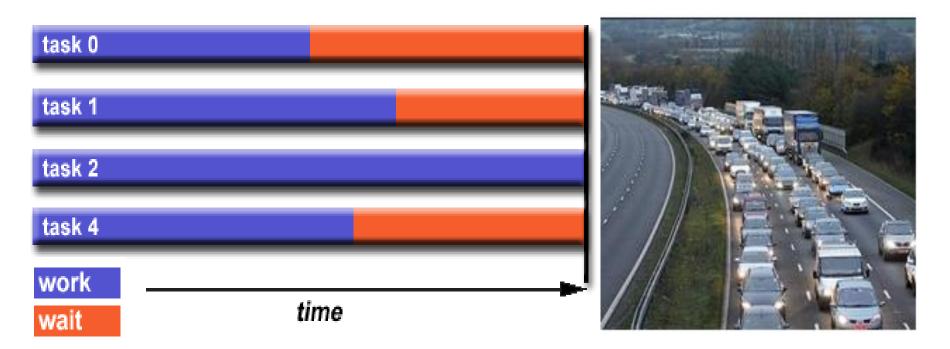
#### **Loop carried data dependence**

#### Loop independent data dependence



# 7. Load Balancing

a minimization of task idle time.





# 7. Load Balancing

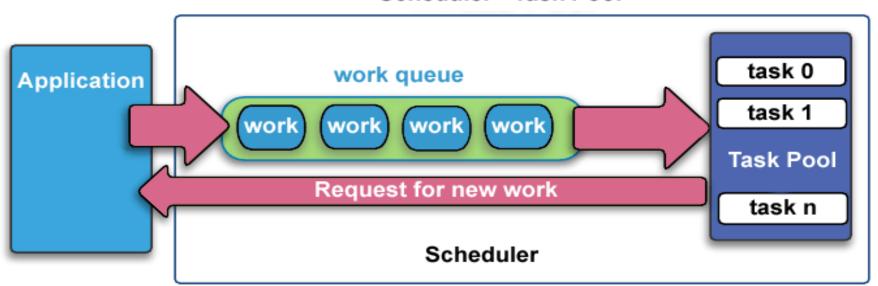
- How to Achieve Load Balance:
  - Equally partition the work each task receives
  - > evenly distribute the data set among the tasks.
  - > evenly distribute the iterations across the tasks.
  - > If a heterogeneous mix of machines with varying performance characteristics are being used, adjust work accordingly.



# 7. Load Balancing

- How to Achieve Load Balance:
  - Use dynamic work assignment

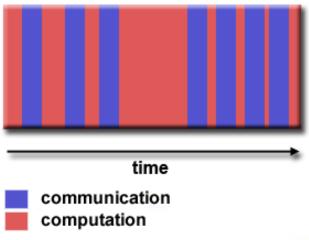
#### Scheduler - Task Pool



scheduler-task pool approach



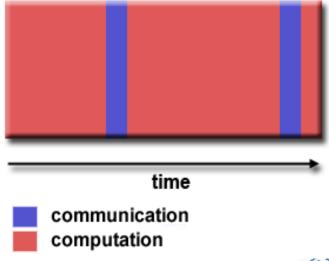
- **8. Granularity**  $\Rightarrow$  Computation / Communication Ratio
  - Fine-grain Parallelism:
    - Low computation to communication ratio
    - Facilitates load balancing
    - high communication overhead and less performance enhancement





# 8. Granularity

- Coarse-grain Parallelism:
  - High computation to communication ratio
  - Implies more opportunity for performance increase
  - Harder to load balance efficiently



# Which is Best? (Fine-grain & Coarse-grain)

- The overhead associated with communications and synchronization is high relative to execution speed, so it is advantageous to have *coarse granularity*.
- *Fine-grain* parallelism can help reduce overheads due to load imbalance.



#### 9. I/O

**■ The Bad News:** 

**Memory Hierarchy** 

Registers	
Cache	
Main memory	
Magnetic disk	
Magnetic tape	

1 ns	1x
10 ns	10x
100 ns	100x
100 ms	100,000,000x
10 s	1e+10x



#### 9. I/O

#### ■ The Good News:

- Parallel file systems are available
- > GPFS : General Parallel File System (IBM)
- > Lustre : for Linux clusters (Intel)
- > HDFS : Hadoop Distributed File System (Apache)
- > PanFS: Panasas ActiveScale File System for Linux clusters (Panasas, Inc.)
- The parallel I/O programming interface specification for MPI is available.



#### 9. I/O

#### A few pointers:

- Reduce overall I/O as much as possible.
- If you have access to a parallel file system, use it.
- Writing large chunks of data rather than small.
- Fewer, larger files performs better than many small files.
- Combine I/O operations across tasks.
- Confine I/O to specific serial portions of the job, and then use parallel communications to distribute data to parallel tasks.

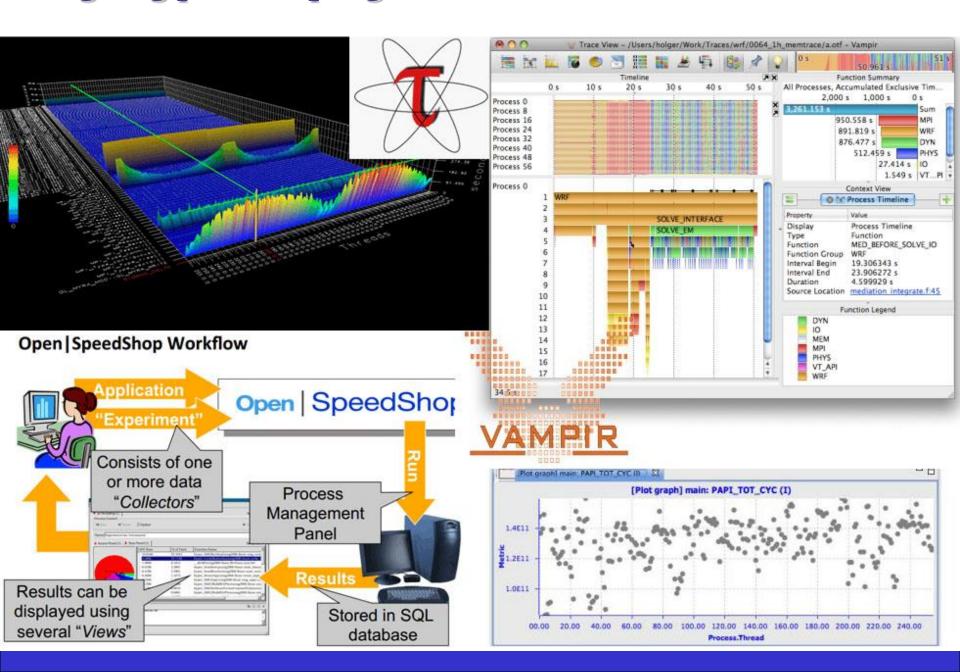


# 10.Debugging

- can be difficult, particularly as codes scale upwards.
- some excellent debuggers available to assist:
  - Threaded pthreads and OpenMP
  - MPI
  - GPU / accelerator
  - Hybrid
- For example:
  - TotalView from RogueWave Software
  - Stack Trace Analysis Tool (STAT)
  - DDT from Allinea
  - Inspector from Intel

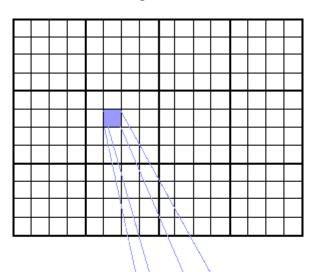








# 1. Array Processing



- The computation on each array element is independent from other array elements.
- The problem is computationally intensive.
- The serial program calculates one element at a time in sequential order
- Serial code could be of the form:

A calculations on 2-dimensional array elements, a function is evaluated on each array element.

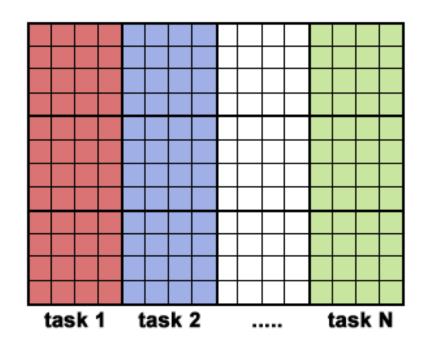
fcn( i, j )



# 1. Array Processing

#### Parallel Solution 1

- The calculation of elements is independent.
- Arrays elements are evenly distributed.



- Independent calculation ensures there is no need for communication or synchronization between tasks.
- There should not be load balance concerns.
- After the array is distributed, each task executes the loop corresponding it owns.



# 1. Array Processing

#### One Possible Solution:

- Implement as a Single Program Multiple Data (SPMD) model - every task executes the same program.
- Master process initializes array, sends info to worker processes and receives results.
- Worker process receives info, performs its share of computation and sends results to master.
- Using the Fortran storage scheme, perform block distribution of the array.



# 1. Array Processing

#### One Possible Solution:

Pseudo code:

red highlights changes for parallelism.

```
find out if I am MASTER or WORKER
if I am MASTER
  initialize the array
  send each WORKER info on part of array it owns
  send each WORKER its portion of initial array
  receive from each WORKER results
else if I am WORKER
  receive from MASTER info on part of array I own
  receive from MASTER my portion of initial array
  # calculate my portion of array
  do j = my first column, my last column
    do i = 1,n
      a(i,j) = fcn(i,j)
    end do
  end do
  send MASTER results
endif
```

# 1. Array Processing

Parallel Solution 2: Pool of Tasks

Solve load balance problem.

- Pool of Tasks Scheme:
  - Master Process:
    - > Holds pool of tasks for worker processes to do
    - > Sends worker a task when requested
    - > Collects results from workers
  - Worker Process: repeatedly does the following
    - Gets task from master process
    - Performs computation
    - > Sends results to master



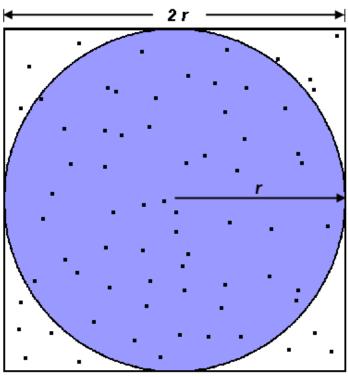
# 1. Array Processing

- Pool of Tasks Scheme:
  - The faster tasks will get more work to do
  - Pseudo code solution: red highlights changes for parallelism.
    find out if I am MASTER or WORKER

```
if I am MASTER
 do until no more jobs
    if request send to WORKER next job
    else receive results from WORKER
  end do
else if I am WORKER
 do until no more jobs
    request job from MASTER
    receive from MASTER next job
    calculate array element: a(i,j) = fcn(i,j)
    send results to MASTER
  end do
endif
```

#### 2. PI Calculation

The value of PI can be calculated in various ways.



$$A_S = (2r)^2 = 4r^2$$
  
 $A_C = \pi r^2$   
 $\pi = 4 \times \frac{A_C}{r}$ 

Monte Carlo method

#### Serial pseudo code

```
npoints = 10000
circle count = 0
do j = 1, npoints
  generate 2 random numbers between 0 and 1
  xcoordinate = random1
  ycoordinate = random2
  if (xcoordinate, ycoordinate) inside circle
  then circle count = circle count + 1
end do
```

PI = 4.0\*circle count/npoints

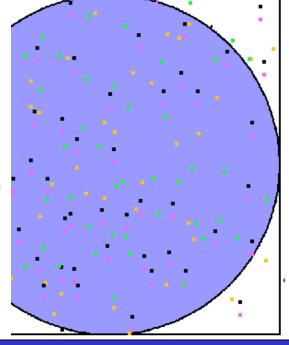


```
npoints = 10000
circle count = 0
p = number of tasks
num = npoints/p
find out if I am MASTER or WORKER
do j = 1, num
  generate 2 random numbers between 0 and 1
  xcoordinate = random1
  ycoordinate = random2
  if (xcoordinate, ycoordinate) inside circle
  then circle count = circle count + 1
end do
if I am MASTER
  receive from WORKERS their circle counts
  compute PI (use MASTER and WORKER calculations)
else if I am WORKER
  send to MASTER circle count
endif
```

ıllelize:

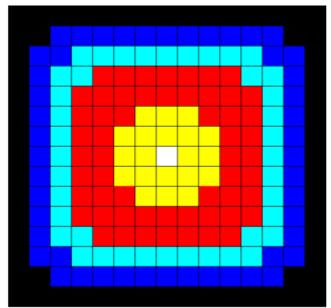
balance concerns

tasks.



# 3. Simple Heat Equation

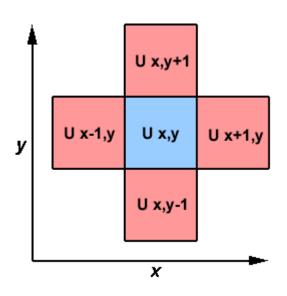
- Describes the temperature change over time, given initial temperature distribution and boundary conditions.
- A finite differencing scheme is employed to solve the heat equation numerically on a square region.





# 3. Simple Heat Equation

• The calculation of an element is *depenent* upon neighbor element values:



```
do iy = 2, ny - 1
  do ix = 2, nx - 1
    u2(ix, iy) = u1(ix, iy) +
        cx * (u1(ix+1,iy) + u1(ix-1,iy) - 2.*u1(ix,iy)) +
        cy * (u1(ix,iy+1) + u1(ix,iy-1) - 2.*u1(ix,iy))
  end do
end do
```

$$U_{x,y} = U_{x,y}$$

$$+ C_x * (U_{x+1,y} + U_{x-1,y} - 2 * U_{xy})$$

$$+ C_y * (U_{x,y+1} + U_{x,y-1} - 2 * U_{x,y})$$

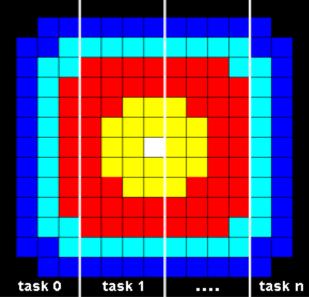


# 3. Simple Heat Equation

```
find out if I am MASTER or WORKER
if I am MASTER
 initialize array
  send each WORKER starting info and subarray
 receive results from each WORKER
else if I am WORKER
 receive from MASTER starting info and subarrayer tasks
 # Perform time steps
  do t = 1, nsteps
   update time
    send neighbors my border info
    receive from neighbors their border info
   update my portion of solution array
  end do
  send MASTER results
endi f
```

outed as subarrays to

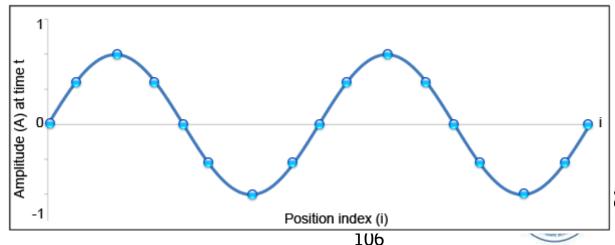
r tasks ighbor task's data,



# 4. 1-D Wave Equation

 the amplitude along a uniform, vibrating string is calculated after a specified amount of time has elapsed.

i as the position index along the x axis, node points imposed along the string, update of the amplitude at discrete time steps.





# 4. 1-D Wave Equation

The equation to be solved is the one-dimensional wave equation:

$$A(i,t+1) = (2.0*A(i,t)) - A(i,t-1) + (c*(A(i-1,t) - (2.0*A(i,t)) + A(i+1,t)))$$

where c is a constant

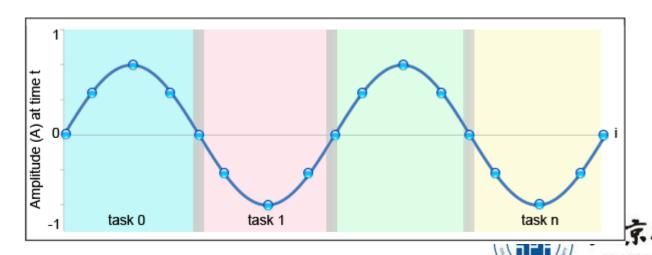


# 4. 1-D Wave Equation

#### Parallel Solution

- Involve communications and synchronization.
- Each task owns an equal portion of the total array.
- Load balancing
- Communication need only occur on data borders.

#### Pseudo code solution



```
find out number of tasks and task identities
#Identify left and right neighbors
left neighbor = mytaskid - 1
right neighbor = mytaskid +1
if mytaskid = first then left neigbor = last
if mytaskid = last then right neighbor = first
find out if I am MASTER or WORKER
if I am MASTER
  initialize array
  send each WORKER starting info and subarray
else if I am WORKER'
  receive starting info and subarray from MASTER
endif
#Perform time steps
#In this example the master participates in calculations
do t = 1, nsteps
  send left endpoint to left neighbor
  receive left endpoint from right neighbor
  send right endpoint to right neighbor
  receive right endpoint from left neighbor
  #Update points along line
  do i = 1, npoints
    newval(i) = (2.0 * values(i)) - oldval(i)
    + (sqtau * (values(i-1) - (2.0 * values(i)) + values(i+1)))
  end do
```

#Collect results and write to file
if I am MASTER
 receive results from each WORKER
 write results to file
else if I am WORKER
 send results to MASTER
endif



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end do

# Thank you!

https://computing.llnl.gov/tutorials/parallel\_comp

