

Chapter 3 Distributed Memory Programming with MPI

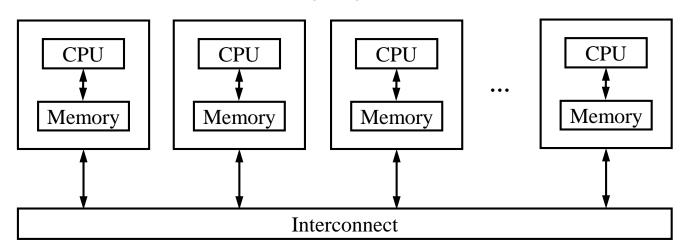
软件学院 邵兵 2022/3/18

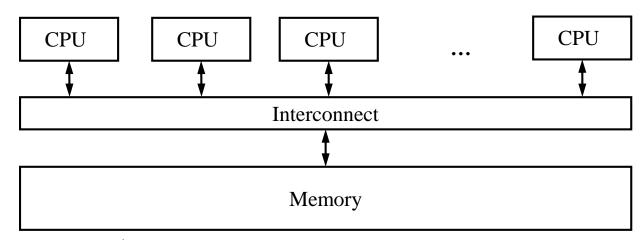
Contents

- 1. Writing our first MPI program
- 2. The Trapezoidal Rule in MPI
- 3. Collective communication
- 4. MPI derived datatypes
- 5. Performance evaluation of MPI programs
- 6. Parallel sorting



A distributed memory system





A shared memory system



1. WRITING OUR FIRST MPI PROGRAM

```
#include <stdio.h>
int main(void) {
    printf("hello, world\n");
    return 0;
}
```



(a classic)



Identifying MPI processes

- Common practice to identify processes by nonnegative integer ranks.
- p processes are numbered 0, 1, 2, ... p-1



Our first MPI program

```
#include <stdio.h>
  #include <string.h> /* For strlen
  #include <mpi.h> /* For MPI functions, etc */
4
  const int MAX STRING = 100;
6
   int main(void) {
8
             greeting[MAX STRING];
      char
                        /* 处理器个数 */
      int
             comm sz;
                        /* 当前进程(My process)序号 */
10
      int
             my rank;
11
12
     MPI Init(NULL, NULL);
      MPI Comm size (MPI COMM WORLD, &comm sz);
13
14
      MPI Comm rank (MPI COMM WORLD, &my rank);
15
16
      if (my rank != 0) {
17
         sprintf(greeting, "Greetings from process %d of %d!",
18
               my rank, comm sz);
19
         MPI Send(greeting, strlen(greeting)+1, MPI CHAR, 0, 0,
20
               MPI COMM WORLD);
21
      } else {
22
        printf("Greetings from process %d of %d!\n", my rank, comm sz);
23
         for (int q = 1; q < comm sz; q++) {
24
            MPI Recv(greeting, MAX STRING, MPI CHAR, q,
               0, MPI COMM WORLD, MPI STATUS IGNORE);
25
26
            printf("%s\n", greeting);
27
         }
28
      }
29
30
     MPI Finalize();
31
      return 0;
32 } /* main */
```



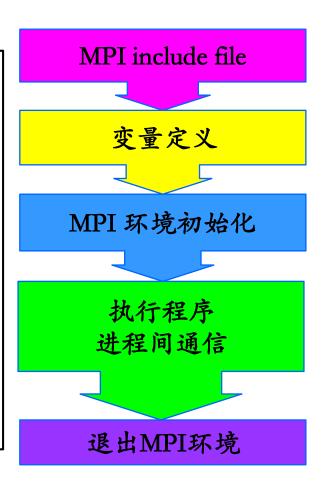
mpi_hello.c

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MPI程序基本结构

```
#include <mpi.h>
void main(int argc, char *argv[]) {
   int comm sz, my rank, ierr;
   ierr = MPI Init(&argc, &argv);
  MPI Comm rank (MPI COMM WORLD, &my_rank);
  MPI Comm size (MPI COMM WORLD, &comm sz);
   /* Do some works */
   ierr = MPI Finalize();
```





Compilation

wrapper script to compile source file -o mpi hello mpi hello.c mpicc -g -Wall produce create this executable file name debugging (as opposed to default a.out) information turns on all warnings



Execution

mpiexec -n <number of processes> <executable>

mpiexec -n 1 ./mpi_hello

run with 1 process

mpiexec -n 4 ./mpi_hello

- run with 4 processes



Execution

```
mpiexec -n 1 ./mpi_hello

Greetings from process 0 of 1 !
```

```
mpiexec -n 4 ./mpi_hello

Greetings from process 0 of 4 !

Greetings from process 1 of 4 !

Greetings from process 2 of 4 !

Greetings from process 3 of 4 !
```



MPI Programs

- Written in C.
 - Has main.
 - Uses stdio.h, string.h, etc.
- Need to add mpi.h header file.
- Identifiers defined by MPI start with "MPI_".
- First letter following underscore is uppercase.
 - For function names and MPI-defined types.
 - Helps to avoid confusion.



MPI Components

- MPI_Init
 - Tells MPI to do all the necessary setup.

```
int MPI_Init(
   int* argc_p /* in/out */,
   char*** argv_p /* in/out */);
```

- MPI_Finalize
 - Tells MPI we're done, so clean up anything allocated for this program.

```
int MPI_Finalize(void);
```



Basic Outline

```
#include <mpi.h>
int main(int argc, char* argv[]) {
   /* No MPI calls before this */
  MPI Init(&argc, &argv);
  MPI Finalize();
   /* No MPI calls after this */
  return 0;
```



Communicators

- A collection of processes that can send messages to each other.
- MPI_Init defines a communicator that consists of all the processes created when the program is started.
- Called MPI_COMM_WORLD.



Communicators

```
int MPI_Comm_size(
    MPI_Comm comm /* in */,
    int* comm_sz_p /* out */);
```

number of processes in the communicator

```
int MPI_Comm_rank(
    MPI_Comm comm /* in */,
    int* my_rank_p /* out */);

my rank

(the process making this call)
```

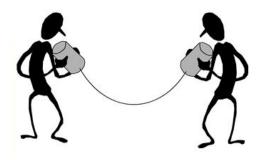


SPMD

- Single-Program Multiple-Data
- **■** We compile <u>one</u> program.
- Process 0 does something different.
 - Receives messages and prints them while the other processes do the work.
- The if-else construct makes our program SPMD.

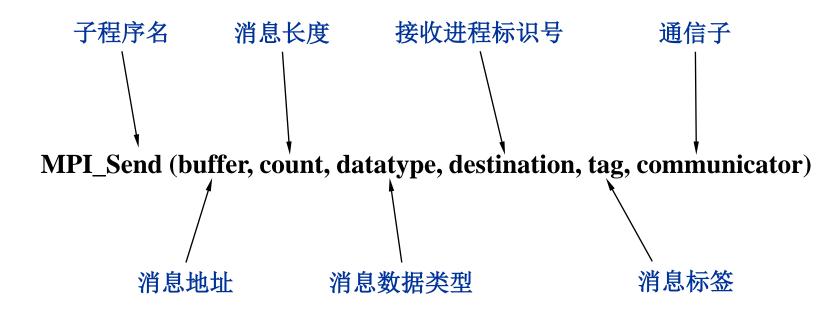


Communication





Message







Data types

MPI datatype	C datatype	
MPI_CHAR	signed char	
MPI_SHORT	signed short int	
MPI_INT	signed int	
MPI_LONG	signed long int	
MPI_LONG_LONG	signed long long int	
MPI_UNSIGNED_CHAR	unsigned char	
MPI_UNSIGNED_SHORT	unsigned short int	
MPI_UNSIGNED	unsigned int	
MPI_UNSIGNED_LONG	unsigned long int	
MPI_FLOAT	float	
MPI_DOUBLE	double	
MPI_LONG_DOUBLE	long double	
MPI_BYTE		
MPI_PACKED	(《八八) 人、、、航空航天大学	
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为什么要使用消息标签(Tag)?

为了说明为什么要用标签,我 们先来看右面一段没有使用标签 的代码。

这段代码打算传送A的前32个字节进入X,传送B的前16个字节进入Y。但是,如果消息B尽管后发送但先到达进程Q,就会被第一个recv()接收在X中。

使用标签可以避免这个错误。

未使用标签

Process P: Process Q:

send(A, 32, Q) recv(X, 32, P) send(B, 16, Q) recv(Y, 16, P)

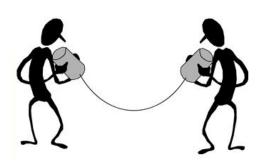
使用了标签

Process P: Process Q:

send(A, 32, Q, tag1) recv (X, 32, P, tag1) recv (Y, 16, P, tag2)



Communication





Message matching

```
MPI Send (send buf p, send buf sz, send type, dest)
                                                     send tag,
          send_comm);
                MPI_Send
                src = q
                                       MPI_Recv
MPI_Recv(recv_buf_p, recv_buf_sz, recv_type,(src)(recv_tag,
           recv comm, &status);
```

Message matching "wildcard" arguments

➤ It's possible that the receiving process doesn't know the order in which the messages will be sent, and one process can receive multiple messages with different tags from another process.



Receiving messages

- A receiver can get a message without knowing:
 - the amount of data in the message,
 - the sender of the message,
 - or the tag of the message.







status_p argument

MPI Status* status;

status.MPI SOURCE

status.MPI TAG

status.MPI_ERROR

MPI_SOURCE
MPI_TAG

MPI Status*

MPI_ERROR



How much data am I receiving?

```
int MPI_Get_count(
     MPI_Status* Status_p /* in */,
     MPI_Datatype type /* in */,
     int* count_p /* out */);
```





Issues with send and receive

- Exact behavior is determined by the MPI implementation.
- MPI_Send may behave differently with regard to buffer size, cutoffs and blocking.
- MPI_Recv always blocks until a matching message is received.
- Know your implementation; don't make assumptions!



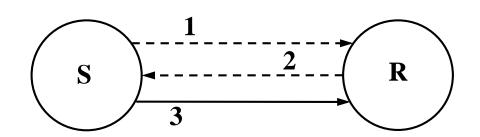
MPI中的四种通信模式 (communication mode)

① 同步的(synchronous)

发送进程直到相应的接收过程已经启动才返回,因此接收端要有存放到达消息的应用缓冲。

注意:在MPI中可以有非阻塞的同步发送,它的返回不意味着消息已经被发出!它的实现不需要在接收端有附加的缓冲,但需要在发送端有一个系统缓冲。为了消除额外的消息拷贝,应使用阻塞的同步发送。

Synchronous

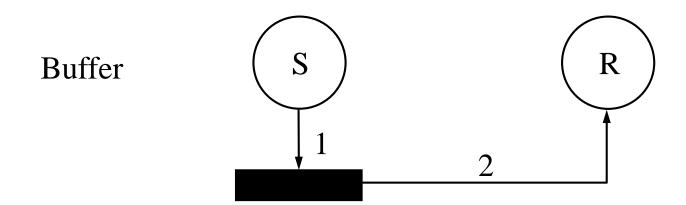




MPI中的四种通信模式

② 缓冲的(buffered)

缓冲的发送假定能得到一定大小的缓冲空间,它必须事先由用户程序通过调用子例程MPI_Buffer_attch(buffer, size)来定义,由它来分配大小为size的用户缓冲。这个缓冲可以用MPI_Buffer_detach(*buffer, *size)来实现。

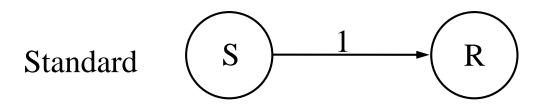




MPI中的四种通信模式

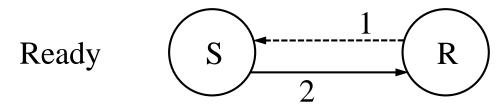
③ 标准的(standard)

发送可以是同步的或缓冲的,取决于实现。



④ 就绪的(ready)

在肯定相应的接收已经开始才进行发送。它不像在同步模式中那样需要等待。这就允许在相同的情况下实际 使用一个更有效的通信协议。

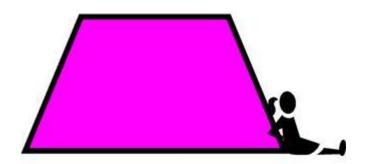




MPI点对点通信中不同的发送和接收操作

通信模式	MPI Primitive	Blocking	Non-Blocking
标准通信	Standard Send	MPI_Send	MPI_Isend
同步通信	Synchronous Send	MPI_ Ssend	MPI_ Issend
缓冲通信	Buffered Send	MPI_ Bsend	MPI_ Ibsend
就绪通信	Ready Send	MPI_ Rsend	MPI_ Irsend
	Receive	MPI_Recv	MPI_Irecv
	Completion Check	MPI_Wait	MPI_Test

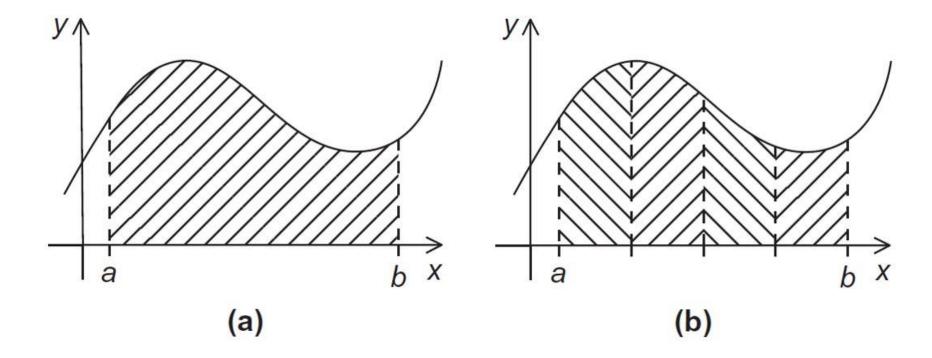




2. TRAPEZOIDAL RULE IN MPI

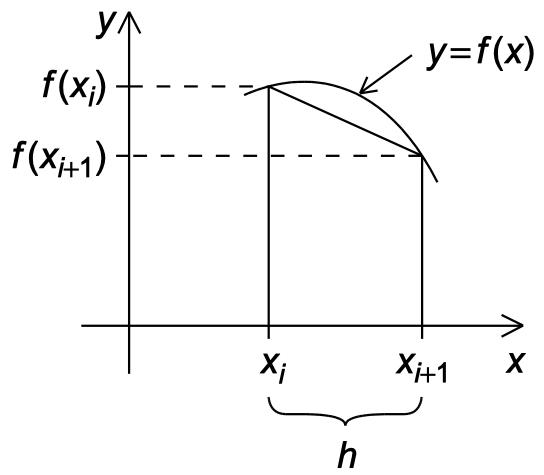


The Trapezoidal Rule





One trapezoid





The Trapezoidal Rule

Area of one trapezoid =
$$\frac{h}{2}[f(x_i) + f(x_{i+1})]$$

$$h = \frac{b - a}{n}$$

$$x_0 = a$$
, $x_1 = a + h$, $x_2 = a + 2h$, ..., $x_{n-1} = a + (n-1)h$, $x_n = b$

Sum of trapezoid areas = $h[f(x_0)/2 + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + f(x_n)/2]$



Pseudo-code for a serial program

```
/* Input: a, b, n */
h = (b - a) / n;
approx = (f(a) + f(b))/2.0;
for (i = 1; i <= n - 1; i++) {
    x_i = a + i * h;
    approx += f(x_i);
}
approx = h * approx;</pre>
```



Parallelizing the Trapezoidal Rule

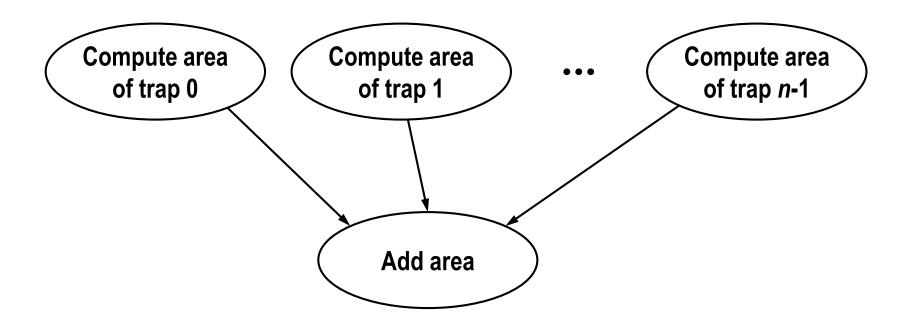
- 1. Partition problem solution into tasks.
- 2. Identify communication channels between tasks.
- 3. Aggregate tasks into composite tasks.
- 4. Map composite tasks to cores.



Parallel pseudo-code

```
Get a, b, n;
    h = (b - a) / n;
    local n = n / comm sz;
    local a = a + my rank * local n * h;
    local b = local a + local n * h;
6
    local integral = Trap(local a, local b, local n, h);
    if (my rank != 0)
8
       Send local integral to process 0;
9
    else /* my rank == 0 */
       total integral = local integral;
10
11
       for (proc = 1; proc < comm sz; proc++) {</pre>
12
          Receive local integral from proc;
13
          total integral += local integral;
14
15
16 if (my rank == 0)
17
       print result;
```

Tasks and communications for Trapezoidal Rule





First version (1)

```
int main(void) {
      int my rank, comm sz, n = 1024, local n;
      double a = 0.0, b = 3.0, h, local a, local b;
     double local int, total int;
      int source;
     MPI Init (NULL, NULL);
     MPI Comm rank (MPI COMM WORLD, &my rank);
      MPI Comm size (MPI COMM WORLD, &comm sz);
9
10
      h = (b - a)/n; /* h is the same for all processes */
11
      local n = n/comm \ sz; /* So is the number of trapezoids */
12
13
14
      local a = a + my rank * local n * h;
15
      local b = local a + local n * h;
      local int = Trap(local a, local b, local n, h);
16
17
      if (my rank != 0) {
18
         MPI Send(&local int, 1, MPI DOUBLE, 0, 0,
19
20
               MPI COMM WORLD);
```

First version (2)

```
} else {
21
22
         total int = local int;
         for (source = 1; source < comm sz; source++) {</pre>
23
            MPI Recv(&local int, 1, MPI DOUBLE, source, 0,
24
25
                   MPI COMM WORLD, MPI STATUS IGNORE);
            total int += local int;
26
27
28
29
30
      if (my rank == 0) {
         printf("With n = %d trapezoids, our estimate\n", n);
31
         printf("of the integral from %f to %f = %.15e\n",
32
33
                a, b, total int);
34
35
      MPI Finalize();
36
      return 0;
      /* main */
37
```



First version (3)

```
double Trap(
        double left endpt /* in */,
        double right endpt /* in */,
         int trap count /* in */,
      double base len    /* in */) {
     double estimate, x;
     int i;
9
     estimate = (f(left endpt) + f(right endpt))/2.0;
     for (i = 1; i <= trap count - 1; i++) {</pre>
10
11
        x = left endpt + i*base len;
        estimate += f(x);
12
13
14
     estimate = estimate*base len;
15
16
     return estimate;
17
    /* Trap */
```

Dealing with I/O

```
#include <stdio.h>
#include <mpi.h>
                                    Each process just
                                    prints a message.
int main(void) {
   int my rank, comm sz;
  MPI Init (NULL, NULL);
  MPI Comm size (MPI COMM WORLD, &comm sz);
  MPI Comm rank (MPI COMM WORLD, &my rank);
   printf("Proc %d of %d > Does anyone have a toothpick?\n",
         My rank, comm sz);
  MPI Finalize();
   return 0;
} /* main */
```



Running with 6 processes

```
Proc 0 of 6 > Does anyone have a toothpick?

Proc 1 of 6 > Does anyone have a toothpick?

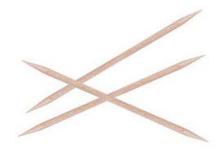
Proc 2 of 6 > Does anyone have a toothpick?

Proc 4 of 6 > Does anyone have a toothpick?

Proc 3 of 6 > Does anyone have a toothpick?

Proc 5 of 6 > Does anyone have a toothpick?
```

unpredictable output





Input

- Most MPI implementations only allow process 0 in MPI_COMM_WORLD access to stdin.
- Process 0 must read the data (scanf) and send to the other processes.

```
MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
Get_input(my_rank, comm_sz, &a, &b, &n);
H = (b - a)/n
...
```



Function for reading user input

```
void Get input(
     int my rank /* in */,
     int dest;
  if (my rank == 0) {
     printf("Enter a, b, and n n);
     scanf("%lf %lf %d", a p, b p, n p);
     for (dest = 1; dest < comm sz; dest++) {</pre>
        MPI Send(a p, 1, MPI DOUBLE, dest, 0, MPI COMM WORLD);
        MPI Send(b p, 1, MPI DOUBLE, dest, 0, MPI COMM WORLD);
        MPI Send(n p, 1, MPI INT, dest, 0, MPI COMM WORLD);
   \} else { /* my rank != 0 */
     MPI Recv(a p, 1, MPI DOUBLE, 0, 0, MPI COMM WORLD,
           MPI STATUS IGNORE);
     MPI Recv(b p, 1, MPI DOUBLE, 0, 0, MPI COMM WORLD,
           MPI STATUS IGNORE);
     MPI Recv(n p, 1, MPI INT, 0, 0, MPI COMM WORLD,
           MPI STATUS IGNORE);
} /* Get input */
```



3. COLLECTIVE COMMUNICATION



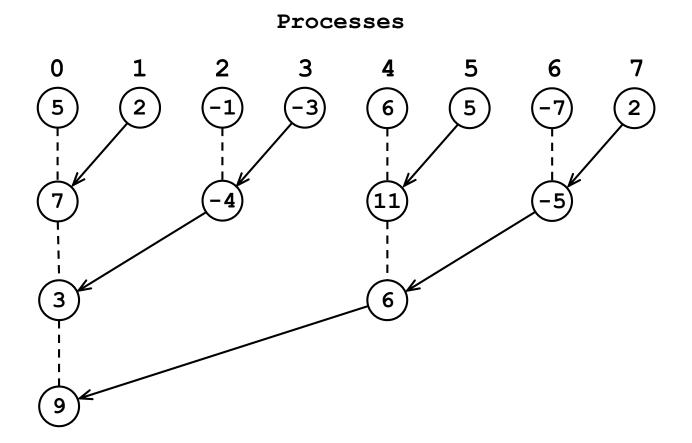


Optimize the trapezoidal rule program

The "global sum" after each process has computed its part of the integral.

```
if (my rank != 0) {
   MPI Send(&local int, 1, MPI DOUBLE, 0, 0,
         MPI COMM WORLD);
} else {
   total int = local int;
   for (source = 1; source < comm sz; source++) {</pre>
      MPI Recv(&local int, 1, MPI DOUBLE, source, 0,
            MPI COMM WORLD, MPI STATUS IGNORE);
      total int += local int;
```

A tree-structured global sum





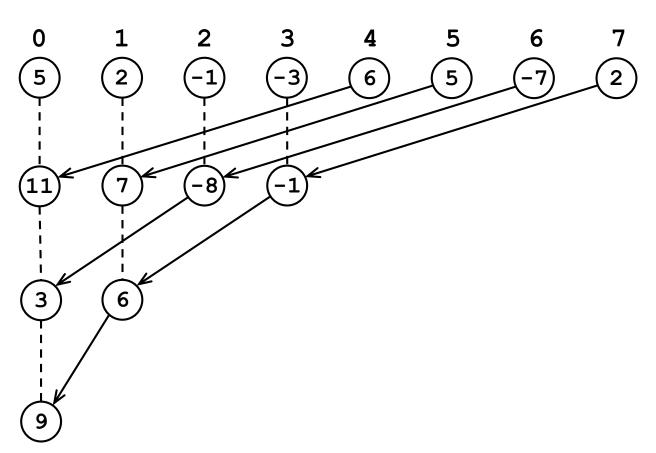
Tree-structured communication

- 1. (a) Process 1 sends to 0, 3 sends to 2, 5 sends to 4, and 7 sends to 6.
 - (b) Processes 0, 2, 4, and 6 add in the received values.
- 2. (a) Processes 2 and 6 send their new values to processes 0 and 4, respectively.
 - (b) Processes 0 and 4 add the received values into their new values.
- 3. (a) Process 4 sends its newest value to process 0.
 - (b) Process 0 adds the received value to its newest value.



An alternative tree-structured global sum

Processes





(1) MPI_Reduce

Predefined reduction operators in MPI

Operation Value	Meaning	3 5 7
MPI_MAX	Maximum	
MPI_MIN	Minimum	
MPI_SUM	Sum	16
MPI_PROD	Product	reduction
MPI_LAND	Logical and	
MPI_BAND	Bitwise and	
MPI_LOR	Logical or	
MPI_BOR	Bitwise or	
MPI_LXOR	Logical exclusive or	
MPI_BXOR	Bitwise exclusive or	
MPI_MAXLOC	Maximum and location of maximum	
MPI_MINLOC	Minimum and location of minimum	Warran 22 02 11 PR
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- All the processes in the communicator must call the same collective function.
- For example, a program that attempts to match a call to MPI_Reduce on one process with a call to MPI_Recv on another process is erroneous, and, in all likelihood, the program will hang or crash.



- The arguments passed by each process to an MPI collective communication must be "compatible."
- For example, if one process passes in 0 as the dest_process and another passes in 1, then the outcome of a call to MPI_Reduce is erroneous, and, once again, the program is likely to hang or crash.



- The output_data_p argument is only used on dest_process.
- However, all of the processes still need to pass in an actual argument corresponding to output_data_p, even if it's just NULL.



- Point-to-point communications are matched on the basis of tags and communicators.
- Collective communications don't use tags. They're matched solely on the basis of the communicator and the order in which they're called.



Example (1)

Time	Process 0	Process 1	Process 2
0	a = 1; c = 2	a = 1; c = 2	a = 1; c = 2
1	MPI_Reduce(&a, &b,	<pre>MPI_Reduce(&c, &d,)</pre>	<pre>MPI_Reduce(&a, &b,)</pre>
2	<pre>MPI_Reduce(&c, &d,)</pre>	<pre>MPI_Reduce(&a, &b,)</pre>	<pre>MPI_Reduce(&c, &d,)</pre>

Multiple calls to MPI_Reduce



Example (2)

- Suppose that each process calls MPI_Reduce with operator MPI_SUM, and destination process 0.
- At first glance, it might seem that after the two calls to MPI_Reduce, the value of b will be 3, and the value of d will be 6.



Example (3)

 However, the names of the memory locations are irrelevant to the matching of the calls to MPI_Reduce.

The order of the calls will determine the matching so the value stored in b will be 1+2+1 = 4, and the value stored in d will be 2+1+2 = 5.



Anther question

```
MPI_Reduce(&x, &x, 1, MPI_DOUBLE, MPI_SUM, 0, comm);
```

- How about using the same buffer as input and output?
- —The result is unpredictable.

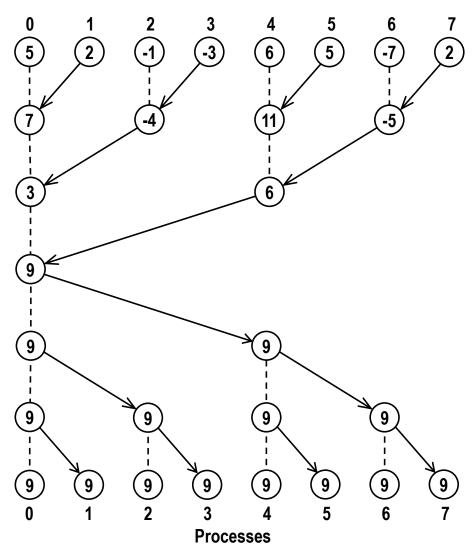


(2) MPI_Allreduce

 Useful in a situation in which all of the processes need the result of a global sum in order to complete some larger computation.



Processes

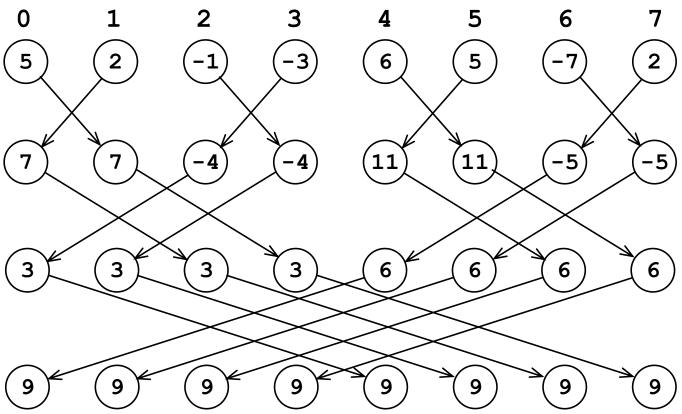


A global sum followed by distribution of the result.



Processes





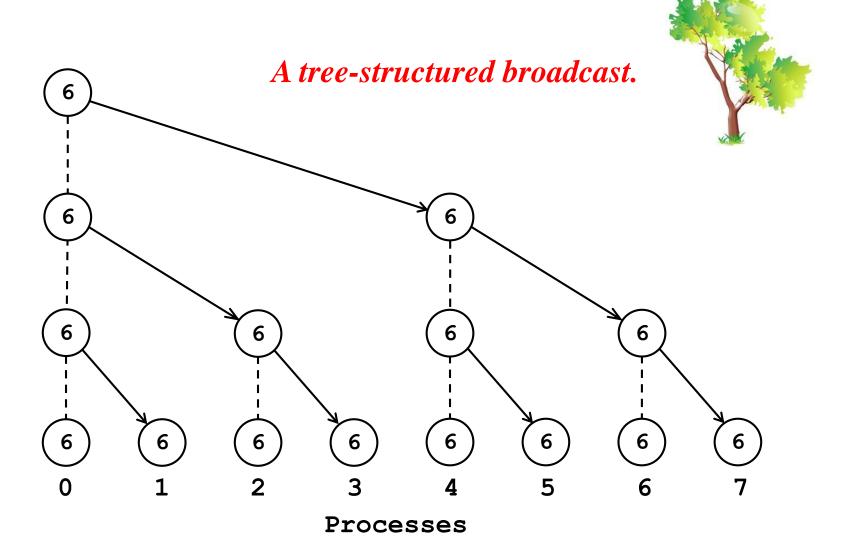
A butterfly-structured global sum.



(3) Broadcast

 Data belonging to a single process is sent to all of the processes in the communicator.







A version of Get_input that uses MPI_Bcast

```
void Get input(
     int my rank /* in */,
     int comm sz /* in */,
     double* a_p /* out */,
     double* b p /* out */,
     if (my rank == 0) {
     printf("Enter a, b, and n n);
     scanf("%lf %lf %d", a_p, b_p, n_p);
  MPI Bcast(a p, 1, MPI DOUBLE, 0, MPI COMM WORLD);
  MPI Bcast (b p, 1, MPI DOUBLE, 0, MPI COMM WORLD);
  MPI Bcast(n p, 1, MPI INT, 0, MPI COMM WORLD);
} /* Get input */
```





Data distributions

$$x + y = (x_0, x_1, ..., x_{n-1}) + (y_0, y_1, ..., y_{n-1})$$

$$= (x_0 + y_0, x_1 + y_1, ..., x_{n-1} + y_{n-1})$$

$$= (z_0, z_1, ..., z_{n-1})$$

$$= z$$

Compute a vector sum.



Serial implementation of vector addition

```
void Vector sum(double x[], double y[], double z[], int n) {
   int i;
   for (i = 0; i < n; i++)</pre>
                                                                 0
       z[i] = x[i] + y[i];
                                                     2
} /* Vector sum */
                                                     8
                                           10
                                                     10
                                                                 10
                                           11
                                                     11
                                                                 11
```



Different partitions of a 12-component vector among 3 processes

Process					(Comp	onent	ts				
		Ble	ock		Cyclic			Block-cyclic Blocksize = 2				
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11



Partitioning options

- Block partitioning
 - Assign blocks of consecutive components to each process.
- Cyclic partitioning
 - Assign components in a round robin fashion.
- Block-cyclic partitioning
 - Use a cyclic distribution of blocks of components.



Parallel implementation of vector addition

```
void Parallel vector sum(
     double local x[] /* in */,
     double local y[] /* in */,
     double local z[] /* out */,
     int local n /* in */) {
   int local i;
   for (local i = 0; local i < local n; local i++)</pre>
     local z[local i] = local x[local i] + local y[local i];
} /* Parallel vector sum */
```



(4) Scatter

MPI_Scatter can be used in a function that reads in an entire vector on process 0 but only sends the needed components to each of the other processes.

```
int MPI Scatter(
      void*
                     send buf p
      int
                     send count
      MPI Datatype
                     send type
                                        in
                                     /* out */,
      void*
                     recv buf p
                                     /* in
      int
                     recv count
      MPI Datatype
                     recv type
                                        in
                                     /* in
                                             */,
      int
                     src proc
                                     /* in */);
      MPI Comm
                     comm
```



Reading and distributing a vector

```
void Read vector(
     double local a[] /* out */,
     int local_n /* in */,
     int n /* in */,
     char vec name[] /* in */,
     int my rank /* in */,
     MPI Comm comm /* in */) {
  double* a = NULL;
  int i;
  if (my rank == 0) {
     a = malloc(n*sizeof(double));
     printf("Enter the vector %s\n", vec name);
     for (i = 0; i < n; i++)
        scanf("%lf", &a[i]);
     MPI Scatter(a, local n, MPI DOUBLE, local a, local n, MPI DOUBLE,
          0, comm);
     free(a);
  } else {
     MPI Scatter(a, local n, MPI DOUBLE, local a, local n, MPI DOUBLE,
          0, comm);
                                                    北京航空航天:
 /* Read vector */
```

(5) Gather

 Collect all of the components of the vector onto process 0, and then process 0 can process all of the components.

```
int MPI Gather (
      void*
                     send buf p
      int
                     send count
                     send type
      MPI Datatype
      void*
                     recv buf p
                                        out
                                     /* in
      int
                     recv count
      MPI Datatype
                                        in
                     recv type
                                     /* in */.
      int
                     dest proc
                                     /* in */);
      MPI Comm
                     comm
```



Print a distributed vector (1)

```
int Print vector(
     double
                 local_b[]
                             /* in */,
     int
                 local n
                             /* in */,
                              /* in */,
     int
                 n
                            /* in */,
     char
                 title[]
                 my rank
     int
                             /* in */,
                               /* in */) {
     MPI Comm
                 comm
  double* b = NULL;
  int i;
```



Print a distributed vector (2)

```
if (my rank == 0) {
  b = malloc(n*sizeof(double));
  MPI Gather (local b, local n, MPI DOUBLE, b, local n,
         MPI DOUBLE, 0, comm);
   printf("%s\n", title);
   for (i = 0; i < n; i++)
      printf("%f ", b[i]);
  printf("\n");
   free(b);
} else {
  MPI_Gather(local_b, local n, MPI DOUBLE, b, local n,
         MPI DOUBLE, 0, comm);
} /* Print vector */
```



(6) Allgather

- Concatenates the contents of each process' send_buf_p and stores this in each process' recv_buf_p.
- As usual, recv_count is the amount of data being received from each process.

```
int MPI Allgather(
      void*
                     send buf p
                                     /* in
      int
                     send count
      MPI Datatype
                     send type
      void*
                     recv buf p
                                      /* out */,
      int
                                      /* in
                     recv count
      MPI Datatype
                     recv type
                                         in
                                      /* in */);
      MPI Comm
                     comm
```



Matrix-vector multiplication

$$A=(a_{ij})$$
 is an $m \times n$ matrix

x is a vector with n components

y = Ax is a vector with m components

$$y_i = a_{i,0}x_0 + a_{i,1}x_1 + a_{i,2}x_2 + \dots + a_{i,n-1}x_{n-1}$$

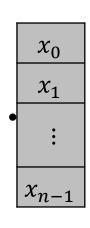
i-th component of y

Dot product of the ith row of A with x.



Matrix-vector multiplication

$a_{0,0}$	$a_{0,1}$	•••	$a_{0,n-1}$
$a_{1,0}$	$a_{1,1}$	•••	$a_{1,n-1}$
:	:		:
$a_{i,0}$	$a_{i,1}$	•••	$a_{i,n-1}$
:	:		:
$a_{m-1,0}$	$a_{m-1,1}$	•••	$a_{m-1,n-1}$



	y_0
	y_1
	:
=	$y_i = a_{i,0}x_0 + a_{i,1}x_1 + \dots + a_{i,n-1}x_{n-1}$
	:
	y_{m-1}



Multiply a matrix by a vector

```
/* For each row of A */
for (i = 0; i < m; i++) {
    /* Form dot product of ith row with x */
    y[i] = 0.0;
    for (j = 0; j < n; j++)
        y[i] += A[i][j] * x[j];
}</pre>
```

Serial pseudo-code



C style arrays



Serial matrix-vector multiplication

```
void Mat vec mult(
     double A[] /* in */,
     double x[] /* in */,
     double y[] /* out */,
     int m /* in */,
     int i, j;
  for (i = 0; i < m; i++) {
     y[i] = 0.0;
     for (j = 0; j < n; j++)
       y[i] += A[i*n+j] * x[j];
} /* Mat vec mult */
```



An MPI matrix-vector multiplication function (1)

```
void Mat vec mult(
    double local A[] /* in */,
    double local x[] /* in */,
    double local_y[] /* out */,
    int local m /* in */,
    int
             n /* in */,
    int local n /* in */,
                    /* in */) {
    MPI Comm
           comm
  double* x;
  int local i, j;
  int local ok = 1;
```

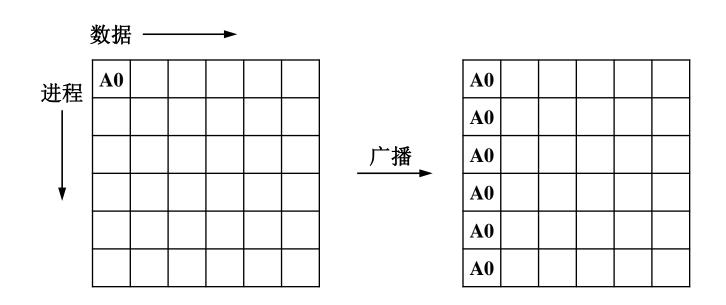


An MPI matrix-vector multiplication function (2)

```
x = malloc(n*sizeof(double));
  MPI Allgather (local x, local n, MPI DOUBLE,
         x, local n, MPI DOUBLE, comm);
   for (local i = 0; local i < local m; local i++) {</pre>
      local y[local i] = 0.0;
      for (j = 0; j < n; j++)
         local y[local i] += local A[local i*n+j] * x[j];
   free(x);
} /* Mat vec mult */
```

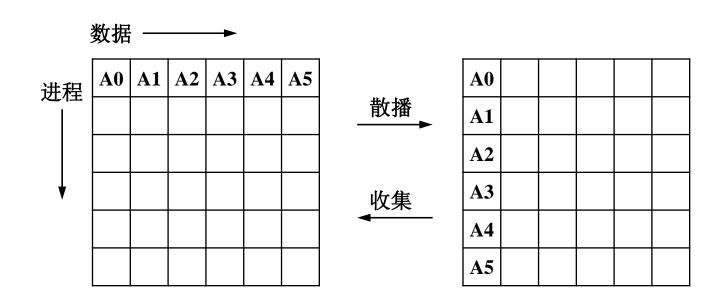


广播 (MPI_Bcast)





收集和散播 (MPI_Gather & MPI_Scatter)





全局收集 (MPI_Allgather)

数据 ───

进程 | |

	A0	В0	CO	D0	E0	F0
•	A0	В0	C0	D0	E0	F0
	A0	В0	C0	D0	E0	F0
	A0	В0	C0	D0	E0	F0
	A0	В0	C0	D0	E0	F0
	A0	В0	C0	D0	E0	FO

全局收集

A0			
В0			
C0			
D0			
E0			
FO			



全局交换 (MPI_Alltoall)

数据 ───

A0	A1	A2	A3	A4	A5
В0	B1	B2	В3	B4	B5
C0	C1	C2	C3	C4	C5
D0	D1	D2	D3	D4	D5
E0	E1	E2	E3	E4	E5
F0	F1	F2	F3	F4	F5
	B0 C0 D0 E0	B0 B1 C0 C1 D0 D1 E0 E1	B0 B1 B2 C0 C1 C2 D0 D1 D2 E0 E1 E2	B0 B1 B2 B3 C0 C1 C2 C3 D0 D1 D2 D3 E0 E1 E2 E3	A0 A1 A2 A3 A4 B0 B1 B2 B3 B4 C0 C1 C2 C3 C4 D0 D1 D2 D3 D4 E0 E1 E2 E3 E4 F0 F1 F2 F3 F4

全局交换

A0	B0	C0	D0	E0	F0
A1	B 1	C1	D1	E 1	F1
A2	B2	C2	D2	E2	F2
A3	В3	C3	D3	E3	F3
A4	B4	C4	D4	E4	F4
A5	B5	C5	D5	E5	F5





4. MPI DERIVED DATATYPES



Derived datatypes



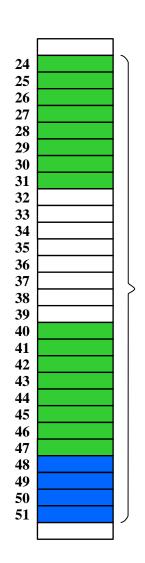
- Used to represent any collection of data items in memory by storing both the types of the items and their relative locations in memory.
- The idea is that if a function that sends data knows this information about a collection of data items, it can collect the items from memory before they are sent.
- Similarly, a function that receives data can distribute the items into their correct destinations in memory when they're received.



Derived datatypes

- Formally, consists of a sequence of basic MPI data types together with a displacement for each of the data types.
- Trapezoidal Rule example:

Variable	Address
a	24
b	40
n	48



{(MPI_DOUBLE, 0), (MPI_DOUBLE, 16), (MPI_INT, 24)}

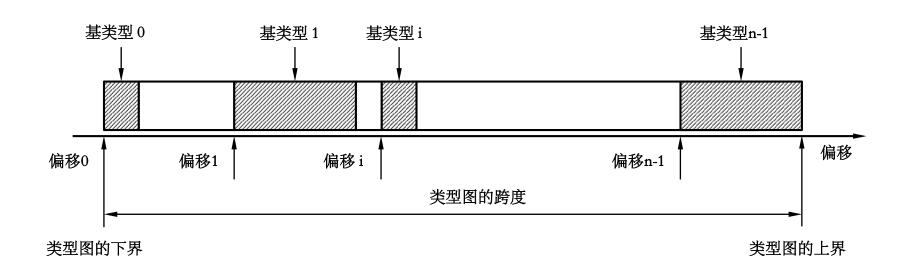


MPI_Type_create_struct

 Builds a derived datatype that consists of individual elements that have different basic types.



派生数据类型的类型图示意结构





MPI_Get_address

- Returns the address of the memory location referenced by location_p.
- The special type MPI_Aint is an integer type that is big enough to store an address on the system.

```
int MPI_Get_address(
     void* location_p /* in */,
     MPI_Aint* address_p /* out */);
```



MPI_Type_commit

 Allows the MPI implementation to optimize its internal representation of the datatype for use in communication functions.

```
int MPI_Type_commit(MPI_Datatype* new_mpi_t_p /* in/out */);
```



MPI_Type_free

When we're finished with our new type, this frees any additional storage used.

```
int MPI_Type_free(MPI_Datatype* old_mpi_t_p /* in/out */);
```



Get_input function with a derived datatype (1)

```
void Build mpi type(
     double* a_p
                               /* in */,
                               /* in */,
     double*
                   b p
                              /* in */,
     int*
             n p
     MPI_Datatype* input mpi t p /* out */) {
  int array of blocklengths[3] = \{1, 1, 1\};
  MPI Datatype array of types[3] = \{MPI DOUBLE,
        MPI DOUBLE, MPI INT };
  MPI Aint a addr, b addr, n addr;
  MPI_Aint array_of displacements[3] = \{0\};
```





Get_input function with a derived datatype (2)

```
MPI Get address(a p, &a addr);
  MPI Get address(b p, &b addr);
  MPI Get address(n p, &n addr);
  array of displacements[1] = b addr - a addr;
  array of displacements[2] = n addr - a addr;
  MPI Type create struct(3, array of blocklengths,
        array of displacements, array of types,
        input mpi t p);
  MPI Type commit(input mpi t p); /*派生出的新数据类型,必须
                            先要经过MPI系统的确认后才能使用。*/
} /* Build mpi type */
```



Get_input function with a derived datatype (3)

```
void Get input(int my rank, int comm sz, double* a p,
      double* b p, int* n p) {
  MPI Datatype input mpi t;
   Build_mpi_type(a_p, b_p, n_p, &input_mpi_t);
   if (my rank == 0) {
      printf("Enter a, b, and n \ ");
      scanf("%lf %lf %d", a p, b p, n p);
   MPI Bcast(a p, 1, input mpi t, 0, MPI COMM WORLD);
  MPI Type free (&input mpi t);
} /* Get input */
```





5. PERFORMANCE EVALUATION



Elapsed parallel time

 Returns the number of seconds that have elapsed since some time in the past.



Elapsed serial time

- In this case, you don't need to link in the MPI libraries.
- Returns time in microseconds elapsed from some point in the past.

```
#include "timer.h"
. . .
double now;
. . .
GET TIME(now);
```





Elapsed serial time

```
#include "timer.h"
. . .
double start, finish;
. . .
GET_TIME(start);
/* Code to be timed */
. . .
GET_TIME(finish);
printf("Elapsed time = %e seconds\n", finish - start);
```



MPI_Barrier

 Ensures that no process will return from calling it until every process in the communicator has started calling it.

```
int MPI Barrier(MPI Comm comm /* in */);
```



MPI_Barrier

```
double local start, local finish, local elapsed, elapsed;
MPI Barrier(comm);
local start = MPI Wtime();
/* Code to be timed */
local finish = MPI Wtime();
local elapsed = local finish - local start;
MPI Reduce (&local elapsed, &elapsed, 1, MPI DOUBLE,
      MPI MAX, 0, comm);
if (my rank == 0)
   printf("Elapsed time = %e seconds\n", elapsed);
```



Run-times of serial and parallel matrix-vector multiplication

comm_sz	Order of Matrix					
	1024	2048	4096	8192	16,384	
1	4.1	16.0	64.0	270	1100	
2	2.3	8.5	33.0	140	560	
4	2.0	5.1	18.0	70	280	
8	1.7	3.3	9.8	36	140	
16	1.7	2.6	5.9	19	71	

(milliseconds)



Speedup

$$S(n, p) = \frac{T_{\text{serial}}(n)}{T_{\text{parallel}}(n, p)}$$



Efficiency

$$E(n, p) = \frac{S(n, p)}{p} = \frac{T_{\text{serial}}(n)}{p \times T_{\text{parallel}}(n, p)}$$



Speedups of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.0	1.0	1.0	1.0	1.0
2	1.8	1.9	1.9	1.9	2.0
4	2.1	3.1	3.6	3.9	3.9
8	2.4	4.8	6.5	7.5	7.9
16	2.4	6.2	10.8	14.2	15.5



Efficiencies of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.00	1.00	1.00	1.00	1.00
2	0.89	0.94	0.97	0.96	0.98
4	0.51	0.78	0.89	0.96	0.98
8	0.30	0.61	0.82	0.94	0.98
16	0.15	0.39	0.68	0.89	0.97



Scalability

• A program is scalable if the problem size can be increased at a rate so that the efficiency doesn't decrease as the number of processes increase.





Scalability

- Programs that can maintain a constant efficiency without increasing the problem size are sometimes said to be strongly scalable.
- Programs that can maintain a constant efficiency if the problem size increases at the same rate as the number of processes are sometimes said to be weakly scalable.



6. A PARALLEL SORTING ALGORITHM



Sorting

- n keys and p = comm_sz processes.
- n/p keys assigned to each process.
- No restrictions on which keys are assigned to which processes.
- When the algorithm terminates:
 - The keys assigned to each process should be sorted in (say) increasing order.
 - If $0 \le q < r < p$, then each key assigned to process q should be less than or equal to every key assigned to process r.



Serial bubble sort

```
void Bubble sort(
     int a[] /* in/out */,
     int n /* in */) {
   int list length, i, temp;
   for (list length = n; list length >= 2; list length--)
     for (i = 0; i < list length-1; i++)
        if (a[i] > a[i+1]) {
           temp = a[i];
           a[i] = a[i+1];
           a[i+1] = temp;
} /* Bubble sort */
```

Odd-even transposition sort

- A sequence of phases.
- Even phases, compare swaps:

$$(a[0], a[1]), (a[2], a[3]), (a[4], a[5]), \cdots$$

Odd phases, compare swaps:

$$(a[1], a[2]), (a[3], a[4]), (a[5], a[6]), \dots$$



Example

Start: 5, 9, 4, 3

Even phase: compare-swap (5,9) and (4,3) getting the list 5, 9, 3, 4

Odd phase: compare-swap (9,3) getting the list 5, 3, 9, 4

Even phase: compare-swap (5,3) and (9,4) getting the list 3, 5, 4, 9

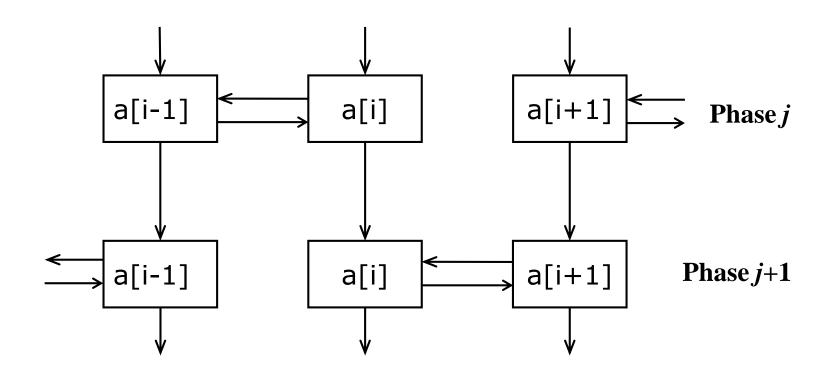
Odd phase: compare-swap (5,4) getting the list 3, 4, 5, 9



Serial odd-even transposition sort

```
void Odd even sort(
     int a[] /* in/out */,
     int n /* in */) {
  int phase, i, temp;
  for (phase = 0; phase < n; phase++)</pre>
     if (phase % 2 == 0) { /* Even phase */
        for (i = 1; i < n; i += 2)
           if (a[i-1] > a[i]) {
              temp = a[i]; a[i] = a[i-1]; a[i-1] = temp;
                          /* Odd phase */
      } else {
        for (i = 1; i < n-1; i += 2)
           if (a[i] > a[i+1]) {
              temp = a[i]; a[i] = a[i+1]; a[i+1] = temp;
} /* Odd even sort */
```

Communications among tasks in odd-even sort



Tasks determining a[i] are labeled with a[i].



Parallel odd-even transposition sort

15, 11, 9, 16, 3, 14, 8, 7, 4, 6, 12, 10, 5, 2, 13, 1

Time	Process				
Time	0	1	2	3	
Start	15, 11, 9, 16	3, 14, 8, 7	4, 6, 12, 10	5, 2, 13, 1	
After Local Sort	9, 11, 15, 16	3, 7, 8, 14	4, 6, 10, 12	1, 2, 5, 13	
After Phase 0	3, 7, 8, 9	11, 14, 15, 16	1, 2, 4, 5	6, 10, 12, 13	
After Phase 1	3, 7, 8, 9	1, 2, 4, 5	11, 14, 15, 16	6, 10, 12, 13	
After Phase 2	1, 2, 3, 4	5, 7, 8, 9	6, 10, 11, 12	13, 14, 15, 16	
After Phase 3	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16	



Pseudo-code

```
Sort local keys;
for (phase = 0; phase < comm sz; phase++) {</pre>
   partner = Compute partner(phase, my rank);
   if (I'm not idle) {
      Send my keys to partner;
      Receive keys from partner;
      if (my rank < partner)</pre>
         Keep smaller keys;
      else
         Keep larger keys;
```

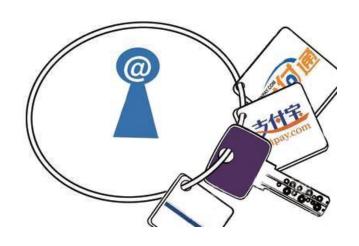


Compute_partner

```
if (phase % 2 == 0)
              /* Even phase */
  partner = my rank - 1;
                      /* Even rank */
  else
    partner = my rank + 1;
else
                      /* Odd phase */
  partner = my rank + 1;
                      /* Even rank */
  else
    partner = my rank - 1;
if (partner == -1 \mid \mid partner == comm sz)
  partner = MPI PROC NULL;
```



SAFETY IN MPI PROGRAMS





Safety in MPI programs

- The MPI standard allows MPI_Send to behave in two different ways:
 - it can simply copy the message into an MPI managed buffer and return,
 - or it can block until the matching call to MPI_Recv starts.



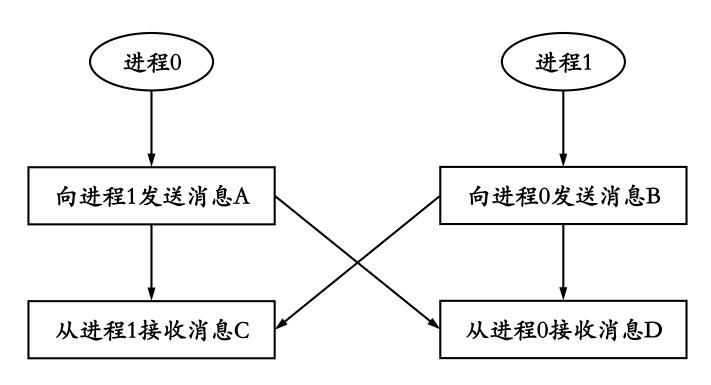
Safety in MPI programs

- Many implementations of MPI set a threshold at which the system switches from buffering to blocking.
- Relatively small messages will be buffered by MPI_Send.
- Larger messages, will cause it to block.



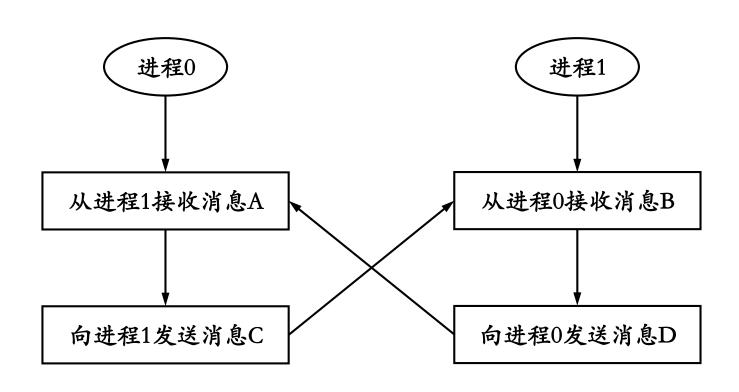
避免死锁deadlock

发送和接收是成对出现的,忽略这个原则很可能会产生死锁



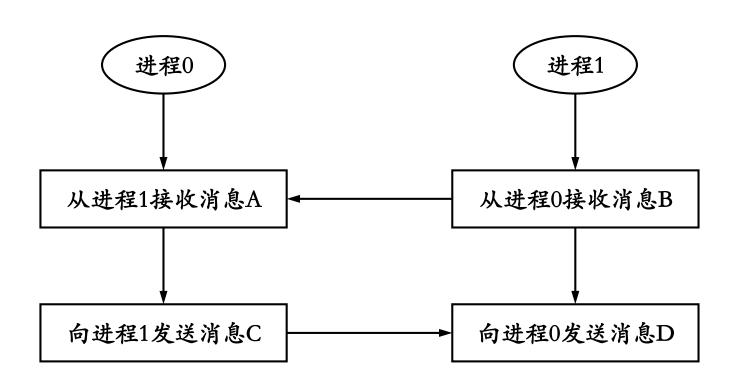


不安全的通信调用次序





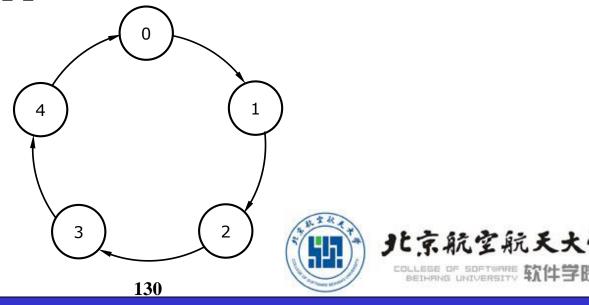
安全的通信调用次序





Safety in MPI programs

- If the MPI_Send executed by each process blocks, no process will be able to start executing a call to MPI_Recv, and the program will hang or deadlock.
- Each process is blocked waiting for an event that will never happen.



Safety in MPI programs

 A program that relies on MPI provided buffering is said to be unsafe.

Such a program may run without problems for various sets of input, but it may hang or crash with other sets.



MPI_Ssend

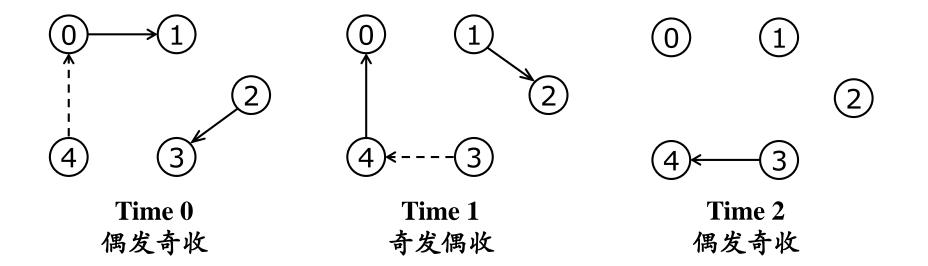
- An alternative to MPI_Send defined by the MPI standard.
- The extra "s" stands for synchronous and MPI_Ssend is guaranteed to block until the matching receive starts.



Restructuring communication

```
MPI Send(msg, size, MPI_INT, (my_rank+1)%comm_sz, 0, comm);
MPI Recv(new msg, size, MPI INT, (my_rank+comm_sz-1)%comm_sz,
      0, comm, MPI STATUS_IGNORE).
if (my rank % 2 == 0) {
   MPI Send(msg, size, MPI INT, (my rank+1)%comm sz, 0, comm);
   MPI Recv(new msg, size, MPI INT, (my rank+comm sz-1)%comm sz,
         0, comm, MPI STATUS IGNORE).
} else {
   MPI Recv(new msg, size, MPI INT, (my rank+comm sz-1)%comm sz,
         0, comm, MPI STATUS IGNORE).
   MPI Send(msg, size, MPI INT, (my rank+1)%comm sz, 0, comm);
```

Safe communication with five processes



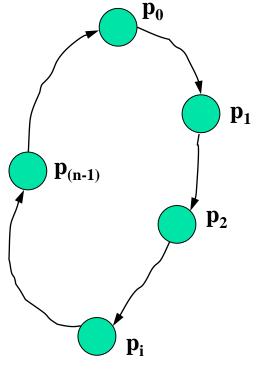
MPI_Sendrecv

- An alternative to scheduling the communications ourselves.
- Carries out a blocking send and a receive in a single call.
- The dest and the source can be the same or different.
- Especially useful because MPI schedules the communications so that the program won't hang or crash.



MPI_Sendrecv

```
int MPI Sendrecv(
     void*
                   send buf p
                                 /* in
     int
                   send buf size
                                 /* in
                                 /* in */,
     MPI Datatype
                   send buf type
     int
                   dest
                                    in
     int
                   send tag
                                 /* in
                                        */,
     void*
                                 /* out */,
                   recv buf p
     int
                                 /* in
                   recv buf size
     MPI Datatype
                                 /* in
                  recv buf type
     int
                   source
                                    in
     int
                                 /* in
                                        */,
                   recv tag
     MPI Comm
                   communicator /* in */,
     MPI Status*
                   status p
                                 /* in */);
```





MPI_Sendrecv用法示意

```
int a, b;

m
MPI_Status status;
int dest = (rank + 1)%p;
int source = (rank + p -1)%p; /* p为进程个数 */
MPI_Sendrecv( &a, 1, MPI_INT, dest, 99, &b, 1, MPI_INT, source, 99, MPI_COMM_WORLD, &status);
```

该函数被每一进程执行一次.



Parallel odd-even transposition sort

```
void Merge low(
     int recv_keys[], /* in */ 都是已排序的列表!
     int temp_keys[], /* scratch */
     int local n /* = n/p, in */) {
  int m i, r_i, t_i;
  m i = r i = t i = 0;
  while (t i < local n) {</pre>
     if (my keys[m i] <= recv keys[r i]) {</pre>
       temp keys[t i] = my keys[m i];
       t i++; m i++;
     } else {
       temp keys[t i] = recv keys[r i];
       t i++; r i++;
  for (m i = 0; m i < local n; m i++)
     my keys[m i] = temp keys[m i];
} /* Merge low */
                                      北京航空航天
```

Run-times of parallel odd-even sort

Processes	Number of Keys (in thousands)				
	200	400	800	1600	3200
1	88	190	390	830	1800
2	43	91	190	410	860
4	22	46	96	200	430
8	12	24	51	110	220
16	7.5	14	29	60	130

(times are in milliseconds)



Concluding Remarks (1)

- MPI or the Message-Passing Interface is a library of functions that can be called from C, C++, or Fortran programs.
- A communicator is a collection of processes that can send messages to each other.
- Many parallel programs use the single-program multiple data or SPMD approach.



Concluding Remarks (2)

- Most serial programs are deterministic: if we run the same program with the same input we'll get the same output.
- Parallel programs often don't possess this property.
- Collective communications involve all the processes in a communicator.



Concluding Remarks (3)

- When we time parallel programs, we're usually interested in elapsed time or "wall clock time".
- Speedup is the ratio of the serial run-time to the parallel run-time.
- Efficiency is the speedup divided by the number of parallel processes.



Concluding Remarks (4)

- If it's possible to increase the problem size (n) so that the efficiency doesn't decrease as p is increased, a parallel program is said to be scalable.
- An MPI program is unsafe if its correct behavior depends on the fact that MPI_Send is buffering its input.



MPI常用例程一览表

1	MPI_Init	
2	MPI_Finalize	
3	MPI_Comm_rank	
4	MPI_Comm_size	
5	MPI_Wtime	
6	MPI_Send	MPI_Ssend, MPI_Isend
7	MPI_Recv	MPI_Irecv
8	MPI_Sendrecv	MPI_Sendrecv_replace
9	MPI_Status	
10	MPI_Barrier	
11	MPI_Bcast	
12	MPI_Gather	MPI_Allgather
13	MPI_Scatter	
14	MPI_Reduce	MPI_Allreduce