

# Backward reasoning

Precondition: ??

```
z = 0;
```

```
if (x != 0) {
```

```
    z = x;
```

```
} else {
```

```
    z = z+1;
```

```
}
```

Postcondition:  $z > 0$ ;

$$\begin{aligned}
& wp(z = 0, (x > 0) \parallel (x == 0 \ \& \ z > -1)) \\
&= \{(x > 0) \parallel (x == 0 \ \& \ 0 > -1)\} \\
&= \{(x > 0) \parallel (x == 0 \ \& \ true)\} \\
&= \{(x > 0) \parallel (x == 0)\} \\
&= \{(x \geq 0)\}
\end{aligned}$$

$z = 0;$

$$\begin{aligned}
& wp(if\ (x \neq 0)\ z = x; else\ z = z + 1; , z > 0) \\
&= \{(x \neq 0 \ \& \ x > 0) \parallel (x == 0 \ \& \ z > -1)\} \\
&= \{(x > 0) \parallel (x == 0 \ \& \ z > -1)\}
\end{aligned}$$

$if\ (x \neq 0)\{$

$$wp(z = x, z > 0) = \{x > 0\}$$

$z = x;$

$\}$

$else\{$

$$wp(z = z + 1, z > 0) = \{z + 1 > 0\} = \{z > -1\}$$

$z = z + 1;$

$\}$

$postcondition : \{z > 0\}$

# Forward reasoning

Precondition:  $x \geq 0$

```
z = 0;
```

```
if (x != 0) {
```

```
    z = x;
```

```
} else {
```

```
    z = z+1;
```

```
}
```

Postcondition: ??;

$$x_0 \geq 0$$

$$z_0 = 0$$

$$\textit{if} (x_0 \neq 0)$$

$$\{x_0 \neq 0 \ \& \ \& x_0 \geq 0 \ \& \ \& z_0 == 0\} = \{x_0 > 0 \ \& \ \& z_0 == 0\}$$

$$z = x;$$

$$\{x_0 > 0 \ \& \ \& z == x_0\}$$

$$\textit{else}$$

$$\{x_0 == 0 \ \& \ \& x_0 \geq 0 \ \& \ \& z_0 == 0\} = \{x_0 == 0 \ \& \ \& z_0 == 0\}$$

$$z = z_0 + 1;$$

$$\{x_0 == 0 \ \& \ \& z == 1\}$$

$$\{x_0 > 0 \ \& \ \& z == x_0\} \parallel \{x_0 == 0 \ \& \ \& z == 1\}$$

$$= \{z > 0\} \parallel \{z == 1\}$$

$$= \{z > 0\}$$