Computer Fundamentals

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Lecture 6

Binary Logic Operations

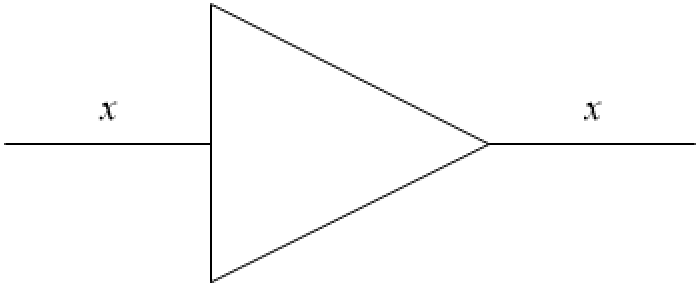
- Also called Boolean logic operations
- Boolean variable can have only two possible values i.e. 0 or 1
- Logic operations or functions of Boolean variables
  - E.g. AND, OR, NOT etc.

Binary Logic Operations

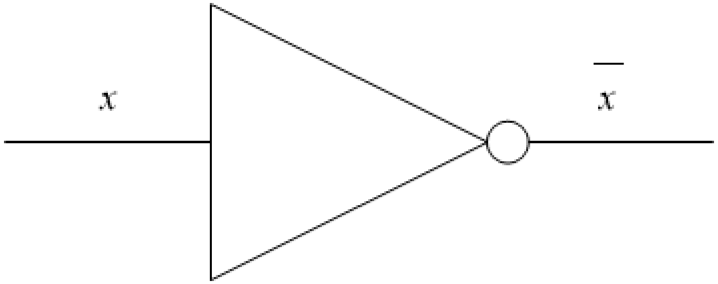
Name	Example	Symbolically
Buffer	y = x	x
NOT	y = NOT(x)	$\overline{x}$
AND	z = x AND y	$x.y$
OR	z = x OR y	$x + y$
XOR	z = x XOR y	$x \oplus y$
NAND	z = x NAND y	$\overline{x.y}$
NOR	z = x NOR y	$\overline{x + y}$
XNOR	z = x XNOR y	$\overline{x \oplus y}$

Diagrammatic Representation

- Buffer

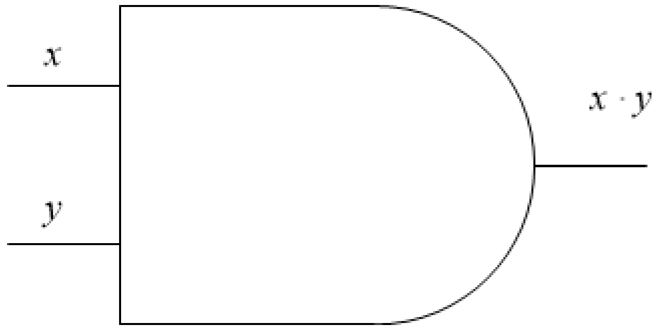


- NOT

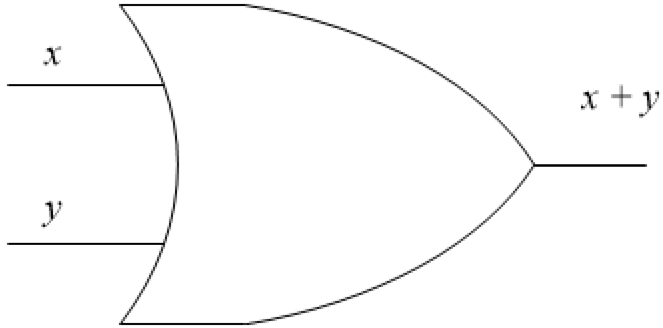


Diagrammatic Representation (cont.)

- AND

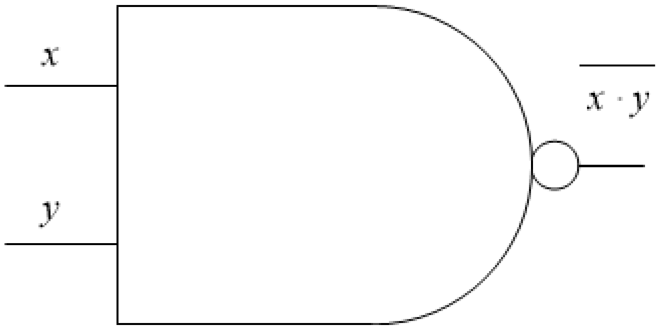


- OR

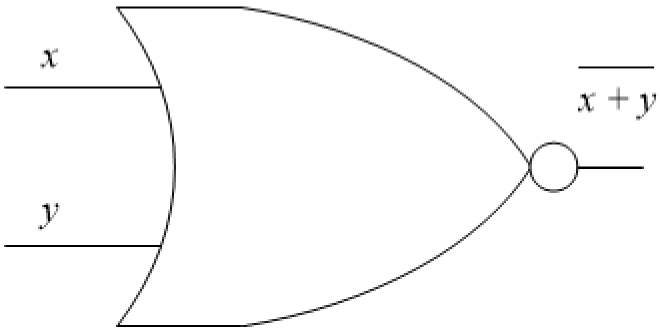


Diagrammatic Representation (cont.)

- NAND

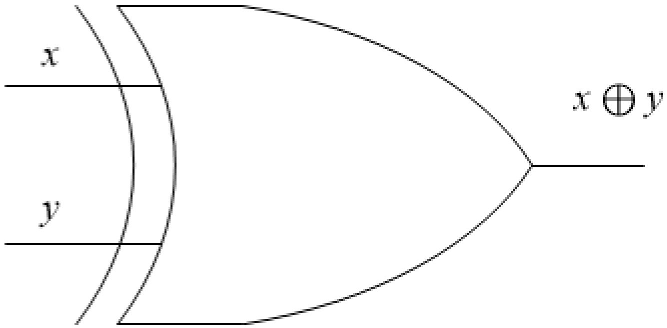


- NOR

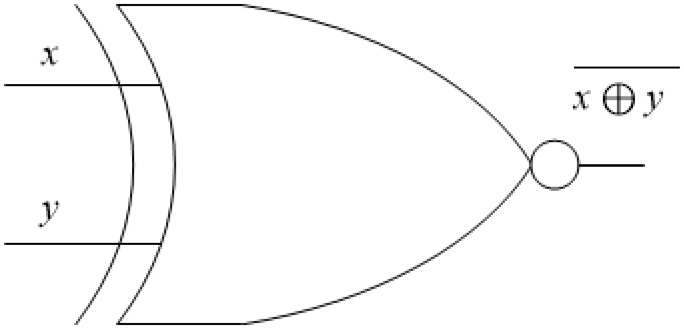


Diagrammatic Representation (cont.)

- XOR



- XNOR



Truth Table

- Defines the output of a logic function for all possible inputs
  - 2^n rows in a truth table (n is the number of inputs)

- Truth table for NOT

x	$\overline{x}$
0	1
1	0

Truth Table (cont.)

- Truth table for OR

x	y	$x+y$
0	0	
0	1	
1	0	
1	1	

Truth Table (cont.)

- Truth table for AND

x	y	$x.y$
0	0	
0	1	
1	0	
1	1	

Truth Table (cont.)

- Truth table for XOR

x	y	$x \oplus y$
0	0	
0	1	
1	0	
1	1	

Truth Table (cont.)

- Truth table for NOR

x	y	$x+y$	$\overline{x+y}$
0	0		
0	1		
1	0		
1	1		

Truth Table (cont.)

- Truth table for NAND

x	y	$x.y$	$\overline{x.y}$
0	0		
0	1		
1	0		
1	1		

Truth Table (cont.)

- Truth table for XNOR

x	y	$x \oplus y$	$\overline{x \oplus y}$
0	0		
0	1		
1	0		
1	1		

Boolean Laws

- AND

$$0 \cdot x = 0$$

$$1 \cdot x = x$$

- OR

$$0 + x = x$$

$$1 + x = 1$$

Boolean Laws (cont.)

- AND

$$x \cdot x = x$$

$$x \cdot \overline{x} = 0$$

- OR

$$x + x = x$$

$$x + \overline{x} = 1$$

Boolean Laws (cont.)

- AND
  - Commutativity

$$x \cdot y = y \cdot x$$

  - Associativity

$$(x \cdot y) \cdot z = x \cdot (y \cdot z)$$

  - Distributivity

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

Boolean Laws (cont.)

- OR
  - Commutativity

$$x + y = y + x$$

  - Associativity

$$(x + y) + z = x + (y + z)$$

  - Distributivity

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

Boolean Laws (cont.)

- DeMorgan's Law

  - NAND

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

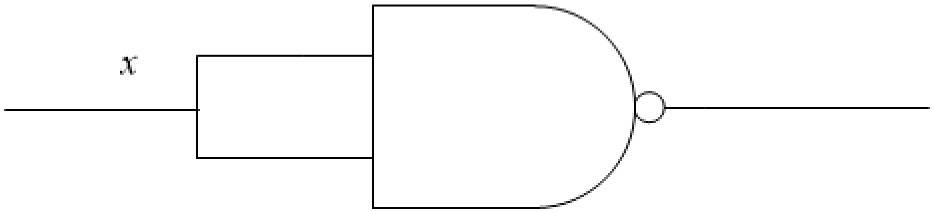
  - NOR

$$\overline{x + y} = \overline{x} \cdot \overline{y}$$

NAND for Every Gate

- Possible to emulate any gate
  - Using combination of NAND gates

- NOT gate with NAND
  - Compare with NOT

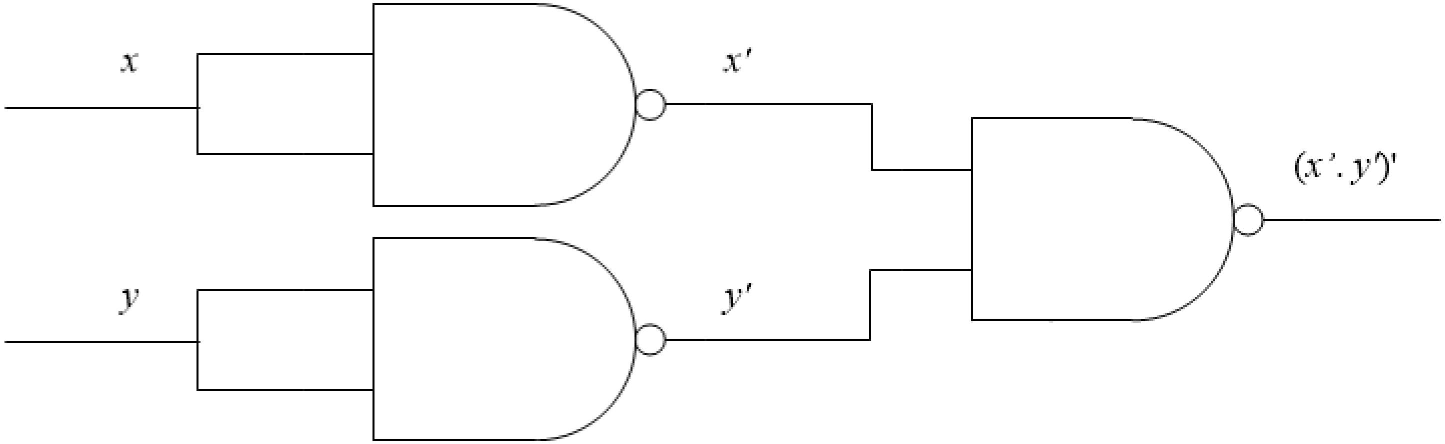


x	$x.x$	$\overline{x.x}$
0	0	1
1	1	0

NAND for Every Gate (cont.)

- OR gate with NAND
  - Recall DeMorgan's Law

$$x + y = \overline{\overline{x}.\overline{y}}$$



NAND for Every Gate (cont.)

- OR gate with NAND
  - Recall DeMorgan's Law

$$x + y = \overline{\overline{x}.\overline{y}}$$

  - Compare with OR

x	y	$\overline{x.x} = \overline{x}$	$\overline{y.y} = \overline{y}$	$\overline{\overline{x}.\overline{y}}$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Exercise

- Design AND gate using only NAND gates