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SECTION : B
ASSIGNMENT NO : 2

Q1. A die is tossed twice and the number of dots facing up is counted and noted in the order of occurance. Let A be the event "number of dots in first toss is not less than number of dots in second toss" and let B be the event "number of dots in first toss is 6". Find P[A|B] and P[B|A].

Solution:

$n = 6$, $k = 2$

size of sample space $= n^k = 6^2 = 36$

$A = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1),(2,2),(3,2),(4,2),(5,2),(6,2),(3,3),(4,3),(5,3),(6,3),(4,4),(5,4),(6,4),(5,5),(6,5),(6,6)\}$

$B = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

$A \cap B = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$$P[A \cap B] = \frac{6}{36} = \frac{1}{6}$$

$$P[B] = \frac{1}{6}$$

$$P[A|B] = \frac{1/6}{1/6} = 1$$

$$P[B|A] = \frac{P[A \cap B]}{P[A]}$$

$$P[A] = \frac{21}{36} = \frac{7}{12}$$

$$P[B|A] = \frac{1/6}{7/12} = \boxed{\frac{2}{7}}$$

(Q2) A number x is selected at random in the interval $[-1, 2]$. Let the events $A = \{x < 0\}$, $B = \{|x - 0.5| < 0.5\}$ and $C = x > 0.75$. Find $P[A|B]$, $P[B|C]$, $P[A|C^c]$ and $P[B|C^c]$.

Solution:

$$P[A] = \frac{\text{length}([-1, 0))}{\text{length}([-1, 2])} = \frac{0 - (-1)}{2 - (-1)} = \frac{1}{3}$$

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$$|x-0.5| < 0.5 \Rightarrow -0.5 < x-0.5 < 0.5$$
$$= 0 < x < 1$$

$$P[B] = \frac{\text{length}((0,1))}{\text{length}([-1,2])} = \frac{1}{3}$$

$$P[C] = \frac{\text{length}((0.75,2])}{\text{length}([-1,2])} = \frac{2-0.75}{3} = \frac{1.25}{3}$$

$$P[C] = \frac{125}{100} \times \frac{1}{3} = \frac{125}{300} = \frac{5}{12}$$

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{0}{1/3} = 0$$

$$P[B|C] = \frac{P[B \cap C]}{P[C]}$$

$$B \cap C = (0.75, 1)$$

$$P[B \cap C] = \frac{\text{length}((0.75,1))}{\text{length}([-1,2])} = \frac{1-0.75}{3} = \frac{0.25}{3} = \frac{1}{12}$$

$$P[B|C] = \frac{\frac{1}{12}}{5/12} = \frac{1}{5}$$

$$P[A|C^c] = \frac{P[A \cap C^c]}{P[C^c]}$$

$$P[C^c] = 1 - P[C] = 1 - \frac{5}{12} = \frac{7}{12}$$

$$P[A \cap C^c] = \frac{\text{length}([-1,0))}{\text{length}([-1,2])} = \frac{1}{3}$$

$$P[A|C^c] = \frac{1/3}{7/12} = \frac{4}{7}$$

$$P[B|C^c] = \frac{P[B \cap C^c]}{P[C^c]}$$

$$P[B \cap C^c] = \frac{\text{length}((0,0.75])}{\text{length}([-1,2])} = \frac{0.75}{3} = \frac{1}{4}$$

$$P[B|C^c] = \frac{1/4}{7/12} = \frac{3}{7}$$

(Q3) Traffic Police checked the CNICs and licenses of all the people driving any vehicle on a particular road on a given day. 85% of the drivers were carrying valid CNICs and 75% of the drivers were carrying valid licenses. 65% were carrying both valid CNICs and licenses. What percent of those who were carrying a valid CNIC were also carrying a valid driving license? What percent of those who were carrying a valid license were also carrying a valid CNIC?

solution:

Let A = Valid CNICs
B = Valid licences
C = Both valid

$P[A] = 0.85$
$P[B] = 0.75$
$P[C] = P[A \cap B] = 0.65$
$P[A|B] = ?$ $P[B|A] = ?$

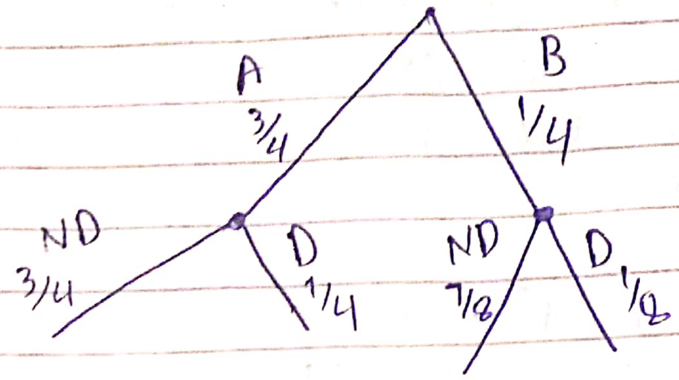
$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{0.65}{0.75} = 0.867 = 86\%$$

$$P[B|A] = \frac{P[A \cap B]}{P[A]} = \frac{0.65}{0.85} = 0.765 = 76\%$$

(Q4) A servay of two new anticlandruff shampoos A and B was conducted by a health care organization. 3/4 of the people who bought shampoo A got rid of dandruff, whereas 7/8 of the people who used shampoo B were satisfied with the shampoo. Among the servayed people, 3/4 of them bought shampoo A while the rest bought shampoo B. What is the probability of people getting rid of dandruff irrespective of which shampoo they used.

Solution:

D = dandruff
ND = No dandruff



$$\text{Total Probability} = P[ND_A]\,P[A] + P[ND_B]\,P[B]$$

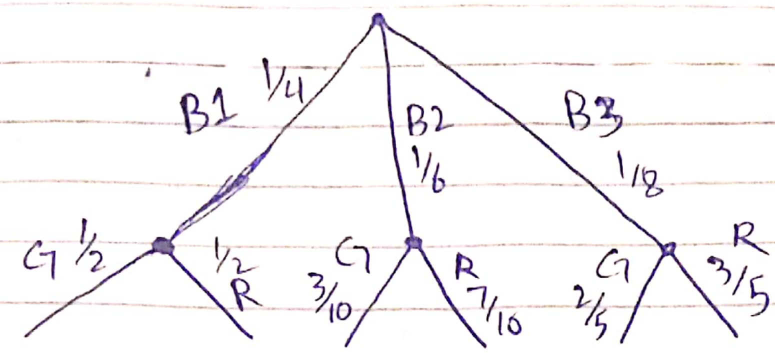
$$= \frac{3}{4} \times \frac{3}{4} + \frac{7}{8} \times \frac{1}{4}$$

$$= \frac{9}{16} + \frac{7}{32} = \frac{18+7}{32} = \frac{25}{32}$$

$$= 0.78$$

(Q5) Three boxes contain red and green balls. Box 1 has 5 red balls and 5 green balls, Box 2 has 7 red balls and 3 green balls and Box 3 contains 6 red balls and 4 green balls. The probability of choosing Box 1 is $1/4$, Box 2 is $1/6$ and Box 3 is $1/8$. What is the probability that the ball choosen is green? What is the probability that the ball choosen is red?

Solution:



Total Probability of Green $= P[G|B1]P[B1] + P[G|B2]P[B2] + P[G|B3]P[B3]$

$$= \frac{1}{2} \times \frac{1}{4} + \frac{3}{10} \times \frac{1}{6} + \frac{2}{5} \times \frac{1}{8}$$

$$= \frac{1}{8} + \frac{1}{20} + \frac{1}{20} = \frac{5+2+2}{40}$$

$$= \frac{9}{40} = \boxed{0.225}$$

Total Probability of Red $= P[R|B1]P[B1] + P[R|B2]P[B2] + P[R|B3]P[B3]$

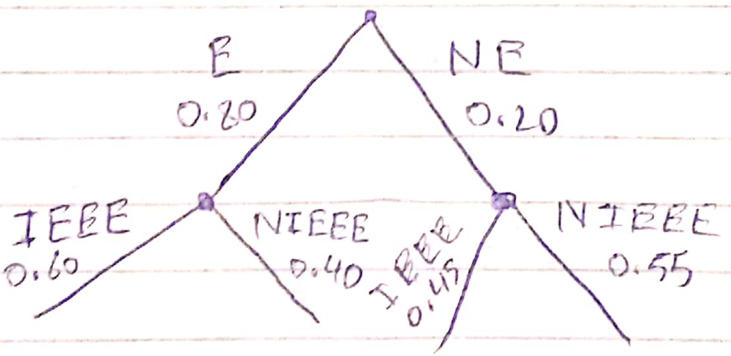
$$= \frac{1}{2} \times \frac{1}{4} + \frac{7}{10} \times \frac{1}{6} + \frac{3}{5} \times \frac{1}{8}$$

$$= \frac{1}{8} + \frac{7}{60} + \frac{3}{40} = \frac{15+14+9}{120}$$

$$= \frac{38}{120} = \boxed{0.317}$$

(Q6) In a University, 80% of the students are studying in various engineering diciplines, and the rest are pursuing non-engineering degrees. 60% of the engineering students ~~are~~ use IEEExplore for research and 45% of non-engineering students avail the IEEExplore service. A student is randomly selected from the university and asked whether he uses IEEExplore service or not. if the student turns out to be the user ov the service, what is the probability that the randomly selected student is an engineering student? What is the probability that the randomly selected student is a non-engineering student?

Solution:



By using Baey's rule:

$$P[E|IEEE] = \frac{P[IEEE|E]\,P[E]}{P[IEEE]}$$

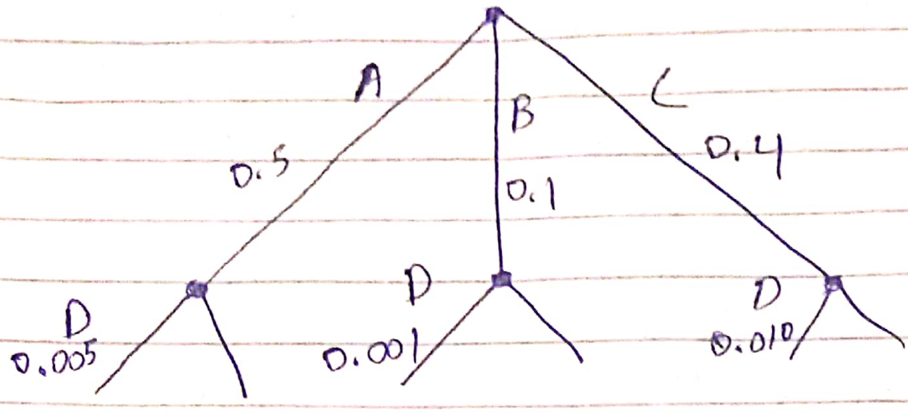
$$P[IEEE] = P[IEEE|E]P[E] + P[IEEE|NE]P[NE]$$
$$= (0.60)(0.80) + (0.45)(0.20)$$
$$= 0.48 + 0.09$$
$$= 0.57$$

$$P[E|IEEE] = \frac{(0.60)\times(0.80)}{0.57}$$
$$= \boxed{0.842}$$

$$P[NE|IEEE] = \frac{P[IEEE|NE]\,P[NE]}{P[IEEE]}$$
$$= \frac{(0.45)\times(0.20)}{0.57}$$
$$= \frac{0.09}{0.57} = \boxed{0.157}$$

(Q7) A computer manufacturer uses Chips from three sources. Chips from sources A, B and C are defective with probabilities 0.005, 0.001 and ~~0.0~~ 0.010 respectively. If a random selected chip is found to be defective Find the probability that the manufacturer was A; that the manufacturer was C. Assume that the proportions of Chips from A, B and C are 0.5, 0.1 and 0.4 respectively.

Solution:



$$P[D|A] = 0.005$$
$$P[D|B] = 0.001$$
$$P[D|C] = 0.010$$

$$P[D] = P[D|A]P[A] + P[D|B]P[B] + P[D|C]P[C]$$
$$= 0.005 \times 0.5 + 0.001 \times 0.1 + 0.010 \times 0.4$$
$$= 0.0025 + 0.0001 + 0.004$$
$$= 0.0066$$

$$P[A|D] = \frac{P[D|A]\,P[A]}{P[D]} = \frac{0.005 \times 0.5}{0.0066}$$
$$= \frac{0.0025}{0.0066} = \boxed{0.378}$$

$$P[C|D] = \frac{P[D|C]\,P[C]}{P[D]} = \frac{0.010 \times 0.4}{0.0066} = \frac{0.004}{0.0066}$$
$$= \boxed{0.606}$$