Assignment #1

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Exercise 1.3

(Q3) $y' = 1 + 0.01y^2$

solution:

$$y' = 1 + 0.01y^2$$

$$\frac{dy}{dx} = 1 + 0.01y^2$$

$$\int \frac{dy}{1+0.01y^2} = \int dx$$

$$\frac{1}{\sqrt{0.01}} \tan^{-1}(\sqrt{0.01}\, y) = x + C$$

$$\tan^{-1}(\sqrt{0.01}\, y) = \sqrt{0.01}\, x + C$$

$$y = \frac{1}{\sqrt{0.01}} \tan(\sqrt{0.01}\, x + C)$$

(Q5) $y' = xy/2$

solution:

$$\frac{dy}{dx} = \frac{xy}{2}$$

$$\int \frac{dy}{y} = \int \frac{x\,dx}{2}$$

$$\ln|y| = \frac{x^2}{4} + C$$

$$\boxed{y = Ce^{x^2/4}}$$

(Q19) $L\frac{dI}{dt} + RI = 0$

$$\frac{dI}{dt} = -\frac{RI}{L}$$

$$\frac{dI}{I} = -\frac{R\,dt}{L}$$

$$\int \frac{dI}{I} + \int \frac{R\,dt}{L} = 0$$

$$\ln|I| + \frac{Rt}{L} = C$$

$$\ln|I_0| + \frac{R(0)}{L} = C$$

$$\ln|I_0| = C$$

$$\ln|I| - \ln|I_0| = -\frac{Rt}{L}$$

$$\ln\left|\frac{I}{I_0}\right| = -\frac{Rt}{L}$$

$$\frac{I}{I_0} = e^{-\frac{Rt}{L}}$$

$$I = I_0 e^{-\frac{Rt}{L}}$$

Exercise 1.5

(Q4) $u = \frac{1}{x^2+y^2}$

Solution:

$$du = 0, \quad du = \frac{du}{dx}dx + \frac{du}{dy}dy$$

$$\Rightarrow \frac{-1}{(x^2+y^2)} 2x\,dx + \frac{-1}{(x^2+y^2)^2} 2y\,dy = 0$$

$$\Rightarrow x\,dx + y\,dy = 0$$

(Q10) $e^{3\theta}(dr + 3r\,d\theta) = 0$

Solution:

$$e^{3\theta}(dr + 3r\,d\theta) = 0$$

$$e^{3\theta}dr + 3re^{3\theta}d\theta = 0$$

$M = e^{3\theta}$

$$\frac{dM}{d\theta} = M_\theta = 3e^{3\theta}$$

$N = 3re^{3\theta}$

$$\frac{dN}{dr} = N_r = 3e^{3\theta}$$

$M_\theta = N_r \Rightarrow$ so this is EDE.

Differentiate M w.r.t γ

$$\int M = \int e^{3\theta} d\gamma$$

$$= \gamma e^{3\theta}$$

Terms in N free of γ = 0

$$\gamma e^{3\theta} + 0 = C$$

$$\boxed{\gamma e^{3\theta} = C} \rightarrow \text{General solution}$$

(Q12) $(\cot y + x^2)dx - x\operatorname{cosec}^2 y\, dy = 0$

$M = \cot y + x^2$

$M_y = -\operatorname{cosec}^2 y$

$N = x\operatorname{cosec}^2 y$

$N_x = -\operatorname{cosec}^2 y$

As $M_y = N_x$ so this is EDE

$$\int M = \int (\cot y + x^2)\, dx$$

$$= x\cot y + \frac{x^3}{3}$$

Terms free of x in $N = x\operatorname{cosec}^2 y$

$$U(x, y) \Rightarrow x\cot y + \frac{x^3}{3} = C$$