# Probability Methods in Engineering

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Lecture 1

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# Resource Material

- Course Book
  - A. Leon-Garcia, "Probability and Random Processes for Electrical Engineering", 3rd Edition, Pearson Prentice Hall, 2008
- Reference Books
  - D. Bertsekas and J. N. Tsitsiklis, "Introduction to Probability", 2nd Edition, Athena Scientific, 2008
  - Hossein Pishro-Nik, "Introduction to Probability, Statistics, and Random Processes", Kappa Research, 2014

# Course Group

- Joining the group is important
  - *https://groups.google.com/forum/#!forum/2020_pme_spring*
- Online group benefits
  - Slides
  - Assignments
  - Announcements
  - Course outline
  - Discussions
  - CLOs and PLOs (OBE)
- Group email
  - *2020_PME_spring@googlegroups.com*
- Teaching method
  - Combination of slides and white board
  - Interaction about concepts encouraged
  - Interruption to ask questions during lectures allowed

# Tentative Grading Criteria

- Exams
  - Final exam: **50%**
  - Mid-term exam: **25%**
- Sessional
  - Attendance: **10%**
  - Assignments: **7.5%**
  - Quizzes: **7.5%**
- All lectures interrelated
  - Each lecture provides base for next lecture
  - Missing any lecture would result in problems in understanding subsequent lectures

- No mobile phone usage during class

# Course Outline

- Introduction to Probability
  - Axioms
  - Probabilities using Counting methods
  - Conditional Probability
  - Law on total Probability
  - Bayes' Rule
  - …
- Random Variables (RVs)
  - Cumulative Distribution Function (CDF)
  - Probability Density Function (PDF)
  - Mean and variance
  - …
- Modern Tools
  - MATLAB
  - Python
  - …

# Course Significance

- Basis for numerous advanced technologies
  - Wave propagation
  - Wireless communication
  - Communication theory
  - Information theory
  - Pattern recognition
  - Radar and sonar signal processing
  - Network design and optimization

# Course Significance (cont.)

- Disadvantages of weak probability concepts
  - No scope in research fields
  - Poor analytical skills
  - Fear of interview questions
  - Inability to conceptualize techniques
  - No major role possible in engineering problem solving
  - Minimum contribution towards nation building
  - Incapability to carry out feasibility studies for mega projects

# Assessment Test

- A fair die is rolled thrice. What is the probability of getting
  - A six in the first attempt
  - Sixes in first two attempts
  - Sixes in all three attempts
  - All odd outcomes
  - All outcomes greater than 4
- What is the number of all possible outcomes?

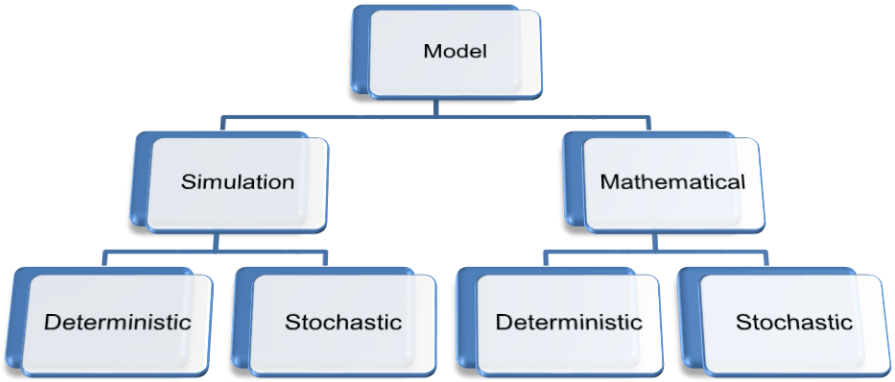
# Randomness

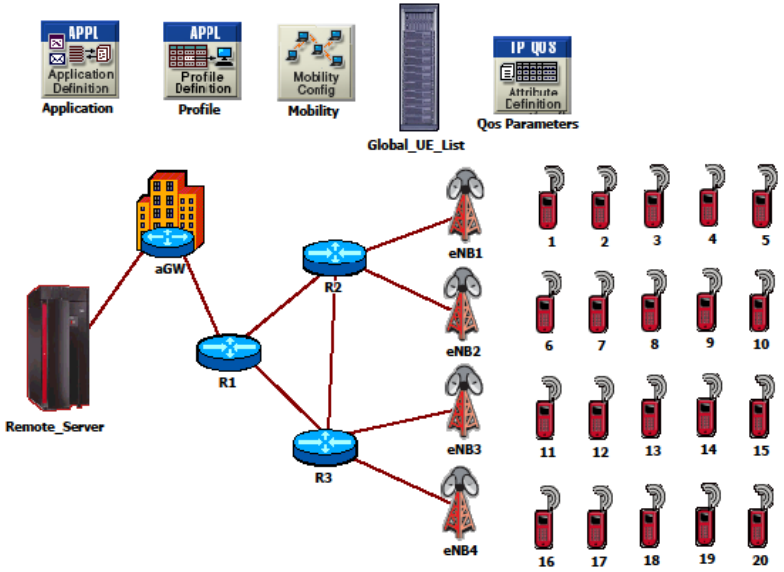
- What is Randomness?
  - Chaos
  - Uncertainty
  - Doubt
- Humans desire some level of 'certainty'
- Examples
  - Solar system
  - Weather forecast at Chitral Airport
  - Traffic situation on University road
- Engineers **quantify** 'certainty'

**Source**: Computer Vision Research Group, *CIIT Lahore*, Pakistan

# Model of a Physical System

- **Model**: Approximate representation of physical situation
  - **Mathematical model**: Set of assumptions about how system works
    - **Deterministic model**: Offers repeatability of results, (e.g. Ohm's Laws)
    - **Stochastic model**: Characterizes randomness and uncertainty
  - **Simulation model**: Imitation of real system
    - **Deterministic model**: No random component involved, (e.g. chemical reaction)
    - **Stochastic model**: Must have random input component




**Source**: S. N. K. Marwat, PhD Thesis, *University of Bremen, Germany*

# Random Experiment

- **Random Experiment**: The result varies in random manner
- **Sample Space**: Set of all possible experiment results
- **Outcome**: A single element of sample space
- **Event**: A subset of sample space

- Example: An urn containing three balls, one is drawn
  - How probable it is that a ball withdrawn at random is labeled '1'?
  - Can you quantify this 'chance'?
  - Everyone of you should be able to write the sample space for this experiment!

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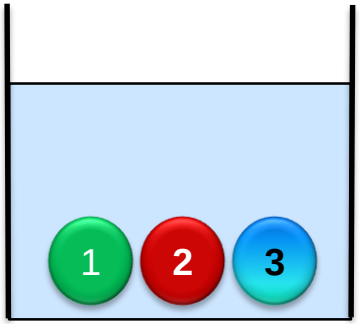


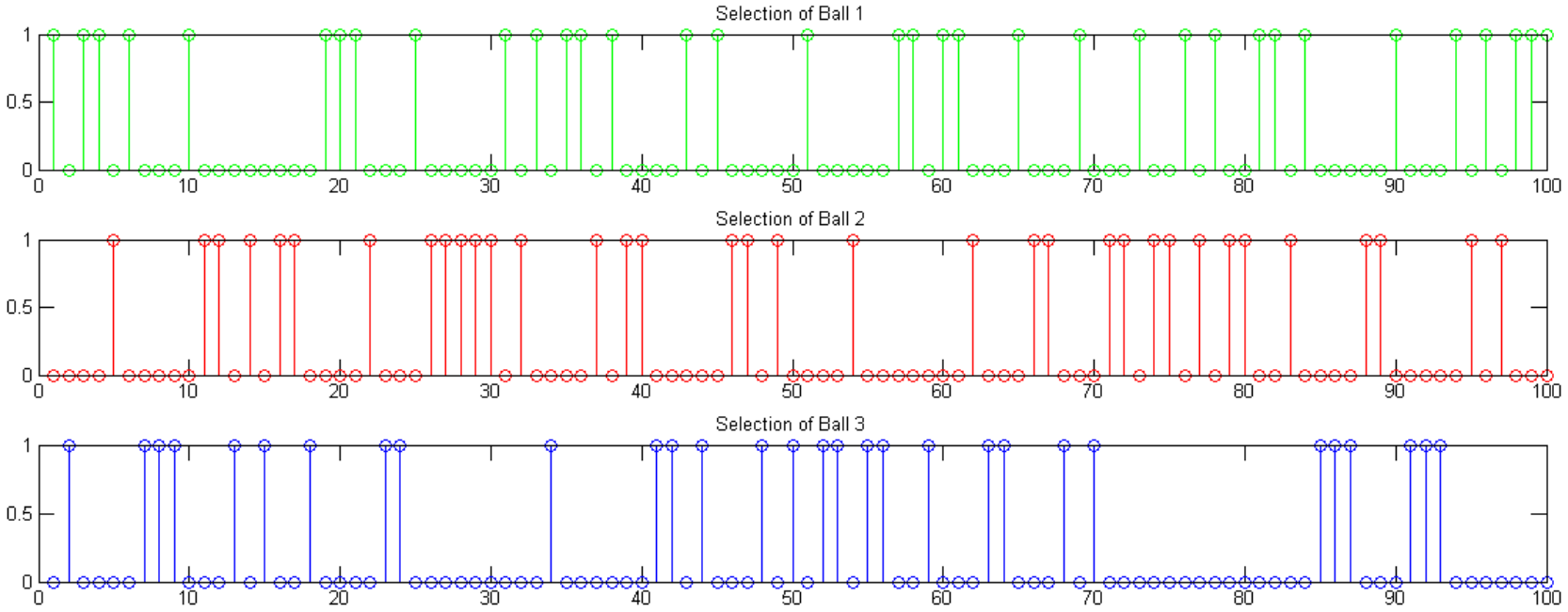
# Random Experiment (cont.)

- Some more questions to answer
  - Is withdrawing all the three balls equi-probable (or is any ball more likely to be drawn)?
  - If '1' means 'sure occurrence' and '0' means 'no chance of occurrence', what number can be given to the chance of getting 'ball 1'?
  - What is the chance of withdrawing an odd-numbered (or even-numbered) ball?

Let the nature answer this

# Random Experiment (cont.)

- Take a ball from the urn
- Record the outcome
- Put it back in the urn
- Do the experiment '$n$' times



# Random Experiment (cont.)

- Number of times $k^{th}$ outcome occurred (or **frequency** of $k$) in a total of $n$ trails

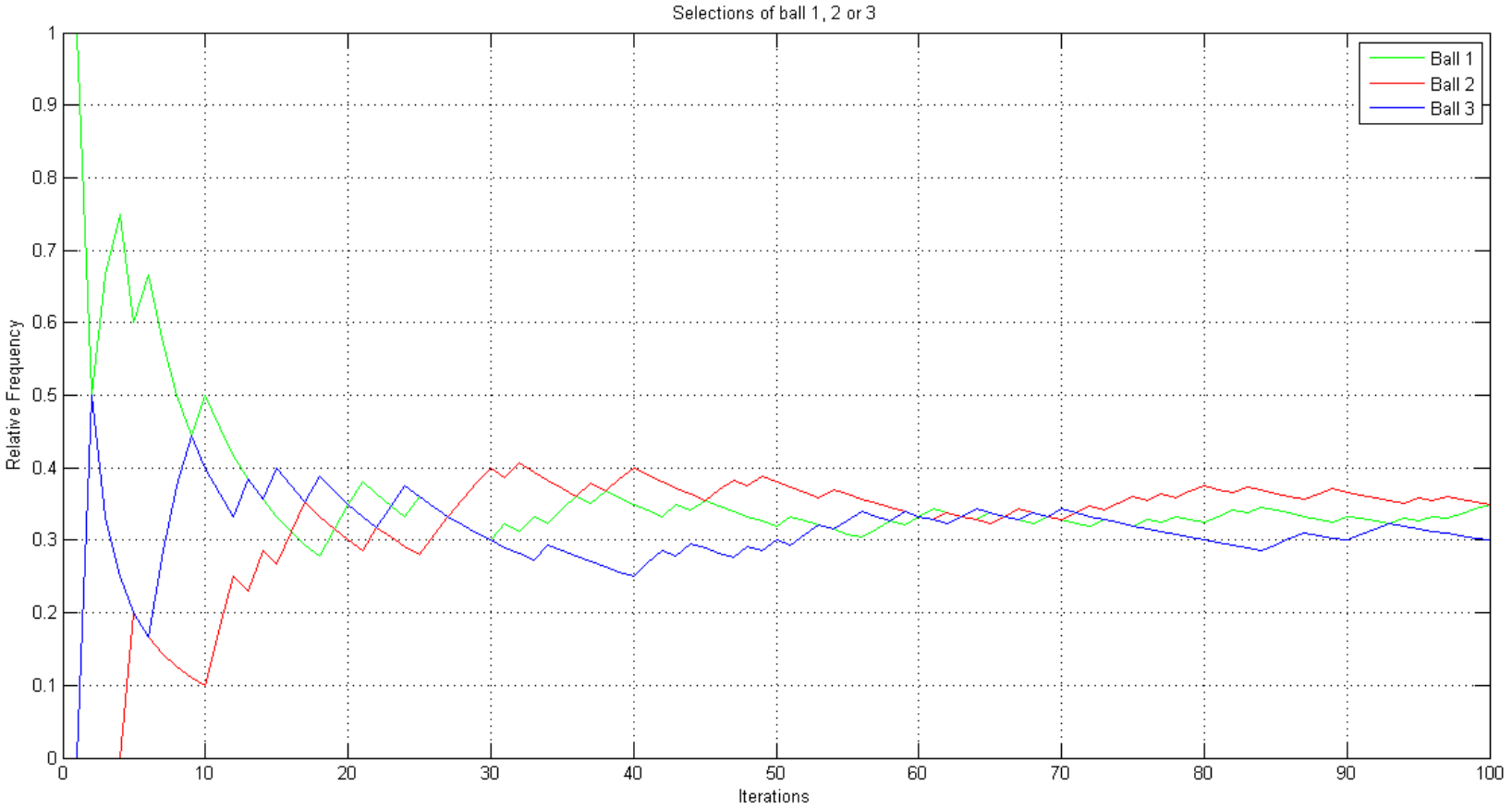
$$N_k(n)$$

- The **relative frequency** of $k^{th}$ outcome

$$f_k(n) = \frac{N_k(n)}{n}$$

# Random Experiment (cont.)

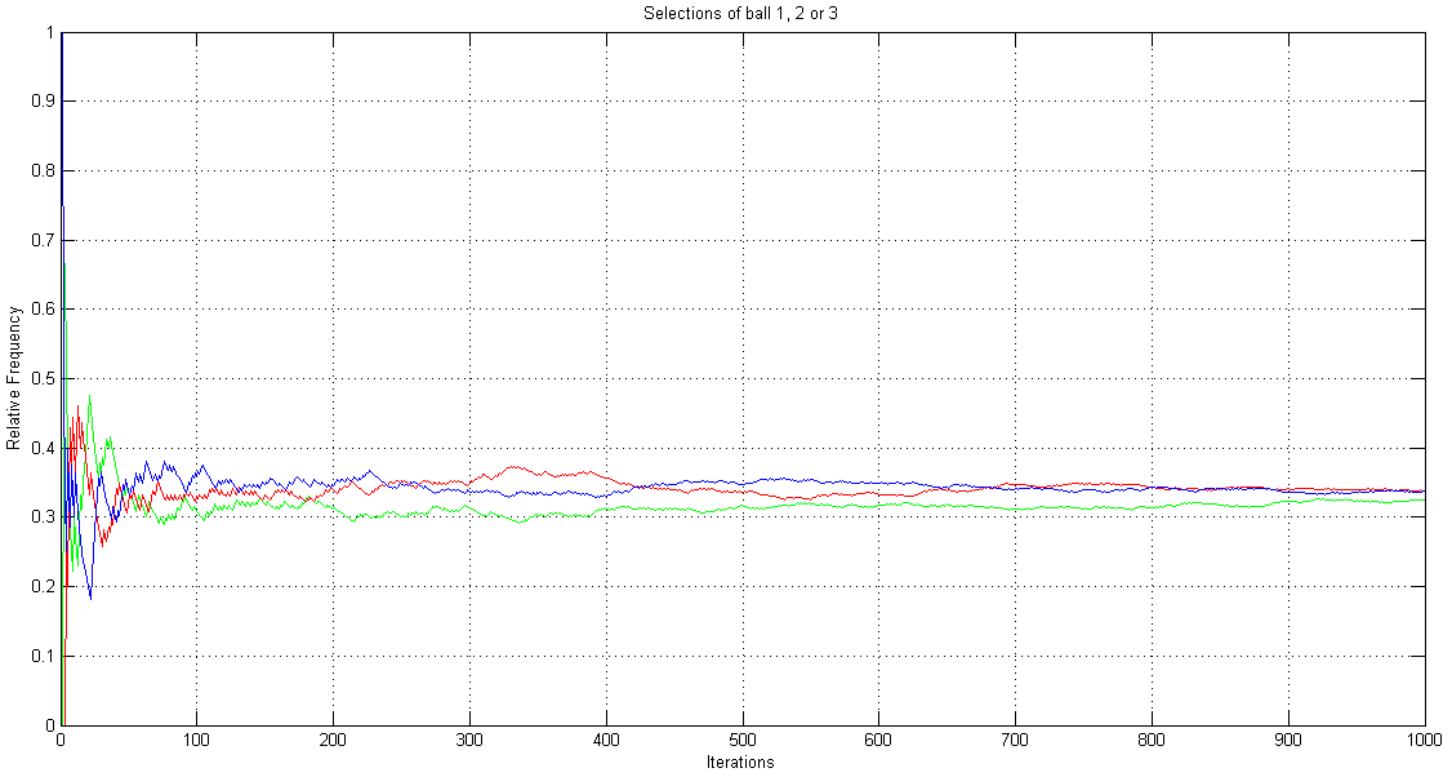
- Statistical Regularity
  - Averages obtained in long sequences yield same value



# Random Experiment (cont.)

- **Probability** defined by *von Mises* as 'limiting case of relative frequency'

$$\lim_{n\to\infty} f_k(n) = \lim_{n\to\infty} \frac{N_k(n)}{n} = p_k$$



# Properties of Relative Frequency

- Number of occurrences of an outcome in $n$ trials
  - A number between zero and $n$

$$0 \leq N_k(n) \leq n$$

- Relative frequencies are
  - A number between zero and one
  - Divide the above equation by $n$ to get

$$0 \leq f_k(n) \leq 1$$

# Properties of Relative Frequency (cont.)

- Sum of number of occurrences of all possible outcomes
  - Must be $n$

$$\sum_{k=1}^{K} N_k(n) = n$$

- Sum of all relative frequencies
  - Must be 1

$$\sum_{k=1}^{K} f_k(n) = 1$$