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SECTION: B
ASSIGNMENT NO: 3

Q1. Let $S = \{1,2,3,4\}$ and $A = \{1,2\}$, $B = \{1,3\}$
$C = \{1,4\}$. Assume the outcomes are equiprobable.
Are A and B independent? Are A and C independent?

Solution:

$P[A] = \frac{1}{2}$

$P[B] = 1/2$

$A \cap B = \{1\}$

$P[A \cap B] = 1/4$

$P[A].P[B] = P[A \cap B]$

$1/2 \times 1/2 = 1/4$

$1/4 = 1/4$ so A and B are Independent.

$P[C] = 1/2$

$A \cap C = \{1\}$

$P[A \cap C] = P[A].P[C]$

$1/4 = 1/2 \times 1/2 = \frac{1}{4}$ so A and C are Independent

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Q2. Player A and B practice shots at penalty in free time. A succeeds with probability P_a and B with probability P_b such that the probabilities are independent. Find the probability of the following outcomes when A and B each take one shot:
A scores; Either A or B scores; both score; both miss.

Solution:

(i) When A scores:

$$P[A] = 1/2$$

(ii) Either A or B scores:

$$P[AB^c \cup A^cB]$$
$$= P[AB^c] + P[A^cB]$$
$$= P[A].P[B^c] + P[A^c]P[B]$$
$$= P[A][1-P[B]] + [1-P[A]]P[B]$$
$$= (1/2)(1-1/2) + (1-1/2)(1/2)$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = 1/2$$

(iii) Both scores:

$$P[AB] = P[A]P[B]$$
$$= 1/2 \times 1/2 = 1/4$$

(iv) Both misses:

$$P[A^cB^c] = P[A^c]\,P[B^c]$$
$$= (1-P[A])\,(1-P[B])$$
$$= (1-\tfrac{1}{2})\,(1-\tfrac{1}{2})$$
$$= \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}$$

Q3. Suppose that five numbers are selected at random from the interval [3, 6]. Find the probability that the first three numbers are less than 5 and the last two numbers are greater than 4.

Solution:

$$S = [3, 6]$$
$$A = 5-3 = 2$$
$$B = 6-4 = 2$$
$$P[A] = \frac{2}{6-3} = \frac{2}{3}$$
$$P[B] = \frac{2}{3}$$

Using sequence of I.E

$$P[A_1 \cap A_2 \cap A_3 \ldots A_n] = P[A_1]P[A_2]P[A_3]\ldots P[A_n]$$
$$= \left(\frac{2}{3}\right)^2 \left(\frac{2}{3}\right)^2$$
$$= \left(\frac{2}{3}\right)^4$$
$$= \boxed{0.131}$$

Q4. Suppose a dice - -- - ?

Solution:

$n = 4$, $p = 1/6$, $k = 0,1,2,3,4$

$P_n(k) = {}^{n}C_{k} (p)^k (1-p)^{n-k}$

$P_4(0) = {}^{4}C_{0} \left(\frac{1}{6}\right)^0 \left(1-\frac{1}{6}\right)^4$

$= 1 \times 1 \times \left(\frac{5}{6}\right)^4 = 0.482$

$P_4(1) = {}^{4}C_{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 = 4 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^3 = 0.385$

$P_4(2) = {}^{4}C_{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = 6 \times \frac{1}{36} \times \left(\frac{5}{6}\right)^2 = 0.115$

$P_4(3) = {}^{4}C_{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) = 4 \times \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) = 0.015$

$P_4(4) = {}^{4}C_{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = 1 \times \left(\frac{1}{6}\right)^4 = 0.00077$

Q5. A student needs eight .....?

Solution:

% of defective chips = 5%

working chips = 100 - 5 = 95%

$P = 0.95$

$P_8(8) = {}^{8}C_{8} (0.95)^8 (0.05)^0 = 0.66$

$P_9(8) = {}^{9}C_{8} (0.95)^8 (0.05) = 0.297$

$$P_9(9) = {}^9_9C\,(0.98)^9(0.05)^0 = 0.63$$

$$P_9(8) + P_9(9) = 0.29 + 0.63 = 0.92$$

Q6. What is the probability - - - - ?

Solution:

By Geometric Probability law

$$P(m) = q^{m-1}\,p$$

$$m = 3$$

$$p = 1/6$$

$$q = 1 - \frac{1}{6} = 5/6$$

$$P(3) = (5/6)^2 (1/6) = 0.1157$$

Q7. Three employees work . . . . . ?

Solution:

C = Cook Available

W = Waiter Available

$P[C] = 0.85$

$P[W] = 0.75$

$$F = (W_1 \cap W_2^c) \cup (W_1^c \cap W_2) \cup (W_1 \cap W_2)$$

$$P[F] = P[W_1 \cap W_2^c] + P[W_1^c \cap W_2] + P[W_1 \cap W_2]$$
$$= 2(0.75)(0.25) + (0.75)^2$$
$$= 0.9375$$

$P[\text{open Restaurant}] = P[C]P[F]$
$= (0.85)(0.93)$
$= 0.7968$

Suppose cook is employed

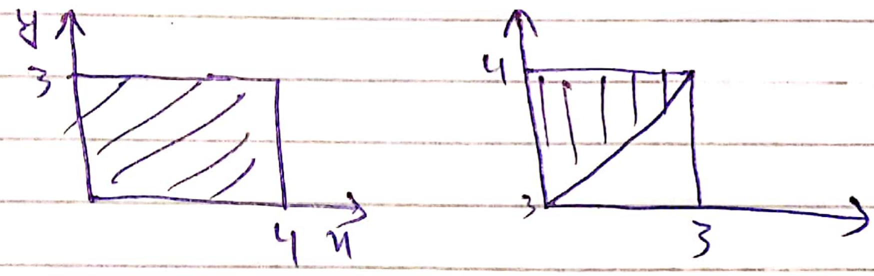
$C = (C_1 \cap C_2^c) \cup (C_1^c \cap C_2) \cup (C_1 \cap C_2)$

$P[C] = P[C_1]P[C_2^c] + P[C_1^c]P[C_2] + P[C_1]P[C_2]$
$= 2(0.85)(0.15) + (0.85)^2$
$= 0.255 + 0.7225$
$= 0.9775$

$P[C]P[F] = (0.97)(0.93) = 0.902$

Q8. In a student hostel ....?

Solution:



$P[A] \neq 0$
$P[B] \neq 0$
But $P[A \cap B] = 0$
so $P[A]P[B] \neq P[A \cap B]$
A and B are not independent.