Assignment No: 1.

Summer - 2020

Dept of Computer System Engineering

Name :- Shah Raza.

Batch :- 20

Date :- 30/09/2020

Question - I.

Find roots of $\sqrt[5]{1-i}$

Solution: let $w = \sqrt[5]{1-i}$

$\Rightarrow w = \sqrt[5]{z}$ — ① $\because z = 1-i$

$\Rightarrow z = w^5$

Converting z into polar form

$r = \sqrt{(1)^2 + (1)^2}$

$r = \sqrt{2}$

and

$\theta = \tan^{-1} y/x \quad \because x > 0$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right)$

$= \tan^{-1}(-1)$

$= -\tan^{-1}(1)$

$\theta = 45$

$\theta = \pi/4$

Now

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Putting in ①

eq ① $\Rightarrow$ $w = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{\frac{1}{5}}$

$w = (\sqrt{2})^{1/5}\left(\cos\frac{\pi}{4\times5} + i\sin\frac{\pi}{4\times5}\right)$

$w = (2)^{\frac{1}{2}\times\frac{1}{5}}\left(\cos\frac{\pi}{20} + i\sin\frac{\pi}{20}\right)$

$w = (2)^{\frac{1}{10}}\left(\cos\pi/20 + i\sin\pi/20\right)$

Now

$$w_k = \sqrt[10]{2}\left(\cos\frac{\pi}{20} + 2k\pi + i\sin\frac{\pi}{20} + 2k\pi\right)$$

$$= \sqrt[10]{2}\left(\cos\frac{\pi + 40k\pi}{20} + i\sin\frac{\pi + 40k\pi}{20}\right)$$

For $k = 0$

$$w_0 = \sqrt[10]{2}\left(\cos\frac{\pi}{20} + i\sin\frac{\pi}{20}\right)$$

For $k=1$

$$w_1 = \sqrt[10]{2}\left(\cos\frac{\pi+40\pi}{20} + i\sin\frac{\pi+40\pi}{20}\right)$$

$$w_1 = \sqrt[10]{2}\left(\cos\frac{41\pi}{20} + i\sin\frac{41\pi}{20}\right)$$

For $k=2$

$$w_2 = \sqrt[10]{2}\left(\cos\frac{\pi+80\pi}{20} + i\sin\frac{\pi+80\pi}{20}\right)$$

$$w_2 = \sqrt[10]{2}\left(\cos\frac{81\pi}{20} + i\sin\frac{81\pi}{20}\right)$$

Q#2:

$$U = e^{-\pi x}\cos ay$$

Find 'a' so given function is harmonic and find harmonic conjugate

Sol:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$$\cos ay \frac{\partial^2 e^{-\pi x}}{\partial x^2} + e^{-\pi x}\frac{\partial^2}{\partial y^2}\cos ay = 0$$

$$\pi^2 e^{-\pi x}\cos ay + (-a^2 e^{-\pi x}\cos ay) = 0$$

$$\pi^2 e^{-\pi x}\cos ay - a^2 e^{-\pi x}\cos ay = 0$$

$$(\pi^2 - a^2)(e^{-\pi x}\cos ay) = 0$$

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$\pi^2 - a^2 = 0$

$a^2 = \pi^2$

$\boxed{a = \pi}$

For $a = \pi$ the function will be harmonic.

$u = e^{-\pi x}\cos\pi y$

Now to find v;

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

$$\frac{\partial v}{\partial y} = \frac{\partial(e^{-\pi x}\cos\pi y)}{\partial x}$$

$$\frac{\partial v}{\partial y} = -\pi\cos(\pi y)e^{-\pi x}$$

$v = -e^{-\pi x}\sin(\pi y) + h(x)$

Now to find $h(x)$;

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$\pi\sin(\pi y)e^{-\pi x} + h'(x) = -(-\pi e^{-\pi x}\sin\pi y)$

$h'(x) = -e^{-\pi x} \times \pi \sin(\pi y) + e^{-\pi x}\pi\sin(\pi y)$

$h'(x) = 0$

$h(x) = 0 + c$

$h(x) = c$

$\boxed{v = -e^{-\pi}\sin(\pi y) + c}$

harmonic conjugate.

Q.73: Find all Solutions in Complex plane.

$\sin z = 1000.$

Sol: As,
$$\sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) = 1000.$$
$$\Rightarrow \frac{1}{i}(e^{iz} - e^{-iz}) = 2000$$
$$\Rightarrow -i(e^{iz} - e^{-iz}) = 2000 \rightarrow (A)$$
×ing and ÷ing by $-i$

let
$$z = x + iy$$
$$\Rightarrow e^{iz} = e^{i(x+yi)} = e^{xi-y}$$
$$= e^{-y}(\cos x + i\sin x)$$
putting in (A)
$$(A) \Rightarrow -i(e^{-y}(\cos x + i\sin x) - e^{y}(\cos x - i\sin x)) = 2000$$
$$\Rightarrow -i(e^{-y}(\cos x + i\sin x) - e^{y}(\cos x + i\sin x)) = 2000$$
$$\Rightarrow -i(\cos x(e^{-y} - e^{-y}) + i\sin(e^{y} + e^{-y}) = 2000$$
$$\Rightarrow -i\cos x(e^{-y} - e^{y}) + \sin x(e^{y} + e^{-y}) = 2000$$
$$\Rightarrow i\cos(e^{y} - e^{-y}) + \sin x(e^{y} + e^{-y}) = 2000$$
Here,
$$\sin x(e^{y} + e^{-y}) = 2000$$
and
$$\cos x(e^{y} - e^{-y}) = 0$$
$\cos x(e^{y} - e^{-y})$ will only be zero

if $e^{y} - e^{-y} = 0 \Rightarrow e^{y} = \frac{1}{e^{y}}$

$\boxed{y = 0}$

if, $\cos x = 0 \Rightarrow x = \frac{\pi}{2} + 2k\pi$

For $y = 0$

$\sin x = 1000$, which is not possible

For $x = \frac{\pi}{2} + 2k\pi$

$\Rightarrow \sin x = 1$

Now,

$e^{y} + e^{-y} = 2000$

$\Rightarrow \frac{e^{y} + e^{-y}}{2} = 1000$

$\Rightarrow \cosh y = 1000$

$\Rightarrow y = \cosh^{-1}(1000)$

Thus $\{ (x, y) : x = \frac{\pi}{2} + 2k\pi, k \, (\exists y) = \cosh^{-1}(1000)$

Q#4:
Find the line Integral $\int_c Im(z^2)\,dz$

c: is shortest path from 0 to 0+i to 2+i

Sol:
$$\int_c z^2\,dz = \int_0^{0+i} z^2\,dz + \int_{0+i}^{2+i} z^2\,dz$$

C_1: $(0,0)$ to $(0,1)$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$$\frac{x-0}{0-0} = \frac{y-0}{1-0} = t$$

$t = y$, $x=0$ for $y=0, t=0$

$y=1, t=1$

$z = 0+it$

$dz = i\,dt$

$$\int_{c_1} z^2\,dz = \int_0^1 (it)^2\, i\,dt$$

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$$\Rightarrow i^2 \times i \int_0^1 t^2 dt$$

$$\Rightarrow -i \left(\frac{i^3}{3} \Big|_0^1 \right)$$

$$= -i \left(\frac{1}{3} - \frac{0}{3} \right)$$

$$\boxed{\int_{c_1} z^2 dz = \frac{-1}{3} i} \rightarrow (1)$$

Now C_2 : $(0,1)$ to $(2,1)$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\frac{x - 0}{2 - 0} = \frac{y - 1}{1 - 0} = t$$

$$\Rightarrow \boxed{\frac{x}{2} = t}, \boxed{y = 0}$$

$$\Rightarrow \boxed{x = 2t} \quad \text{for} \quad \begin{matrix} x = 0, t = 0 \\ x = 2, t = 1 \end{matrix}$$

$$z = 2t$$

$$dz = 2dt$$

$$\int_{c_2} z^2 dz = \int_0^1 (2t)^2 2dt$$

$$\Rightarrow 8 \int_0^1 t^2 dt$$

$$= 8 \left(\frac{t^3}{3} \Big|_0^1 \right)$$

$\Rightarrow 8\left(\frac{1}{3} - \frac{0}{3}\right)$

$= 8/3$

$$\boxed{\int_{c_2} z^2\,dz = 8/3}$$

$\Rightarrow \int_c \mathrm{Im}(z)^2\,dz = \frac{8}{3} - \frac{1}{3}i$