Probability Methods in Engineering

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Lecture 9

Independence of Events

- An event occurs
  - But probability of a second event remains unchanged
  - Then the second event is independent of first
- Occurrence of one has no impact on probability of other

$$\boxed{P[A] = P[A \mid B] = \frac{P[A \cap B]}{P[B]}}$$

- What if $P[B] = 0$?

$$\boxed{P[A]P[B] = P[A \cap B]}$$

- Event A and B are said to be independent

Examples (cont.)

- A ball is selected from an urn containing two black balls, numbered 1 and 2, and two white balls, numbered 3 and 4. Let the events A, B, and C be defined as follows:

  A = {(1, b), (2, b)}, "black ball selected,"

  B = {(2, b), (4, w)}, "even-numbered ball selected," and

  C = {(3, w), (4, w)}, "number of ball is greater than 2."

- Are events A and B independent? Are events A and C independent?

Examples (cont.)

- Two numbers x and y are selected at random between zero and one. Let the events A, B, and C be defined as follows:

$$A = \{x > 0.5\},\ B = \{y > 0.5\},\ \text{and}\ C = \{x > y\}$$

- Are the events A and B independent? Are A and C independent?

Examples (cont.)

- Suppose a fair coin is tossed three times and we observe the resulting sequence of heads and tails. Find the probability of the elementary events.

Examples (cont.)

- A system consists of a controller and three peripheral units. The system is said to be "up" if the controller and at least two of the peripherals are functioning. Find the probability that the system is up, assuming that all components fail independently with a failure probability of 0.1 for peripherals and 0.2 for the controller. Find the probability that the system is up, if another controller is added.

Sequential Experiments

- Random experiment as sequence of subexperiments
  - Subexperiments usually simpler and easy to perceive
  - Subexperiments may be independent

- Goal
  - Obtain probabilities of events in sequential experiments

Sequences of Independent Experiments

- Assume A_k as independent subexperiments, $k = 1$ to n
- Consider each subexperiment as an event
- The probability of independent events would be

$$\boxed{P[A_1 \cap A_2 \cap \ldots \cap A_n] = P[A_1]P[A_2]\ldots P[A_n]}$$

Examples (cont.)

- Suppose that 10 numbers are selected at random from the interval [0, 1]. Find the probability that the first 5 numbers are less than 1/4 and the last 5 numbers are greater than 1/2.