

(Q1)(a) Find all yords in the complex plane \$ -2 solution: Z = -2 + 0iY = \((-1)^2 = \(\frac{1}{4} = 2\) NCO and 4=0  $\theta = \pi + 2 \kappa \pi$ Q= T(1+2K) KEZ  $-2 = 2(\cos(1+2k)\pi + 2\sin(1+2k)\pi)$  $[-2]^{1/5} = (2)^{1/5} ((05(1+2k)\pi + 2\sin(1+2k)\pi)^{1/5}$ =  $(2)^{5}$  (  $(05(1+2K)\Pi + 25^{\circ}n(1+2K)\Pi$ ) Put K=0(2) 15 (  $\cos \pi + i \sin \pi$ ) =  $2^{15}$  (is  $\pi$ ) Put K=1  $(2)^{5} \left( (05377 + 25in377) - 25(is377) \right)$ 5 K = 2  $2^{15}(\cos \pi + 2\sin \pi) = 2^{15}(\sin \pi)$ Put

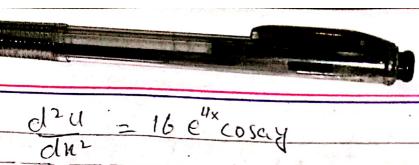
Put 
$$K=3$$

$$2^{45} \left( \cos 7\pi + i \sin 7\pi \right) = 2^{15} \left( i \sin 7\pi \right)$$

Thus the roots on 5,-2 are;

(b) Find c'a" so that the given function is harmonic and find a harmonic conjugate  $U = e^{4x} (osay)$ 

solution:



$$16-a^{2}=0$$

$$a^{2}=16$$

$$a=4$$

Thus 
$$u = e^{4x} \cos 4y$$

$$\frac{dV}{dy} = \frac{du}{dn}$$

$$\frac{dV}{dy} = 4e^{4x} \cos 4y$$

$$h'(x) = 0$$

$$h(x) = C \quad So \quad V = e^{4x} in4y + C$$

## (c) Find all solutions in the complex plane coshz=D

solutions

$$Coshncosy = 0$$
  
 $Cosy = 0$   
 $y = (2k+1)T/2$ 

$$Z = D + i^{\circ}(2k+1)T_{2}$$

[Q2)(a) Find the line integral 
$$\int_{\mathbb{R}^2} \mathbb{R}^2 (Z^2) dZ$$
  
(is the shortest path from 0 to 0+i)

Solutions
(0,0), (0,1)

 $\frac{X-0}{D-D} = \frac{Y-0}{1-D} = t$ 
 $X=D$ ,  $Y=t$ 
 $Z=X+i^2Y=0+i^2t$ 
 $Z=X^2-Y^2+2i^2XY=D-t^2+0=-t^2$ 
 $Re(Z^2)=-t^2$ 

01Z= 2 0+0i => t = 0 when when

(21)(b) Find the integral of sinh2z dz

( consists of 
$$|z|=1$$
 (cw)

Volution:

 $\int_{C} \frac{\text{dinh2z}}{z^2-3z} dz$ 
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On  $z=0$  there is singularity so,

 $\int_{C} \frac{\text{sinh2z}}{z^2-3z} dz$ 
 $\int_{C} \frac{\text{sinh2z}}{z^2-3z} dz$ 

## (Q3)(c) Find the integral of sinb2 dz $(z-4)^3$ (consists of |z|=5 (ccw) and $(z-3)=\frac{3}{2}$ (cw) Solution.

The integral sinbZ is not analytic

Region does not contain Z=4

sio by using (auchy's integral formula

f sinbz dz=0

(Z-4)3

