Probability Methods in Engineering

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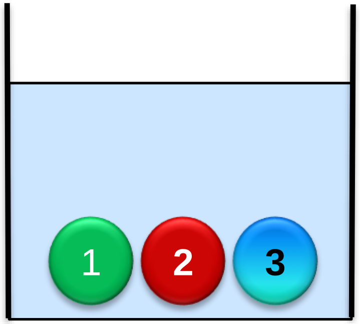
Lecture 2

Properties of Relative Frequency

- Number of occurrences of an outcome in n trials
  - A number between zero and n
  - Also called frequency

$$\boxed{0 \le N_k(n) \le n}$$



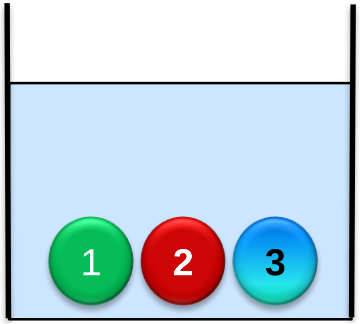
- Relative frequencies are
  - A number between zero and one
  - Divide the above equation by n to get

$$\boxed{0 \le f_k(n) \le 1}$$

Properties of Relative Frequency (cont.)

- Sum of number of occurrences of all possible outcomes
  - Must be n
  - n, sum all frequencies

$$\sum_{k=1}^{K} N_k(n) = n$$



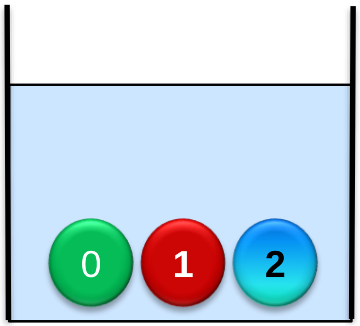
- Sum of all relative frequencies
  - Must be 1

$$\sum_{k=1}^{K} f_k(n) = 1$$

Properties of Relative Frequency (cont.)

- E is an event that the outcome is even

$$f_E(n) = f_0(n) + f_2(n)$$



- If C is an event such that, "either A **or** B occurs" but simultaneous occurrence not possible

$$\boxed{f_C(n) = f_A(n) + f_B(n)}$$

Axioms of Probability

- Axiom
  - Universally accepted principle or rule
- If A and B are two events from a sample space S
- In modern theory of probability, probability of event A

$$\boxed{0 \le P[A] \le 1}$$

- Probability of S

$$\boxed{P[S] = 1}$$

- If A and B cannot occur simultaneously

$$\boxed{P[A \text{ or } B] = P[A] + P[B]}$$

Problems

- Consider the following three random experiments:
  - Experiment 1: Toss a coin.
  - Experiment 2: Roll a die.
  - Experiment 3: Select a ball at random from an urn containing balls numbered 0 to 9.

a) Specify the sample space of each experiment.
b) Find the relative frequency of each outcome in each of the above experiments in a large number of repetitions of the experiment. Explain your answer.

Problems (cont.)

- Explain how the following experiments are equivalent to random urn experiments:
  a) Flip a fair coin twice.
  b) Toss a pair of fair dice.
  c) Draw two cards from a deck of 52 distinct cards, with replacement after the first draw; without replacement after the first draw.

Problems (cont.)

- Explain under what conditions the following experiments are equivalent to a random coin toss. What is the probability of heads in the experiment?
  a) Observe a pixel (dot) in a scanned black-and-white document.
  b) Receive a binary signal in a communication system.
  c) Test whether a device is working.
  d) Determine whether your friend Joe is online.
  e) Determine whether a bit error has occurred in a transmission over a noisy communication channel.

Problems (cont.)

- An urn contains three electronically labelled balls with labels 00, 01, 10. Lisa, Homer, and Bart are asked to characterize the random experiment that involves selecting a ball at random and reading the label. Lisa's label reader works fine; Homer's label reader has the most significant digit stuck at 1; Bart's label reader's least significant digit is stuck at 0.
  a) What is the sample space determined by Lisa, Homer, and Bart?
  b) What are the relative frequencies observed by Lisa, Homer, and Bart in a large number of repetitions of the experiment?

Problems (cont.)

- A random experiment has sample space $S = \{1, 2, 3, 4\}$ with probabilities $p_1 = 1/2$, $p_2 = 1/4$, $p_3 = 1/8$ and $p_4 = 1/8$.
  a) Describe how this random experiment can be simulated using tosses of a fair coin.
  b) Describe how this random experiment can be simulated using an urn experiment.
  c) Describe how this experiment can be simulated using a deck of 52 distinct cards.

Problems (cont.)

- A random experiment consists of selecting two balls in succession from an urn containing two black balls and one white ball.
  a) Specify the sample space for this experiment.
  b) Suppose that the experiment is modified so that the ball is immediately put back into the urn after the first selection. What is the sample space now?
  c) What is the relative frequency of the outcome (white, white) in a large number of repetitions of the experiment in part a? In part b?
  d) Does the outcome of the second draw from the urn depend in any way on the outcome of the first draw in either of these experiments?

Problems (cont.)

- Let A be an event associated with outcomes of a random experiment, and let the event B be defined as "event A does not occur." Show that

$$f_B(n) = 1 - f_A(n)$$

Problems (cont.)

- Let A, B, and C be events that cannot occur simultaneously as pairs or triplets, and let D be the event "A or B or C occurs." Show that

$$f_D(n) = f_A(n) + f_B(n) + f_C(n)$$