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(Q1)

Solution:

$S_v = \{-a, b, c, d\}$

where a, b, c and d are numbers of my choice between 1 and 9.

So $S_v = \{-1, 4, 7, 2\}$

$Z = V^3 \quad S_Z = \{-1, 64, 343, 8\}$

$P_X(-1) = 1/4$

$P_X(4) = 1/4$

$P_X(7) = 1/4$

$P_X(2) = 1/4$

$$E[Z] = -1 \times \frac{1}{4} + 64 \times \frac{1}{4} + 343 \times \frac{1}{4} + 8 \times \frac{1}{4}$$

$$= -\frac{1}{4} + 16 + 85.75 + 2$$

$$\boxed{E[Z] = 103.5}$$

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(Q2)

Solution:

$S_V = \{-3, 1, 3\}$

$P_X(-3) = P_X(1) = P_X(3) = 1/3$

$$P = \frac{V^2}{R} = \frac{V^2}{1/2} = 2V^2$$

$S_P = \{2, 18\}$

(a) $E[P] = E[2V^2] = 2E[V^2]$ —— (1)

$E[V^2] = 9\times 1/3 + 1\times 1/3 + 9\times 1/3 = 19/3$

$E[P] = 2(19/3)$

$\boxed{E[P] = 12.66}$

(b)

$STD[P] = \sqrt{VAR[P]}$

$VAR[P] = E[(P - E[P])^2]$

$= E[P^2] - E^2[P]$

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$S_{p^2} = \{4, 324\}$

$E[P^2] = 4 \times \frac{1}{3} + 324 \times \frac{2}{3}$

$= 217.34$

$VAR[P] = 217.34 - (12.66)^2$

$= 57.07$

$STD[P] = \sqrt{VAR[P]} = \sqrt{57.07}$

$$\boxed{STD[P] = 7.55}$$

(Q3)

solution:

$\lambda = 3/1\,min = 3/60 = 1/20\,s$

$\alpha = \lambda t$

(i) More than 1 customer in 30 seconds:

$t = 30s$

$\alpha = \frac{1}{20} \times 30 = 3/2$

$P[N>1] = 1 - P[N \leq 1]$

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$$P[N \leq 1] = \sum_{K=0}^{1} \frac{(\alpha)^K}{K!} e^{-\alpha}$$

$$= \sum_{K=0}^{1} \frac{(3/2)^K}{K!} e^{-3/2}$$

$$= \frac{(3/2)^0}{0!} e^{-3/2} + \frac{(3/2)}{1!} e^{-3/2}$$

$$= 1 e^{-3/2} + \frac{3}{2} e^{-3/2}$$

$$= 0.223 + \left(\frac{3}{2}\right)(0.223)$$

$$P[N \leq 1] = 0.5575$$

$$P[N > 1] = 1 - 0.5575$$

$$\boxed{P[N>1] = 0.4425}$$

(ii) less than or equal to 1 customer in 2 mins:

$t = 120s$

$$\alpha = \frac{1}{20} \times 120 = 6$$

$$P[N \leq 1] = \sum_{K=0}^{1} \frac{(\alpha)^K}{K!} e^{-\alpha}$$

$$= \frac{(6)^0}{0!} e^{-6} + \frac{6}{1!} e^{-6}$$

$$= e^{-6} + 6 e^{-6}$$

$$= 0.0025 + 0.1482$$

$$\boxed{P[N<1] = 0.01729}$$

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(Q4)

Solution:

Characteristic Function:

$$E[e^{j\omega x}] = \int_0^{\infty} \lambda e^{-\lambda x} \cdot e^{j\omega x} dx$$

$$= \lambda \int_0^{\infty} e^{-\lambda x + j\omega x} dx$$

$$= \lambda \int_0^{\infty} e^{-x(\lambda - \omega j)} dx$$

$$= \lambda \left[ \frac{e^{-x(\lambda - \omega j)}}{-(\lambda - j\omega)} \right]_0^{\infty}$$

$$= 0 + \frac{\lambda}{\lambda - j\omega}$$

$$\phi_x(\omega) = \frac{\lambda}{\lambda - j\omega} \longrightarrow (1)$$

First Moment:

$$E[\lambda'] = \frac{1}{j} \frac{d}{d\omega} \phi_\omega(\omega) \Big|_{\omega=0}$$

$$= \frac{1}{j} \frac{d}{d\omega} \left( \frac{\lambda}{\lambda - j\omega} \right)$$

$$= \frac{1}{j} \left[ \frac{(\lambda - j\omega)\frac{d}{d\omega}\lambda - \lambda \frac{d}{d\omega}(\lambda - j\omega)}{(\lambda - j\omega)^2} \right]$$

$$= \frac{1}{j} \left[ \frac{0 - \lambda(-j)}{(\lambda - j\omega)^2} \right]$$

$$= \frac{1}{j} \left[ \frac{\lambda j}{(\lambda - j\omega)^2} \right]_{\omega=0}$$

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$$E[x] = \frac{1}{j}\,\phi_x'(\omega)\Big|_{\omega=0}$$

$$= \frac{1}{j}\left[\frac{\lambda j}{(\lambda - j\omega)^2}\right]_{\omega=0} = \frac{1}{\cancel{j}}\left(\frac{\cancel{\lambda}\cancel{j}}{\lambda^{\cancel{2}}}\right)$$

$$E[X] = \frac{1}{\lambda}$$

(ii) Second Moment:

$$E[X^2] = \phi_x''(\omega)$$

$$= 0 - \frac{d}{d\omega}\,\frac{(\lambda^2 - 2\lambda j\omega - \omega^2)(\lambda j)}{(\lambda - j\omega)^4}$$

$$= -\frac{[(0) - 2\lambda j - 2\omega](\lambda j)}{(\lambda - j\omega)^4}$$

$$= \frac{2\lambda^2 j^2 + 2\lambda j\omega}{(\lambda - j\omega)^4}$$

$$= \frac{2\lambda j\omega - 2\lambda^2}{(\lambda - j\omega)^4}$$

$$= \frac{-2\lambda(\lambda - j\omega)}{(\lambda - j\omega)^4}$$

$$= \frac{-2\lambda}{(\lambda - j\omega)^3}$$

$$E[x^2] = \frac{1}{j^2}\,\phi_x''(\omega)\Big|_{\omega=0}$$

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$$E[X^2] = \frac{1}{j^2}\left(\frac{-2\lambda}{(\lambda - jw)^3}\right)\Bigg|_{w=0}$$

$$= \frac{1}{j^2}\left(\frac{-2\lambda}{\lambda^3}\right)$$

$$= \frac{1}{-1}\left(\frac{-2\lambda}{\lambda^3}\right) = \frac{2}{\lambda^2}$$

$$\boxed{E[X^2] = \frac{2}{\lambda^2}}$$

(Q5)

(Ans)

Probability Generating function

$S_V = \{-2, -1, 1, 2\}$

PGF is only for Non-Negative value and integers.

$$G_X(Z) = E[Z^X] = \sum_{K=0}^{\infty} P_X(K) Z^K$$

$$\sum_{K=1}^{2} = Z^1 \times \frac{1}{4} + Z^2 \times \frac{1}{4}$$

$$G_X(Z) = \frac{1}{4}(Z^1 + Z^2).$$

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