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SECTION: B
ASSIGNMENT NO: 1

Q1. Let A, B and C be events that can not occur simultaneously as pairs or triplets and let D be the event "A or B or C". Show that

$$f_D(n) = f_A(n) + f_B(n) + f_C(n)$$

Solution:

we know that

$$N_D(n) = N_A(n) + N_B(n) + N_C(n)$$

Dividing both sides by n

$$\frac{N_D(n)}{n} = \frac{N_A(n)}{n} + \frac{N_B(n)}{n} + \frac{N_C(n)}{n}$$

$$\Rightarrow f_D(n) = f_A(n) + f_B(n) + f_C(n)$$

(Q2) A fair dice is rolled thrice:

(a) What is the size of the sample space for this experiment?

Size of the sample space $= n^k$

$= 6^3$

$= 216$

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(b) What is the probability that the sum of First and Second outcome is equal to third outcome?

Solution:

For $n = 1 = 0 \Rightarrow P = 0$

$n = 2 = (1,1) \Rightarrow P = \frac{1}{216}$

$n = 3 = (1,2)(2,1) \Rightarrow P = \frac{2}{216}$

$n = 4 = (2,2)(3,1)(1,3) = \frac{3}{216}$

$n = 5 = (1,4)(4,1)(2,3)(3,2) \Rightarrow P = \frac{4}{216}$

$n = 6 = (1,5)(5,1)(3,3)(4,2)(2,4) \Rightarrow P = \frac{5}{216}$

$$\frac{0+1+2+3+4+5}{216} = \frac{15}{216} = \boxed{0.069}$$

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Q3. A number X is selected at random from the unit interval. Let the events A and B be:

A = "X differs from 1/2 by more than 1/4"

B = "1 − X is less than 1/2".

Find the events $A \cap B$, $A^c \cap B$, $A \cup B$.

Solution:

$$A = \left\{ \left| X - \frac{1}{2} \right| > \frac{1}{4} \right\}$$

$$x - \frac{1}{2} > \frac{1}{4} \quad \text{or} \quad x - \frac{1}{2} = \frac{-1}{4}$$

$$x > \frac{3}{4} \quad \text{or} \quad x < \frac{1}{4}$$

$$A = \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)$$

$$B = \left\{ 1 - X < \frac{1}{2} \right\}$$

$$1 - X < \frac{1}{2}$$

$$X > 1 - \frac{1}{2}$$

$$X > \frac{1}{2}$$

$$B = \left(\frac{1}{2}, 1\right)$$

$A^c \cap B$:

$$A^c = \left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)^c$$

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$$A^c = \left[\frac{1}{4}, \frac{3}{4}\right]$$

$$A^c \cap B = \left[\frac{1}{4}, \frac{3}{4}\right] \cap \left(\frac{1}{2}, 1\right)$$

$$= \left(\frac{1}{2}, \frac{3}{4}\right)$$

$$A \cap B = \left(\left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)\right) \cap \left(\frac{1}{2}, 1\right)$$

$$= \left(\left(0, \frac{1}{4}\right) \cap \left(\frac{1}{2}, 1\right)\right) \cup \left(\left(\frac{3}{4}, 1\right) \cap \left(\frac{1}{2}, 1\right)\right)$$

$$= \left(\frac{3}{4}, 1\right)$$

$$A \cup B = \left(\left(0, \frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right)\right) \cup \left(\frac{1}{2}, 1\right)$$

$$= \left(0, \frac{1}{4}\right)\left(\frac{1}{2}, 1\right)$$

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Q4. Among seven cards numbered 1, 2 ... 7, two are drawn with replacement what is the probability that:

(a) Both cards are different.

Solution:

$$n = 7 \quad , \quad k = 2$$

Size of sample space $= n^k = 7^2 = 49$

Let A be the event when both cards are different

Size of sample space of $A = 49 - 7 = 42$

$$P[A] = \frac{42}{49} = 0.85$$

(b) Second card has a number larger than first card.

Solution:

Let Event $B =$ "2nd card > first card"

Size of $B = {}^{n}C_{k} = \frac{n!}{k!(n-k)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{2(5 \times 4 \times 3 \times 2)} = \frac{42}{2}$

$= 21$

$$P[B] = \frac{21}{49} = 0.43$$

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Q5. A team of 3 players has to be selected from amoung 9 players. What is the probability that two particular players will be included in the team.

Solution:

No of possible arrangements where 2 particular players are selected $= {}^{3}P_{2}$

$$= \frac{3!}{(3-2)!} = 3 \times 2 = \boxed{6}$$

Let A = 1st particular player
Let B = 2nd particular player
And C = Any other player

$$A \times B \times C = \frac{1}{9} \times \frac{1}{8} \times \frac{7}{7} = \frac{1}{72}$$

Probability of 2 particular players being selected

$$= 6 \times \frac{1}{72} = \boxed{\frac{1}{12}}$$

(Ans)

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(Q6) An Urn consists of 8 balls including 5 white balls and 3 black balls. 4 balls are drawn at random. What is the probability that exactly 2 balls are black.

Solution:

$$n = 8, \quad k = 4$$

Probability of the given event $= \dfrac{{}^{3}C_{2} \times {}^{5}C_{2}}{{}^{8}C_{4}}$

$$= \frac{\frac{3!}{2!} \times \frac{5!}{2!(3)!}}{\frac{8!}{4!\,4!}}$$

$$= \frac{3 \times 10}{70} = \frac{30}{70}$$

$$= \frac{3}{7} = \boxed{0.428}$$

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(Q7) A multiple choice test has 5 questions with 3 choices each. How many ways are there to answer the test; what is the probability that all answers are same?

Solution:

$$n = 3, \quad K = 5$$

$$\text{No of ways} = n^k = 3^5$$

$$= 243$$

Probability that all answers are same:

$$= \frac{3}{243}$$

$$= \frac{1}{81} = \boxed{0.012}$$

(Q8) An urn contains 5 balls - - - - ?

Solution:

$$n = 5, \quad k = 6$$

$$\text{Total no of outcomes} = {}^{n-1+k}C_k = {}^{10}C_6$$

$$= 210$$

No of outcomes where all numbers are same = 5

$$P = \frac{5}{210} = \boxed{0.0238}$$

(Ans)

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