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QUIZ #2

REG NO: 18PWCE1858

Q3) $u = e^{x^2/y}$

$$\frac{d}{dx} e^{x^2/y} = e^{x^2/y} \frac{2x}{y}$$

$$\frac{d}{dy} e^{x^2/y} = e^{x^2/y} \left(-\frac{x^2}{y^2} \right)$$

$$\frac{du}{dx} \cdot dx + \frac{du}{dy} \cdot dy = 0$$

$$\frac{2x}{y} e^{x^2/y} dx - \frac{x^2}{y^2} e^{x^2/y} dy = 0$$

$$\Rightarrow \frac{2xy e^{x^2/y} dx - x^2 e^{x^2/y} dy}{y^2} = 0$$

$$\Rightarrow x(2y e^{x^2/y} dx - x e^{x^2/y} dy) = 0$$

$$= 2y e^{x^2/y} dx - x e^{x^2/y} dy = 0$$

$$= e^{x^2/y} (2y dx - x dy) = 0$$

$$\Rightarrow \boxed{2y dx - x dy = 0}$$

This is the exact DE for $u = e^{x^2/y}$

$$(Q10) \quad e^{3\theta} (dx + 3x d\theta) = 0$$

Solution:

$$e^{3\theta} dx + 3xe^{3\theta} d\theta = 0$$

$$\text{Here } M = e^{3\theta}, \quad N = 3xe^{3\theta}$$

$$\frac{dM}{d\theta} = 3e^{3\theta}$$

$$\frac{dN}{dx} = 3e^{3\theta}$$

$$\frac{dM}{d\theta} = \frac{dN}{dx}, \text{ so the given equation}$$

is exact Differential equation.

$$\int M = \int e^{3\theta} dx$$

$$= xe^{3\theta} + C$$

* Terms in $N(x, \theta)$ free of $x \neq 0$

$$xe^{3\theta} + 0 = C$$

$xe^{3\theta} = C$ is the General solution.

$$(Q.14) \quad 2y^{-1} \cos 2x dx = y^{-2} \sin 2x dy, \quad y\left(\frac{\pi}{4}\right) = 3.8$$

Solution:

$$\frac{2}{y} \cos 2x dx - \frac{\sin 2x dy}{y^2} = 0$$

$$M(x, y) = \frac{2 \cos 2x}{y}, \quad N(x, y) = -\frac{\sin 2x}{y^2}$$

$$M_y = -\frac{2 \cos 2x}{y^2}$$

$$N_x = -\frac{2 \cos 2x}{y^2}$$

$M_y = N_x$ so the given equation is EDE.

$$\begin{aligned} \int M(x, y) dx &= \int \frac{2 \cos 2x}{y} dx \\ &= \frac{\sin 2x}{y} \end{aligned}$$

Terms free of x in $N(x, y) = 0$

$$\text{so } \int 0 dy = C_1$$

$$\frac{\sin 2x}{y} + C_1 = C$$

$$\frac{\sin 2x}{y} = C \quad \therefore C = C - C_1$$

$$y\left(\frac{\pi}{4}\right) = 3.8$$

$$\frac{\sin 2\left(\frac{\pi}{4}\right)}{3.8} = C \Rightarrow C = \frac{1}{3.8}$$

$$\boxed{y = 3.8 \sin 2x}$$