FOURIER SERIES REPRESENTATION OF CONTINUOUS TIME SIGNALS

LAB # 10

CSE301L Signals & Systems Lab

Submitted by: **Shah Raza**

Registration No: **18PWCSE1658**

Class Section: **B**

"On my honor, as a student of University of Engineering and Technology, I have neither given nor received unauthorized assistance on this academic work."

Student Signature: ______________

Submitted to: **Engr. Durr-e-Nayab**

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Department of Computer Systems Engineering

University of Engineering and Technology, Peshawar

Lab Objectives:

Objectives of this lab are as follows:

- Fourier Series Representation of Continuous Time Period Signals
- Convergence of CT Fourier Series
- Properties of CT Fourier Series
    - Linearity
    - Time Shifting
    - Frequency Shifting
    - Time Reversal
    - Time Scaling

Task # 1:

In the above example, ak's are chosen to be symmetric about the index k=0, i.e. ak = a-k. Select new ak's on your own to alter this symmetry and form the new signal. What do you observe? Is x(t) a real signal when coefficients are not symmetric?

Observation:

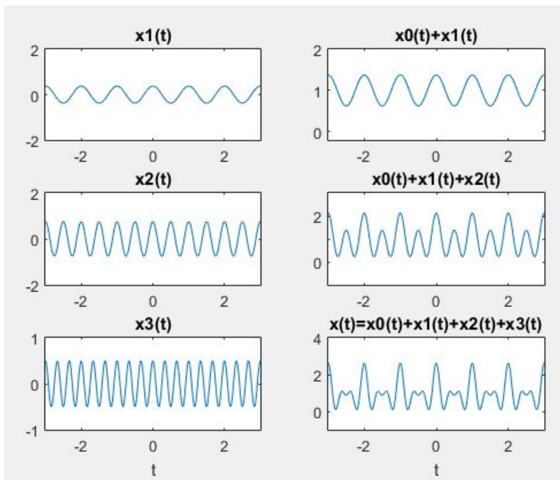
x(t) is a real signal when coefficients are not symmetric.

Code:

```
t = -3:0.01:3; % duration of signal
% dc component for k=0
x0 = 1;
% first harmonic components for k=-1 and k=1
x1 = (1/4)*exp(j*(-1)*2*pi*t)+(1/8)*exp(j*(1)*2*pi*t);
y1 = x0 + x1; % sum of dc component and first harmonic
% second harmonic components for k=-2 and k=2
x2 = (1/2)*exp(j*(-2)*2*pi*t)+(1/4)*exp(j*(2)*2*pi*t);
y2 = y1 + x2; % sum of all components until second harmonic
% third harmonic components for k=-3 and k=3
x3 = (1/3)*exp(j*(-3)*2*pi*t)+(1/6)*exp(j*(3)*2*pi*t);
x = x0 + x1 + x2 + x3; % sum of all components until third harmonic
figure;
subplot(3,2,1);
plot(t,x1);
axis([-3 3 -2 2]);
title('x1(t)');
subplot(3,2,2);
plot(t,y1);
axis([-3 3 -0.2 2]);
title('x0(t)+x1(t)');
subplot(3,2,3);
plot(t,x2);
axis([-3 3 -2 2]);
title('x2(t)');
```

```
subplot(3,2,4);
plot(t,y2);
axis([-3 3 -1 3]);
title('x0(t)+x1(t)+x2(t)');
subplot(3,2,5);
plot(t,x3);
xlabel('t');
axis([-3 3 -1 1]);
title('x3(t)');
subplot(3,2,6);
plot(t,x);
xlabel('t');
axis([-3 3 -1 4]);
title('x(t)=x0(t)+x1(t)+x2(t)+x3(t)');
```

Output:



Task # 2:

A discrete-time periodic signal x[n] is real valued and has a fundamental period of N = 5. The non-zero Fourier series coefficients for x[n] are

$$a_0 = 1, a_2 = a_{-2} = e^{j\,pi/4}, a_4 = a_{-4} = 2e^{j\,pi/3}$$

Express x[n] as a linear combination of given coefficients.

Problem Analysis:

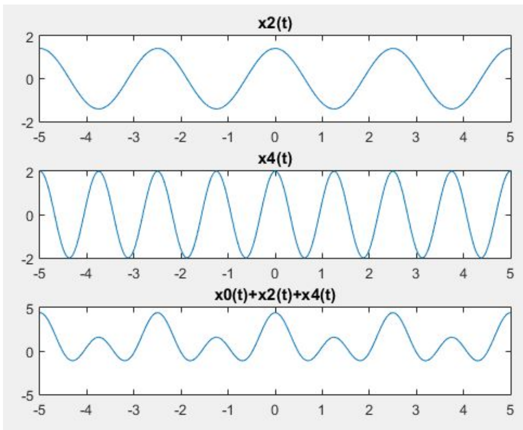
Change the coefficients of a_0 to 1 and a_2, a_{-2} equal to $e^{j\,pi/4}$ and a_4, a_{-4} equal to $2e^{j\,pi/3}$

Code:

```
t = -5:0.01:5; % duration of signal
% dc component for k=0
x0 = 1;
% second harmonic components for k=?2 and k=2
x2 = (exp(j*(pi/4)))*exp(j*(-2)*(2*pi/5)*t)+(exp(j*(pi/4)))*exp(j*(2)*(2*pi/5)*t);
y2 = x0 + x2; % sum of all components until second harmonic
% fourth harmonic components for k=?4 and k=4
```

```
x4 = (2*exp(j*(pi/3)))*exp(j*(-4)*(2*pi/5)*t)+(2*exp(j*(pi/3)))*exp(j*(4)*(2*pi/5)*t);
x = x0 + x2 + x4; % sum of all components until fourth harmonic
figure;
subplot(3,1,1);
plot(t,x2);
title('x2(t)');
subplot(3,1,2);
plot(t,x4);
title('x4(t)');
subplot(3,1,3);
plot(t,x);
title('x0(t)+x2(t)+x4(t)');
```

Output:



Task # 3:

Considering the FS coefficients plot given below, what do you observe happens to the envelope of the coefficients when T1 is reduced from 1/4 to 1/16 with constant time period T?

Observation:

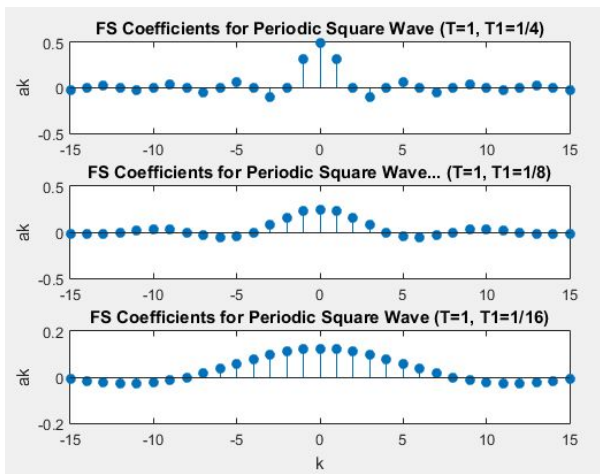
As T1 is changed from 1/4 to 1/16 and we observe that the frequency of the wave decreases and time period increases.

Code:

```
k = -15:15; %number of square wave coefficients
T = 1; %time period of square wave
T1 = 1/4; %duty cycle of square wave
ak1 = sin(k*2*pi*(T1/T))./(k*pi); %square wave Fourier series coefficients
% Ignore the "divide by zero" warning that happens
% because k in the denominator hits 0. We will now do
% a manual correction for a0 ?> ak1(16)
ak1(16) = 2*T1/T;
subplot(3,1,1);
stem(k,ak1,'filled');
ylabel('ak');
```

```
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/4)');
T1 = 1/8;
ak2 = sin(k*2*pi*(T1/T))./(k*pi);
ak2(16) = 2*T1/T; % Manual correction for a0 ?> ak2(16)
subplot(3,1,2);
stem(k,ak2,'filled');
ylabel('ak');
title('FS Coefficients for Periodic Square Wave... (T=1, T1=1/8)');
T1 = 1/16;
ak3 = sin(k*2*pi*(T1/T))./(k*pi);
ak3(16) = 2*T1/T; % Manual correction for a0 ?> ak3(16)
subplot(3,1,3);
stem(k,ak3,'filled');
xlabel('k');
ylabel('ak');
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/16)');
```

Output:



Task # 4:

Considering the plots of square wave reconstructed using M = 10, 20, & 100 terms above, what do you observe about Gibb's phenomenon?

Observation:

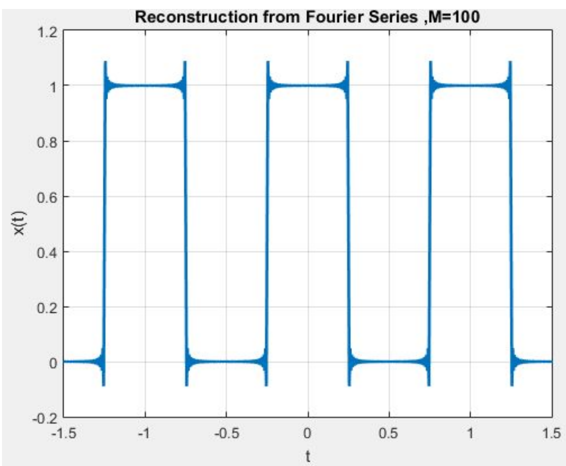
As the number of signals to be added increases from 10 to 100 the square wave smoothens in form and is more defined.

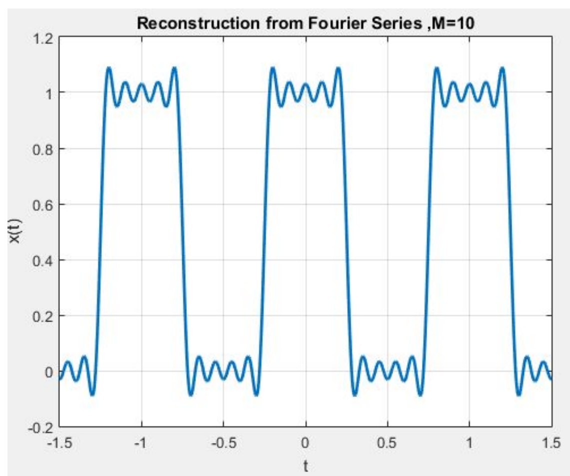
Code:

```
t = -1.5:0.005:1.5; %square wave duration
T = 1; %time period of square wave
T1 = 1/4; %duty cycle of square wave
```

```
w0 = 2*pi/T; %fundamental radian frequency of square wave
M = 100; %number of coefficients
k = -M:M; %2M+1 total coefficients to construct square wave
ak = sin(k*2*pi*(T1/T))./(k*pi);
ak(M+1) = 2*T1/T; % Manual correction for a0 ?> ak(M+1)
x = zeros(1,length(t));
for k = -M:M
x = x + ak(k+M+1)*exp(j*k*w0*t);
end
plot(t,x,'lineWidth',2);
grid;
xlabel('t');
ylabel('x(t)');
title('Reconstruction from Fourier Series');
```

Output:





Task # 5:

Given the following FS coefficients:

$$a_k = \begin{cases} 1, & k \ \ even \\ 2, & k \ \ odd \end{cases}$$

Plot the coefficients & reconstructed signal. Take the terms for reconstructed signal to be M = 10, 20, & 50. What effect do you see when M is varied?

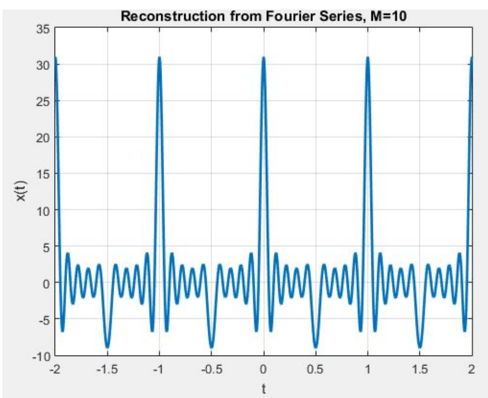
Observation:

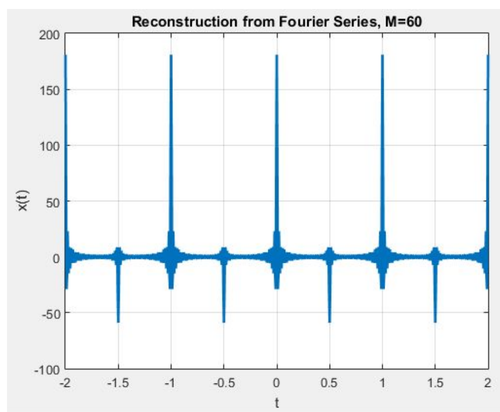
As the number of signals to be added increases from 10 to 60 the wave smoothens in form.

Code:

```
t = -2:0.005:2; %square wave duration
T = 1; %time period of square wave
T1 = 1/4; %duty cycle of square wave
w0 = 2*pi/T; %fundamental radian frequency of square wave
M = 60; %number of coefficients
x = zeros(1,length(t));
for k = -M:M; %2M+1 total coefficients to construct square wave
if(mod(k,2)==0)
ak = 1;
x = x + ak*exp(j*k*w0*t);
else
ak = 2;
x = x + ak*exp(j*k*w0*t);
end
end
plot(t,x,'lineWidth',2);
grid;
xlabel('t');
ylabel('x(t)');
title('Reconstruction from Fourier Series, M=60');
```

Output:





Task # 6:

Given the following FS coefficients:

$$a_k = \begin{cases} jk, & |k| < 3 \\ 0, & otherwise \end{cases}$$

Plot the coefficients & reconstructed signal. Take 10 terms (M=10) for reconstructed signal.

Problem Analysis:

Take M equal to 10 and plot the signal.

Code:

```
t = -5:0.005:5; %square wave duration
T = 1; %time period of square wave
T1 = 1/4; %duty cycle of square wave
w0 = 2*pi/T; %fundamental radian frequency of square wave
M = 10; %number of coefficients
x = zeros(1,length(t));
for k = -M:M; %2M+1 total coefficients to construct square wave
if(abs(k)< 3)
ak = j*k;
x = x + ak*exp(j*k*w0*t);
else
ak = 0;
x = x + ak*exp(j*k*w0*t);
end
end
plot(t,x,'lineWidth',2);
grid;
xlabel('t');
ylabel('x(t)');
title('Reconstruction from Fourier Series, M=10');
```

Output: