Probability Methods in Engineering

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Lecture 3

Sample Space

- Same experimental procedure but different sample spaces
  - Toss a coin three times and note the outcomes
  - Toss a coin three times and note the number of heads

$$S = \{HHH, HHT, HTH, THH, HTT, THT\ TTH, TTT\}$$

$$S = \{0, 1, 2, 3\}$$

- Same sample space but different representations
  - Pick a number at random between zero and one
  - Measure the time between page requests in a Web server

$$S = \{x : 0 \leq x \leq 1\}$$

$$S = \{x : x \geq 0\}$$

$$[0, 1]$$

$$[0, \infty)$$

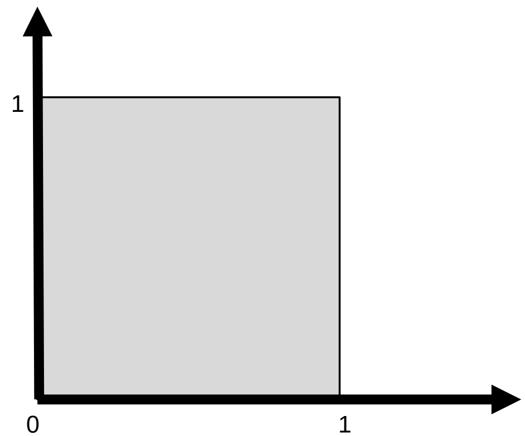
Sample Space (cont.)

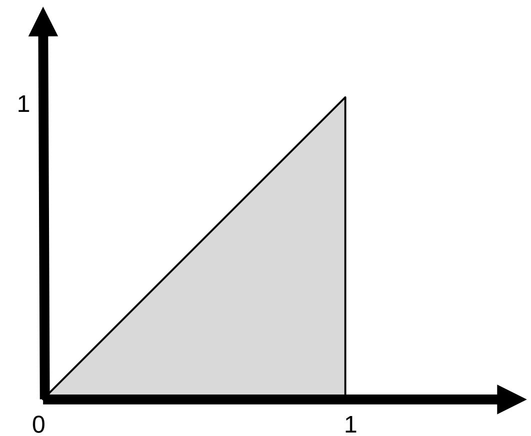
- Multidimensional sample spaces
  - Pick two numbers at random between zero and one
  - Pick a number x at random between zero and one, then pick a number y at random between zero and y

$$S = \{(x, y) : 0 \le x \le 1 \text{ and } 0 \le y \le 1\}$$

$$S = \{(x, y) : 0 \le x \le 1 \text{ and } 0 \le y \le x\}$$

$$S = \{(x, y) : 0 \le y \le x \le 1\}$$



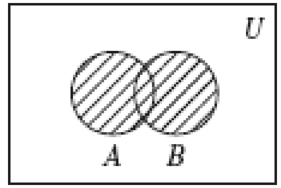


Set Theory

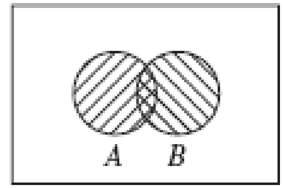
- Representation of events by sets
- Capital letters for names $S, A, B, \ldots$
- Small letters for elements $a, b, x, y, \ldots$
- Venn diagram illustrates sets and their interrelationship

Source: C. Görg, *Communication Networks II*, University of Bremen, Germany

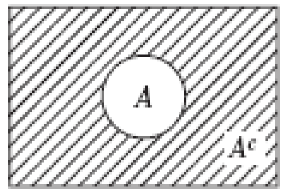
Set Theory (cont.)



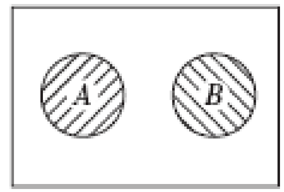
(a) $A \cup B$



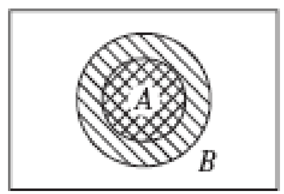
(b) $A \cap B$



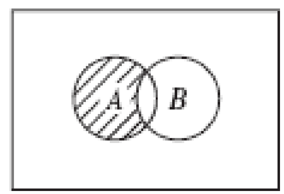
(c) A^c



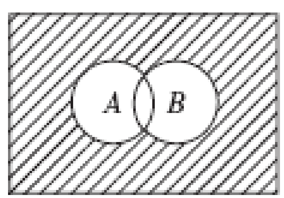
(d) $A \cap B = \varnothing$



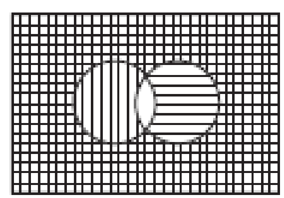
(e) $A \subset B$



(f) $A - B$



(g) $(A \cup B)^c$



(h) $A^c \cap B^c$

Axioms of Probability

$$\boxed{0 \le P[A] \le 1}$$

$$\boxed{P[S] = 1}$$

- If $A \cap B = \varnothing$

$$\boxed{P[A \cup B] = P[A] + P[B]}$$

- If $A_i \cap A_j = \varnothing$ for all $i \neq j$

$$P\left[\bigcup_{k=1}^{\infty} A_k\right] = \sum_{k=1}^{\infty} P[A_k]$$

Corollaries

- Rules or propositions based on or derived from axiom(s)

$$\boxed{P[A^c] = 1 - P[A]}$$

$$\boxed{P[\emptyset] = 0}$$

$$\boxed{P[A \cup B] = P[A] + P[B] - P[A \cap B]}$$

$$\boxed{\begin{aligned} P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] \\ - P[B \cap C] - P[A \cap C] + P[A \cap B \cap C] \end{aligned}}$$

Examples

Discrete countably finite sample space

- An urn contains 10 identical balls numbered 0, 1, ... , 9. A random experiment involves selecting a ball from the urn and noting the number of the ball. Find the probability of the following events:
  - A = "number of ball selected is odd,"
  - B = "number of ball selected is a non-zero multiple of 3,"
  - C = "number of ball selected is less than 5,"
  - $D = A \cup B$
  - $E = A \cup B \cup C$

Examples (cont.)

Discrete countably infinite sample space

- A fair coin is tossed repeatedly until the first heads shows up; the outcome of the experiment is the number of tosses required until the first heads occurs. Find a probability law for this experiment.

Examples (cont.)

Continuous uncountably infinite sample space

- Consider the random experiment "pick a number x at random between zero and one." Let the probability that the outcome falls in a subinterval of S be proportional to the length of the subinterval.
  - What is the sample space of this random experiment?
  - What is the probability that the outcome falls in the interval [0, 0.5], [0.5, 1]?
  - What is the probability that the outcome is 0.5?
  - What is the probability that the outcome falls in either [0, 0.2] or [0.8, 1]?
  - What is the probability that the outcome falls in either [0.3, 0.6] or [0.5, 0.8]?