Question 1

Sol/- As $-\pi i$ is singularity

So here $g(z) = \cosh z$

$$g(z) = \cosh(-\pi i) = -1$$

$$g'(z) = \sinh(-\pi j) = 0$$

$$g''(z) = \cosh(-\pi i) = -1$$

$$g'''(z) = \sinh(-\pi j) = 0$$

$$g''''(z) = \cos(-\pi j) = -1$$

$$\frac{1}{(z+\pi j)^2}[\cosh z] = \frac{1}{(z+\pi j)^2}\left[-1 - \frac{1}{2}(z+\pi j)^2 - \frac{1}{24}(z+\pi j)^4 \ldots\right]$$

$$\frac{-1}{(z+\pi j)^2} - \frac{1}{2} - \frac{1}{24}(z+\pi j)^2 \ldots$$

$$\frac{\cosh z}{z+\pi j} = \frac{-1}{(z+\pi j)^2} - \frac{1}{2} - \frac{1}{24}(z+\pi j)^2$$

Question 2

$\cosh 4z$

for zeros of $f(z)$ we take $f(z)=0$

$\cosh 4z = 0$

$$\frac{e^{4z} + e^{-4z}}{2} = 0$$

$$e^{4z} + e^{-4z} = 0$$

$$e^{8z} + 1 = 0$$

$$e^{8z} = -1$$

$$\Rightarrow 8z = \ln(-1)$$

$$8z = \ln(1) + j(\pi + 2k\pi)$$

$$8z = \pi j(2k+1)$$

Thus zeros of $\cosh 4z$ are

$$\frac{\pi}{8} j (2k+1) \quad k \in \mathbb{Z}$$

for order of zero we have

$$f'(z) = 4\sinh(4z)$$

$$f'(z) = 4\sinh\left(\frac{4}{8}\pi(2k+1)j\right)$$

here as sinh function has value '0' only on $n\pi j$

and $4(2k+1)$ will never be divisible to give $\frac{nj}{8}$ or $n\pi j$

So

$$\sinh\left(\frac{4(2k+1)}{8}\pi j\right) \neq 0$$

so 4hz is of order 1

Question 3

$$\oint \frac{z+1}{z^3(z-2)} \, dz$$

two singularities

$z=0$, $z=2$

we will will consider $z=0$ as $z=2$ is outside circle

According to Laurent Series

$$\frac{(z+1)}{(z-2)} = \frac{-1}{2} - \frac{3}{4}z - \frac{3}{8}z^2 - \frac{3}{16}z^3$$

$$\frac{(z+1)}{z^3(z-2)} = \frac{-1}{2z^3} - \frac{3}{4z^2} - \frac{3}{8z} - \frac{3}{16}$$

residue is $\frac{-3}{16}$

Clockwise integration yeilds

$$\oint_C \frac{z+1}{z^4-2z^3} = -2\pi j \operatorname*{Res}_{z\to 0} f(z)$$

$$= -\cancel{2}\pi j \left(-3/\cancel{8}_{4}\right)$$

$$\Rightarrow \oint_C \frac{z+1}{z^4-2z^3} = \frac{3}{4}\pi j$$

Question 4

$$\int_0^{2\pi} \frac{1+\sin\theta}{3+\cos\theta}\, d\theta$$

$$\cos\theta = \frac{1}{2}\left(z+\frac{1}{z}\right)$$

$$\sin\theta = \frac{1}{2j}\left(z-\frac{1}{z}\right)$$

$$d\theta = \frac{dz}{jz}$$

$$\cos\theta = \frac{1}{2z}\left(z^2+1\right)$$

$$\cos\theta = \frac{z^2+1}{2z}$$

Similarly

$$\sin\theta = \frac{1}{2zj}\left(z^2-1\right)$$

$$\sin\theta = \frac{z^2-1}{2zj}$$

$$\Rightarrow \int_0^{2\pi} \frac{1+\sin\theta}{3+\cos\theta}\, d\theta = \oint_c \left(\frac{1+\frac{z^2-1}{2zj}}{3+\frac{z^2+1}{2z}}\right)\frac{dz}{jz}$$

$$= \oint_c \left(\frac{z^2+2zj-1}{2zj}\right) \div \left(\frac{z^2+6z+1}{2z}\right)\frac{dz}{jz}$$

$$= \oint_c \frac{z^2+2zj-1}{\cancel{2z}j} \times \frac{\cancel{2z}}{z^2+6z+1}\Bigg)\frac{dz}{jz}$$

$$= \oint_c \frac{z^2+2zj-1}{j^2(z^2+6z+1)z}\, dz$$

$$= -\oint_c \frac{z^2+2zj-1}{j(z^2+6z+1)}\, dz$$

$$= -\oint_c \frac{z^2+2zj-1}{z(z^2+6z+1)}\, dz$$

here C is unit circle $|z|=1$
Thus 3 singularities formed.

$$z(z^2+6z+1)=0$$
$$z=0 \quad , z = -3 \pm \sqrt{8}$$

for $z=0 \quad |0|<1$
$$z = |-3-\sqrt{8}| > 1$$
$$z = |-3+\sqrt{8}| < 1$$

as $(-3-\sqrt{8})$ is out of circle, it is ignored

Now Residue at $z=0$

$$f(z) = \frac{-(z^2+2zj-1)}{z(z^2+6z+1)}$$

$$f(z) = \frac{(z^2+2zj-1)}{z(z+3+\sqrt{8})(z+3-\sqrt{8})}$$

$$Res(f(z)) = \frac{1}{(1-1)!}\frac{d^{1-1}}{dz^{1-1}}\left[z-0\left[\frac{-z^2+2zj-1}{z(z+3+\sqrt{8})(z+3-\sqrt{8})}\right]\right]$$

$$= \frac{1}{0!}\left(\cancel{z}\left(\frac{-(z^2+2zj-1)}{\cancel{z}(z+3+\sqrt{8})(z+3-\sqrt{8})}\right)\right)$$

$$= \lim_{z\to 0}\left(\frac{-z^2+2zj-1}{(z+3+\sqrt{8})(z+3-\sqrt{8})}\right)$$

putting limits

$$\frac{-(0+0-1)}{(0+3+\sqrt{8})(0+3-\sqrt{8})}$$

$$= \frac{-1(-1)}{3^2 - \sqrt{8}^2}$$

$$= \frac{1}{9-8}$$

$$= 1$$

Now Residue at $-3+\sqrt{8}$ is

$$\text{Res } f(z) = \frac{1}{0!} \lim_{z \to -3+\sqrt{8}} \left(\frac{(z+3-\sqrt{8})(-(z^2+2zj-1))}{z(z+3+\sqrt{8})(z+3-\sqrt{8})} \right)$$

$$\lim_{z \to -3+\sqrt{8}} \left(\frac{-z^2+2zj-1}{z(z+3+\sqrt{8})} \right)$$

putting limits

$$= \frac{-16-6\sqrt{8} - j(6-2\sqrt{8})}{-6\sqrt{8}+16}$$

$$= \frac{-16+6\sqrt{8}}{-6\sqrt{8}+16} + \frac{(6-2\sqrt{8})j}{-6\sqrt{8}+16}$$

$$= \frac{-\cancel{(-6\sqrt{8}+16)}}{\cancel{-6\sqrt{8}+16}} + \frac{\cancel{2}(3-\sqrt{8})}{\cancel{2}(8-3\sqrt{8})} j$$

$$= -1 + \frac{3-\sqrt{8}}{8-3\sqrt{8}} j$$

$$Res = -1 - \frac{\sqrt{2}}{4} j$$

$$Res\ f(z) = \operatorname*{Res}_{z=0} f(z) + \operatorname*{Res}_{z=-3+\sqrt{8}} f(z)$$

$$= -1 -1 - \frac{\sqrt{2}}{4}$$

$$Res\ f(z) = \frac{-\sqrt{2}}{4} j$$

Now $\oint \frac{1+\sin\theta}{3+\cos\theta} = 2\pi j b$

$$= 2\pi j \left(\frac{-\sqrt{2}}{4} j \right)$$

$$= \frac{-\sqrt{2}\pi j^2}{2}$$

$$\oint_c \frac{1+\sin\theta}{3+\cos\theta} = \frac{\sqrt{2}\pi}{2}$$

$$\oint \frac{1+\sin\theta}{3+\cos\theta} = \frac{\pi}{\sqrt{2}}$$

Question 5.

finding Roots of $z^4 + 16$

$z^4 + 16 = 0$

$z^4 = -16$

$z = (-16)^{1/4}$

$(-16)^{1/4} = r \cos\theta + j\sin\theta)^{1/n}$

$r = \sqrt{(-16)^2 + (0)^2}$

$r = 16$.

for θ $\quad x < 0$ and $y = 0$

$\theta = \pi$

$\theta = \pi + 2k\pi$

putting values

$(-16)^{1/4} = 16\left(\cos\pi(2k+1) + j\sin(2k+1)\pi\right)^{1/4}$

$(-16)^{1/4} = 16\left(\cos\left(\frac{2k+1}{4}\right)\pi + j\sin\left(\frac{2k+1}{4}\pi\right)\right)$

$\rightarrow e^{\cancel{16}\left(\frac{2k+1}{4}\right)\pi j}$

$e^{4(2k+1)\pi j}$

we have $e^{4\pi j}, e^{12\pi j}, e^{-12\pi j}, e^{-4\pi j}$

$z_1 = e^{2\pi j} \quad z_2 = e^{2\pi j}$

$\underset{z=z_1}{\text{Res}} f(z) = \left[\frac{1}{z^2+16}\right]_{z=z_1}$

$= \left[\frac{1}{4z^3}\right]_{z=z_1}$

$= \frac{1}{4(e^{2\pi j})^3}$

$$\operatorname{Res} f(z) = \frac{1}{4e^{6\pi j}}$$

$$= \frac{1}{4e^{2\pi j}}$$

$$\operatorname*{Res}_{z_1 = z} f(z) = \frac{1}{4} e^{-2\pi j}$$

for $z_e = z_2$

$$\operatorname{Res} f(z) = \left[\frac{1}{4z^3}\right]_{z = z_2}$$

$$= \frac{1}{4(e^{\pi j})^3}$$

$$= \frac{1}{4e^{6\pi j}}$$

$$= \frac{1}{4} e^{-2\pi j}$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+16} = \frac{2\pi j}{4}\left(e^{-2\pi j} + e^{-2\pi j}\right)$$

$$= \frac{2\pi j}{4}\left(2e^{-2\pi j}\right)$$

$$= \pi j\, e^{-2\pi j}$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+16} = \frac{\pi j}{e^{2\pi j}}$$

Question#6

$x_1 = 0 \qquad x_2 = 10$

$T_1 = 100 deg \qquad T_2 = 0 deg$

By Laplace Eq.

$$\frac{\partial \phi}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

as it doest depend on y.

$$\frac{\partial^2 \phi}{\partial x^2} = 0$$

$$\frac{d^2 \theta}{d x^2} = 0$$

Integrating

$$\frac{d\theta}{dx} = A$$

$$\frac{d\theta}{dx} = A$$

$$\phi = Ax + B$$

at $x_1 = 0,\ T = 100$

$100 = B$

at $x_2 = 10$, $T_2 = 0$

$$0 = 10A + B$$

$$10A = -100$$

$$A = -10$$

$$\phi = -10x + 100$$

for complex conjgate we have

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

$$\Rightarrow \frac{\partial \psi}{\partial y} = -10 \quad \text{and} \quad \psi = -10y + h(x)$$

$$\Rightarrow \frac{\partial \psi}{\partial x} = h'(x) \qquad \frac{\partial \phi}{\partial y} = 0$$

$$\Rightarrow h'(x) = \cancel{-}(0)$$

$$h'(x) = 0$$

$$h(x) = c$$

$$\psi = -10y + c$$

for complex temp

$$F = (-10x + 100) + j(-10y + c)$$