Probability Methods in Engineering

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Lecture 18

Important Discrete RVs

- Poisson Random Variable

$S_X = \{0, 1, 2, \ldots\}$

$P_k = (\alpha^k/k!)e^{-\alpha}$

$E[X] = \alpha,\ VAR[X] = \alpha$

- Counting number of occurrences of an event in a time period
- Arises in situations where events occur at random
  - E.g. counts of emissions from radioactive substances
- Here, α is number of arrivals in time interval of length t
  - α is unitless and given as $\alpha = \lambda t$
  - λ is arrival rate with unit of jobs/time (e.g. packets/sec)

Examples

- The number N of queries arriving in t seconds at a call centre is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in queries/second. Assume that the arrival rate is 4 queries per minute. Find the probability of the following events:
  - 4 queries in 10 seconds
  - More than 4 queries in 10 seconds
  - Less than or equal to 5 queries in 2 minutes

Examples (cont.)

- The number N of packet arrivals in t seconds at a multiplexer is a Poisson random variable with $\alpha = \lambda t$ where λ is the average arrival rate in packets/second. Find the probability that there are no packet arrivals in t seconds.

The Cumulative Distribution Function

- The cdf is the probability of event $\{X \le x\}$

$$F_X(x) = P[X \le x] = P\left[\left\{\zeta : X(\zeta) \le x\right\}\right]$$

- Probability that RV X takes on a value in set $(-\infty, x]$

Examples

- Find the cdf of RV X, such that X is the outcome of rolling a fair dice.

Examples (cont.)

- Find the cdf of RV X, where X is the number of heads in three tosses of a fair coin.