Name: Shah Raza   Reg no: 18PWCSE1658

(Q2) (a) Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 6t - 8$ by Laplace transform $y(0) = 0$, $y'(0) = 0$

Solution:

$$y'' - 4y' + 3y = 6t - 8$$

Applying LT on BHS

$$\mathcal{L}(y'' - 4y' + 3y) = \mathcal{L}(6t - 8)$$

$$\mathcal{L}(y'') - \mathcal{L}(4y') + \mathcal{L}(3y) = \mathcal{L}(6t) - \mathcal{L}(8)$$

$$s^2Y(s) - sY(0) - y'(0) - 4[sY(s) - Y(0)] + 3Y(s) = \frac{6}{s^2} - \frac{8}{s}$$

$$[s^2Y(s) - 4sY(s) + 3Y(s)] = \frac{6}{s^2} - \frac{8}{s}$$

$$(s^2 - 4s + 3)Y(s) = \frac{2}{s}\left(\frac{3}{s} - 4\right)$$

$$= \frac{2}{s^2}(3 - 4s)$$

$$Y(s) = \frac{2(3 - 4s)}{s^2(s^2 - 4s + 3)} = \frac{(6 - 8s)}{s^2(s^2 - 4s + 3)}$$

$$Y(s) = \frac{A}{s^2} + \frac{B}{(s^2 - 4s + 3}$$

$$Y(s) = \frac{As^2 - 4As + 3A + Bs^2}{s^2(s^2-4s+3)}$$

$A + B = 0$, $-4A = -8$, $3A = 6$

$A = -B$ $\quad \boxed{A = 2}$ $\quad \boxed{A = 2}$

$2 = -B$

$\boxed{B = -2}$

$$Y(s) = \frac{2}{s^2} - \frac{2}{(s^2 - 4s + 3)}$$

$$\mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{2}{s^2}\right] - \mathcal{L}^{-1}\left[\frac{2}{(s^2 - 4s + 3)}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 - 4s + 3}\right] = \mathcal{L}^{-1}\left[\frac{1}{(s-3)(s-1)}\right]$$

$$\frac{A}{(s-3)} + \frac{B}{(s-1)}$$

$$\frac{(A+B)s - A - 3B}{(s-3)(s-1)} = \frac{1}{(s-3)(s-1)}$$

$A + B = 0$ $\quad$ $-A - 3B = 1$

$A = -B$ $\quad$ $B - 3B = 1$; $\boxed{B = -\frac{1}{2}}$

$\boxed{A = \frac{1}{2}}$

$$\mathcal{L}^{-1}\left[\frac{1}{2(s-3)} - \frac{1}{2(s-1)}\right] = \frac{e^{3t}}{2} - \frac{e^{t}}{2}$$

$$\mathcal{L}^{-1}(Y(s)] = \mathcal{L}^{-1}\left(\frac{2}{s^2}\right] - \mathcal{L}^{-1}\left[\frac{2}{s^2-4s+3}\right]$$

$$= 2t - 2\left(\frac{e^{3t}}{2} - \frac{e^{t}}{2}\right)$$

$$y(t) = 2t - e^{3t} + e^{t}$$

(Q2)(b) solve $x\dfrac{dy}{dx} - 3y = k$

solution:

$$x \cdot y' - 3y = k$$

Using Power series method:

Let $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$$y' = 0 + a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \dots$$

$$\cancel{x(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots) -}$$

$$x(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + 6a_6 x^5 + \dots) - 3(a_0 + a_1 x$$
$$+ a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots) = k$$

$$-3a_0 = K$$

$$a_0 = -\frac{K}{3}$$

$\boxed{y = -\frac{K}{3}}$ is the only solution.

(Q1)(b) $y''' - 4y' = \cos x$

solution:

A linear Non homogeneous ODE with constant coefficients has the form of

$a_n y^{(n)} + \dots + a_1 y' + a_0 y = g(x)$

G.S : $y = y_h + y_p$

$y_h = C_1 + C_2 e^{-2x} + C_3 e^{2x}$

$y_p = -\frac{1}{5}\sin(x)$

$$\boxed{y = C_1 + C_2 e^{-2x} + C_3 e^{2x} - \frac{1}{5}\sin(x)}$$

Scanned with CamScanner