

```

In[490]:= polix =  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ;
poliy =  $\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$ ;
poliz =  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ;
taum = {polix, poliy, poliz};
I2m =  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ;
rotationQ[a_, x_, y_, z_] :=

$$\begin{pmatrix} \cos[a] + (1 - \cos[a]) x^2 & (1 - \cos[a]) x y - \sin[a] z & (1 - \cos[a]) x z + \sin[a] y \\ (1 - \cos[a]) x y + \sin[a] z & \cos[a] + (1 - \cos[a]) y^2 & (1 - \cos[a]) y z - \sin[a] x \\ (1 - \cos[a]) x z - \sin[a] y & (1 - \cos[a]) z y + \sin[a] x & \cos[a] + (1 - \cos[a]) z^2 \end{pmatrix}$$
;
rotationP[a_, x_, y_, z_] := Cos[ $\frac{a}{2}$ ] I2m + Sin[ $\frac{a}{2}$ ] (-x I polix - y I poliy - z I poliz);
dv3[a_, b_] := {Sin[a] Cos[b], Sin[a] Sin[b], Cos[a]};
dv2[a_, b_] := {Cos[ $\frac{a}{2}$ ], Exp[I b] Sin[ $\frac{a}{2}$ ]};
reflectXYQ =  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ ;
reflectXYP = -1;
(*Test Rotation, rotate a vector {Sin[a]Cos[b],Sin[a]Sin[b],Cos[a]} around x,
y,z axis and dot z*)

In[187]:= dv3[0, 0]

Out[187]= {0, 0, 1}

In[496]:= dv2[0, 0]

Out[496]= {1, 0}

In[489]:= (rotationQ[w, 0, 1, 0].dv3[a, b]).dv3[0, 0]

Out[489]= cos(a) cos(w) - sin(a) cos(b) sin(w)

In[497]:= FullSimplify[FullSimplify[Conjugate[rotationP[w, 0, 1, 0].dv2[a, b].{1, 0}]
(rotationP[w, 0, 1, 0].dv2[a, b].{1, 0}), Assumptions -> w > 0 && a > 0 && b > 0] -
FullSimplify[Conjugate[rotationP[w, 0, 1, 0].dv2[a, b].{0, 1}]
(rotationP[w, 0, 1, 0].dv2[a, b].{0, 1}), Assumptions -> w > 0 && a > 0 && b > 0] -
(rotationQ[w, 0, 1, 0].dv3[a, b]).dv3[0, 0], Assumptions -> w > 0 && a > 0 && b > 0]
FullSimplify[FullSimplify[Conjugate[rotationP[w, 1, 0, 0].dv2[a, b].{1, 0}]
(rotationP[w, 1, 0, 0].dv2[a, b].{1, 0}), Assumptions -> w > 0 && a > 0 && b > 0] -
FullSimplify[Conjugate[rotationP[w, 1, 0, 0].dv2[a, b].{0, 1}]
(rotationP[w, 1, 0, 0].dv2[a, b].{0, 1}), Assumptions -> w > 0 && a > 0 && b > 0] -
(rotationQ[w, 1, 0, 0].dv3[a, b]).dv3[0, 0], Assumptions -> w > 0 && a > 0 && b > 0]
FullSimplify[FullSimplify[Conjugate[rotationP[w, 0, 0, 1].dv2[a, b].{1, 0}]
(rotationP[w, 0, 0, 1].dv2[a, b].{1, 0}), Assumptions -> w > 0 && a > 0 && b > 0] -
FullSimplify[Conjugate[rotationP[w, 0, 0, 1].dv2[a, b].{0, 1}]
(rotationP[w, 0, 0, 1].dv2[a, b].{0, 1}), Assumptions -> w > 0 && a > 0 && b > 0] -
(rotationQ[w, 0, 0, 1].dv3[a, b]).dv3[0, 0], Assumptions -> w > 0 && a > 0 && b > 0]

Out[497]= 0

Out[498]= 0

Out[499]= 0

(*Test Rotation, rotate a vector {Sin[a]Cos[b],Sin[a]Sin[b],Cos[a]} around x,
y,z axis and dot x*)

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```
In[319]:= dv3[ $\frac{\pi}{2}, 0$ ]
```

$$\text{dv3}\left[-\frac{\pi}{2}, 0\right]$$

```
Out[319]= {1, 0, 0}
```

```
Out[320]= {-1, 0, 0}
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \left\{ \cos\left[\frac{a}{2}\right], i \exp[i b] \sin\left[\frac{a}{2}\right] \right\}$$

```
In[500]:= FullSimplify[FullSimplify[Conjugate[rotationP[w, 0, 1, 0].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 0, 1, 0].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]), Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] -
  FullSimplify[Conjugate[rotationP[w, 0, 1, 0].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 0, 1, 0].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]),
  Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] - (rotationQ[w, 0, 1, 0].dv3[a, b]).
  dv3[ $\frac{\pi}{2}, 0$ ], Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ]
  FullSimplify[FullSimplify[Conjugate[rotationP[w, 1, 0, 0].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 1, 0, 0].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]), Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] -
  FullSimplify[Conjugate[rotationP[w, 1, 0, 0].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 1, 0, 0].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]),
  Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] - (rotationQ[w, 1, 0, 0].dv3[a, b]).
  dv3[ $\frac{\pi}{2}, 0$ ], Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ]
  FullSimplify[FullSimplify[Conjugate[rotationP[w, 0, 0, 1].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 0, 0, 1].dv2[a, b].Conjugate[dv2[ $\frac{\pi}{2}, 0$ ]]), Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] -
  FullSimplify[Conjugate[rotationP[w, 0, 0, 1].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]]
  (rotationP[w, 0, 0, 1].dv2[a, b].Conjugate[dv2[- $\frac{\pi}{2}, 0$ ]]),
  Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ] - (rotationQ[w, 0, 0, 1].dv3[a, b]).
  dv3[ $\frac{\pi}{2}, 0$ ], Assumptions  $\rightarrow w > 0 \ \&\& \ a > 0 \ \&\& \ b > 0$ ]
```

```
Out[500]= 0
```

```
Out[501]= 0
```

```
Out[502]= 0
```

```
(*Test Rotation, rotate a vector {Sin[a]Cos[b],Sin[a]Sin[b],Cos[a]} around
  {Sin[a1]Cos[b1],Sin[a1]Sin[b1],Cos[a1]} axis and dot ANY VECTOR not (dv1.dv2)^2+
  (dv1.dv2')^2=1,so (dv1.dv2)^2-(dv1.dv2')^2 = 2(dv1.dv2)^2-1*)
```

```

In[503]:= 2 Conjugate[rotationP[w, Sin[a1] Cos[b1], Sin[a1] Sin[b1], Cos[a1]].dv2[a, b].
          Conjugate[dv2[a3, b3]]] (rotationP[w, Sin[a1] Cos[b1], Sin[a1] Sin[b1], Cos[a1]].
          dv2[a, b].Conjugate[dv2[a3, b3]]) - 1 -
          (rotationQ[w, Sin[a1] Cos[b1], Sin[a1] Sin[b1], Cos[a1]].dv3[a, b]).dv3[a3, b3] /.
          w -> RandomReal[{0, 2 Pi}] /. a -> RandomReal[{0, 2 Pi}] /. b -> RandomReal[{0, 2 Pi}] /.
          a1 -> RandomReal[{0, 2 Pi}] /. b1 -> RandomReal[{0, 2 Pi}] /.
          a3 -> RandomReal[{0, 2 Pi}] /. b3 -> RandomReal[
          {0, 2 Pi}]

```

```

Out[503]= -2.77556 × 10-16 + 0. i

```

```

(*)

```

Tetrahedron

```

*)

```

```

(* Examples of Tetrahedron *)

```

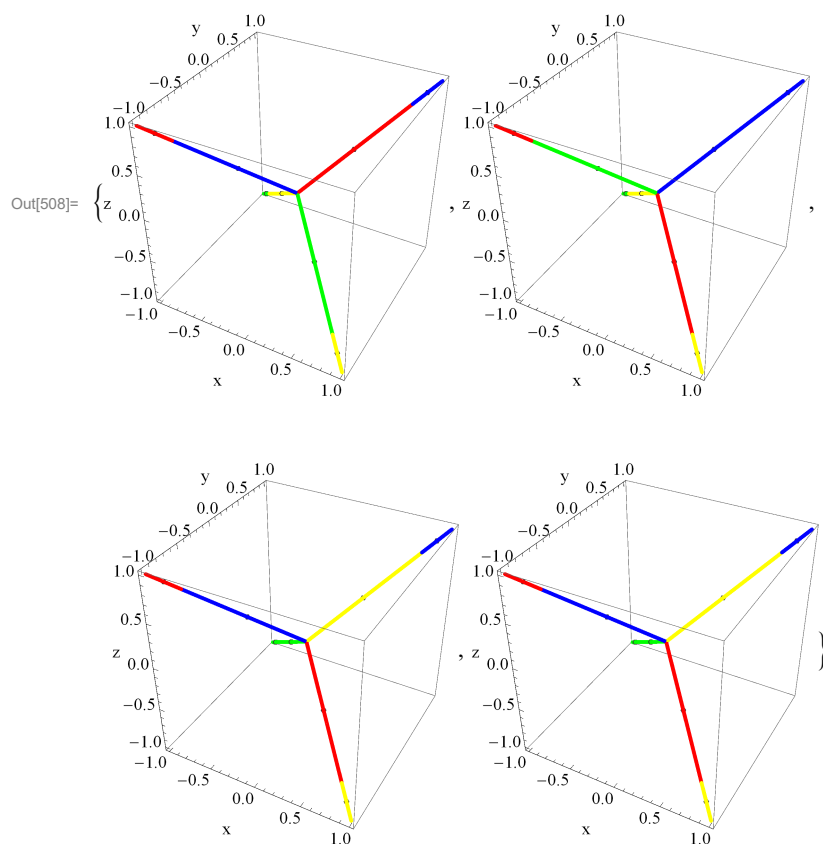
```

In[504]:= u3t1 = rotationQ[Pi, 0, 0, 1];
u3t2 = rotationQ[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ];
u3t3 = rotationQ[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ];
u3t4 = rotationQ[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ].rotationQ[ $\frac{\text{Pi}}{2}$ , 0, 0, 1].reflectXYQ; (* Td *)

{Show[
  ParametricPlot3D[u3t1.{1, 1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[u3t1.{-1, -1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[u3t1.{-1, 1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[u3t1.{1, -1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  ParametricPlot3D[{1, 1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[{-1, -1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[{-1, 1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[{1, -1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  PlotRange -> All, AxesLabel -> {"x", "y", "z"}
],
Show[
  ParametricPlot3D[u3t2.{1, 1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[u3t2.{-1, -1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[u3t2.{-1, 1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[u3t2.{1, -1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  ParametricPlot3D[{1, 1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[{-1, -1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[{-1, 1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[{1, -1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  PlotRange -> All, AxesLabel -> {"x", "y", "z"}
],

```

```
Show[
  ParametricPlot3D[u3t3.{1, 1, 1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[u3t3.{-1, -1, 1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[u3t3.{-1, 1, -1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[u3t3.{1, -1, -1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Yellow, Yellow}],
  ParametricPlot3D[{1, 1, 1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[{-1, -1, 1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[{-1, 1, -1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[{1, -1, -1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Yellow, Yellow}],
  PlotRange → All, AxesLabel → {"x", "y", "z"}
],
Show[
  ParametricPlot3D[u3t3.{1, 1, 1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[u3t3.{-1, -1, 1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[u3t3.{-1, 1, -1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[u3t3.{1, -1, -1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Yellow, Yellow}],
  ParametricPlot3D[{1, 1, 1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[{-1, -1, 1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[{-1, 1, -1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[{1, -1, -1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Yellow, Yellow}],
  PlotRange → All, AxesLabel → {"x", "y", "z"}
]]
```



(*They are not self-inverse*)

```

In[509]:= rotationP[Pi, 0, 0, 1].rotationP[Pi, 0, 0, 1]
rotationP[Pi, 0, 1, 0].rotationP[Pi, 0, 1, 0]
rotationP[Pi, 1, 0, 0].rotationP[Pi, 1, 0, 0]

Out[509]=  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 

Out[510]=  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 

Out[511]=  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 

(*those are IG*)

In[557]:= {1 -  $\frac{1}{4}$  Re[Tr[rotationP[Pi, 0, 0, 1] + reflectXYP rotationP[Pi, 0, 0, 1]]] // FullSimplify,
1 -  $\frac{1}{4}$  Re[Tr[rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ] +
reflectXYP rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ]]] // FullSimplify,
1 -  $\frac{1}{4}$  Re[Tr[rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ] + reflectXYP
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ]]] // FullSimplify}]

Out[557]=  $\left\{1, \frac{1}{2}, \frac{3}{2}\right\}$ 

(* It is closed *)

In[527]:= {rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ].rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ] -
reflectXYP rotationP[Pi, 1, 0, 0],
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ].rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ] -
reflectXYP rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ]}

Out[527]=  $\begin{pmatrix} \{0, 0\} & \{0, 0\} \\ \{0, 0\} & \{0, 0\} \end{pmatrix}$ 

(* Bulid the tables for program *)

```

```

In[546]:= Gprime = {-1 I2m, rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ], rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],
  rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ], rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],
  rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ], rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ],
  rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ], rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ],
  rotationP[Pi, 0, 0, 1], rotationP[Pi, 0, 1, 0], rotationP[Pi, 1, 0, 0], I2m,
  -1 rotationP[Pi, 1, 0, 0], -1 rotationP[Pi, 0, 1, 0], -1 rotationP[Pi, 0, 0, 1],
  -1 rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ], -1 rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ],
  -1 rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ], -1 rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ],
  -1 rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ], -1 rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],
  -1 rotationP[ $\frac{2 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ], -1 rotationP[ $\frac{4 \text{ Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ], -1 I2m};

In[547]:= For[i = 1, i ≤ 13, i++, Print[Gprime[[i]].Gprime[[26 - i]]]]

```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```

In[554]:= retk = Table[gk = Gprime[[ii]].Gprime[[jj]];
          retk = 0;
          For[kk = 1, kk < 26, kk++,
            If[gk == Gprime[[kk]] && 0 == retk, retk = kk,];
          ];
          retk, {ii, 1, 25}, {jj, 1, 25}]

```

Out[554]=

13	23	24	21	22	19	20	17	18	16	15	14	1	12	11	10	8	9	6	7	4	5	2	3	13
23	3	1	20	15	18	14	22	16	19	17	21	2	4	8	6	5	10	9	12	7	11	24	13	23
24	1	23	12	8	10	4	11	6	9	5	7	3	20	22	18	15	19	16	21	14	17	13	2	24
21	18	11	5	1	24	16	20	12	17	6	2	4	23	19	8	7	14	3	10	22	13	9	15	21
22	14	6	1	21	15	8	10	2	7	24	18	5	9	3	20	16	23	11	17	13	4	12	19	22
19	22	12	18	10	7	1	24	15	2	21	8	6	17	4	23	3	11	20	13	9	16	5	14	19
20	16	8	11	2	1	19	14	4	22	9	24	7	3	18	5	12	21	13	6	15	23	10	17	20
17	20	10	24	14	22	11	9	1	4	2	19	8	6	23	21	18	13	5	15	3	12	7	16	17
18	15	4	16	6	12	2	1	17	24	20	5	9	22	7	3	13	8	14	23	10	19	11	21	18
16	17	7	19	9	4	24	2	22	1	14	11	10	15	12	13	23	5	21	3	6	18	8	20	16
15	21	9	2	20	17	5	6	24	12	1	16	11	10	13	14	19	3	8	22	23	7	4	18	15
14	19	5	8	24	2	18	21	7	15	10	1	12	13	16	11	4	20	23	9	17	3	6	22	14
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1
12	6	22	17	3	23	9	4	20	11	16	13	14	1	10	15	21	7	2	18	8	24	19	5	12
11	4	18	23	7	8	22	19	3	14	13	10	15	16	1	12	6	24	17	5	2	20	21	9	11
10	8	20	6	18	21	3	23	5	13	12	15	16	11	14	1	2	22	4	24	19	9	17	7	10
8	7	16	3	12	5	15	18	13	21	23	6	17	19	2	4	9	1	22	11	24	14	20	10	8
9	11	21	10	19	24	23	13	8	3	7	22	18	5	20	24	1	17	12	2	16	6	15	4	9
6	5	14	9	16	20	13	3	11	23	4	17	19	8	21	2	24	15	7	1	18	10	22	12	6
7	10	17	15	23	13	6	12	21	5	18	3	20	24	9	22	14	4	1	19	11	2	16	8	7
4	9	15	22	13	3	10	7	14	8	19	23	21	2	6	17	20	12	24	16	5	1	18	11	4
5	12	19	13	4	11	17	16	23	20	3	9	22	18	24	7	10	2	15	8	1	21	14	6	5
2	24	13	7	11	9	12	5	10	6	8	4	23	21	17	19	22	16	18	14	20	15	3	1	2
3	13	2	14	17	16	21	15	19	18	22	20	24	7	5	9	11	6	10	4	12	8	1	23	3
13	23	24	21	22	19	20	17	18	16	15	14	1	12	11	10	8	9	6	7	4	5	2	3	13

(* Note it is not Ablien*)

```

In[577]:= retk = Transpose[retk]

```

Out[577]=

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	2	1	-4	-2	2	1	2	-4	2	0	-2	4	-2	-2	-1	4	2	-2	-1	0	0	0	0
0	0	0	1	2	-2	-4	1	2	2	-4	2	0	-2	4	-2	-1	-2	2	4	-1	-2	0	0	0	0
0	-2	-1	0	0	6	5	-4	-4	-2	4	-6	0	6	-4	2	4	4	-6	-5	0	0	2	1	0	0
0	-1	-2	0	0	5	6	-4	-4	-2	4	-6	0	6	-4	2	4	4	-5	-6	0	0	1	2	0	0
0	4	2	-6	-5	0	0	2	3	-2	4	6	0	-6	-4	2	-2	-3	0	0	6	5	-4	-2	0	0
0	2	4	-5	-6	0	0	3	2	-2	4	6	0	-6	-4	2	-3	-2	0	0	5	6	-2	-4	0	0
0	-2	-1	4	4	-2	-3	0	0	2	-4	-2	0	2	4	-2	0	0	2	3	-4	-4	2	1	0	0
0	-1	-2	4	4	-3	-2	0	0	2	-4	-2	0	2	4	-2	0	0	3	2	-4	-4	1	2	0	0
0	-2	-2	2	2	2	2	-2	-2	0	2	-4	0	4	-2	0	2	2	-2	-2	-2	-2	2	2	0	0
0	4	4	-4	-4	-4	-4	4	4	-2	0	6	0	-6	0	2	-4	-4	4	4	4	4	-4	-4	0	0
0	-2	-2	6	6	-6	-6	2	2	4	-6	0	0	0	6	-4	-2	-2	6	6	-6	-6	2	2	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	2	2	-6	-6	6	6	-2	-2	-4	6	0	0	0	-6	4	2	2	-6	-6	6	6	-2	-2	0	0
0	-4	-4	4	4	4	4	-4	-4	2	0	-6	0	6	0	-2	4	4	-4	-4	-4	-4	4	4	0	0
0	2	2	-2	-2	-2	-2	2	2	0	-2	4	0	-4	2	0	-2	-2	2	2	2	2	-2	-2	0	0
0	2	1	-4	-4	2	3	0	0	-2	4	2	0	-2	-4	2	0	0	-2	-3	4	4	-2	-1	0	0
0	1	2	-4	-4	3	2	0	0	-2	4	2	0	-2	-4	2	0	0	-3	-2	4	4	-1	-2	0	0
0	-4	-2	6	5	0	0	-2	-3	2	-4	-6	0	6	4	-2	2	3	0	0	-6	-5	4	2	0	0
0	-2	-4	5	6	0	0	-3	-2	2	-4	-6	0	6	4	-2	3	2	0	0	-5	-6	2	4	0	0
0	2	1	0	0	-6	-5	4	4	2	-4	6	0	-6	4	-2	-4	-4	6	5	0	0	-2	-1	0	0
0	1	2	0	0	-5	-6	4	4	2	-4	6	0	-6	4	-2	-4	-4	5	6	0	0	-1	-2	0	0
0	0	0	-2	-1	4	2	-2	-1	-2	4	-2	0	2	-4	2	2	1	-4	-2	2	1	0	0	0	0
0	0	0	-1	-2	2	4	-1	-2	-2	4	-2	0	2	-4	2	1	2	-2	-4	1	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

```

In[556]:= et = Table[1 - 1/4 Re[Tr[Gprime[[ii]] + Gprime[[26 - ii]]]], {ii, 1, 24}]

```

Out[556]= $\left\{2, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, 1, 1, 1, 0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{3}{2}\right\}$

ig = {2, 4, 6, 8, 18, 20, 22, 24} (*8*)

(*

Cube

Octahedron

rotation elements are 2*edge, Cube has 12 edges,
Octahedron also has 12 edges, so they are the same

*)

```
In[567]:= u3o1 = rotationQ[Pi,  $\frac{1}{\sqrt{2}}$ ,  $\frac{1}{\sqrt{2}}$ , 0];

u3o2 = rotationQ[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ];

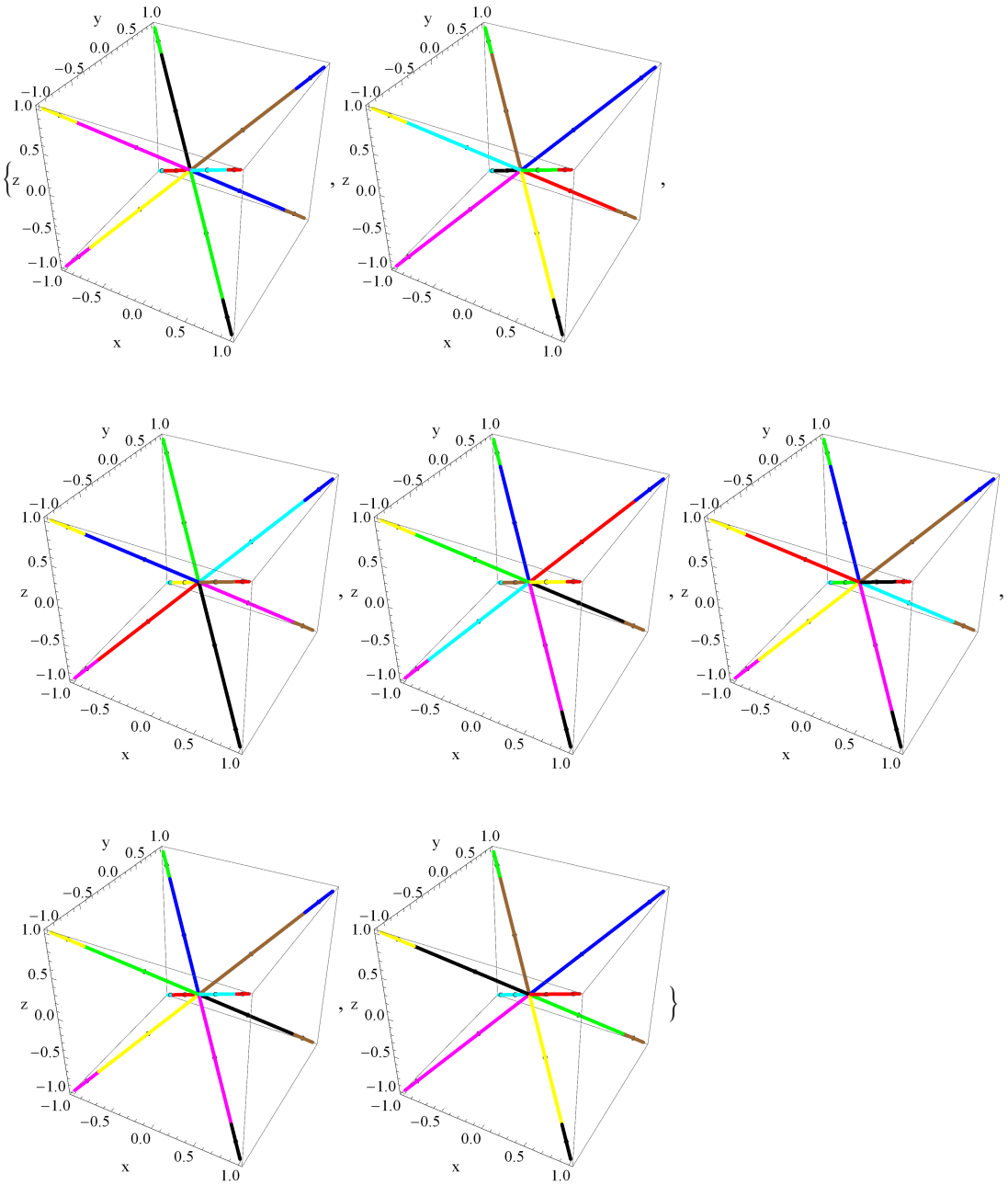
u3o3 = rotationQ[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ];

u3o4 = rotationQ[ $\frac{\text{Pi}}{2}$ , 0, 0, 1];

u3o5 = rotationQ[ $\frac{3\text{Pi}}{2}$ , 0, 1, 0];

u3o6 = u3o2.reflectXYQ; (* Oh *)
u3o7 = u3o5.reflectXYQ;
u3o = {u3o1, u3o2, u3o3, u3o4, u3o5, u3o6, u3o7};
Table[Show[
  ParametricPlot3D[u3o[[ii]].{1, 1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[u3o[[ii]].{1, 1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Brown, Brown}],
  ParametricPlot3D[u3o[[ii]].{1, -1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[u3o[[ii]].{1, -1, -1} v,
    {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Black, Black}],
  ParametricPlot3D[u3o[[ii]].{-1, 1, 1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[u3o[[ii]].{-1, 1, -1} v, {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Cyan, Cyan}],
  ParametricPlot3D[u3o[[ii]].{-1, -1, 1} v,
    {v, 0, 0.8}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  ParametricPlot3D[u3o[[ii]].{-1, -1, -1} v, {v, 0, 0.8},
    Mesh -> 1, MeshShading -> {Magenta, Magenta}],
  ParametricPlot3D[{1, 1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Blue, Blue}],
  ParametricPlot3D[{1, 1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Brown, Brown}],
  ParametricPlot3D[{1, -1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Red, Red}],
  ParametricPlot3D[{1, -1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Black, Black}],
  ParametricPlot3D[{-1, 1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Green, Green}],
  ParametricPlot3D[{-1, 1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Cyan, Cyan}],
  ParametricPlot3D[{-1, -1, 1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Yellow, Yellow}],
  ParametricPlot3D[{-1, -1, -1} v, {v, 0.8, 1}, Mesh -> 1, MeshShading -> {Magenta, Magenta}],
  PlotRange -> All, AxesLabel -> {"x", "y", "z"}
], {ii, 1, 7}]
```

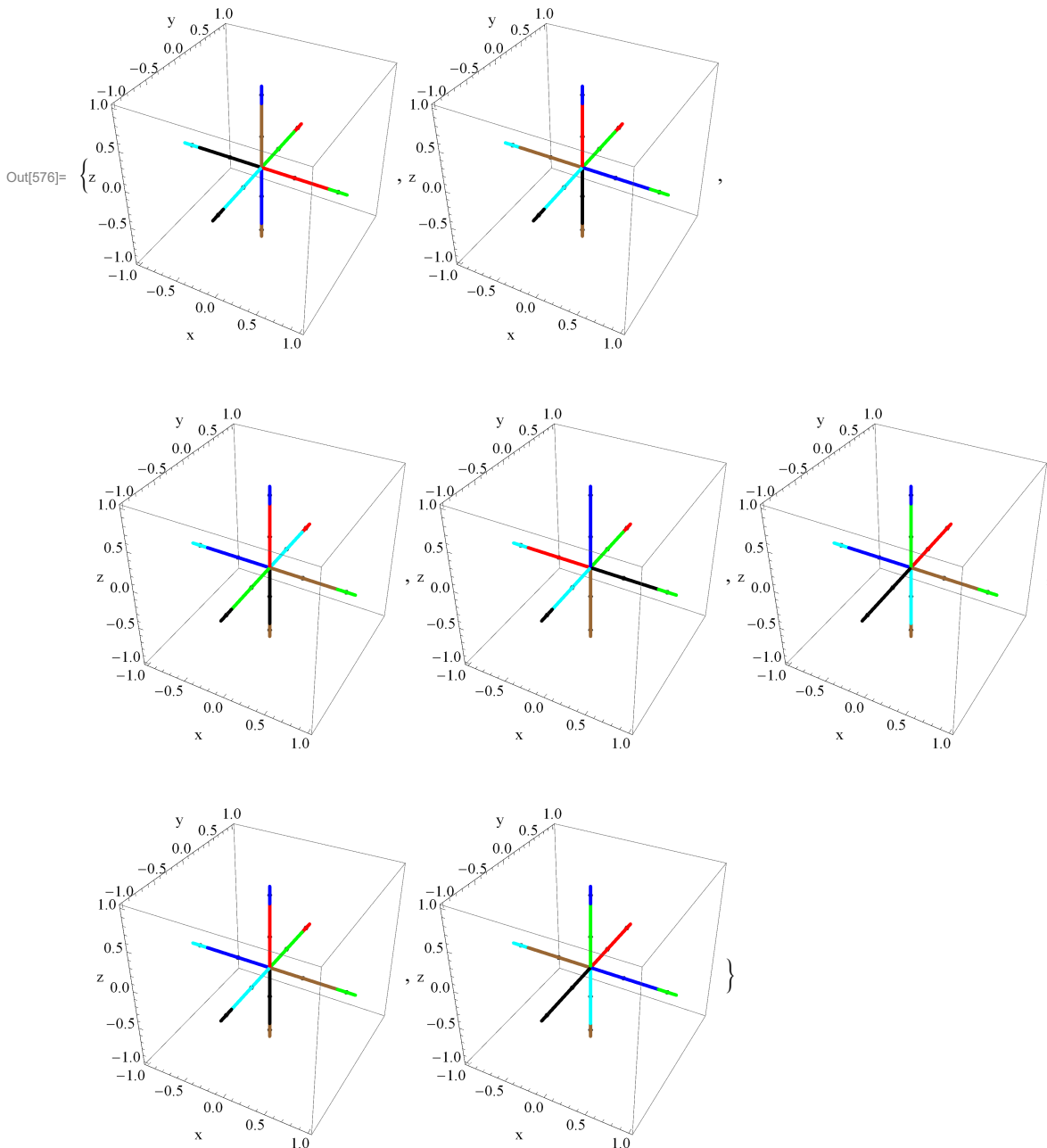

Out[575]=



```

In[576]:= Table[Show[
  ParametricPlot3D[u3o[[ii]].{0, 0, 1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[u3o[[ii]].{0, 0, -1} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Brown, Brown}],
  ParametricPlot3D[u3o[[ii]].{0, 1, 0} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[u3o[[ii]].{0, -1, 0} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Black, Black}],
  ParametricPlot3D[u3o[[ii]].{1, 0, 0} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[u3o[[ii]].{-1, 0, 0} v, {v, 0, 0.8}, Mesh → 1, MeshShading → {Cyan, Cyan}],
  ParametricPlot3D[{0, 0, 1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Blue, Blue}],
  ParametricPlot3D[{0, 0, -1} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Brown, Brown}],
  ParametricPlot3D[{0, 1, 0} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Red, Red}],
  ParametricPlot3D[{0, -1, 0} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Black, Black}],
  ParametricPlot3D[{1, 0, 0} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Green, Green}],
  ParametricPlot3D[{-1, 0, 0} v, {v, 0.8, 1}, Mesh → 1, MeshShading → {Cyan, Cyan}],
  PlotRange → All, AxesLabel → {"x", "y", "z"}
], {ii, 1, 7}]

```



(* They are not self inverse *)

```
In[581]:= rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ].rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ]  
rotationP[ $\frac{3\text{Pi}}{2}$ , 0, 1, 0].rotationP[ $\frac{\text{Pi}}{2}$ , 0, 1, 0]
```

```
Out[581]=  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 
```

```
Out[582]=  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ 
```

```
In[588]:= Gprime2 = {-1 I2m,  
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{2\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{4\text{Pi}}{3}$ ,  $\frac{1}{\sqrt{3}}$ ,  $\frac{-1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{3}}$ ],  
rotationP[ $\frac{\text{Pi}}{2}$ , 0, 0, 1],  
rotationP[Pi, 0, 0, 1],  
rotationP[ $\frac{3\text{Pi}}{2}$ , 0, 0, 1],  
rotationP[ $\frac{\text{Pi}}{2}$ , 0, 1, 0],  
rotationP[Pi, 0, 1, 0],  
rotationP[ $\frac{3\text{Pi}}{2}$ , 0, 1, 0],  
rotationP[ $\frac{\text{Pi}}{2}$ , 1, 0, 0],  
rotationP[Pi, 1, 0, 0],  
rotationP[ $\frac{3\text{Pi}}{2}$ , 1, 0, 0],  
rotationP[Pi,  $\frac{1}{\sqrt{2}}$ ,  $\frac{1}{\sqrt{2}}$ , 0],  
rotationP[Pi,  $\frac{1}{\sqrt{2}}$ , 0,  $\frac{1}{\sqrt{2}}$ ],  
rotationP[Pi, 0,  $\frac{1}{\sqrt{2}}$ ,  $\frac{1}{\sqrt{2}}$ ],
```

$$\text{rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right],$$

$$\text{rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right],$$

$$\text{rotationP}\left[\text{Pi}, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right],$$

I2m,

$$-1 \text{ rotationP}\left[\text{Pi}, 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right],$$

$$-1 \text{ rotationP}\left[\frac{\text{Pi}}{2}, 1, 0, 0\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, 1, 0, 0\right],$$

$$-1 \text{ rotationP}\left[\frac{3 \text{ Pi}}{2}, 1, 0, 0\right],$$

$$-1 \text{ rotationP}\left[\frac{\text{Pi}}{2}, 0, 1, 0\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, 0, 1, 0\right],$$

$$-1 \text{ rotationP}\left[\frac{3 \text{ Pi}}{2}, 0, 1, 0\right],$$

$$-1 \text{ rotationP}\left[\frac{\text{Pi}}{2}, 0, 0, 1\right],$$

$$-1 \text{ rotationP}\left[\text{Pi}, 0, 0, 1\right],$$

$$-1 \text{ rotationP}\left[\frac{3 \text{ Pi}}{2}, 0, 0, 1\right],$$

$$-1 \text{ rotationP}\left[\frac{2 \text{ Pi}}{3}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{4 \text{ Pi}}{3}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{2 \text{ Pi}}{3}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{4 \text{ Pi}}{3}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{2 \text{ Pi}}{3}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{4 \text{ Pi}}{3}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP}\left[\frac{2 \text{ Pi}}{3}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right],$$

$$-1 \text{ rotationP} \left[\frac{4 \text{ Pi}}{3}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right],$$

$$-1 \text{ I2m} \};$$

```
In[589]:= For [i = 1, i ≤ 25, i++, Print[Gprime2[[i]].Gprime2[[50 - i]]]]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
In[1445]:= retk2 = Table[gk = Gprime2[[ii]].Gprime2[[jj]];

```

```
retk = 0;

```

```
For[kk = 1, kk < 50, kk++,

```

```
  If[gk == Gprime2[[kk]] && 0 == retk, retk = kk,];

```

```
];

```

```
retk, {ii, 1, 49}, {jj, 1, 49}]

```

```
Out[1445]=
25 47 48 45 46 43 44 41 42 38 39 40 35 36 37 32 33 34 31 30 29 28 27 26 1 24 23 22 21 20
47 3 1 8 36 11 5 17 43 20 9 35 21 7 32 19 45 38 15 18 12 23 24 28 2 22 26 27 40 34
48 1 47 17 44 9 36 45 39 18 43 29 12 5 31 15 41 30 32 38 35 24 28 27 3 23 22 26 13 10
45 6 14 5 1 33 41 11 3 27 7 15 34 42 24 10 2 19 13 21 28 20 16 40 4 12 32 30 22 29
46 33 42 1 45 47 39 7 14 32 41 24 31 48 40 27 6 13 34 28 30 21 10 37 5 15 38 29 20 22
43 14 45 11 9 7 1 2 17 21 3 18 28 41 38 13 46 23 24 19 15 16 40 30 6 20 12 32 37 31
44 41 39 3 17 1 43 14 46 15 45 23 32 47 29 28 42 40 30 24 38 13 34 31 7 19 18 35 10 26
41 11 7 36 47 4 33 9 1 26 5 32 10 6 28 20 3 15 21 12 27 18 19 13 8 35 31 34 23 40
42 5 33 43 39 36 48 1 41 35 47 30 26 4 34 12 7 28 27 32 31 15 21 10 9 38 29 37 19 16
38 21 15 26 35 27 32 20 18 11 12 1 6 28 41 2 19 45 14 3 7 17 46 42 10 9 5 33 44 48
39 7 41 9 43 5 47 3 45 12 1 38 27 33 30 21 14 24 28 15 32 19 13 34 11 18 35 31 16 37
40 32 30 18 23 35 29 15 24 1 38 39 5 31 48 7 28 42 33 41 47 14 6 4 12 45 43 36 2 8
35 19 18 10 26 21 12 16 22 2 20 9 14 15 1 46 23 43 45 17 3 44 39 41 13 8 11 7 48 33
36 45 43 2 8 3 9 46 44 19 17 22 15 1 35 24 39 29 38 23 18 40 30 32 14 16 20 12 34 27
37 38 29 19 16 18 22 24 40 45 23 44 1 35 36 41 30 48 47 39 43 42 33 5 15 46 17 9 6 11
32 20 12 37 31 10 27 22 35 8 26 5 2 21 7 17 18 1 3 9 11 43 45 14 16 36 4 6 39 42
33 9 5 44 48 8 4 43 47 22 36 31 20 11 27 18 1 32 12 35 26 38 15 21 17 29 37 10 24 13
34 35 31 23 40 22 37 38 30 43 29 48 9 26 4 1 32 33 5 47 36 41 7 11 18 39 44 8 14 2
31 18 35 16 37 20 26 23 29 17 22 36 3 12 5 45 38 47 1 43 9 39 41 7 19 44 8 11 42 6
30 12 32 22 29 26 31 18 38 9 35 47 11 27 33 3 15 41 7 1 5 45 14 6 20 43 36 4 46 25
29 15 38 20 22 12 35 19 23 3 18 43 7 32 47 14 24 39 41 45 1 46 42 33 21 17 9 5 25 4
28 26 27 29 30 37 34 35 32 36 31 33 8 10 6 9 12 7 11 5 4 1 3 2 22 47 48 25 45 46
27 22 26 40 34 16 10 29 31 44 37 4 17 20 11 43 35 5 9 36 8 47 1 3 23 48 25 2 41 14
26 23 22 13 10 19 20 40 37 46 16 8 45 18 9 39 29 36 43 44 17 48 47 1 24 25 2 3 33 7
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
24 27 28 35 38 31 30 12 15 5 32 41 4 34 42 11 21 14 6 7 33 3 2 25 26 1 47 48 17 44
23 28 24 12 18 32 38 21 19 7 15 45 33 30 39 6 13 46 42 14 41 2 25 48 27 3 1 47 8 36
22 24 23 21 20 15 18 13 16 14 19 17 41 38 43 42 40 44 39 46 45 25 48 47 28 2 3 1 4 5
21 37 10 30 28 40 13 31 27 48 34 6 44 16 2 36 26 11 8 4 25 5 9 17 29 33 42 46 1 45
20 40 16 28 21 24 19 34 10 42 13 2 39 23 17 48 37 8 44 25 46 4 36 43 30 6 14 45 5 1
19 34 13 32 15 30 24 27 21 33 28 14 48 40 46 4 10 2 25 6 42 11 8 44 31 7 41 39 9 43
16 30 40 15 19 38 23 28 13 41 24 46 47 29 44 33 34 25 48 42 39 6 4 36 32 14 45 43 11 9
17 42 46 7 3 41 45 6 2 28 14 19 30 39 23 34 25 16 40 13 24 10 37 29 33 21 15 38 26 35
18 13 19 27 12 28 15 10 20 6 21 3 42 24 45 25 16 17 46 2 14 8 44 39 34 11 7 41 36 47
13 31 34 38 24 29 40 32 28 47 30 42 36 37 25 5 27 6 4 33 48 7 11 8 35 41 39 44 3 17
14 4 6 47 41 48 42 5 7 31 33 28 37 25 13 26 11 21 10 27 34 12 20 16 36 32 30 40 18 23
15 10 21 31 32 34 28 26 12 4 27 7 25 13 14 8 20 3 2 11 6 9 17 46 37 5 33 42 43 39
10 29 37 24 13 23 16 30 34 39 40 25 43 22 8 47 31 4 36 48 44 33 5 9 38 42 46 17 7 3
11 44 8 42 6 46 2 48 4 40 25 10 23 17 20 29 36 26 22 37 16 31 35 18 39 34 13 19 32 15
12 16 20 34 27 13 21 37 26 25 10 11 46 19 3 44 22 9 17 8 2 36 43 45 40 4 6 14 47 41
8 39 44 14 2 45 17 42 25 24 46 16 38 43 22 30 48 37 29 40 23 34 31 35 41 13 19 18 27 12
9 46 17 6 11 14 3 25 8 13 2 20 24 45 18 40 44 22 23 16 19 37 29 38 42 10 21 15 31 32
6 36 4 39 42 44 25 47 33 29 48 34 22 8 10 35 5 27 26 31 37 32 12 20 43 30 40 16 15 19
7 8 11 48 33 25 6 36 5 37 4 27 16 2 21 22 9 12 20 26 10 35 18 19 44 31 34 13 38 24
4 43 36 46 25 17 8 39 48 23 44 37 18 9 26 38 47 31 35 29 22 30 32 12 45 40 16 20 28 21
5 17 9 25 4 2 11 44 36 16 8 26 19 3 12 23 43 35 18 22 20 29 38 15 46 37 10 21 30 28
2 48 25 41 14 39 46 33 6 30 42 13 29 44 16 31 4 10 37 34 40 27 26 22 47 28 24 23 12 18
3 25 2 33 7 42 14 4 11 34 6 21 40 46 19 37 8 20 16 10 13 26 22 23 48 27 28 24 35 38
25 47 48 45 46 43 44 41 42 38 39 40 35 36 37 32 33 34 31 30 29 28 27 26 1 24 23 22 21 20

```

```
In[591]:= et2 = Table[1 - 1/4 Re[Tr[Gprime2[[ii]] + Gprime2[[50 - ii]]]], {ii, 1, 48}]

```

```
Out[591]= {2, 1/2, 3/2, 1/2, 3/2, 1/2, 3/2, 1/2, 3/2, 1 - 1/sqrt(2), 1, 1 + 1/sqrt(2), 1 - 1/sqrt(2), 1, 1 + 1/sqrt(2), 1 - 1/sqrt(2), 1, 1 + 1/sqrt(2), 1, 1, 1, 1, 1, 1, 0, 1,
1, 1, 1, 1, 1, 1 + 1/sqrt(2), 1, 1 - 1/sqrt(2), 1 + 1/sqrt(2), 1, 1 - 1/sqrt(2), 1 + 1/sqrt(2), 1, 1 - 1/sqrt(2), 3/2, 1/2, 3/2, 1/2, 3/2, 1/2, 3/2, 1/2}
```

```

In[592]:= % // N

Out[592]= {2., 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.292893, 1., 1.70711, 0.292893, 1.,
  1.70711, 0.292893, 1., 1.70711, 1., 1., 1., 1., 1., 0., 1., 1., 1., 1., 1., 1.70711, 1.,
  0.292893, 1.70711, 1., 0.292893, 1.70711, 1., 0.292893, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5}

ig2 = {10, 13, 16, 34, 37, 40}

(*

Icosahedron
E (1)
6 C5 (24)
10 C3 (20)
15 C2 (15)

*)

In[596]:= Join[{a, b, c}, {d, e}]

Out[596]= {a, b, c, d, e}

In[595]:= Reverse[{1, 2, 3}]

Out[595]= {3, 2, 1}

In[599]:= Sin[ArcTan[2]] // FullSimplify

$$\frac{2}{\sqrt{5}}$$


In[1179]:= vertTable1 = Table[{Sin[ArcTan[2]] Sin[ $\frac{2 \text{ Pi } (ii - 1)}{5}$ ],
  Sin[ArcTan[2]] Cos[ $\frac{2 \text{ Pi } (ii - 1)}{5}$ ], Cos[ArcTan[2]]}, {ii, 1, 5}];

vertTable2 = Table[{Sin[ArcTan[2]] Sin[ $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$ ],
  Sin[ArcTan[2]] Cos[ $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$ ], -Cos[ArcTan[2]]}, {ii, 1, 5}];

c5roTV = Join[{0, 0, 1}], vertTable1];
vertTable = Join[{0, 0, 1}], vertTable1, vertTable2, {{0, 0, -1}}];
showTable1 = Table[ParametricPlot3D[vertTable1[[ii]] (1 - v) + v {0, 0, 1},
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];
showTable2 = Table[ParametricPlot3D[vertTable2[[ii]] (1 - v) + v {0, 0, -1},
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];
showTable3 = Table[ParametricPlot3D[vertTable1[[ii]] (1 - v) +
  v vertTable2[[If[5 == ii, 1, ii + 1]]],
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];
showTable4 = Table[ParametricPlot3D[vertTable1[[ii]] (1 - v) +
  v vertTable1[[If[5 == ii, 1, ii + 1]]],
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];
showTable5 = Table[ParametricPlot3D[vertTable2[[ii]] (1 - v) +
  v vertTable2[[If[5 == ii, 1, ii + 1]]],
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];
showTable6 = Table[ParametricPlot3D[vertTable1[[ii]] (1 - v) + v vertTable2[[ii]],
  {v, 0, 1}, Mesh -> 1, MeshShading -> {Gray, Gray}], {ii, 1, 5}];

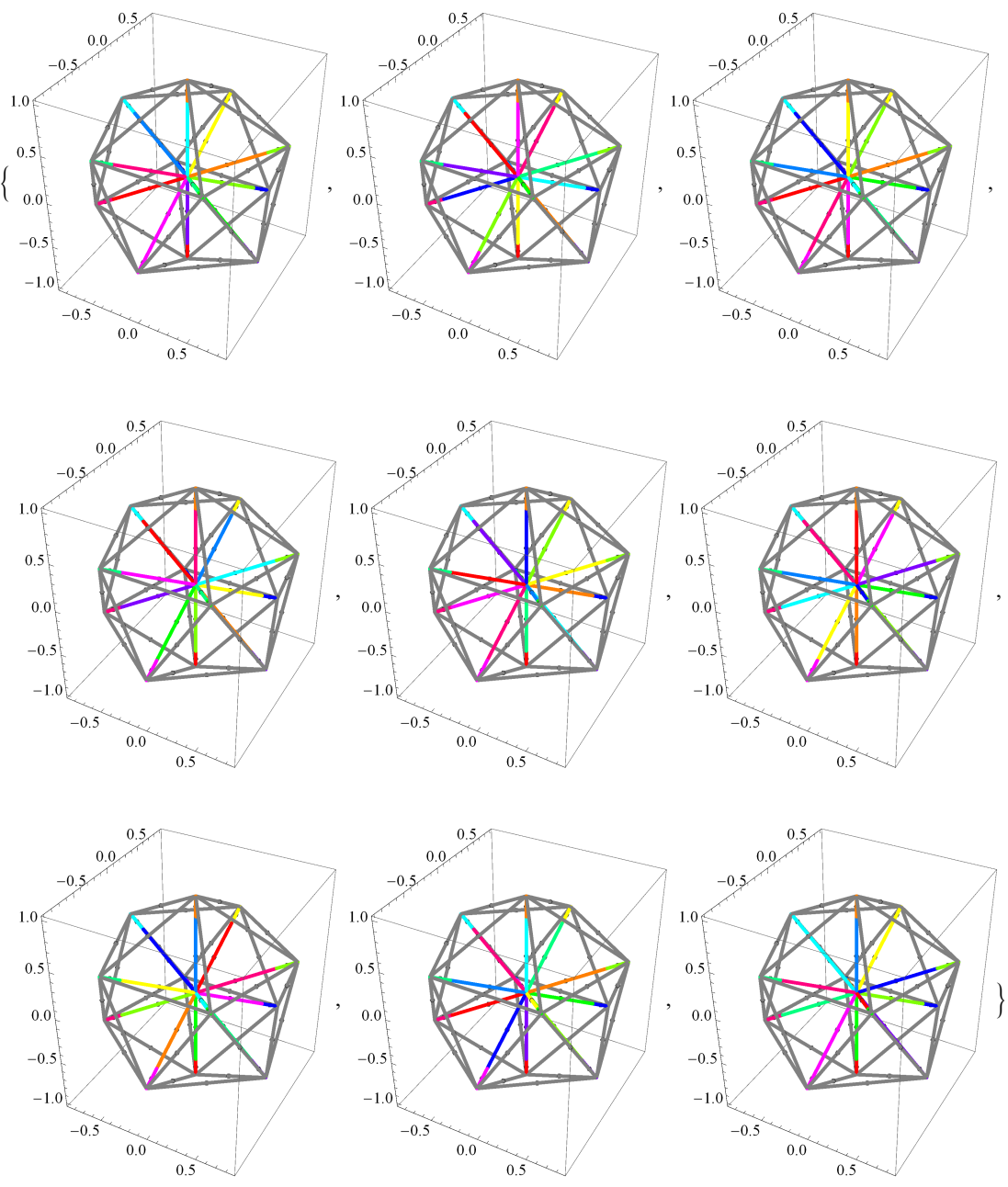
```

```

c3rotv1 = Table[ { Sin[ ArcTan[ 3 - Sqrt[5] ] ] Sin[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ],
  Sin[ ArcTan[ 3 - Sqrt[5] ] ] Cos[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ], Cos[ ArcTan[ 3 - Sqrt[5] ] ] }, { ii, 1, 5 } ];
c3rotv2 = Table[ { Sin[ ArcTan[ 3 + Sqrt[5] ] ] Sin[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ],
  Sin[ ArcTan[ 3 + Sqrt[5] ] ] Cos[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ], Cos[ ArcTan[ 3 + Sqrt[5] ] ] }, { ii, 1, 5 } ];
c3rotv = Join[ c3rotv1, c3rotv2 ];
c2rotv1 = Table[ { Sin[ ArcTan[  $\frac{\sqrt{5} - 1}{2}$  ] ] Sin[  $\frac{2 \text{ Pi } (ii - 1)}{5}$  ],
  Sin[ ArcTan[  $\frac{\sqrt{5} - 1}{2}$  ] ] Cos[  $\frac{2 \text{ Pi } (ii - 1)}{5}$  ], Cos[ ArcTan[  $\frac{\sqrt{5} - 1}{2}$  ] ] }, { ii, 1, 5 } ];
c2rotv2 = Table[ { Sin[ ArcTan[  $\frac{\sqrt{5} + 1}{2}$  ] ] Sin[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ],
  Sin[ ArcTan[  $\frac{\sqrt{5} + 1}{2}$  ] ] Cos[  $\frac{2 \text{ Pi } (ii - 1)}{5} - \frac{\text{Pi}}{5}$  ], Cos[ ArcTan[  $\frac{\sqrt{5} + 1}{2}$  ] ] }, { ii, 1, 5 } ];
c2rotv3 = Table[ { Sin[  $\frac{\text{Pi } (4 ii - 3)}{10}$  ], Cos[  $\frac{\text{Pi } (4 ii - 3)}{10}$  ], 0 }, { ii, 1, 5 } ];
c2rotv = Join[ c2rotv1, c2rotv2, c2rotv3 ];
showTableall = Table[ ParametricPlot3D[ vertTable[ [ ii ] ] v,
  { v, 0.8, 1 }, Mesh → 1, MeshShading → { Hue[ ii/12 ], Hue[ ii/12 ] }, { ii, 1, 12 } ];
(*C5*)
u3i1 = rotationQ[  $\frac{2 \text{ Pi}}{5}$ , c5rotv[[2]][[1]], c5rotv[[2]][[2]], c5rotv[[2]][[3]] ];
u3i2 = rotationQ[  $3 \frac{2 \text{ Pi}}{5}$ , c5rotv[[4]][[1]], c5rotv[[4]][[2]], c5rotv[[4]][[3]] ];
(*C3*)
u3i3 = rotationQ[  $\frac{2 \text{ Pi}}{3}$ , c3rotv[[2]][[1]], c3rotv[[2]][[2]], c3rotv[[2]][[3]] ];
u3i4 = rotationQ[  $\frac{4 \text{ Pi}}{3}$ , c3rotv[[9]][[1]], c3rotv[[9]][[2]], c3rotv[[9]][[3]] ];
(*C2*)
u3i5 = rotationQ[ Pi, c2rotv[[7]][[1]], c2rotv[[7]][[2]], c2rotv[[7]][[3]] ];
u3i6 = rotationQ[ Pi, c2rotv[[12]][[1]], c2rotv[[12]][[2]], c2rotv[[12]][[3]] ];
(*reflect*)
u3i7 = rotationQ[  $4 \frac{2 \text{ Pi}}{5}$ , c5rotv[[3]][[1]], c5rotv[[3]][[2]], c5rotv[[3]][[3]] ].
  rotationQ[ Pi, 0, 0, 1 ].reflectXYQ;
u3i8 = rotationQ[  $\frac{2 \text{ Pi}}{3}$ , c3rotv[[9]][[1]], c3rotv[[9]][[2]], c3rotv[[9]][[3]] ].
  rotationQ[ Pi, 0, 0, 1 ].reflectXYQ;
u3i9 = rotationQ[ Pi, c2rotv[[3]][[1]], c2rotv[[3]][[2]], c2rotv[[3]][[3]] ].
  rotationQ[ Pi, 0, 0, 1 ].reflectXYQ;
u3is = { u3i1, u3i2, u3i3, u3i4, u3i5, u3i6, u3i7, u3i8, u3i9 };
Table[ Show[ Join[ showTable1, showTable2, showTable3, showTable4, showTable5, showTable6,
  showTableall, Table[ ParametricPlot3D[ u3is[[jj]].vertTable[ [ ii ] ] v, { v, 0, 0.8 }, Mesh → 1,
    MeshShading → { Hue[ ii/12 ], Hue[ ii/12 ] }, { ii, 1, 12 } ], PlotRange → All ], { jj, 1, 9 } ]

```

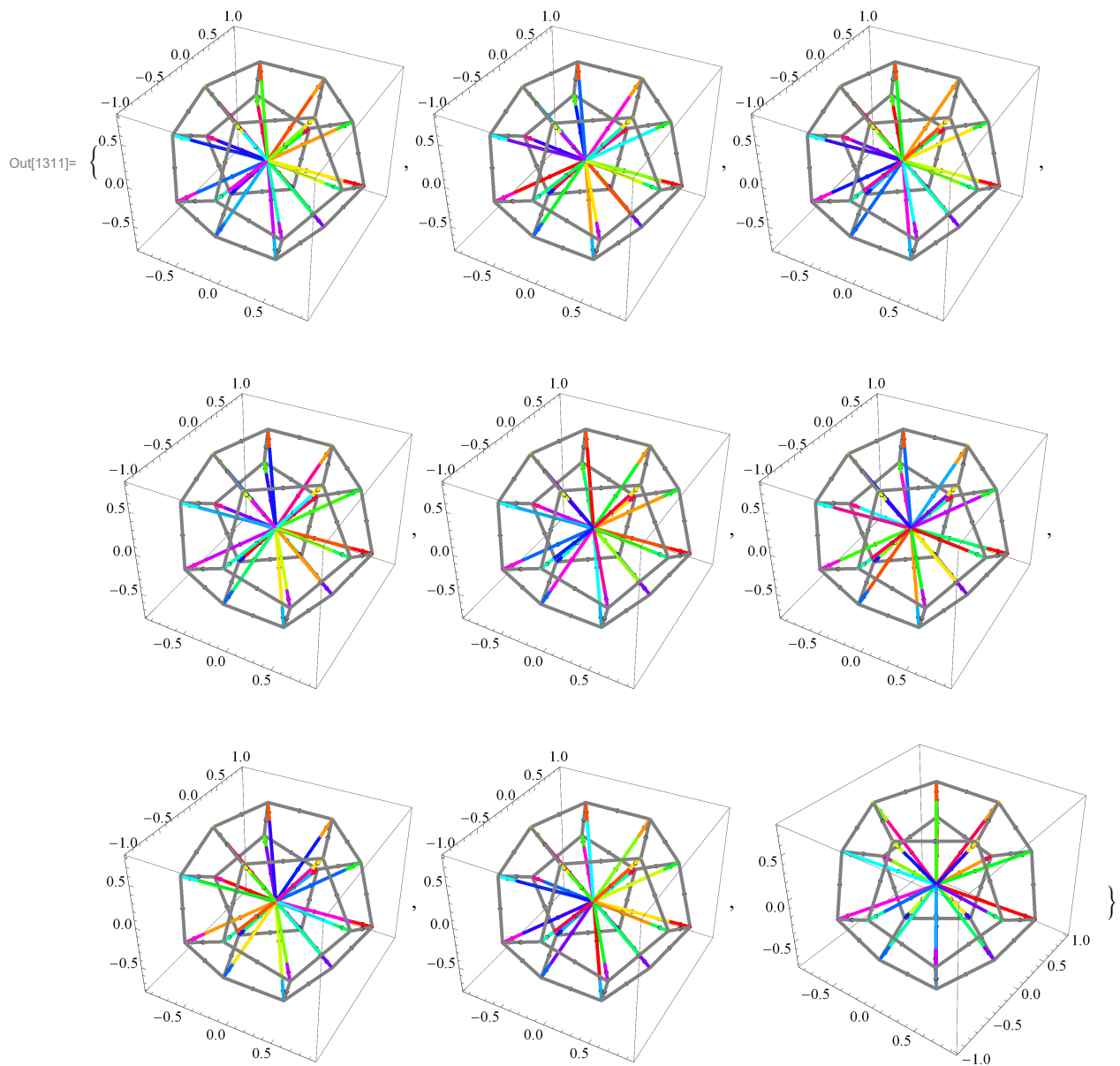

Out[1207]= {



```

In[1303]:= vertable3 = Join[c3rotrv, -c3rotrv];
showedge1 = Table[ParametricPlot3D[c3rotrv1[[ii]] (1 - v) + c3rotrv1[[If[ii == 5, 1, ii + 1]]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showedge2 = Table[ParametricPlot3D[c3rotrv1[[ii]] (1 - v) + c3rotrv2[[ii]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showedge3 = Table[ParametricPlot3D[-c3rotrv2[[ii]] (1 - v) +
  c3rotrv2[[If[ii ≤ 2, 5 + ii - 2, ii - 2]]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showedge4 = Table[ParametricPlot3D[-c3rotrv2[[ii]] (1 - v) +
  c3rotrv2[[If[ii ≥ 4, ii + 2 - 5, ii + 2]]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showedge5 = Table[ParametricPlot3D[-c3rotrv1[[ii]] (1 - v) - c3rotrv2[[ii]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showedge6 = Table[ParametricPlot3D[-c3rotrv1[[ii]] (1 - v) - c3rotrv1[[If[ii == 5, 1, ii + 1]]] v,
  {v, 0, 1}, Mesh → 1, MeshShading → {Gray, Gray}], {ii, 1, 5}];
showTableall2 = Table[ParametricPlot3D[vertable3[[ii]] v, {v, 0.8, 1},
  Mesh → 1, MeshShading → {Hue[ii/20], Hue[ii/20]}], {ii, 1, 20}];
Table[Show[Join[showedge1, showedge2, showedge3, showedge4, showedge5, showedge6,
  showTableall2, Table[ParametricPlot3D[u3is[[jj]].vertable3[[ii]] v, {v, 0, 0.8}, Mesh → 1,
    MeshShading → {Hue[ii/20], Hue[ii/20]}], {ii, 1, 20}]], PlotRange → All], {jj, 1, 9}]

```



In[1319]:= **Mod**[5, 4]

Out[1319]= 1

In[1322]:= **Floor**[5 / 4]

Out[1322]= 1

In[1376]:= **Clear**[r5table]
Clear[reVERR5table]
Clear[r3table]
Clear[reVERR3table]
Clear[r2table]
Clear[reVERR2table]

```

In[1382]:= r5table = Table[rotationP[ $\frac{2\pi}{5}$  (Mod[ii - 1, 4] + 1), c5rotrv[[Floor[(ii - 1)/4] + 1]][[1]],
    c5rotrv[[Floor[(ii - 1)/4] + 1]][[2]], c5rotrv[[Floor[(ii - 1)/4] + 1]][[3]]], {ii, 1, 24}];
reVERR5table = Reverse[Table[-1 rotationP[ $2\pi - \frac{2\pi}{5}$  (Mod[ii - 1, 4] + 1),
    c5rotrv[[Floor[(ii - 1)/4] + 1]][[1]], c5rotrv[[Floor[(ii - 1)/4] + 1]][[2]],
    c5rotrv[[Floor[(ii - 1)/4] + 1]][[3]]], {ii, 1, 24}]];
r3table = Table[rotationP[ $\frac{2\pi}{3}$  (Mod[ii - 1, 2] + 1), c3rotrv[[Floor[(ii - 1)/2] + 1]][[1]],
    c3rotrv[[Floor[(ii - 1)/2] + 1]][[2]], c3rotrv[[Floor[(ii - 1)/2] + 1]][[3]]], {ii, 1, 20}];
reVERR3table = Reverse[Table[-1 rotationP[ $2\pi - \frac{2\pi}{3}$  (Mod[ii - 1, 2] + 1),
    c3rotrv[[Floor[(ii - 1)/2] + 1]][[1]], c3rotrv[[Floor[(ii - 1)/2] + 1]][[2]],
    c3rotrv[[Floor[(ii - 1)/2] + 1]][[3]]], {ii, 1, 20}]];
r2table = Table[rotationP[Pi, c2rotrv[[ii]][[1]], c2rotrv[[ii]][[2]], c2rotrv[[ii]][[3]]],
    {ii, 1, 15}];
reVERR2table = Reverse[Table[-1 rotationP[Pi, c2rotrv[[ii]][[1]],
    c2rotrv[[ii]][[2]], c2rotrv[[ii]][[3]]], {ii, 1, 15}]];

In[1388]:= r5table[[1]].reVERR5table[[24]] // FullSimplify
r3table[[1]].reVERR3table[[20]] // FullSimplify
r2table[[1]].reVERR2table[[15]] // FullSimplify

Out[1388]=  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 
Out[1389]=  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 
Out[1390]=  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

In[1391]:= 1 + 24 + 20 + 15 + 1 + 15 + 20 + 24 + 1

Out[1391]= 121

In[1392]:= Clear[Gprime3]

In[1393]:= Gprime3 = Join[{-1 I2m}, r5table, r3table,
    r2table, {I2m}, reVERR2table, reVERR3table, reVERR5table, {-1 I2m}];

```

```
In[1397]:= Table[Gprime3[[ii]].Gprime3[[122 - ii]] // FullSimplify, {ii, 1, 61}] // FullSimplify
```

[illegible]

```
In[1440]:= Gprime3n = N[Gprime3];
```

```

In[1442]:= count = 0; ProgressIndicator[Dynamic[count]]
retk3 = Table[gk = Gprime3n[ii].Gprime3n[jj];
  retk = 0;
  For[kk = 1, kk < 122, kk++,
    If[Abs[gk - Gprime3n[kk]].{1, 1}.{1, 1} < 0.0000000001 && 0 == retk, retk = kk,];
  ];
  count = (ii*121 + jj)/(121*121);
  retk, {ii, 1, 121}, {jj, 1, 121}];

```

Out[1442]=

In[1444]:= retk3

61	117	118	119	120	113	114	115	116	109	110	111	112	105	106	107	108	101	102	103	104	97	98	99
117	3	4	5	1	26	51	37	105	28	52	39	101	30	53	41	97	32	54	43	113	34	55	4
118	4	5	1	117	50	24	81	91	46	8	79	89	47	12	77	87	48	16	85	95	49	20	8
119	5	1	117	118	35	45	110	75	27	37	106	74	29	39	102	73	31	41	98	72	33	43	11
120	1	117	118	119	21	83	70	94	25	81	69	92	9	79	68	90	13	77	67	88	17	85	7
113	28	47	31	17	7	8	9	1	38	63	77	97	10	40	62	85	2	32	54	43	26	50	3
114	52	12	41	85	8	9	1	113	56	102	87	95	38	78	99	71	28	30	15	58	46	4	3
115	39	77	98	71	9	1	113	114	79	89	118	76	56	103	88	96	52	11	40	86	29	31	10
116	101	87	72	96	1	113	114	115	105	91	75	94	79	68	90	120	39	63	78	100	13	41	6
109	30	48	33	21	28	46	27	25	11	12	13	1	40	62	85	113	14	42	66	83	2	34	5
110	53	16	43	83	47	4	35	45	12	13	1	109	57	98	95	93	40	86	115	70	30	32	1
111	41	85	114	70	31	33	20	59	13	1	109	110	77	87	118	75	57	99	96	94	53	15	4
112	97	95	76	94	17	43	66	84	1	109	110	111	101	89	74	92	77	67	88	120	41	62	8
105	32	49	35	25	2	26	51	37	30	47	29	9	15	16	17	1	42	66	83	109	18	44	6
106	54	20	45	81	32	34	23	60	48	4	27	37	16	17	1	105	58	114	93	91	42	84	11
107	43	83	110	69	54	19	44	82	33	35	24	60	17	1	105	106	85	95	118	74	58	115	9
108	113	93	75	92	43	66	84	112	21	45	65	82	1	105	106	107	97	87	73	90	85	71	9
101	34	50	27	9	22	36	64	79	2	28	52	39	32	48	31	13	19	20	21	1	44	65	8
102	55	24	37	79	44	82	107	68	34	26	7	56	49	4	29	39	20	21	1	101	59	110	9
103	45	81	106	68	59	111	92	90	55	23	36	80	35	27	8	56	21	1	101	102	83	93	11
104	109	91	74	90	83	70	94	120	45	65	82	108	25	37	64	80	1	101	102	103	113	95	7
97	26	46	29	13	36	64	79	101	6	38	63	77	2	30	53	41	34	49	33	17	23	24	2
98	51	8	39	77	60	106	89	87	36	80	103	67	26	28	11	57	50	4	31	41	24	25	1
99	37	79	102	67	81	91	118	72	60	107	90	88	51	7	38	78	27	29	12	57	25	1	9
100	105	89	73	88	109	93	76	96	81	69	92	120	37	64	80	104	9	39	63	78	1	97	9
95	46	29	13	97	51	37	105	117	7	56	102	87	28	11	57	98	3	48	16	85	50	35	2
96	9	101	87	72	25	109	93	76	37	106	74	119	8	56	103	88	29	12	57	99	5	17	8
93	47	31	17	113	46	27	25	109	52	39	101	117	11	57	98	95	30	15	58	114	3	49	2
94	13	97	95	76	5	21	83	70	9	105	91	75	39	102	73	119	12	57	99	96	31	16	5
91	48	33	21	109	3	50	24	81	47	29	9	105	53	41	97	117	15	58	114	93	32	19	5
92	17	113	93	75	33	20	59	111	5	25	81	69	13	101	89	74	41	98	72	119	16	58	11
89	49	35	25	105	34	23	60	106	3	46	8	79	48	31	13	101	54	43	113	117	19	59	11
90	21	109	91	74	20	59	111	92	35	24	60	107	5	9	79	68	17	97	87	73	43	114	7
87	50	27	9	101	23	60	106	89	26	7	56	102	3	47	12	77	49	33	17	97	55	45	10
88	25	105	89	73	45	110	75	119	24	60	107	90	27	8	56	103	5	13	77	67	21	113	9
85	7	52	12	41	64	79	101	97	80	103	67	98	6	10	40	62	26	3	48	16	51	27	5
86	79	102	67	99	105	117	95	71	106	74	119	96	64	80	104	100	8	52	11	40	9	13	4
83	11	53	16	43	52	29	5	21	63	77	97	113	78	99	71	114	10	14	42	66	28	3	4
84	77	98	71	115	13	17	43	66	101	117	93	70	102	73	119	94	63	78	100	116	12	53	1
81	15	54	20	45	30	3	50	24	53	31	5	25	62	85	113	109	86	115	70	110	14	18	4
82	85	114	70	111	16	54	19	44	17	21	45	65	97	117	91	69	98	72	119	92	62	86	11
79	19	55	24	37	18	22	36	64	32	3	46	8	54	33	5	9	66	83	109	105	84	111	6
80	83	110	69	107	66	84	112	108	20	55	23	36	21	25	37	64	113	117	89	68	114	76	11
77	23	51	8	39	82	107	68	102	22	6	38	63	34	3	47	12	55	35	5	13	65	81	10
78	81	106	68	103	110	75	119	88	65	82	108	104	24	51	7	38	25	9	39	63	109	117	8
76	29	13	97	95	27	25	109	93	8	79	89	118	52	63	67	72	47	53	62	71	4	33	4
75	31	17	113	93	4	35	45	110	29	9	105	91	12	77	87	118	53	62	71	76	48	54	6
74	33	21	109	91	49	55	65	69	4	27	37	106	31	13	101	89	16	85	95	118	54	66	7
73	35	25	105	89	55	65	69	74	50	51	64	68	4	29	39	102	33	17	97	87	20	83	9
72	27	9	101	87	24	81	91	118	51	64	68	73	46	52	63	67	4	31	41	98	35	21	11
71	8	39	77	98	37	105	117	95	64	68	73	72	7	38	78	99	46	47	53	62	27	5	1
70	12	41	85	114	29	5	21	83	39	101	117	93	63	67	72	76	11	40	86	115	47	48	5
69	16	43	83	110	48	49	55	65	31	5	25	81	41	97	117	91	62	71	76	75	15	42	8
68	20	45	81	106	19	44	82	107	49	50	51	64	33	5	9	79	43	113	117	89	66	70	7
67	24	37	79	102	65	69	74	73	23	36	80	103	50	46	52	63	35	5	13	77	45	109	11
66	63	57	62	58	39	13	17	43	102	87	95	114	103	88	96	115	38	10	14	42	52	47	4
65	62	58	66	59	53	48	49	55	41	17	21	45	98	95	93	110	99	96	94	111	40	14	1
64	66	59	65	60	47	18	22	36	54	49	50	51	43	21	25	37	114	93	91	106	115	94	9

Out[1444]=

63	65	60	64	56	111	92	90	103	44	22	6	38	55	50	46	52	45	25	9	39	110	91	8
62	64	56	63	57	106	89	87	98	107	90	88	99	36	6	10	40	51	46	47	53	37	9	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	2
60	58	66	59	65	15	32	34	23	16	33	35	24	85	113	109	81	71	76	75	69	86	116	11
59	57	62	58	66	12	31	33	20	77	97	113	83	67	72	76	70	78	100	116	84	11	30	3
58	56	63	57	62	79	101	97	85	68	73	72	71	80	104	100	86	7	28	30	15	8	29	3
57	60	64	56	63	69	74	73	67	82	108	104	78	23	26	28	11	24	27	29	12	81	105	10
56	59	65	60	64	84	112	108	80	19	34	26	7	20	35	27	8	83	109	105	79	70	75	7
55	99	86	42	19	57	53	48	49	98	85	43	20	72	76	70	59	88	120	112	44	78	10	2
54	103	78	40	15	102	77	41	16	73	72	71	58	90	120	116	42	80	6	2	32	56	52	4
53	107	80	38	11	74	73	67	57	92	120	100	40	82	22	2	30	60	51	46	47	106	79	3
52	111	82	36	7	94	120	104	38	84	18	2	28	59	55	50	46	110	81	37	8	75	74	6
51	115	84	44	23	86	14	2	26	58	54	49	50	114	83	45	24	76	75	69	60	96	120	10
50	96	116	18	34	99	40	30	3	71	58	54	49	76	70	59	55	119	92	82	23	88	104	6
49	88	100	14	32	67	57	53	48	72	71	58	54	119	94	84	19	90	108	22	34	103	38	2
48	90	104	10	30	73	67	57	53	119	96	86	15	92	112	18	32	107	36	26	3	68	56	5
47	92	108	6	28	119	88	78	11	94	116	14	30	111	44	34	3	69	60	51	46	74	68	5
46	94	112	22	26	96	100	10	28	115	42	32	3	70	59	55	50	75	69	60	51	119	90	8
44	98	71	115	84	41	16	54	19	97	113	83	59	87	118	75	111	67	88	120	112	57	40	1
45	40	15	54	20	11	47	4	35	57	41	17	21	99	71	114	83	100	116	84	59	10	2	3
42	102	67	99	86	101	97	85	58	89	118	76	115	68	90	120	116	56	38	10	14	39	12	5
43	38	11	53	16	56	39	13	17	103	67	98	85	104	100	86	58	6	2	32	54	7	46	4
40	106	68	103	78	91	118	72	99	69	92	120	100	60	36	6	10	37	8	52	11	105	101	7
41	36	7	52	12	107	68	102	77	108	104	78	57	22	2	30	53	23	50	4	31	60	37	9
38	110	69	107	80	70	94	120	104	59	44	22	6	45	24	51	7	109	105	79	56	93	118	7
39	44	23	51	8	112	108	80	56	18	2	28	52	19	49	4	29	59	45	25	9	111	69	10
36	114	70	111	82	58	42	18	22	43	20	55	23	113	109	81	60	95	118	74	107	71	96	12
37	42	19	55	24	14	2	26	51	15	48	4	27	58	43	21	25	115	70	110	81	116	112	8
34	72	96	116	18	98	62	15	32	95	114	66	19	118	75	111	44	73	90	108	22	67	78	1
35	100	14	32	49	78	11	47	4	99	62	16	33	96	115	66	20	120	112	44	55	104	6	2
32	73	88	100	14	87	98	62	15	118	76	115	42	74	92	112	18	68	80	6	2	102	63	1
33	104	10	30	48	103	63	12	31	88	99	62	16	120	116	42	54	108	22	34	49	80	7	4
30	74	90	104	10	118	72	99	40	75	94	116	14	69	82	22	2	106	64	7	28	89	102	6
31	108	6	28	47	90	103	63	12	120	100	40	53	112	18	32	48	82	23	50	4	107	64	8
28	75	92	108	6	76	96	100	10	70	84	18	2	110	65	23	26	91	106	64	7	118	73	10
29	112	22	26	46	120	104	38	52	116	14	30	47	84	19	49	4	111	65	24	27	92	107	6
26	76	94	112	22	71	86	14	2	114	66	19	34	93	110	65	23	118	74	107	36	72	88	10
27	116	18	34	50	100	10	28	46	86	15	48	4	115	66	20	35	94	111	65	24	120	108	3
22	95	76	94	112	85	58	42	18	113	83	59	44	117	91	69	82	87	73	90	108	98	99	10
23	71	115	84	44	62	15	32	34	85	43	20	55	95	93	110	65	72	119	92	82	99	100	6
24	86	42	19	55	40	30	3	50	62	16	33	35	71	114	83	45	96	94	111	65	100	61	2
25	14	32	49	35	10	28	46	27	40	53	31	5	86	58	43	21	116	84	59	45	61	22	2
18	87	72	96	116	97	85	58	42	117	93	70	84	89	74	92	112	102	103	104	61	77	57	4
19	67	99	86	42	77	41	16	54	87	95	114	66	73	119	94	84	103	104	61	18	63	11	3
20	78	40	15	54	63	12	31	33	67	98	85	43	88	96	115	66	104	61	18	19	38	28	3
21	10	30	48	33	38	52	29	5	78	57	41	17	100	86	58	43	61	18	19	20	6	26	5
14	89	73	88	100	117	95	71	86	91	75	94	116	106	107	108	61	79	56	38	10	101	77	5
15	68	103	78	40	89	87	98	62	74	119	96	86	107	108	61	14	64	7	28	30	79	39	1
16	80	38	11	53	68	102	77	41	90	88	99	62	108	61	14	15	36	26	3	48	64	8	2
17	6	28	47	31	80	56	39	13	104	78	57	41	61	14	15	16	22	34	49	33	36	51	2
10	91	74	90	104	93	76	96	100	110	111	112	61	81	60	36	6	105	79	56	38	117	87	6
11	69	107	80	38	75	119	88	78	111	112	61	10	65	23	26	28	81	37	8	52	91	89	10
12	82	36	7	52	92	90	103	63	112	61	10	11	44	34	3	47	65	24	27	29	69	106	7
13	22	26	46	29	108	80	56	39	61	10	11	12	18	32	48	31	44	55	35	5	82	60	3
6	93	75	92	108	114	115	116	61	83	59	44	22	109	81	60	36	117	89	68	80	95	72	8
7	70	111	82	36	115	116	61	6	66	19	34	26	83	45	24	51	93	91	106	64	76	119	9
8	84	44	23	51	116	61	6	7	42	32	3	46	66	20	35	27	70	110	81	37	94	92	10
9	18	34	50	27	61	6	7	8	14	30	47	29	42	54	33	5	84	59	45	25	112	82	6
2	118	119	120	61	95	71	86	14	93	70	84	18	91	69	82	22	89	68	80	6	87	67	7
3	119	120	61	2	72	99	40	30	76	115	42	32	75	111	44	34	74	107	36	26	73	103	3
4	120	61	2	3	88	78	11	47	96	86	15	48	94	84	19	49	92	82	23	50	90	80	7
5	61	2	3	4	104	38	52	29	100	40	53	31	116	42	54	33	112	44	55	35	108	36	5
61	117	118	119	120	113	114	115	116	109	110	111	112	105	106	107	108	101	102	103	104	97	98	9

In[1446]:= $\text{et3} = \text{Table}\left[1 - \frac{1}{4} \text{Re}\left[\text{Tr}\left[\text{Gprime3}\left[\left[\text{ii}\right]\right] + \text{Gprime3}\left[\left[122 - \text{ii}\right]\right]\right]\right], \{\text{ii}, 1, 120\}\right]$

```
Out[1446]= {2, 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ ,
  1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ ,
  1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ ,
  1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ ,
   $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ , 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0,
  1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,
  1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ ,
  1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ ,
  1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ ,
  1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 + \sqrt{5})$ , 1 +  $\frac{1}{4}(1 - \sqrt{5})$ , 1 +  $\frac{1}{4}(-1 - \sqrt{5})$ }
```

```
In[1447]:= % // N
```

```
Out[1447]= {2., 0.190983, 0.690983, 1.30902, 1.80902, 0.190983, 0.690983, 1.30902, 1.80902, 0.190983, 0.690983, 1.30902,
  1.80902, 0.190983, 0.690983, 1.30902, 1.80902, 0.190983, 0.690983, 1.30902, 1.80902, 0.190983, 0.690983,
  1.30902, 1.80902, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5,
  1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.5,
  0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5,
  0.690983, 0.190983, 1.80902, 1.30902, 0.690983, 0.190983, 1.80902, 1.30902, 0.690983, 0.190983, 1.80902,
  1.30902, 0.690983, 0.190983, 1.80902, 1.30902, 0.690983, 0.190983, 1.80902, 1.30902, 0.690983, 0.190983}

(* ig table has 12 elements *)
ig3 = {2, 6, 10, 14, 18, 22, 104, 108, 112, 116, 120}
```

```
In[1449]:= et3[[104]] // N
```

```
Out[1449]= 0.190983
```