

STAT303-1

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Table of contents

Preface	5
1 Introduction to Python and Jupyter Notebooks	6
1.1 Jupyter notebook	6
1.1.1 Introduction	6
1.1.2 Writing and executing code	7
1.1.3 Saving and loading notebooks	7
1.1.4 Rendering notebook as HTML	8
In-class exercise	8
1.2 Python language basics	8
1.2.1 Object Oriented Programming	8
1.2.2 Assigning variable name to object	9
1.2.3 Importing libraries	10
1.2.4 Built-in objects	10
1.2.5 Control flow	12
2 Data structures	14
2.0.1 Concatenating tuples	15
2.0.2 Unpacking tuples	15
2.0.3 Tuple methods	16
2.1 List	17
2.1.1 Adding and removing elements in a list	17
2.1.2 List comprehensions	19
2.1.3 Concatenating lists	19
2.1.4 Sorting a list	20
2.1.5 Slicing a list	20
2.1.6 Practice exercise	22
2.2 Dictionary	24
2.2.1 Adding and removing elements in a dictionary	24
2.2.2 Iterating over elements of a dictionary	25
2.2.3 Practice exercise	26
2.3 Functions	26
2.3.1 Global and local variables with respect to a function	28
2.3.2 Practice exercise	28
2.4 In-class exercise	29

3	Reading data	30
3.1	Reading a <i>csv</i> file with <i>Pandas</i>	30
3.1.1	Using the <i>read_csv</i> function	31
3.1.2	Specifying the working directory	31
3.1.3	Data overview and summary statistics	32
3.2	Reading other data formats - <i>txt</i> , <i>html</i> , <i>json</i>	34
3.2.1	Reading <i>txt</i> files	34
3.2.2	Reading HTML data	35
3.2.3	Reading JSON data	37
3.2.4	Reading data from web APIs	38
3.3	Writing data	39
3.4	Sub-setting data: <i>loc</i> and <i>iloc</i>	40
4	NumPy	42
4.1	Vectorized computation with NumPy	43
	In-class exercise	46
4.2	Pseudorandom number generation	47
	In-class exercise	49
5	Pandas	50
5.1	Data manipulations with Pandas	51
5.1.1	Sub-setting data	51
5.1.2	Sorting data	52
5.1.3	Unique values, value counts and membership	53
5.2	Operations between DataFrame and Series	54
5.3	Correlation	59
6	Data visualization	61
6.0.1	Scatterplots and trendline	62
6.0.2	Subplots	64
6.0.3	Overlapping plots with legend	66
6.1	Pandas	67
6.1.1	Scatterplots and lineplots	67
6.1.2	Bar plots	69
6.2	Seaborn	73
6.2.1	Bar plots with confidence intervals	74
6.2.2	Facetgrid: Multi-plot grid for plotting conditional relationships	75
6.2.3	Histogram and density plots	78
6.2.4	Boxplots	80
7	Data cleaning and preparation	82
7.0.1	Types of missing values	82
7.0.2	Missing Completely at Random (MCAR)	82

7.0.3	Missing at Random (MAR)	82
7.0.4	Missing Not at Random (MNAR)	83
8	Data wrangling	84
9	Data aggregation	85
10	Datasets	86
	References	87

Preface

This book is developed for the course STAT303-1 (Data Science with Python-1).

1 Introduction to Python and Jupyter Notebooks

This chapter is a very brief introduction to python and Jupyter notebooks. We only discuss the content relevant for applying python to analyze data.

Anaconda: If you are new to python, we recommend downloading the [Anaconda installer](#) and following the instructions for installation. Once installed, we'll use the Jupyter Notebook interface to write code.

Quarto: We'll use Quarto to publish the `.ipynb` file containing text, python code, and the output. Download and install Quarto from [here](#).

1.1 Jupyter notebook

1.1.1 Introduction

Jupyter notebook is an interactive platform, where you can write code and text, and make visualizations. You can access Jupyter notebook from the Anaconda Navigator, or directly open the Jupyter Notebook application itself. It should automatically open up in your default browser. The figure below shows a Jupyter Notebook opened with Google Chrome. This page is called the *landing page* of the notebook.

<IPython.core.display.Image object>

To create a new notebook, click on the **New** button and select the **Python 3** option. You should see a blank notebook as in the figure below.

<IPython.core.display.Image object>

1.1.2 Writing and executing code

Code cell: By default, a cell is of type *Code*, i.e., for typing code, as seen as the default choice in the dropdown menu below the *Widgets* tab. Try typing a line of python code (say, `2+3`) in an empty code cell and execute it by pressing *Shift+Enter*. This should execute the code, and create an new code cell. Pressing *Ctrl+Enter* for *Windows* (or *Cmd+Enter* for *Mac*) will execute the code without creating a new cell.

Commenting code in a code cell: Comments should be made while writing the code to explain the purpose of the code or a brief explanation of the tasks being performed by the code. A comment can be added in a code cell by preceding it with a `#` sign. For example, see the comment in the code below.

Writing comments will help other users understand your code. It is also useful for the coder to keep track of the tasks being performed by their code.

```
#This code adds 3 and 5
3+5
```

8

Markdown cell: Although a comment can be written in a code cell, a code cell cannot be used for writing headings/sub-headings, and is not appropriate for writing lengthy chunks of text. In such cases, change the cell type to *Markdown* from the dropdown menu below the *Widgets* tab. Use any markdown cheat sheet found online, for example, [this one](#) to format text in the markdown cells.

Give a name to the notebook by clicking on the text, which says 'Untitled'.

1.1.3 Saving and loading notebooks

Save the notebook by clicking on **File**, and selecting **Save as**, or clicking on the **Save and Checkpoint** icon (below the **File** tab). Your notebook will be saved as a file with an extension *ipynb*. This file will contain all the code as well as the outputs, and can be loaded and edited by a Jupyter user. To load an existing Jupyter notebook, navigate to the folder of the notebook on the *landing page*, and then click on the file to open it.

1.1.4 Rendering notebook as HTML

We'll use Quarto to print the `**ipynb*` file as HTML. Check the procedure for rendering a notebook as HTML [here](#). You have several options to format the file.

You will need to open the command prompt, navigate to the directory containing the file, and use the command: `quarto render filename.ipynb --to html`.

In-class exercise

1. Create a new notebook.
2. Save the file as `In_class_exercise1`.
3. Give a heading to the file - `First HTML file`.
4. Print `Today is day 1 of class`.
5. Compute and print the number of hours of this course in the quarter (that will be 10 weeks x 2 classes per week x 1.33 hours per class).

The HTML file should look like the picture below.

```
<IPython.core.display.Image object>
```

1.2 Python language basics

1.2.1 Object Oriented Programming

Python is an object-oriented programming language. In layman terms, it means that every number, string, data structure, function, class, module, etc., exists in the python interpreter as a python object. An object may have attributes and methods associated with it. For example, let us define a variable that stores an integer:

```
var = 2
```

The variable `var` is an object that has attributed and methods associated with it. For example a couple of its attributes are `real` and `imag`, which store the real and imaginary parts respectively, of the object `var`:

```
print("Real part of 'var': ",var.real)
print("Real part of 'var': ",var.imag)
```



```
Real part of 'var': 2
Real part of 'var': 0
```

Attribute: An attribute is a value associated with an object, defined within the class of the object.

Method: A method is a function associated with an object, defined within the class of the object, and has access to the attributes associated with the object.

For looking at attributes and methods associated with an object, say `obj`, press tab key after typing `obj..`

Consider the example below of a class *example_class*:

```
class example_class:
    class_name = 'My Class'
    def my_method(self):
        print('Hello World!')

e = example_class()
```

In the above class, `class_name` is an attribute, while `my_method` is a method.

1.2.2 Assigning variable name to object

When an object is assigned to a variable name, the variable name serves as a reference to the object. For example, consider the following assignment:

```
x = [5,3]
```

The variable name `x` is a reference to the memory location where the object `[5, 3]` is stored. Now, suppose assign `x` to a new variable `y`:

```
y = x
```

In the above statement the variable name `y` now refers to the same object `[5,3]`. The object `[5,3]` does **not** get copied to a new memory location referred by `y`. To prove this, let us add an element to `y`:

```
y.append(4)
print(y)
```

```
[5, 3, 4]
```

```
print(x)
```

```
[5, 3, 4]
```

When we changed `y`, note that `x` also changed to the same object, showing that `x` and `y` refer to the same object, instead of referring to different copies of the same object.

1.2.3 Importing libraries

There are several [built-in functions](#) in python like `print()`, `abs()`, `max()`, `sum()` etc., which do not require importing any library. However, these functions will typically be insufficient for analyzing data. Some of the popular libraries and their primary purposes are as follows:

1. NumPy: Performing numerical operations and efficiently storing numerical data.
2. Pandas: Reading, cleaning and manipulating data.
3. Matplotlib, Seaborn: Visualizing data.
4. SciPy: Performing scientific computing such as solving differential equations, optimization, statistical tests, etc.
5. Scikit-learn: Data pre-processing and machine learning, with a focus on prediction.
6. Statsmodels: Developing statistical models with a focus on inference

A library can be imported using the `import` keyword. For example, a NumPy library can be imported as:

```
import numpy as np
```

Using the `as` keyword, the NumPy library has been given the name `np`. All the functions and attributes of the library can be called using the `'np.'` prefix. For example, let us generate a sequence of whole numbers upto 10 using the NumPy function `arange()`:

```
np.arange(8)
```

```
array([0, 1, 2, 3, 4, 5, 6, 7])
```

1.2.4 Built-in objects

There are several [built-in objects, modules and functions in python](#). Below are a few examples:

Scalar objects: Python has some built-in datatypes for handling scalar objects such as number, string, boolean values, and date/time. The built-in function `type()` function can be used to determine the datatype of an object:

```
var = 2.2
type(var)
```

float

range(): The `range()` function returns a sequence of evenly-spaced integer values. It is commonly used in `for` loops to define the sequence of elements over which the iterations are performed.

Below is an example where the `range()` function is used to create a sequence of whole numbers upto 10:

```
print(list(range(1,10)))
```

[1, 2, 3, 4, 5, 6, 7, 8, 9]

Date time: Python has a built-in `datetime` module for handling date/time objects:

```
import datetime as dt

#Defining a date-time object
dt_object = dt.datetime(2022, 9, 20, 11,30,0)
```

Information about date and time can be accessed with the relevant attribute of the `datetime` object.

```
dt_object.day
```

20

```
dt_object.year
```

2022

The `strftime` method of the `datetime` module formats a `datetime` object as a string. There are several types of formats for representing date as a string:

```
dt_object.strftime('%m/%d/%Y')
```

```
'09/20/2022'
```

```
dt_object.strftime('%m/%d/%y %H:%M')
```

```
'09/20/22 11:30'
```

```
dt_object.strftime('%h-%d-%Y')
```

```
'Sep-20-2022'
```

1.2.5 Control flow

As in other languages, python has [built-in keywords](#) that provide conditional flow of control in the code.

If-elif-else: The `if-elif-else` statement can check several conditions, and execute the code corresponding to the condition that is true. Note that there can be as many `elif` statements as required.

```
#Example of if-elif-else
x = 5
if x>0:
    print("x is positive")
elif x==0:
    print("x is zero")
else:
    print("X is negative")
    print("This was the last condition checked")
```

```
x is positive
```

for loop: A `for` loop iterates over the elements of an object, and executes the statements within the loop in each iteration. For example, below is a `for` loop that prints odd natural numbers upto 10:

```
for i in range(10):  
    if i%2!=0:  
        print(i)
```

1
3
5
7
9

while loop: A **while** loop iterates over a set of statements *while* a condition is satisfied. For example, below is a **while** loop that prints odd numbers upto 10:

```
i=0  
while i<10:  
    if i%2!=0:  
        print(i)  
    i=i+1
```

1
3
5
7
9

2 Data structures

In this chapter we'll learn about the python data structures that are often used or appear while analyzing data.

Tuple is a sequence of python objects, with two key characteristics: (1) the number of objects are fixed, and (2) the objects are immutable, i.e., they cannot be changed.

Tuple can be defined as a sequence of python objects separated by commas, and enclosed in rounded brackets (). For example, below is a tuple containing three integers.

```
tuple_example = (2,7,4)
```

We can check the data type of a python object using the *type()* function. Let us check the data type of the object *tuple_example*.

```
type(tuple_example)
```

tuple

Elements of a tuple can be extracted using their index within square brackets. For example the second element of the tuple *tuple_example* can be extracted as follows:

```
tuple_example[1]
```

7

Note that an object of a tuple cannot be modified. For example, consider the following attempt in changing the second element of the tuple *tuple_example*.

```
tuple_example[1] = 8
```

TypeError: 'tuple' object does not support item assignment

The above code results in an error as tuple elements cannot be modified.

2.0.1 Concatenating tuples

Tuples can be concatenated using the + operator to produce a longer tuple:

```
(2,7,4) + ("another", "tuple") + ("mixed","datatypes",5)
```

```
(2, 7, 4, 'another', 'tuple', 'mixed', 'datatypes', 5)
```

Multiplying a tuple by an integer results in repetition of the tuple:

```
(2,7,"hi") * 3
```

```
(2, 7, 'hi', 2, 7, 'hi', 2, 7, 'hi')
```

2.0.2 Unpacking tuples

If tuples are assigned to an expression containing multiple variables, the tuple will be unpacked and each variable will be assigned a value as per the order in which it appears. See the example below.

```
x,y,z = (4.5, "this is a string", ("Nested tuple",5))
```

```
x
```

```
4.5
```

```
y
```

```
'this is a string'
```

```
z
```

```
('Nested tuple', 5)
```

If we are interested in retrieving only some values of the tuple, the expression `*_` can be used to discard the other values. Let's say we are interested in retrieving only the first and the last two values of the tuple:

```
x,*_,y,z = (4.5, "this is a string", (("Nested tuple",5)), "99",99)
```

```
x
```

```
4.5
```

```
y
```

```
'99'
```

```
z
```

```
99
```

2.0.3 Tuple methods

A couple of useful tuple methods are `count`, which counts the occurrences of an element in the tuple and `index`, which returns the position of the first occurrence of an element in the tuple:

```
tuple_example = (2,5,64,7,2,2)
```

```
tuple_example.count(2)
```

```
3
```

```
tuple_example.index(2)
```

```
0
```

Now that we have an idea about tuple, let us try to think where it can be used.

```
<IPython.core.display.HTML object>
```


2.1 List

List is a sequence of python objects, with two key characteristics that differentiate them from tuples: (1) the number of objects are variable, i.e., objects can be added or removed from a list, and (2) the objects are mutable, i.e., they can be changed.

List can be defined as a sequence of python objects separated by commas, and enclosed in square brackets []. For example, below is a list containing three integers.

```
list_example = [2,7,4]
```

2.1.1 Adding and removing elements in a list

We can add elements at the end of the list using the *append* method. For example, we append the string 'red' to the list *list_example* below.

```
list_example.append('red')
```

```
list_example
```

```
[2, 7, 4, 'red']
```

Note that the objects of a list or tuple can be of different datatypes.

An element can be added at a specific location of the list using the *insert* method. For example, if we wish to insert the number 2.32 as the second element of the list *list_example*, we can do it as follows:

```
list_example.insert(1,2.32)
```

```
list_example
```

```
[2, 2.32, 7, 4, 'red']
```

For removing an element from the list, the *pop* and *remove* methods may be used. The *pop* method removes an element at particular index, while the *remove* method removes the element's first occurrence in the list by its value. See the examples below.

Let us say, we need to remove the third element of the list.

```
list_example.pop(2)
```

7

```
list_example
```

```
[2, 2.32, 4, 'red']
```

Let us say, we need to remove the element 'red'.

```
list_example.remove('red')
```

```
list_example
```

```
[2, 2.32, 4]
```

```
#If there are multiple occurrences of an element in the list, the first occurrence will be removed
list_example2 = [2,3,2,4,4]
list_example2.remove(2)
list_example2
```

```
[3, 2, 4, 4]
```

For removing multiple elements in a list, either `pop` or `remove` can be used in a for loop, or a for loop can be used with a condition. See the examples below.

Let's say we need to remove integers less than 100 from the following list.

```
list_example3 = list(range(95,106))
list_example3
```

```
[95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105]
```

```
#Method 1: For loop with remove
list_example3_filtered = list(list_example3) #
```

```

for element in list_example3:
    #print(element)
    if element<100:
        list_example3_filtered.remove(element)
print(list_example3_filtered)

```

[100, 101, 102, 103, 104, 105]

Q: What's the need to define a new variable 'list_example3_filtered' in the above code?

Replace list_example3_filtered with list_example3 and identify the issue.

```

#Method 2: For loop with condition
[element for element in list_example3 if element>100]

```

[101, 102, 103, 104, 105]

2.1.2 List comprehensions

List comprehension is a compact way to create new lists based on elements of an existing list.

Example: Create a list that has squares of natural numbers from 5 to 15.

```

sqrt_natural_no_5_15 = [(x**2) for x in range(5,16)]
print(sqrt_natural_no_5_15)

```

[25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225]

Example: Create a list of tuples, where each tuple consists of a natural number and its square, for natural numbers ranging from 5 to 15.

```

sqrt_natural_no_5_15 = [(x,x**2) for x in range(5,16)]
print(sqrt_natural_no_5_15)

```

[(5, 25), (6, 36), (7, 49), (8, 64), (9, 81), (10, 100), (11, 121), (12, 144), (13, 169), (14, 196), (15, 225)]

2.1.3 Concatenating lists

As in tuples, lists can be concatenated using the + operator:

```
import time as tm
```

```
list_example4 = [5,'hi',4]  
list_example4 = list_example4 + [None,'7',9]  
list_example4
```

```
[5, 'hi', 4, None, '7', 9]
```

For adding elements to a list, the **extend** method is preferred over the **+** operator. This is because using the **+** operator creates a new list, while the **extend** method adds elements to an existing list.

```
list_example4 = [5,'hi',4]  
list_example4.extend([None, '7', 9])  
list_example4
```

```
[5, 'hi', 4, None, '7', 9]
```

2.1.4 Sorting a list

A list can be sorted using the **sort** method:

```
list_example5 = [6,78,9]  
list_example5.sort(reverse=True) #the reverse argument is used to specify if the sorting i  
list_example5
```

```
[78, 9, 6]
```

2.1.5 Slicing a list

We may extract or update a section of the list by passing the starting index (say **start**) and the stopping index (say **stop**) as **start:stop** to the index operator **[]**. This is called *slicing* a list. For example, see the following example.

```
list_example6 = [4,7,3,5,7,1,5,87,5]
```

Let us extract a slice containing all the elements starting from the the 3rd position upto the 7th position.

```
list_example6[2:7]
```

```
[3, 5, 7, 1, 5]
```

Note that while the element at the **start** index is included, the element with the **stop** index is excluded in the above slice.

If either the **start** or **stop** index is not mentioned, the slicing will be done from the beginning or upto the end of the list, respectively.

```
list_example6[:7]
```

```
[4, 7, 3, 5, 7, 1, 5]
```

```
list_example6[2:]
```

```
[3, 5, 7, 1, 5, 87, 5]
```

To slice the list relative to the end, we can use negative indices:

```
list_example6[-4:]
```

```
[1, 5, 87, 5]
```

```
list_example6[-4:-2:]
```

```
[1, 5]
```

An extra colon (':') can be used to slice every *i*th element of a list.

```
#Selecting every 3rd element of a list  
list_example6[::3]
```

```
[4, 5, 5]
```

```
#Selecting every 3rd element of a list from the end
list_example6[::-3]
```

[5, 1, 3]

```
#Selecting every element of a list from the end or reversing a list
list_example6[::-1]
```

[5, 87, 5, 1, 7, 5, 3, 7, 4]

2.1.6 Practice exercise

Start with the list [8,9,10]. Do the following: 1. Set the second entry (index 1) to 17 2. Add 4, 5, and 6 to the end of the list 3. Remove the first entry from the list 4. Sort the list 5. Double the list (concatenate the list to itself) 6. Insert 25 at index 3

The final list should equal [4,5,6,25,10,17,4,5,6,10,17]

[4, 5, 6, 25, 10, 17, 4, 5, 6, 10, 17]

Now that we have an idea about lists, let us try to think where it can be used.

<IPython.core.display.HTML object>

Now that we have learned about lists and tuples, let us compare them.

Q: A list seems to be much more flexible than tuple, and can replace a tuple almost everywhere. Then why use tuples?

A: The additional flexibility of a list comes at the cost of efficiency. Some of the advantages of a tuple over a list are as follows:

1. Since a list can be extended, space is over-allocated when creating a list. A tuple takes less storage space as compared to a list of the same length.
2. Tuples are not copied. If a tuple is assigned to another tuple, both tuples point to the same memory location. However, if a list is assigned to another list, a new list is created consuming the same memory space as the original list.

3. Tuples refer to their element directly, while in a list, there is an extra layer of pointers that refers to their elements. Thus it is faster to retrieve elements from a tuple.

The examples below illustrate the above advantages of a tuple.

```
#Example showing tuples take less storage space than lists for the same elements
tuple_ex = (1, 2, 'Obama')
list_ex = [1, 2, 'Obama']
print("Space taken by tuple =",tuple_ex.__sizeof__()," bytes")
print("Space taken by list =",list_ex.__sizeof__()," bytes")
```

Space taken by tuple = 48 bytes

Space taken by list = 64 bytes

```
#Examples showing that a tuples are not copied, while lists can be copied
tuple_copy = tuple(tuple_ex)
print("Is tuple_copy same as tuple_ex?", tuple_ex is tuple_copy)
list_copy = list(list_ex)
print("Is list_copy same as list_ex?",list_ex is list_copy)
```

Is tuple_copy same as tuple_ex? True

Is list_copy same as list_ex? False

```
#Examples showing tuples takes lesser time to retrieve elements
import time as tm
tt = tm.time()
list_ex = list(range(1000000)) #List containinig whole numbers upto 1 million
a=(list_ex[::-2])
print("Time take to retrieve every 2nd element from a list = ", tm.time()-tt)

tt = tm.time()
tuple_ex = tuple(range(1000000)) #tuple containinig whole numbers upto 1 million
a=(tuple_ex[::-2])
print("Time take to retrieve every 2nd element from a tuple = ", tm.time()-tt)
```

Time take to retrieve every 2nd element from a list = 0.03579902648925781

Time take to retrieve every 2nd element from a tuple = 0.02684164047241211

2.2 Dictionary

A dictionary consists of key-value pairs, where the keys and values are python objects. While values can be any python object, keys need to be immutable python objects, like strings, intergers, tuples, etc. Thus, a list can be a value, but not a key, as a elements of list can be changed. A dictionary is defined using the keyword `dict` along with curly braces, colons to separate keys and values, and commas to separate elements of a dictionary:

```
dict_example = {'USA':'Joe Biden', 'India':'Narendra Modi', 'China':'Xi Jinping'}
```

Elements of a dictionary can be retrieved by using the corresponding key.

```
dict_example['India']
```

```
'Narendra Modi'
```

2.2.1 Adding and removing elements in a dictionary

New elements can be added to a dictionary by defining a key in square brackets and assiging it to a value:

```
dict_example['Japan'] = 'Fumio Kishida'
dict_example['Countries'] = 4
dict_example
```

```
{'USA': 'Joe Biden',
 'India': 'Narendra Modi',
 'China': 'Xi Jinping',
 'Japan': 'Fumio Kishida',
 'Countries': 4}
```

Elements can be removed from the dictionary using the `del` method or the `pop` method:

```
#Removing the element having key as 'Countries'
del dict_example['Countries']

dict_example
```



```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Japan': 'Fumio Kishida'}
```

```
#Removing the element having key as 'USA'  
dict_example.pop('USA')
```

```
'Joe Biden'
```

```
dict_example
```

```
{'India': 'Narendra Modi', 'China': 'Xi Jinping', 'Japan': 'Fumio Kishida'}
```

New elements can be added, and values of existing keys can be changed using the `update` method:

```
dict_example = {'USA': 'Joe Biden', 'India': 'Narendra Modi', 'China': 'Xi Jinping', 'Countries': 3}  
dict_example
```

```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Countries': 3}
```

```
dict_example.update({'Countries': 4, 'Japan': 'Fumio Kishida'})
```

```
dict_example
```

```
{'USA': 'Joe Biden',  
 'India': 'Narendra Modi',  
 'China': 'Xi Jinping',  
 'Countries': 4,  
 'Japan': 'Fumio Kishida'}
```

2.2.2 Iterating over elements of a dictionary

The `items()` attribute of a dictionary can be used to iterate over elements of a dictionary.

```
for key,value in dict_example.items():
    print("The Head of State of",key,"is",value)
```

```
The Head of State of USA is Joe Biden
The Head of State of India is Narendra Modi
The Head of State of China is Xi Jinping
The Head of State of Countries is 4
The Head of State of Japan is Fumio Kishida
```

2.2.3 Practice exercise

The GDP per capita of USA for most years from 1960 to 2021 is given by the dictionary D given in the code cell below.

Find:

1. The GDP per capita in 2015
2. The GDP per capita of 2014 is missing. Update the dictionary to include the GDP per capita of 2014 as the average of the GDP per capita of 2013 and 2015.
3. Impute the GDP per capita of other missing years in the same manner as in (2), i.e., as the average GDP per capita of the previous year and the next year. Note that the GDP per capita is not missing for any two consecutive years.

```
D = {'1960':3007,'1961':3067,'1962':3244,'1963':3375,'1964':3574,'1965':3828,'1966':4146,'
```

2.3 Functions

If an algorithm or block of code is being used several times in a code, then it can be separately defined as a function. This makes the code more organized and readable. For example, let us define a function that prints prime numbers between **a** and **b**, and returns the number of prime numbers found.

```
#Function definition
def prime_numbers (a,b=100):
    num_prime_nos = 0

    #Iterating over all numbers between a and b
    for i in range(a,b):
        num_divisors=0
```

```

#Checking if the ith number has any factors
for j in range(2, i):
    if i%j == 0:
        num_divisors=1;break;

#If there are no factors, then printing and counting the number as prime
if num_divisors==0:
    print(i)
    num_prime_nos = num_prime_nos+1

#Return count of the number of prime numbers
return num_prime_nos

```

In the above function, the keyword `def` is used to define the function, `prime_numbers` is the name of the function, `a` and `b` are the arguments that the function uses to compute the output.

Let us use the defined function to print and count the prime numbers between 40 and 60.

```

#Printing prime numbers between 40 and 60
num_prime_nos_found = prime_numbers(40,60)

```

```

41
43
47
53
59

```

```

num_prime_nos_found

```

```

5

```

If the user calls the function without specifying the value of the argument `b`, then it will take the default value of 100, as mentioned in the function definition. However, for the argument `a`, the user will need to specify a value, as there is no value defined as a default value in the function definition.

2.3.1 Global and local variables with respect to a function

A variable defined within a function is local to that function, while a variable defined outside the function is global with respect to that function. In case a variable with the same name is defined both outside and inside a function, it will refer to its global or local value, depending on where it occurs.

The example below shows a variable with the name `var` referring to its local value when called within the function, and global value when called outside the function.

```
var = 5
def sample_function(var):
    print("Local value of 'var' within 'sample_function()' = ",var)

sample_function(4)
print("Global value of 'var' outside 'sample_function()' = ",var)
```

```
Local value of 'var' within 'sample_function()' = 4
Global value of 'var' outside 'sample_function()' = 5
```

2.3.2 Practice exercise

The object `deck` defined below corresponds to a deck of cards. Estimate the probability that a five card hand will be a [flush](#), as follows:

1. Write a function that accepts a hand of 5 cards as argument, and returns whether the hand is a flush or not.
2. Randomly pull a hand of 5 cards from the deck. Call the function developed in (1) to determine if the hand is a flush.
3. Repeat (2) 10,000 times.
4. Estimate the probability of the hand being a flush from the results of the 10,000 simulations.

You may use the function [shuffle\(\)](#) from the `random` library to shuffle the deck everytime before pulling a hand of 5 cards.

```
deck = [{'value':i, 'suit':c}
for c in ['spades', 'clubs', 'hearts', 'diamonds']
for i in range(2,15)]
```

2.4 In-class exercise

The code cell below defines an object having the nutrition information of drinks in starbucks.

```
, 'value': 1}, {'starbucks_drinks_distribution':{'value':10},'starbucks_drinks_distribution_type':'c'}
```

Use the object above to answer the following questions: 1. What is the datatype of the object? 2. What is the datatype of the elements of the object? 3. What is the datatype of the elements withing the elements of the object? 4. How many calories are there in `Iced Coffee`? 5. Which drink(s) have the highest amount of protein in them, and what is that protein amount? 6. Which drink(s) have a fat content of more than 10g, and what is their fat content?

3 Reading data

Reading data is the first step to extract information from it. Data can exist broadly in two formats:

- (1) Structured data and,
- (2) Unstructured data.

Structured data is typically stored in a tabular form, where rows in the data correspond to “observations” and columns correspond to “variables”. For example, the following dataset contains 5 observations, where each observation (or row) consists of information about a movie. The variables (or columns) contain different pieces of information about a given movie. As all variables for a given row are related to the same movie, the data below is also called as relational data.

	Title	US Gross	Production Budget	Release Date	Major Genre
0	The Shawshank Redemption	28241469	25000000	Sep 23 1994	Drama
1	Inception	285630280	160000000	Jul 16 2010	Horror/Thriller
2	One Flew Over the Cuckoo's Nest	108981275	4400000	Nov 19 1975	Comedy
3	The Dark Knight	533345358	185000000	Jul 18 2008	Action/Adventure
4	Schindler's List	96067179	25000000	Dec 15 1993	Drama

Unstructured data is data that is not organized in any pre-defined manner. Examples of unstructured data can be text files, audio/video files, images, Internet of Things (IoT) data, etc. Unstructured data is relatively harder to analyze as most of the analytical methods and tools are oriented towards structured data. However, an unstructured data can be used to obtain structured data, which in turn can be analyzed. For example, an image can be converted to an array of pixels - which will be structured data. Machine learning algorithms can then be used on the array to classify the image as that of a dog or a cat.

In this course, we will focus on analyzing structured data.

3.1 Reading a *csv* file with *Pandas*

Structured data can be stored in a variety of formats. The most popular format is *data_file_name.csv*, where the extension *csv* stands for comma separated values. The variable

values of each observation are separated by a comma in a *.csv* file. In other words, the **delimiter** is a comma in a *csv* file. However, the comma is not visible when a *.csv* file is opened with Microsoft Excel.

3.1.1 Using the *read_csv* function

We will use functions from the *Pandas* library of *Python* to read data. Let us import *Pandas* to use its functions.

```
import pandas as pd
```

Note that *pd* is the acronym that we will use to call a *Pandas* function. This acronym can be anything as desired by the user.

The function to read a *csv* file is `read_csv()`. It reads the dataset into an object of type *Pandas DataFrame*. Let us read the dataset *movie_ratings.csv* in Python.

```
movie_ratings = pd.read_csv('movie_ratings.csv')
```

The built-in python function `type` can be used to check the datatype of an object:

```
type(movie_ratings)
```

```
pandas.core.frame.DataFrame
```

Note that the file *movie_ratings.csv* is stored at the same location as the python script containing the above code. If that is not the case, we'll need to specify the location of the file as in the following code.

```
movie_ratings = pd.read_csv('D:/Books/DataScience_Intro_python/movie_ratings.csv')
```

Note that forward slash is used instead of backslash while specifying the path of the data file. Another option is to use two consecutive backslashes instead of a single forward slash.

3.1.2 Specifying the working directory

In case we need to read several datasets from a given location, it may be inconvenient to specify the path every time. In such a case we can change the current working directory to the location where the datasets are located.

We'll use the *os* library of *Python* to view and/or change the current working directory.

```
import os #Importing the 'os' library
os.getcwd() #Getting the path to the current working directory
```

```
'C:\\Users\\akl0407\\Desktop\\STAT303-1\\Quarto Book\\DataScience_Intro_python'
```

The function `getcwd()` stands for get current working directory.

Suppose the dataset to be read is located at 'D:\\Books\\DataScience_Intro_python\\Datasets'. Then, we'll use the function `chdir` to change the current working directory to this location.

```
os.chdir('D:/Books/DataScience_Intro_python/Datasets')
```

Now we can read the dataset from this location without mentioning the entire path as shown below.

```
movie_ratings = pd.read_csv('movie_ratings.csv')
```

3.1.3 Data overview and summary statistics

Once the data has been read, we may want to see what the data looks like. We'll use another *Pandas* function `head()` to view the first few rows of the data.

```
movie_ratings.head()
```

	Title	US Gross	Worldwide Gross	Production Budget	Release Date	MPAA Rating
0	Opal Dreams	14443	14443	9000000	Nov 22 2006	PG/PG-13
1	Major Dundee	14873	14873	3800000	Apr 07 1965	PG/PG-13
2	The Informers	315000	315000	18000000	Apr 24 2009	R
3	Buffalo Soldiers	353743	353743	15000000	Jul 25 2003	R
4	The Last Sin Eater	388390	388390	2200000	Feb 09 2007	PG/PG-13

Row Indices and column names (axis labels):

The bold integers on the left are the indices of the DataFrame. Each index refers to a distinct row. For example, the index *2* corresponds to the row of the movie *The Informers*. By default, the indices are integers starting from 0. However, they can be changed (to even non-integer values) if desired by the user.

The bold text on top of the DataFrame refers to column names. For example, the column *US Gross* consists of the gross revenue of a movie in the US.

Collectively, the indices and column names are referred as **axis labels**.

Shape of DataFrame:

For finding the number of rows and columns in the data, you may use the `shape()` function.

```
#Finding the shape of movie_ratings dataset
movie_ratings.shape
```

(2228, 11)

The *movie_ratings* dataset contains 2,809 observations (or rows) and 15 variables (or columns).

For obtaining summary statistics of data, you may use the `describe()` function with a Pandas DataFrame.

```
#Finding summary statistics of movie_ratings dataset
movie_ratings.describe()
```

	US Gross	Worldwide Gross	Production Budget	IMDB Rating	IMDB Votes
count	2.228000e+03	2.228000e+03	2.228000e+03	2228.000000	2228.000000
mean	5.076370e+07	1.019370e+08	3.816055e+07	6.239004	33585.154847
std	6.643081e+07	1.648589e+08	3.782604e+07	1.243285	47325.651561
min	0.000000e+00	8.840000e+02	2.180000e+02	1.400000	18.000000
25%	9.646188e+06	1.320737e+07	1.200000e+07	5.500000	6659.250000
50%	2.838649e+07	4.266892e+07	2.600000e+07	6.400000	18169.000000
75%	6.453140e+07	1.200000e+08	5.300000e+07	7.100000	40092.750000
max	7.601676e+08	2.767891e+09	3.000000e+08	9.200000	519541.000000

Answer the following questions based on the above table.

<IPython.core.display.HTML object>

<IPython.core.display.HTML object>

The Pandas DataFrame class also has functions such as `sum()` and `mean()` to compute sum over rows or columns of a DataFrame.

Let us compute the mean of all the numeric columns of the data:

```
movie_ratings.mean(axis = 0)
```

```
US Gross          5.076370e+07
Worldwide Gross   1.019370e+08
Production Budget  3.816055e+07
IMDB Rating       6.239004e+00
IMDB Votes        3.358515e+04
dtype: float64
```

The argument `axis=0` denotes that the mean is taken over all the rows of the DataFrame. For computing a statistic across column the argument `axis=1` should be used.

If mean over individual columns is desired, then they should be subset from the data. For example, let us compute the mean IMDB rating of all the movies:

```
movie_ratings['IMDB Rating'].mean(axis = 0)
```

```
6.2390035906642725
```

3.2 Reading other data formats - txt, html, json

Although *csv* is a very popular format for structured data, data is found in several other formats as well. Some of the other data formats are *txt*, *html* and *json*.

3.2.1 Reading *txt* files

The *txt* format offers some additional flexibility as compared to the *csv* format. In the *csv* format, the delimiter is a comma (or the column values are separated by a comma). However, in a *txt* file, the delimiter can be anything as desired by the user. Let us read the file *movie_ratings.txt*, where the variable values are separated by a tab character.

```
movie_ratings_txt = pd.read_csv('movie_ratings.txt', sep='\t')
```

We use the function `read_csv` to read a *txt* file. However, we mention the tab character (`'\t'`) as a separator of variable values.

Note that there is no need to remember the argument name - *sep* for specifying the delimiter. You can always refer to the [read_csv\(\)](#) documentation to find the relevant argument.

3.2.2 Reading HTML data

The *Pandas* function `read_html` searches for tabular data, i.e., data contained within the `<table>` tags of an html file. Let us read the tables in the GDP per capita [page](#) on Wikipedia.

```
#Reading all the tables from the Wikipedia page on GDP per capita
tables = pd.read_html('https://en.wikipedia.org/wiki/List_of_countries_by_GDP_(nominal)_pe
```

All the tables will be read and stored in the variable named as *tables*. Let us find the datatype of the variable *tables*.

```
#Finidng datatype of the variable - tables
type(tables)
```

list

The variable - *tables* is a list of all the tables read from the HTML data.

```
#Number of tables read from the page
len(tables)
```

6

The in-built function *len* can be used to find the length of the list - *tables* or the number of tables read from the Wikipedia page. Let us check out the first table.

```
#Checking out the first table. Note that the index of the first table will be 0.
tables[0]
```

	0	1	2
0	.mw-parser-output .legend{page-break-inside:av...	\$20,000 - \$30,000 \$10,000 - \$20,000 \$5,000 - \$...	\$1,000 - \$2,000

The above table doesn't seem to be useful. Let us check out the second table.

```
#Checking out the second table. Note that the index of the first table will be 1.
tables[1]
```

	Country/Territory	UN Region	IMF[4][5]		United Nations[6]		World Bank[7]	
	Country/Territory	UN Region	Estimate	Year	Estimate	Year	Estimate	Year
0	Liechtenstein *	Europe	—	—	180227	2020	169049	2019
1	Monaco *	Europe	—	—	173696	2020	173688	2020
2	Luxembourg *	Europe	135046	2022	117182	2020	135683	2021
3	Bermuda *	Americas	—	—	123945	2020	110870	2021
4	Ireland *	Europe	101509	2022	86251	2020	85268	2020
...
212	Central African Republic *	Africa	527	2022	481	2020	477	2020
213	Sierra Leone *	Africa	513	2022	475	2020	485	2020
214	Madagascar *	Africa	504	2022	470	2020	496	2020
215	South Sudan *	Africa	393	2022	1421	2020	1120	2015
216	Burundi *	Africa	272	2022	286	2020	274	2020

The above table contains the estimated GDP per capita of all countries. This is the table that is likely to be relevant to a user interested in analyzing GDP per capita of countries. Instead of reading all tables of an HTML file, we can focus the search to tables containing certain relevant keywords. Let us try searching all table containing the word ‘Country’.

```
#Reading all the tables from the Wikipedia page on GDP per capita, containing the word 'Country'
tables = pd.read_html('https://en.wikipedia.org/wiki/List_of_countries_by_GDP_(nominal)_per_capita')
```

The *match* argument can be used to specify the keywords to be present in the table to be read.

```
len(tables)
```

```
1
```

Only one table contains the keyword - ‘Country’. Let us check out the table obtained.

```
#Table having the keyword - 'Country' from the HTML page
tables[0]
```

	Country/Territory	UN Region	IMF[4][5]		United Nations[6]	World Bank[7]		
	Country/Territory	UN Region	Estimate	Year	Estimate	Year	Estimate	Year
0	Liechtenstein *	Europe	—	—	180227	2020	169049	2019
1	Monaco *	Europe	—	—	173696	2020	173688	2020
2	Luxembourg *	Europe	135046	2022	117182	2020	135683	2021
3	Bermuda *	Americas	—	—	123945	2020	110870	2021
4	Ireland *	Europe	101509	2022	86251	2020	85268	2020
...
212	Central African Republic *	Africa	527	2022	481	2020	477	2020
213	Sierra Leone *	Africa	513	2022	475	2020	485	2020
214	Madagascar *	Africa	504	2022	470	2020	496	2020
215	South Sudan *	Africa	393	2022	1421	2020	1120	2015
216	Burundi *	Africa	272	2022	286	2020	274	2020

The argument *match* helps with a more focussed search, and helps us discard irrelevant tables.

3.2.3 Reading JSON data

JSON stands for JavaScript Object Notation, in which the data is stored and transmitted as plain text. Since the format is text only, JSON data can easily be exchanged between web applications, and used by any programming language. Unlike the *csv* format, JSON supports a hierarchical data structure, and is easier to integrate with APIs.

Lets read JSON data on Ted Talks. The *Pandas* function `read_json` converts JSON data to a dataframe.

```
tedtalks_data = pd.read_json('https://raw.githubusercontent.com/cwkenwaysun/TEDmap/master/
tedtalks_data.head()
```

	id	speaker	headline	URL	description
0	7	David Pogue	Simplicity sells	http://www.ted.com/talks/view/id/7	New York
1	6	Craig Venter	Sampling the ocean's DNA	http://www.ted.com/talks/view/id/6	Genomic
2	4	Burt Rutan	The real future of space exploration	http://www.ted.com/talks/view/id/4	In this p
3	3	Ashraf Ghani	How to rebuild a broken state	http://www.ted.com/talks/view/id/3	Ashraf G
4	5	Chris Bangle	Great cars are great art	http://www.ted.com/talks/view/id/5	America

<IPython.core.display.HTML object>

3.2.4 Reading data from web APIs

[API](#), an acronym for Application programming interface, is a way of transferring information between systems. Many websites have public APIs that provide data via JSON or other formats. For example, the [IMDb-API](#) is a web service for receiving movies, serial, and cast information. API results are in the JSON format and include items such as movie specifications, ratings, Wikipedia page content, etc. One of these APIs contains ratings of the top 250 movies on IMDB. Let us read this data using the IMDB API.

We'll use the *get* function from the python library *requests* to request data from the API and obtain a response code. The response code will let us know if our request to pull data from this API was successful.

```
#Importing the requests library
import requests as rq

# Downloading imdb top 250 movie's data
url = 'https://imdb-api.com/en/API/Top250Movies/k_v6gf8ppf' #URL of the API containing top
response = rq.get(url) #Requesting data from the API
response
```

<Response [200]>

We have a response code of 200, which indicates that the request was successful.

The response object's JSON method will return a dictionary containing JSON parsed into native Python objects.

```
movie_data = response.json()
```

```
movie_data.keys()
```

```
dict_keys(['items', 'errorMessage'])
```

The *movie_data* contains only two keys. The *items* key seems likely to contain information about the top 250 movies. Let us convert the values of the *items* key (which is list of dictionaries) to a dataframe, so that we can view it in a tabular form.

```
#Converting a list of dictionaries to a dataframe
movie_data_df = pd.DataFrame(movie_data['items'])
```

```
#Checking the movie data pulled using the API
movie_data_df.head()
```

	id	rank	title	fullTitle	year	image
0	tt0111161	1	The Shawshank Redemption	The Shawshank Redemption (1994)	1994	https://m.r
1	tt0068646	2	The Godfather	The Godfather (1972)	1972	https://m.r
2	tt0468569	3	The Dark Knight	The Dark Knight (2008)	2008	https://m.r
3	tt0071562	4	The Godfather Part II	The Godfather Part II (1974)	1974	https://m.r
4	tt0050083	5	12 Angry Men	12 Angry Men (1957)	1957	https://m.r

```
#Rows and columns of the movie data
movie_data_df.shape
```

(250, 9)

This API provides the names of the top 250 movies along with the year of release, IMDB ratings, and cast information.

3.3 Writing data

The *Pandas* function `to_csv` can be used to write (or export) data to a *csv* or *txt* file. Below are some examples.

Example 1: Let us export the movies data of the top 250 movies to a *csv* file.

```
#Exporting the data of the top 250 movies to a csv file
movie_data_df.to_csv('movie_data_exported.csv')
```

The file *movie_data_exported.csv* will appear in the working directory.

Example 2: Let us export the movies data of the top 250 movies to a *txt* file with a semi-colon as the delimiter.

```
movie_data_df.to_csv('movie_data_exported.txt',sep=';')
```

Example 3: Let us export the movies data of the top 250 movies to a *JSON* file.

```
with open('movie_data.json', 'w') as f:
    json.dump(movie_data, f)
```

3.4 Sub-setting data: loc and iloc

Sometimes we may be interested in working with a subset of rows and columns of the data, instead of working with the entire dataset. The indexing operators `loc` and `iloc` provide a convenient of selecting a subset of desired rows and columns. The operator `loc` uses axis labels (row indices and column names) to subset the data, while `iloc` uses the index (this is different from the row index) corresponding to the position of the row or columns. Note that the index of the position for both the row and column starts from 0.

Let us read the file *movie_IMDBratings_sorted.csv*, which has movies sorted in the descending order of their IMDB ratings.

```
movies_sorted = pd.read_csv('./Datasets/movie_IMDBratings_sorted.csv', index_col = 0)
```

The argument `index_col=0` assigns the first column of the file as the row indices of the DataFrame.

```
movies_sorted.head()
```

	Title	US Gross	Worldwide Gross	Production Budget	Release Date	M
Rank						
1	The Shawshank Redemption	28241469	28241469	25000000	Sep 23 1994	R
2	Inception	285630280	753830280	160000000	Jul 16 2010	PG
3	The Dark Knight	533345358	1022345358	185000000	Jul 18 2008	PG
4	Schindler's List	96067179	321200000	25000000	Dec 15 1993	R
5	Pulp Fiction	107928762	212928762	8000000	Oct 14 1994	R

Let us say, we wish to subset the title, worldwide gross, production budget, and IMDB rating of top 3 movies.

```
# Subsetting the DataFrame by loc - using axis labels
movies_subset = movies_sorted.loc[1:3,['Title','Worldwide Gross','Production Budget','IMDB
movies_subset
```


Rank	Title	Worldwide Gross	Production Budget	IMDB Rating
1	The Shawshank Redemption	28241469	25000000	9.2
2	Inception	753830280	160000000	9.1
3	The Dark Knight	1022345358	185000000	8.9

```
# Subsetting the DataFrame by iloc - using index of the position of rows and columns
movies_subset = movies_sorted.iloc[0:3,[0,2,3,9]]
movies_subset
```

Rank	Title	Worldwide Gross	Production Budget	IMDB Rating
1	The Shawshank Redemption	28241469	25000000	9.2
2	Inception	753830280	160000000	9.1
3	The Dark Knight	1022345358	185000000	8.9

4 NumPy

NumPy, short for Numerical Python is used to analyze numeric data with Python. Although numeric operations may be performed without NumPy, NumPy is preferred for its efficiency, especially when working with large arrays of data. A couple of reasons that make NumPy more efficient are:

1. NumPy arrays use much less memory than other built-in Python data structures. This is because a NumPy array is densely packed due to the homogenous nature of data stored in it. This helps retrieve the data faster as well, thereby making computations faster.
2. With NumPy, vectorized computations can replace the relatively more expensive python for loops.

We'll see the above two advantages of NumPy with the examples below.

Let us import the NumPy library to use its methods and functions.

```
import numpy as np
```

Example 1: This example shows that computations using NumPy arrays are typically much faster than computations with other data structures such as a list.

Q: Multiply whole numbers upto 1 million by an integer, say 2. Compare the time taken for the computation if the numbers are stored in a NumPy array vs a list.

Use the numpy function `arange()` to define a one-dimensional NumPy array.

```
import time as tm
start_time = tm.time()
list_ex = list(range(1000000)) #List containinig whole numbers upto 1 million
a=(list_ex*2)
print("Time take to multiply numbers in a list = ", tm.time()-start_time)

start_time = tm.time()
tuple_ex = tuple(range(1000000)) #List containinig whole numbers upto 1 million
a=(tuple_ex*2)
print("Time take to multiply numbers in a tuple = ", tm.time()-start_time)
```

```

start_time = tm.time()
numpy_ex = np.arange(1000000) #tuple containinig whole numbers upto 1 million
a=(numpy_ex*2)
print("Time take to multiply numbers in a NumPy array = ", tm.time()-start_time)

```

```

Time take to multiply numbers in a list = 0.04031014442443848
Time take to multiply numbers in a tuple = 0.03827619552612305
Time take to multiply numbers in a NumPy array = 0.0

```

4.1 Vectorized computation with NumPy

Several matrix algebra operations such as multiplications, decompositions, determinants, etc. can be performed conveniently with NumPy. However, we'll focus on matrix multiplication as it is very commonly used to avoid python for loops and make computations faster. The `dot` function is used to multiply matrices:

```

#Defining a 2x3 matrix
a = np.array([[0,1],[3,4]])
a

```

```

array([[0, 1],
       [3, 4]])

```

```

#Defining a 3x2 matrix
b = np.array([[6,-1],[2,1]])
b

```

```

array([[ 6, -1],
       [ 2,  1]])

```

```

#Multiplying matrices 'a' and 'b' using the dot function
a.dot(b)

```

```

array([[ 2,  1],
       [26,  1]])

```

```
#Note that * results in element-wise multiplication
a*b
```

```
array([[ 0, -1],
       [ 6,  4]])
```

Example 2: This example will show vectorized computations with NumPy. Vectorized computations help perform computations more efficiently, and also make the code concise.

Q: Read the (1) quantities of roll, bun, cake and bread required by 3 people - Ben, Barbara & Beth, from *food_quantity.csv*, (2) price of these food items in two shops - Target and Kroger, from *price.csv*. Find out which shop should each person go to minimize their expenses.

```
#Reading the datasets on food quantity and price
import pandas as pd
food_qty = pd.read_csv('./Datasets/food_quantity.csv')
price = pd.read_csv('./Datasets/price.csv')
```

```
food_qty
```

	Person	roll	bun	cake	bread
0	Ben	6	5	3	1
1	Barbara	3	6	2	2
2	Beth	3	4	3	1

```
price
```

	Item	Target	Kroger
0	roll	1.5	1.0
1	bun	2.0	2.5
2	cake	5.0	4.5
3	bread	16.0	17.0

First, let's start from a simple problem. We'll compute the expenses of Ben if he prefers to buy all food items from Target

```
#Method 1: Using loop
bens_target_expense = 0 #Initializing Ben's expenses to 0
for k in range(4):      #Iterating over all the four desired food items
    bens_target_expense += food_qty.iloc[0,k+1]*price.iloc[k,1] #Total expenses on the kth
bens_target_expense      #Total expenses for Ben if he goes to Target
```

50.0

```
#Method 2: Using NumPy array
food_num = food_qty.iloc[0,1:].to_numpy() #Converting food quantity (for Ben) dataframe to NumPy array
price_num = price.iloc[:,1].to_numpy()    #Converting price (for Target) dataframe to NumPy array
food_num.dot(price_num) #Matrix multiplication of the quantity vector with the price vector
```

50.0

Ben will spend \$50 if he goes to Target

Now, let's add another layer of complication. We'll compute Ben's expenses for both stores - Target and Kroger

```
#Method 1: Using loops

#Initializing a Series of length two to store the expenses in Target and Kroger for Ben
bens_store_expense = pd.Series(0.0,index=price.columns[1:3])
for j in range(2):      #Iterating over both the stores - Target and Kroger
    for k in range(4):   #Iterating over all the four desired food items
        bens_store_expense[j] += food_qty.iloc[0,k+1]*price.iloc[k,j+1]
bens_store_expense
```

```
Target    50.0
Kroger    49.0
dtype: float64
```

```
#Method 2: Using NumPy array
food_num = food_qty.iloc[0,1:].to_numpy() #Converting food quantity (for Ben) dataframe to NumPy array
price_num = price.iloc[:,1:].to_numpy()   #Converting price dataframe to NumPy array
food_num.dot(price_num) #Matrix multiplication of the quantity vector with the price vector
```

```
array([50.0, 49.0], dtype=object)
```

Ben will spend \ \$50 if he goes to Target, and \$49 if he goes to Kroger. Thus, he should choose Kroger.

Now, let's add the final layer of complication, and solve the problem. We'll compute everyone's expenses for both stores - Target and Kroger

```
#Method 1: Using loops
store_expense = pd.DataFrame(0.0,index=price.columns[1:3],columns = food_qty['Person'])
for i in range(3):    #Iterating over all the three people - Ben, Barbara, and Beth
    for j in range(2):    #Iterating over both the stores - Target and Kroger
        for k in range(4):    #Iterating over all the four desired food items
            store_expense.iloc[j,i] += food_qty.iloc[i,k+1]*price.iloc[k,j+1]
store_expense
```

Person	Ben	Barbara	Beth
Target	50.0	58.5	43.5
Kroger	49.0	61.0	43.5

```
#Method 2: Using NumPy array
food_num = food_qty.iloc[:,1:].to_numpy() #Converting food quantity dataframe to NumPy array
price_num = price.iloc[:,1:].to_numpy() #Converting price dataframe to NumPy array
food_num.dot(price_num) #Matrix multiplication of the quantity matrix with the price matrix
```

```
array([[50. , 49. ],
       [58.5, 61. ],
       [43.5, 43.5]])
```

Based on the above table, Ben should go to Kroger, Barbara to Target and Beth can go to either store.

Note that, with each layer of complication, the number of for loops keep increasing, thereby increasing the complexity of Method 1, while the method with NumPy array does not change much. Vectorized computations with arrays are much more efficient.

In-class exercise

Use matrix multiplication to find the average IMDB rating and average Rotten tomatoes rating for each genre - comedy, action, drama and horror. Use the data: *movies_cleaned.csv*. Which

is the most preferred genre for IMDB users, and which is the least preferred genre for Rotten Tomatoes users?

Hint: 1. Create two matrices - one containing the IMDB and Rotten Tomatoes ratings, and the other containing the genre flags (comedy/action/drama/horror). 2. Multiply the two matrices created in 1. 3. Divide each row/column of the resulting matrix by a vector having the number of ratings in each genre to get the average rating for the genre.

4.2 Pseudorandom number generation

Random numbers often need to be generated to analyze processes or systems, especially in cases when these processes or systems are governed by known probability distributions. For example, the number of personnel required to answer calls at a call center can be analyzed by simulating occurrence and duration of calls.

NumPy's `random` module can be used to generate arrays of random numbers from several different probability distributions. For example, a `3x5` array of uniformly distributed random numbers can be generated using the `uniform` function of the `random` module.

```
np.random.uniform(size = (3,5))
```

```
array([[0.69256322, 0.69259973, 0.03515058, 0.45186048, 0.43513769],  
       [0.07373366, 0.07465425, 0.92195975, 0.72915895, 0.8906299 ],  
       [0.15816734, 0.88144978, 0.05954028, 0.81403832, 0.97725557]])
```

Random numbers can also be generated by Python's built-in `random` module. However, it generates one random number at a time, which makes it much slower than NumPy's `random` module.

Example 3: Suppose 500 people eat at Mod Pizza, and another 500 eat at Viet nom nom, everyday.

The waiting time at Viet nom nom has a normal distribution with mean 8 minutes and standard deviation 3 minutes, while the waiting time at Mod Pizza has a uniform distribution with minimum 5 minutes and maximum 25 minutes.

Simulate a dataset containing waiting times for 500 ppl for 30 days in each of the food joints. Assume that the waiting time is measured simultaneously at a certain time in both places, i.e., the observations are paired.

On how many days is the average waiting time at Viet Nom Nom higher than that at Mod Pizza?

What percentage of times the waiting time at Viet nom nom was higher than the waiting time at Mod Pizza?

Try both approaches: (1) Using loops to generate data, (2) numpy array to generate data. Compare the time taken in both approaches.

```
import time as tm

#Method 1: Using loops
start_time = tm.time() #Current system time

#Initializing waiting times for 500 ppl over 30 days
waiting_times_MOD = pd.DataFrame(0,index=range(500),columns=range(30)) #Mod pizza
waiting_times_Vnom = pd.DataFrame(0,index=range(500),columns=range(30)) #Viet nom nom
import random as rm
for i in range(500): #Iterating over 500 ppl
    for j in range(30): #Iterating over 30 days
        waiting_times_Vnom.iloc[i,j] = rm.gauss(8,3) #Simulating waiting time in Viet nom
        waiting_times_MOD.iloc[i,j] = rm.uniform(5,25) #Simulating waiting time in Mod pizza
time_diff = waiting_times_Vnom-waiting_times_MOD

print("On ",sum(time_diff.mean(>0))," days, the average waiting time at Viet Nom Nom higher")
print("Percentage of times waiting time at Viet nom nom was greater than that at Mod Pizza")
end_time = tm.time() #Current system time
print("Time taken = ", end_time-start_time)
```

```
On 0 days, the average waiting time at Viet Nom Nom higher than that at Mod Pizza
Percentage of times waiting time at Viet nom nom was greater than that at Mod Pizza = 16.58
Time taken = 3.5454351902008057
```

```
#Method 2: Using NumPy arrays
start_time = tm.time()
waiting_time_Vnom = np.random.normal(8,3,size = (500,30)) #Simultaneously generating the w
waiting_time_MOD = np.random.uniform(5,25,size = (500,30)) #Simultaneously generating the
time_diff = waiting_time_Vnom-waiting_time_MOD
print("On ",(time_diff.mean(>0)).sum()," days, the average waiting time at Viet Nom Nom hi")
print("Percentage of times waiting time at Viet nom nom was greater than that at Mod Pizza")
end_time = tm.time()
print("Time taken = ", end_time-start_time)
```

```
On 0 days, the average waiting time at Viet Nom Nom higher than that at Mod Pizza
```



```
Percentage of times waiting time at Viet nom nom was greater than that at Mod Pizza = 16.48
Time taken = 0.001995563507080078
```

The approach with NumPy is much faster than the one with loops.

In-class exercise

Lab Question: Bootstrapping

Question) Find the 95% confidence interval of Profit for 'Action' movies, using Bootstrapping
Answer) Bootstrapping is a non-parametric method for obtaining confidence interval. The Bootstrapping method for finding the confidence interval is as follows.

- (a) Find the profit for each of the 'Action' movies. Suppose there are N such movies. We will have a *Profit* column with N values.
- (b) Randomly sample N values with replacement from the *Profit* column
- (c) Find the mean of the N values obtained in (b)
- (d) Repeat steps (b) and (c) $M=1000$ times
- (e) The 95% Confidence interval is the range between the 2.5% and 97.5% percentile values of the 1000 means obtained in (c)

Use the *movies_cleaned.csv* dataset.

Go ahead, code this up, and find the confidence interval!

5 Pandas

The Pandas library contains several methods and functions for cleaning, manipulating and analyzing data. While NumPy is suited for working with homogenous numerical array data, Pandas is designed for working with tabular or heterogenous data.

Let us import the Pandas library to use its methods and functions.

```
import pandas as pd
```

A DataFrame is a two-dimensional object - comprising of tabular data organized in rows and columns, where individual columns can be of different value types (numeric / string / boolean etc.). A DataFrame has row indices which refer to individual rows, and column names that refer to individual columns. By default, the row indices are integers starting from zero. However, both the row indices and column names can be customized by the user.

Let us read the spotify data - *spotify_data.csv*, using the Pandas function `read_csv()`.

```
spotify_data = pd.read_csv('./Datasets/spotify_data.csv')
spotify_data.head()
```

	artist_followers	genres	artist_name	artist_popularity	track_name	track_popularity
0	16996777	rap	Juice WRLD	96	All Girls Are The Same	0
1	16996777	rap	Juice WRLD	96	Lucid Dreams	0
2	16996777	rap	Juice WRLD	96	Hear Me Calling	0
3	16996777	rap	Juice WRLD	96	Robbery	0
4	5988689	rap	Roddy Ricch	88	Big Stepper	0

The object `spotify_data` is a pandas DataFrame:

```
type(spotify_data)
```

```
pandas.core.frame.DataFrame
```

A Series is a one-dimensional object, containing a sequence of values, where each value has an index. Each column of a DataFrame is Series as shown in the example below.

```
#Extracting movie titles from the movie_ratings DataFrame
spotify_songs = movie_ratings['track_name']
spotify_songs
```

```
0          Eno Ide
1      Ee Tanuvu Ninnade
2      Munjaane Manjalli
3      Gudugudiya Sedi Nodo
4          Ambar
...
245618      Coming Up Roses
245619      Young Kid
245620      Apricots
245621      Time I Love to Waste
245622      Call me
Name: track_name, Length: 245623, dtype: object
```

```
#The object movie_titles is a Series
type(spotify_songs)
```

pandas.core.series.Series

5.1 Data manipulations with Pandas

5.1.1 Sub-setting data

In the chapter on reading data, we learned about operators `loc` and `iloc` that can be used to subset data based on axis labels and position of rows/columns respectively. However, usually we are not aware of the relevant row indices, and we may want to subset data based on some condition(s). For example, suppose we wish to analyze only those songs whose track popularity is higher than 50.

Q: Do we need to subset rows or columns in this case?

A: Rows, as songs correspond to rows, while features of songs correspond to columns.

As we need to subset rows, the filter must be applied at the starting index. As we don't need to subset any specific features of the songs, there is no subsetting to be done on the columns. A : at the ending index means that all columns need to be selected.

```
popular_songs = spotify_data.loc[spotify_data.track_popularity>=50,:]
popular_songs.head()
```

	artist_followers	genres	artist_name	artist_popularity	track_name	track_popularity
181	1277325	hip hop	Dave	77	Titanium	69
191	1123869	rap	Jay Wheeler	85	Viendo el Techo	64
208	3657199	rap	Polo G	91	RAPSTAR	89
263	1461700	pop & rock	Teoman	67	Gecenin Sonuna Yolculuk	52
293	299746	pop & rock	Lars Winnerbäck	62	Själ och hjärta	55

Suppose we wish to analyze only *track_name*, *release_year* and *track_popularity* of songs. Then, we can subset the relevant columns:

```
relevant_columns = spotify_data.loc[:,['track_name','release_year','track_popularity']]
relevant_columns.head()
```

	track_name	release_year	track_popularity
0	All Girls Are The Same	2021	0
1	Lucid Dreams	2021	0
2	Hear Me Calling	2021	0
3	Robbery	2021	0
4	Big Stepper	2021	0

5.1.2 Sorting data

Sorting dataset is a very common operation. The `sort_values()` function of Pandas can be used to sort a Pandas DataFrame or Series. Let us sort the spotify data in decreasing order of *track_popularity*:

```
spotify_sorted = spotify_data.sort_values(by = 'track_popularity', ascending = False)
spotify_sorted.head()
```

	artist_followers	genres	artist_name	artist_popularity	track_name	track_popularity
2398	1444702	pop	Olivia Rodrigo	88	drivers license	99
2442	177401	hip hop	Masked Wolf	85	Astronaut In The Ocean	98
3133	1698014	pop	Kali Uchis	88	telepatía	97
6702	31308207	pop	The Weeknd	96	Save Your Tears	97

	artist_followers	genres	artist_name	artist_popularity	track_name	track_popularity
6703	31308207	pop	The Weeknd	96	Blinding Lights	96

Drivers license is the most popular song!

<IPython.core.display.HTML object>

5.1.3 Unique values, value counts and membership

The Pandas function `unique` provides the unique values of a Series. For example, let us find the number of unique genres of songs in the spotify dataset:

```
spotify_data.genres.unique()
```

```
array(['rap', 'pop', 'miscellaneous', 'metal', 'hip hop', 'rock',
      'pop & rock', 'hoerspiel', 'folk', 'electronic', 'jazz', 'country',
      'latin'], dtype=object)
```

The Pandas function `value_counts()` provides the number of observations of each value of a Series. For example, let us find the number of songs of each genre in the spotify dataset:

```
spotify_data.genres.value_counts()
```

```
pop          70441
rock         49785
pop & rock   43437
miscellaneous 35848
jazz        13363
hoerspiel    12514
hip hop      7373
folk         2821
latin        2125
rap          1798
metal        1659
country      1236
electronic    790
Name: genres, dtype: int64
```

More than half the songs in the dataset are *pop*, *rock* or *pop & rock*.

The Pandas function `isin()` provides a boolean Series indicating the position of certain values in a Series. The function is helpful in sub-setting data. For example, let us subset the songs that are either *latin*, *rap*, or *metal*:

```
latin_rap_metal_songs = spotify_data.loc[spotify_data.genres.isin(['latin','rap','metal'])]
latin_rap_metal_songs.head()
```

	artist_followers	genres	artist_name	artist_popularity	track_name	track_popularity
0	16996777	rap	Juice WRLD	96	All Girls Are The Same	0
1	16996777	rap	Juice WRLD	96	Lucid Dreams	0
2	16996777	rap	Juice WRLD	96	Hear Me Calling	0
3	16996777	rap	Juice WRLD	96	Robbery	0
4	5988689	rap	Roddy Ricch	88	Big Stepper	0

5.2 Operations between DataFrame and Series

Let us learn arithmetic operations between DataFrame and Series with the help of an example.

Example: Spotify recommends songs based on songs listened by the user. Suppose you have listened to the song *drivers license*. Spotify intends to recommend you 5 songs that are *similar* to *drivers license*. Which songs should it recommend?

Let us see what information do we have about songs that can help us identify songs similar to *drivers license*. The `columns` attribute of DataFrame will display all the columns names. The description of some of the column names relating to audio features is [here](#).

```
spotify_data.columns
```

```
Index(['artist_followers', 'genres', 'artist_name', 'artist_popularity',
      'track_name', 'track_popularity', 'duration_ms', 'explicit',
      'release_year', 'danceability', 'energy', 'key', 'loudness', 'mode',
      'speechiness', 'acousticness', 'instrumentalness', 'liveness',
      'valence', 'tempo', 'time_signature'],
      dtype='object')
```

Solution approach: We have several features of a song. Let us find songs similar to *drivers license* in terms of *danceability*, *energy*, *key*, *loudness*, *mode*, *speechiness*, *acousticness*, *instrumentalness*, *liveness*, *valence*, *time_signature* and *tempo*. Note that we are considering only audio features for simplicity.

To find the songs most similar to *drivers license*, we need to define a measure that quantifies the similarity. Let us define similarity of a song with *drivers license* as the Euclidean distance of the song from *drivers license*, where the coordinates of a song are: (danceability, energy, key, loudness, mode, speechiness, acousticness, instrumentalness, liveness, valence, time_signature, tempo). Thus, similarity can be formulated as:

$$Similarity_{DL-S} = \sqrt{(danceability_{DL} - danceability_S)^2 + (energy_{DL} - energy_S)^2 + \dots + (tempo_{DL} - tempo_S)^2}$$

where the subscript *DL* stands for *drivers license* and *S* stands for any song. The top 5 songs with the least value of $Similarity_{DL-S}$ will be the most similar to *drivers license* and should be recommended.

Let us subset the columns that we need to use to compute the Euclidean distance.

```
audio_features = spotify_data[['danceability', 'energy', 'key', 'loudness', 'mode', 'speechiness', 'acousticness', 'instrumentalness', 'liveness', 'valence', 'tempo']]
```

```
audio_features.head()
```

	danceability	energy	key	loudness	mode	speechiness	acousticness	instrumentalness	liveness	valence	tempo
0	0.673	0.529	0	-7.226	1	0.3060	0.0769	0.000338	0.0856	0.4215	120.008
1	0.511	0.566	6	-7.230	0	0.2000	0.3490	0.000000	0.3400	0.2793	120.008
2	0.699	0.687	7	-3.997	0	0.1060	0.3080	0.000036	0.1210	0.4046	120.008
3	0.708	0.690	2	-5.181	1	0.0442	0.3480	0.000000	0.2220	0.3919	120.008
4	0.753	0.597	8	-8.469	1	0.2920	0.0477	0.000000	0.1970	0.3801	120.008

```
#Distribution of values of audio_features
audio_features.describe()
```

	danceability	energy	key	loudness	mode	speechiness	acousticness	instrumentalness	liveness	valence	tempo
count	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000
mean	0.568357	0.580633	5.240326	-9.432548	0.670928	0.111984	0.198068	0.000000	0.198068	0.380100	120.008
std	0.159444	0.236631	3.532546	4.449731	0.469877	0.198068	0.198068	0.000000	0.198068	0.380100	120.008

	danceability	energy	key	loudness	mode	speechiness	acousticness
min	0.000000	0.000000	0.000000	-60.000000	0.000000	0.000000	0.000000
25%	0.462000	0.405000	2.000000	-11.990000	0.000000	0.033200	0.000000
50%	0.579000	0.591000	5.000000	-8.645000	1.000000	0.043100	0.000000
75%	0.685000	0.776000	8.000000	-6.131000	1.000000	0.075300	0.000000
max	0.988000	1.000000	11.000000	3.744000	1.000000	0.969000	0.000000

Note that the audio features differ in terms of scale. Some features like *key* have a wide range of $[0,11]$, while others like *danceability* have a very narrow range of $[0,0.988]$. If we use them directly, features like *danceability* will have a much higher influence on $Similarity_{DL-S}$ as compared to features like *key*. Assuming we wish all the features to have equal weight in quantifying a song's similarity to *drivers license*, we should scale the features, so that their values are comparable.

Let us scale the value of each column to a standard uniform distribution: $U[0,1]$.

For scaling the values of a column to $U[0,1]$, we need to subtract the minimum value of the column from each value, and divide by the range of values of the column. For example, *danceability* can be standardized as follows:

```
#Scaling danceability to U[0,1]
danceability_value_range = audio_features.danceability.max()-audio_features.danceability.min()
danceability_std = (audio_features.danceability-audio_features.danceability.min())/danceability_value_range
danceability_std
```

```
0      0.681174
1      0.517206
2      0.707490
3      0.716599
4      0.762146
```

```
...
243185  0.621457
243186  0.797571
243187  0.533401
243188  0.565789
243189  0.750000
```

```
Name: danceability, Length: 243190, dtype: float64
```

However, it will be cumbersome to repeat the above code for each audio feature. We can instead write a function that scales values of a column to $U[0,1]$, and apply the function on all the audio features.


```
#Function to scale a column to U[0,1]
def scale_uniform(x):
    return (x-x.min())/(x.max()-x.min())
```

We will use the Pandas function `apply()` to apply the above function to the DataFrame `audio_features`.

```
#Scaling all audio features to U[0,1]
audio_features_scaled = audio_features.apply(scale_uniform)
```

lambda function: Note that one line functions can be conveniently written as lambda functions in Python. These functions do not require a name, and can be defined using the keyword `lambda`. The above two blocks of code can be concisely written as:

```
audio_features_scaled = audio_features.apply(lambda x: (x-x.min())/(x.max()-x.min()))

#All the audio features are scaled to U[0,1]
audio_features_scaled.describe()
```

	danceability	energy	key	loudness	mode	speechiness	acousticness
count	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000	243190.000000
mean	0.575260	0.580633	0.476393	0.793290	0.670928	0.115566	0.006622
std	0.161380	0.236631	0.321141	0.069806	0.469877	0.204405	0.013573
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	0.467611	0.405000	0.181818	0.753169	0.000000	0.034262	0.000000
50%	0.586032	0.591000	0.454545	0.805644	1.000000	0.044479	0.000000
75%	0.693320	0.776000	0.727273	0.845083	1.000000	0.077709	0.000000
max	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Since we need to find the Euclidean distance from the song *drivers license*, let us find the index of the row containing features of **drivers license*.

```
drivers_license_index = spotify_data[spotify_data.track_name=='drivers license'].index[0]
```

Now, we'll subtract the audio features of *drivers license* from all other songs:

```
songs_minus_DL = audio_features_scaled-audio_features_scaled.loc[drivers_license_index,:]
```

Now, let us square the difference computed above. We'll use the in-built python function `pow()` to square the difference:

```
songs_minus_DL_sq = songs_minus_DL.pow(2)
songs_minus_DL_sq.head()
```

	danceability	energy	key	loudness	mode	speechiness	acousticness	instrumentalness	liveness
0	0.007933	0.008649	0.826446	0.000580	0.0	0.064398	0.418204	1.055600e-07	0.006751
1	0.005610	0.016900	0.132231	0.000577	1.0	0.020844	0.139498	1.716100e-10	0.053531
2	0.013314	0.063001	0.074380	0.005586	1.0	0.002244	0.171942	5.382400e-10	0.008751
3	0.015499	0.064516	0.528926	0.003154	0.0	0.000269	0.140249	1.716100e-10	0.013607
4	0.028914	0.025921	0.033058	0.000021	0.0	0.057274	0.456981	1.716100e-10	0.008751

Now, we'll sum the squares of differences from all audio features to compute the similarity of all songs to *drivers license*.

```
distance_squared = songs_minus_DL_sq.sum(axis = 1)
distance_squared.head()
```

```
0    1.337163
1    1.438935
2    1.516317
3    1.004043
4    0.920316
dtype: float64
```

Now, we'll sort these distances to find the top 5 songs closest to drivers's license.

```
distances_sorted = distance_squared.sort_values()
distances_sorted.head()
```

```
2398    0.000000
81844    0.008633
4397    0.011160
130789    0.015018
143744    0.015058
dtype: float64
```

Using the indices of the top 5 distances, we will identify the top 5 songs most similar to *drivers license*:

```
spotify_data.loc[distances_sorted.index[0:6],:]
```

	artist_followers	genres	artist_name	artist_popularity	track_name
2398	1444702	pop	Olivia Rodrigo	88	drivers license
81844	2264501	pop	Jay Chou	74	
4397	25457	pop	Terence Lam	60	in Bb major
130789	176266	pop	Alan Tam	54	
143744	396326	pop & rock	Laura Branigan	64	How Am I Supposed to Live With You
35627	1600562	pop	Tiziano Ferro	68	Non Me Lo So Spiegare

We can see the top 5 songs most similar to *drivers license* in the *track_name* column above. Interestingly, three of the five songs are Asian! These songs indeed sound similar to *drivers license*!

5.3 Correlation

Correlation may refer to any kind of association between two random variables. However, in this book, we will always consider correlation as the linear association between two random variables, or the Pearson's correlation coefficient. Note that correlation does not imply causality and vice-versa.

The Pandas function `corr()` provides the pairwise correlation between all columns of a DataFrame, or between two Series. The function `corrwith()` provides the pairwise correlation of a DataFrame with another DataFrame or Series.

```
#Pairwise correlation amongst all columns
spotify_data.corr()
```

	artist_followers	artist_popularity	track_popularity	duration_ms	explicit	release_year
artist_followers	1.000000	0.577861	0.197426	0.040435	0.082857	0.098589
artist_popularity	0.577861	1.000000	0.285565	-0.097996	0.092147	0.062007
track_popularity	0.197426	0.285565	1.000000	0.060474	0.193685	0.568329
duration_ms	0.040435	-0.097996	0.060474	1.000000	-0.024226	0.067665
explicit	0.082857	0.092147	0.193685	-0.024226	1.000000	0.215656
release_year	0.098589	0.062007	0.568329	0.067665	0.215656	1.000000
danceability	-0.010120	0.038784	0.158507	-0.145779	0.138522	0.204119
energy	0.080085	0.039583	0.217342	0.075990	0.104734	0.338008
key	-0.000119	-0.011005	0.013369	0.007710	0.011818	0.021434

	artist_followers	artist_popularity	track_popularity	duration_ms	explicit	release_date
loudness	0.123771	0.045165	0.296350	0.078586	0.124410	0.430000
mode	0.004313	0.018758	-0.022486	-0.034818	-0.060350	-0.071000
speechiness	-0.059933	0.236942	-0.056537	-0.332585	0.077268	-0.032000
acousticness	-0.107475	-0.075715	-0.284433	-0.133960	-0.129363	-0.369000
instrumentalness	-0.033986	-0.066679	-0.124283	0.067055	-0.039472	-0.149000
liveness	0.002425	0.099678	-0.090479	-0.034631	-0.024283	-0.045000
valence	-0.053317	-0.034501	-0.038859	-0.155354	-0.032549	-0.070000
tempo	0.016524	-0.032036	0.058408	0.051046	0.006585	0.079000
time_signature	0.030826	-0.033423	0.071741	0.085015	0.043538	0.089000

Q: Which audio feature is the most correlated with *track_popularity*?

```
spotify_data.corrwith(spotify_data.track_popularity).sort_values(ascending = False)
```

```
track_popularity    1.000000
release_year        0.568329
loudness             0.296350
artist_popularity    0.285565
energy              0.217342
artist_followers     0.197426
explicit             0.193685
danceability         0.158507
time_signature       0.071741
duration_ms          0.060474
tempo                0.058408
key                  0.013369
mode                 -0.022486
valence              -0.038859
speechiness          -0.056537
liveness             -0.090479
instrumentalness     -0.124283
acousticness         -0.284433
dtype: float64
```

Loudness is the audio feature having the highest correlation with *track_popularity*.

6 Data visualization

It is generally easier for humans to comprehend information with plots, diagrams and pictures, rather than with text and numbers. This makes data visualizations a vital part of data science. Some of the key purposes of data visualization are:

1. Data visualization is the first step towards exploratory data analysis (EDA), which reveals trends, patterns, insights, or even irregularities in data.
2. Data visualization can help explain the workings of complex mathematical models.
3. Data visualization are an elegant way to summarise the findings of a data analysis project.
4. Data visualizations (especially interactive ones such as those on Tableau) may be the end-product of data analytics project, where the stakeholders make decisions based on the visualizations.

We'll use a couple of libraries for making data visualizations - [matplotlib](#) and [seaborn](#). Matplotlib is mostly used for creating relatively simple two-dimensional plots. Its plotting interface that is similar to the `plot()` function in MATLAB, so those who have used MATLAB should find it familiar. Seaborn is a recently developed data visualization library based on matplotlib. It is more oriented towards visualizing data with Pandas DataFrame and NumPy arrays. While matplotlib may also be used to create complex plots, seaborn has some built-in themes that may make it more convenient to make complex plots. Seaborn also has color schemes and plot styles that improve the readability and aesthetics of matplotlib plots. However, preferences depend on the user and their coding style, and it is perfectly fine to use either library for making the same visualization.

Let's visualize the life expectancy of different countries with GDP per capita. We'll read the data file *gdp_lifeExpectancy.csv*, which contains the GDP per capita and life expectancy of countries from 1952 to 2007.

```
import pandas as pd
import numpy as np

gdp_data = pd.read_csv('./Datasets/gdp_lifeExpectancy.csv')
gdp_data.head()
```

	country	continent	year	lifeExp	pop	gdpPercap
0	Afghanistan	Asia	1952	28.801	8425333	779.445314
1	Afghanistan	Asia	1957	30.332	9240934	820.853030
2	Afghanistan	Asia	1962	31.997	10267083	853.100710
3	Afghanistan	Asia	1967	34.020	11537966	836.197138
4	Afghanistan	Asia	1972	36.088	13079460	739.981106

6.0.1 Scatterplots and trendline

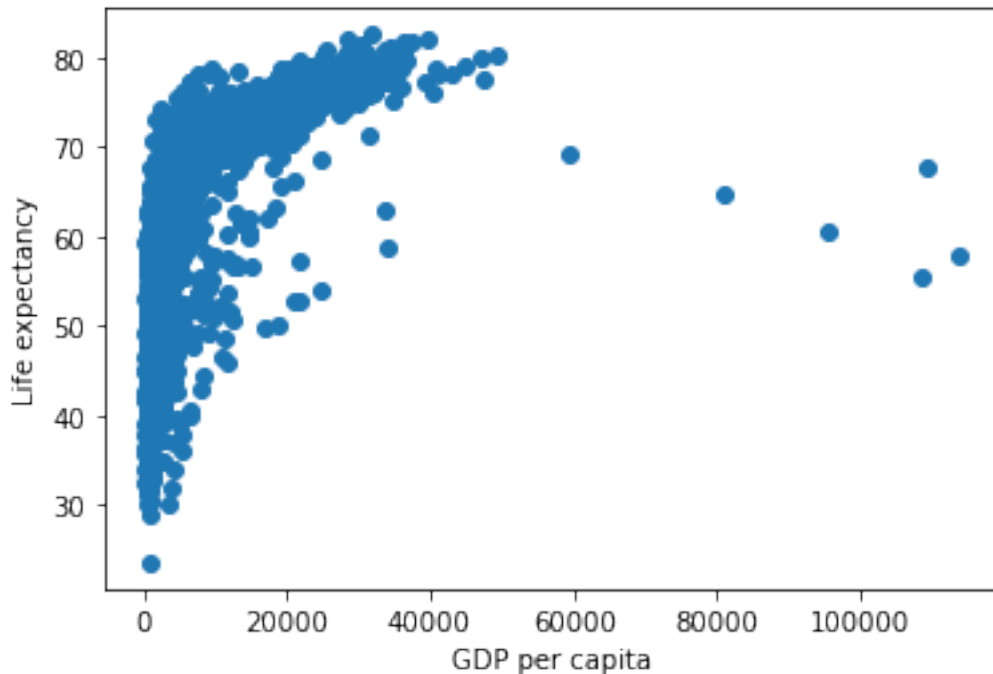
Now, we'll import the `pyplot` module of `matplotlib` to make plots. We'll use the `plot()` function to make the scatter plot, and the functions `xlabel()` and `ylabel()` for labelling the plot axes.

```
import matplotlib.pyplot as plt
```

Q: Make a scatterplot of Life expectancy vs GDP per capita.

```
#Making a scatterplot of Life expectancy vs GDP per capita
x = gdp_data.gdpPercap
y = gdp_data.lifeExp
plt.plot(x,y,'o') #By default, the plot() function makes a lineplot. The 'o' arguments spe
plt.xlabel('GDP per capita') #Labelling the horizontal X-axis
plt.ylabel('Life expectancy') #Labelling the verical Y-axis
```

```
Text(0, 0.5, 'Life expectancy')
```



From the above plot, we observe that life expectancy seems to be positively correlated with the GDP per capita of the country, as one may expect. However, there are a few outliers in the data - which are countries having extremely high GDP per capita, but not a correspondingly high life expectancy.

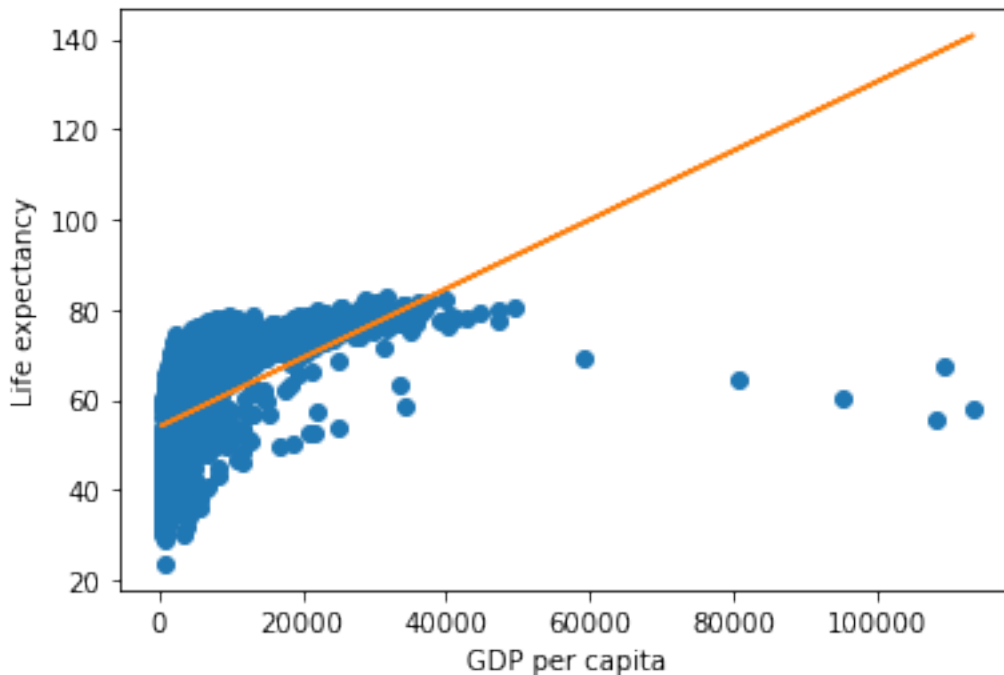
Sometimes it is difficult to get an idea of the overall trend (positive or negative correlation). In such cases, it may help to add a trendline to the scatter plot. In the plot below we add a trendline over the scatterplot showing that the life expectancy on an average increases with increasing GDP per capita. The trendline is actually a linear regression of life expectancy on GDP per capita. However, we'll not discuss linear regression in this book.

Q: Add a trendline over the scatterplot of life expectancy vs GDP per capita.

```
#Making a scatterplot of Life expectancy vs GDP per capita
x = gdp_data.gdpPerCap
y = gdp_data.lifeExp
plt.plot(x,y,'o') #By default, the plot() function makes a lineplot. The 'o' arguments spe
plt.xlabel('GDP per capita') #Labelling the horizontal X-axis
plt.ylabel('Life expectancy') #Labelling the verical Y-axis

#Plotting a trendline (linear regression) on the scatterplot
slope_intercept_trendline = np.polyfit(x,y,1) #Finding the slope and intercept for the t
```

```
compute_y_given_x = np.poly1d(slope_intercept_trendline) #Defining a function that compute
plt.plot(x,compute_y_given_x(x)) #Plotting the trendline
```



The above plot shows that our earlier intuition of a positive correlation between Life expectancy and GDP per capita was correct.

We used the NumPy function `polyfit()` to compute the slope and intercept of the trendline. Then, we defined an object `compute_y_given_x` of `poly1d` class and used it to compute the trendline.

6.0.2 Subplots

There is often a need to make a few plots together to compare them. See the example below.

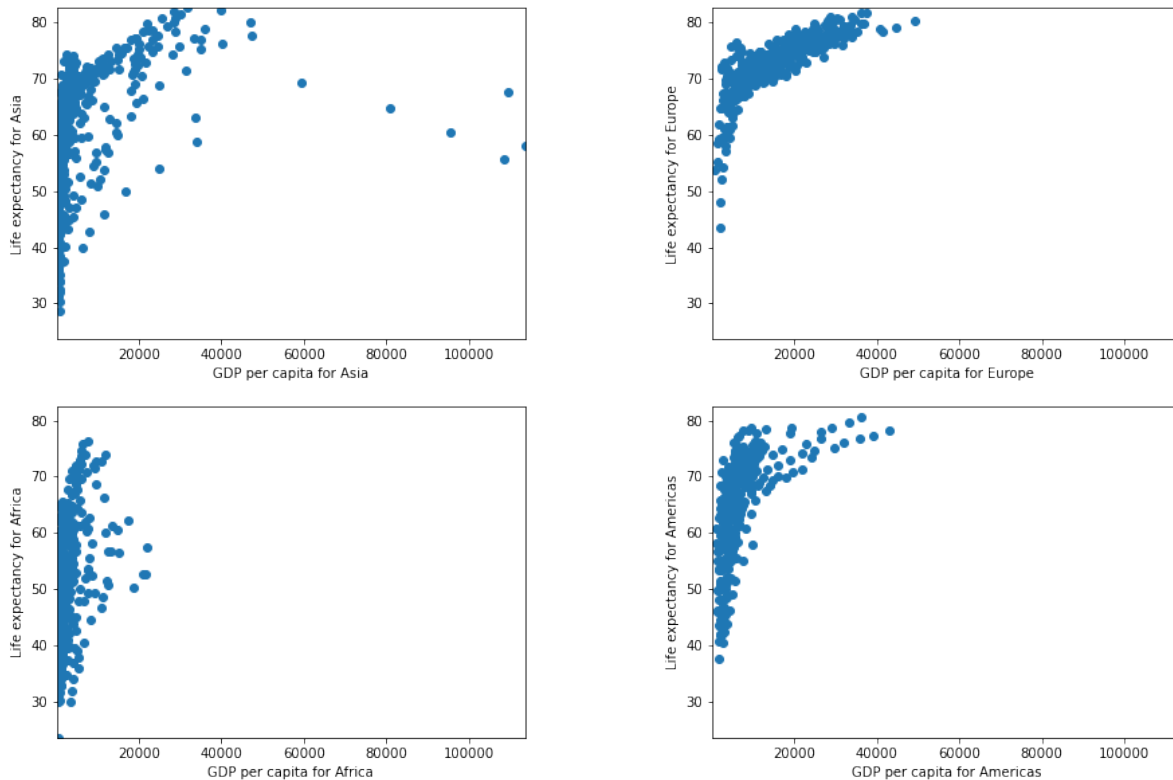
Q: Make scatterplots of life expectancy vs GDP per capita separately for each of the 4 continents of Asia, Europe, Africa and America. Arrange the plots in a 2 x 2 grid.

```
#Defining a 2x2 grid of subplots
fig, axes = plt.subplots(2,2,figsize=(15,10))
plt.subplots_adjust(wspace=0.4) #adjusting white space between individual plots
```



```
#Making a scatterplot of Life expectancy vs GDP per capita for each continent
continents = np.array(['Asia', 'Europe'], ['Africa', 'Americas'])

#Looping over the 2x2 grid
for i in range(2):
    for j in range(2):
        x = gdp_data.loc[gdp_data.continent==continents[i,j],:].gdpPercap
        y = gdp_data.loc[gdp_data.continent==continents[i,j],:].lifeExp
        axes[i,j].plot(x,y,'o')
        axes[i,j].set_xlim([gdp_data.gdpPercap.min(), gdp_data.gdpPercap.max()])
        axes[i,j].set_ylim([gdp_data.lifeExp.min(), gdp_data.lifeExp.max()])
        axes[i,j].set_xlabel('GDP per capita for ' + continents[i,j])
        axes[i,j].set_ylabel('Life expectancy for ' + continents[i,j])
```



We observe that for each continent, except Africa, initially life expectancy increases rapidly with increasing GDP per capita. However, after a certain threshold of GDP per capita, life expectancy increases slowly. Several countries in Europe enjoy a relatively high GDP per capita as well as high life expectancy. Some countries in Asia have an extremely high GDP

per capita, but a relatively low life expectancy. It will be interesting to see the proportion of GDP associated with healthcare for these outlying Asian countries, and European countries.

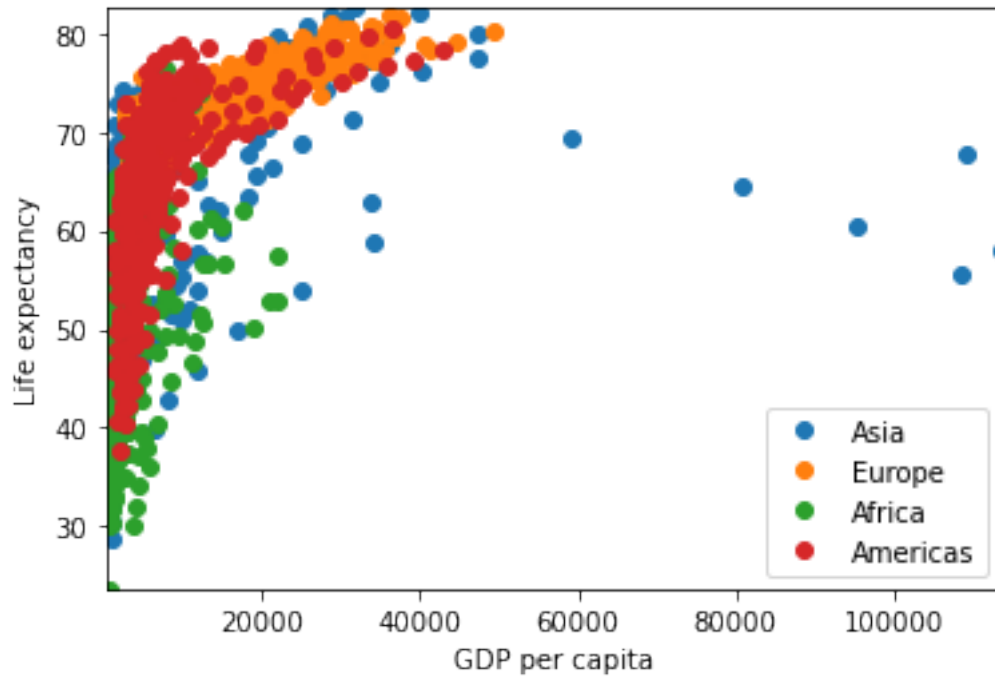
We used the `subplot` function of matplotlib to define the 2x2 grid of subplots. The function `subplots_adjust()` can be used to adjust white spaces around the plot. We used a `for` loop to iterate over each subplot. The `axes` object returned by the `subplot()` function was used to refer to individual subplots.

6.0.3 Overlapping plots with legend

We can also have the scatterplot of all the continents on the sample plot, with a distinct color for each continent. A legend will be required to identify the continent's color.

```
continents = np.array(['Asia', 'Europe'], ['Africa', 'Americas'])
for i in range(2):
    for j in range(2):
        x = gdp_data.loc[gdp_data.continent==continents[i,j],:].gdpPercap
        y = gdp_data.loc[gdp_data.continent==continents[i,j],:].lifeExp
        plt.plot(x,y,'o',label = continents[i,j])
        plt.xlim([gdp_data.gdpPercap.min(), gdp_data.gdpPercap.max()])
        plt.ylim([gdp_data.lifeExp.min(), gdp_data.lifeExp.max()])
        plt.xlabel('GDP per capita')
        plt.ylabel('Life expectancy')
plt.legend()
```

<matplotlib.legend.Legend at 0x1d09bf00040>



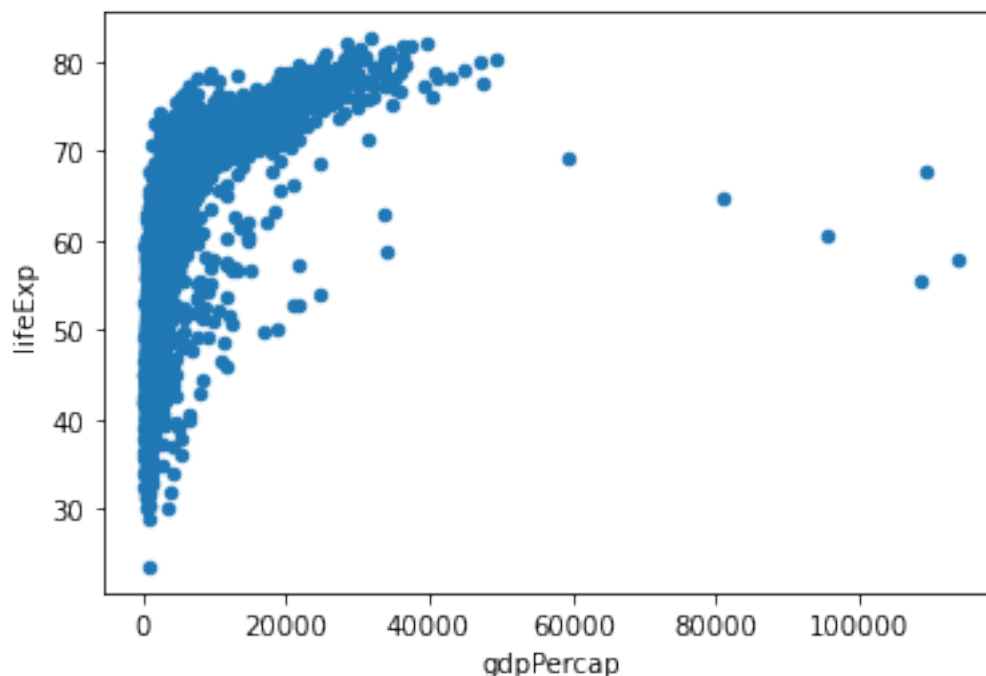
6.1 Pandas

Matplotlib is low-level tool, in which different components of the plot, such as points, legend, axis titles, etc. need to be specified separately. The Pandas `plot()` function can be used directly with a DataFrame or Series to make plots.

6.1.1 Scatterplots and lineplots

```
#Plotting life expectancy vs GDP per capita using the Pandas plot() function
gdp_data.plot(x = 'gdpPercap', y = 'lifeExp', kind = 'scatter')
```

```
<AxesSubplot:xlabel='gdpPercap', ylabel='lifeExp'>
```



Note that with matplotlib, it will take 3 lines to make the same plot - one for the scatterplot, and two for the axis titles.

Let us re-arrange the data to show other benefits of the Pandas `plot()` function. Note that data reshaping is explained in Chapter 8 of the book, so you may ignore the code block below that uses the `pivot_table()` function.

```
#You may ignore this code block until Chapter 8.
mean_gdp_per_capita = gdp_data.pivot_table(index = 'year', columns = 'continent', values =
mean_gdp_per_capita.head()
```

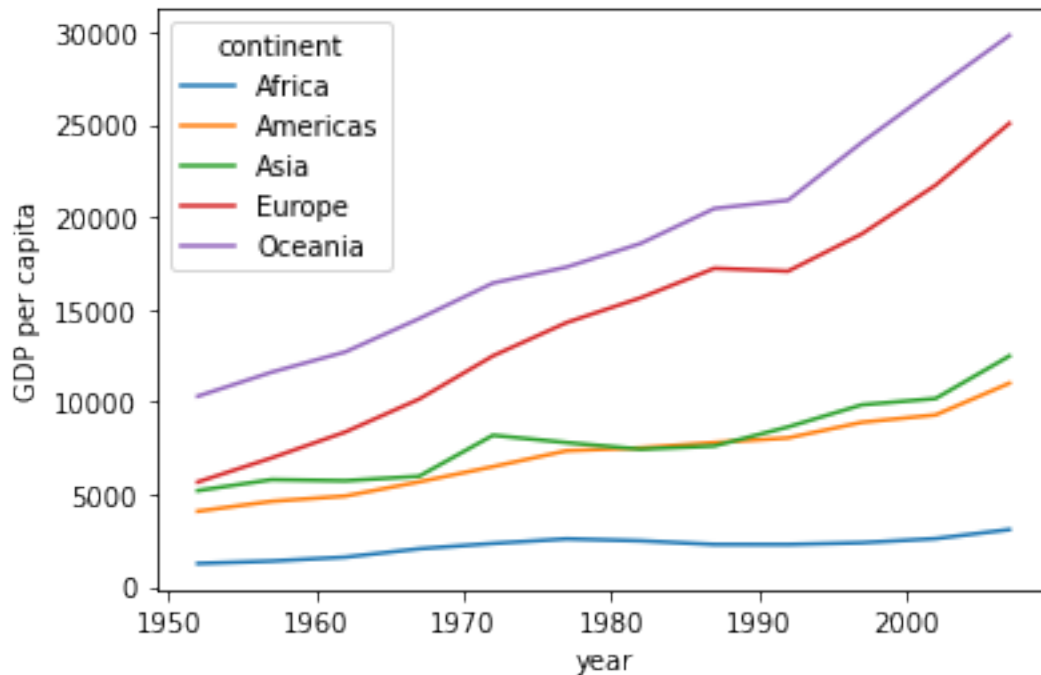
continent	Africa	Americas	Asia	Europe	Oceania
year					
1952	1252.572466	4079.062552	5195.484004	5661.057435	10298.085650
1957	1385.236062	4616.043733	5787.732940	6963.012816	11598.522455
1962	1598.078825	4901.541870	5729.369625	8365.486814	12696.452430
1967	2050.363801	5668.253496	5971.173374	10143.823757	14495.021790
1972	2339.615674	6491.334139	8187.468699	12479.575246	16417.333380

We have reshaped the data to obtain the mean GDP per capita of each continent for each year.

The pandas `plot()` function can be directly used with this DataFrame to create line plots showing mean GDP per capita of each continent with year.

```
mean_gdp_per_capita.plot(ylabel = 'GDP per capita')
```

```
<AxesSubplot:xlabel='year', ylabel='GDP per capita'>
```



We observe that the mean GDP per capita of Europe and Oceania have increased rapidly, while that for Africa is increasing very slowly.

The above plot will take several lines of code if developed using only matplotlib. The pandas `plot()` function has a framework to conveniently make commonly used plots.

6.1.2 Bar plots

Bar plots can be made using the pandas `bar` function with the DataFrame or Series, just like the line plots and scatterplots.

Below, we are reading the dataset of noise complaints of type *Loud music/Party* received the police in New York City in 2016.

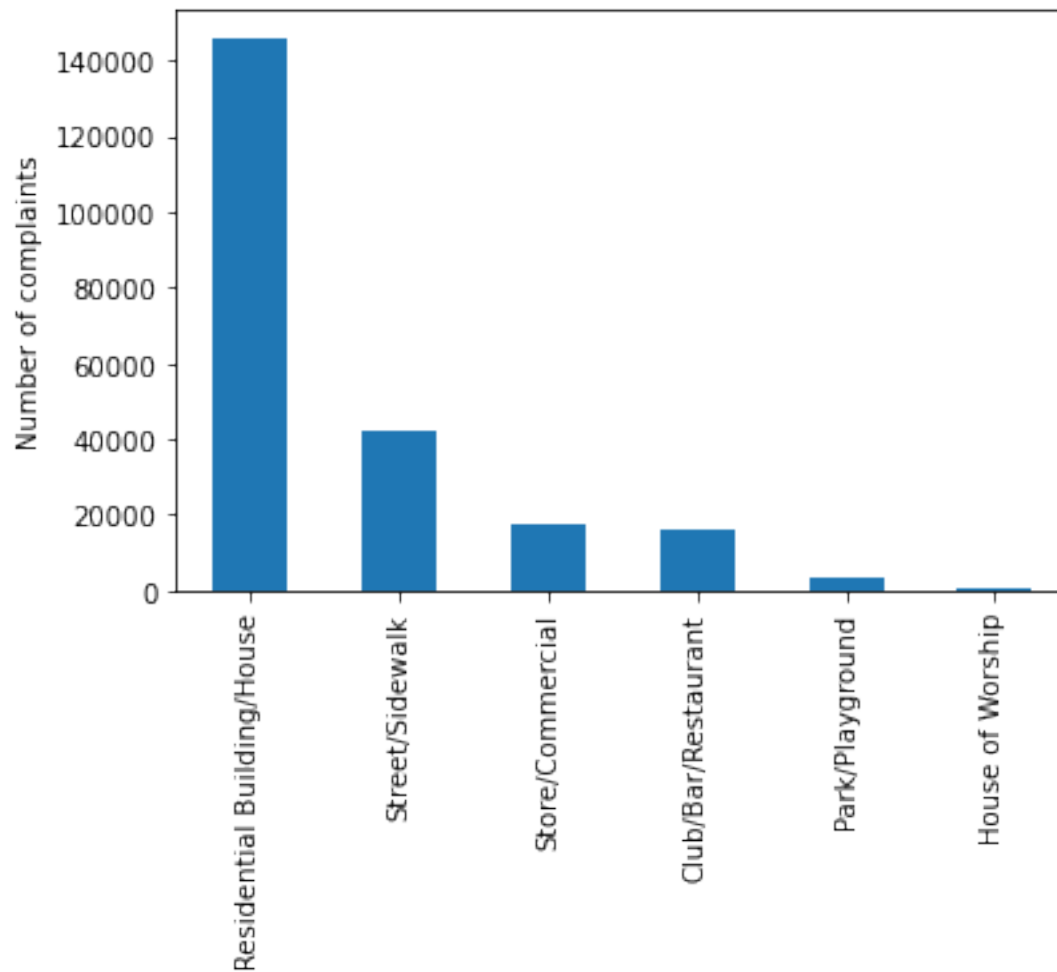
```
nyc_party_complaints = pd.read_csv('./Datasets/party_nyc.csv')
nyc_party_complaints.head()
```

	Created Date	Closed Date	Location Type	Incident Zip	City	Borough
0	12/31/2015 0:01	12/31/2015 3:48	Store/Commercial	10034.0	NEW YORK	MANHAT
1	12/31/2015 0:02	12/31/2015 4:36	Store/Commercial	10040.0	NEW YORK	MANHAT
2	12/31/2015 0:03	12/31/2015 0:40	Residential Building/House	10026.0	NEW YORK	MANHAT
3	12/31/2015 0:03	12/31/2015 1:53	Residential Building/House	11231.0	BROOKLYN	BROOKLYN
4	12/31/2015 0:05	12/31/2015 3:49	Residential Building/House	10033.0	NEW YORK	MANHAT

Let us visualise the locations from where the the complaints are coming.

```
#Using the pandas function bar() to create bar plot
nyc_party_complaints['Location Type'].value_counts().plot.bar(ylabel = 'Number of complain
```

```
<AxesSubplot:ylabel='Number of complaints'>
```

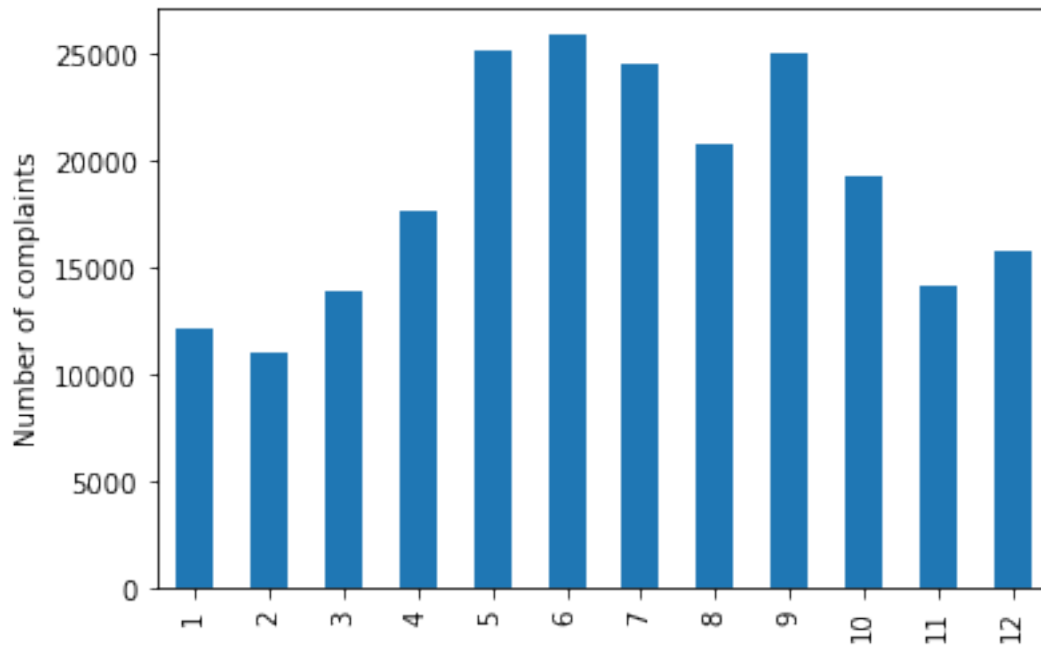


From the above plot, we observe that most of the complaints come from residential buildings and houses, as one may expect.

Let us visualize the time of the year when most complaints occur.

```
#Using the pandas function bar() to create bar plot
nyc_party_complaints['Month_of_the_year'].value_counts().sort_index().plot.bar(ylabel = 'N
```

```
<AxesSubplot:ylabel='Number of complaints'>
```



Try executing the code without `sort_index()` to figure out the purpose of using the function.

From the above plot, we observe that most of the complaints occur during summer and early Fall.

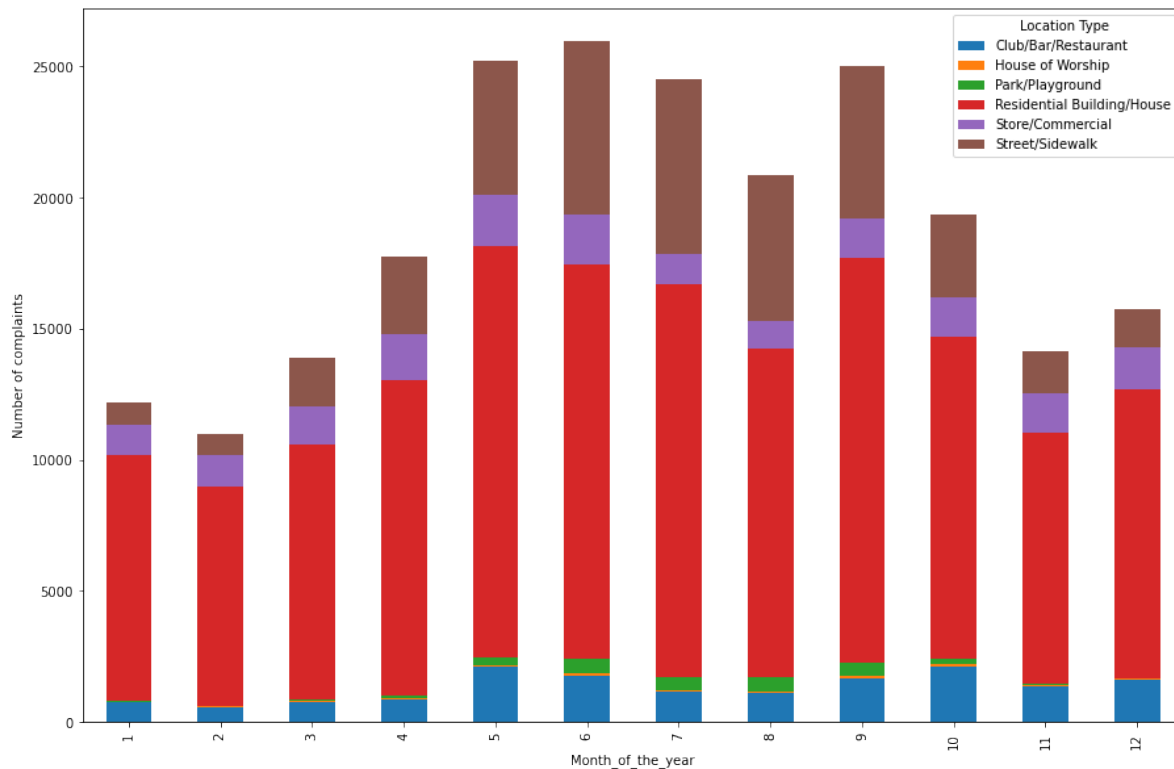
Let us create a stacked bar chart that combines both the above plots into a single plot. You may ignore the code used for re-shaping the data until Chapter 8. The purpose here is to show the utility of the pandas `bar()` function.

```
#Reshaping the data to make it suitable for a stacked barplot - ignore this code until chapter 8
complaints_location=pd.crosstab(nyc_party_complaints.Month_of_the_year, nyc_party_complaints.Location_Type)
complaints_location.head()
```

Location Type	Club/Bar/Restaurant	House of Worship	Park/Playground	Residential Building/House
Month_of_the_year				
1	748	24	17	9393
2	570	29	16	8383
3	747	39	90	9689
4	848	53	129	11984
5	2091	72	322	15676


```
#Stacked bar plot showing number of complaints at different months of the year, and from d
complaints_location.plot.bar(stacked=True,ylabel = 'Number of complaints',figsize=(15, 10))
```

```
<AxesSubplot:xlabel='Month_of_the_year', ylabel='Number of complaints'>
```



The above plots gives the insights about location and day of the year simultaneously that were previously separately obtained by the individual plots.

6.2 Seaborn

Seaborn offers the flexibility of simultaneously visualizing multiple variables in a single plot, and offers several themes to develop plots.

```
#Importing the seaborn library
import seaborn as sns
```

6.2.1 Bar plots with confidence intervals

We'll group the data to obtain the total complaints for each *Location Type*, *Borough*, *Month_of_the_year*, and *Hour_of_the_day*. Note that you'll learn grouping data in Chapter 9, so you may ignore the next code block. The grouping is done to shape the data in a suitable form for visualization.

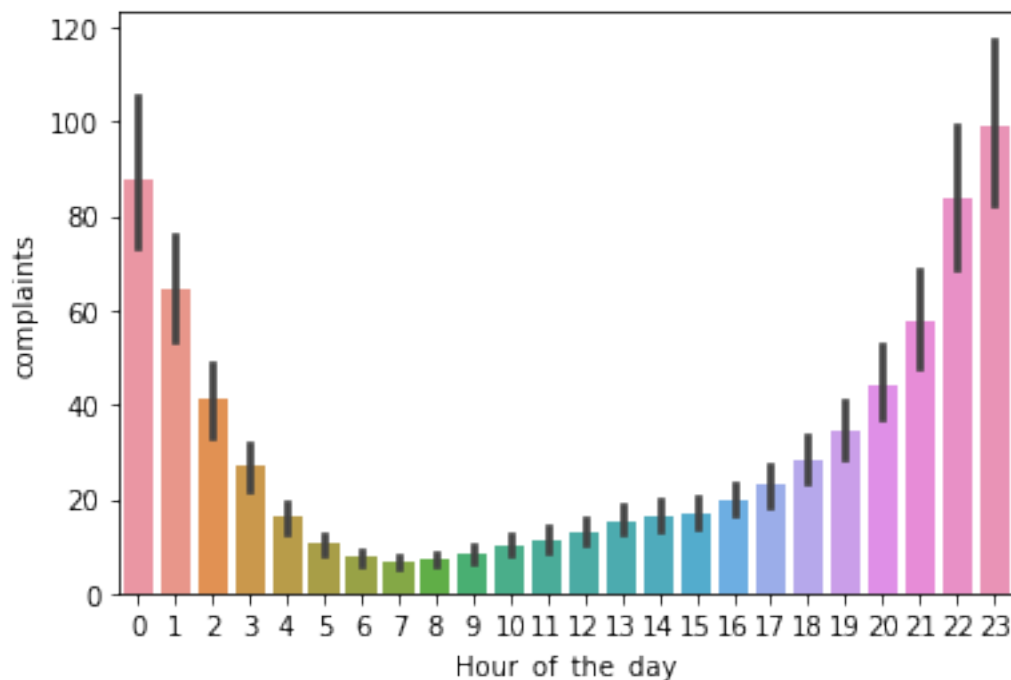
```
#Grouping the data to make it suitable for visualization using Seaborn. Ignore this code b
nyc_complaints_grouped = nyc_party_complaints[['Location Type', 'Borough', 'Month_of_the_yea
nyc_complaints_grouped.head()
```

	Location Type	Borough	Month_of_the_year	Hour_of_the_day	complaints
0	Club/Bar/Restaurant	BRONX	1	0	10
1	Club/Bar/Restaurant	BRONX	1	1	10
2	Club/Bar/Restaurant	BRONX	1	2	6
3	Club/Bar/Restaurant	BRONX	1	3	6
4	Club/Bar/Restaurant	BRONX	1	4	3

Let us create a bar plot visualizing the average number of complaints with the time of the day.

```
sns.barplot(x="Hour_of_the_day", y = 'complaints', data=nyc_complaints_grouped)
```

```
<AxesSubplot:xlabel='Hour_of_the_day', ylabel='complaints'>
```



From the above plot, we observe that most of the complaints are made around midnight. However, interestingly, there are some complaints at each hour of the day.

Note that the above barplot shows the mean number of complaints in a month at each hour of the day. The black lines are the 95% confidence intervals of the mean number of complaints.

6.2.2 Facetgrid: Multi-plot grid for plotting conditional relationships

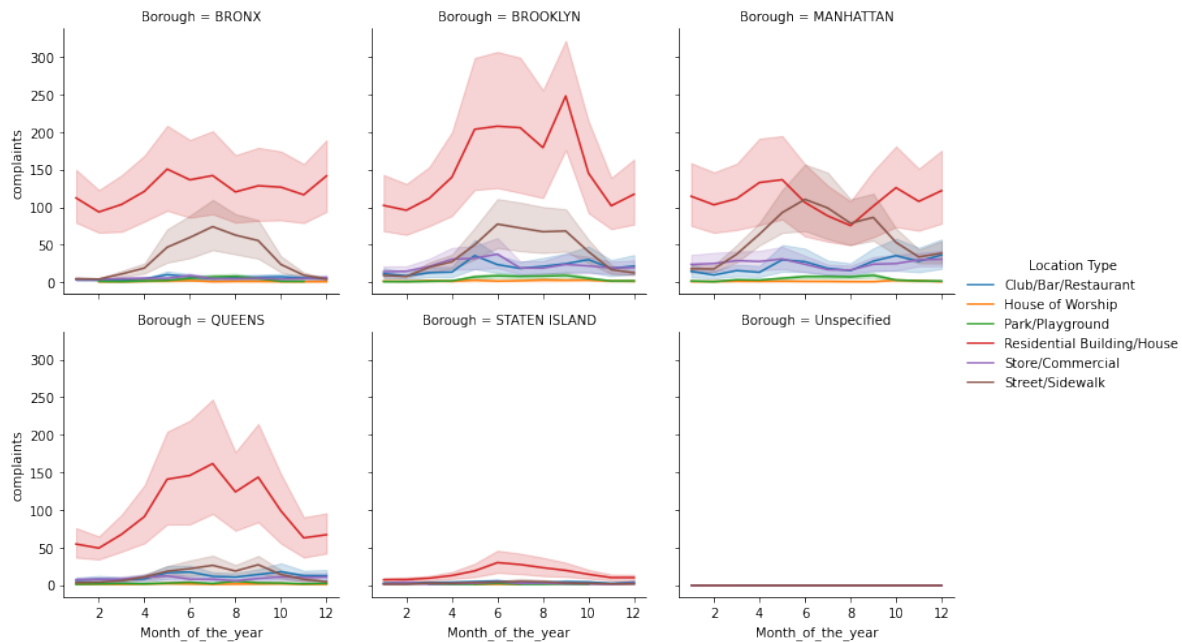
With pandas, we simultaneously visualized the number of complaints with month of the year and location type. We'll use Seaborn to add another variable - Borough to the visualization.

Q: Visualize the number of complaints with *Month_of_the_year*, *Location Type*, and *Borough*.

The seaborn class `FacetGrid` is used to design the plot, i.e., specify the way the data will be divided in mutually exclusive subsets for visualization. Then the `[map]` function of the `FacetGrid` class is used to apply a plotting function to each subset of the data.

```
#Visualizing the number of complaints with Month_of_the_year, Location Type, and Borough.
a = sns.FacetGrid(nyc_complaints_grouped, hue = 'Location Type', col = 'Borough',col_wrap=
a.map(sns.lineplot,'Month_of_the_year','complaints')
a.add_legend()
```

<seaborn.axisgrid.FacetGrid at 0x1d0e52ff580>



From the above plot, we get a couple of interesting insights: 1. For Queens and Staten Island, most of the complaints occur in summer, for Manhattan and Bronx it is mostly during late spring, while Brooklyn has a spike of complaints in early Fall. 2. In most of the Boroughs, the majority complaints always occur in residential areas. However, for Manhattan, the number of street/sidewalk complaints in the summer are comparable to those from residential areas.

We have visualized 4 variables simultaneously in the above plot.

Let us consider another example, where we will visualize the weather in a few cities of Australia. The file *Australia_weather.csv* consists of weather details of Sydney, Canberra, and Melbourne from 2007 to 2017.

```
aussie_weather = pd.read_csv('./Datasets/Australia_weather.csv')
aussie_weather.head()
```

	Date	Location	MinTemp	MaxTemp	Rainfall	Evaporation	Sunshine	WindGustDir	WindSpeed
0	10/20/2010	Sydney	12.9	20.3	0.2	3.0	10.9	ENE	37
1	10/21/2010	Sydney	13.3	21.5	0.0	6.6	11.0	ENE	41
2	10/22/2010	Sydney	15.3	23.0	0.0	5.6	11.0	NNE	41
3	10/26/2010	Sydney	12.9	26.7	0.2	3.8	12.1	NE	33

	Date	Location	MinTemp	MaxTemp	Rainfall	Evaporation	Sunshine	WindGustDir	Wind
4	10/27/2010	Sydney	14.8	23.8	0.0	6.8	9.6	SSE	54

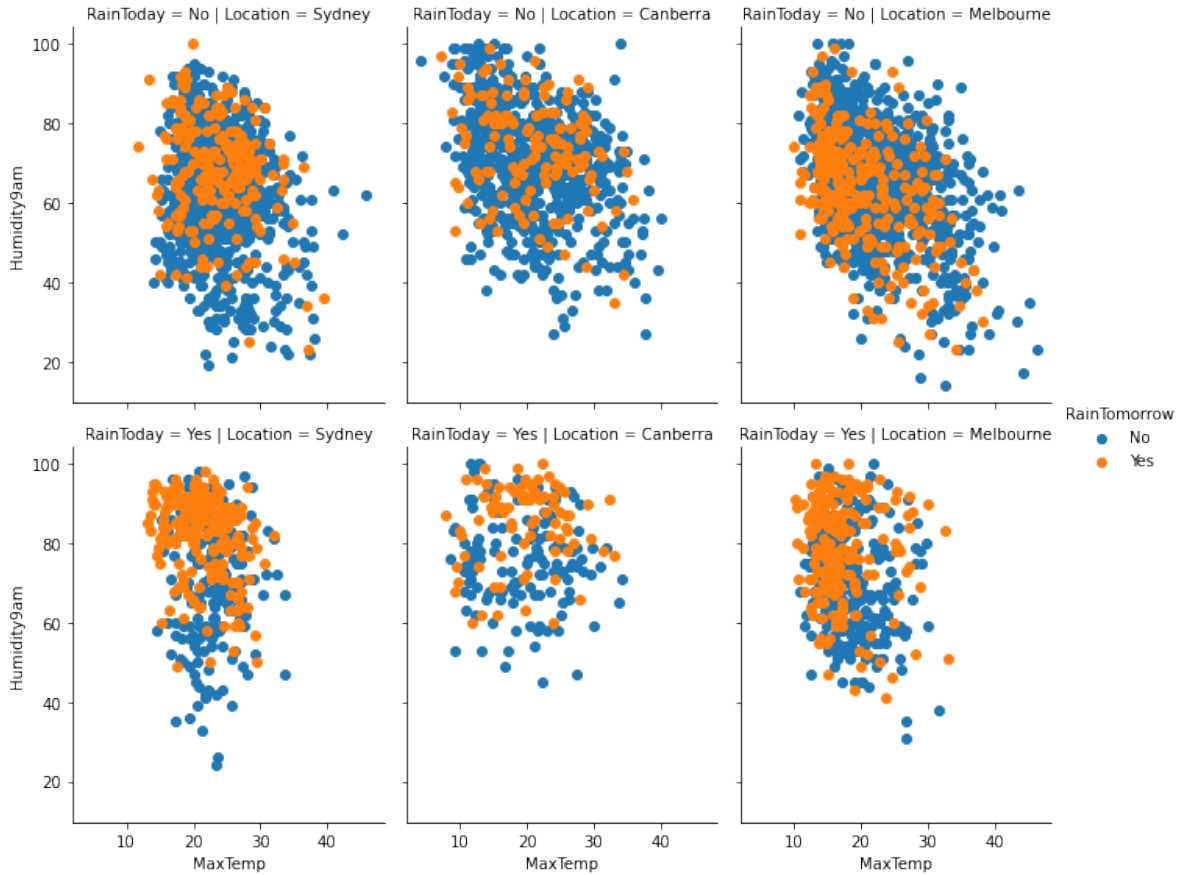
```
aussie_weather.shape
```

```
(4666, 24)
```

Q: Visualize if it rains the next day (*RainTomorrow*) given whether it has rained today (*RainToday*), the current day's humidity (*Humidity9am*), maximum temperature (*MaxTemp*) and the city (*Location*).

```
a = sns.FacetGrid(aussie_weather,col='Location',row='RainToday',height = 4,aspect = 0.8,hu
a.map(plt.scatter,'MaxTemp','Humidity9am')
a.add_legend()
```

```
<seaborn.axisgrid.FacetGrid at 0x1d0e77b0610>
```



Humidity tends to be higher when it is going to rain the next day. However, the correlation is much more pronounced for Sydney. In case it is not raining on the current day, humidity seems to be slightly negatively correlated with temperature.

6.2.3 Histogram and density plots

Histogram and density plots visualize the data distribution. A histogram plots the number of observations occurring within discrete, evenly spaced bins of a random variable, to visualize the distribution of the variable. It may be considered a special case of bar plot as bars are used to plot the observation counts.

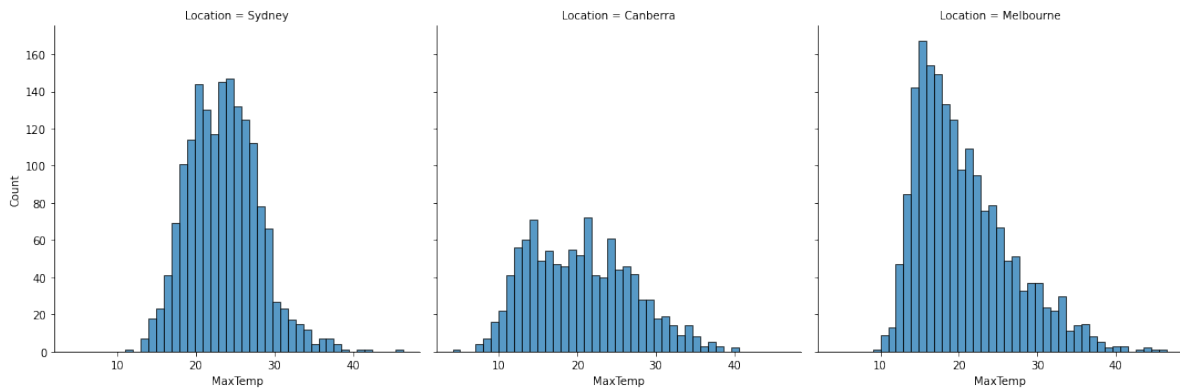
A density plot uses a kernel density estimate to approximate the distribution of random variable.

We can use the Seaborn `displot()` function to make both kinds of plots - histogram or density plot.

Example: Make a histogram showing the distributions of maximum temperature in Sydney, Canberra and Melbourne.

```
sns.displot(data = aussie_weather, x = 'MaxTemp', kind = 'hist', col='Location')
```

<seaborn.axisgrid.FacetGrid at 0x1d0ec989e50>

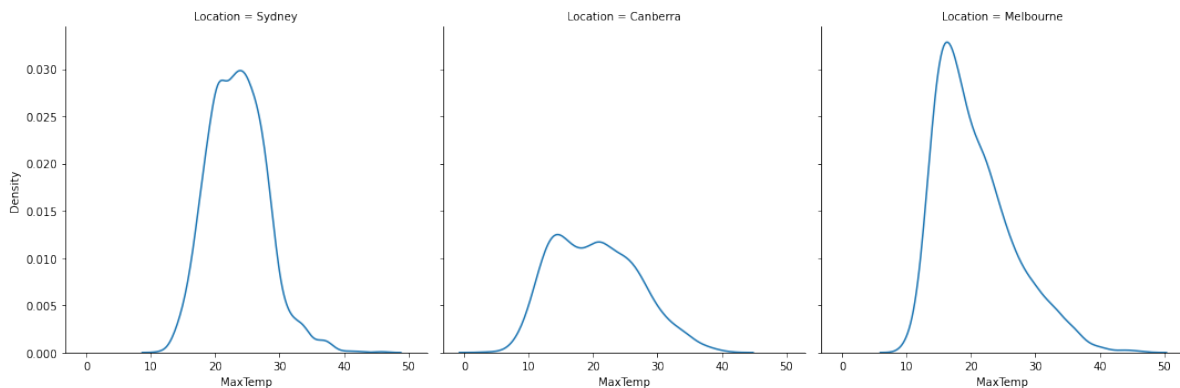


From the above plot, we observe that: 1. Melbourne has a right skewed distribution with the median temperature being smaller than the mean. 2. Canberra seems to have the highest variation in the temperature.

Example: Make a density plot showing the distributions of maximum temperature in Sydney, Canberra and Melbourne.

```
sns.displot(data = aussie_weather, x = 'MaxTemp', kind = 'kde', col = 'Location')
```

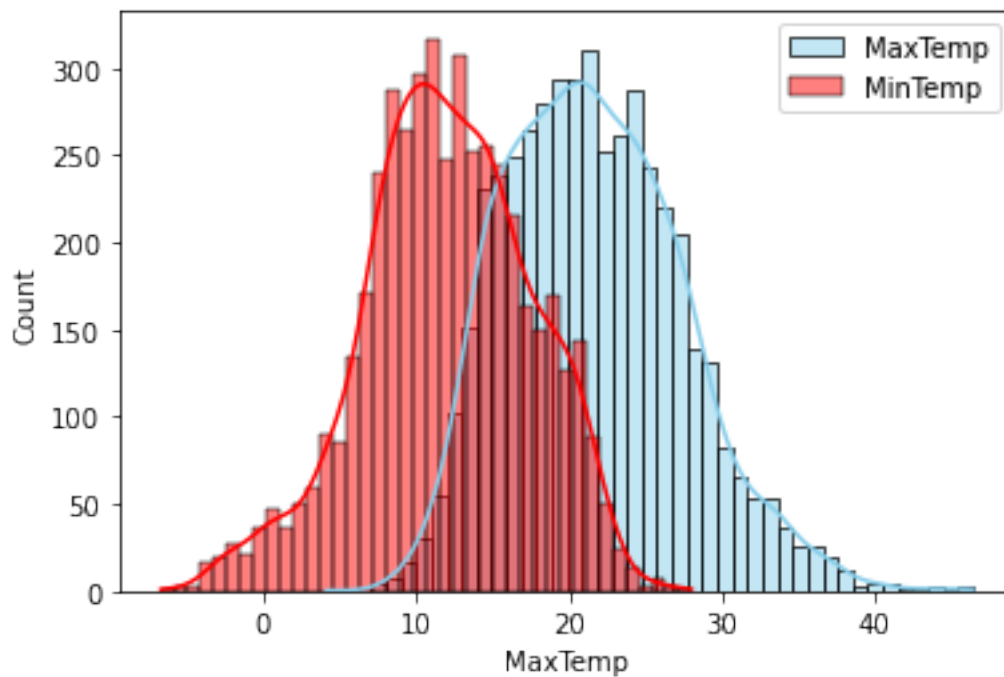
<seaborn.axisgrid.FacetGrid at 0x1d0e963f8e0>



Example: Show the distributions of the maximum and minimum temperatures in a single plot.

```
sns.histplot(data=aussie_weather, x="MaxTemp", color="skyblue", label="MaxTemp", kde=True)
sns.histplot(data=aussie_weather, x="MinTemp", color="red", label="MinTemp", kde=True)
plt.legend()
```

<matplotlib.legend.Legend at 0x1d0eb6b9790>



The Seaborn function `histplot()` can be used to make a density plot overlapping on a histogram.

6.2.4 Boxplots

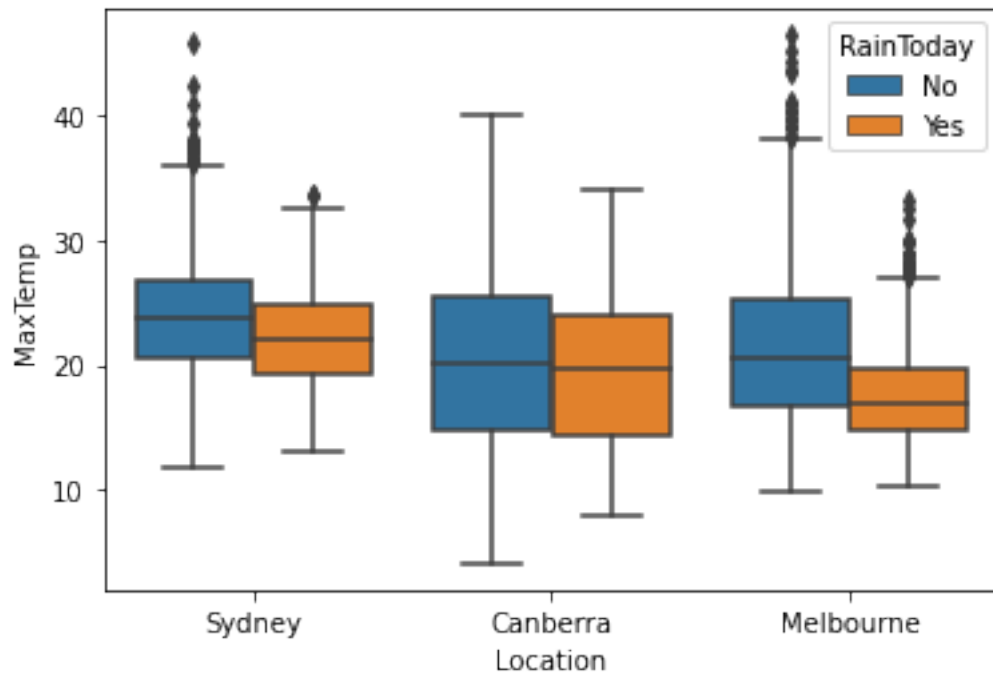
Boxplots is a standardized way of visualizing the data distribution. They show five key metrics that describe the data distribution - median, 25th percentile value, 75th percentile value, minimum and maximum, as shown in the figure below. Note that the minimum and maximum exclude the outliers.

<IPython.core.display.Image object>

Example: Make a boxplot comparing the distributions of maximum temperatures of Sydney, Canberra and Melbourne, given whether or not it has rained on the day.

```
sns.boxplot(data = aussie_weather, x = 'Location', y = 'MaxTemp', hue = 'RainToday')
```

```
<AxesSubplot:xlabel='Location', ylabel='MaxTemp'>
```



From the above plot, we observe that: 1. The maximum temperature of the day, on an average, is lower if it rained on the day. 2. Sydney and Melbourne have some extremely high outlying values of maximum temperature.

We have used the Seaborn `boxplot()` function for the above plot.

7 Data cleaning and preparation

Missing values in a dataset can occur due to several reasons such as breakdown of measuring equipment, accidental removal of observations, lack of response by respondents, error on the part of the researcher, etc.

Removing all rows / columns with even a single missing value results in loss of data that is non-missing in the respective rows/columns. In this section, we'll see how to make a smart guess for the missing values, so that we can (hopefully) maximize the information that can be extracted from the data.

7.0.1 Types of missing values

Rubin (1976) classified missing values into three categories, depending on how the values could have been missing.

7.0.2 Missing Completely at Random (MCAR)

If the probability that a random variable's value is missing is the same for all the observations, then the data is said to be missing completely at random. An example of MCAR is a weighing scale that ran out of batteries. Some of the data will be missing simply because of bad luck.

7.0.3 Missing at Random (MAR)

If the probability of being missing is the same only within groups defined by the observed data, then the data are missing at random (MAR). MAR is a much broader class than MCAR. For example, when placed on a soft surface, a weighing scale may produce more missing values than when placed on a hard surface. Such data are thus not MCAR. If, however, we know surface type and if we can assume MCAR within the type of surface, then the data are MAR.

7.0.4 Missing Not at Random (MNAR)

MNAR means that the probability of being missing varies for reasons that are unknown to us. For example, the weighing scale mechanism may wear out over time, producing more missing data as time progresses, but we may fail to note this. If the heavier objects are measured later in time, then we obtain a distribution of the measurements that will be distorted. MNAR includes the possibility that the scale produces more missing values for the heavier objects (as above), a situation that might be difficult to recognize and handle.

Source: <https://stefvanbuuren.name/fmd/sec-MCAR.html>

8 Data wrangling

Data wrangling

9 Data aggregation

Data aggregation

10 Datasets

Datasets used in the book can be found [here](#)

References

Rubin, Donald B. 1976. “Inference and Missing Data.” *Biometrika* 63 (3): 581–92.