Computational Radiometry Work Package

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Subject

03-Introduction-to-Radiometry

Distribution

Conclusions/Decisions/Amendments

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1 3 Brief Introduction to Radiometry

This notebook forms part of a series on computational optical radiometry[1]

The date of this document and module versions used in this document are given at the end of the file.

Feedback is appreciated: neliswillers at gmail dot com.

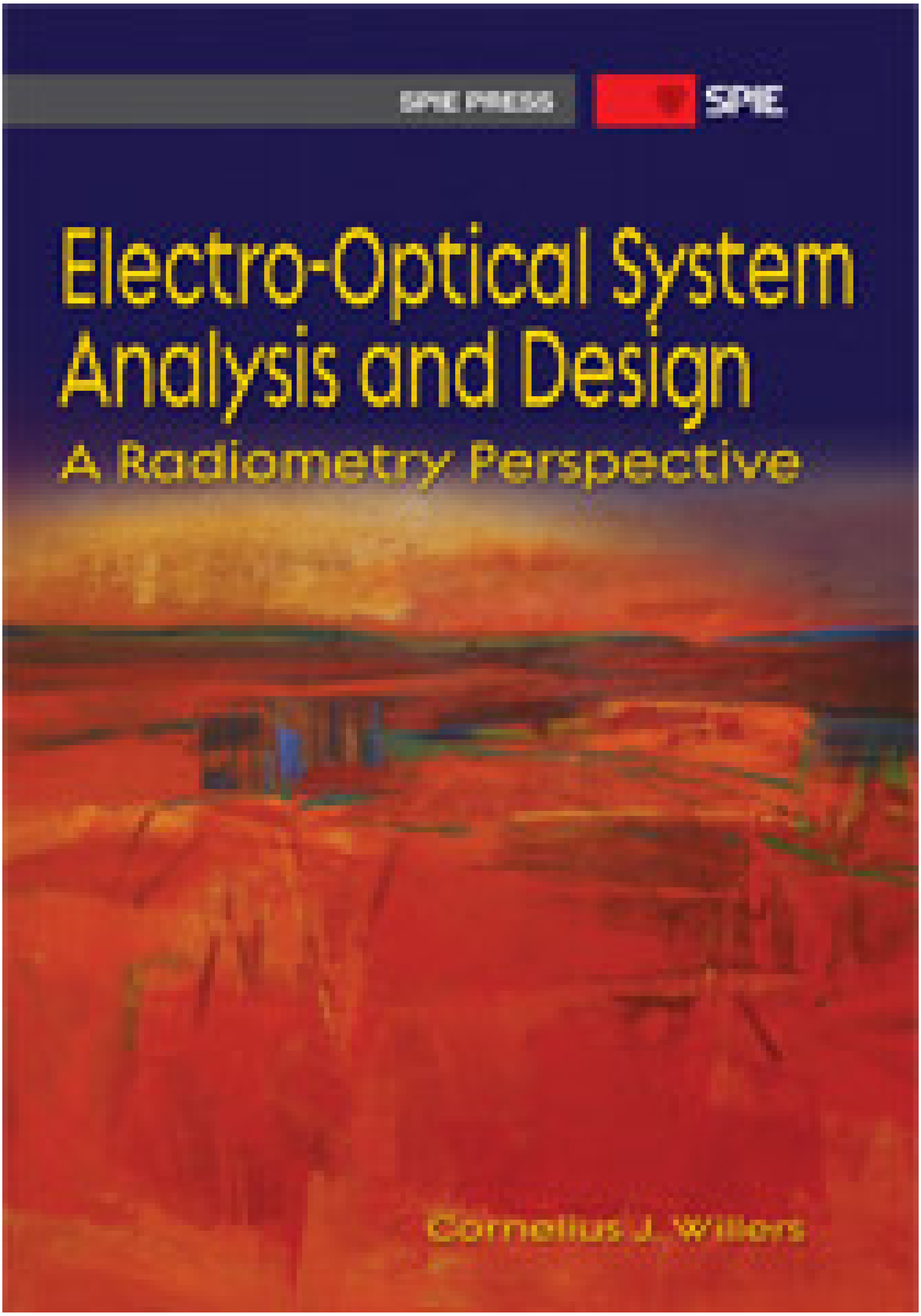
1.1 Overview

```
from IPython.display import display
from IPython.display import Image
from IPython.display import HTML
```

The [2] toolkit is a Python toolkit to perform optical and infrared computational radiometry (flux flow) calculations. Radiometry is the measurement and calculation of electromagnetic flux transfer for systems operating in the spectral region ranging from ultraviolet to microwaves. Indeed, these principles can be applied to electromagnetic radiation of any wavelength. This book only considers ray-based radiometry for incoherent radiation fields.

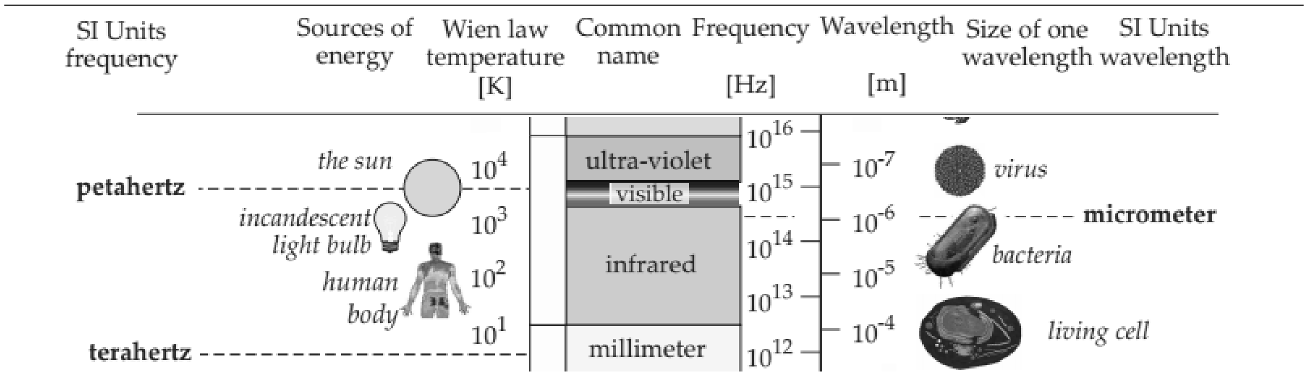
The briefly summarised information in this notebook is taken from my book[3], see the book for more details.

```
display(Image(filename='images/PM236.jpg'))
```



Electromagnetic radiation can be modeled as a number of different phenomena: rays, electromagnetic waves, wavefronts, or particles. All of these models are mathematically related. The appropriate model to use depends on the task at hand. Either the electromagnetic wave model(developed by Maxwell) or the particle model (developed by Einstein) are used when most appropriate. The part of the electromagnetic spectrum normally considered in optical radiometry is as follows:

```
display(Image(filename='images/radiometry03.png'))
```



The photon is a massless elementary particle and acts as the energy carrier for the electromagnetic wave. Photon particles have discrete energy quanta proportional to the frequency of the electromagnetic energy, $Q = h\nu = hc/\lambda$, where h is Planck's constant.

1.2 Definitions

The following figure (expanded from Pinson) and table defines the key radiometry units. The difference operator 'd' is used to denote 'a small quantity of ...'. This 'small quantity' of one variable is almost always related to a 'small quantity' of another variable in some physical dependency. For example, irradiance is defined as $E = d\Phi/dA$, which means that a small amount of flux $d\Phi$ impinges on a small area dA, resulting in an irradiance of E. 'Small' is defined as the extent or domain over which the quantity, or any of its dependent quantities, does not vary significantly. Because any finite-sized quantity varies over a finite-sized domain, the d operation is only valid over an infinitely small domain $dA = \lim_{\Delta A \to 0} \Delta A$. The difference operator, written in the form of a differential such as $E = d\Phi/dA$, is not primarily meant to mean differentiation in the mathematical sense. Rather, it is used to indicate something that can be integrated (or summed).

In practice, it is impossible to consider infinitely many, infinitely small domains. Following the reductionist approach, any real system can, however, be assembled as the sum of a set of these small domains, by integration over the physical domain as in $A = \int dA$. Hence, the 'small-quantity' approach proves very useful to describe and understand the problem, whereas the real-world solution can be obtained as the sum of a set of such small quantities. In almost all of the cases in this notebook, it is implied that such 'small-quantity' domains will be integrated (or summed) over the (larger) problem domain.

Photon rates are measured in quanta per second. The 'second' is an SI unit, whereas quanta is a unitless count: the number of photons. Photon rate therefore has units of [1/s] or [s^{-1}]. This form tends to lose track of the fact that the number of quanta per second is described. The notebook may occasionally contain units of the form [q/s] to emphasize the photon count. In this case, the 'q' is not a formal unit, it is merely a reminder of 'counts.' In dimensional analysis the 'q' is handled the same as any other unit.

Radiometric quantities can be defined in terms of three different but related units: radiant power (watts), photon rates (quanta per second), or photometric luminosity (lumen). Photometry is radiometry applied to human visual perception. The conversion from radiometric to photometric quantities is covered in more detail in my book. It is important to realize that the underlying concepts are the same, irrespective of the nature of the quantity. All of the derivations and examples presented in this book are equally valid for radiant, photon, or photometric quantities.

Flux is the amount of optical power, a photon rate, or photometric luminous flux, flowing between two surfaces. There is always a source area and a receiving area, with the flux flowing between them.

All quantities of flux are denoted by the symbol Φ. The units are [W], [q/s], or [lm], depending on the nature of the quantity.

Irradiance (areance) is the areal density of flux on the receiving surface area. The flux flows inward onto the surface with no regard to incoming angular density. All quantities of irradiance are denoted by the symbol E. The units are [W/m^2], [q/(s·m^2)], or [lm/m^2], depending on the nature of the quantity.

Exitance (areance) is the areal density of flux on the source surface area. The flux flows outward from the surface with no regard to angular density. The exitance leaving a surface can be due to reflected light, transmitted light, emitted light, or any combination thereof. All quantities of exitance are denoted by the symbol M. The units are [W/m^2], [q/(s·m^2)], or [lm/m^2], depending on the nature of the quantity.

Intensity (pointance) is the density of flux over solid angle. The flux flows outward from the source with no regard for surface area. Intensity is denoted by the symbol I. The human perception of a point source (e.g., a star at long range) 'brightness' is an intensity measurement. The units are [W/sr], [q/(s·sr)], or [lm/sr], depending on the nature of the quantity.

Radiance (sterance) is the density of flux per unit source surface area and unit solid angle. Radiance is a property of the electromagnetic field irrespective of spatial location (in a lossless medium). For a radiating surface, the radiance may comprise transmitted light, reflected light, emitted light, or any combination thereof. The radiance in a field created by a Lambertian source is conserved: the radiance is constant anywhere in space, also on the receiving surface. All radiance quantities are denoted by the symbol L. The human perception of 'brightness' of a large surface can be likened to a radiance experience (beware of the nonlinear response in the eye, however). The units are [W/(m^2 ·sr)], [q/(s·m^2 ·sr)], or [lm/(m^2 ·sr)], depending on the nature of the quantity.

```
display(Image(filename='images/radiometry01.png'))
```

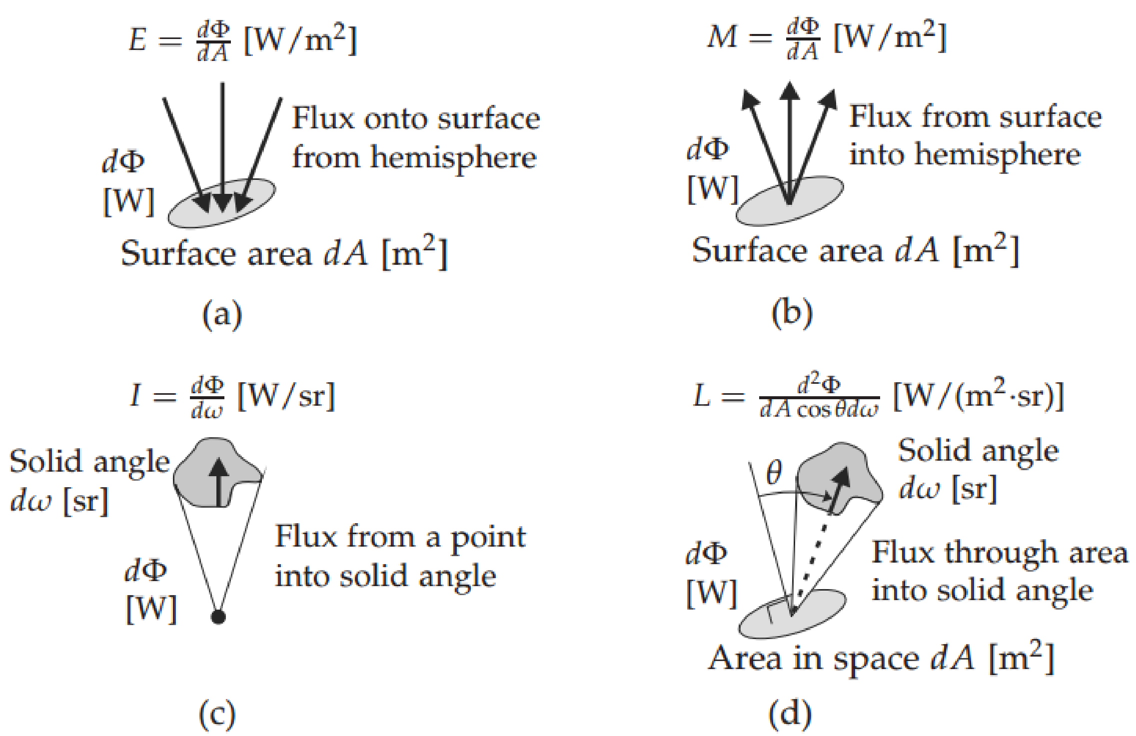


Figure 2.3 Graphic representation of radiometric quantities: (a) irradiance, (b) exitance, (c) intensity, and (d) radiance.

```
display(Image(filename='images/radiometry02.png'))
```

Table 2.1 Nomenclature for radiometric quantities.

Basic Quantity	Radiant	Photon	Photometry
Energy	Radiant energy Q_e joule [J]	Photon count Q_p,Q_q photon count [q]	Luminous energy Q_ν lumen-second [lm·s]
Flux	Radiant flux Φ_e watt [W]	photon flux Φ_p,Φ_q photon rate [q/s]	Luminous flux Φ_ν lumen [lm]
Exitance	Radiant exitance M_e [W/m^2]	Photon exitance M_p,M_q [q/(s·m^2)]	Luminous exitance M_ν [lm/m^2]
Intensity	Radiant intensity I_e [W/sr]	Photon intensity I_p,I_q [q/(s·sr)]	Luminous intensity I_ν candela=cd=[lm/sr]
Radiance	Radiance L_e [W/(m^2·sr)]	Photon radiance L_p,L_q [q/(s·m^2·sr)]	Luminance L_ν nit=nt=[lm/(m^2·sr)]
Irradiance	Radiant irradiance E_e [W/m^2]	Photon irradiance E_p,E_q [q/(s·m^2)]	Illuminance E_ν lux=lx=[lm/m^2]
Subscript	e	p or q	ν

1.3 Spectral quantities

See notebook 4 in this series, [4], for a detailed description of spectral quantities.

Three spectral domains are commonly used: wavelength λ in [m], frequency ν in [Hz], and wavenumber $\tilde{\nu}$ in [cm^{-1}] (the number of waves that will fit into a 1-cm length). Spectral quantities indicate an amount of the quantity within a small spectral width $d\lambda$ around the value of λ: it is a spectral density. Spectral density quantity symbols are subscripted with a λ or ν, i.e., L_λ or L_ν. The dimensional units of a spectral density quantity are indicated as [μm^{-1}] or [(cm^{-1})$^{-1}$], i.e., [W/(m^2 ·sr· μm)].

The relationship between the wavelength and wavenumber spectral domains is $\tilde{\nu}=10^4/\lambda$, where λ is in units of μm. The conversion of a spectral density quantity such as [W/(m^2 ·sr·cm^{-1})] requires the derivative, $\%d\tilde{\nu}=-\frac{10^4}{\lambda^2}d\lambda=-\frac{\tilde{\nu}^2}{10^4}d\lambda.$ $d\tilde{\nu}=-10^4 d\lambda/\lambda^2=-\tilde{\nu}^2 d\lambda/10^4$. The derivative relationship converts between the spectral *widths*, and hence the spectral densities, in the two respective domains. The conversion from a wavelength spectral density quantity to a wavenumber spectral density quantity is $dL_{\tilde{\nu}}=dL_\lambda\lambda^2/10^4=dL_\lambda 10^4/\tilde{\nu}^2$.

Spectral quantities denote the amount in a small spectral width $d\lambda$ around a wavelength λ. It follows that the total quantity over a spectral range can be determined by integration (summation) over the spectral range of interest:

$$L = \int_{\lambda_1}^{\lambda_2} L_\lambda d\lambda. \tag{1.1}$$

The above integral satisfies the requirements of dimensional analysis (see my book) because the units of L_λ are [W/(m^2 ·sr· μm)], whereas $d\lambda$ has the units of [μm], and L has units of [W/(m^2 ·sr)].

1.4 Solid Angle

The *geometric* solid angle ω of any arbitrary surface P from the *reference point* is given by

$$\omega = \iint^{P} \frac{d^2P\cos\theta_1}{R^2}, \tag{1.2}$$

where $d^2P\cos\theta_1$ is the projected surface area of the surface P in the direction of the reference point, and R is the distance from d^2P to the reference point. The integral is independent of the viewing direction (θ_0, α_0) from the reference point. Hence, a given area at a given distance will always have the same geometric solid angle irrespective of the direction of the area.

The geometric solid angle of a cone is $\omega = 4\pi\sin^2\left(\frac{\Theta}{2}\right)$, where Θ is the cone half-apex angle.

The *projected* solid angle Ω of any arbitrary surface P from the *reference area* dA_0 is given by

$$\Omega = \iint^{P} \frac{d^2P\cos\theta_0\cos\theta_1}{R^2}, \tag{1.3}$$

where $d^2P\cos\theta_1$ is the projected surface area of the surface P in the direction of the reference area, and R is the distance from d^2P to the reference area. The integral depends on the viewing direction (θ_0, α_0) from the reference area, by the projected area $(dA_0\cos\theta_0)$ of dA_0 in the direction of d^2P. Hence, a given area at a given distance will always have a different projected solid angle in different directions.

The *projected* solid angle of a cone is $\omega = \pi\sin^2(\Theta)$, where Θ is the cone half-apex angle.

```
display(Image(filename='images/radiometry04.png'))
```

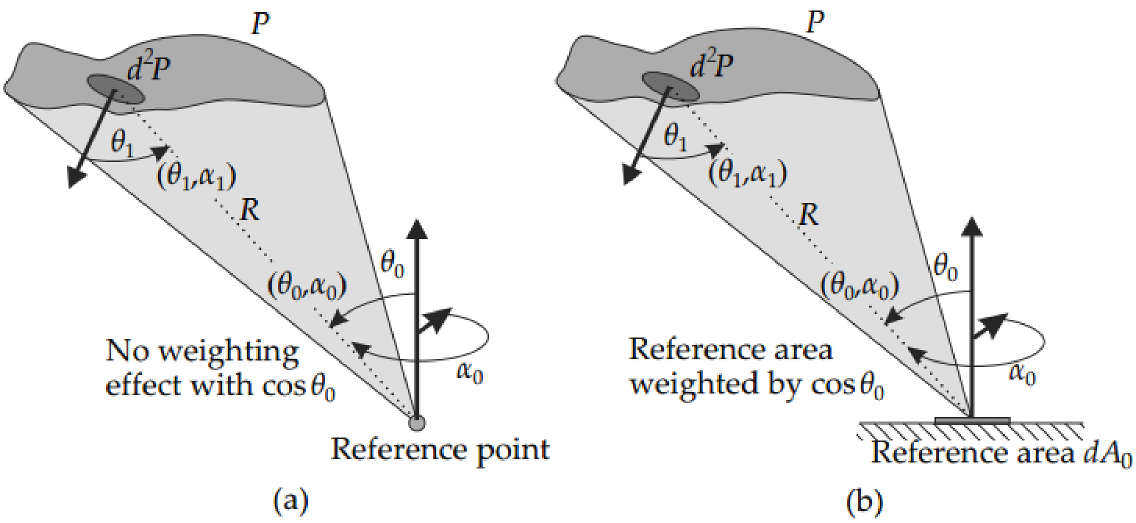


Figure 2.6 Solid angle definitions: (a) geometric solid angle, and (b) projected solid angle.

1.5 Lambertian radiators

A Lambertian source is, by definition, one whose radiance is completely independent of viewing angle. Many (but not all) rough and natural surfaces produce radiation whose radiance is approximately independent of the angle of observation. These surfaces generally have a rough texture at microscopic scales. Planck-law blackbody radiators are also Lambertian sources (see my book). Any Lambertian radiator is completely described by its scalar radiance magnitude only, with no angular dependence in radiance.

The relationship between the exitance and radiance for such a Lambertian surface can be easily derived. If the flux radiated from a Lambertian surface Φ [W] is known, it is a simple matter to calculate the exitance $M = \Phi/A$ [W/m^2], where A is the radiating surface area. The exitance of a Lambertian radiator is related to radiance by the projected solid angle of π sr, *not* the geometric solid angle of 2π sr as one might expect. The details are given in my book.

1.6 Conservation of radiance

Radiance is conserved for flux from a Lambertian surface propagation through a lossless optical medium. Consider the construction below: two elemental areas dA_0 and dA_1 are separated by a distance R_{01}, with the angles between the normal vector of each surface and the line of sight given by θ_0 and θ_1. A total flux of $d^2\Phi$ is flowing through both the surfaces. It can be shown (see my book) that *for a Lambertian radiator* the radiance in an arbitrary dA_n is the same as the radiance in dA_1.

As light propagates through mediums with different refractive indices n such as air, water, glass, etc., the entity called *basic radiance*, defined by L/n^2, is invariant. It can be shown that for light propagating from a medium with refractive index n_1 to a medium with refractive index n_2, the basic radiance is conserved:

$$\frac{L_1}{n_1^2} = \frac{L_2}{n_2^2}. \tag{1.4}$$

```
display(Image(filename='images/radiometry05.png'))
```

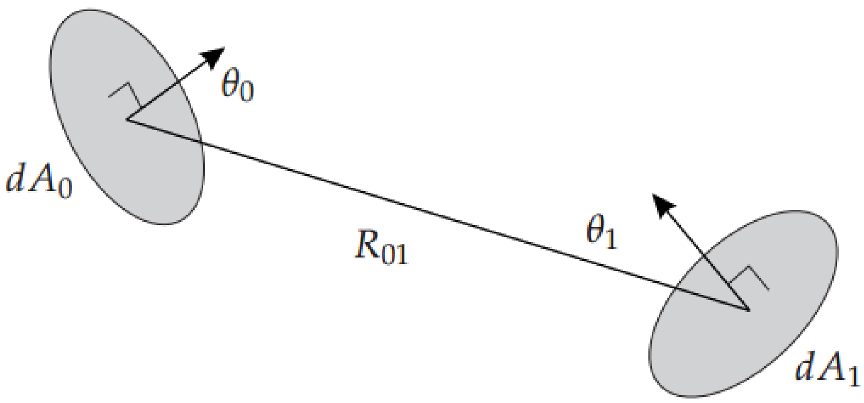


Figure 2.12 Geometrical construction for radiative flux between two elemental areas.

1.7 Flux transfer through lossless and lossy mediums

A lossless medium is defined as a medium with no losses between the source and the receiver, such as a complete vacuum. This implies that no absorption, scattering, or any other attenuating mechanism is present in the medium. For a lossless medium the flux that flow between both dA_0 and dA_1 is given by

$$d^2\Phi = \frac{L_{01}\,dA_0\cos\theta_0\,dA_1\cos\theta_1}{R_{01}^2}. \tag{1.5}$$

If the medium has loss, the loss effect is accounted for by including a 'transmittance' factor $\tau_{01} = \Phi_1/\Phi_0 = L_{10}/L_{01}$, i.e., the fraction of the flux from A_0 that arrives at A_1, then

$$d^2\Phi = \frac{L_{01}\,dA_0\cos\theta_0\,dA_1\cos\theta_1\tau_{01}}{R_{01}^2}. \tag{1.6}$$

1.8 Sources and receivers of arbitrary shape

The above equation calculates the flux flowing between two infinitely small areas. The flux flowing between two arbitrary shapes can be calculated by integrating the equation over the source surface and the receiving surface. In the general case, the radiance L cannot be assumed constant over A_0, introducing the spatial radiance distribution $L(dA_0)$ as a factor into the spatial integral. Likewise, the medium transmittance between any two areas dA_0 and dA_1 varies with the spatial locations of dA_0 and dA_1 — hence $\tau_{01}(dA_0, dA_1)$ should also be included in the spatial integral.

The integral can be performed over any arbitrary shape, as shown in the following figure, supporting the solution with complex geometries. Clearly matters such as obscuration and occlusion should be considered when performing this integral:

$$\Phi = \int_{A_0}\int_{A_1}\frac{L(dA_0)\,dA_0\cos\theta_0\,dA_1\cos\theta_1\,\tau_{01}(dA_0,dA_1)}{R_{01}^2}. \tag{1.7}$$

```
display(Image(filename='images/radiometry06.png'))
```

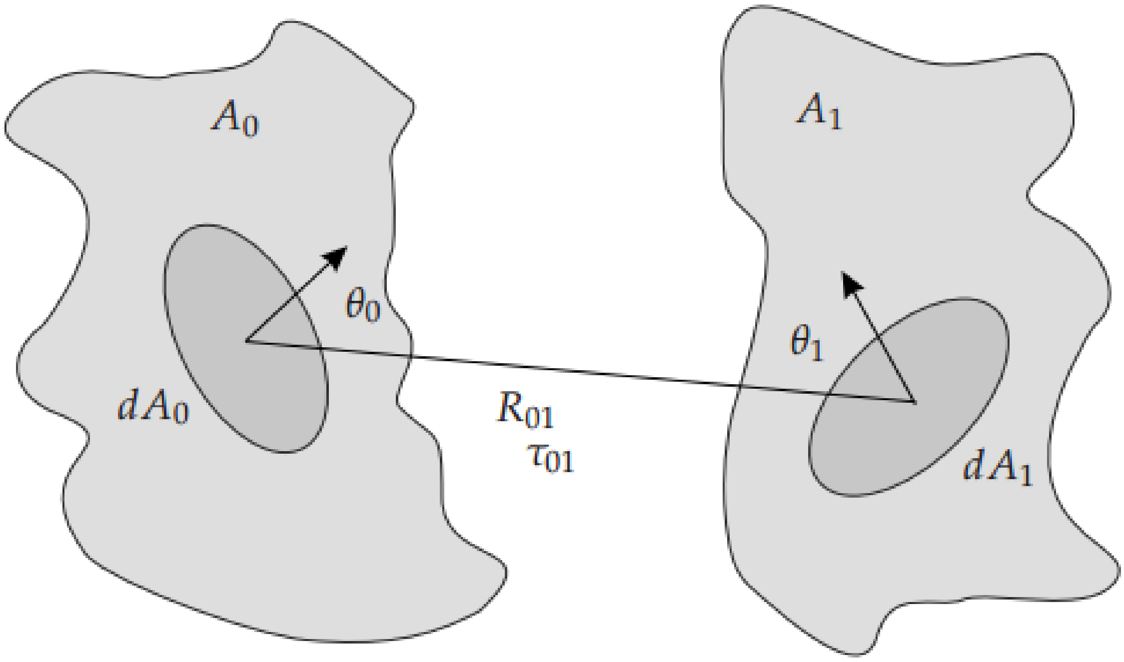


Figure 2.13 Radiative flux between areas of arbitrary shape.

1.9 Multi-spectral flux transfer

The optical power leaving a source undergoes a succession of scaling or 'spectral filtering' processes as the flux propagates through the system, as shown below. This filtering varies with wavelength. Examples of such filters are source emissivity, atmospheric transmittance, optical filter transmittance, and detector responsivity. The multi-spectral filter approach described here is conceptually simple but fundamental to the calculation of radiometric flux.

```
display(Image(filename='images/radiometry07.png'))
```

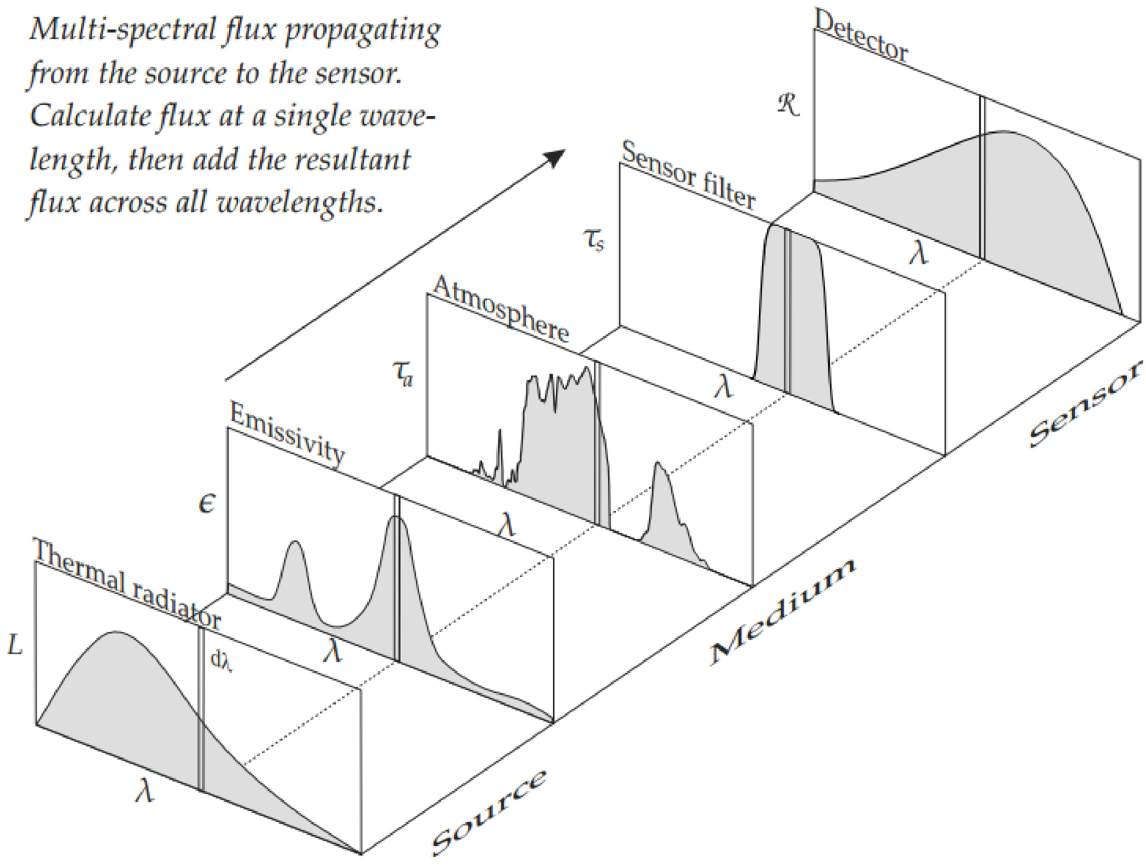


Figure 2.14 Describing the electro-optical system as a thermal source and a series of spectral filters.

Extend the above flux-transfer equation for multi-spectral calculations by noting that over a spectral width $d\lambda$ the radiance is given by $L = L_\lambda d\lambda$:

$$d^3\Phi_\lambda = \frac{L_{01\lambda}\, dA_0\, \cos\theta_0\, dA_1\, \cos\theta_1\, \tau_{01}\, d\lambda}{R_{01}^2}, \tag{1.8}$$

where $d^3\Phi_\lambda$ is the total flux in [W] or [q/s] flowing in a spectral width $d\lambda$ at wavelength λ, from a radiator with radiance $L_{0\lambda}$ with units [W/(m^2 ·sr· μm)] and projected surface area $dA_0 \cos\theta_0$, through a receiver with projected surface area $dA_1 \cos\theta_1$ at a distance R_{01}, with a transmittance of τ_{01} between the two surfaces. The transmittance τ_{01} now includes all of the spectral variables in the path between the source and the receiver.

To determine the *total flux* flowing from elemental area dA_0 through dA_1 over a wide spectral width, divide the wide spectral band into a large number N of narrow widths $\Delta\lambda$ at wavelengths λ_n and add the flux for all of these narrow bandwidths together as follows:

$$d^2\Phi = \sum_{n=0}^{N} \left(\frac{L_{01\lambda_n}\, dA_0\, \cos\theta_0\, dA_1\, \cos\theta_1\, \tau_{01\lambda_n} \Delta\lambda}{R_{01}^2} \right). \tag{1.9}$$

By the Riemann–Stieltjes theorem in reverse, if now $\Delta\lambda \to 0$ and $N \to \infty$, the summation becomes the integral

$$d^2\Phi = \int_{\lambda_1}^{\lambda_2} \frac{L_{01\lambda}\, dA_0\, \cos\theta_0\, dA_1\, \cos\theta_1\, \tau_{01\lambda} d\lambda}{R_{01}^2}\ . \tag{1.10}$$

This equation describes the total flux at all wavelengths in the spectral range λ_1 to λ_2 passing through the system. This equation is developed further in my book.

1.10 Conclusion

The flux transfer between any two arbitrary surfaces, over any spectral band can be calculated by

$$\Phi = \int_{A_0}\int_{A_1}\int_{\lambda_1}^{\lambda_2} \frac{L_{01\lambda}\, dA_0 \cos\theta_0 \, dA_1 \cos\theta_1 \, \tau_{01\lambda} d\lambda}{R_{01}^2} \,. \tag{1.11}$$

In practice these integrals are performed by finite sums of small elemental areas and spectral widths. Any arbitrary problem can be solved using this approach. For a simple example see the flame sensor[5] and the other pages of this notebook series[6].

1.11 Python and module versions, and dates

```
try:
    import pyradi.ryutils as ryutils
    print(ryutils.VersionInformation('matplotlib,numpy,pyradi,scipy,↵
       pandas'))
except:
    print("pyradi.ryutils not found")
```

```
Software versions
Python:   3.8.3 64bit [MSC v.1916 64 bit (AMD64)]
IPython:   7.26.0
OS:   Windows 10 10.0.19041 SP0
matplotlib:   3.4.3
numpy:   1.20.3
pyradi:   1.1.4
scipy:   1.7.1
pandas:   1.3.2
Mon Aug 23 12:02:06 2021 South Africa Standard Time
```

BIBLIOGRAPHY

[1] [Online]. Available: https://github.com/NelisW/ComputationalRadiometry#computational-optical-radiometry-with-pyradi

[2] [Online]. Available: http://nelisw.github.io/pyradi-docs/_build/html/index.html

[3] [Online]. Available: http://spie.org/Publications/Book/2021423?origin_id=x646

[4] [Online]. Available: http://nbviewer.ipython.org/github/NelisW/ComputationalRadiometry/blob/master/04-IntroductionToComputationalRadiometryWithPyradi.ipynb

[5] [Online]. Available: http://nbviewer.ipython.org/github/NelisW/ComputationalRadiometry/blob/master/12a-FlameSensorAnalysis.ipynb

[6] [Online]. Available: https://github.com/NelisW/ComputationalRadiometry#computational-optical-radiometry-with-pyradi