CSE 321 – Introduction to Algorithm Design

Homework 01

Deadline: 21 October 23:55

PLEASE DO NOT FORGET TO READ THE NOTES CAREFULLY

1. Consider the following functions:

$$T_1(n) = 3n^4 + 3n^3 + 1$$
$$T_2(n) = 3^n$$
$$T_3(n) = (n-2)!$$
$$T_4(n) = ln^2\, n$$
$$T_5(n) = 2^{2n}$$
$$T_6(n) = \sqrt[3]{n}$$

   Order them according to their asymptotic complexity. Then prove with using limits that, every function has more complexity from the function to its left one. For example, if you suggest to order functions as $T_2 < T_4 < T_1 < T_3 < T_6 < T_5$, then you have to prove that $T_2 = O(T_4)$, $T_4 = O(T_1)$, $T_1 = O(T_3)$, $T_3 = O(T_6)$ and $T_6 = O(T_5)$ by using the method of taking limits.

2. The implementation of delicious algorithm in Python is given below. The input given to algorithm is a single dimensional array of integers. Array elements are sorted and shifted by a random amount. For example: the array [71, 83, 99, 1, 4, 15, 36] has been shifted 3 times to right.

```python
def delicious(fruits):
    plum = sys.maxsize # you can think sys.maxsize as positive infinity
    watermelon = 0
    orange = 0
    orangeTime = False
    while not orangeTime:
        for fruit in fruits:
            if fruit > watermelon:
                watermelon = fruit
            if fruit < plum:
                plum = fruit
                break
        else:
            orangeTime = True
    for fruit in fruits:
        if abs(fruit - (watermelon+plum)//2) < abs(orange - (watermelon+plum)//2):
            orange = fruit
    return orange
```

a) Explain what this algorithm does. Tell the role of all variables.
b) Show this algorithm's number of operations in terms of input array size (n) and complexity using the asymptotic notations for the following cases:

i. Worst case scenario
j. Best case scenario
k. Average case scenario

Explain and prove all of them.

3. Summation representation of some algorithms are given below. Find the order of growth of each of them. Write corresponding C or Python code of first (a) and the last (d) one.
   a) $\sum_{i=0}^{n-1}(i^2+1)^2$
   b) $\sum_{i=2}^{n-1}\log i^2$
   c) $\sum_{i=1}^{n}(i+1)2^{i-1}$
   d) $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i+j)$

4. Show summation representation of the C code given below. Show its complexity using asymptotic notations.

```
int fun(int n)
{
  int count = 0;
  for (int i = n; i > 0; i /= 2)
     for (int j = 0; j < i; j++)
        count += 1;
  return count;
}
```

5. Prove or disprove the following statements.
   a) $n^3 \in O(3^{2n})$
   b) $n \in o(\log\log n)$
   c) $n^2\log^2 n \in O(n!)$
   d) $\sqrt{10n^2+7n+3} \in \theta(n)$

IMPORTANT NOTES

1. Homeworks will be submitted to moodle in PDF format.
2. Cheating will be punished. (Grade will be -100)
3. Use homework question forum on moodle if you have any questions about homework.
4. No late submissions.