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# Mosaic

## RUNNING META-BLOCKCHAINS TO SCALE DECENTRALIZED APPLICATIONS

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### Abstract

This is a working draft summary of notes and whiteboard sessions as we have been iterating towards the implementation for Mosaic v0.10. This is not the version intended for publication, and you are invited to review critically all content. The code is work in progress at [github.com/openstfoundation/mosaic-contracts](https://github.com/openstfoundation/mosaic-contracts).

## 1. Introduction

Mosaic is a parallelisation schema for decentralized applications. Mosaic introduces a set of Byzantine fault-tolerant (BFT) consensus rules to compose heterogeneous blockchain systems into one another. Decentralized applications can use Mosaic to compute over a composed network of multiple blockchain systems in parallel.

A decentralized application is an application for which the computation is requested, performed, and paid for by independent actors. To date, Ethereum is the leading network of nodes that enables developers to write and deploy code which can be called by independent users to collectively construct a shared state of the application.

The Ethereum network achieves agreement on the collective state by constructing a blockchain. A blockchain derives correctness from full replication of the computation by nodes. Furthermore, to ensure correct execution of blockchain computations, an order on the inputs of execution must be determinable. In its current implementation, the Ethereum chain is appended through Proof-of-Work consensus. Proof-of-Work (PoW) introduces a block reward incentive for nodes to keep producing blocks and thereby securing the chain of historical blocks. However, PoW produces a serial execution model and nodes cannot be collectively rewarded for the computation they all have to perform, which renders the system computationally inefficient[1].

Active research and implementation work is ongoing to scale Ethereum's transaction throughput by dividing the verification work into multiple sections, also referred to as *shards*. At the same time, Ethereum is committed to moving from the probabilistic Proof-of-Work consensus engine to a BFT based Proof-of-Stake (PoS) consensus engine<sup>1</sup>. The outset of a BFT consensus engine is to collectively produce a blockchain which provably — under certain honesty assumptions — cannot finalize conflicting blocks.

Thus far a vision for a decentralized web has been a major driver of innovation. However, to power global networks of billions of users and decentralized computation on vast decentralized data stores, it is reasonable to assume no single blockchain network will suffice as a backbone for all types of applications. Much like the internet is a composed network of networks, we can conceive decentralized applications to transcend any single blockchain and execute across a network of blockchain networks.

We detail two protocols in this work. The first, a layer-2 protocol, constructs *meta-blockchains* on top of existing blockchain networks to extend their state space and transaction throughput capacity. The second protocol, called a *gateway*, acts as a message bus to send typed messages atomically between the underlying layer-1 blockchain and the meta-blockchain running on top of it.

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<sup>1</sup><https://github.com/ethereum/eth2.0-specs/blob/master/specs/beacon-chain.md>

Together these two building blocks can be used by decentralized applications to construct a parallelisation schema to increase computational capacity at lower transaction cost. In its simplest form, a decentralized application can offload transaction processing to a single meta-blockchain. More advanced applications can deploy logic across many meta-blockchains and process transactions in parallel while they can send asynchronous messages across meta-blockchains.

## 2. Composing Meta-Blockchains

A meta-blockchain composes transactions executed on an auxiliary blockchain system into an origin blockchain, such that the capacity of the origin blockchain extends and the auxiliary blockchain inherits the security properties of the origin blockchain.

To this end consider two blockchain systems, an origin blockchain  $O$  with state transition rules  $t_O$  which progress the state  $\Sigma_O$

$$t_O : \Sigma_O \rightarrow \Sigma'_O \quad (1)$$

and similarly consider an auxiliary system  $A$  with state transition rules  $t_A$  such that  $t_A : \Sigma_A \rightarrow \Sigma'_A$ .

Transition rules for the origin and auxiliary system can be similar but do not have to be. In our discussion, when we need an example, we will refer to the origin blockchain as Ethereum (Byzantium), running Proof-of-Work, and for the auxiliary system we consider Go-Ethereum, running “clique” Proof-of-Authority (PoA). We deliberately use PoA for our considerations for the auxiliary system to emphasize that the security for the composed auxiliary system is not derived from the consensus engine of the auxiliary system. Rather the security properties of the composed system must rely only on the security properties of the consensus rules of the origin blockchain and on the Mosaic BFT consensus rules composing the auxiliary system into the origin system.<sup>2</sup>

We define a meta-blockchain  $\mathcal{B}$  with a staked validator set  $\mathcal{V}_{\mathcal{B}}$  on the origin blockchain. The blocks  $B_i$  of  $\mathcal{B}$  are committed to a *core* contract on origin  $O$ . For a given history of  $O$ ,<sup>3</sup> the meta-blockchain cannot fork if we enforce that block  $B_{n+1}$  can only be (proposed and) committed after block  $B_n$  has been committed.

We define a meta-block  $B_i(K, T, S) : \Sigma_A \rightarrow \Sigma'_A$  to progress the state of the auxiliary system  $A$ , where we call  $K$  the *kernel*,  $T$  the *transition object* and  $S$  the *seal*. A block  $B_i$  at height  $i$  of the meta-blockchain is committed with a seal  $S$  when a  $+2/3$  supermajority of the weighted votes has been verified.<sup>4</sup> We will detail later how the meta-block is constructed, but first we will describe the vote messages which, combined, seal a meta-block.

### 2.1. Checkpoint Finalisation - Casper FFG

The reader is assumed at this point to be familiar with the work presented in *Casper the Friendly Finality Gadget*[2] (FFG), as we will build on the logic and proofs presented there. We will intend to align with definitions and notations where possible and highlight deviations where appropriate. This section briefly (and incompletely) summarizes core concepts from the paper, so that the shared concepts are established for the reader.

Casper FFG presents an algorithm to build up an overlay network of vote messages presented to a smart contract about the blocks already produced by the underlying network of nodes running the blockchain. The overlay network aims to repeatedly economically finalize a single branch of the underlying network. It does so by allowing staked validators to cast votes which together can construct a *justified chain*.

A justified chain is formed by a sequence of *supermajority links*, where a link identifies a *source* blockhash  $s$  and its height  $h_s$  and a *target* blockhash  $t$  and its height  $h_t$ . A vote message in Casper FFG is a link

<sup>2</sup>The block proposers of the auxiliary system do have the ability to censor transactions from the auxiliary system if they have an ability to collude.

<sup>3</sup>The origin blockchain  $O$  might be probabilistically finalized, in which case a transaction to a contract can always assert that it is valid only on the intended branch of history by referencing a historical blockhash.

<sup>4</sup>While a vote counted on a BFT system itself is already BFT for a simple majority, we still require a supermajority, because the votes to commit a meta-block will be collected on the auxiliary system for which there is no assumption it is BFT, and we want Mosaic to be able to finalize transactions on the auxiliary system before they are committed to origin.

$\langle s, t, h_s, h_t \rangle_v$  signed by a validator  $v$ . A  $+2/3$  supermajority signing of the same link makes it a supermajority link which justifies the target block if the source block is itself already justified.<sup>5</sup>

In the work it is shown that checkpoints along a justified chain can additionally be considered *finalized*, if and only if they are justified themselves and their direct child checkpoint is justified. It is then shown that, should validators finalize a checkpoint on a contradicting branch of the history of the underlying blockchain, minimally  $1/3$  of the validator weight must have signed vote messages that violate either one of two rules: *the Casper slashing conditions*.

These slashing conditions can be intrinsically validated given two signed vote messages from a validator  $v$ . As such a validator must never sign two vote messages  $\langle s_1, t_1, h_{s_1}, h_{t_1} \rangle_v$  and  $\langle s_2, t_2, h_{s_2}, h_{t_2} \rangle_v$  for which either  $h_{t_1} = h_{t_2}$  or  $h_{s_1} < h_{s_2} < h_{t_2} < h_{t_1}$  holds.

It is additionally shown that validators can always finalize a new checkpoint, without being required to violate the slashing conditions, given that the block proposers propose blocks to append to the finalized fork, i.e. follow the fork selection rule of the overlay network.

## 2.2. Finalizing Auxiliary Systems

Given a validator set  $\mathcal{V}_B$  staked for the meta-blockchain on origin, validators  $v \in \mathcal{V}_B$  can submit, on the auxiliary system, vote messages about the auxiliary system of the form

$$\langle \tilde{T}, s, t, h_s, h_t \rangle_v \quad (2)$$

where  $\tilde{T}$  is the keccak256 hash of the transition object  $T$ .

We introduce a transition object hash in the vote message to externalize otherwise intrinsic properties of the auxiliary system. This transition object allows the Mosaic validators to coarse-grain and abstract the auxiliary system into a meta-blockchain when proposing meta-blocks on origin.

We require that any transition object is calculable by a smart contract on the blockchain in question for any finalized checkpoint along a justified chain. For a given link, we define the transition object to refer to the state of the blockchain at source block  $s$ . It then follows that for a given source blockhash  $s$ ,  $\tilde{T}$  is uniquely determined by  $s$  and the same properties of accountable safety and plausible liveness hold for a finalized checkpoint on a justified chain constructed by such *externalized vote messages* (2), if we extend the slashing conditions to accommodate for the transition object hash.

A validator  $v \in \mathcal{V}_B$  must never sign two externalized vote messages  $\langle \tilde{T}_1, s_1, t_1, h_{s_1}, h_{t_1} \rangle_v$  and  $\langle \tilde{T}_2, s_2, t_2, h_{s_2}, h_{t_2} \rangle_v$  such that any of the following three conditions holds:

1.  $h_{t_1} = h_{t_2}$ ,
2.  $h_{s_1} < h_{s_2} < h_{t_2} < h_{t_1}$ ,
3.  $s_1 = s_2 \wedge T_1 \neq T_2$ .

As the slashing conditions can be intrinsically asserted to have been violated given two externalized vote messages by the same validator, there is no communication overhead to assert a possible violation of these conditions on the origin blockchain; even if the justified chain is being constructed on the auxiliary system. As a result, the validators in  $\mathcal{V}_B$  can be held accountable on the origin blockchain for their voting actions on the auxiliary system. The slashing condition can be asserted on both systems and there is a clear incentive for the honest validators to assert any violation without delay on both the origin and auxiliary system, naturally synchronising the validator weights when such an event occurs.

We will later in this work address a dynamic validator set  $\mathcal{V}_B$ , where validators can join and log out on the origin system. However, already with a static validator set, we can observe that the finality gadget first introduced for (re)defining economic finality in layer-2 of Ethereum's PoW – turning miners into mere block proposers and introducing a PoS (partial) consensus engine in the smart contract layer – can also be applied to finalize an independent (auxiliary) system with a validator set whose stake is held on an external (origin) blockchain.

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<sup>5</sup>The genesis block of the auxiliary blockchain is considered justified by definition.

### 2.3. Observing Origin

Per construction, the finalization of the auxiliary system by the Mosaic validators economically binds the block proposers of the auxiliary system to the Mosaic fork selection. By finalizing the auxiliary system the Mosaic validators reach consensus about the auxiliary system itself. However, to be able to pass messages bidirectionally between chains, the Mosaic validators are additionally required to reach consensus about their observation of the origin blockchain on the auxiliary system.

To achieve this, the Mosaic validators construct a justified chain and finalize checkpoints along it for reported blocks of the origin blockchain on the auxiliary system. The incentive structure is now reversed and the origin blockchain is in no way incentivized (nor should it be) to follow the fork selection rule of how a meta-blockchain's validator set finalized its observation of the origin blockchain.

In case the origin blockchain is probabilistically finalized, then it is always possible that the Mosaic validators of a given meta-blockchain running on top of the origin blockchain would economically finalize an observation of origin on the auxiliary system which is (later) reverted by the origin blockchain - even if they sufficiently trailed the head of origin. Note that the validators cannot revert their finalized observation, because they would be required to sign vote messages which would violate the slashing conditions.

Under this scenario we must force the meta-blockchain to halt at the highest finalized checkpoint of the auxiliary system which was still consistent with its observation of the (now reverted) history of the origin blockchain. This property can be enforced by including in the transition object of a justified chain of the auxiliary system  $T^A$  information about the finalization of the observation of the origin blockchain  $T^O$ ,

$$T_i^A = (O_f^j, \dots),$$

where  $O_f^j = f(T^O)$  is the function that returns the highest finalized block number  $j$  and blockhash of the origin system as observed by the Mosaic validators for this meta-blockchain.

The origin blockchain can inspect  $T^A$  to assert that the highest finalized observation of itself on the auxiliary system is within its current history. Should this not be the case, then the origin blockchain must reject meta-blocks containing contradictory observations.

However, origin cannot directly assert that prior to the last finalized observation, there was no checkpoint finalized from a contradictory branch of its history, as the nodes on origin should not fully verify the transactions executed on the auxiliary system. This is resolved by introducing an option to challenge a proposed finalized observation.

Assume an observed checkpoint  $a$  was finalized on a contradictory branch of origin relative to the last finalized observed checkpoint  $b$  included in  $T^A$ . Any honest node can challenge the finalization of  $b$  on origin by presenting the finalization of  $a$  and demonstrating that  $a \notin \text{history}(b)$ . Note that if validators would want to alter the finalized observation from  $o \rightarrow a \rightarrow b$  to  $o \rightarrow b$  they would have to produce vote messages violating the slashing conditions given the already existing vote messages.

Upon success, the challenger is rewarded from the stake of the offending validators and the core contract must declare the meta-blockchain halted.

### 2.4. Calculating the Transition Objects

We construct a transition object  $T^A$  to coarse-grain a vast amount of transactions processed on the auxiliary system under the state transition rules  $t_A$  into a single, abstracted state transition that can be validated by the core contract on the origin blockchain (under the state transition rules of the origin blockchain  $t_O$ , eq. 1).

On the auxiliary chain with every block running parameters are tracked and these consist of the latest finalized observation of the origin blockchain  $O_f^j = f(T^O)$ , the *accumulated transaction root*  $r_i$ , the *accumulated gas*  $g_i$  consumed, and the current *dynasty number*  $d$  on auxiliary. The transition object  $T^A$ , however, is updated at every justified checkpoint  $s$  with height  $h_s$  and we write for a given meta-block height  $n$ :

$$T_{n,d}^A(s) = (d, O_f^{j_d}, r_d, g_d, \tilde{K}_n)$$

where  $\tilde{K}_n$  is the keccak256 hash of the kernel  $K_n$ .

A smart contract calculates the parameters that go into  $T^A$  for every block. Therefore, the validators have to report the block header of every block to the smart contract. If the reported block is within the most recent

256 blocks of the auxiliary blockchain,<sup>6</sup> then the smart contract can verify its correctness by accessing the corresponding block hash. If the validators fall behind more than 256 blocks in reporting, they can report more than one block per new block in order to catch up. The smart contract will record all reports, but only mark them as valid if they build a chain that reconnects to a block hash within the most recent 256 blocks of the current branch.

Tracking of  $T^A$  begins at the genesis checkpoint. For the genesis checkpoint, the accumulated transaction root is defined as the transaction root of the block, the accumulated gas consumed equals the gas consumed in the block, and the current dynasty number is 0. For all subsequent blocks, the accumulated transaction root  $r_i$  at block height  $i$  is  $\text{keccak256}(r_{i-1}, R_i)$ , where  $R_i$  is the transaction root of the block at height  $i$ . The accumulated gas consumed  $g_i$  at block height  $i$  is  $g_{i-1} + G_i$ , where  $G_i$  is the gas consumed in the block at height  $i$ . The dynasty number equals the number of finalized checkpoints in the chain from the root checkpoint to the parent block, carrying over the definition of dynasty number as in Casper FFG [2].

Further, we introduce a constant *core identifier*, a 256-bit string where the first 12 bytes are a constant specifying the origin blockchain and the rightmost 20 bytes are determined by the smart contract address of the core contract on the origin blockchain. Rather than storing the core identifier in the transaction object, it can be included as a constant in the signing string for vote messages to ensure vote messages are valid only about the intended meta-blockchain. An origin blockchain also has a core identifier to identify vote messages about origin. For core identifiers that identify an origin blockchain, the rightmost 20 bytes are all zero.

The constant that specifies Ethereum main net as the origin blockchain is `bytes12(1)`.

## 2.5. Proposing Meta-Blocks On Origin

For a meta-blockchain  $\mathcal{B}$  the chain can be appended by proposing and committing new meta-blocks  $B_n(K_n, T_n^A, S_n)$  in the core contract on the origin blockchain. The genesis meta-block  $B_0$  is considered committed per definition.

The kernel  $K_n$  is a tuple  $(n, \tilde{p}_n, \Delta\mathcal{V}_{\mathcal{B}_n}, gt_n)$  fully determined by and stored in the core contract on the origin blockchain. On a given branch of the origin blockchain a meta-blockchain cannot fork. The act of committing a meta-block  $B_{n-1}(K_{n-1}, T_{n-1}^A, S_{n-1})$  activates the new kernel  $K_n$  at height  $n$  and *opens* the core contract to accept proposals of the form  $B_n(K_n, \cdot, \cdot)$ . The kernel further specifies the *parent hash*  $\tilde{p}_n$  and the *updated validator weights*  $\Delta\mathcal{V}_{\mathcal{B}_n}$  to declare new validators joining and existing validators logging out.

**gas target** Lastly  $gt_n$  is the *gas target* which introduces a forcing function on the total amount of gas that can be consumed in the meta-block  $B_n$ . The earliest finalized checkpoint at which the gas target has been consumed by the meta-block must be committed to the origin blockchain by the validators. We highlight that this is not a maximum gas limit, because it cannot be guaranteed that a checkpoint can be finalized before a hard gas limit would be surpassed. Rather validators would gradually lose part of their stake as a function of the number of finalized checkpoints they failed to commit after the gas target has been surpassed.

To show that validators can be held accountable, assume the finalized checkpoint  $s_d$  has been committed which surpassed the gas target and has dynasty number  $d$ . Now assume there exists a finalized checkpoint  $s_{d'}$  which also has surpassed the gas target but has a lower dynasty number  $d' < d$ , but was not committed before  $s_d$ . Any honest node can present the  $s_{d'}$  and the core contract can assert the gas target had been consumed at a lower dynasty number than what is committed and reward the challenger.

**committing a meta-block** Given a committed meta-block  $B_{n-1}$  at height  $n-1$  a proposal for a meta-block  $B_n(K_n, T_n^A, \cdot)$  at height  $n$  is a valid proposal if validity assertions for the transition object  $T_n^A$  hold. These validity assertions require that the dynasty number and the gas consumed are strictly increasing compared to the transition object committed  $T_{n-1}^A$ . The transition object  $T_n^A$  must reference the correct kernel  $\tilde{K}_n$ . It must be checked that the latest finalized observation of origin is within the history of the current state. Furthermore a brief challenge period exists where the transition object can be contested on the grounds that it contains a prior observation of origin that contradicts the current history of origin, as explained above in section (2.3).

It now follows that for a given active kernel  $K_n$  multiple proposals  $T_{n,d}^A$  can be submitted to the core contract for different dynasty numbers  $d$ . These proposals are no longer contradicting versions of the history, rather they are ordered, sequenced snapshots of possible state transitions  $B_{n,d_n}(\Sigma_{A,d_{n-1}})$  that can be committed onto origin as a new meta-block.

<sup>6</sup>This is specific to the Ethereum virtual machine, but the logic can be applied for other blockchain systems as well.

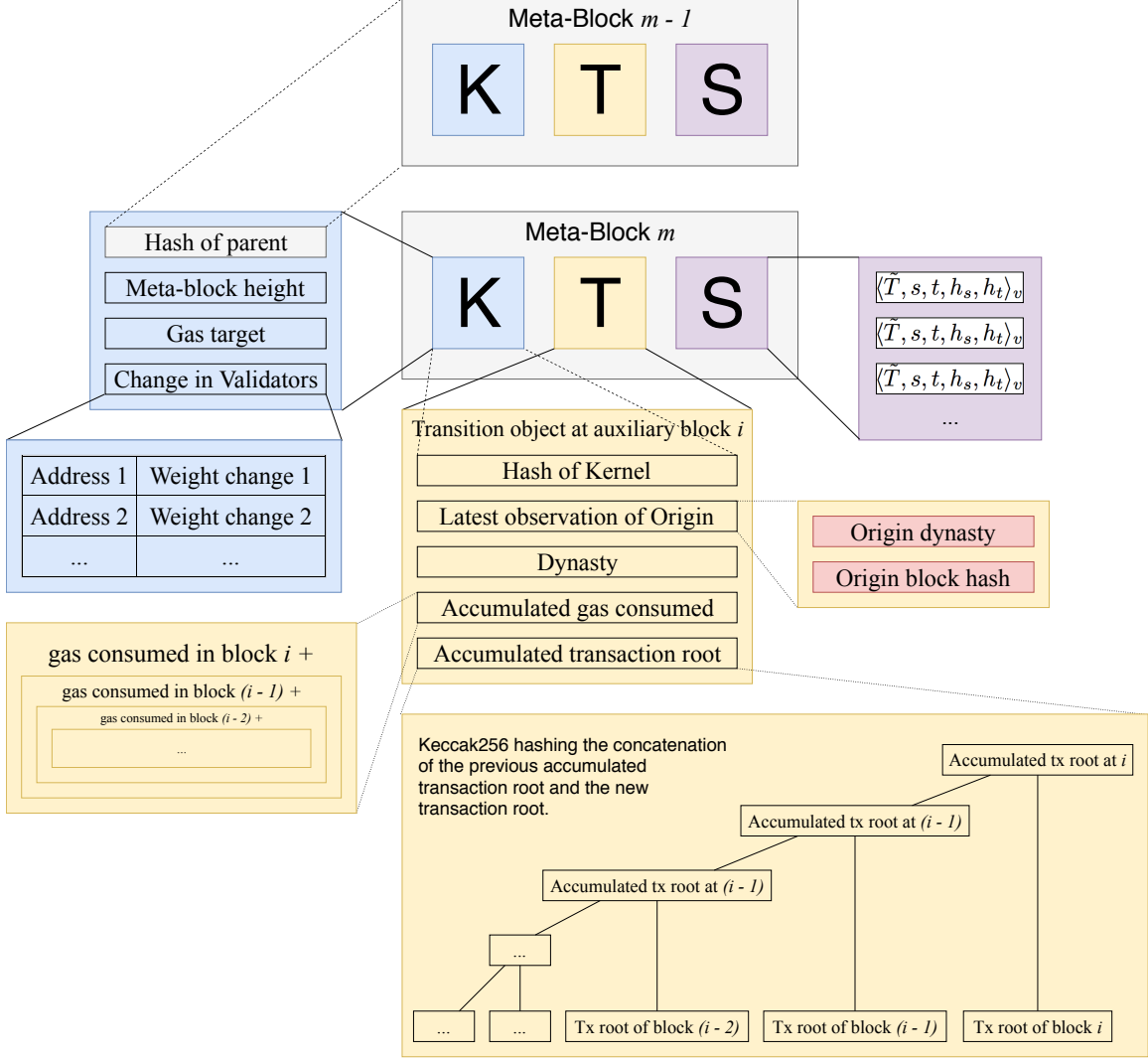


Figure 1: **Anatomy of a meta-block.** The meta-block consists of a kernel, a transition object, and a seal.

To commit a meta-block then requires selecting among the valid proposals a dynasty number  $d_n$  for which the meta-block will be closed. This selection occurs automatically by sequentially verifying the vote messages on the origin blockchain. The meta-block  $B_n(K_n, T_n^A, S_n)$  will be committed with the seal  $S_n$  which first asserts a  $+2/3$  supermajority of the voting weight  $\mathcal{V}_{B_n}$  for a valid proposal in the set of valid proposals

$$\{S_{n,d}\} = \left\{ \left\{ \langle \tilde{T}_{n,d}^A, s, t, h_s, h_t \rangle_v \right\} \right\}$$

where it is required that the supermajority link finalizes the source  $s$ . As such we require that  $h_t - h_s = 1$  and the source  $s$  must have been justified.

Following the same reasoning as presented in section (2.3) we can assert whether or not the finalized checkpoint  $s$  was obtained through an unbroken justified chain leading up to the finalized checkpoint, by allowing for a challenge<sup>7</sup> to be raised against a proposed transition object  $T^A$  included in a vote message  $\langle \tilde{T}^A, s, t, h_s, h_t \rangle_v$ .

However, recall that in the case of verifying observations of origin (2.3), origin itself can act as an arbiter because origin can rely on its own history. Our concern now is to assert that the state transition to  $s$  from the last committed state  $B_{n-1}$  is a valid state transition according to the state transition rules  $t_A$  of the auxiliary system.

<sup>7</sup>A challenger must always put forward a sufficient stake to underwrite the cost implied by her challenge.

**proof** Assume first a contradictory finalisation  $s'$  of the auxiliary system relative to  $s$  exists. This implies that more than  $1/3$  of the validator weight can be slashed. The core contract on origin must declare the meta-blockchain halted when it can slash more than the safety threshold of  $\mathcal{V}_{B_i}$  at a single meta-blockchain height  $i$ .<sup>8</sup>

Alternatively, no contradictory finalisation  $s'$  of the auxiliary system relative to  $s$  exists. Under the security model assuming a  $+2/3$  supermajority of the validator weight is honest, it follows that the proposed state transition to  $s$  has been obtained through honest verification of all transactions under the state transition rules  $t_A$ .

Before continuing we summarize that the proposal mechanism to commit a meta-block is asynchronous, requires no leadership-selection and is proven to be Byzantine fault-tolerant under the security model of an honest supermajority.

## 2.6. Beyond An Honest Supermajority

While the origin blockchain protects assets from being double-spent, users in a meta-blockchain could see their funds stolen if more than a supermajority of the open validator set would collude to violate the transition rules  $t_A$ .

We identify at least three active research topics which can allow to push the security model radically further in an efficient way; namely that (up to) a *single* honest observer can hold all validators accountable when a proposal for committing a meta-block to the origin blockchain would violate the state transition rules  $t_A$ . It is outside the scope of the current work to explore in-depth, but they are worth summarising below as future work paths.

**observer network** In *A Guide to 99% Fault Tolerant Consensus*[3] V. Buterin recaptures an existing algorithm by L. Lamport (1982) to describe how a *latency-dependent* network of observers can be retrofitted on top of a *threshold-dependent* consensus algorithm - in casu our here work as derived from Casper FFG - to introduce a *strong-finality* upon the finalized history constructed by the Mosaic validators arbitrarily pushing up the fault-tolerance at the cost of requiring more observers and a longer time to eventually commit (a strongly finalized proposal).

**truebit** A second worthwhile path is to apply the *TrueBit*[4] protocol to challenge a state transition  $T^A$  proposed by a subset of the validators  $\mathcal{V}_{B_i}$ . The validators  $\{v\}$  who have signed vote messages  $\langle \tilde{T}^A, s, t, h_s, h_t \rangle_v$  can be seen as the (collective) *solvers* of the off-(of-origin-)chain computation to transition the state  $B_{n-1}$  to the proposed solution  $s$  given the set of transactions specified by the accumulated transaction root  $r_{h_s}$ . The computation at hand is the execution of the virtual machine that implements the state transition rules  $t_A$ .

If less than  $1/3$  of the validator weight loses the verification game to a challenger, their stake can be taken as the reward for the challenger, on top of the shared interest of all participants in the meta-blockchain for only correct proposals to be committed to the origin chain - removing the need for a jackpot and a *forced-error*. The verification game can be seen as an extension to the slashing conditions strengthening the accountable safety of the meta-blockchain to hold a minority of the validators accountable for violating the state transition rules  $t_A$ . This is made possible because the validator stake has been externalized on the origin blockchain with an independent (assumed correct) consensus mechanism.

If more than  $2/3$  of the validator weight signed the vote message which was challenged and lost the verification game, their stake is the reward for the challenger, and the meta-blockchain must be declared halted by the core contract.

**argument of knowledge** A third strategy can be constructed by requiring any of the challenged validators of a proposal  $T^A$  to present an *argument of knowledge*, eg. using zkSNARKs generated after being challenged, to cryptographically prove that the block  $s$  has been obtained by correctly applying the state transition rules  $t_A$  recursively on each parent block building up from the committed block found in  $B_{n-1}$ .

The costs both in time and expense to generate this proof by a validator scale at first approximation linearly with the gas consumed in the proposed meta-block  $B_n$ . The transition object  $T^A$  to commit this meta-block

<sup>8</sup>When a meta-blockchain has been halted, assets and stateful objects can be recovered on the origin blockchain at any later time with Merkle proofs against the last committed state root of the meta-blockchain. We will detail the conditions under which a meta-blockchain must halt and how the recovery processes can work later in this writing.

tracks the gas consumed, so the core contract can require that the challenger puts forward a sufficient stake to substantiate her claim.

There are open considerations regarding the (economic) feasibility of requiring all validators to be able to generate such a proof on commodity hardware. The task of generating these proofs could be designated to a specialized node, but an incentive problem arises when it is economically unattractive to run such a node considering it is unlikely that a proposal would be challenged.

We have gone beyond the assumption of an honest supermajority of validators by considering three strategies discussed in the field to radically improve the security model; up to the point where a single honest observer can cost-effectively challenge the validator set on the origin blockchain.

## 2.7. Dynamic Set of Validators

The set of validators for the meta-blockchain  $\mathcal{B}$  must be dynamic to allow new validators to join and for validators to safely exit from their responsibilities. It is defined for each meta-block height  $n$  and we note  $\mathcal{V}_{\mathcal{B}_n}$ .

A validator  $v$  is defined as an address which has a strictly-positive validator weight  $w_v$  associated with it on the origin blockchain. A validator can join the set of validators by sending a *deposit message* to the origin blockchain and putting forward a stake. A validator can initiate log out by sending a *logout message* to origin.

The weight of a validator  $w_v$  at meta-block height  $n$  is the product of the stake of the validator at that height  $s_v(n)$  multiplied with a *reputation function*  $R(n; v, n_0)$

$$w_v(n) = s_v(n) \cdot R(n; v, n_0) \quad (3)$$

where the reputation function  $R$  has values in the range  $[0, 1]$  and  $n_0$  is the meta-block height where the validator  $v$  sent the deposit message.

The reputation function allows for different security models to mitigate collusion attempts of malicious actors within the validator set of the meta-blockchain. As examples, the reputation function can require initial work by the validator before obtaining a non-zero reputation. The reputation function can also enforce a finite (probabilistic) lifetime for all validators, honest and dishonest alike. Inducing churn in the (honest) validator set adds an important stochastic tool to prevent collusion of more than  $1/3$  of the validator weight.

Changes to the validator weights  $w_v(n)$  are included in the kernel  $K_n$  as  $\Delta \mathcal{V}_{\mathcal{B}_n}$ . When tallying votes both the origin blockchain and the auxiliary system must use the *effective weight*  $W_v(n)$ ,

$$W_v(n) = w_v(n) \cdot \theta_v$$

where  $\theta_v$  is initially 1 for all validators, but irreversibly and immediately set to 0 when a validator  $v$  is shown to have violated any slashing condition. The effective weight allows for the voting power of an offending validator to be revoked on both blockchains instantaneously without waiting for the meta-block to be committed.

The stake  $s_v(n)$  of a validator can only be preserved or decrease for increasing  $n$  when the validator does not fulfill its responsibilities. The *loss function*  $L$  recalculates the stake of a validator for the new meta-block height

$$s_v(n) = L(s_v(n-1); v). \quad (4)$$

The loss function reduces the actual value the validator can possibly withdraw in the future. The loss function can reward challengers for presenting proofs of bad behaviour by the validator from his stake and it can burn the remaining reduction.

When a validator violates any slashing condition the loss function returns the validator stake to zero.

Validators can see their stake reduced when they become inactive in the voting process (*inactivity leak*<sup>9</sup>), or when they fail to commit a meta-block back to origin according to the gas target requirements (section 2.5).

When a validator successfully logs out the reputation is set to zero, but the stake remains locked in the contract for a fixed amount of time to protect against long range revision attacks.<sup>10</sup> After this time the validator can withdraw his stake (and remaining rewards).

<sup>9</sup>The inactivity leak forces validators to participate in the voting process and be available. The stake of inactive validators can gradually be diluted if they fail to verify their vote on a committed meta-block. We credit the term to Casper FFG[2] for the original argument.

<sup>10</sup>By holding on to the stake the validator can still be held accountable for evidence of wrongdoing that may surface later or new attempts of revising the history. We follow Casper FFG's proposal of approximating four months delay before withdrawal is allowed.



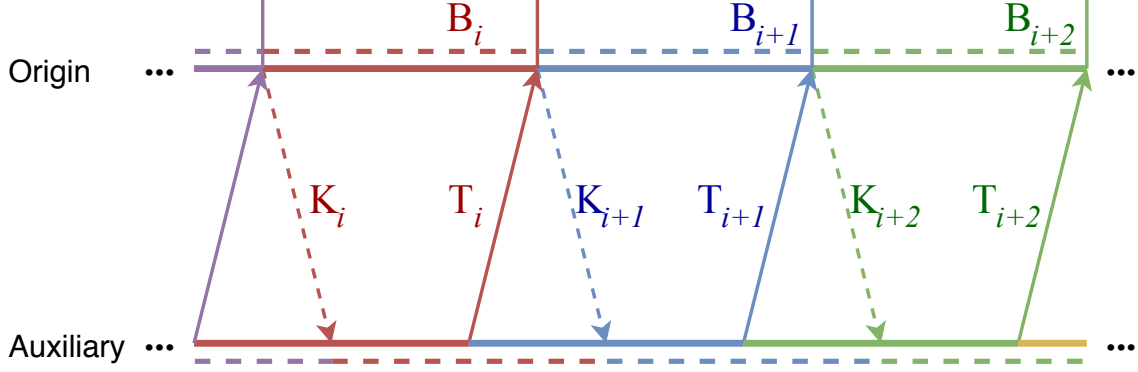


Figure 2: **Asynchronous composition.** When a proposed transition object  $T_i$  is committed as meta-block  $B_i$  on origin (solid red line), the new kernel  $K_{i+1}$  is opened on origin. The validator set  $\mathcal{V}_{B_i}$  casts votes on the auxiliary system to finalise both the checkpoints of the auxiliary system and its observation of the origin blockchain on the auxiliary system (dashed red line on aux). The commit of  $B_i$  and opening of  $K_{i+1}$  is observed by  $\mathcal{V}_{B_i}$  and  $K_{i+1}$  is confirmed on the auxiliary system transitioning to the new validator set  $\mathcal{V}_{B_{i+1}}$  (dashed blue line on aux). On the auxiliary system all processes continue uninterrupted. At each confirmation of a new kernel  $K_j$  the changes to the validator set  $\Delta\mathcal{V}_{B_j}$ , and the new gas target  $gt_j$  are enacted on the auxiliary system.

**asynchronous composition of blockchains** We refresh for the reader (section 2.5) that a meta-block  $B_i$  is committed on origin with a valid<sup>11</sup> transition object  $T_i^A(\bar{K}_i, \dots)$  for which a seal  $S_i$  was created on origin by verifying a supermajority of the voting weight  $\mathcal{V}_{B_i}$  on vote messages of the form  $\langle \bar{T}_i^A, s, t, h_s, h_t \rangle_v$ . As the hash of the kernel  $\bar{K}_i$  is included in the transition object, and as the hash of the transition object  $\bar{T}_i^A$  is included in the vote messages, the order to construct a meta-block  $B_i(K_i, T_i^A, S_i)$  is logically enforced.

When a proposed transition object  $T_i^A$  is committed as meta-block  $B_i(K_i T_i^A, S_i)$  then the state root of the auxiliary system is immutably anchored into the origin blockchain under the blockhash  $s$ <sup>12</sup>. It follows that the state of the meta-blockchain is first knowable on the auxiliary system and causally passed to the origin blockchain upon committing  $B_i$  (see solid lines on figure 2).

A new kernel  $K_{i+1}$  is defined by the committed meta-block  $B_i$ , the deposit, logout and slashing messages<sup>13</sup> that were included on origin during height  $i$ <sup>14</sup> and a new gas target  $gt_{i+1}$  for closing  $B_{i+1}$ . These messages combined with the recalculation of the validator weights (eq. 3) update the validator set

$$\mathcal{V}_{B_{i+1}} = \Delta\mathcal{V}_{B_{i+1}}(\mathcal{V}_{B_i}) \quad (5)$$

which is included in  $K_{i+1}$  as the operator  $\Delta\mathcal{V}_{B_{i+1}}$ .

The validator set  $\mathcal{V}_{B_i}$  is activated on the auxiliary system when the kernel  $K_i$  is *confirmed* on the auxiliary system. We define the confirmation of a kernel to require two conditions to be met. First, a Merkle proof that  $K_i$  exists on the origin blockchain must be presented against a finalised observation of (the state of) the origin blockchain as finalised by the validators set  $\mathcal{V}_{B_{i-1}}$ . Second, if this Merkle proof of the kernel  $K_i$  was included on the auxiliary system at dynasty  $d$  then we consider  $K_i$  confirmed when this branch of history reaches dynasty  $d + 2$ . The dashed lines on figure 2 indicate the activity period of  $\mathcal{V}_{B_i}$  and start at opening of  $K_i$  on origin and the confirmation of  $K_i$  on the auxiliary system.

<sup>11</sup>A valid proposal is a transition object  $T^A$  which satisfies the validity checks and has been unchallenged, or has been successfully proven upon being challenged.

<sup>12</sup>We assume here without loss of generality that the blockheader of the auxiliary system includes the state root of the system at height  $h_s$  and that the blockhash is the hash of an encoding of the blockheader.

<sup>13</sup>Slashing messages can concern both violations of slashing conditions reducing the stake of a validator to zero, or a proof of inactivity gradually reducing the validator stake.

<sup>14</sup>We say a transaction on origin is included during height  $i$  if it is included in the history of origin before  $B_i$  is committed and after  $B_{i-1}$  was committed.

For the auxiliary system all other processes continue uninterrupted, such as the calculation of accumulated gas consumed and the accumulated transaction root. At each confirmation of a new kernel  $K_j$  the changes to the validator set  $\Delta\mathcal{V}_{B_j}$ , and the new gas target  $gt_j$  are enacted on the auxiliary system.

On the origin blockchain the state for the meta-block  $B_i$  is voted on by  $\mathcal{V}_{B_i}$  and the activity period of the validator set  $\mathcal{V}_{B_i}$  coincides with the duration of the meta-block height  $i$  on origin. On the auxiliary system the activity period of the validator set  $\mathcal{V}_{B_i}$  is delayed relative to the state that will be included in the meta-block  $B_i$ , as  $T_{i-1}$  must be committed on origin and  $K_i$  confirmed. It is guaranteed that the confirmation of  $K_i$  is included in the state transition of  $B_i$  if  $T_i$  is calculated by a smart contract on the auxiliary system.

**forward and rear validator set** To ensure that a major change to the validator set cannot introduce an attack vector, we apply an analogous defence strategy as described in Casper FFG[2] as the “stitching mechanism”.

When a validator’s deposit message is included on origin in meta-block height  $i$ , the *start meta-block height*  $n_0(v)$  is equal to  $i + 1$  and the validator joins in the next validator set  $\mathcal{V}_{B_{i+1}}$ <sup>15</sup>. When a validator  $v$  sees its logout message included on origin in meta-block height  $j \geq i + 1$ , then the *end meta-block height*  $n_e(v)$  is equal to  $j + 1$  and the validator participates for a last time in the validator set  $\mathcal{V}_{B_{j+1}}$ . Before a logout message for a validator is included the end meta-block height is considered infinite.

We use the start and end meta-block height of a validator to define two subsets of the validator set  $\mathcal{V}_{B_n}$ . The *forward validator set*  $\mathcal{V}_{B_{f,n}}$  includes all active validators, except those who are in their end meta-block height  $n_e(v)$ :

$$\mathcal{V}_{B_{f,n}} = \{v \in \mathcal{V}_{B_n} : n < n_e(v)\} = \mathcal{V}_{B_n} \setminus \{v : n = n_e(v)\} \quad (6)$$

and the *rear validator set*  $\mathcal{V}_{B_{r,n}}$  includes all active validators, except those who have commenced in their start meta-block height  $n_0(v)$ :

$$\mathcal{V}_{B_{r,n}} = \{v \in \mathcal{V}_{B_n} : n_0(v) < n\} = \mathcal{V}_{B_n} \setminus \{v : n = n_0(v)\}. \quad (7)$$

We redefine that a supermajority link over the validator set  $\mathcal{V}_{B_n}$  henceforth means that a  $+2/3$  supermajority of validator weight has voted for the link counted over both the rear and the forward validator set separately:

$$\mathcal{S}(\mathcal{V}_{B_n}) \doteq \mathcal{S}(\mathcal{V}_{B_{f,n}}) \wedge \mathcal{S}(\mathcal{V}_{B_{r,n}}). \quad (8)$$

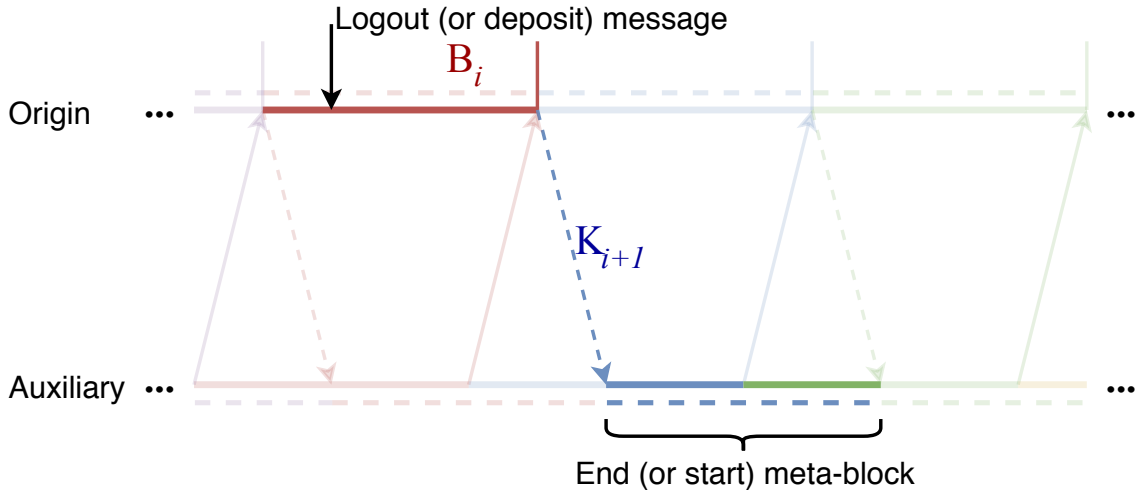


Figure 3: **Dynamic set of validators.** The validator sends a log out (or deposit) message in  $B_i$ . The change in the set of validators is transferred to auxiliary as part of kernel  $K_{i+1}$  when it gets confirmed. The validator does its last (first) round in the rear (forward) validator set of  $\mathcal{V}_{B_{i+1}}$  ( $\mathcal{V}_{B_{r,i+1}}$  or  $\mathcal{V}_{B_{f,i+1}}$  respectively).

<sup>15</sup>We note that in Casper FFG[2] the analogous *start dynasty* of a validator is defined as  $d + 2$ . The confirmation of the kernel requires a full dynasty increase, i.e.  $d + 2$ , for it to be confirmed on the auxiliary system, hence it is sufficient for validators to join in the validator set of the next meta-block, i.e.  $\mathcal{V}_{B_{i+1}}$

### 3. A Mosaic of Cores

summary: By constructing asynchronous BFT consensus rules to compose a heterogeneous auxiliary blockchain into the origin blockchain, we leverage the security of the origin blockchain to asynchronously finalize the transactions on the auxiliary blockchain. As there are no time constraints or time-outs in the Mosaic consensus rules, the limited processing capacity of the origin blockchain does not undercut the ability to compose many auxiliary systems into the same origin blockchain. This enables Mosaic to run many meta-blockchains in parallel, additively improving the total transaction capacity, but keeping a bound on the cost imposed on the origin blockchain.

### 4. Gateway

Gateway protocol allows decentralized applications to move tokens between two homogeneous or heterogeneous blockchain systems. This protocol can be used to move token(ERC20, later non-fungible tokens) between origin blockchain and auxiliary system supporting a specific meta-blockchain where transactions can be finalized faster, and at a lower transaction cost. Gateway protocol leverages the idea of message bus which defines a standard to asynchronously transfer typed messages between two chains.

We define message bus  $M_b$  as a state machine with messages  $m_s$  where  $s$  denotes state, outbox  $o_b$  and inbox  $i_b$  for each blockchain. Messages from source chain are kept in the outbox and once verified on target chain using Merkle proof, they are saved in the inbox of target chain.

$$M_b = (\{m_s^1, m_s^2, \dots, m_s^n\}, o_b, i_b) \quad (9)$$

Gateway contract is deployed on origin chain and twin contract co-gateway is deployed on an auxiliary chain. These two contracts are linked before initiating any message passing. Linking ensures that both contracts are set up to transfer the same token and use the same version of the message bus. Gateway and Co-gateway contracts defines different message types i.e. linking, stake & mint and redeem & unstake. Any facilitator can transfer messages between source and target blockchain by staking bounty as a security amount. Facilitator is rewarded by message initiator along with the bounty on successful completion of message transfers.

#### 4.1. Message Bus

The Message Bus is a messaging framework that allows to atomically communicate typed messages between two homogeneous or heterogeneous blockchain systems. This communication is a two-phased commit on each blockchain to move a message between blockchains. Message bus acts as a state machine where message state is changed from one state to another in response to interaction with message bus. If message  $M$  is intended to move from blockchain  $A$  to blockchain  $B$ , then in this case the  $A$  is termed as the source and  $B$  is termed as target. The state of message  $M$  is stored in outbox of source blockchain and in the inbox of target blockchain.

Similarly if the message is intended to move from blockchain  $B$  to blockchain  $A$ , then the source will be  $B$  and target will be  $A$ . Throughout the paper the terms source and target blockchain will be used.

#### Message

We introduce message  $M$  consists of message type intent hash  $i_t$  where  $t$  defines type of message, nonce  $n$ , gas price  $g_p$ , gas limit  $g_l$ , sender  $s$ , hash lock  $h_l$ . Nonce  $n$  is the count of message moved from source blockchain to target blockchain by the sender  $s$ . Gas price  $g_p$  and gas limit  $g_l$  is the price at which the reward is calculated for facilitation of message transfer, refer 4.8. Hash lock  $h_l$  is a hash that is set by the facilitator<sup>16</sup>, refer section 4.2.4

$$M = (i, n, g_p, g_l, s, h_l) \quad (10)$$

Message hash  $m_h$  is a unique identifier of a message  $M$ . It can be calculated as the hash of message type hash<sup>17</sup>  $t$ , intent hash  $i$ , nonce  $n$ , gas price  $g_p$ , gas limit  $g_l$ , sender  $s$  and  $K$  is *keccak256* hashing function.

<sup>16</sup>refer facilitator section

<sup>17</sup>For more details of facilitator please refer section 4.5

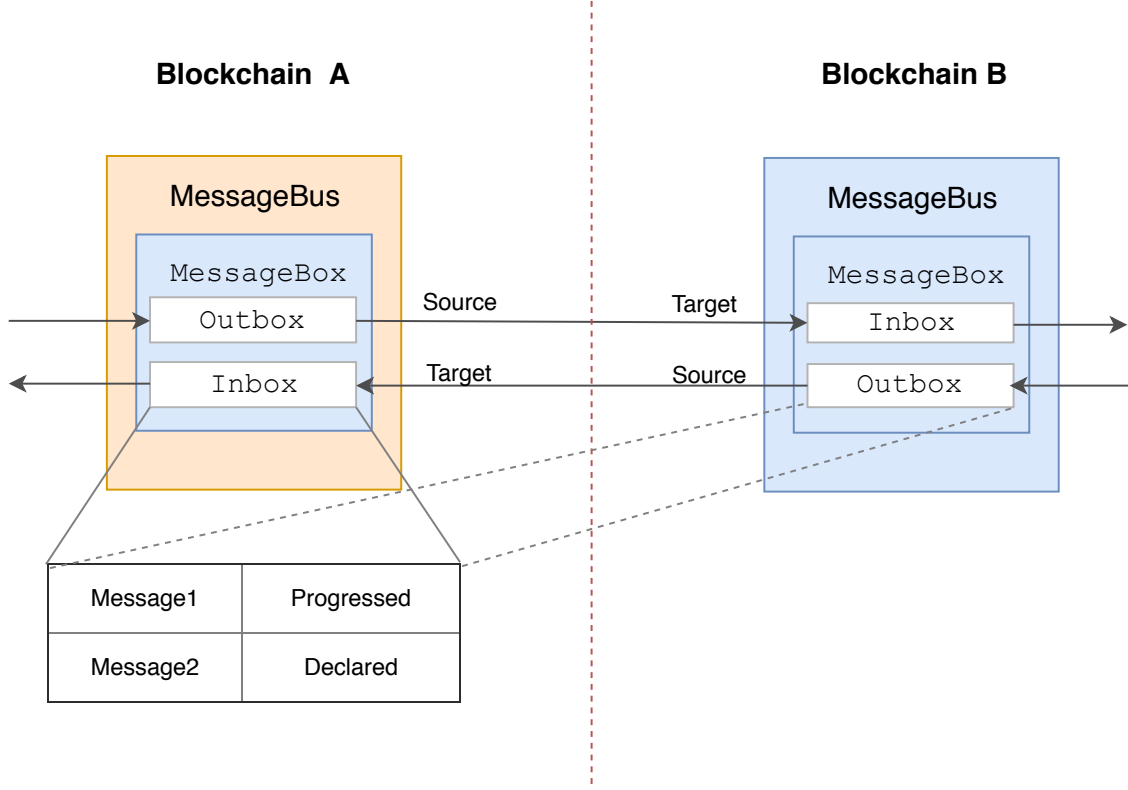


Figure 4: MessageBox of the MessageBus

## State Machine

Message bus acts as a state machine, the transition of message state is deterministic. The possible states  $S$  of the message can be Undeclared  $S_u$ , Declared  $S_d$ , Progressed  $S_p$ , RevocationDeclared  $S_{rd}$  and Revoked  $S_r$ .

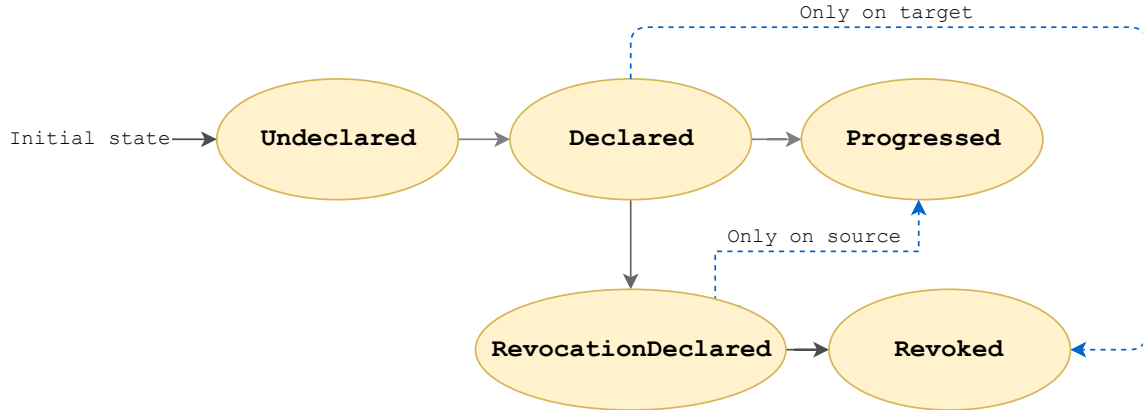


Figure 5: States of the message

Message  $M$  can be moved from source blockchain to target blockchain in four steps consisting of two phased commit transactions in both blockchains each. This four steps are

1. Declare
2. Confirm

3. Progress outbox
4. Progress inbox

## Declare

This is the first step for the message transfer and is done on the source blockchain. For message transfer the message initiator<sup>18</sup> should signs the message hash  $m_h$  of the message  $M$ . The state of message in the outbox must be undeclared. Message bus verifies the state of message and the signature of message initiator. On successful declaration of message  $M$  in outbox, the message  $M$  is state is changed to declared.

## Confirm

This is the second step for the message transfer which is done on the target blockchain. In this step the message bus confirms the declaration of message in the source chain using the merkle proof. The declared message  $M$  along with the merkle proof is presented to the target blockchain by the facilitator for verification. On successful verification of proof the message state is declared in the inbox.

## Progress Outbox

Once the message  $M$  is declared in both source and target blockchain. A merkle proof specifying the state of message as declared in the inbox of target chain can be presented to the source blockchain. On successful verification of merkle proof, the message state is progressed.

## Progress Inbox

Progress inbox can be done on target blockchain by presenting a merkle proof specifying the state of message on source blockchain as declared or progressed. If the current state of message in the inbox of the target chain is declared and the merkle proof is valid, then the message bus changes the state of message to progressed.

This complete the four steps of two phased commit transactions in both blockchains each.

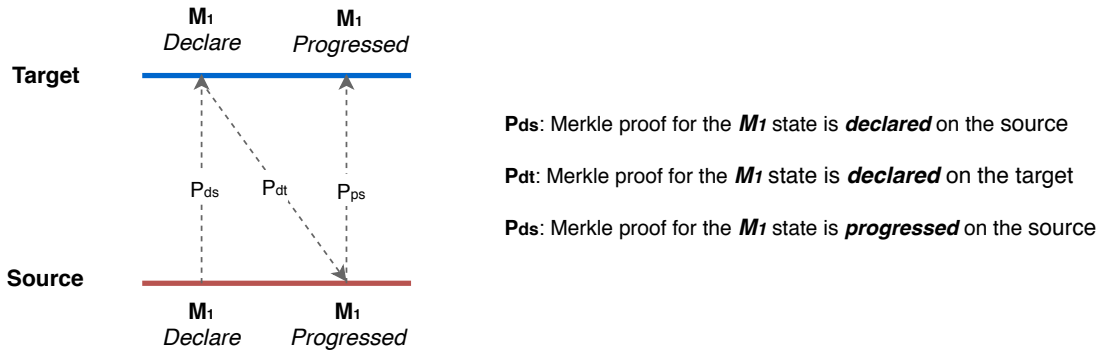


Figure 6: Progress with proof

## Progress with HashLock

As merkle proof verification is costly process, so an alternative mechanism to complete progress inbox and progress outbox is progress with hash lock in both source and target blockchains. While declaring the message  $M$ , an hashlock  $h_l$  can be provided in the message. We define hash lock  $h_l$  as

$$h_l = K(s) \quad (11)$$

where  $K$  is hashing function like *keccak256*,  $s$  is a unlock secret

<sup>18</sup>some text for initiator

Hash lock is added while declaring and confirming message on source and target chain. Progress with hash lock can be used by presenting unlock secret on both chains.

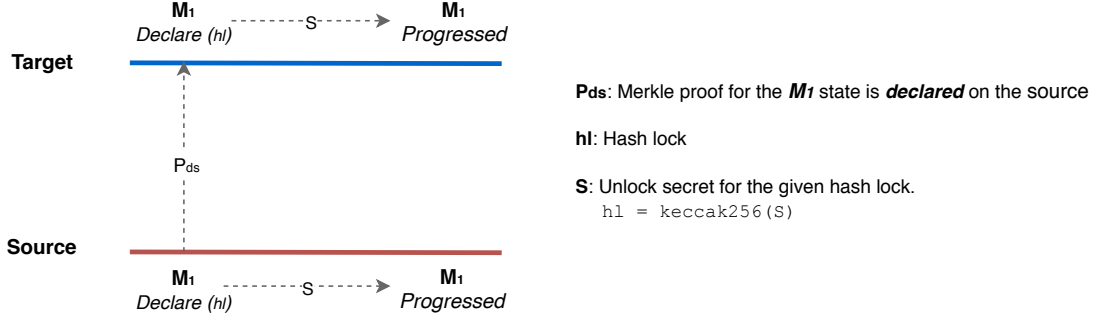


Figure 7: **Progress with hash lock**

## Revocation

Any declared message  $M$  can be revoked if its current state is not already progressed in any of the two blockchains. In the source blockchain the only case to change the state to revocation declared is that current state in the outbox should be declared. For the target blockchain if the current status of the message  $M$  in inbox is declared then a merkle proof can be presented specifying the status as revocation declared in outbox of the source blockchain. If the merkle proof is valid then the status can be changed to revoked in the inbox of target blockchain. Incase the message  $M$  is not declared in the inbox of target blockchain then it needs to be declared first. If the status of message  $M$  is already progressed in the inbox then state cannot be changed.

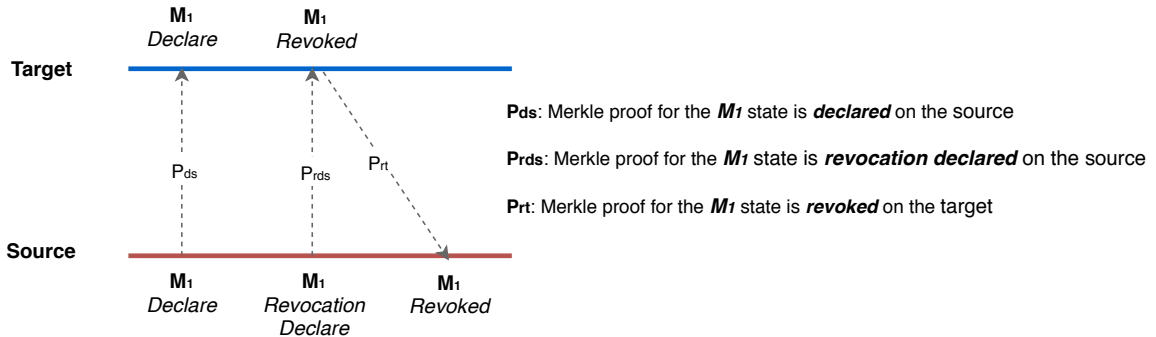


Figure 8: **Revocation of message**

On the source blockchain if the message state of message  $M$  is revocation declared but the status of message in target blockchain is progressed, in this case the status on source blockchain can be changed only to progressed by presenting the merkle proof for message  $M$  specifying the state as progressed in inbox of target chain.

## 4.2. Linking of Gateway and Co-Gateway

Gateway is defined on origin blockchain where as Co-gateway defined on auxiliary blockchain and can co-exists only in pair to be functional. Both the contracts are inactive by default i.e both the contracts cannot transfer any messages except linking messages.

Linking of Gateway and Co-gateway is a special type of message transfer which cryptographically verify uniformity of token and message bus on origin and auxiliary blockchain. As explained in section 4.1, each message contains message type intent hash. For linking messages, we define gateway linking intent hash  $i_{gl}$  as a hash of gateway address  $g_a$ , co-gateway address  $c_a$ , message bus code hash  $m_{ch}$ , ERC20 token name  $t_n$ , token symbol  $t_s$ , token decimal  $t_d$ , nonce  $n$ , token address  $t_a$  and  $K$  is  $keccak256$  hashing function.

$$i_{gl} = K(g_a, c_a, m_{ch}, t_n, t_s, t_d, n, t_a) \quad (12)$$

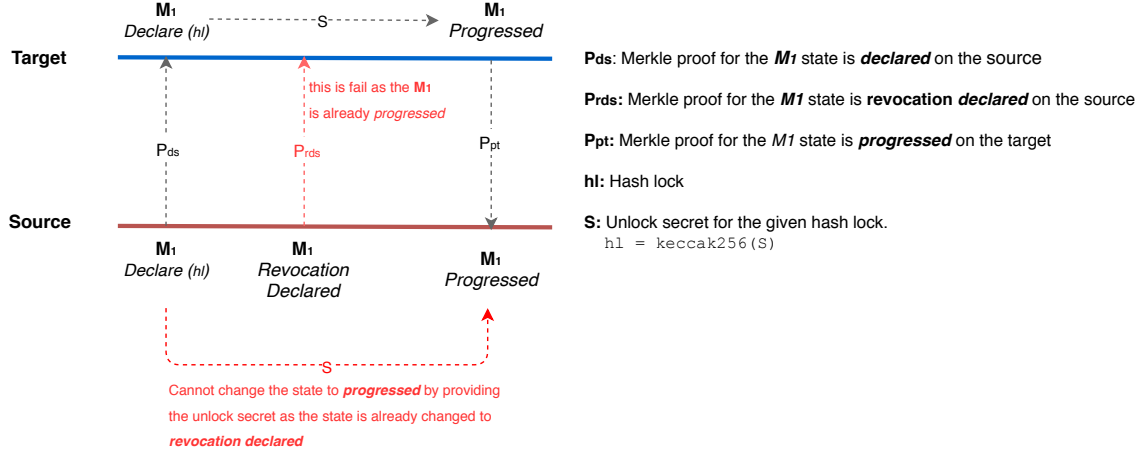


Figure 9: **Progress after revocation declared.**

Once the linking message is declared in gateway, it can be confirmed on co-gateway by merkle proof. After completion of the progress the Gateway and Co-gateway are activated and can perform the other message transfers like stake & mint or redeem & unstake.

### 4.3. Stake and Mint

Gateway protocol supports transfer of stake and mint type typed message from origin to auxiliary chain. Staking happens on origin chain which locks staker's tokens into the escrow and minting happens on auxiliary chain which creates new tokens at beneficiary address. By combining meta-blockchain based layer 2 scaling solution with this message type, a token economy can be designed with the high transaction throughput and low gas cost on auxiliary chain.

Staker needs to submit a signed message hash to facilitator to initiate message transfers. As described in section 4.2, message hash generation needs message type specific intent hash which is staking intent hash  $i_s$  in this scenario. We define staking intent  $i_s$  as :

$$s = K(a_s, b_a, s_a, n_s, g_p, g_l, t_a) \quad (13)$$

Where  $K$  is *keccak256* hashing function,  $a_s$  is staking amount,  $b_a$  is beneficiary address,  $s_a$  is staker address,  $n_s$  is staker nonce,  $g_p$  and  $g_l$  are gas price and gas limit respectively as described in section 4.8,  $t_a$  is ERC20 staking token address.

Gateway along with message bus verifies staker's signature generated by signing message hash and initiates message transfer. Facilitator must stake bounty to gateway during message declaration on source chain as described in section 4.7.

Message declared on origin chain with staking intent hash is verified and confirmed on auxiliary chain with merkle proof. Tokens are minted for beneficiary on auxiliary blockchain after successful progress. On successful mint of token, reward is awarded to facilitator from the minted token refer 4.8 section reward.

### 4.4. Redeem & Unstake

### 4.5. Bounty

In Gateway protocol, facilitator pool is an open network of nodes which are responsible for message transfer between source and target chain. Any node/machine can act as facilitator by staking bounty to gateway during message transfer.

Bounty is the fixed amount staked by facilitator as a security deposit during message transfer. This amount should be just sufficient to maintain the accountability for message transfers by facilitator. Facilitator must approve gateway for bounty amount before declaring message. Bounty enforces facilitator to progress on source chain. It is returned back to facilitator who reveals unlock secret or progresses with the proof.

If message initiator(staker or redeemer) decides to revoke message transfer, it has to pay penalty described in 4.7. Both bounty and penalty are burned on progress revocation.

#### 4.6. Reward

Gateway protocol has reward structure for facilitator for successful completion of message transfers. Section 4.1 explains the four step process to transfer message from source to target blockchain. Facilitator presents hash lock during message declaration on source blockchain. When the correct unlock secret is presented on target blockchain during progress, facilitator is rewarded with reward  $r$ .

We define reward  $r$  as following:

$$r = g_p * \min(g_l, g_c) + K \quad (14)$$

Where  $g_p$  is gas price,  $g_l$  is gas limit,  $K$  is constant and  $g_c$  is total gas consumption.  $g_c$  is the sum of gas spent in declaring inbox message  $g_d$  and progressing inbox  $g_p$  and can be defined as :

$$g_c = g_d + g_p \quad (15)$$

$g_p$  and  $g_l$  is decided by message initiator which determines maximum reward amount  $r_{max}$

$$r_{max} = g_p * g_l \quad (16)$$

#### 4.7. Penalty

Penalty is the amount charged to message initiator for revocation of message transfer. Penalty is charged when initiator decides to revoke message after declaration and before progress. In order to ensure a fair and honest process between the facilitator and the initiator, the penalty charge must always be greater than the bounty amount.

Penalty  $P$  for message transfer revocation is:

$$P = n * b_f \quad (17)$$

Where  $n \geq 1$  and  $b_f$  is bounty staked by facilitator.

#### 4.8. Gateway Deactivating

Gateway deactivating is an ability of Gateway and Co-gateway to restrict the movement of messages from source blockchain to target blockchain. If Gateway is deactivated then no new stake and mint process can be initiated, but this does not restrict the redeem and unstake flow. Users can choose to redeem the tokens even if the Gateway is deactivated. Similarly if a Co-gateway is deactivated then no new redemption process can be initiated, but the stake and mint flow can work. If both the Gateway and Co-gateway is deactivated then this will cause the Gateway to halt.

#### 4.9. Chain Halting

### 5. Outlook

### 6. Conclusion

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