Machine Learning for Neuroscience

Slides and notebooks: `https://github.com/PBarnaghi/ML4NS`

Basics of Matrix Algebra

Matrix: A matrix is an arrangement of elements (e.g., numbers) in rows and columns. A matrix has a dimension which represents the number of rows and columns in that matrix. For example, a 2×3 matrix has 2 rows and 3 columns. A sample matrix is shown below:

$$A = \begin{bmatrix} 11 & 27 & 11 \\ 32 & 6 & -1 \end{bmatrix}$$

Transpose of a Matrix: Transpose of a matrix, shown as A^T, flips the rows and columns of the matrix over its diagonal. For the matrix A show above:

$$A^T = \begin{bmatrix} 11 & 32 \\ 27 & 6 \\ 11 & -1 \end{bmatrix}$$

If a matrix is 2×3 then its transpose will be a 3×2 matrix.

Vector: A vector is a list of numbers (or other elements). A column vector is a $m \times 1$ matrix. A row vector is a $1 \times m$ matrix. For example x is a column vector, and y is a row vector.

$$x = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathrm{y} = \begin{bmatrix} -5 & 0 & 5 \end{bmatrix}$$

Matrix Multiplication: In linear algebra, the matrix multiplication operation produces a matrix from two matrices. To multiply two matrices, the number of columns in the first matrix must be equal to the number of rows in the second matrix. For example, a 3×2 matrix can be multiplied by a 2×5 matrix, and the result will be a 3×5 matrix. But, for example, a 1×3 cannot be multiplied by 1×3 matrix as the number of columns in the first matrix is not equal to the number of rows in the second matrix.

Multiplication of two matrices is done by creating *"dot product"* of rows and columns. For example:

$$A = \begin{bmatrix} -3 & -1 & -2 \\ 3 & 2 & -1 \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & -1 \\ -3 & -1 & -3 \end{bmatrix}$$

$$A \times B = C$$

C_{ij} *refers to the element in row* i *and column* j

$C_{11} = -3 \times 1 + -1 \times 3 + -2 \times -3 = 0$
$C_{12} = -3 \times 2 + -1 \times 6 + -2 \times -1 = -10$
$C_{13} = -3 \times 1 + -1 \times -1 + -2 \times -3 = 4$

$C_{21} = 3 \times 1 + 2 \times 3 + -1 \times -3 = 12$
$C_{22} = 3 \times 2 + 2 \times 6 + -1 \times -1 = 19$
$C_{23} = 3 \times 1 + 2 \times -1 + -1 \times -3 = 4$

$$A \times B = C = \begin{bmatrix} 0 & -10 & 4 \\ 12 & 19 & 4 \end{bmatrix}$$

Basic Matrix Algebra Rules:

$(AB)^T = B^T A^T$

$AA^{-1} = A^{-1}A = I$
A^{-1} is the inverse matrix, and I is the identity matrix (see the mathematical notations).

Matrix Derivative: The Derivative of a matrix with respect to a scalar parameter x is calculated via element-by-element derivative of that matrix with respect to the parameter x. For example:

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$\frac{\partial A}{\partial x} = \begin{bmatrix} \frac{\partial a_{11}}{\partial x} & \frac{\partial a_{12}}{\partial x} & \frac{\partial a_{13}}{\partial x} \\ \frac{\partial a_{21}}{\partial x} & \frac{\partial a_{22}}{\partial x} & \frac{\partial a_{23}}{\partial x} \\ \frac{\partial a_{31}}{\partial x} & \frac{\partial a_{32}}{\partial x} & \frac{\partial a_{33}}{\partial x} \end{bmatrix}$$