Question 1:

Entropy of the target taste

$$P(\text{taste} = \text{meh}) = 5/10$$

$$P(\text{taste} = \text{yummy}) = 5/10$$

$$H(\text{taste}) = -5/10 \log_2 5/10 - 5/10 \log_2 5/10$$

$$= 1.0 \text{ information gain}$$

Question 2:

Entropy of Visual Defects at root

$P(\text{visual defects} = \text{some}) = 3/10$

$P(\text{visual defects} = \text{none}) = 4/10$

$P(\text{visual defects} = \text{many}) = 3/10$

$\text{Taste} = [\text{meh}: 5, \text{yummy}: 5]$

$\text{Taste}_{\text{some}} = [\text{meh}: 3, \text{yummy}: 0]$

$\text{Taste}_{\text{none}} = [\text{meh}: 2, \text{yummy}: 2]$

$\text{Taste}_{\text{many}} = [\text{meh}: 0, \text{yummy}: 3]$

$H(\text{Taste} \mid \text{visual defects}) =$

$$\text{Entropy}(\text{Taste}) - (3/10\ \text{Entropy}(\text{Taste}_{\text{some}}) + 4/10\ \text{Entropy}(\text{Taste}_{\text{none}}) + 3/10\ \text{Entropy}(\text{Taste}_{\text{many}}))$$

$$\text{Entropy}(\text{Taste}_{\text{some}}) = -\left(\tfrac{3}{3}\log_2\tfrac{3}{3} + \tfrac{0}{3}\log_2\tfrac{0}{3}\right) = 0$$

$$\text{Entropy}(\text{Taste}_{\text{none}}) = -\left(\tfrac{2}{4}\log_2\tfrac{2}{4} + \tfrac{2}{4}\log_2\tfrac{2}{4}\right) = 1.0$$

$$\text{Entropy}(\text{Taste}_{\text{many}}) = -\left(\tfrac{0}{3}\log_2\tfrac{0}{3} + \tfrac{3}{3}\log_2\tfrac{3}{3}\right) = 0$$

$$1.0 - \left(\tfrac{3}{10}(0) + \tfrac{4}{10}(1.0) + \tfrac{3}{10}(0)\right)$$

$$1.0 - (0 + .4 + 0)$$

$$1.0 - .4 = .6$$

Information Gain (Visual Defects) = .6

Question 3:

entropy of visual defects some & none w respect to taste

$H(\text{Taste} \mid \text{Visual defect: some}) =$

$$-\left(\tfrac{3}{3}\log_2\tfrac{3}{3} + \tfrac{0}{3}\log_2\tfrac{0}{3}\right) = 0$$

$H(\text{Taste} \mid \text{Visual defect: none}) =$

$$-\left(\tfrac{2}{4}\log_2\tfrac{2}{4} + \tfrac{2}{4}\log_2\tfrac{2}{4}\right) = 1.0$$