Linear Regression

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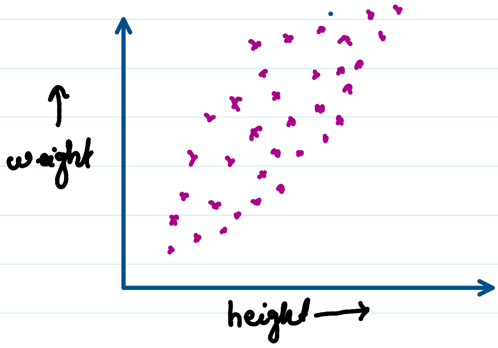
Assumption: x and y has a linear Relation

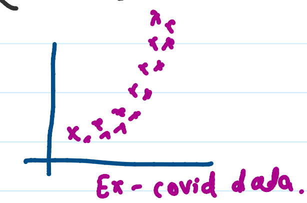


Linear Regression

- Easy to learn → Intuition
- Application in other Algo

LR (Supervised) — Regression / classification

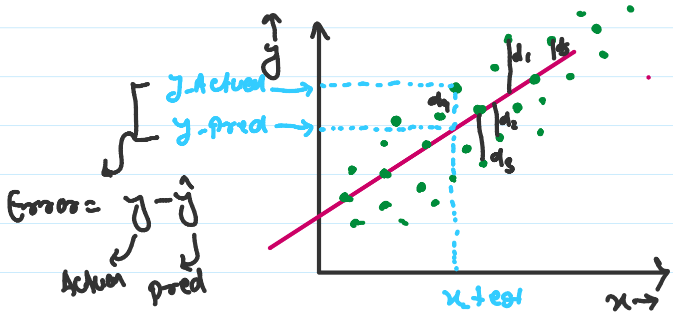
- **Linear LR** (when data is linear)
  - **Simple**: 1 - input, 1 - Output
  - **Multiple**: 1 < input, 0 - output.
- **Polynomial LR** (when data is not linear)



Linear Regression

→ Aim is to find best fit line → Minimum Error.

Error = Actual value − Prediction value.



$$\text{Error} = d_1 + d_2 + d_3 + \dots d_n$$

$$= (y_1 - \hat{y}_1) + (y_2 - \hat{y}_2) + (y_3 - \hat{y}_3) + \dots$$

$$= \sum_{i=1}^{n} (y_i - \hat{y}_i)$$

But to manage +ve and −ve Error we have to square the Error.

$$\boxed{\therefore \text{Error}(J) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \quad \text{— eq (1)}$$

(Loss function)

We known $\hat{y} = mx + c$

↳ by putting in eq (1)

$$\boxed{\text{Loss function } (J) = \sum_{i=1}^{n} (y_i - mx_i - c)^2}$$

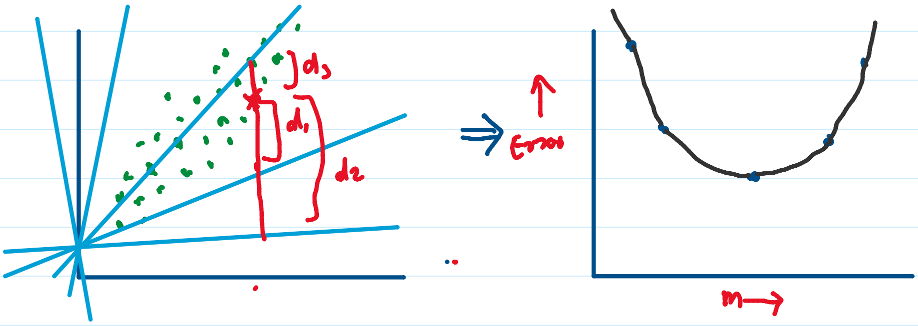
Here,

$y_i, x_i \longrightarrow$ Const, we get in training time

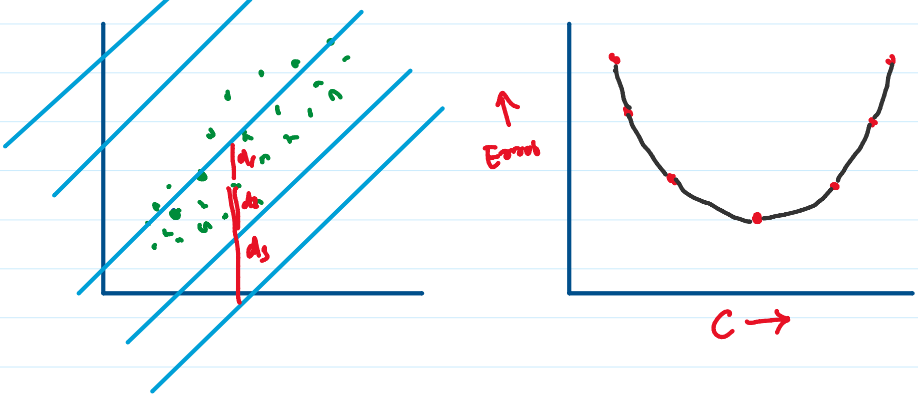
$m, c \longrightarrow$ variable, we have to calculate.

Let's Analyse the M and C w.r.t Loss-function

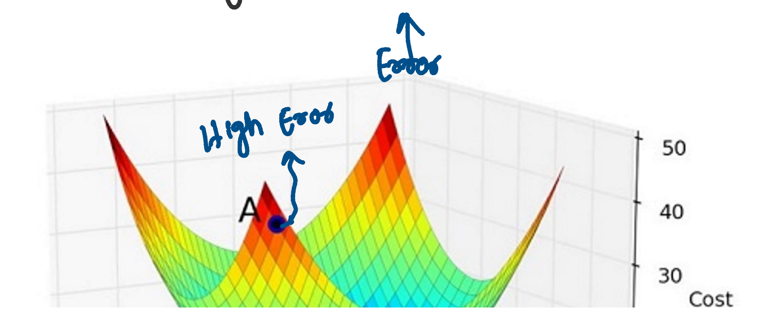
Relation bet M & loss-fun. (C = const)

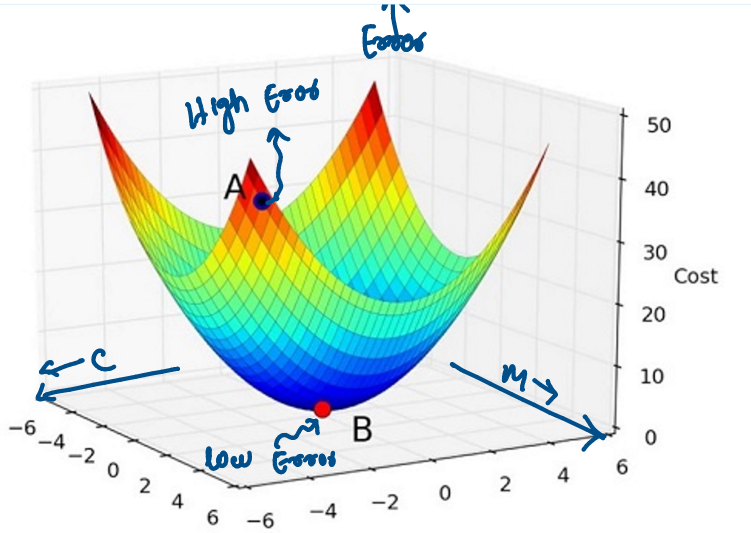


Relation bet C and loss-fun (M = const)



If we Analyse M and C both together we will get the figure like below.





we can find the point of low Error by differentiating the loss Function

Since for minima $\longrightarrow$ $f'(x) = 0$

$$f(x) = \text{loss Function}(J) = \sum_{i=1}^{n} (y_i - mx_i - c)^2$$

variable

Since we have 2 variable we have to do partial derivative $\rightarrow$ $\frac{dJ}{dc} = 0$ and $\frac{dJ}{dm} = 0$

$$\frac{dJ}{dc} = \frac{d}{dc}\left(\sum (y_i - mx_i - c)^2\right) = 0$$

$$\Rightarrow \sum \frac{d}{dc} (y_i - mx_i - c)^2 = 0$$

$$\Rightarrow \sum 2(y_i - mx_i - c) \times -1 = 0$$

$$\Rightarrow \sum (y_i - mx_i - c) = 0$$

$$\Rightarrow \sum y_i - \sum mx_i - \sum c = 0$$

$$\Rightarrow \sum y_i - \sum mx_i = \sum c$$

$$\Rightarrow \frac{\sum y_i}{n} - \frac{\sum mx_i}{n} = \frac{\sum c}{n} \quad \text{(deviding both side)}$$

$$\Rightarrow \bar{y} - m\bar{x} = c \quad , \quad \bar{x}, \bar{y} \rightarrow \text{mean}$$

$$\Rightarrow \boxed{c = \bar{y} - m\bar{x}} \quad \text{eq 2}$$

2) $$\frac{dJ}{dm} = \frac{d}{dm}\left(\sum (y_i - mx_i - c)^2\right) = 0$$

$$\Rightarrow \frac{d}{dm}\left(\sum (y_i - mx_i - \bar{y} + m\bar{x})^2\right) = 0$$

$$\Rightarrow \sum \frac{d}{dm} (y_i - mx_i - \bar{y} + m\bar{x})^2 = 0$$

$$\Rightarrow \sum 2(y_i - mx_i - \bar{y} + m\bar{x}) \times (-x_i + \bar{x}) = 0$$

$$\Rightarrow \sum (y_i - mx_i - \bar{y} + m\bar{x}) \times (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum (y_i - \bar{y} - m(x_i - \bar{x})) \times (x_i - \bar{x}) = 0$$

$$\Rightarrow \sum \left[(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2\right] = 0$$

$$\Rightarrow \sum m(x_i - \bar{x})^2 = \sum (y_i - \bar{y})(x_i - \bar{x})$$

$$\Rightarrow \boxed{m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}} \quad \text{eq 3}$$

Finally we got our m and c

now we can calculate y-pred. by

$$\boxed{\hat{y} = mx + c}$$

we got from training data

y-pred

x-test