

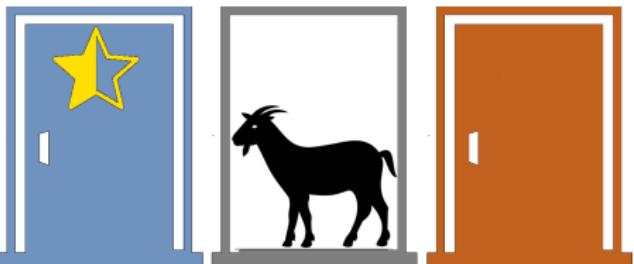
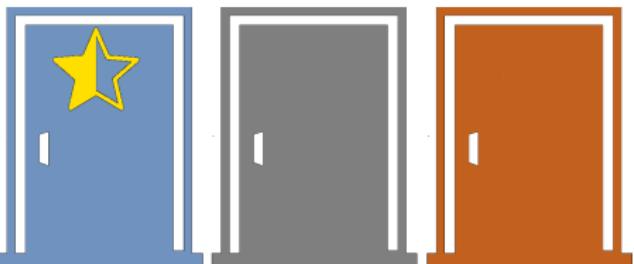
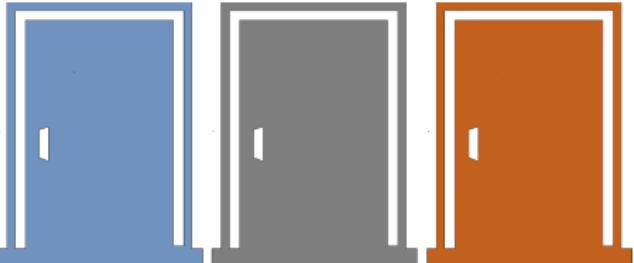
Probability Puzzles

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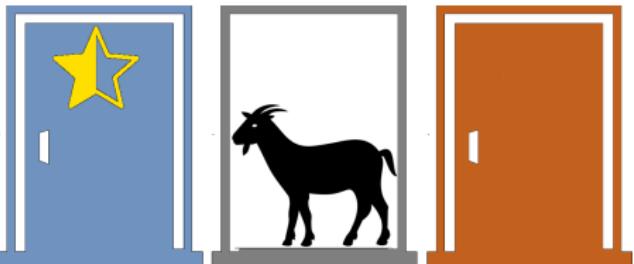
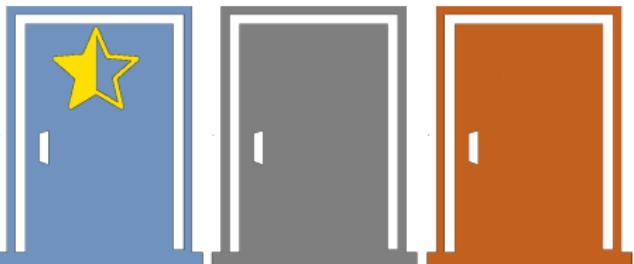
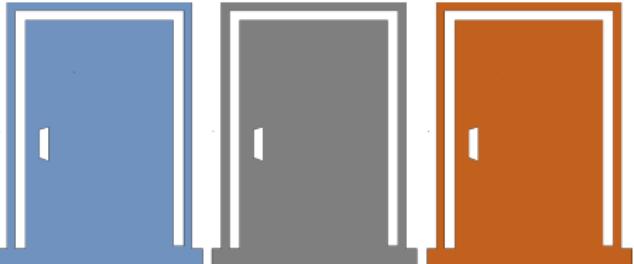
August 5, 2025

The Monty-Hall problem

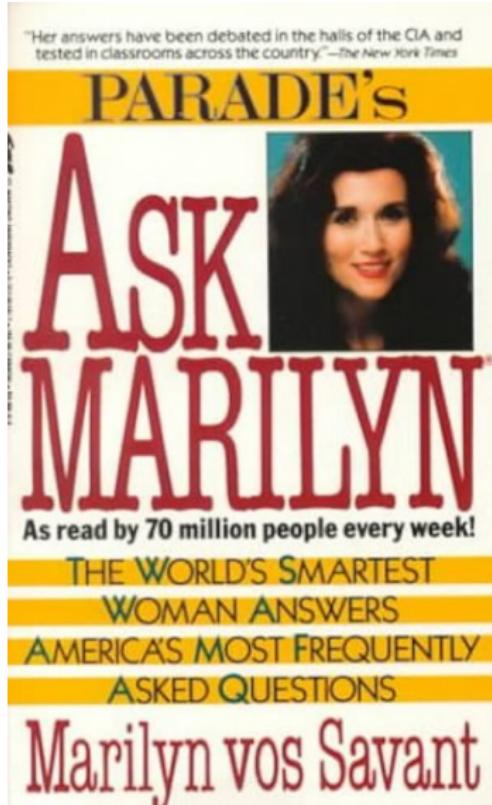
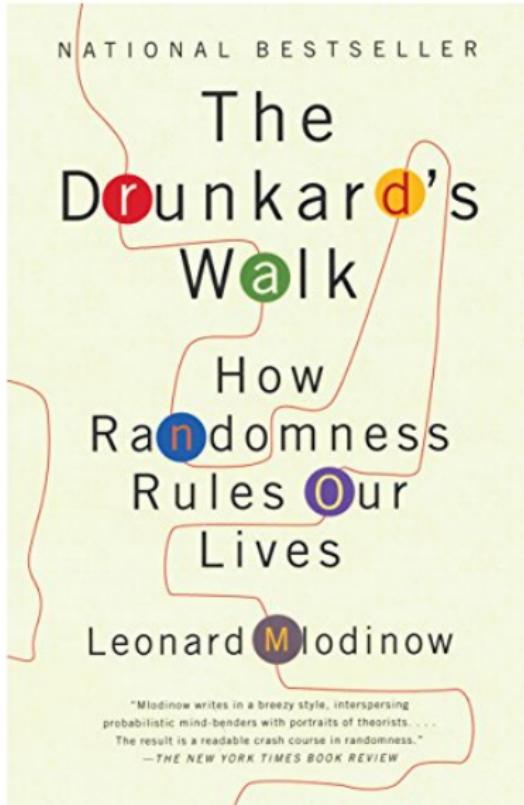


The Monty-Hall problem

DO YOU WANT TO SWITCH?



Books suggestion

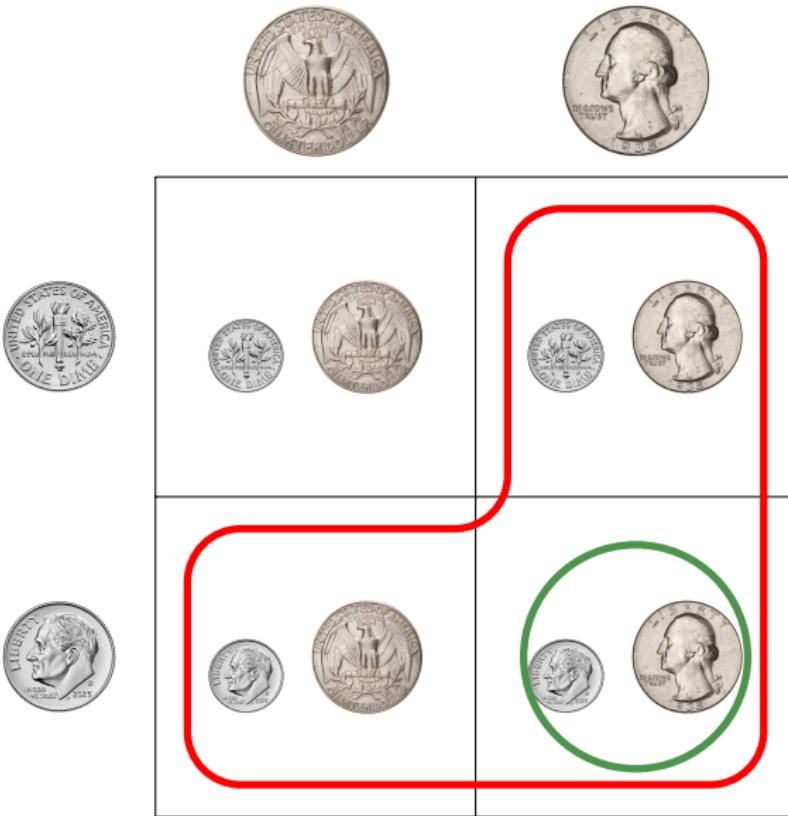


Puzzle #1 – Two heads problem



- You flip two coins without seeing the result.
- Someone informs you that one of the coins landed on heads.
- What is the probability that both coins landed on heads?

Puzzle #1 – Two heads problem


$$\frac{1}{3}$$

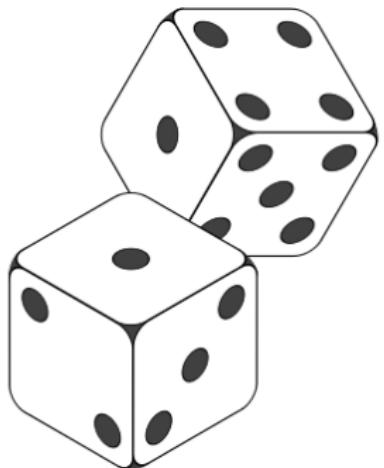
Puzzle #1 – Two heads problem

“The quarter is heads”



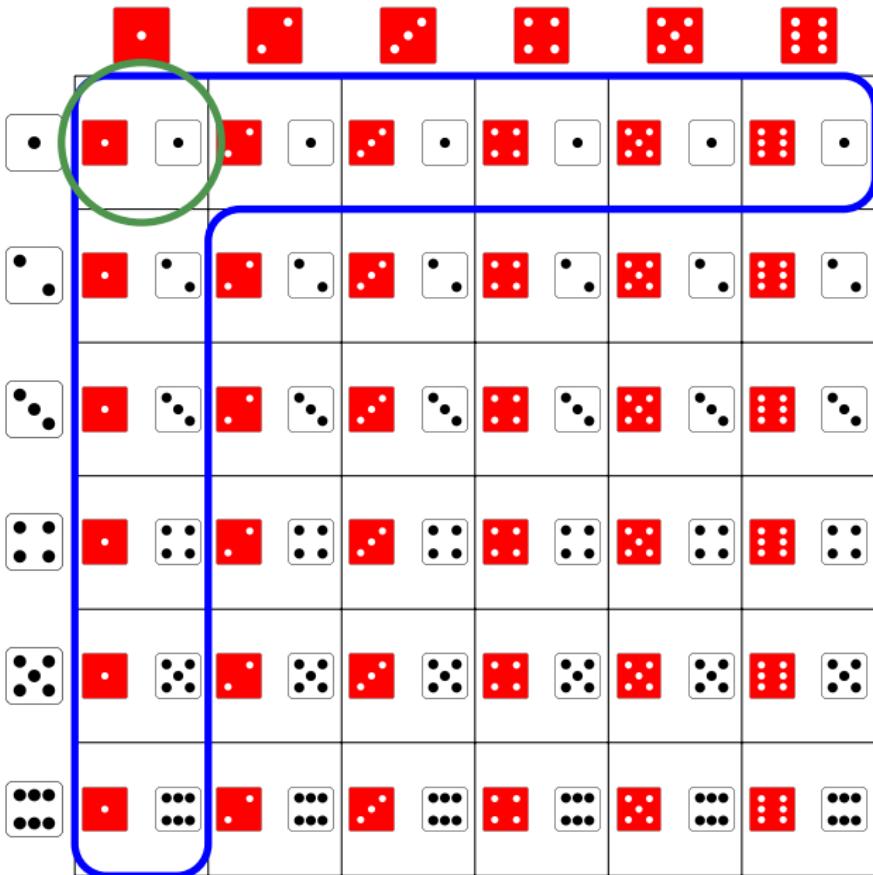
$\frac{1}{2}$

Puzzle #2 – Snake eyes problem



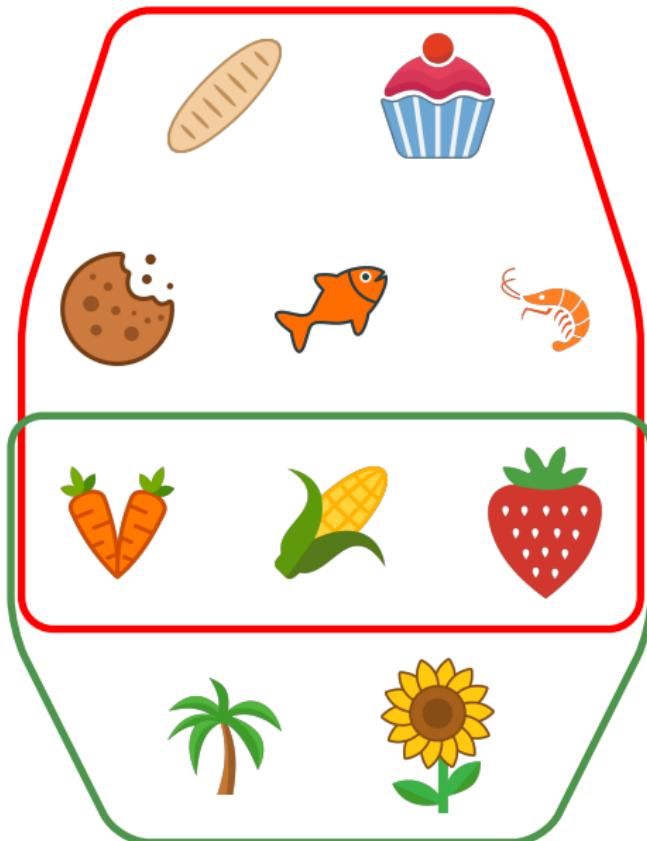
- You roll a pair of fair six-sided dice without looking.
- The dealer says that one of the dice is showing a one.
- What is the probability of rolling “snake eyes” (where both dice land on the number 1)?

Puzzle #2 – Snake eyes problem



$\frac{1}{11}$

Conditional Probability



$$P(\text{Food}) = \frac{8}{10} \quad P(\text{Plant}) = \frac{5}{10}$$

$$P(\text{Food and Plant}) = \frac{3}{10}$$

Conditional Probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

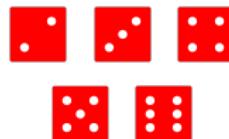
$$P(\text{Food|Plant}) = \frac{3/10}{5/10} = \frac{3}{5}$$

$$P(\text{Plant|Food}) = \frac{3/10}{8/10} = \frac{3}{8}$$

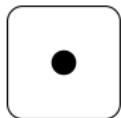
Puzzle #2 – Snake eyes problem (revisited)



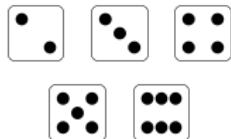
$$\frac{1}{6}$$



$$\frac{5}{6}$$



$$\frac{1}{6}$$



$$\frac{5}{6}$$

$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$	$\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$
$\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$	$\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

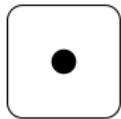
Puzzle #2 – Snake eyes problem (revisited)



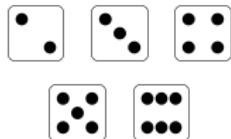
$$\frac{1}{6}$$



$$\frac{5}{6}$$



$$\frac{1}{6}$$



$$\frac{5}{6}$$

$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$	$\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$
$\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$	$\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$$\frac{\frac{1}{36}}{\frac{11}{36}} = \frac{1}{11}$$

Puzzle #? (reskinned) - The 2 child paradox

- A family has two children.
- One of them is a girl.
- What is the probability that the kids are both girls?

Working on the naive model where a child is either a boy or a girl with independent and equal probability.

Puzzle #1 (reskinned)– The 2 child paradox

	Boy	Girl
Boy	BB	BG
Girl	GB	GG

A 3x3 grid representing the sample space of two children. The columns are labeled "Boy" and "Girl" at the top, and the rows are labeled "Boy" and "Girl" on the left. The four outcomes are: BB (top-left, both boys), BG (top-right, boy then girl), GB (bottom-left, girl then boy), and GG (bottom-right, both girls). A red L-shaped bracket highlights the first three outcomes (BB, BG, GB). A green circle highlights the bottom-right outcome (GG).

$\frac{1}{3}$

Puzzle #3 – The girl named Florida

- A family has two children.
- One of them is a girl **named Florida**.
- What is the probability that the kids are both girls?



Puzzle #3 – The girl named Florida

	Boy	Girl (not Florida)	Girl (Florida)
Boy	$\frac{1}{2}$	$\left(\frac{1}{2} - f\right)$	f
Girl (not Florida)	$\frac{1}{4}$	$\frac{1}{4} - \frac{f}{2}$	$\frac{f}{2}$
Girl (Florida)	$\left(\frac{1}{2} - f\right)$	$\frac{1}{4} - \frac{f}{2}$	$\frac{f}{2} - f^2$

The table shows the joint probability distribution of three variables: Boy, Girl (not Florida), and Girl (Florida). The probabilities are as follows:

- Probability of Boy: $\frac{1}{2}$
- Probability of Girl (not Florida) given Boy: $\left(\frac{1}{2} - f\right)$
- Probability of Girl (Florida) given Boy: f
- Probability of Boy and Girl (not Florida): $\frac{1}{4}$
- Probability of Boy and Girl (Florida): $\frac{f}{2}$
- Probability of Girl (not Florida) and Girl (Florida): $\frac{1}{4} - \frac{f}{2}$
- Probability of all three variables: $\frac{1}{4} - f + f^2$
- Probability of Girl (not Florida) and Girl (Florida): $\frac{f}{2} - f^2$
- Probability of Girl (Florida) only: f^2

Red outlines highlight the first two rows (Boy and Girl (not Florida)) and the last two columns (Girl (not Florida) and Girl (Florida)). A green outline highlights the bottom-right cell (f^2).

Puzzle #3 – The girl named Florida

- A family has two children.
- One of them is a girl **named Florida**.
- What is the probability that the kids are both girls?

$$P(\text{Both girls} | \text{One girl named Florida}) = \frac{1-f}{2-f}$$

US census data: 3000 Floridas $f \approx \frac{1}{100000} = 0.00001$

$$P(\text{Both girls} | \text{One girl named Florida}) \approx \frac{0.99999}{1.99999} \approx \frac{1}{2}$$

Puzzle #4 – Mr F. problem

- A health survey was conducted in a representative sample of adult males.
- Mr. F. was selected randomly from the list of participants.
- Which is more probable?
 - A) Mr. F. has had one or more heart attacks.
 - B) Mr. F. has had one or more heart attacks and he is over 55 years old.

$$P(A \text{ and } B) \leq P(A)$$

85% of undergraduates in UBC got this wrong [Tversky & Kahneman (1981)]

Puzzle #4 – Mr F. problem (revisited)

- A health survey was conducted in a representative sample of adult males.
- Mr. F. was selected randomly from the list of participants.
- **In this data we observe that Mr. F. had a heart attack.**
- Which is more probable?
 - A) Mr. F. is under 55 years old.
 - B) Mr. F. is over 55 years old.

$$P(\text{Is over 55} | \text{Had a heart attack}) > P(\text{Is under 55} | \text{Had a heart attack})$$

Why do we get this wrong?

Our **gut** instincts are not prepared to deal with incomplete information.

WEIGH YOUR
PROPOSITIONS CAREFULLY.

Probability matching



70%



30%

Human

$$70\% \times 70\% = 49\%$$

$$30\% \times 30\% = 9\%$$

Total 58%

Probability matching



70%



30%

“Irrational” animals

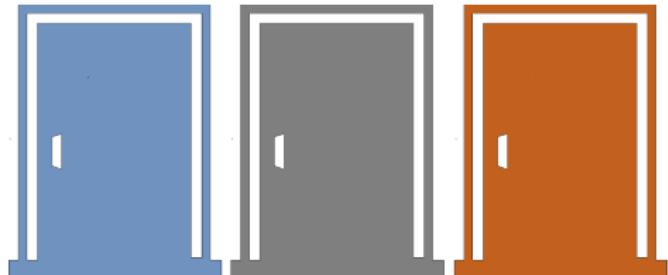
$$100\% \times 70\% = 70\%$$

$$0\% \times 30\% = 0\%$$

Total 70%

The “irrational” animals get a better outcome, because they don’t try to find a pattern.

Puzzle #5 – The Monty-Hall problem



- You are in a game show. The host gives you the choice of three doors.
- Behind one door is a brand-new car and behind the others are goats.
- You pick a door. The host (who already knows what is behind the doors) opens a different door and reveals a goat.
- The host then asks you: “Do you want to switch?”

What should you answer?

Puzzle #5 – The Monty-Hall problem

Correct door	Wrong door
$\frac{1}{3}$	$\frac{2}{3}$



	Lose	Win
Switch		
Keep	Win	Lose

$$P(\text{Win}|\text{Switch}) = \frac{2}{3}$$

$$P(\text{Win}|\text{Keep}) = \frac{1}{3}$$

YOU SHOULD SWITCH

Puzzle #6 – Medical diagnosis

- Imagine a medical test for a rare disease.
- The test is 99% accurate, meaning that it gives a positive diagnosis in 99% of tests people who have it and gives a negative diagnosis in 99% of the time for people who don't.
- If someone tests positive for it, what is the probability that they actually have the disease?

We can not answer with the information we have here.

Puzzle #6 –Medical diagnosis (complete)

- Imagine a medical test for a disease that affects $\frac{1}{1000}$ of people.
- The test is 99% accurate.
- If someone tests positive for it, what is the probability that they actually have the disease?

		Correct result	Wrong result
		99/100	1/100
Disease $\frac{1}{1000}$	Positive	$\frac{99}{100000}$	$\frac{1}{100000}$
	Negative		
Health $\frac{999}{1000}$	Negative	$\frac{98901}{100000}$	$\frac{999}{100000}$
	Positive		

$$P(\text{Positive and Disease}) = \frac{99}{100000}$$

$$P(\text{Positive}) = \frac{1098}{10000}$$

$$P(\text{Disease|Positive}) = \frac{99}{1098} \approx 0.09$$

Conclusion



- Probability theory is not intuitive.
- Formulating the question appropriately, meaning accord to the information you do have, is of utmost importance.
- Our mental biases directly affect our decisions both in advanced scientific endeavors and in day-to-day tasks.

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