Probability Puzzles

Pedro Pessoa

Arizona State University
Center for Biological Physics

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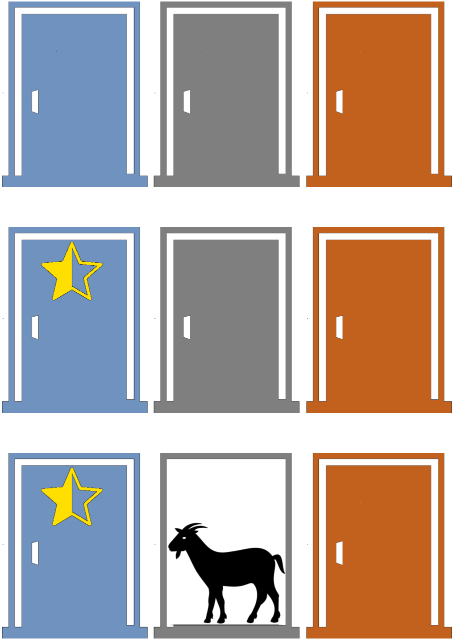
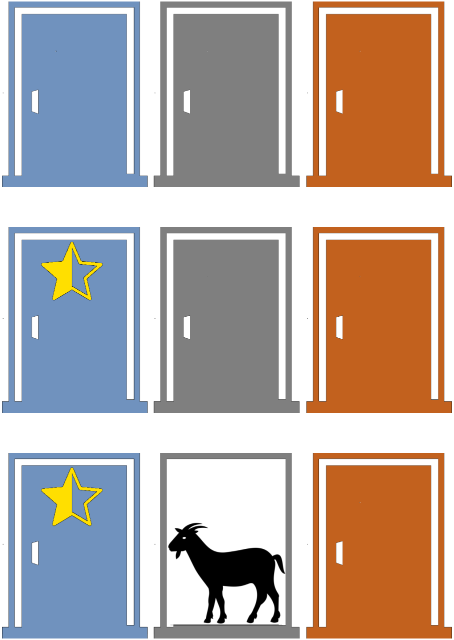
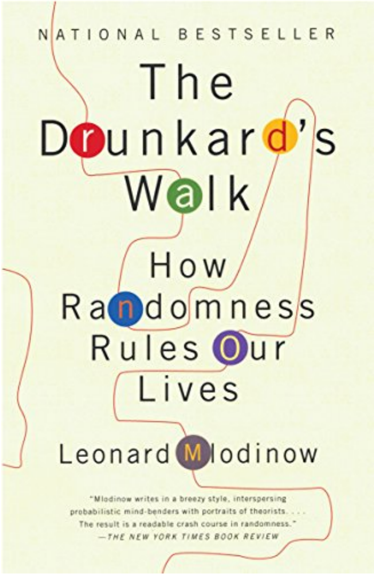
The Monty-Hall problem

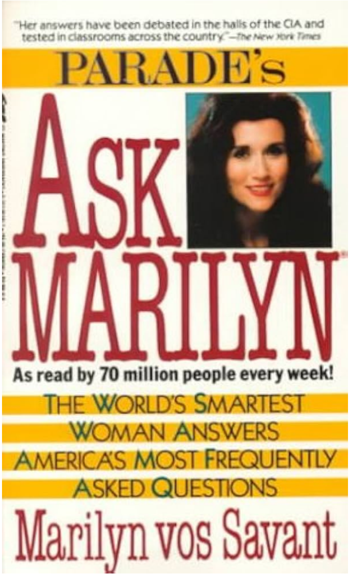


The Monty-Hall problem



Books suggestion

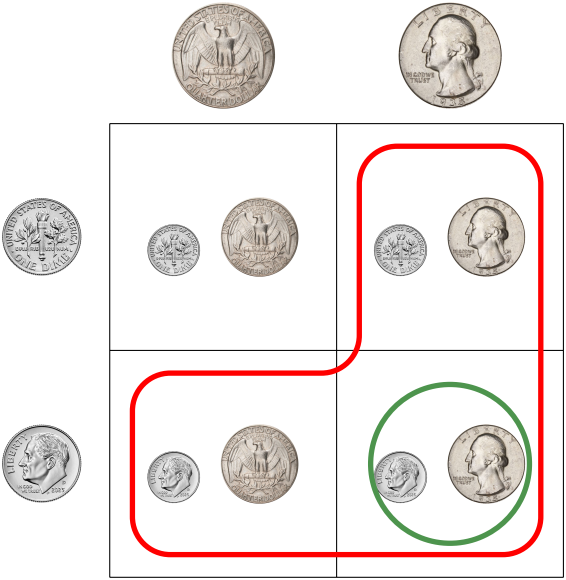




Puzzle #1 – Two heads problem

- You flip two coins without seeing the result.
- Someone informs you that one of the coins landed on heads.
- What is the probability that both coins landed on heads?

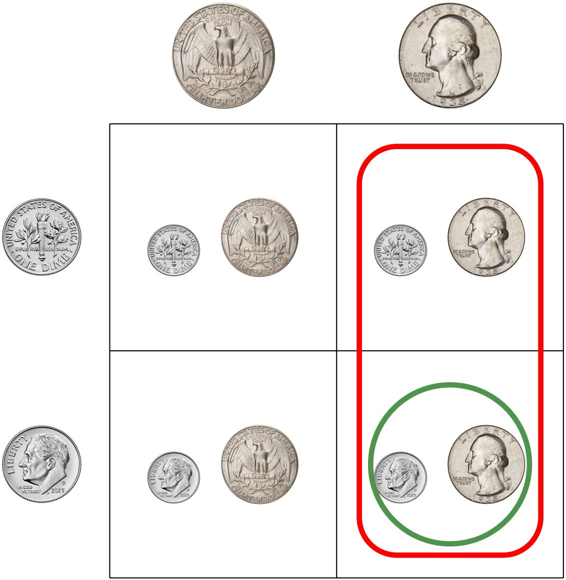
Puzzle #1 – Two heads problem



$$\frac{1}{3}$$

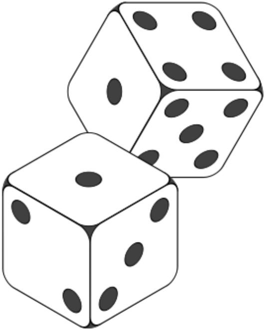
Puzzle #1 – Two heads problem

"The quarter is heads"



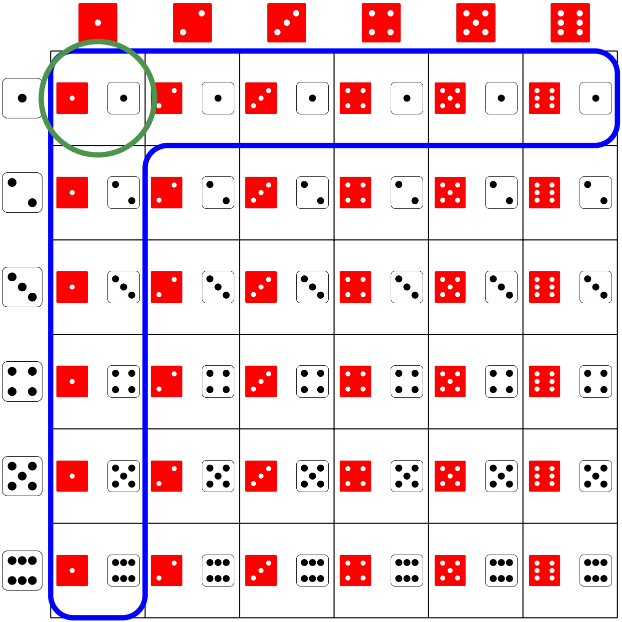
$\frac{1}{2}$

Puzzle #2 – Snake eyes problem



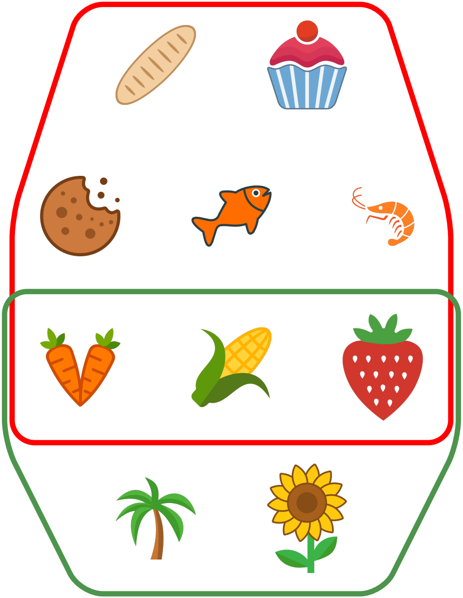
- You roll a pair of fair six-sided dice without looking.
- The dealer says that one of the dice is showing a one.
- What is the the probability of rolling "snake eyes" (where both dice land on the number 1)?

Puzzle #2 – Snake eyes problem



$$\frac{1}{11}$$

Conditional Probability



$$P(\text{Food}) = \frac{8}{10} \quad P(\text{Plant}) = \frac{5}{10}$$

$$P(\text{Food and Plant}) = \frac{3}{10}$$

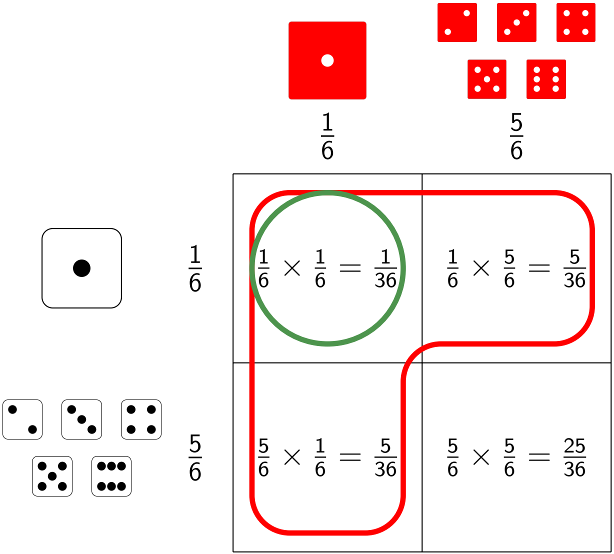
Conditional Probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

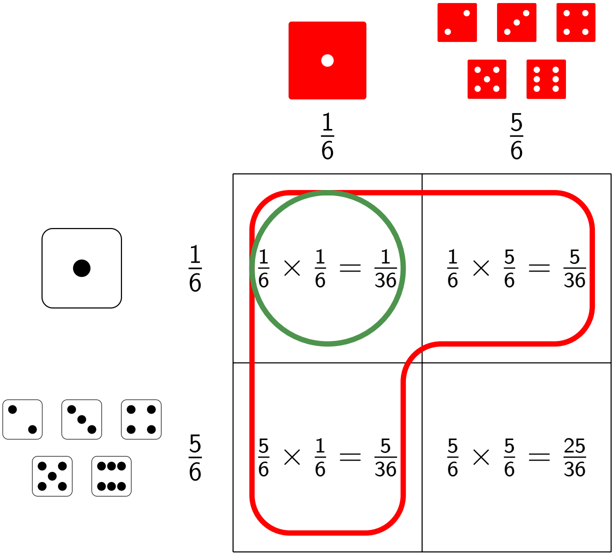
$$P(\text{Food}|\text{Plant}) = \frac{3/10}{5/10} = \frac{3}{5}$$

$$P(\text{Plant}|\text{Food}) = \frac{3/10}{8/10} = \frac{3}{8}$$

Puzzle #2 – Snake eyes problem (revisited)



Puzzle #2 – Snake eyes problem (revisited)



$$\frac{1/36}{11/36} = \frac{1}{11}$$

Puzzle #? (reskinned) - The 2 child paradox

- A family has two children.
- One of them is a girl.
- What is the probability that the kids are both girls?

Working on the naive model where a child is either a boy or a girl with independent and equal probability.

Puzzle #1 (reskinned)– The 2 child paradox

	Boy	Girl
Boy	BB	BG
Girl	GB	GG

$$\frac{1}{3}$$

Puzzle #3 – The girl named Florida

- A family has two children.
- One of them is a girl **named Florida**.
- What is the probability that the kids are both girls?

Puzzle #3 – The girl named Florida

		Boy	Girl (not Florida)	Girl (Florida)
		$\frac{1}{2}$	$\left(\frac{1}{2} - f\right)$	f
Boy	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4} - \frac{f}{2}$	$\frac{f}{2}$
Girl (not Florida)	$\left(\frac{1}{2} - f\right)$	$\frac{1}{4} - \frac{f}{2}$	$\frac{1}{4} - f + f^2$	$\frac{f}{2} - f^2$
Girl (Florida)	f	$\frac{f}{2}$	$\frac{f}{2} - f^2$	f^2

Puzzle #3 – The girl named Florida

- A family has two children.
- One of them is a girl **named Florida**.
- What is the probability that the kids are both girls?

$$P(\text{Both girls}|\text{One girl named Florida}) = \frac{1-f}{2-f}$$

US census data: 3000 Floridas $\quad f \approx \frac{1}{100000} = 0.00001$

$$P(\text{Both girls}|\text{One girl named Florida}) \approx \frac{0.99999}{1.99999} \approx \frac{1}{2}$$

Puzzle #4 – Mr F. problem

- A health survey was conducted in a representative sample of adult males.
- Mr. F. was selected randomly from the list of participants.
- Which is more probable?

A) Mr. F. has had one or more heart attacks.

B) Mr. F. has had one or more heart attacks and he is over 55 years old.

$$P(A \text{ and } B) \leq P(A)$$

85% of undergraduates in UBC got this wrong [Tversky & Kahneman (1981)]

Puzzle #4 – Mr F. problem (revisited)

- A health survey was conducted in a representative sample of adult males.
- Mr. F. was selected randomly from the list of participants.
- **In this data we observe that Mr. F. had a heart attack.**
- Which is more probable?

A) Mr. F. is under 55 years old.

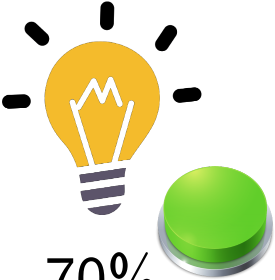
B) Mr. F. is over 55 years old.

$$P(\text{Is over }55|\text{Had a heart attack}) > P(\text{Is under }55|\text{Had a heart attack})$$

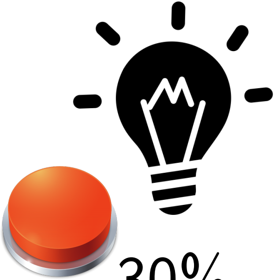
Why do we get this wrong?

Our **gut** instincts are not prepared to deal with incomplete information.

Probability matching



70%



30%

Human

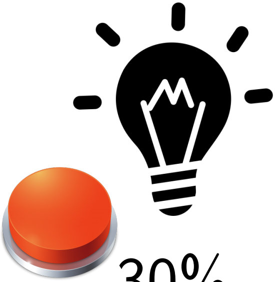
$70\% \times 70\% = 49\%$

$30\% \times 30\% = 9\%$

Total 58%

Probability matching

70%



30%

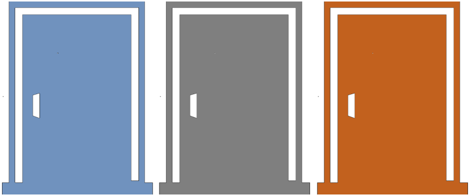
“Irrational” animals

$100\% \times 70\% = 70\%$ $0\% \times 30\% = 0\%$

Total 70%

The “irrational” animals get a better outcome, because they don’t try to find a pattern.

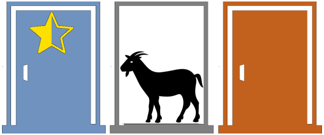
Puzzle #5 – The Monty-Hall problem



- You are in a game show. The host gives you the choice of three doors.
- Behind one door is a brand-new car and behind the others are goats.
- You pick a door. The host (who already knows what is behind the doors) opens a different door and reveals a goat.
- The host then asks you: “Do you want to switch?”

What should you answer?

Puzzle #5 – The Monty-Hall problem



	Correct door $^1/_3$	Wrong door $^2/_3$
Switch	Lose	Win
Keep	Win	Lose

$$P(\text{Win}|\text{Switch}) = \frac{2}{3}$$

$$P(\text{Win}|\text{Keep}) = \frac{1}{3}$$

YOU SHOULD SWITCH

Puzzle #6 – Medical diagnosis

- Imagine a medical test for a rare disease.
- The test is 99% accurate, meaning that it gives a positive diagnosis in 99% of tests people who have it and gives a negative diagnosis in 99% of the time for people who don't.
- If someone tests positive for it, what is the probability that they actually have the disease?

We can not answer with the information we have here.

Puzzle #6 –Medical diagnosis (complete)

- Imagine a medical test for a disease that affects $\frac{1}{1000}$ of people.
- The test is 99% accurate.
- If someone tests positive for it, what is the probability that they actually have the disease?

	Correct result $^{99}/_{100}$	Wrong result $^{1}/_{100}$
Disease $^{1}/_{1000}$	$\frac{99}{100000}$ **Positive**	$\frac{1}{100000}$ **Negative**
Health $^{999}/_{1000}$	**Negative** $\frac{98901}{100000}$	**Positive** $\frac{999}{100000}$

$$P(\textbf{Positive} \text{ and Disease}) = \frac{99}{100000}$$

$$P(\textbf{Positive}) = \frac{1098}{10000}$$

$$P(\text{Disease}|\textbf{Positive}) = \frac{99}{1098} \approx 0.09$$

Conclusion

WEBSITE

- Probability theory is not intuitive.
- Formulating the question appropriately, meaning accord to the information you do have, is of utmost importance.
- Our mental biases directly affect our decisions both in advanced scientific endeavors and in day-to-day tasks.

ppessoa@asu.edu

ppessoa

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