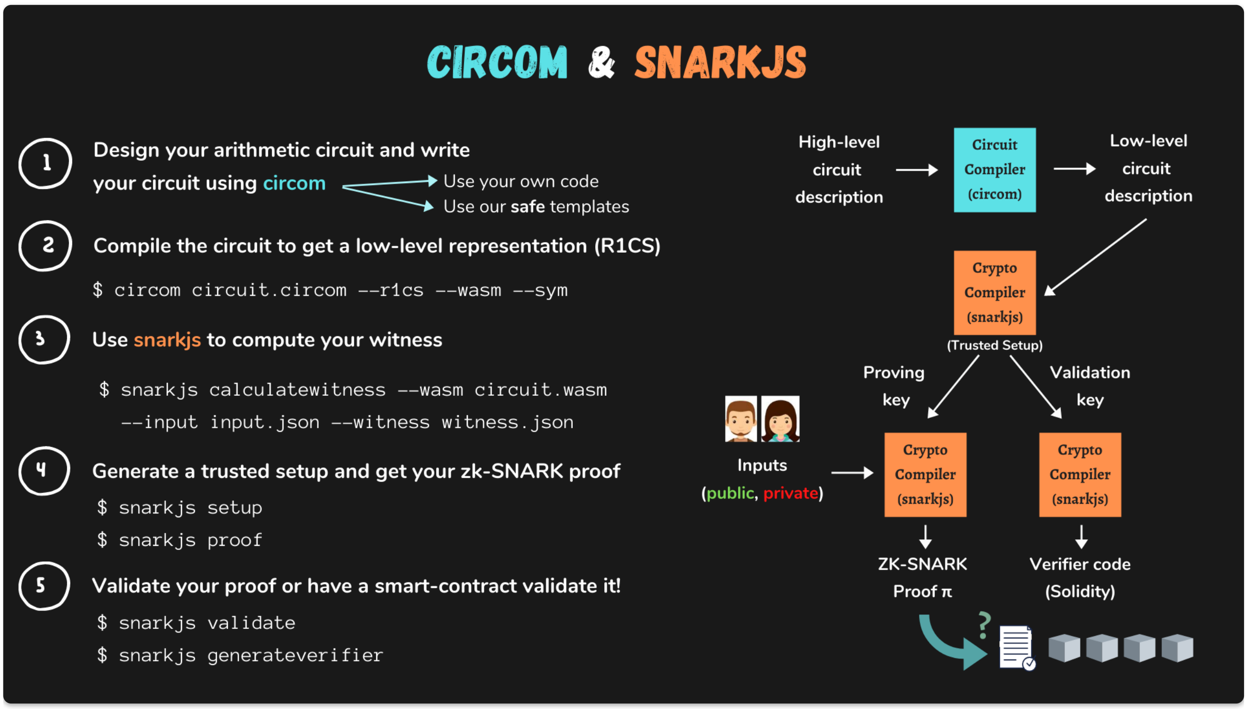
Lesson 12 - Circom / SNARK theory / PLONK

Quote from Remco Bloemen remco@0x.org

> *Disclaimer: contains maths*
>
> *If you don’t understand something*
>
> - *Not your fault, this stuff is hard*
> - *Nobody understands it fully*
>
> *If you don’t understand anything*
>
> - *My fault, anything can be explained at some level*
>
> *If you do understand everything*
> ** Collect your Turing Award & Fields Medal*

Circom

See Documentation



CIRCOM & SNARKJS

1 Design your arithmetic circuit and write your circuit using circom → Use your own code / Use our **safe** templates

2 Compile the circuit to get a low-level representation (R1CS)

```
$ circom circuit.circom --r1cs --wasm --sym
```

3 Use snarkjs to compute your witness

```
$ snarkjs calculatewitness --wasm circuit.wasm
  --input input.json --witness witness.json
```

4 Generate a trusted setup and get your zk-SNARK proof

```
$ snarkjs setup
$ snarkjs proof
```

5 Validate your proof or have a smart-contract validate it!

```
$ snarkjs validate
$ snarkjs generateverifier
```

High-level circuit description
Circuit Compiler (circom)
Low-level circuit description
Crypto Compiler (snarkjs)
(Trusted Setup)
Proving key
Validation key
Inputs (public, private)
Crypto Compiler (snarkjs)
Crypto Compiler (snarkjs)
ZK-SNARK Proof π
Verifier code (Solidity)
?

Installation

Dependencies

> - Rust - use rustup
>
>   ```
>   curl --proto '=https' --tlsv1.2 https://sh.rustup.rs -sSf | sh
>   ```
>
> - Circom
>
>   ```
>   git clone https://github.com/iden3/circom.git
>   cd circom
>   cargo build --release
>   cargo install --path circom
>   ```
>
> - Snarkjs
>
>   ```
>   npm install -g snarkjs
>   ```

Circom coding

Writing circuits
Taken from the documentation

Circom allows programmers to define the constraints that define the arithmetic circuit. All constraints must be of the form `A*B + C = 0`, where `A`, `B` and `C` are linear combinations of signals.

You can define contraints in this way

```
pragma circom 2.0.0;

/*This circuit template checks that c is the multiplication of a and b.*/

template Multiplier2 () {

   // Declaration of signals.
   signal input a;
   signal input b;
   signal output c;

   // Constraints.
   c <== a * b;
}
```

Signals

The arithmetic circuits built using circom operate on signals
Signals can be named with an identifier or can be stored in arrays and declared using the keyword signal.
Signals can be defined as input or output, and are considered intermediate signals otherwise.
Signals are by default private.
The programmer can distinguish between public and private signals only when defining the main component, by providing the list of public input signals.

```
pragma circom 2.0.0;

template Multiplier2(){
   //Declaration of signals
   signal input in1;
   signal input in2;
   signal output out;
   out <== in1 * in2;
}

component main {public [in1,in2]} = Multiplier2();
```

Circom data types

- Field Element
  Integers mod the max field value, these are the default type.
- Arrays
  These hold items of the same type

```
var x[3] = [2,8,4];
var z[n+1];  // where n is a parameter of a template
var dbl[16][2] = base;
var y[5] = someFunction(n);
```

Templates and components

The mechanism to create generic circuits in Circom is the so-called templates.

They are normally parametric on some values that must be instantiated when the template is used.
The instantiation of a template is a new circuit object, which can be used to compose other circuits, so as part of larger circuits.

```
template tempid ( param_1, ... , param_n ) {
 signal input a;
 signal output b;

 .....

}
```

The instantiation of a template is made using the keyword component and by providing the necessary parameters.

`component c = tempid(v1,...,vn);`
The values of the parameters should be known constants at compile time.

Components

A component defines an arithmetic circuit has input signals, output signals and intermediate signals, and can have a set of constraints.
Components are immutable once instantiated.

```
template A(N){
   signal input in;
   signal output out;
   out <== in;
}

template C(N){
   signal output out;
   out <== N;
}
template B(N){
  signal output out;
  component a;
  if(N > 0){
     a = A(N);
  }
  else{
     a = A(0);
  }
}

component main = B(1);
```

We can create arrays of components.

```
template MultiAND(n) {
    signal input in[n];
    signal output out;
    component and;
    component ands[2];
    var i;
    if (n==1) {
        out <== in[0];
    } else if (n==2) {
          and = AND();
        and.a <== in[0];
        and.b <== in[1];
        out <== and.out;
    } else {
        and = AND();
    var n1 = n\2;
        var n2 = n-n\2;
        ands[0] = MultiAND(n1);
        ands[1] = MultiAND(n2);
        for (i=0; i<n1; i++) ands[0].in[i] <== in[i];
        for (i=0; i<n2; i++) ands[1].in[i] <== in[n1+i];
        and.a <== ands[0].out;
        and.b <== ands[1].out;
        out <== and.out;
    }
}
```

The main component

In order to start the execution, an initial component has to be given. By default, the name of this component is “main”, and hence the component main needs to be instantiated with some template.

This is a special initial component needed to create a circuit and it defines the global input and output signals of a circuit. For this reason, compared to the other components, it has a special attribute: the list of public input signals. The syntax of the creation of the main component is:

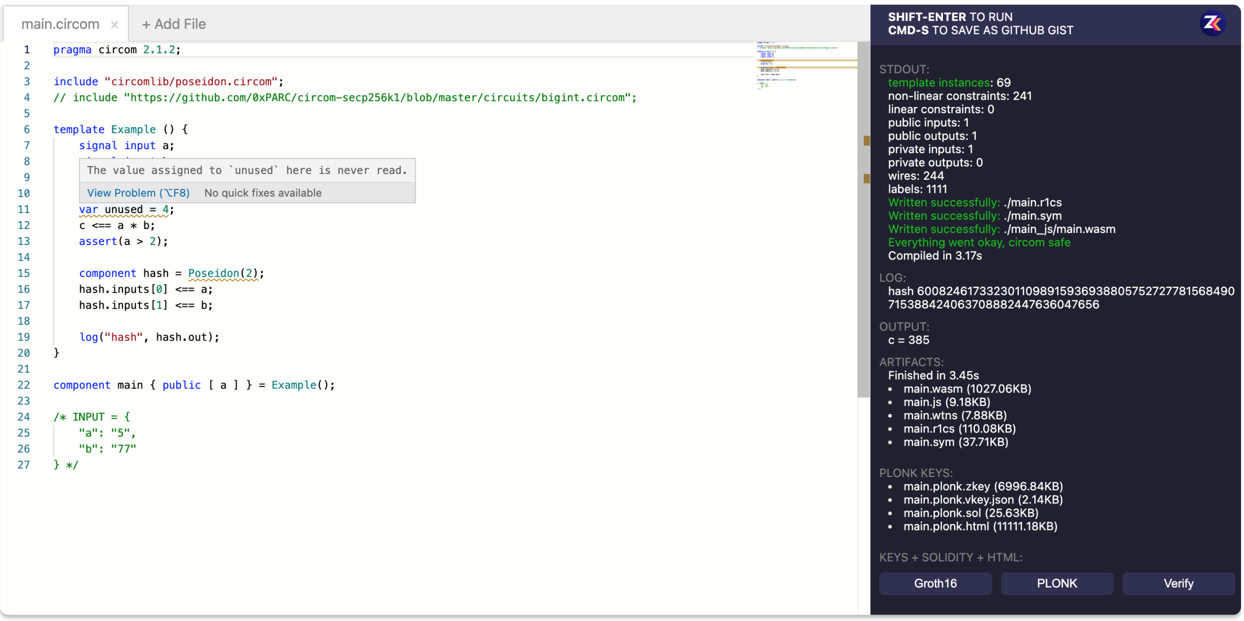
```
component main {public [signal_list]} = tempid(v1,...,vn);
```

```
pragma circom 2.0.0;

template A(){
    signal input in1;
    signal input in2;
    signal output out;
    out <== in1 * in2;
}

component main {public [in1]}= A();
```

Useful Tool

zkRepl. from 0xPARC

REPL for circom



```
pragma circom 2.1.2;

include "circomlib/poseidon.circom";
// include "https://github.com/0xPARC/circom-secp256k1/blob/master/circuits/bigint.circom";

template Example () {
    signal input a;
    var unused = 4;
    c <== a * b;
    assert(a > 2);

    component hash = Poseidon(2);
    hash.inputs[0] <== a;
    hash.inputs[1] <== b;

    log("hash", hash.out);
}

component main { public [ a ] } = Example();

/* INPUT = {
    "a": "5",
    "b": "77"
} */
```

The value assigned to `unused` here is never read.

```
STDOUT:
  template instances: 69
  non-linear constraints: 241
  linear constraints: 0
  public inputs: 1
  public outputs: 1
  private inputs: 1
  private outputs: 0
  wires: 244
  labels: 1111
  Written successfully: ./main.r1cs
  Written successfully: ./main.sym
  Written successfully: ./main_js/main.wasm
  Everything went okay, circom safe
  Compiled in 3.17s
LOG:
  hash 6008246173323011098915936938805752727781568490
  715388424063708882447636047656
OUTPUT:
  c = 385
ARTIFACTS:
  Finished in 3.45s
  • main.wasm (1027.06KB)
  • main.js (9.18KB)
  • main.wtns (7.88KB)
  • main.r1cs (110.08KB)
  • main.sym (37.71KB)

PLONK KEYS:
  • main.plonk.zkey (6996.84KB)
  • main.plonk.vkey.json (2.14KB)
  • main.plonk.sol (25.63KB)
  • main.plonk.html (11111.18KB)

KEYS + SOLIDITY + HTML:
```

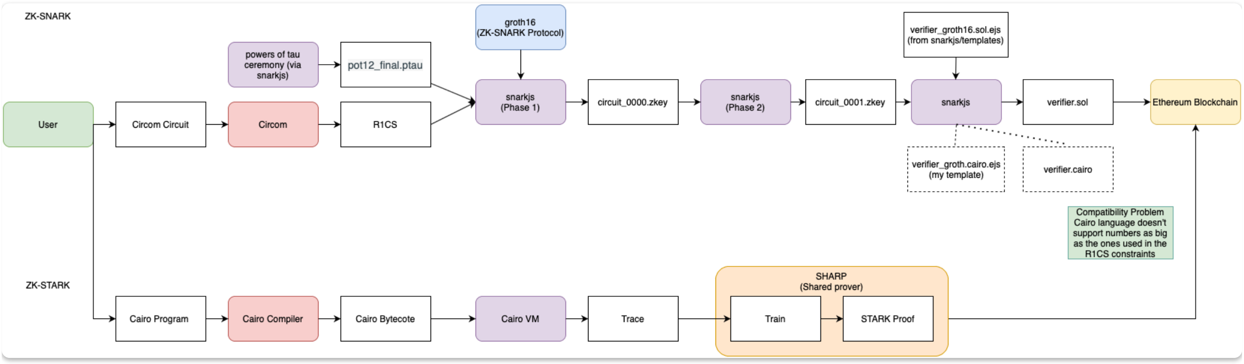
Circom -> Cairo

See repo and take note of the caveats

Allows circom projects to be verified on Ethereum by exporting to Cairo

First write and compile a circuit and compute the witness through circom, then generate a validation key through snarkjs (this process is properly explained at https://docs.circom.io/getting-started/installation/), this will yield a .zkey, which we can use to generate a solidity verifier through the command:

```
snarkjs zkey export solidityverifier [name of your key].zkey [nme of the verifier produced]
```



Polynomial Introduction

A polynomial is an expression that can be built from constants and variables by means of addition, multiplication and exponentiation to a non-negative integer power.

e.g. $3x^2 + 4x + 3$

Quote from Vitalik Buterin
"There are many things that are fascinating about polynomials. But here we are going to zoom in on a particular one: polynomials are a single mathematical object that can contain an unbounded amount of information (think of them as a list of integers and this is obvious)."
Furthermore, a single equation between polynomials can represent an unbounded number of equations between numbers.
For example, consider the equation A(x)+B(x)=C(x). If this equation is true, then it's also true that:

- A(0)+B(0)=C(0)
- A(1)+B(1)=C(1)
- A(2)+B(2)=C(2)
- A(3)+B(3)=C(3)

Adding, multiplying and dividing polynomials

We can add, multiply and divide polynomials, for examples see https://en.wikipedia.org/wiki/Polynomial_arithmetic

Roots

For a polynomial P of a single variable x in a field K and with coefficients in that field, the root r of P is an element of K such that $P(r) = 0$

B is said to divide another polynomial A when the latter can be written as

$A = BC$

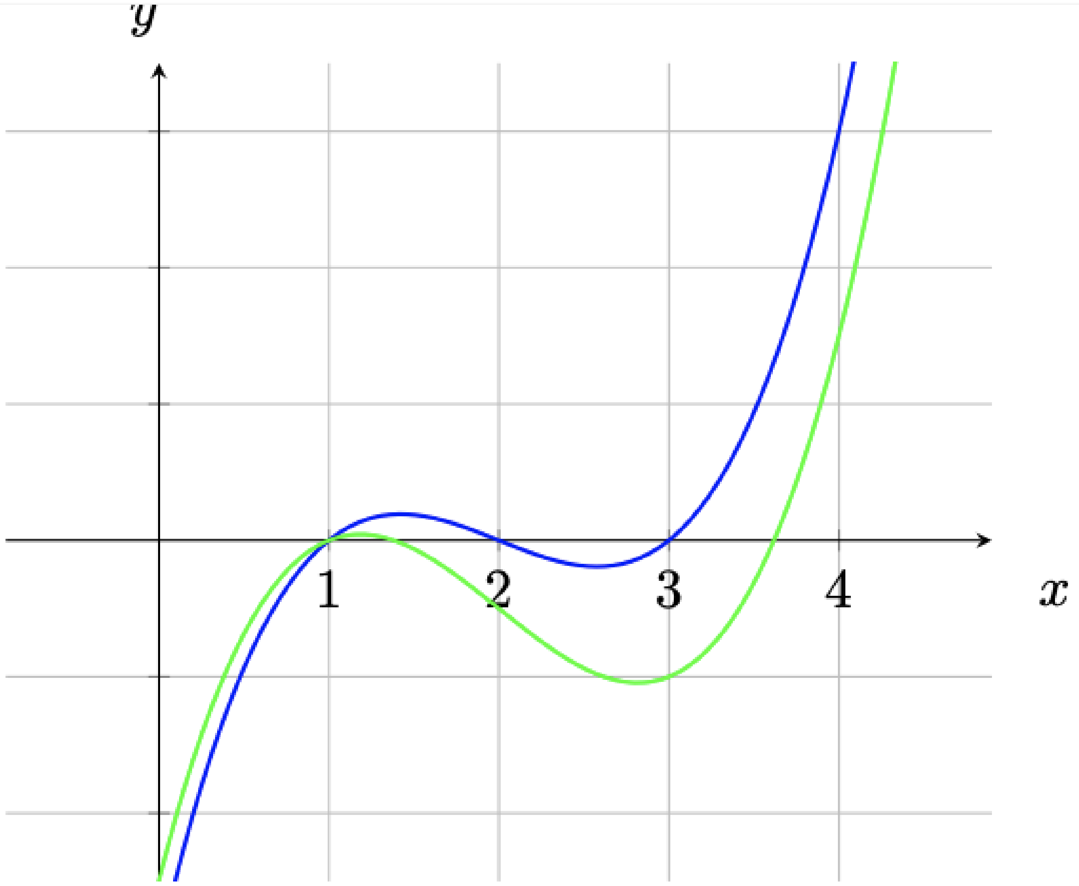
with C also a polynomial,the fact that B divides A is denoted $B|A$

If one root r of a polynomial $P(x)$ of degree n is known then polynomial long division can be used to factor $P(x)$ into the form
$(x - r)(Q(x))$
where
$Q(x)$ is a polynomial of degree $n - 1$.
$Q(x)$ is simply the quotient obtained from the division process; since r is known to be a root of $P(x)$, it is known that the remainder must be zero.

Schwartz-Zippel Lemma

“different polynomials are different at most points”.

Polynomials have an advantageous property, namely, if we have two non-equal polynomials of degree at most d, they can intersect at no more than d points.

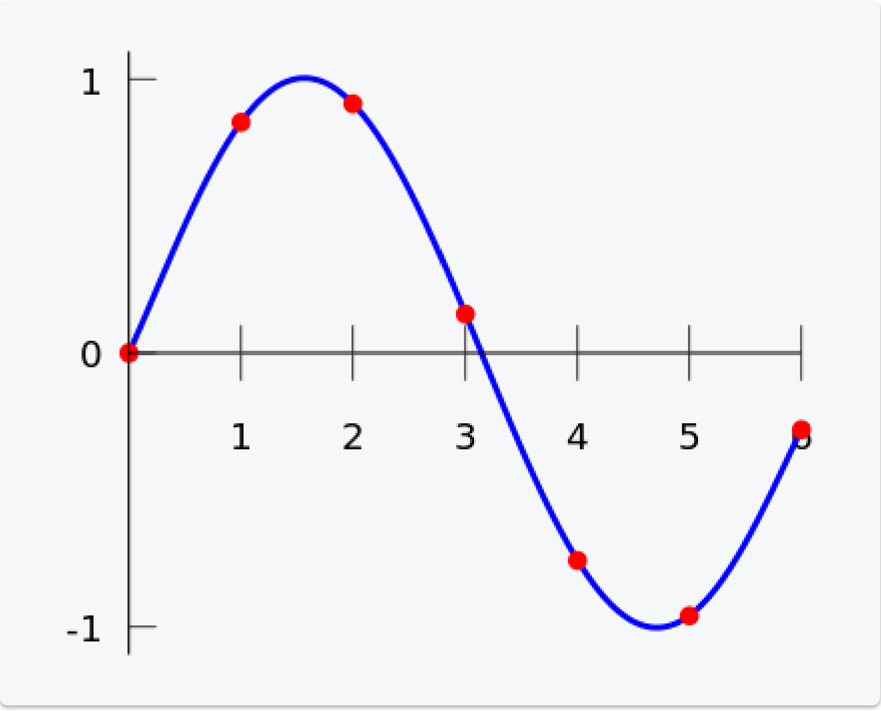


Lagrange Interpolation

If you have a set of points then doing a Lagrange interpolation on those points gives you a polynomial that passes through all of those points.

If you have two points on a plane, you can define a single straight line that passes through both, for 3 points, a single 2nd-degree curve (e.g. $5x^2 + 2x + 1$) will go through them etc.
For n points, you can create a n-1 degree polynomial that will go through all of the points.

(We can use this in all sorts of interesting schemes as well as zkps)


1
0
-1
1
2
3
4
5
6

> ## Polynomial Commitments
>
> A polynomial commitment is a short object that "represents" a polynomial, and allows you to verify evaluations of that polynomial, without needing to actually contain all of the data in the polynomial.
>
> That is, if someone gives you a commitment c representing $P(x)$, they can give you a proof that can convince you, for some specific z, what the value of $P(z)$ is. There is a further mathematical result that says that, over a sufficiently big field, if certain kinds of equations (chosen before z is known) about polynomials evaluated at a random z are true, those same equations are true about the whole polynomial as well.
>
> For example, if $P(z). Q(z) + R(z) = S(z) + 5$ for a particular z, then we know that it's overwhelmingly likely that $P(x). Q(x) + R(x) = S(x) + 5$ in general.
> Using such polynomial commitments, we could very easily check all of the above polynomial equations above - make the commitments, use them as input to generate z, prove what the evaluations are of each polynomial at z, and then run the equations with these evaluations instead of the original polynomials.

Idealised proving system

There is much missed out, and assumed here, this is just to show a general process.

Steps

1. Prover claims Statement S
2. Verifier provides some constraints about the polynomials
3. Prover provides (or commits to) $P_i..P_k$: polynomials
4. Verifier provides $z \in 0,..p-1$
5. Prover provides evaluations of polynomials: $P_1(z)...P_k(z)$
6. Verifier decides whether to accept S

The degree expected are typically about 10^6 (still considered low degree)
Note the probability of accepting a false proof is
$< 10.d/p$
where p is the size of the field, so of the order of 2^{-230}
if our finite field has p of ~ 2^{256}

typically the number of queries is 3 - 10, much less than the degree

The only randomness we use here is sampling z from $0,..p-1$, in general the randomness we use in the process is essential for both succinctness and zero knowledge

Why doesn't the verifier evaluate the polynomials themselves ?
- because, the prover doesn't actually send all the polynomials to the verifier, if they did we would lose succinctness, they contain more information than our original statement, so the prover just provides a commitment to the polynomials

What are the properties of polynomials that are important here ?

1. Polynomials are good error correcting codes

If we have polynomials of degree d over an encoding domain D, and two messages $m1$ and $m2$, then $m1$ and $m2$ will differ at $|D| - d$ points
This is important because we want the difference between a correct and an incorrect statement to be large, so easily found.
This leads to good sampling, which helps succinctness, we need only sample a few values to be sure that the probability of error is low enough to be negligible.

2. Have efficient batch zero testing
   This also helps with succinctness

> Imagine we want to prove that a large degree polynomial $P(x)$ (degree ~ 10 million) evaluates to zero at points $1...1$ million, but we want to do this with only one query.
>
> Imagine that our statement is that P vanishes on these points.
> If the verifier just uses sampling the prover could easily cheat by providing a point that evaluates to zero, but the other 999,999 could be non zero.
>
> We solve this by
>
> take a set S = $1....10^6$
>
> define V as the unique polynomial that vanishes on these points
> i.e.
> $(x-1)(x-2)(x-3)....$
> the degree of V = size of S
>
> this is good because
>
> $P(x)$ vanishes on S iff
> there exists $P'(x)$ such that
> 1. $P(x) = P'(x).V(x)$
> 2. degree of P' = degree of P - size of S
>
> It is the introduction of $V(x)$ that allows us to check across the whole domain
>
> 3. Have "multiplication" property
>   We can 'wrap' a constraint around a polynomial
>   For example if we have the constraint C, that our evaluation will always be a zero or a one, we could write this as $C(x) = x.(x-1)$
>   You could imagine this constraining an output to be a boolean, something that may be useful for computational integrity.
>
> But here instead of x being just a point it could be the evaluation of a polynomial $P_1(x)$ at a point
> i.e.
> $C(P_1(x)) = P_1(x).(P_1(x)-1)$
>
> and the degrees of the polynomials produced by the multiplication then are additive so degree of $C(x)$ = 2. degree of $P_1(x)$
>
> We can then make the claim, that if $P_1(x)$ does indeed obey this constraint for our set S then as before we can say that there is some polynomial $P'(x)$ such that
>
> $C(P_1(x)) = P'(x).V(x)$
>
> If $P_1(x)$ didn't obey the constraint (for example if for one value of x,$P_1(x) = 93$) then we wouldn't be able to find such polynomials, the equality wouldn't hold and there

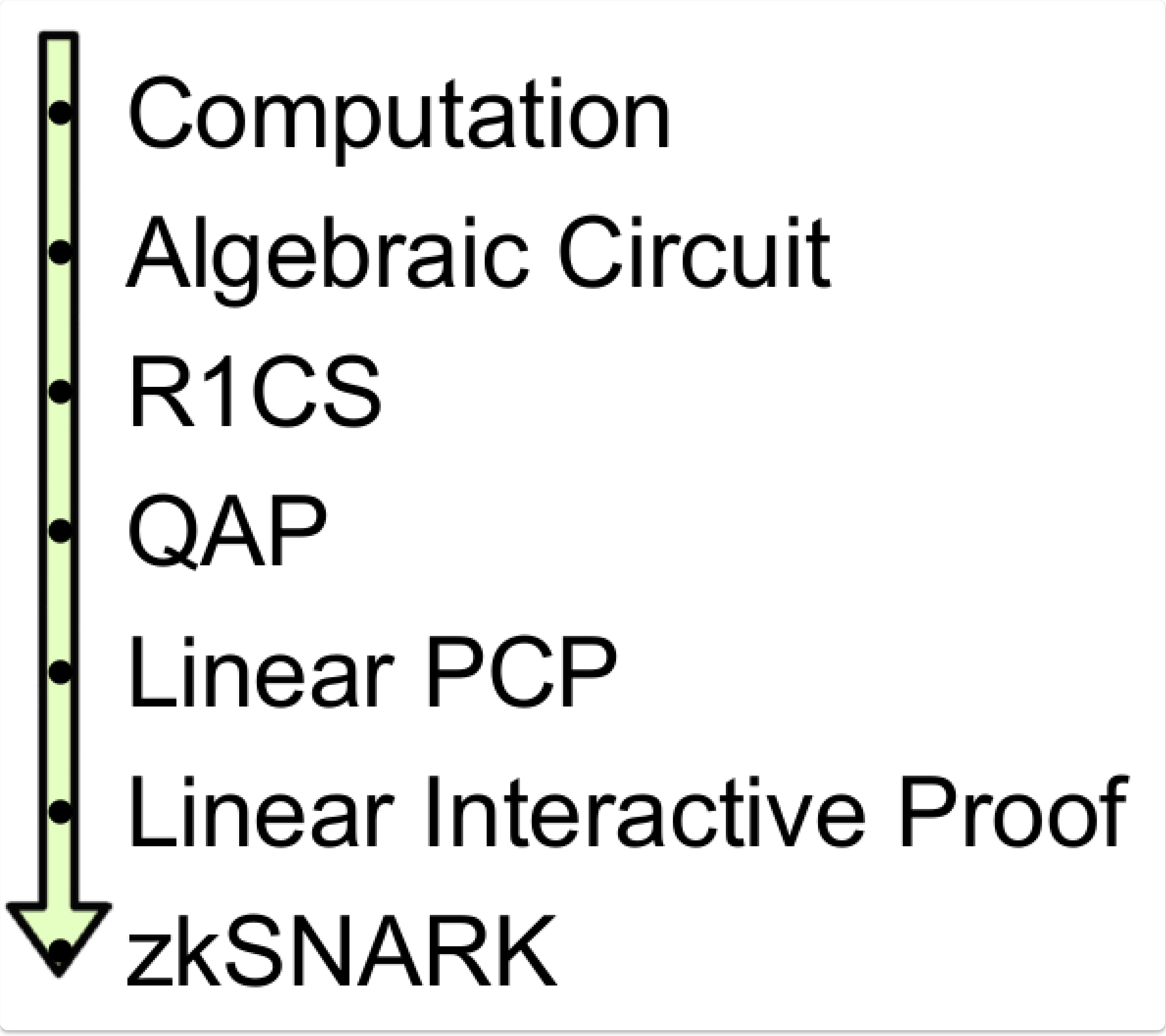
> would effectively be a remainder in the preceding equation.
>
> ### Use of randomness
>
> The prover uses randomness to achieve zero knowledge, the verifier uses randomness when generating queries to the prover, to detect cheating by the prover.

ZKSnark Process

General Process

- Arithmetisation
  - Flatten code
  - Arithmetic Circuit
  - R1CS
  - QAP
- Polynomials
- Polynomial Commitment Scheme
- Inner product argument
- Cryptographic proving system
- Make non interactive

Transformations in SNARKS



Computation

Algebraic Circuit

R1CS

QAP

Linear PCP

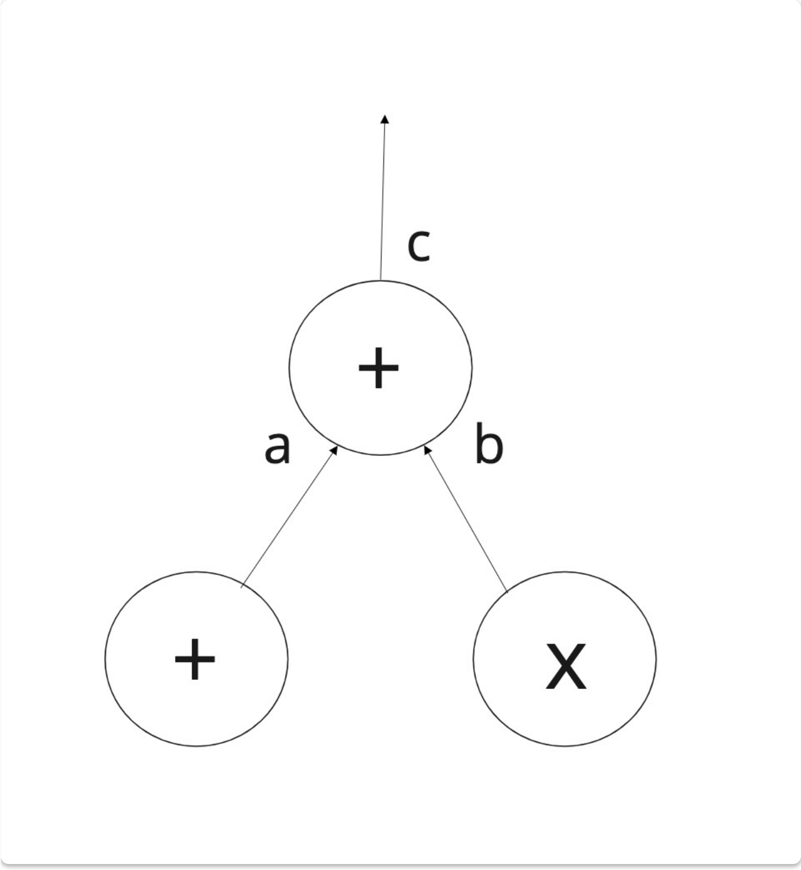
Linear Interactive Proof

zkSNARK

Process in PLONK

1. Trusted Setup
   ZKSNarks require a one off set up step to produce prover and verifier keys.
   This step is generally seen as a drawback to zkSNARKS, it requires an amount of trust, if details of the setup are later leaked it would be possible to create false proofs.

2. A High Level description is turned into an arithmetic circuit
   The creator of the zkSNARK uses a high level language to specify the algorithm that constitutes and tests the proof.
   This high level specification is compiled into an arithmetic circuit.
   An arithmetic circuit can be thought of as similar to a physical electrical circuit consisting of logical gates and wires. This circuit constrains the allowed inputs that will lead to a correct proof.



3. Further Mathematical refinement
   The circuit is then turned into a an R1CS, and then a series of formulae called a Quadratic Arithmetic Program (QAP).
   The QAP is then further refined to ensure the privacy aspect of the process.
   The end result is a proof in the form of series of bytes that is given to the

verifier. The verifier can pass this proof through a verifier function to receive a true or false result.
There is no information in the proof that the verifier can use to learn any further information about the prover or their witness.

Trusted Setups

From ZCash explanation :

"SNARKs require something called “the public parameters”. The SNARK public parameters are numbers with a specific cryptographic structure that are known to all of the participants in the system. They are baked into the protocol and the software from the beginning.

The obvious way to construct SNARK public parameters is just to have someone generate a public/private keypair, similar to an ECDSA keypair, (See ZCash explanation) and then destroy the private key.

The problem is that private key. Anybody who gets a copy of it can use it to counterfeit money. (However, it cannot violate any user’s privacy — the privacy of transactions is not at risk from this.)"

ZCash used a *secure multiparty computation* in which multiple people each generate a “shard” of the public/private keypair, then they each destroy their shard of the toxic waste private key, and then they all bring together their shards of the public key to to form the SNARK public parameters.
If that process works — i.e. if *at least one of the participants* successfully destroys their private key shard — then the toxic waste byproduct never comes into existence at all.

They have recently introduced Halo2 which eliminates the need for a trusted setup

Halo2 uses the curves Pallas and Vesta (collectively Pasta) which are also used by Mina

Pallas: y^2 = x^3 + 5y2=x3+5 over GF(0x40000000000000000000000000000000224698fc094cf91b992d30ed00000001)

Vesta: y^2 = x^3 + 5y2=x3+5 over GF(0x40000000000000000000000000000000224698fc0994a8dd8c46eb2100000001)

The use “nested amortization”— repeatedly solving over cycles of elliptic curves so that computational proofs can be used to reason about themselves efficiently, which eliminates the need for a trusted setup.

Transforming our problem into a QAP

Lets look first at transforming the problem into a QAP, there are
3 steps :

- code flattening,
- conversion to a rank-1 constraint system (R1CS)
- formulation of the QAP.

Code Flattening

We are aiming to create arithmetic and / or boolean circuits from our code, so we change the high level language into a sequence of statements that are of two forms

x = y (where y can be a variable or a number)
and
x = y (op) z
(where op can be +, -, $*$, / and y and z can be variables, numbers or themselves sub-expressions).

For example we go from

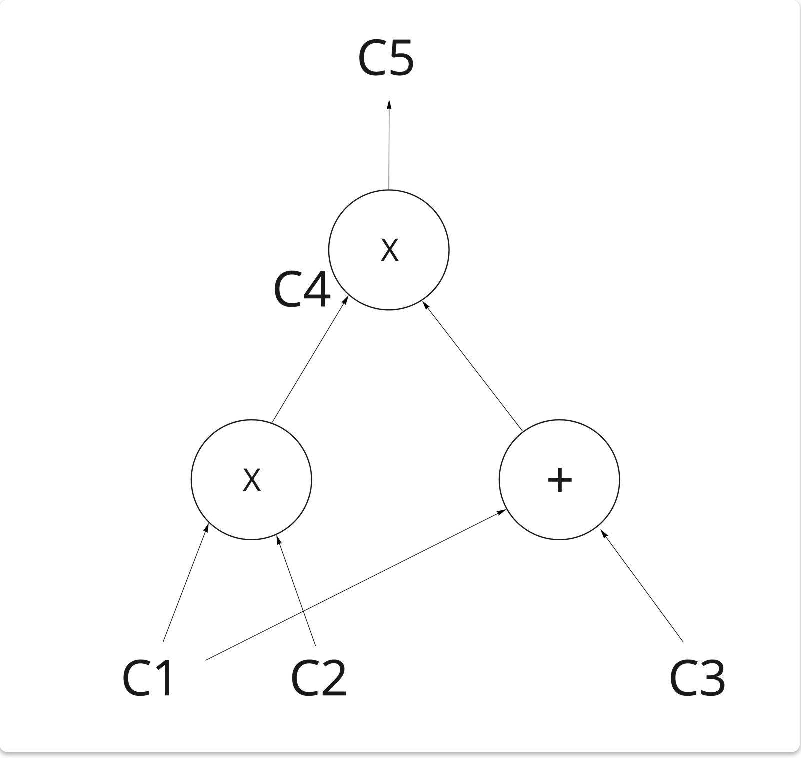
```
def qeval(x):
    y = x**3
    return x + y + 5
```

to

```
sym_1 = x * x
y = sym_1 * x
sym_2 = y + x
~out = sym_2 + 5
```

ARITHMETIC CIRCUIT

This is a collection of multiplication and addition gates



Rank 1 Constraint Systems

Constraint languages can be viewed as a generalization of functional languages:

- everything is referentially transparent and side-effect free
- there is no ordering of constraints
- composing two R1CS programs just means that their constraints are simultaneously satisfied.

(From http://coders-errand.com/constraint-systems-for-zk-snarks/)

The important thing to understand is that a R1CS is not a computer program, you are not asking it to produce a value from certain inputs. Instead, a R1CS is more of a verifier, it shows that an already complete computation is correct .

The arithmetc circuit is a compositon of multplicatve sub-circuits (a single multiplcation gate and mutiple addition gates)

A rank 1 constraint system is a set of these sub-circuits expressed as constraints, each of the form:
$AXB = C$
where A, B, C are each linear combinations c1· v1+ c2· v2+ ...
The c_i are constant field elements, and the v_i are instance or witness variables (or 1).

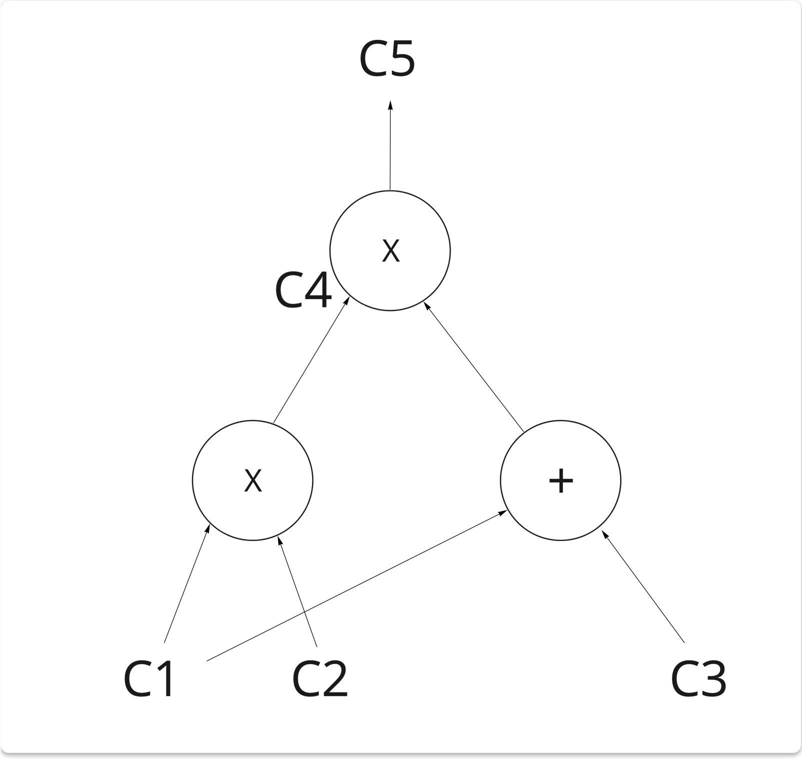
- $AXB = C$ doesn't mean C is computed from A and B just that A, B, C are consistent.

> More generally, an implementation of $x = f(a, b)$ doesn't mean that x is computed from a and b, just that x, a, and b are consistent.
>
> Thus our R1CS contains :
>
> - the constant 1
> - all public inputs
> - outputs of the function
> - private inputs
> - auxilliary variables
>
> The R1CS has
>
> - one constraint per gate;
> - one constraint per circuit output.

EXAMPLE

Assume Peggy wants to prove to Victor that she knows $c1, c2, c3$ such that $(c1 \cdot c2) \cdot (c1 + c3) = 7$

We transform the expression above into an arithmetic circuit as depicted below



A legal assignment for the circuit is of the form:

(c1, . . . , c5), where c4 = c1 · c2 and c5 = c4 · (c1 + c3).

SNARK Process continued

From R1CS to QAP

The next step is taking this R1CS and converting it into QAP form, which implements the exact same logic except using polynomials instead of dot products.

To create the polynomials we can use interpolation of the values in our R1CS

Then instead of checking the constraints in the R1CS individually, we can now check *all of the constraints at the same time* by doing the dot product check *on the polynomials*.

Because in this case the dot product check is a series of additions and multiplications of polynomials, the result is itself going to be a polynomial. If the resulting polynomial, evaluated at every x coordinate that we used above to represent a logic gate, is equal to zero, then that means that all of the checks pass; if the resulting polynomial evaluated at at least one of the x coordinate representing a logic gate gives a nonzero value, then that means that the values going into and out of that logic gate are inconsistent

How having polynomials helps us

We can change the problem into that of knowing a polynomial with certain properties

This paper gives a reasonable explanation of how the polynomials are used to prevent the prover 'cheating'

We converted a set of vectors into polynomials that generate them when evaluated at certain fixed points.
We used these fixed points to generate a vanishing polynomial that divides any polynomial that evaluates to 0 at least on all those points.
We created a new polynomial that summarizes all constraints and a particular assignment, and the consequence is that we can verify all constraints at once if we can divide that polynomial by the vanishing one without remainder.
This division is complicated, but there are methods (the Fast Fourier Transform) that can perform if efficiently.

From our QAP we have

$$L := \sum_{i=1}^{m} ci \cdot Li, R := \sum_{i=1}^{m} ci \cdot Ri, O := \sum_{i=1}^{m} ci \cdot Oi$$

and we define the polynomial P
$P := L \cdot R - O$
Defining the target polynomial $V(x) := (x-1) \cdot (x-2) \ldots$,
This will be zero at the points that correspond to our gates, but the P polynomial, having all the constraints information would be a some multiple of this if

- it is also zero at those points
- to be zero at those points, $L \cdot R - O$ must equate to zero, which will only happen if our constraints are met.

So we want V to divide P with no remainder, which would show that P is indeed zero at the points.

If Peggy has a satisfying assignment it means that, defining L,R,O,P as above, there exists a polynomial P' such that
$P = P'.V$

In particular, for any $z \in \mathbb{F}p$ we have $P(z) = P'(z) \cdot V(z)$

Suppose now that Peggy doesn't have a satisfying witness, but she still constructs L, R, O, P as above from some unsatisfying assignment (c1,…,cm)(c1,…,cm).
Then we are guaranteed that V does not divide P.
This means that for any polynomial V of degree at most $d-2$, P and L, R, O, V will be different polynomials.
Note that P here is of degree at most $2(d-1)$, L, R, O here are of degree at most $d-1$ and V here is degree at most $d-2$.

Remember the Schwartz-Zippel Lemma tells us that two different polynomials of degree at most d can agree on at most d points.

Homomorphic Hiding Review

If $E(x)$ is a function with the following properties

- Given $E(x)$ it is hard to find x
- Different inputs lead to different outputs so if $x \neq y$ $E(x) \neq E(y)$
- We can compute $E(x + y)$ given $E(x)$ and $E(y)$

The group $\mathbb{Z}_p^*$ with operations addition and multiplication allows this.

Here's a toy example of why Homomorphic Hiding is useful for Zero-Knowledge proofs: Suppose Alice wants to prove to Bob she knows numbers x,y such that $x + y = 7$

1. Alice sends $E(x)$ and $E(y)$ to Bob.
2. Bob computes $E(x + y)$ from these values (which he is able to do since E is an HH).
3. Bob also computes $E(7)$, and now checks whether $E(x + y) = E(7)$. He accepts Alice's proof only if equality holds.

As different inputs are mapped by E to different hidings, Bob indeed accepts the proof only if Alice sent hidings of x, y such that $x + y = 7$. On the other hand, Bob does not learn x and y as he just has access to their hidings 2(https://electriccoin.co/blog/snark-explain/#id5).

Creating a non interactive proof and adding zero knowledge

BLIND EVALUATION OF A POLYNOMIAL USING HOMOMORPHIC HIDING

Suppose Peggy has a polynomial P of degree d , and Victor has a point $z \in \mathbb{F}_p$that he chose randomly.
Victor wishes to learn $E(P(z))$, i.e., the Homomorphic Hiding of the evaluation of P at z
Two simple ways to do this are:

1. Peggy sends P to Victor, and he computes $E(P(z))$ by himself.
2. Victor sends z to Peggy; she computes $E(P(z))$ and sends it to Victor.

However, in the blind evaluation problem we want Victor to learn $E(P(z))$ without learning P
which precludes the first option; and, most importantly,
we don't want Peggy to learn z, which rules out the second

Using homomorphic hiding, we can perform blind evaluation as follows.

1. Victor sends to Peggy the hidings $E(1), E(z_1), \ldots, E(z_d)$

2. Peggy computes $E(P(z))$ from the elements sent in the first step, and sends $E(P(z))$ to Victor. (Peggy can do this since E supports linear combinations, and $P(z)$ is a linear combination of $1, z_1, \ldots, z_d$)

Note that, as only hidings were sent, neither Peggy learned z nor Victor learned P

The rough intuition is that the verifier has a “correct” polynomial in mind, and wishes to check the prover knows it. Making the prover blindly evaluate their polynomial at a random point not known to them, ensures the prover will give the wrong answer with high probability if their polynomial is not the correct one (Schwartz-Zippel Lemma).

However
The fact that Peggy is able to compute $E(P(z))$ does not guarantee she will indeed send $E(P(z))$ to Victor, rather than some completely unrelated value.

Our process then becomes

1. Peggy chooses polynomials L,R,O,P, P'
2. Victor chooses a random point $z \in \mathbb{F}p$, and computes $E(P(z))$
3. Peggy sends Victor the hidings of all these polynomials evaluated at z, i.e. $E(L(z)), E(R(z)), E(O(z)), E(P(z), E(P'(z)))$

Furthermore we use

- Random values added to our z to conceal the z value
- The Knowledge of Coefficient Assumption to prove Peggy can produce a linear combination of the polynomials.

If Peggy does not have a satisfying assignment, she will end up using polynomials where the equation does not hold identically, and thus does not hold at most choices of z Therefore, Victor will reject with high probability over his choice of z.

We now need to make our proof non interactive, for this we use the Common Reference String from Victor’s first message

Non-interactive proofs in the common reference string model

In the CRS model, before any proofs are constructed, there is a setup phase where a string is constructed according to a certain randomized process and broadcast to all parties. This string is called the CRS and is then used to help construct and verify proofs. The assumption is that the randomness used in the creation of the CRS is not known to any party – as knowledge of this randomness might enable constructing proofs of false claims.

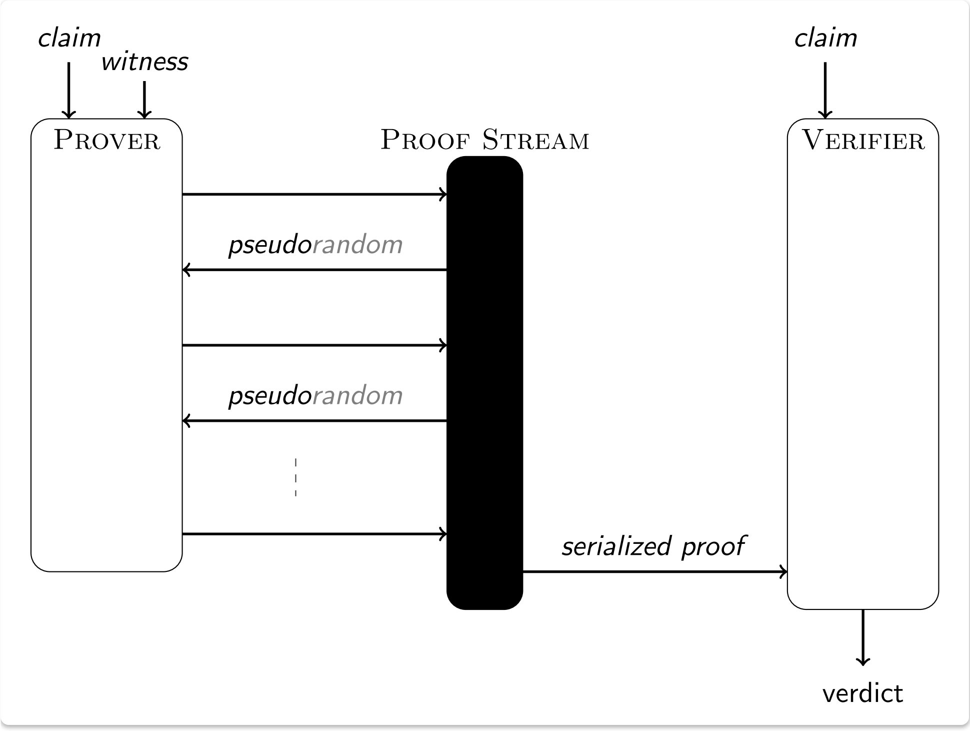
Why do we need this randomness

> Victor is sending challenges to Peggy, if Peggy could know what exactly the challenge is going to be, she could choose its randomness in such a way that it could satisfy the challenge, even if she did not know the correct solution for the instance (that is, faking the proof).
>
> So, Victor must only issue the challenge after Peggy has already fixed her randomness. This is why Peggy first *commits* to her randomness, and implicitly reveals it only after the challenge, when she uses that value to compute the proof. That ensures two things:
>
> 1. Victor cannot *guess* what value Peggy committed to;
> 2. Peggy cannot *change* the value she committed to.
>
> ---

Fiat-Shamir heuristic

See https://aszepieniec.github.io/stark-anatomy/basic-tools

This is a process by which we can make an interactive proof non-interactive.
It works by providing commitments to the messages that would form the interaction.
The hash functions are used as a source of randomness.



Resources

Quadratic Arithmetic Programs: from Zero to Hero

How Plonk Works

Plonkish protocols

(fflonk, turbo PLONK, ultra PLONK, plonkup, and recently plonky2.)

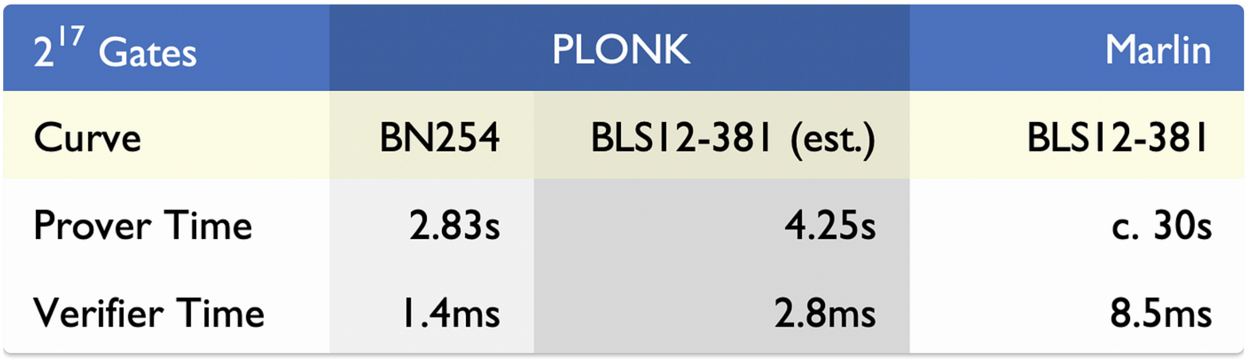
Before PLONK

Early SNARK implementations such as Groth16 depend on a common reference string, this is a large set of points on an elliptic curve.
Whilst these numbers are created out of randomness, internally the numbers in this list have strong algebraic relationships to one another. These relationships are used as short-cuts for the complex mathematics required to create proofs.
Knowledge of the randomness could give an attacker the ability to create false proofs.

A trusted-setup procedure generates a set of elliptic curve points $G, G \cdot s, G \cdot s^2 .... G \cdot s^n$, as well as $G2 \cdot s$, where G and $G2$ are the generators of two elliptic curve groups and s is a secret that is forgotten once the procedure is finished (note that there is a multi-party version of this setup, which is secure as long as at least one of the participants forgets their share of the secret).
(The Aztec reference string goes up to the 10066396th power)

A problem remains that if you change your program and introduce a new circuit you require a fresh trusted setup.

In January 2019 Mary Maller, Sean Bowe et al released SONIC that has a universal setup, with just one setup, it could validate any conceivable circuit (up to a predefined level of complexity).
This was unfortunately not very efficient, PLONK managed to optimise the process to make the proof process feasible.



2^{17} Gates	PLONK		Marlin
Curve	BN254	BLS12-381 (est.)	BLS12-381
Prover Time	2.83s	4.25s	c. 30s
Verifier Time	1.4ms	2.8ms	8.5ms

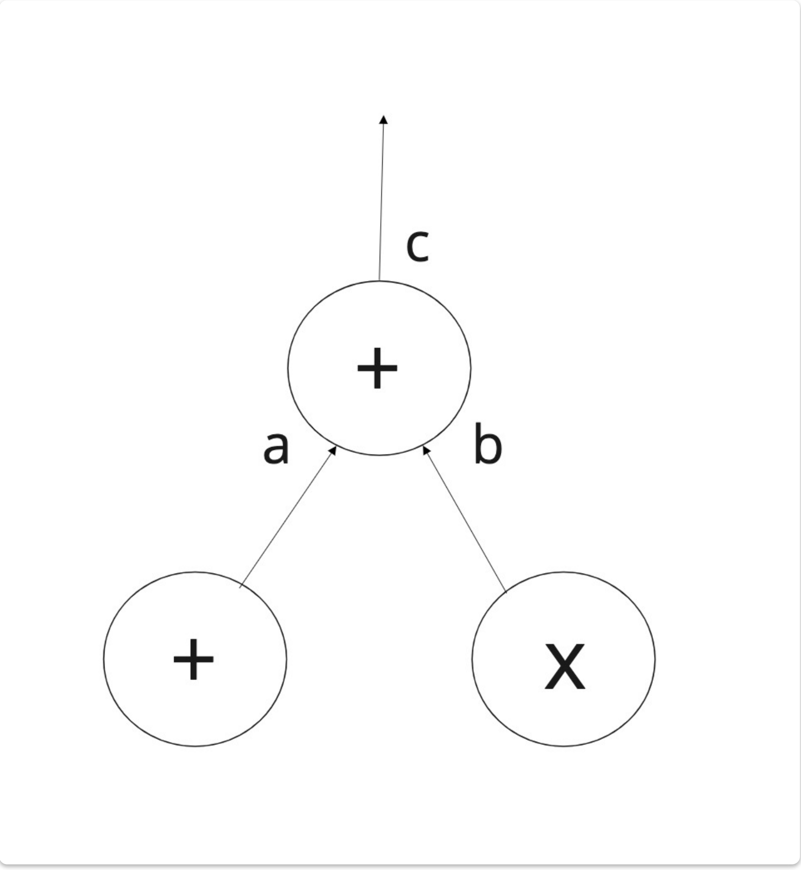
See PLONK: Permutations over Lagrange-bases for Oecumenical Noninteractive arguments of Knowledge
Also see Understanding PLONK

Trusted Setup

This is still needed, but it is a "universal and updateable" trusted setup.

- There is one single trusted setup for the whole scheme after which you can use the scheme with any program (up to some maximum size chosen when making the setup).
- There is a way for multiple parties to participate in the trusted setup such that it is secure as long as any one of them is honest, and this multi-party procedure is fully sequential:

Arithmetic Circuits



Developing a circuit

Once we have a (potentially large)circuit, we want to get it into a more usable form, so we can put the values into a table detailing the inputs and outputs for a gate.

So for gates 1 to i we can represent the a, b and c values as
a_i, b_i, c_i

And if the circuit is correct then for an addition gate

a_i+ b_i = c_i

or
a_i+ b_i - c_i = 0

and for a multiplication gate

a_i.b_i - c_i = 0

We would end up with a table like this

	a	b	c	S
1	a_1	a_1	a_1	1
2	a_2	b_2	c_2	1
3	a_3	b_3	c_3	0

But we would also want to know what type of gate it is, there is a useful trick where we introduce a selector S, which is 1 for an addition gate and 0 for a multiplication gate.

We can then generalise our equation as

$$S_i(a_i + b_i) + (1 - S_i)a_i.b_i - c_i = 0$$

These are called the *gate constraints* because they refer to the equalities for a particular gate.

We can also have *copy constraints* where we have a relationship between values that are not on the same gate, for example it may be the case that

a_7 = b_5 for a particular circuit, in fact this is how we link the gates together.

In PLONK we also have constant gates and more specialised gates.
For example representing a hash function as a series of generic gates (addition, multiplication and constant) would be inefficient.

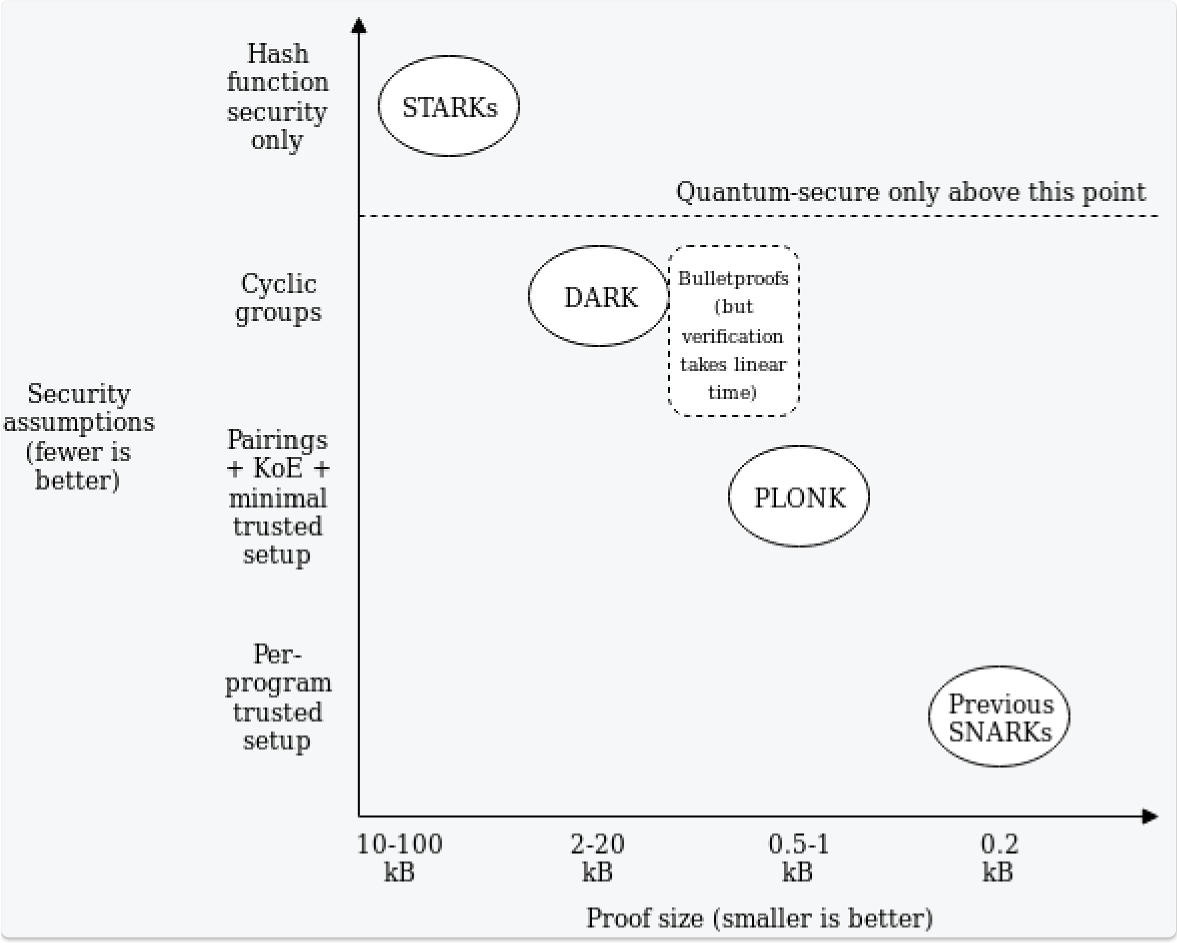
> From Zac Williamson
>
> "PLONK’s strength is these things we call custom gates. It’s basically you can define your own custom bit arithmetic operations. I guess you can call them mini gadgets.
> That are extremely efficient to evaluate inside a PLONK circuit.
> So you can do things like do elliptical curve point addition in a gate.
> You can do things like efficient Poseidon hashes, Pedersen hashes. You can do parts of a SHA-256 hash. You can do things like 8-bit logical XOR operations.
> All these like explicit instructions which are needed for real world circuits, but they’re all quite custom."

Plookup

Some operations such as boolean operations do not need to be computed, but can be put into a lookup table. Similarly public data can be put in a lookup table.
For example we could use a lookup table for the XOR operation, or include common values as we saw in the range proofs in Aztec.

Polynomial Commitments in PLONK

PLONK uses Kate commitments based on trusted setup and elliptic curve pairings, but these can be swapped out with other schemes, such as FRI (which would turn PLONK into a kind of STARK


Security assumptions (fewer is better)
Hash function security only
Cyclic groups
Pairings + KoE + minimal trusted setup
Per-program trusted setup
STARKs
Quantum-secure only above this point
DARK
Bulletproofs (but verification takes linear time)
PLONK
Previous SNARKs
10-100 kB
2-20 kB
0.5-1 kB
0.2 kB
Proof size (smaller is better)

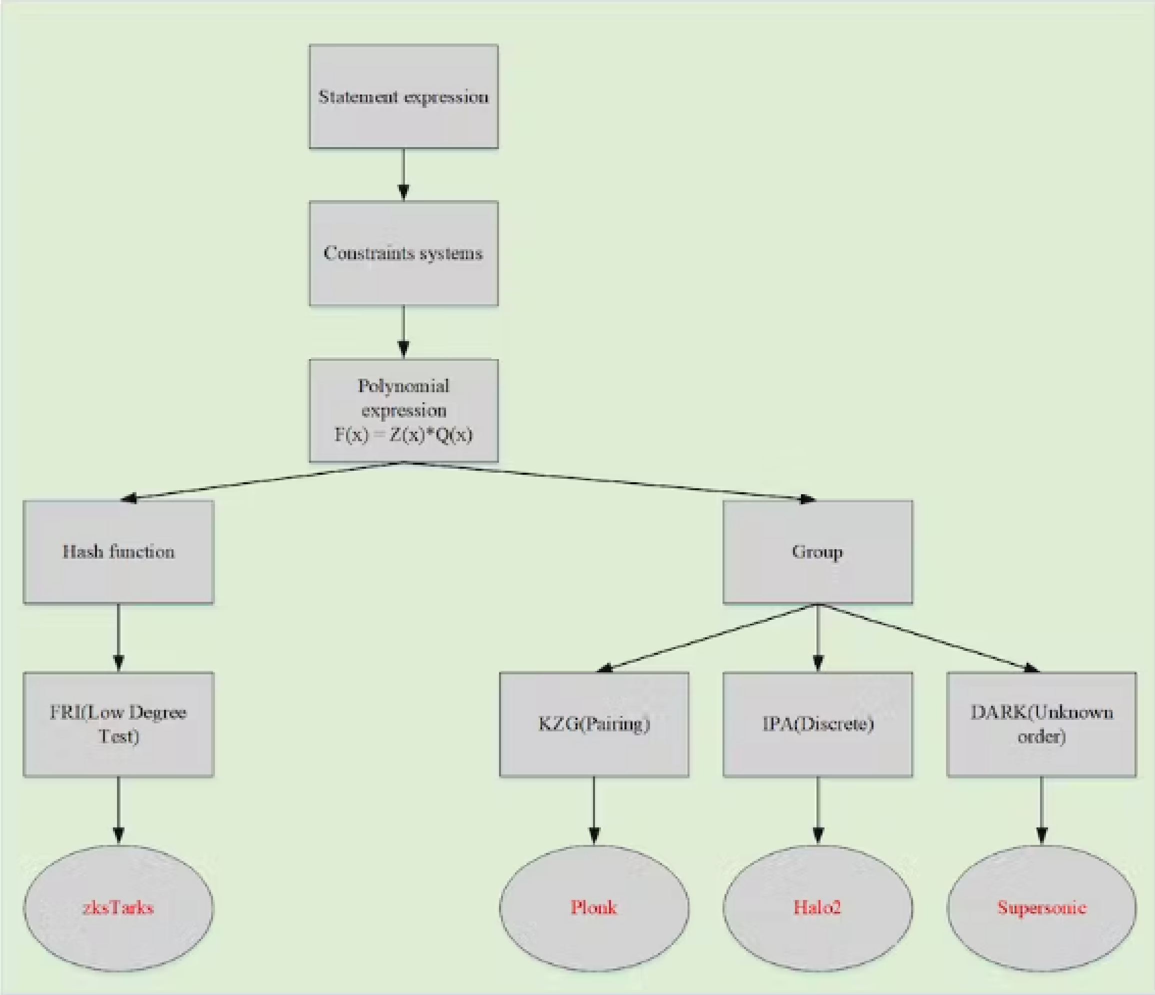
This means the arithmetisation - the process for converting a program into a set of polynomial equations can be the same in a number of schemes.
If this kind of scheme becomes widely adopted, we can thus expect rapid progress in improving shared arithmetisation techniques.

For a detailed talk about some of the custom gates used in Plonk see this video

For more information about the KZG commitment scheme see this introduction

Other commitment schemes are available :
Dory
Dark

For an analysis of polynomial commitment schemes, see this article


Statement expression
Constraints systems
Polynomial expression F(x) = Z(x)*Q(x)
Hash function
Group
FRI(Low Degree Test)
KZG(Pairing)
IPA(Discrete)
DARK(Unknown order)
zksTarks
Plonk
Halo2
Supersonic

Plonky2

From article

Plonky2 also allows us to speed up proving times for proofs that don’t involve recursion. With FRI, you can either have fast proofs that are big (so they’re more expensive to verify on Ethereum), or you can have slow proofs that are small. Constructions that use FRI, like the STARKs that Starkware uses in their ZK-rollups, have to choose; they can’t have maximally fast proving times and proof sizes that are small enough to reasonably verify on Ethereum.

Plonky2 eliminates this tradeoff. In cases where proving time matters, we can optimize for maximally fast proofs. When these proofs are recursively aggregated, we’re left with a single proof that can be verified in a small circuit. At this point, we can optimize for proof size. We can shrink our proof sizes down to 45kb with only 20s of proving time (not a big deal since we only generate when we submit to Ethereum), dramatically reducing costs relative to Starkware.

Plonky2 is natively compatible with Ethereum. Plonky2 requires only keccak-256 to verify a proof. We’ve estimated that the gas cost to verify a plonky2 size-optimized proof on Ethereum will be approximately 1 million gas.

However, this cost is dominated by the CALLDATA costs to publish the proof on Ethereum. Since CALLDATA was repriced in EIP-4488, the verification cost of a plonky2 proof has dropped to between 170-200k gas, which could make it not only the fastest proving system, but also the cheapest to verify on Ethereum.

Circuit / QAP process in PLONK

What the prover and verifier can calculate ahead of time

The program-specific polynomials that the prover and verifier need to compute ahead of time are:

$Q_L(x), Q_R(x), Q_O(x), Q_M(x), Q_C(x)$, which together represent the gates in the circuit (note that $Q_C(x)$ encodes public inputs, so it may need to be computed or modified at runtime)

The "permutation polynomials" $\sigma_a(x), \sigma_b(x)$ and $\sigma_c(x)$, which encode the copy constraints between the $a, b,$and c wires

Given a program P, you convert it into a circuit, and generate a set of equations that look like this:
Each equation is of the following form (L = left, R = right, O = output, M = multiplication, C = constant):
and a_i, b_i are the wire values

> Each $Q $ value is a constant; the constants in each equation (and the number of equations) will be different for each program.
>
> $$(Q_{L_i})a_i + (Q_{R_i})b_i + (Q_{O_i})c_i + (Q_{M_i})a_i b_i + Q_{C_i} = 0$$
>
> You then convert this set of equations into a single polynomial equation:
>
> $$Q_L(x)a(x) + Q_R(x)b(x) + Q_O(x)c(x) + Q_M(x)a(x)b(x) + Q_C(x) = 0$$
>
> You also generate from the circuit a list of copy constraints. From these copy constraints you generate the three polynomials representing the permuted wire indices:
> $\sigma_a(x), \sigma_b(x), \sigma_c(x)$
>
> To generate a proof, you compute the values of all the wires and convert them into three polynomials:
> $a(x), b(x), c(x)$
> You also compute six "coordinate pair accumulator" polynomials as part of the permutation-check argument.
>
> Finally you compute the cofactors $H_i(x)$
>
> There is a set of equations between the polynomials that need to be checked; you can do this by making commitments to the polynomials, opening them at some random z (along with proofs that the openings are correct), and running the equations on these evaluations instead of the original polynomials.
> The proof itself is just a few commitments and openings and can be checked with a few equations.