Solutions to selected exercises in complement of the book

Principles of Abstract Interpretation

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Solution to exercise 3.40 Consider $P = \{0\}$. We have $P \subseteq \{z \mid z \leqslant 0\} = \gamma_\pm(\{z \mid z \leqslant 0\})$ and $P \subseteq \{z \mid z \geqslant 0\} = \gamma_\pm(\{z \mid z \geqslant 0\})$ and so, by def. of a Galois connection, $\alpha_\pm(P) \sqsubseteq_\pm \{z \mid z \leqslant\}$ and $\alpha_\pm(P) \sqsubseteq_\pm \{z \mid z \geqslant 0\}$. The only element with this property in $\langle \mathbb{P}^\pm, \sqsubseteq_\pm \rangle$ is $\varnothing$ so we must have $\alpha_\pm(P) = \varnothing$ so $\alpha_\pm(\{0\}) \sqsubseteq_\pm \varnothing$ by reflexivity. By def. of a Galois connection, it follows that $\{0\} \subseteq \gamma_\pm(\varnothing) = \varnothing$, a contradiction. □

1 Solutions to selected exercises of chapter 4

Solution to exercise 4.4 An `if (B)` $\mathtt{S}_t$ `else` $\mathtt{S}_f$ can be replaced with $((\mathtt{B}) \uparrow (\mathtt{B})$ is $\neg(\mathtt{B}))$

$$\texttt{while (B) \{ } \mathtt{S}_t \texttt{ break ; \} while ((B)} \uparrow \texttt{(B)) \{ } \mathtt{S}_f \texttt{ break ; \}}.$$ □

Solution to exercise 4.7

$$\begin{array}{ll}
\mathsf{after}[\![\mathtt{P}_{15}]\!] = \mathsf{after}[\![\mathtt{Sl}_{14}]\!] = \ell_7 & \mathsf{after}[\![\mathtt{S}_9]\!] = \ell_2 \\
\mathsf{after}[\![\mathtt{Sl}_{12}]\!] = \mathsf{at}[\![\mathtt{S}_{13}]\!] = \ell_6 & \mathsf{after}[\![\mathtt{Sl}_6]\!] = \mathsf{after}[\![\mathtt{S}_9]\!] = \ell_2 \\
\mathsf{after}[\![\mathtt{S}_{13}]\!] = \mathsf{after}[\![\mathtt{Sl}_{14}]\!] = \ell_7 & \mathsf{after}[\![\mathtt{Sl}_4]\!] = \mathsf{at}[\![\mathtt{S}_5]\!] = \ell_4 \\
\mathsf{after}[\![\mathtt{Sl}_{10}]\!] = \mathsf{at}[\![\mathtt{S}_{11}]\!] = \ell_2 & \mathsf{after}[\![\mathtt{S}_5]\!] = \mathsf{after}[\![\mathtt{Sl}_6]\!] = \ell_2 \\
\mathsf{after}[\![\mathtt{S}_{11}]\!] = \mathsf{after}[\![\mathtt{Sl}_{12}]\!] = \ell_6 & \mathsf{after}[\![\mathtt{Sl}_1]\!] = \mathsf{at}[\![\mathtt{S}_2]\!] = \ell_3 \\
\mathsf{after}[\![\mathtt{Sl}_7]\!] = \mathsf{at}[\![\mathtt{S}_8]\!] = \ell_1 & \mathsf{after}[\![\mathtt{S}_2]\!] = \mathsf{after}[\![\mathtt{Sl}_4]\!] = \ell_4 \\
\mathsf{after}[\![\mathtt{S}_8]\!] = \mathsf{after}[\![\mathtt{Sl}_{10}]\!] = \ell_2 & \mathsf{after}[\![\mathtt{S}_3]\!] = \mathsf{after}[\![\mathtt{S}_5]\!] = \ell_2
\end{array}$$

So $\mathsf{after}[\![\mathtt{S}]\!]$ is the label where execution goes on when `S` terminates without a **`break ;`**. □

Solution to exercise 4.12

$$\begin{array}{l}
\mathsf{break\text{-}to}[\![\mathtt{P}_{15}]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_{14}]\!] = \ell_7 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_{12}]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_{13}]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_{14}]\!] = \ell_7 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_{10}]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_{11}]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_{12}]\!] = \ell_7 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_7]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_8]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_{10}]\!] = \ell_7 \\
\mathsf{break\text{-}to}[\![\mathtt{S}_9]\!] = \mathsf{after}[\![\mathtt{S}_{11}]\!] = \ell_6 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_6]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_9]\!] = \ell_6 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_4]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_5]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_6]\!] = \ell_6 \\
\mathsf{break\text{-}to}[\![\mathtt{Sl}_1]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_2]\!] = \mathsf{break\text{-}to}[\![\mathtt{Sl}_4]\!] = \ell_6 \\
\mathsf{break\text{-}to}[\![\mathtt{S}_3]\!] = \mathsf{break\text{-}to}[\![\mathtt{S}_5]\!] = \ell_6
\end{array}$$

so a `break` before the `while` loop would terminate the program at ℓ_7 while a `break` inside the `while` loop (like ℓ_5**`break ;`**) terminates this loop at ℓ_6. □

2 Solutions to selected exercises of chapter 5

Solution to exercise 5.1 Given $\mathtt{a} \in \mathcal{A}$ and the empty string ϵ, a regular expression has syntax $R ::= \mathtt{a} \mid \epsilon \mid R? \mid R_1 \mid R_2 \mid R^+ \mid R^*$, and semantics $\mathcal{S}[\![\mathtt{a}]\!] \triangleq \{\mathtt{a}\}$, $\mathcal{S}[\![\epsilon]\!] \triangleq \{\epsilon\}$, $\mathcal{S}[\![R?]\!] \triangleq \mathcal{S}[\![R]\!] \cup \{\epsilon\}$, $\mathcal{S}[\![R_1 \mid R_2]\!] \triangleq \mathcal{S}[\![R_1]\!] \cup \mathcal{S}[\![R_2]\!]$, $\mathcal{S}[\![R^0]\!] \triangleq \{\epsilon\}$, $\mathcal{S}[\![R^1]\!] \triangleq \mathcal{S}[\![R]\!]$, $\mathcal{S}[\![R^{n+1}]\!] \triangleq \mathcal{S}[\![R]\!]\mathcal{S}[\![R^n]\!]$, $\mathcal{S}[\![R^+]\!] \triangleq \bigcup_{n>0} \mathcal{S}[\![R^n]\!]$, $\mathcal{S}[\![R^*]\!] \triangleq \bigcup_{n\geqslant 0} \mathcal{S}[\![R^n]\!]$ and language concatenation $\mathcal{L}_1\mathcal{L}_2 \triangleq \{\sigma_1\sigma_2 \mid \sigma_1 \in \mathcal{L}_1 \wedge \sigma_2 \in \mathcal{L}_2\}$ is the concatenation of strings in the languages. □

Solution to exercise 5.4

```
aterm:
  | NUM
  | IDENT
  | MINUS aterm
```