

Solutions to selected exercises
in complement of the book

Principles of Abstract Interpretation

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Solution to exercise 3.40 Consider $P = \{0\}$. We have $P \subseteq \{z \mid z \leq 0\} = \gamma_+(\{z \mid z \leq 0\})$ and $P \subseteq \{z \mid z \geq 0\} = \gamma_+(\{z \mid z \geq 0\})$ and so, by def. of a Galois connection, $\alpha_+(P) \sqsubseteq_+ \{z \mid z \leq 0\}$ and $\alpha_+(P) \sqsubseteq_+ \{z \mid z \geq 0\}$. The only element with this property in $\langle \mathbb{P}^+, \sqsubseteq_+ \rangle$ is \emptyset so we must have $\alpha_+(P) = \emptyset$ so $\alpha_+(\{0\}) \sqsubseteq_+ \emptyset$ by reflexivity. By def. of a Galois connection, it follows that $\{0\} \subseteq \gamma_+(\emptyset) = \emptyset$, a contradiction. \square

1 Solutions to selected exercises of chapter 4

Solution to exercise 4.4 An `if (B) St else Sf` can be replaced with `((B) ↑ (B) is ¬(B))`

`while (B) { St break ; } while ((B) ↑ (B)) { Sf break ; }.` \square

Solution to exercise 4.7

$$\begin{array}{ll}
\text{after}[\![P_{15}]\!] = \text{after}[\![SL_{14}]\!] = \ell_7 & \text{after}[\![S_9]\!] = \ell_2 \\
\text{after}[\![SL_{12}]\!] = \text{at}[\![S_{13}]\!] = \ell_6 & \text{after}[\![SL_6]\!] = \text{after}[\![S_9]\!] = \ell_2 \\
\text{after}[\![S_{13}]\!] = \text{after}[\![SL_{14}]\!] = \ell_7 & \text{after}[\![SL_4]\!] = \text{at}[\![S_5]\!] = \ell_4 \\
\text{after}[\![SL_{10}]\!] = \text{at}[\![S_{11}]\!] = \ell_2 & \text{after}[\![S_5]\!] = \text{after}[\![SL_6]\!] = \ell_2 \\
\text{after}[\![S_{11}]\!] = \text{after}[\![SL_{12}]\!] = \ell_6 & \text{after}[\![SL_1]\!] = \text{at}[\![S_2]\!] = \ell_3 \\
\text{after}[\![SL_7]\!] = \text{at}[\![S_8]\!] = \ell_1 & \text{after}[\![S_2]\!] = \text{after}[\![SL_4]\!] = \ell_4 \\
\text{after}[\![S_8]\!] = \text{after}[\![SL_{10}]\!] = \ell_2 & \text{after}[\![S_3]\!] = \text{after}[\![S_5]\!] = \ell_2
\end{array}$$

So $\text{after}[\![S]\!]$ is the label where execution goes on when S terminates without a **break** ;. \square

Solution to exercise 4.12

$$\begin{array}{l}
\text{break-to}[\![P_{15}]\!] = \text{break-to}[\![SL_{14}]\!] = \ell_7 \\
\text{break-to}[\![SL_{12}]\!] = \text{break-to}[\![S_{13}]\!] = \text{break-to}[\![SL_{14}]\!] = \ell_7 \\
\text{break-to}[\![SL_{10}]\!] = \text{break-to}[\![S_{11}]\!] = \text{break-to}[\![SL_{12}]\!] = \ell_7 \\
\text{break-to}[\![SL_7]\!] = \text{break-to}[\![S_8]\!] = \text{break-to}[\![SL_{10}]\!] = \ell_7 \\
\text{break-to}[\![S_9]\!] = \text{after}[\![S_{11}]\!] = \ell_6 \\
\text{break-to}[\![SL_6]\!] = \text{break-to}[\![S_9]\!] = \ell_6 \\
\text{break-to}[\![SL_4]\!] = \text{break-to}[\![S_5]\!] = \text{break-to}[\![SL_6]\!] = \ell_6 \\
\text{break-to}[\![SL_1]\!] = \text{break-to}[\![S_2]\!] = \text{break-to}[\![SL_4]\!] = \ell_6 \\
\text{break-to}[\![S_3]\!] = \text{break-to}[\![S_5]\!] = \ell_6
\end{array}$$

so a **break** before the **while** loop would terminate the program at ℓ_7 while a **break** inside the **while** loop (like ℓ_5 **break** ;) terminates this loop at ℓ_6 . \square

2 Solutions to selected exercises of chapter 5

Solution to exercise 5.1 Given $a \in \mathcal{A}$ and the empty string ϵ , a regular expression has syntax $R ::= a \mid \epsilon \mid R? \mid R_1 \mid R_2 \mid R^+ \mid R^*$, and semantics $\mathcal{S}[\![a]\!] \triangleq \{a\}$, $\mathcal{S}[\![\epsilon]\!] \triangleq \{\epsilon\}$, $\mathcal{S}[\![R?]\!] \triangleq \mathcal{S}[\![R]\!] \cup \{\epsilon\}$, $\mathcal{S}[\![R_1 \mid R_2]\!] \triangleq \mathcal{S}[\![R_1]\!] \cup \mathcal{S}[\![R_2]\!]$, $\mathcal{S}[\![R^0]\!] \triangleq \{\epsilon\}$, $\mathcal{S}[\![R^1]\!] \triangleq \mathcal{S}[\![R]\!]$, $\mathcal{S}[\![R^{n+1}]\!] \triangleq \mathcal{S}[\![R]\!]\mathcal{S}[\![R^n]\!]$, $\mathcal{S}[\![R^+]\!] \triangleq \bigcup_{n>0} \mathcal{S}[\![R^n]\!]$, $\mathcal{S}[\![R^*]\!] \triangleq \bigcup_{n \geq 0} \mathcal{S}[\![R^n]\!]$ and language concatenation $\mathcal{L}_1\mathcal{L}_2 \triangleq \{\sigma_1\sigma_2 \mid \sigma_1 \in \mathcal{L}_1 \wedge \sigma_2 \in \mathcal{L}_2\}$ is the concatenation of strings in the languages. \square

Solution to exercise 5.4

```

aterm:
| NUM
| IDENT
| MINUS aterm

```