Project Assignment 2

General Rules:

- You may work in teams of up to three students. Cooperation between members of different teams is not allowed and any sign of it will result in reduction of points for both sides.

Finding the extrema of a given function $f(x)$ is an important numerical problem. You have to develop a C-program to find local minima and maxima of $f(x)$ in case of a polynomial fraction, that is,

$$f(x) = \frac{a_m x^m + \cdots + a_1 x + a_0}{b_n x^n + \cdots + b_1 x + b_0}$$

for certain rationals $a_m, \ldots, a_1, a_0, b_n, \ldots, b_1, b_0$. Follow the subsequent list of individual steps to approach this exercise:

1. Create a datatype for a polynomial fraction as given above. You can assume that m and n are limited by 100.

   Create a `main` procedure that reads an input polynomial fraction $f(x)$ from the standard input stream and stores it as your new datatype. Valid input specifies first the two integers m and n followed by a space separated sequence of $m + 1 + n + 1$ rational numbers giving the values of the coefficients a_i and b_j. For instance, the input

   ```
   2 1 2.0 0.0 0.0 1.0 1.0
   ```

   describes exactly the polynomial fraction $f(x)$ used in the example below the exercise.

2. Create a function `valueat` that, given a polynomial fraction $f(x)$ and a location $x_0 \in \mathbb{R}$, computes the value $f(x_0)$.

3. Implement a function `differentiate` that, given a polynomial fraction $f(x)$, determines the first derivative $f'(x)$ of $f(x)$, which, if it exists, is also a polynomial fraction. See

   `https://en.wikipedia.org/wiki/Differentiation_rules`

   for differentiation rules.

4. Develop a function `newton` that uses Newton's method to compute a real root of a given polynomial fraction $f(x)$ and a rational starting value $x_0 \in \mathbb{R}$. You can use

   `https://en.wikipedia.org/wiki/Newton's_method`

   to get more information on this numerical procedure. Use your new function in yet another function, called `root`, which computes real roots of a given polynomial fraction $f(x)$ by guessing meaningful initial values $x_0 \in \mathbb{R}$ itself.

5. Complete your `main` procedure by combining your functions in an appropriate way. After reading the input $f(x)$, your program should search for local extrema x_e using the fact that $f'(x_e) = 0$ and $f''(x_e) \neq 0$. The output of your program should be a list of found positions x_e, each attended with a hint whether it is a local minimum or maximum.

6. (Extra Task - you can get 5 bonus points) Extend your program in such a way that the input function $f(x)$ can be specified by an arbitrary bracket term, which we define inductively as follows:
   - Every rational constant is a bracket term.
   - For all `n` $\in \mathbb{N}$ the expression `x^n` is a bracket term.
   - If `a`,`b` are bracket terms then `(a+b)`, `(a-b)`, `(a*b)`, `(a/b)` are also bracket terms.

   For instance, the bracket term

   `((1/(x^1+1))*(2*x^2))`

   describes exactly the polynomial fraction $f(x)$ used in the example below.

Example:

Consider the polynomial fraction $f(x)$ and the corresponding first and second derivative:

$$\begin{aligned} f(x) &= \frac{2x^2}{x+1}, \\ f'(x) &= \frac{2x^2+4x}{x^2+2x+1}, \\ f''(x) &= \frac{4x+4}{x^4+4x^3+6x^2+4x+1}. \end{aligned}$$

Possible extrema of $f(x)$ are located at real roots of $f'(x)$, which can be found at $x_1 = 0$ and $x_2 = -2$ by Newton's method, for instance. As $f''(x_1) = 4$ and $f''(x_2) = -4$, we have found a local minimum in x_1 and a local maximum in x_2.