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There are numerous ways to classify numbers, but for the sake of brevity only a few

classifications (the ones covered in college entrance tests) will be discussed in this

reviewer. In addition to this, we will touch on factors and multiples, which are also an

integral part of basic arithmetic.

CLASSIFICATION OF NUMBERS

One of the simplest ways numbers can be classified is through grouping them into

integers and non-integers. An integer are defined as a number that can be

represented without a component such as a fraction, radical, or decimal. A few

examples of integers and non-integers are shown below:

Integers: -1, 0, 1, -24, 379, -3.0, $6\frac{2}{3}$

Non-integers: $\frac{4}{5}$, $2\frac{1}{2}$, $\sqrt{5}$, $2\sqrt{2}$, 4.6

Through the above examples, we can observe that although integers may have a

fractional component, such as -3.0 and $6\frac{2}{2}$, they can be simplified into a number

without this component. In these cases, -3.0 is equal to -3, and $6\frac{2}{2}$ is equal to 7, both

of which are integers.

Integers can further be divided into three categories: positive integers, negative

integers, and 0. Positive integers are just integers greater than 0, while negative

integers are those less than 0. As negative integers are part of the set of negative

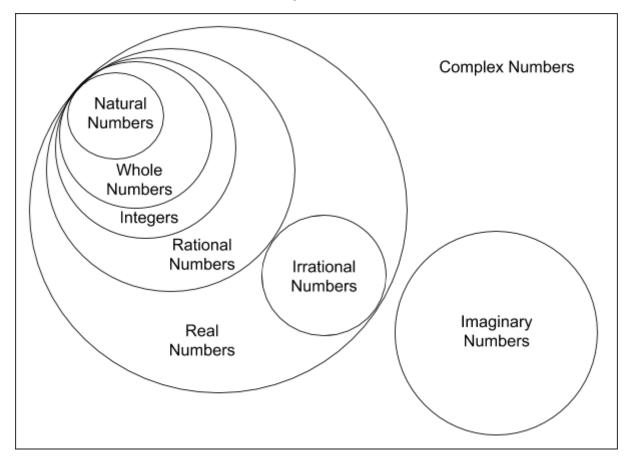
numbers, they are also denoted with a negative (-) sign. O is considered the only

integer that is neither positive nor negative, and as such warrants its own third

category.

To make things more specific, we can place all numbers in the set of complex numbers, which can further be divided into two: real and imaginary numbers. The definitions for all the sets are given below:

- The set of natural numbers consists of positive integers only, beginning from 1 onwards.
- The set of **whole numbers** consists of all natural numbers and 0.
- The set of integers, as mentioned previously, consists of all whole numbers as well as all negative integers.
- The set of **rational numbers** is the set of numbers that can be represented as a ratio between two integers. This set includes positive and negative fractions where both numerator and denominator are integers, as well as all **terminating** decimals (3.42293823 is rational, but $\pi = 3.141592...$ is not). This set also includes the entire set of integers as well.
- The set of **irrational numbers** is the set of all numbers that naturally cannot be represented as the ratio of two integers. This set consists of numbers such as $\sqrt{2}$ or Euler's number, the constant e.
- The set of **real numbers** consists of two main subsets: rational and irrational numbers. As a result, this also means natural numbers, whole numbers, and integers are also considered real numbers.
- The set of **imaginary numbers** is comprised of imaginary numbers whose foundation are negative numbers placed inside a square root. These numbers are often represented as bi, where b is a real number coefficient and $i = \sqrt{-1}$
- The set of complex numbers consists of two sets: that of real and imaginary numbers. Complex numbers are usually represented in the form a + bi, where a and b are both real numbers.



We can summarize this information using the set of numbers shown below:

ODD AND EVEN NUMBERS

Odd numbers are integers that leave a remainder of one when divided by 2, and are represented in the form 2k+1. On the other hand, **even numbers** are integers that do not leave a remainder when divided by 2, and represented in the form 2k. Aside from these basic properties, odd and even numbers can help us predict the sum, difference, or product of two numbers.

1st Number	2nd Number	Sum	Difference	Product
Odd	Odd	Even	Even	Odd
Odd	Even	Odd	Odd	Even
Even	Odd	Odd	Odd	Even
Even	Even	Even	Even	Even

FACTORS AND MULTIPLES

Factors of a particular integer, Q, are defined as integers that can be multiplied to another integer to equal Q. A **multiple** of a number R, on the other hand, is R multiplied to various positive integers (R, 2R, 3R... are all multiples of R). The **greatest common factor**, commonly written as gcd or gcf, is the greatest factor shared by a set of numbers, while the **least common multiple**, commonly written as lcm, is the smallest multiple shared by a set of integer.

To concretise, gcd(12,18) = 6 as 6 is the biggest integer such that when 12 and 18 are divided by it, no remainder occurs, and lcm(12,18) = 36, as 36 is the smallest integer that is a multiple of both 12 and 18 (12 * 3 = 36, 18 * 2 = 36).

IMAGINARY NUMBERS

As mentioned previously, imaginary numbers can be represented with i, or sqrt(-1). Due to this, we can deduce the values of i raised to various exponents.

$$i^{1} = i$$

$$i^{2} = -1$$

$$i^{3} = -i$$

$$i^{4} = 1$$

We can see that as $i^4 = 1$ and we know 1 * x = x for all x, then this cycle repeats every 4 terms. We can generalise below that for larger exponents:

Given the exponent x,

- if x = 4k + 1 for some integer k, i.e. x = (1,5,9,...), $i^x = i$.
- if x = 4k + 2 for some integer k, i.e. x = (2,6,10...), $i^x = -1$.
- if x = 4k + 3 for some integer k, i.e. x = (3,7,11..), $i^x = -i$.
- if x = 4k for some integer k, i.e. x = (4,8,12...), $i^x = 1$.

Thus, to determine the value of i^a for some integer a, one simply needs to get the remainder of a when divided by 4.