Sets

- A set is a collection of things. This can be anything from numbers to words to symbols.
- Sets are normally denoted by **elements** separated by a comma, contained inside a curly bracket. For example, we define the set A as:

  A = { a, b, c, d, e, f, g}

  Each letter within this set is an **element.** If instead, the set were {a,b,c,d,e,fg}, then neither f nor g will be elements, but instead fg. Elements must be **distinct**; the set {a,a} is invalid, for example.

  Notice that this set ends, and thus is called a **finite set**.

  - To denote that an element is a member of a certain set, we can use the symbol $\in$
  - In the above set A, we can therefore say that $a \in A$, but as w, for example, is not in the set, we can therefore state that $w \notin A$ (read as w is not an element of A).
- Some sets are unending and are called **infinite sets.** These sets are denoted with ellipsis (three dots) following its list of elements as so:
  - Set of prime numbers: {2, 3, 5, 7, 11, 13, 17, ...}
    - Notice how the ellipsis follow the elements in the infinite set and not in between the elements. It is indeed possible that a finite set makes use of the ellipsis to simply shorthand the list. Below is an example.

      Alphabet: {a, b, c, ..., x, y, z}
- A subset of a certain set one where **all** its elements are also present in the larger set. The subset is called a **proper subset** if and only if the larger set contains some element not present in the subset. Using the sets below:

  A = {a, b, c, d, e, f}

  B = { a, b, e}

C = { a, b, x}

D = {b, a, d, c, f, e}

We can observe that B is a subset of A or $B \subset A$.

We can also see that C is not a subset of A or $C \not\subset A$.

We can say that D is not a proper subset of A but is only a subset of A;

$D \subseteq A$, **but not** $D \subset A$. The former can be thought of as “less than or equal to”, while the latter is “strictly less than” (as an analogy; do not actually use these terms for sets!)

- We say that two sets are equal when they have the same elements **regardless of order**.

So, in the example given above, we can say that D = A

The universal set is a set that contains everything of interest, including the sets that are to be part of this universal set. A possible universal set, ***U,*** that contains sets A, B, C, and D as described above, for instance is ***U*** = {a, b, c, d, e, f, x}. Note that we can arbitrarily add elements to this universal set and it would still be universal. Thus, all sets contained in the universal set are **subsets**, not necessarily proper.

- The **cardinality** or **order** of a set is the number of elements within the set. Example:

  Q = { a, b, c, d}

  $\therefore$ A has an order of 4 or |A| = 4. Oftentimes this can also be written as n(A) = 4.

The set containing all weekdays which we can denote as W, for example, has cardinality 5 (Mon - Fri).

Set Notations

Roster Notation: List elements of a set inside the curly brackets are separated by commas. For example:

Set of all multiples of three: T = {3, 6, 9, 12, 15...}

Set-Builder Notation: Elements are described and not listed. The same set T above can be rewritten as {x | x is a multiple of 3}. Moreover, the set {1,2,3,4,5} can be rewritten as {y | y is an integer between 1 and 5, inclusive} or any other equivalent statement. This notation is more useful when **sets are large and have high cardinality**.

Intersection

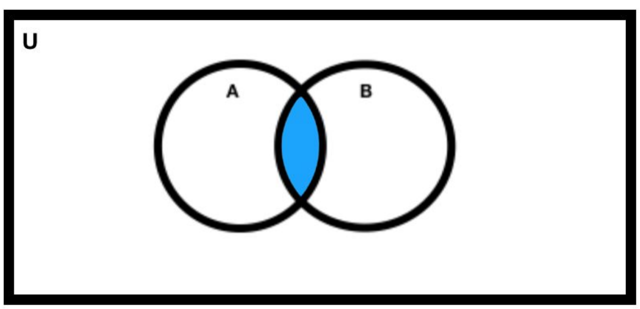
- The intersection of two or more sets pertains to the set containing the elements common in all sets; it is denoted by ∩. If, for example:

  A = { a, b, c, d, e, f}

  B = { a, b, c, x, y, z}

Then, A ∩ B = {a, b, c}.

This can also be shown in a Venn Diagram where each circle represents a set.



Union

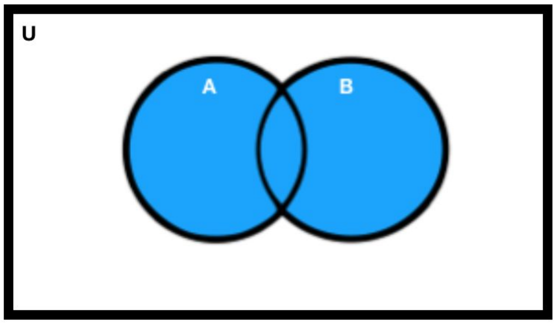
- The union pertains to all the elements in both sets; it is denoted by ∪. For example:

  A = { a, b, c, d, e, f}

  B = { a, b, c, x, y, z}

Then, A ∪ B = {a, b, c, d, e, f, x, y ,z}

Again, this can be shown through means of a Venn diagram.



Complement

- The complement of a set contains all the elements in the universal set not present in that set. This is often denoted as A', or in some other cases, A^c.

For example:

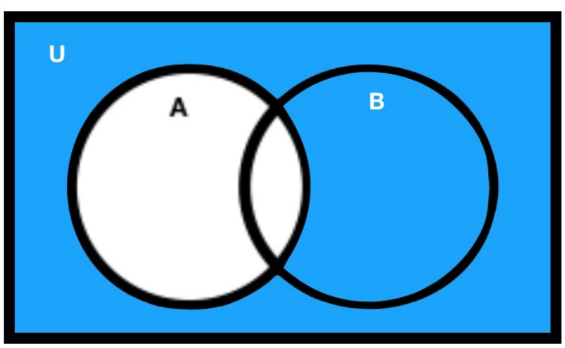
If A = {a, b, c, d, e, f}

And U = { a, b, c, d, e, f, x, y, z}

Then, A^c = {x, y ,z}

- In short, the complement is ***U*** - A.

This can also be shown in a Venn Diagram where each circle represents a set.



Some Useful Simple Theorems in the Algebra of Sets

Let A, B and C be sets. Proving these identities to hold true is left as an exercise to the curious reader! Hint: Use Venn Diagrams to demonstrate each situation.

- $A \cap B \subseteq A$

- $A \cap B \subseteq B$
- $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$
- $(A \cap B)^c = A^c \cup B^c$
- $(A \cup B)^c = A^c \cap B^c$
- $(A \cup B) + (A \cap B) = A + B$
- $A \cup B \cup C = A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C)$