

# DIFFUSION MONTE CARLO

## ROLLING THE DICE BUT THE DICE ARE MOLECULES

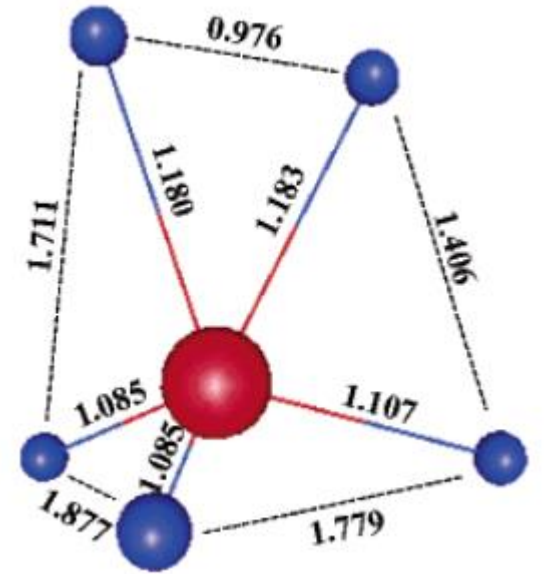
Meredith Fore  
Physics PhD student @UW

# CARBON WITH FIVE HYDROGEN ATOMS?

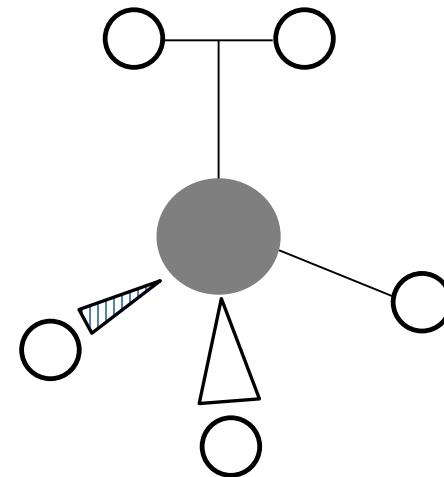
It can exist!

An  $\text{H}_2$  bound to a  $\text{CH}_3^+$

But that  $\text{H}_2$  is not the same two  
hydrogen atoms all the time...  
can we even say it has a structure?



Minimum energy configuration  
McCoy et al., J. Phys. Chem. A, 2004.



# ASTROCHEMISTRY: CHEMICALS IN SPACE

Is the space between stars really empty?

What are stars/nebulae made of?

Do exoplanets have water/atmospheres?

Can we detect life on other planets?



Artist's View of Extrasolar Planet HD 189733b  
NASA, ESA, and G. Bacon (STScI) • STScI-PRC08-11

# ASTROCHEMISTRY: CHEMICALS IN SPACE

## Scientists Detect Abundant Methane in Space

By **Thomas O'Toole**

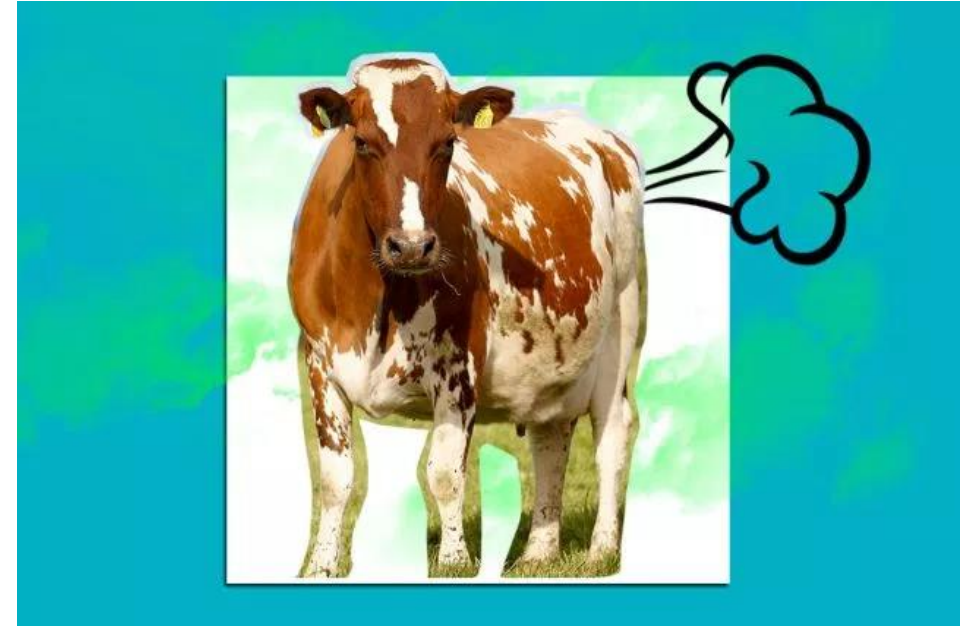
July 4, 1978

Methane, the major constituent of natural gas and a prominent product of biologic decay, has been found to be even more abundant in distant space than on earth.

-Washington Post, 1978

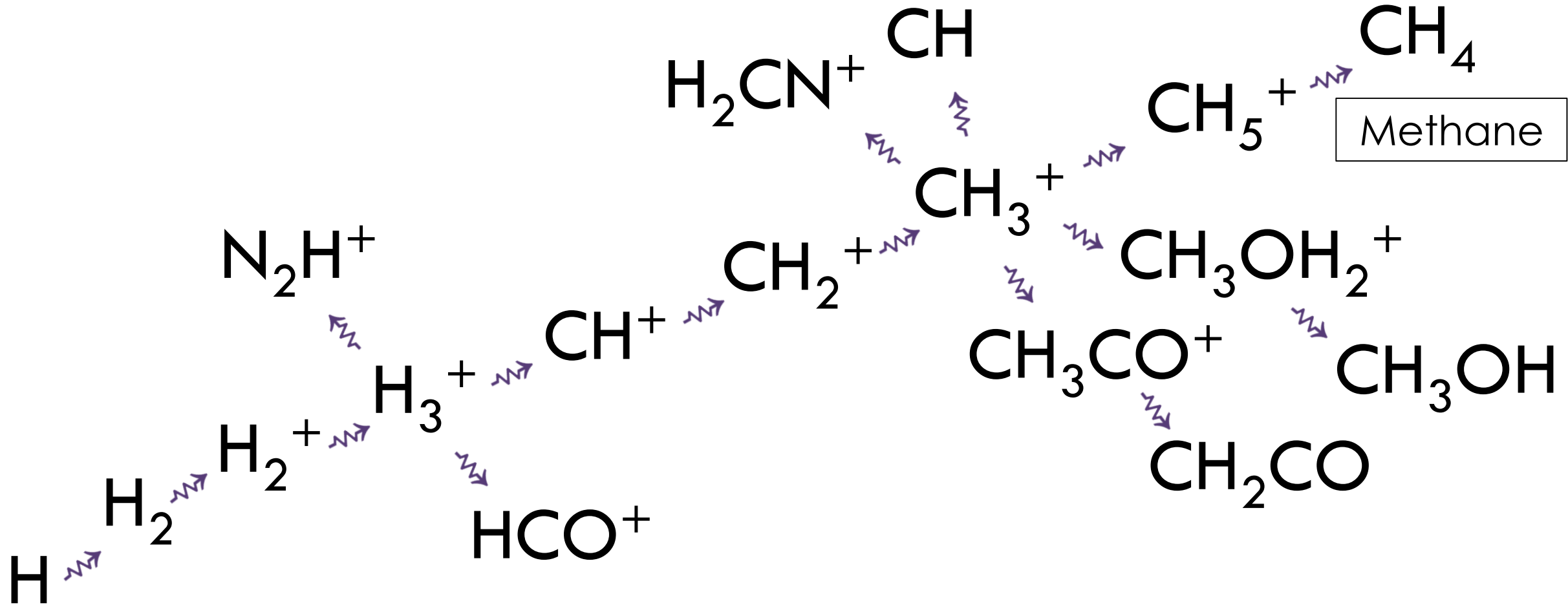
One of the “building blocks” of life

How did it get there?

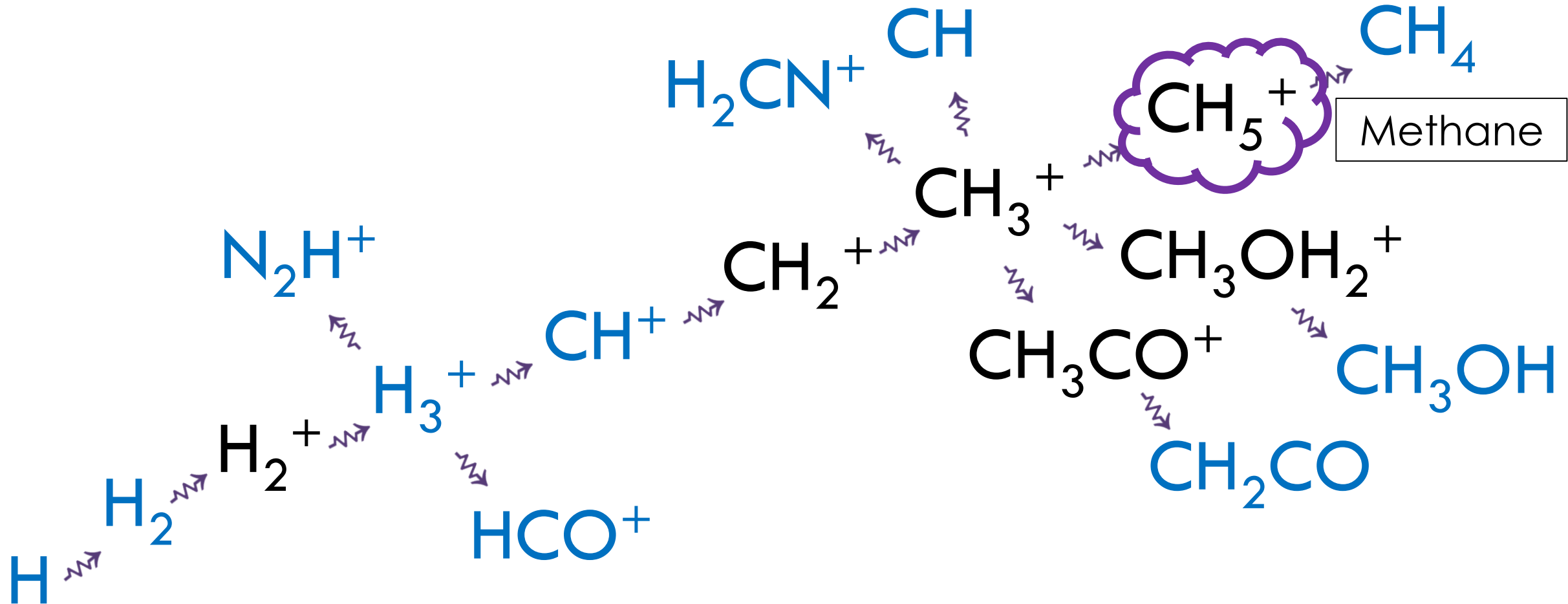
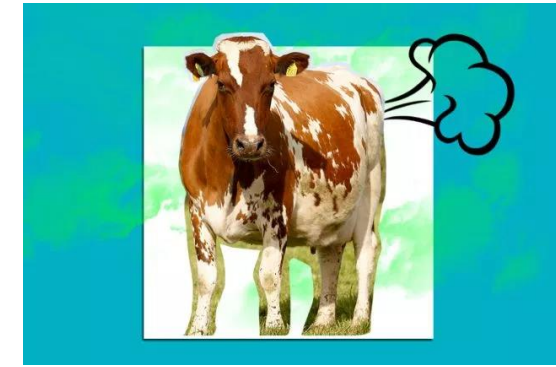


Methane

# ASTROCHEMISTRY: CHEMICALS IN SPACE

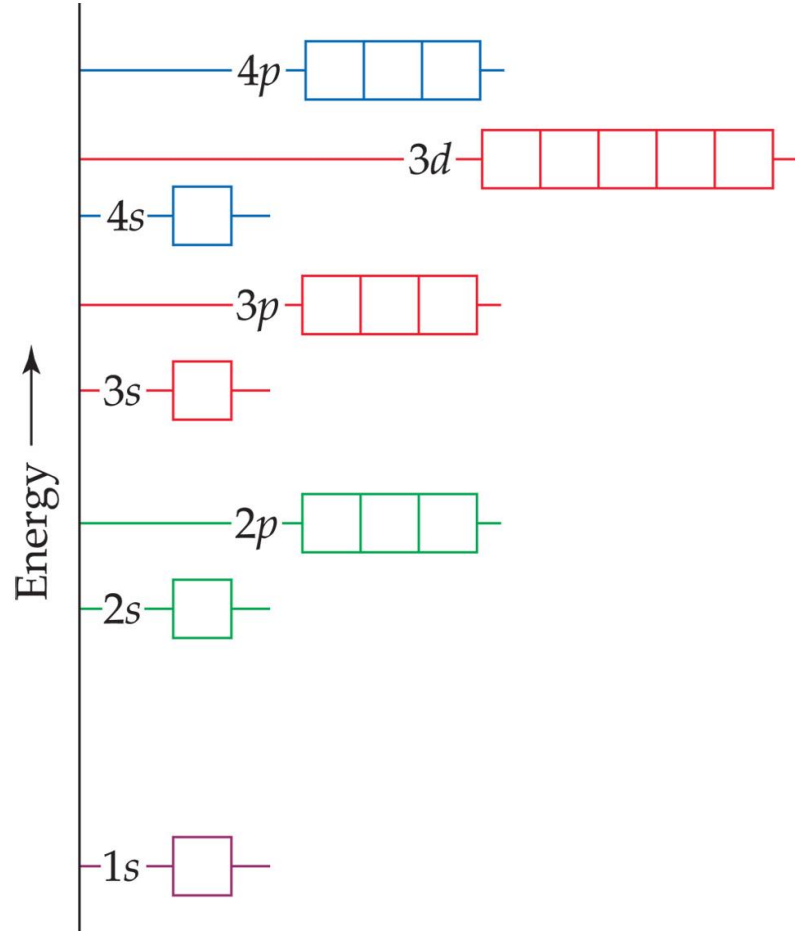


# ASTROCHEMISTRY: CHEMICALS IN SPACE



"quantized" = quantum mechanics!

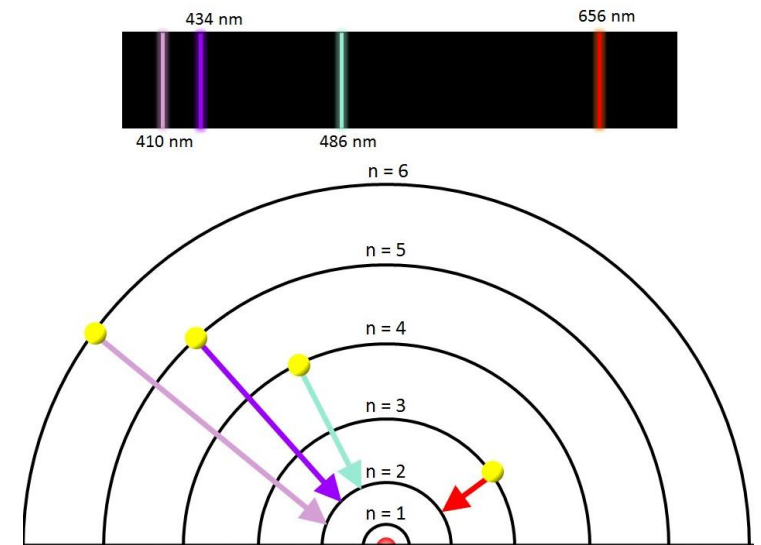
# ELECTRONIC ENERGY



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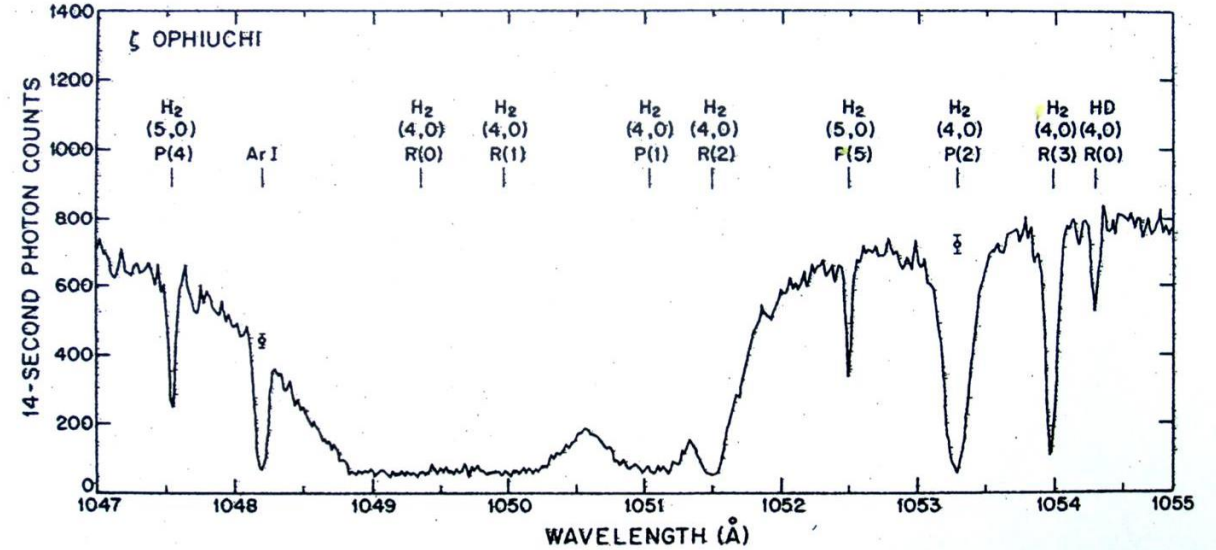
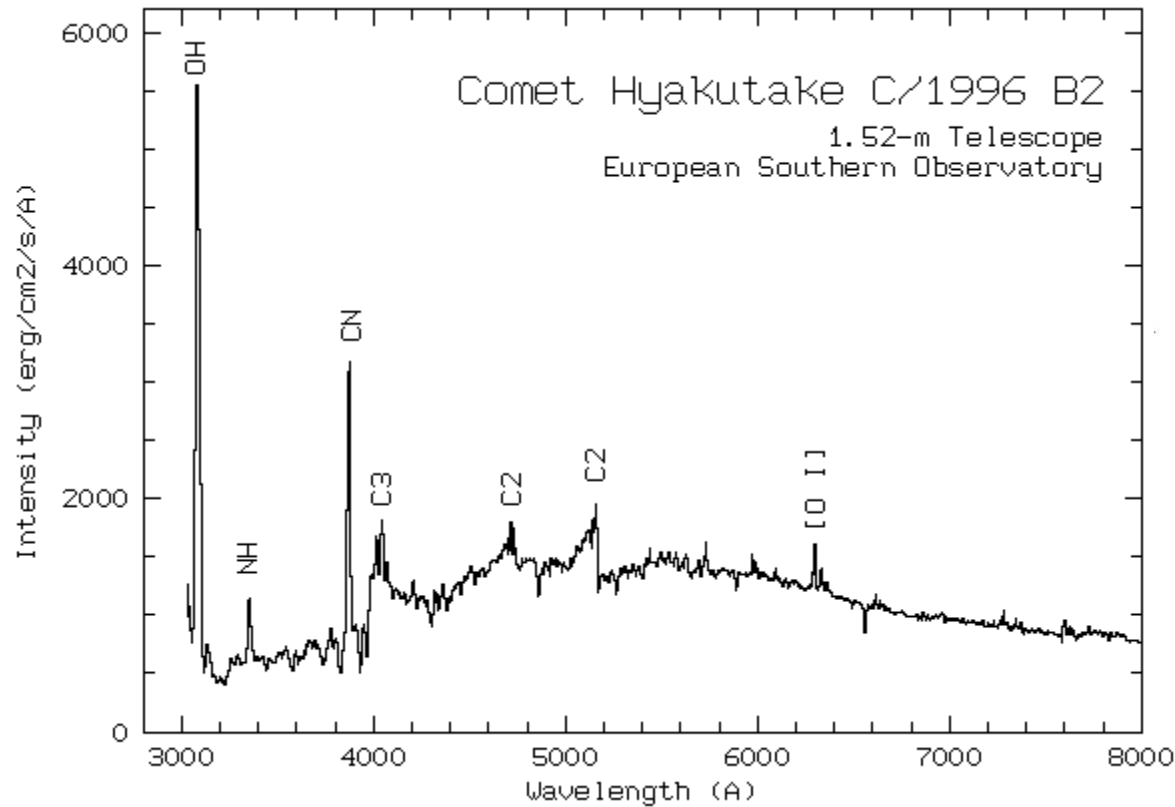
- Atoms/molecules are only allowed to have certain energies
- So they can only absorb or emit light (photons) of certain energies
- Electron(s) can absorb or emit the energy to change orbitals

This pattern of energies is a **unique fingerprint**





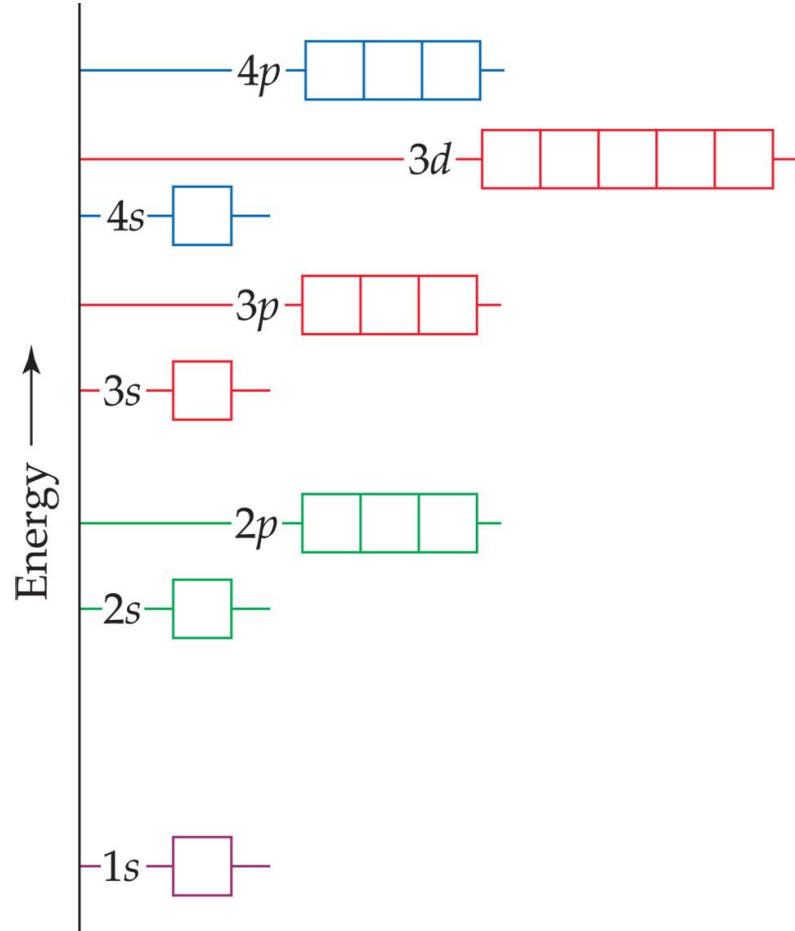
# RECOGNIZE CHEMICALS IN SPACE!





"quantized" = quantum mechanics!

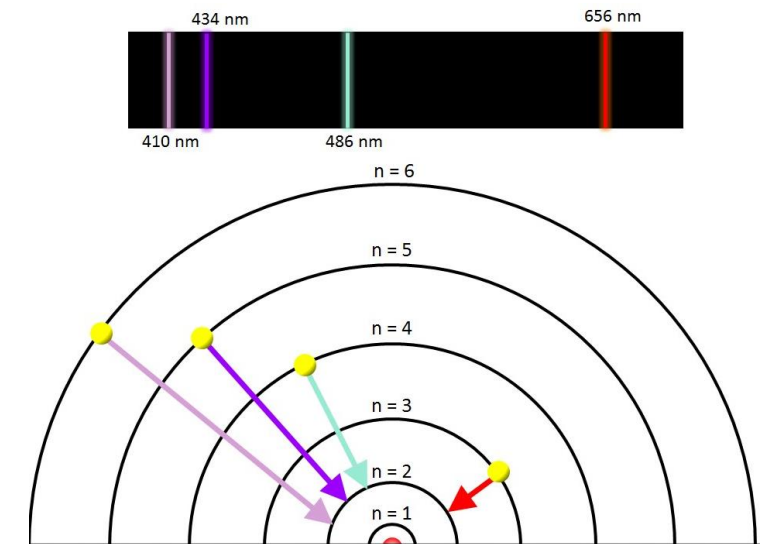
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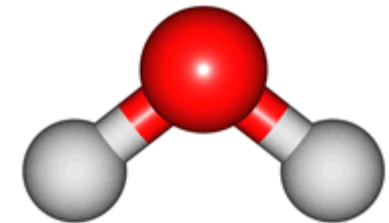
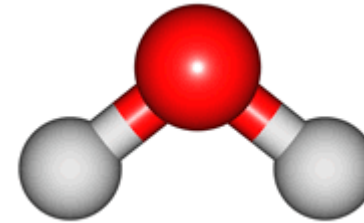
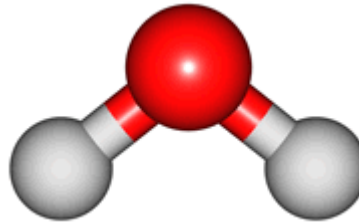
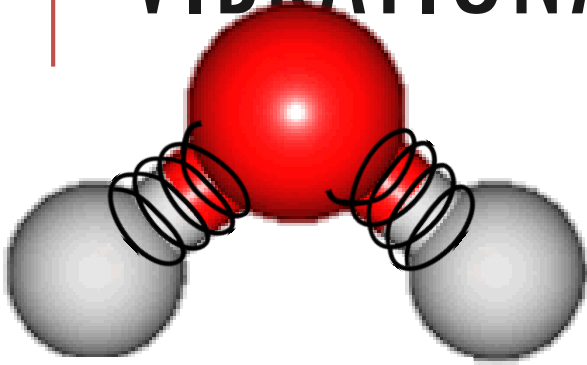
- Atoms/molecules are only allowed to have certain energies
- So they can only absorb or emit light (photons) of certain energies
- Electron(s) can absorb or emit the energy to change orbitals

This pattern of energies is a **unique fingerprint**



also quantized!

# VIBRATIONAL ENERGY



This also gives us a unique fingerprint of energies

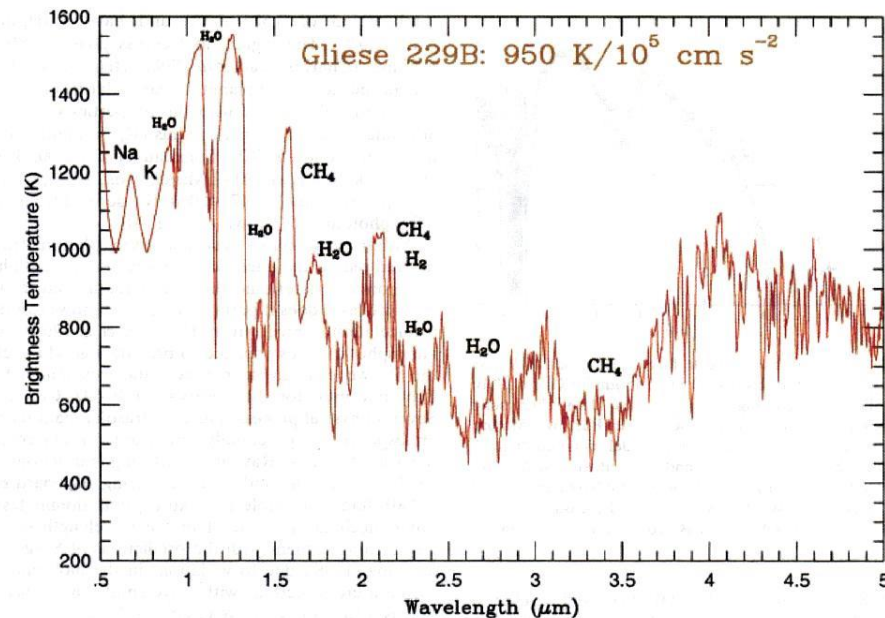
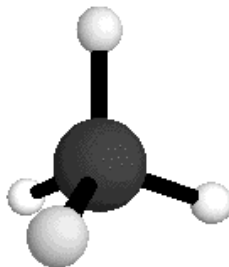
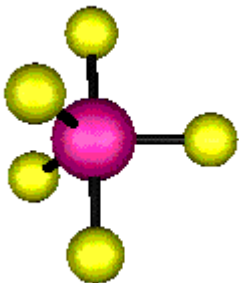
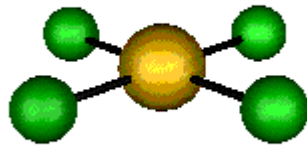
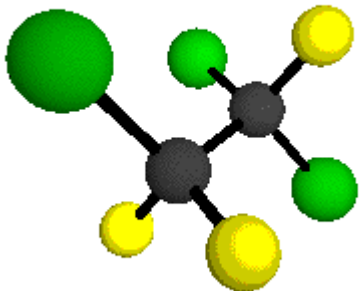
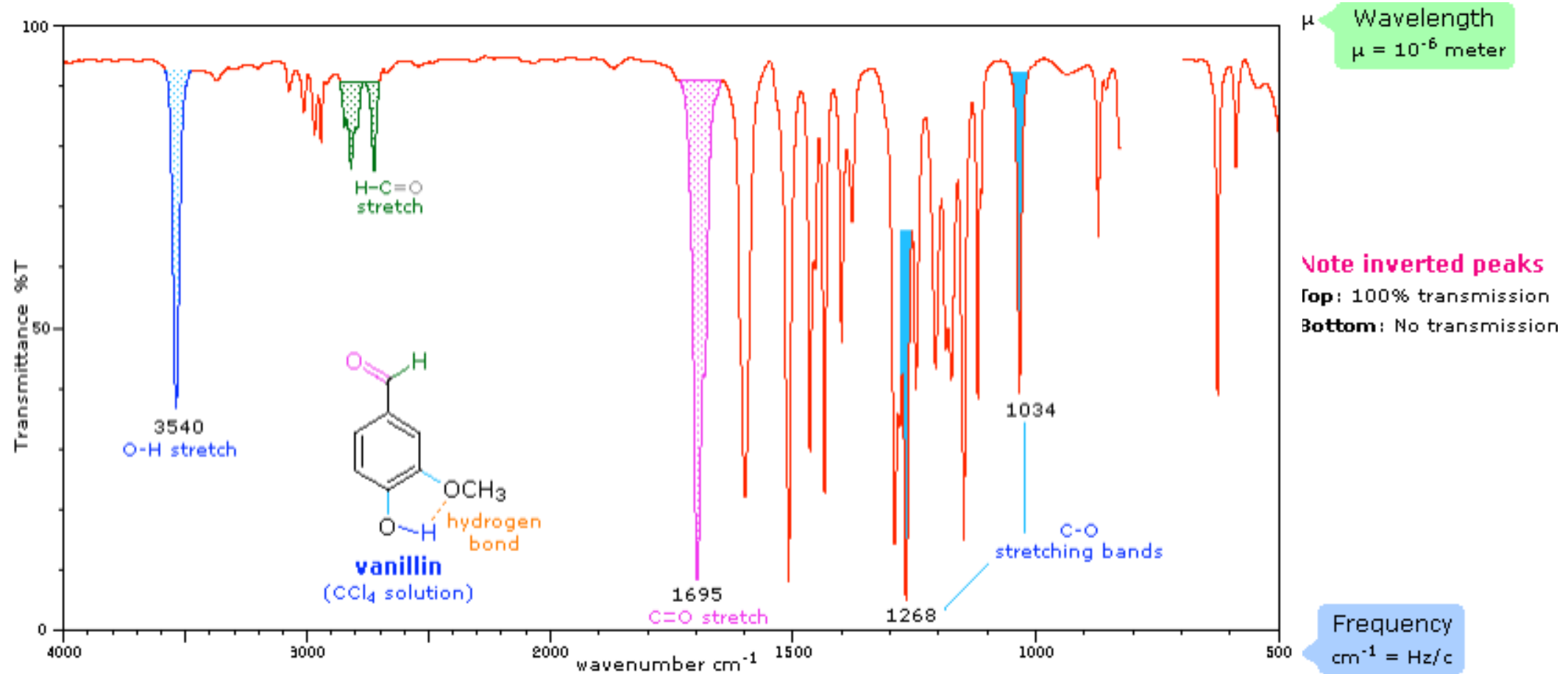


FIG. 28. The "brightness" temperature (in K) vs wavelength (in microns) from 0.5 to 5.0  $\mu\text{m}$  for a representative model of Gliese

# VIBRATIONAL ENERGY

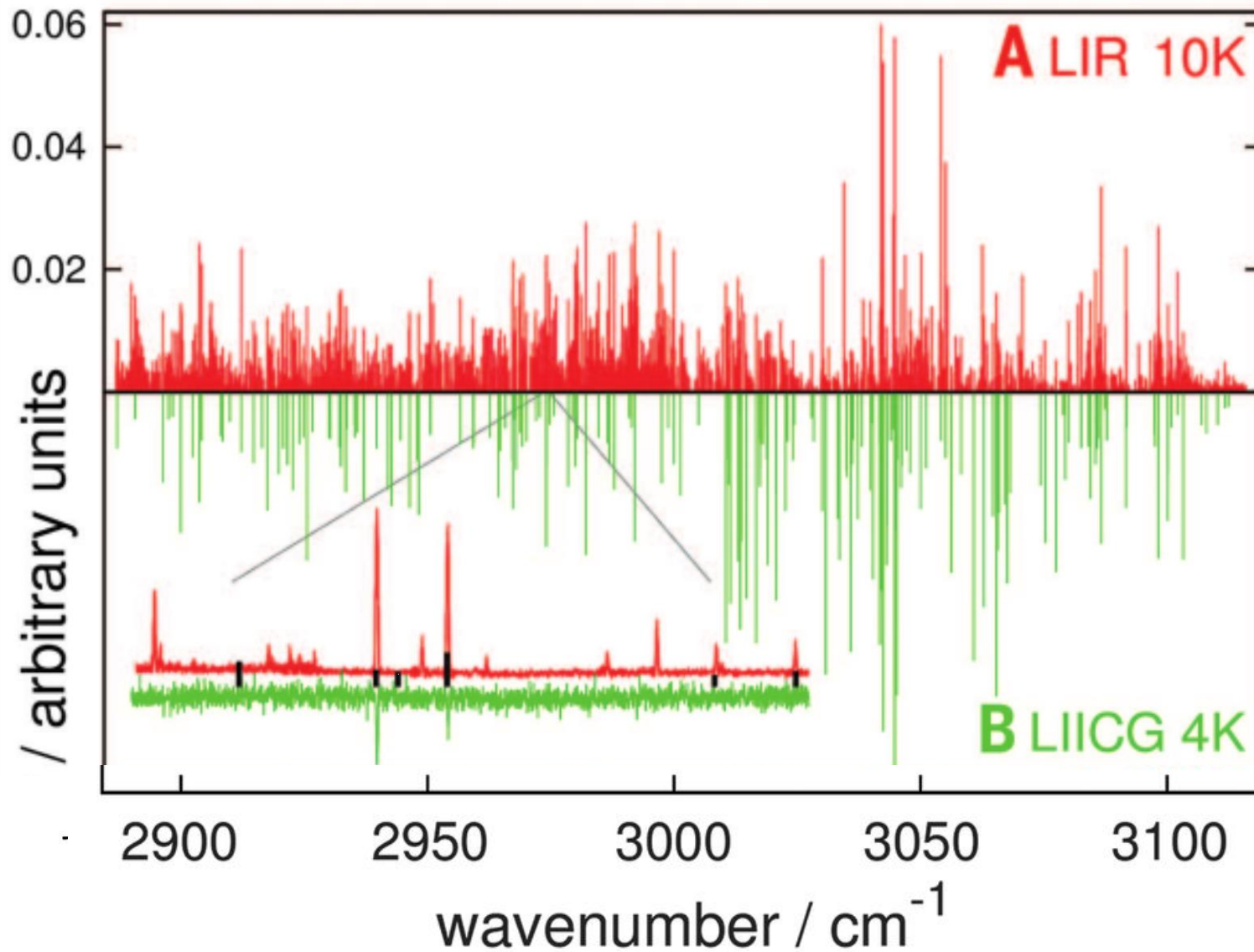


Asvany, Yamada, Brunken, Potapov, Schlemmer,  
Science (2015)

2897 lines

“enfant terrible”

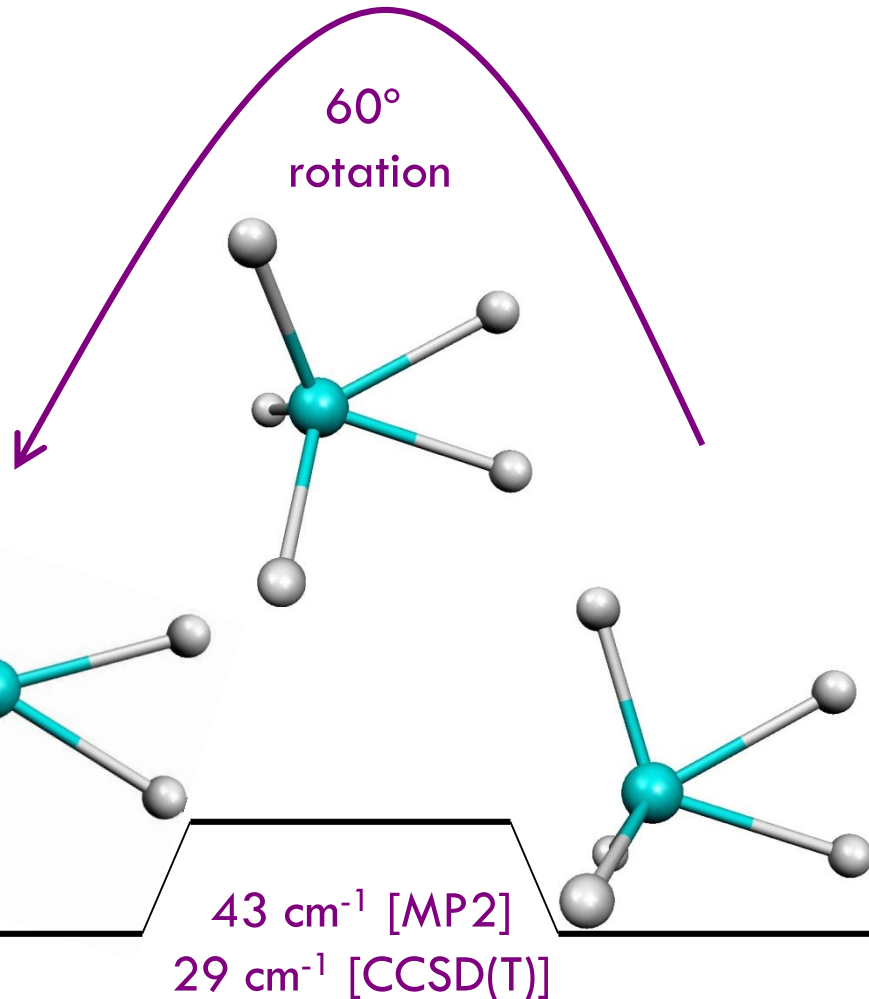
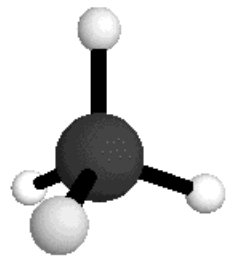
It isn't enough to have  
this spectrum; we must  
understand it



White, Tang and Oka, Science (1999)

\*Based on the surfaces of A. Brown, J. Zhong, B. J. Braams and J. M. Bowman

# $\text{CH}_5^+$ : WEIRD AND WIGGLY



193  $\text{cm}^{-1}$  [MP2]  
341  $\text{cm}^{-1}$  [CCSD(T)]

Molecule can access all  
120 permutations of  
five hydrogen atoms in  
its ground state.

**No** well-defined  
structure!

What if we had a  
theoretical tool that  
didn't need one?

# MONTE CARLO

**Monte Carlo** refers to a class of computational algorithms that use random sampling to achieve numerical results

To implement **Monte Carlo**, you will need:

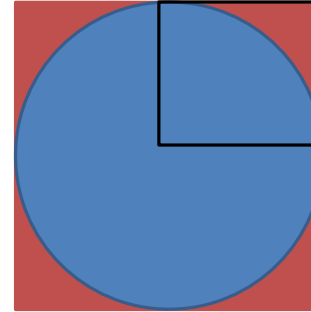
1. A range or domain from which to pull sample points (optional: a probability distribution within that domain)
2. A deterministic calculation to perform on your points
3. To collect all of your results





# MONTE CARLO

$$\begin{array}{l} A_C = \pi r^2 \\ A_S = 4r^2 \end{array} \longrightarrow \pi = 4 \frac{A_C}{A_S}$$



What is my domain?

$[0,1]$  in  $x$  and  $y$ .

What is my deterministic calculation?

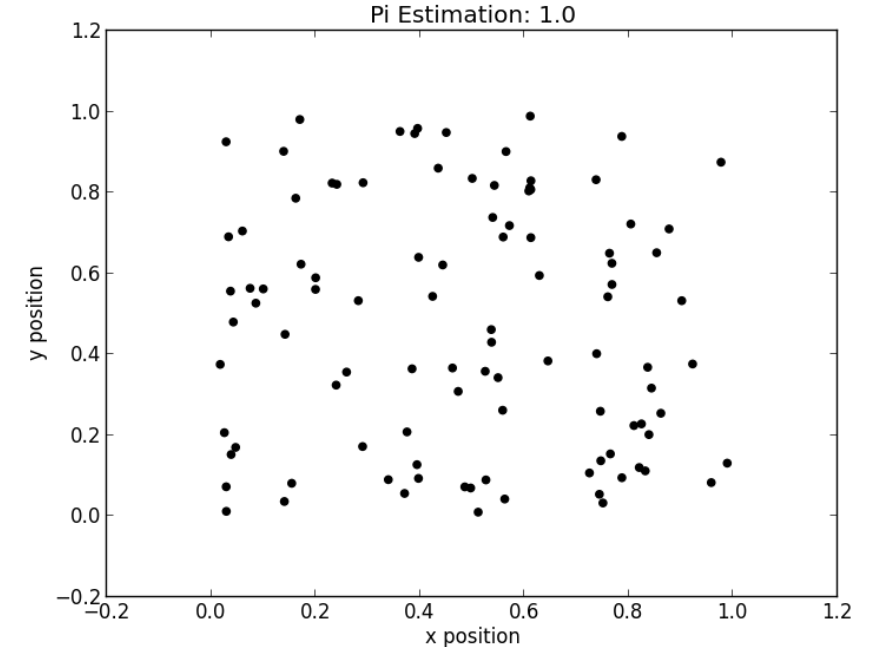
Whether my point is in the area of the circle or not.

Mathematically:

$$\begin{array}{l} x^2 + y^2 > 1 \\ x^2 + y^2 < 1 \end{array}$$

Collect my results:

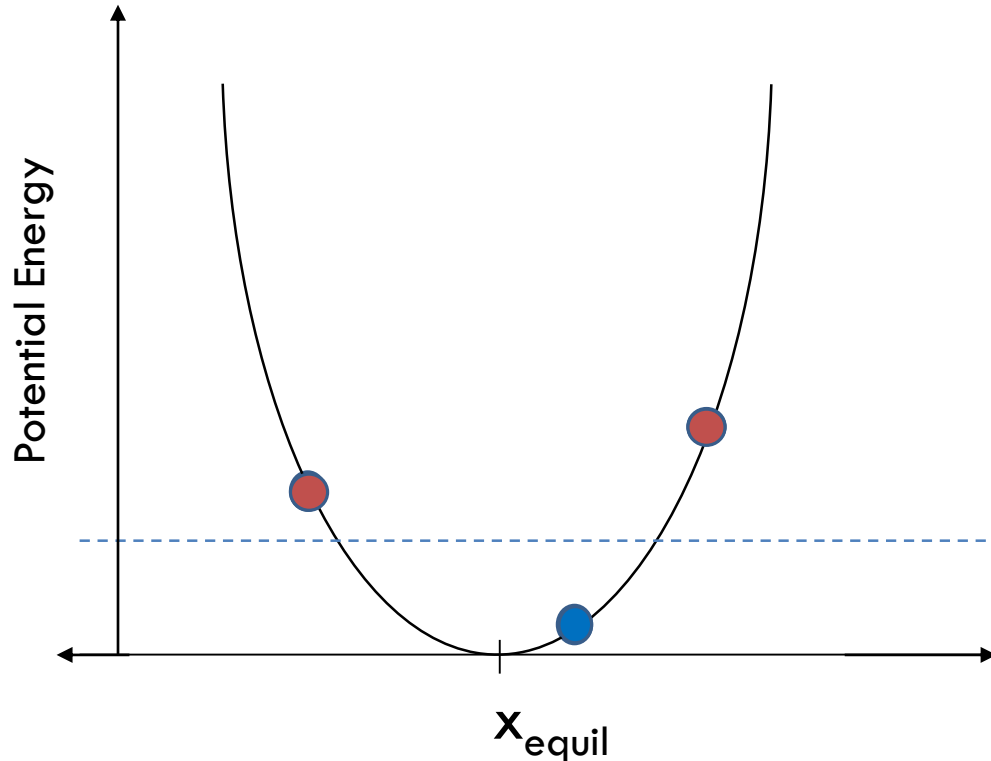
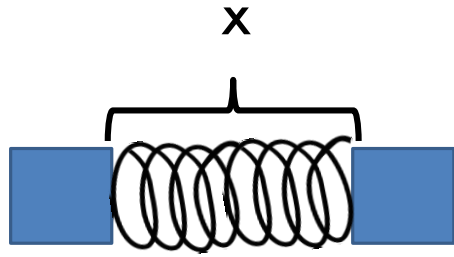
$$\pi = 4 \frac{N_{in\ circle}}{N_{total}}$$



Animation credit to Dr. Lindsey Madison



# DIFFUSION MONTE CARLO



Avg Energy of  
all my points

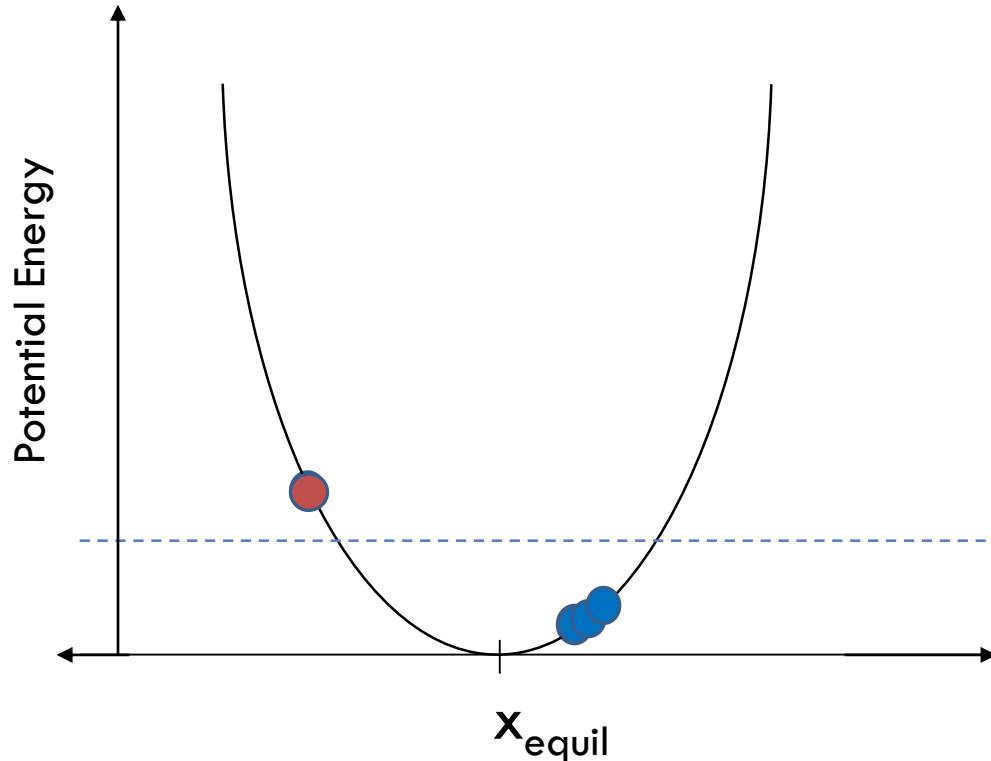
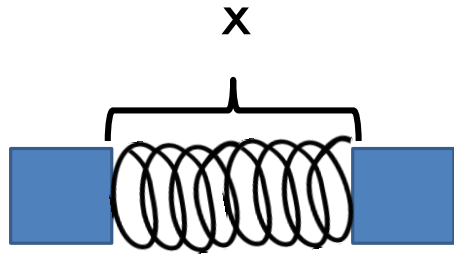
Represent our wavefunction with points that we call “walkers”

Tag them: below or above average?  
(Classically-allowed or classically-forbidden?)

Roll the dice! Do they survive?  
Do they get to replicate?



# DIFFUSION MONTE CARLO



Avg Energy of  
all my points

Represent our wavefunction with points that  
we call “walkers”

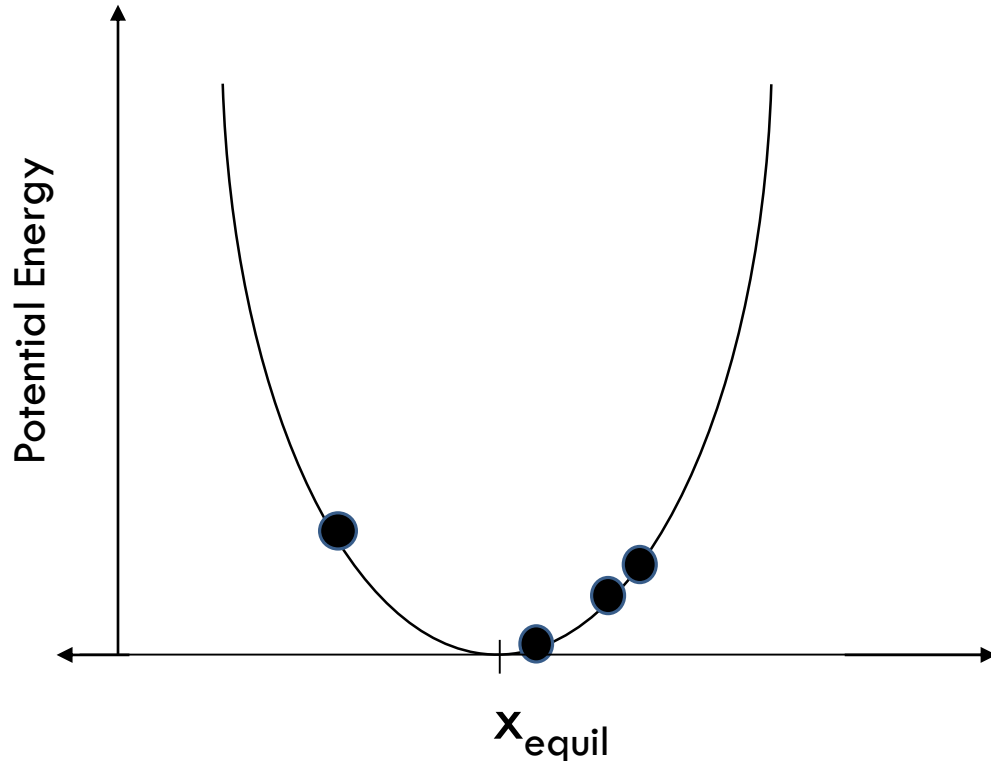
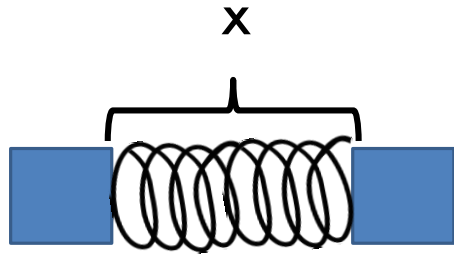
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Diffuse your new set of points



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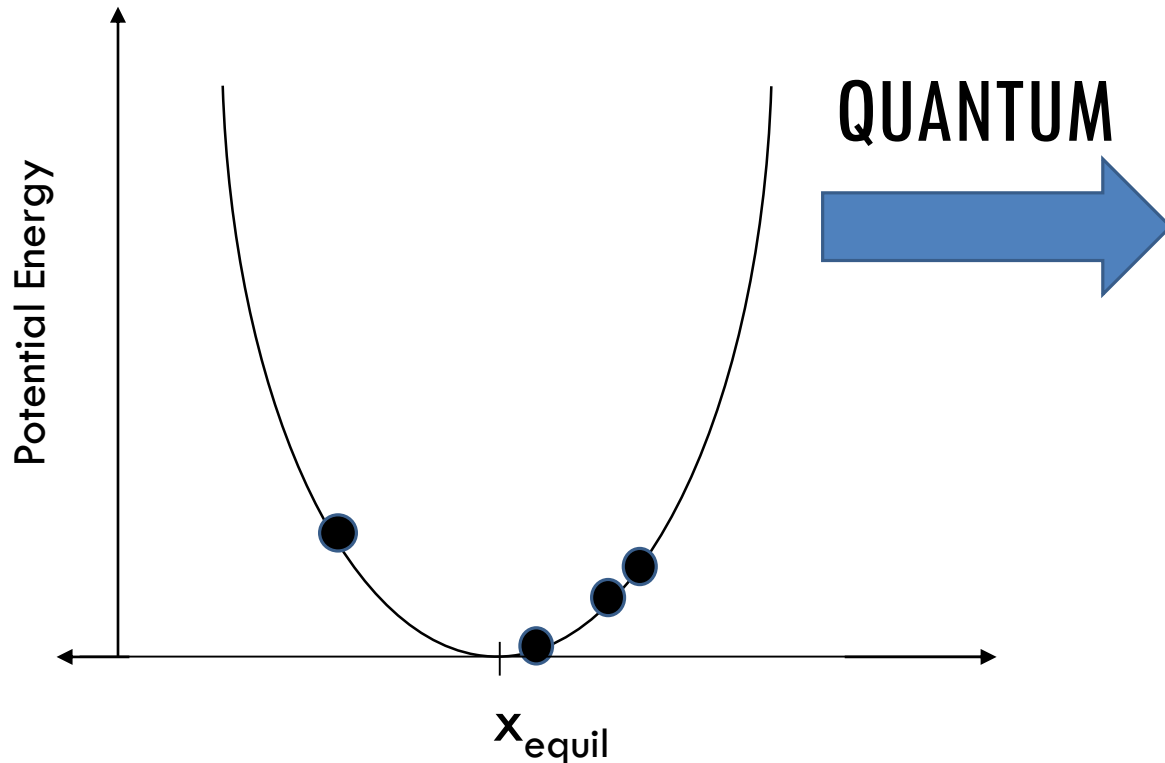
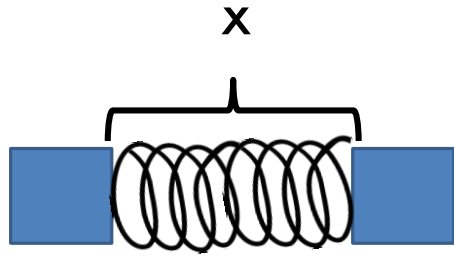
Roll the dice! Do they survive?  
Do they get to replicate?

Diffuse your new set of points



Take your new average energy, start over!

# DIFFUSION MONTE CARLO



Represent our wavefunction with points that we call “walkers”

Tag them: below or above average?  
(Classically-allowed or classically-forbidden?)

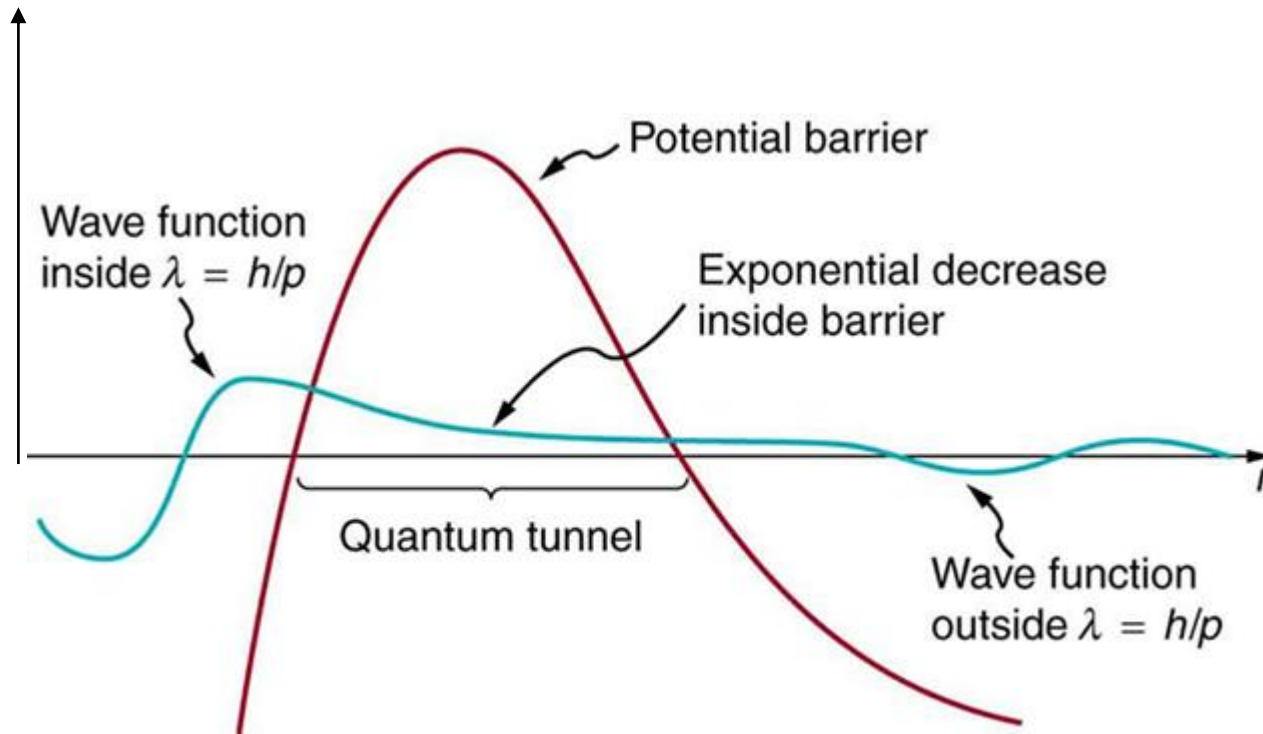
Roll the dice! Do they survive?  
Do they get to replicate?

Diffuse your new set of points



Take your new average energy, start over!

# QUANTUM TUNNELING



Non-zero probability of being in a region where the potential energy is greater than the energy of the system

**Quantum mechanics and Monte Carlo: both inherently probabilistic!**

# QUANTUM MONTE CARLO

There are **many** different flavors of quantum Monte Carlo.  
They all have essentially one goal: solve the Schrödinger equation.

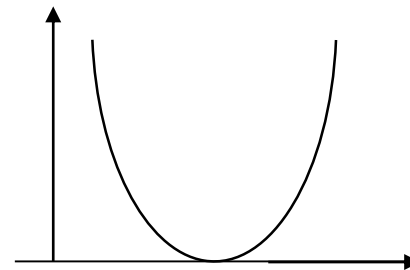
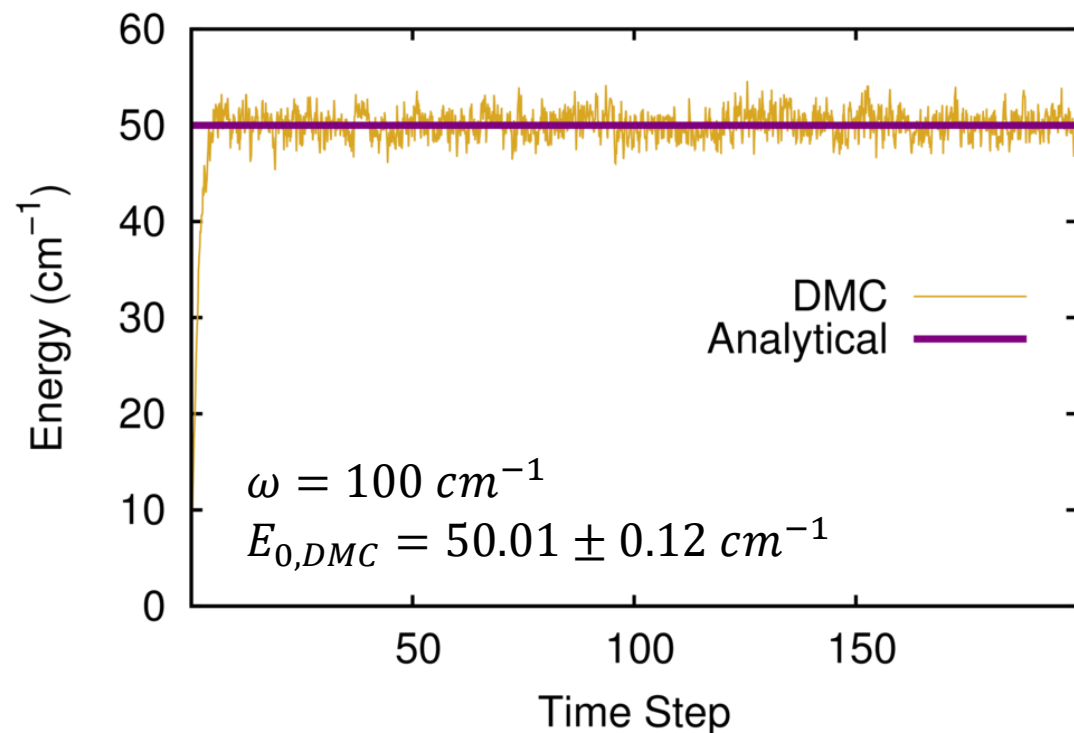
$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

Get the wavefunction:  $\psi$

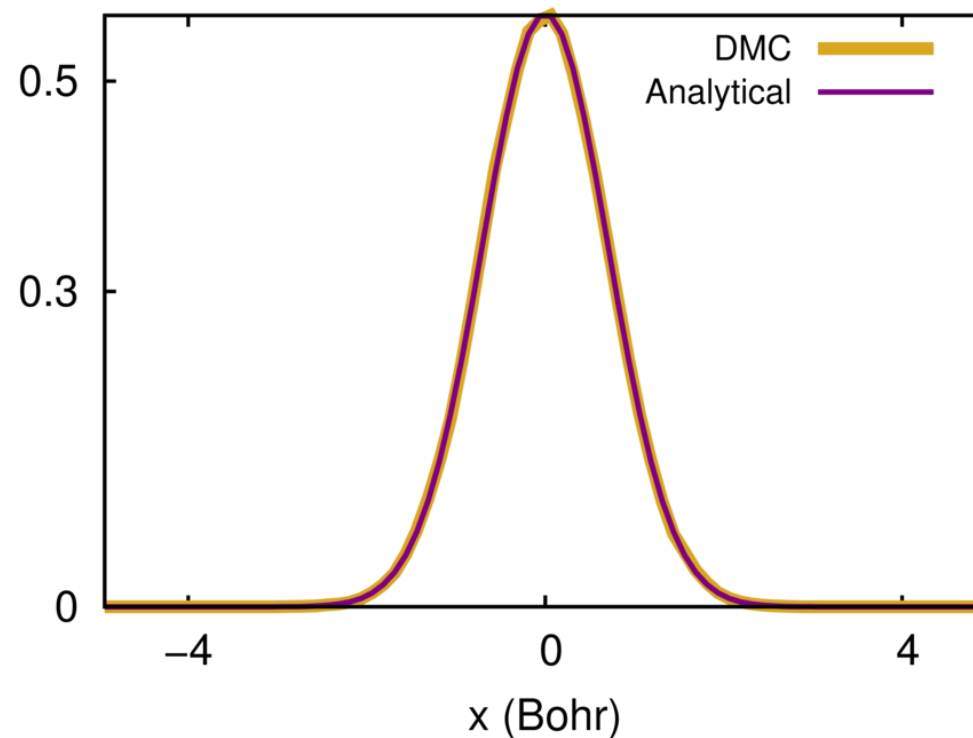
Usual data run:  
25,000 walkers  
10,000 timesteps  
x10 simulations  
Time: ~10 hours

# DMC CAN CALCULATE:

Zero-point energy



Wavefunction as a density of walkers



But how do I look at a multi-dimensional density?

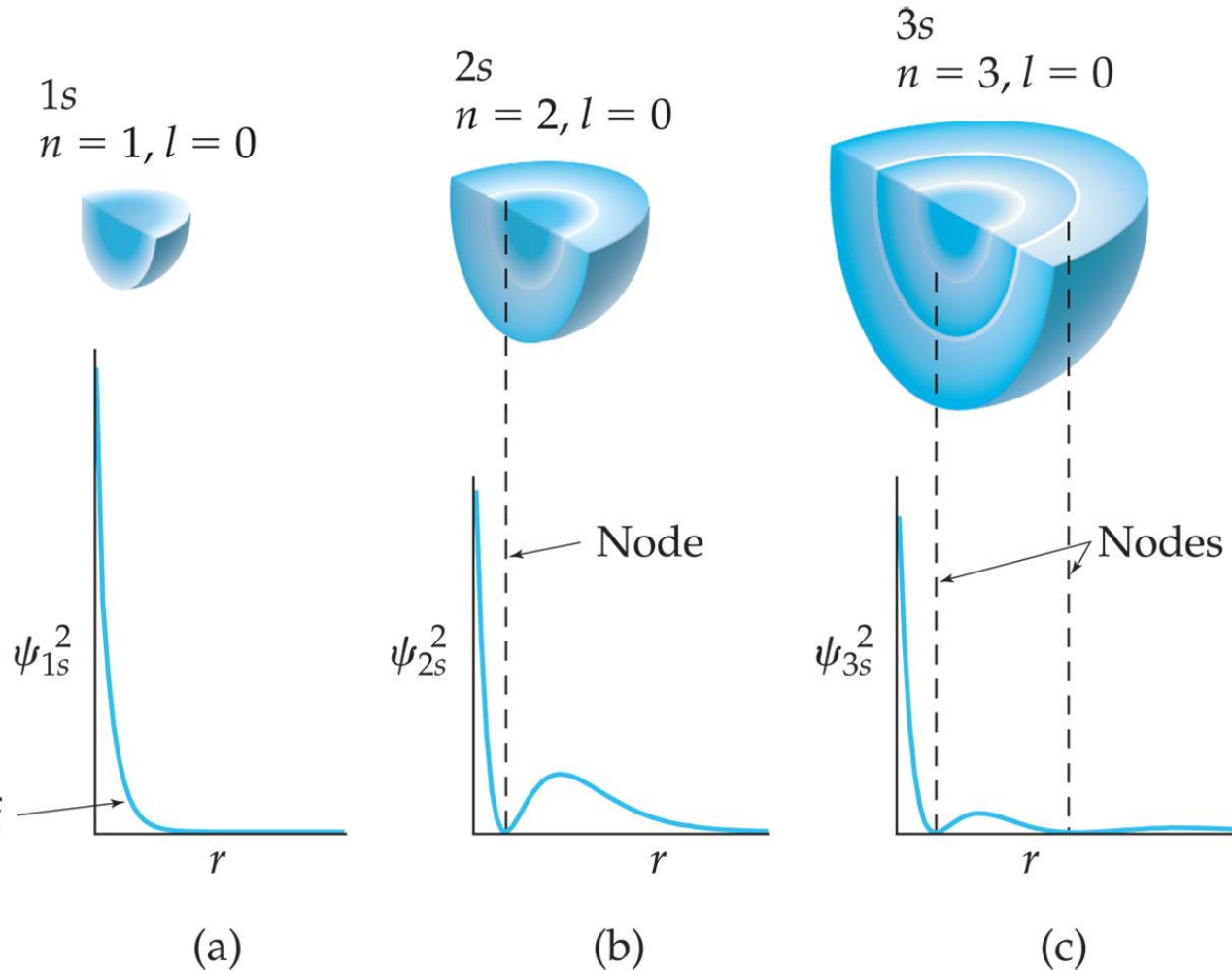


# PROBABILITY DISTRIBUTION: $|\psi|^2$

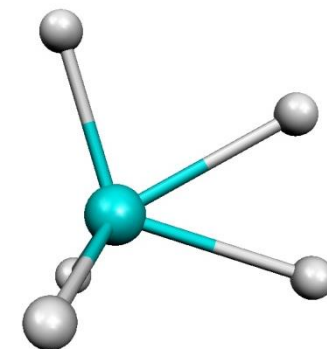
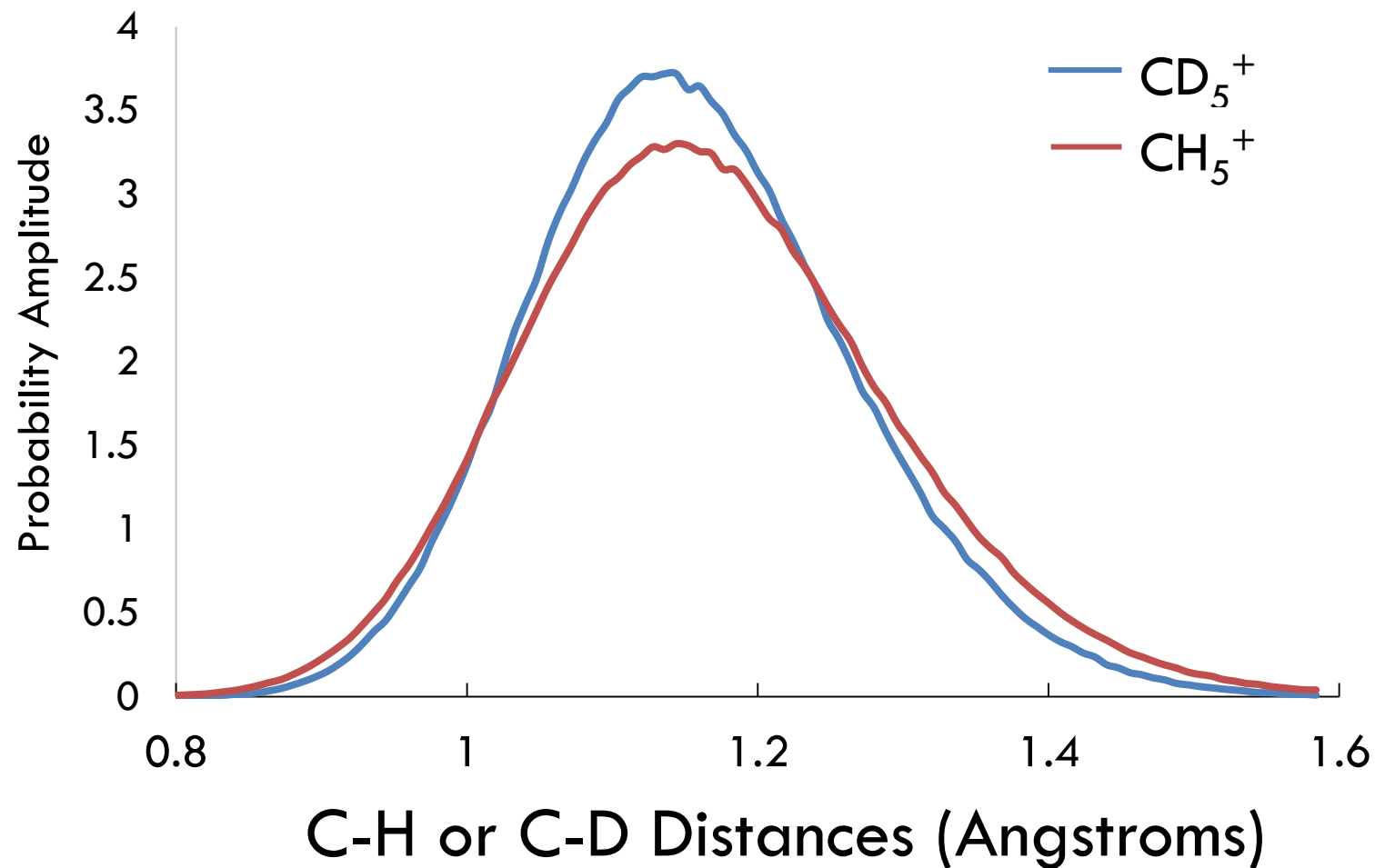
We have  $\psi$ , obtain  
another representation of  
it to get  $|\psi|^2$

Use  $|\psi|^2$  to get a  
probability distribution  
over **one** dimension of  
the wavefunction

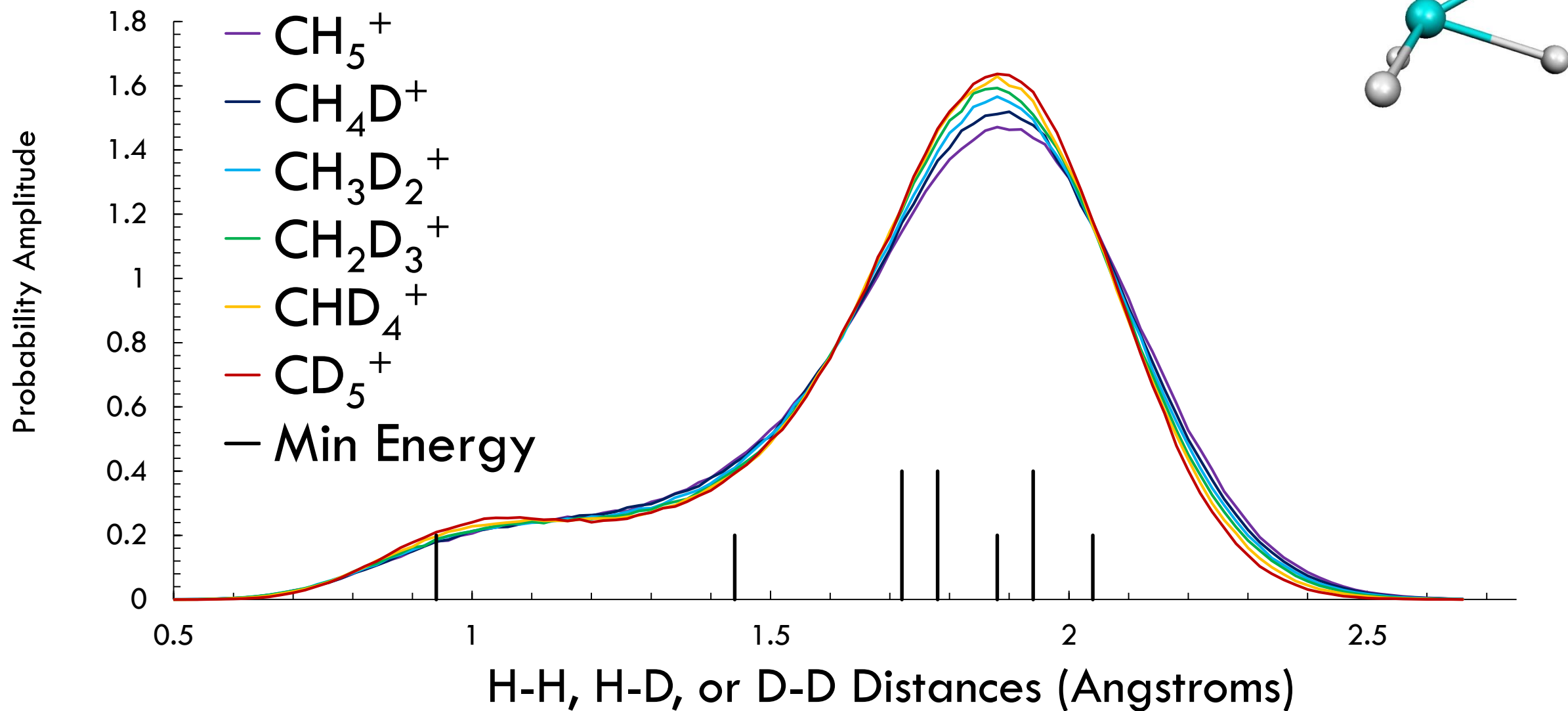
Height of graph  
indicates density of  
dots as we move  
from origin



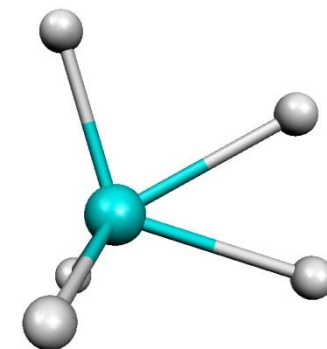
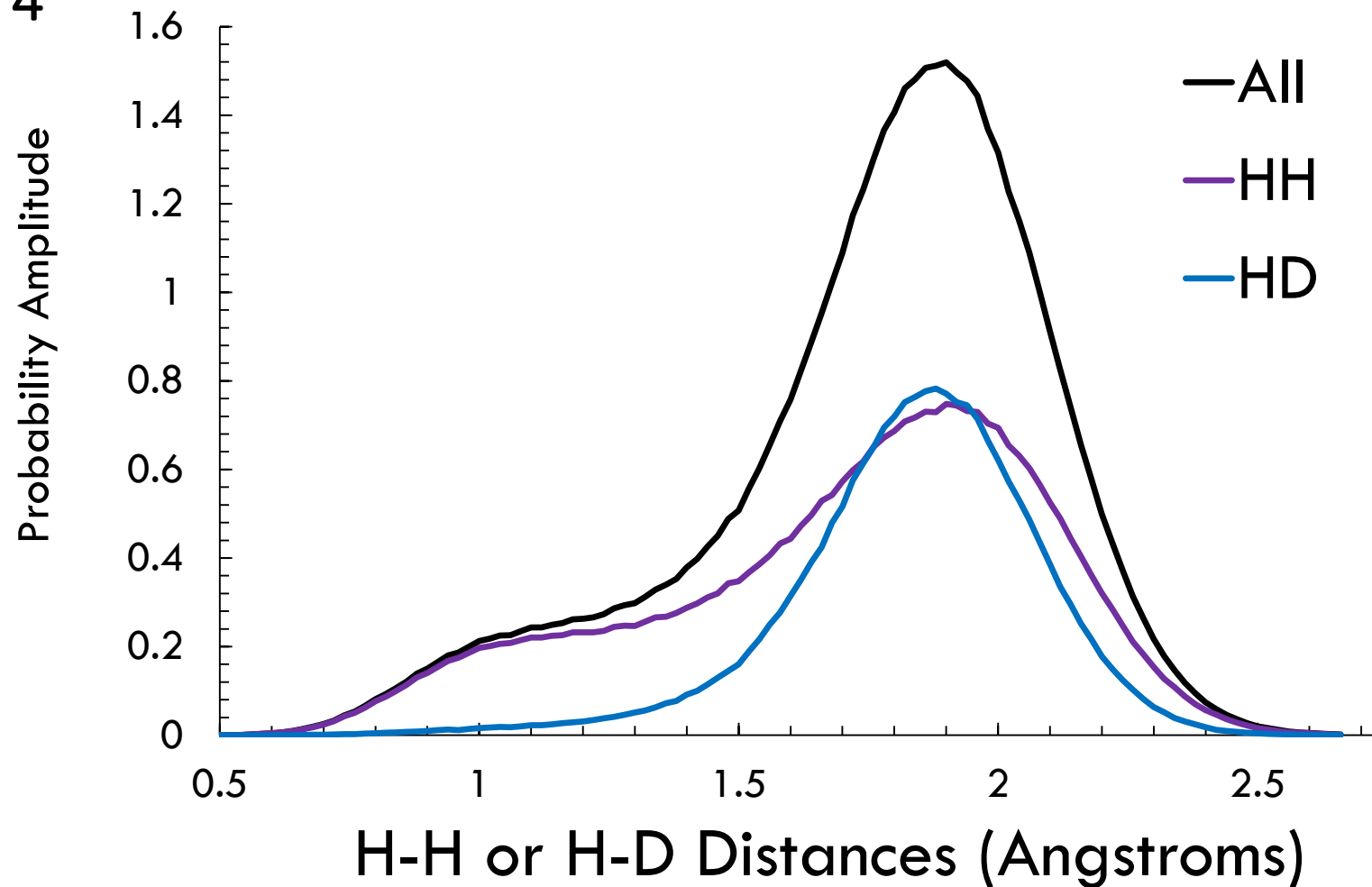
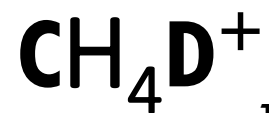
# C-H, C-D DISTRIBUTIONS



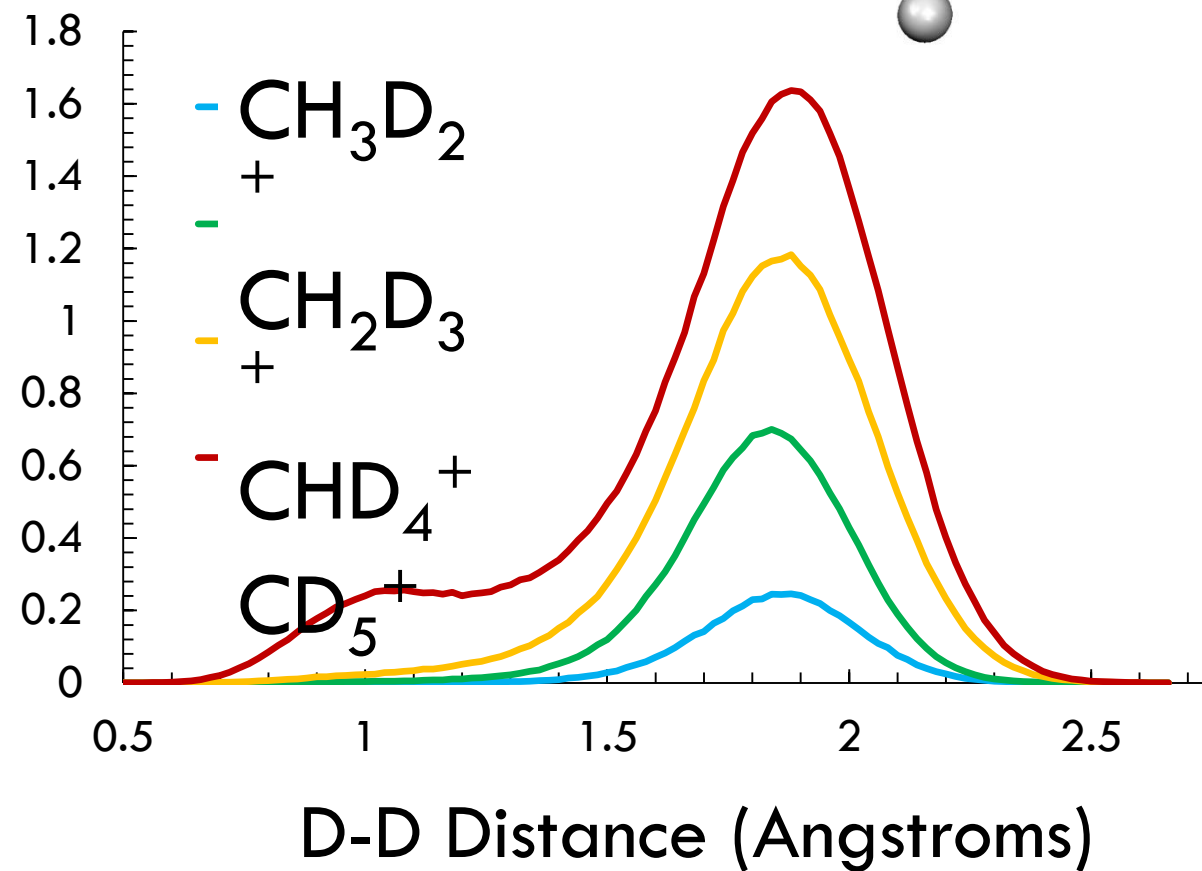
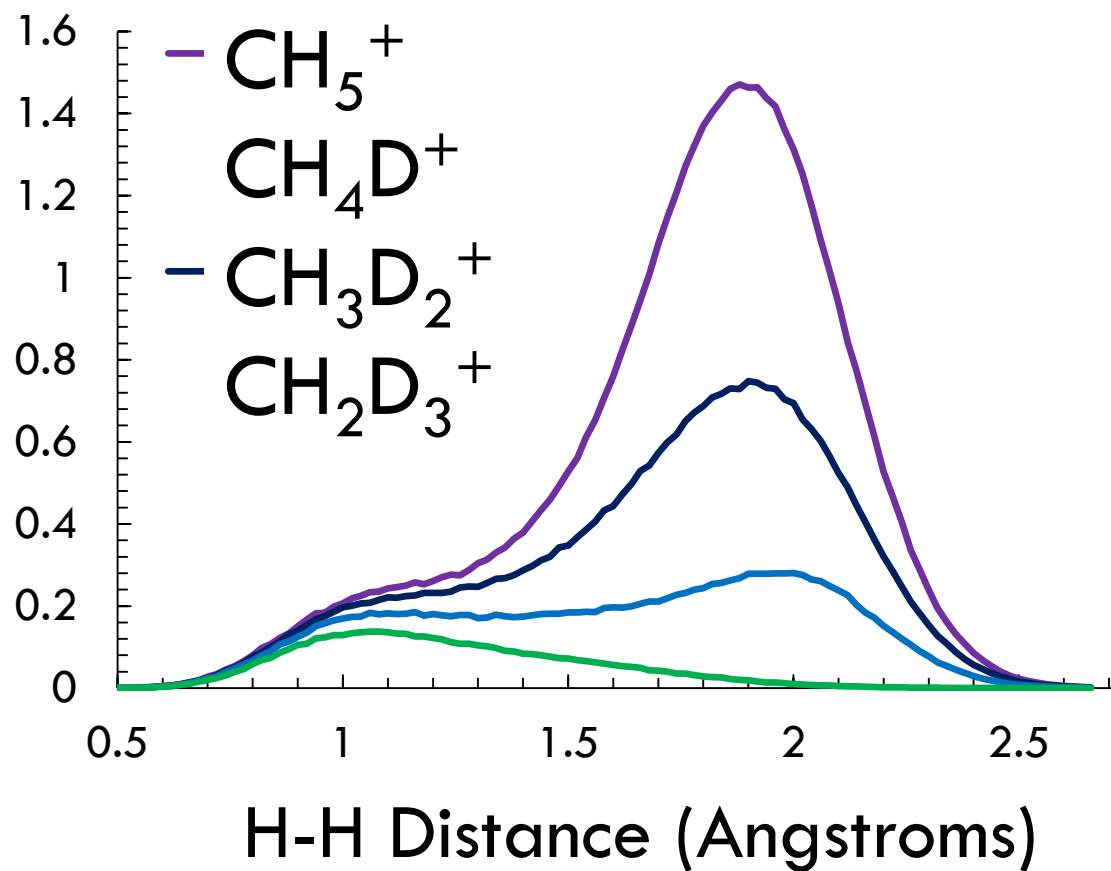
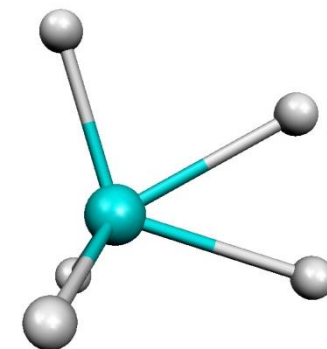
# Internuclear 1D Probability Distributions



# Internuclear 1D Probability Distributions

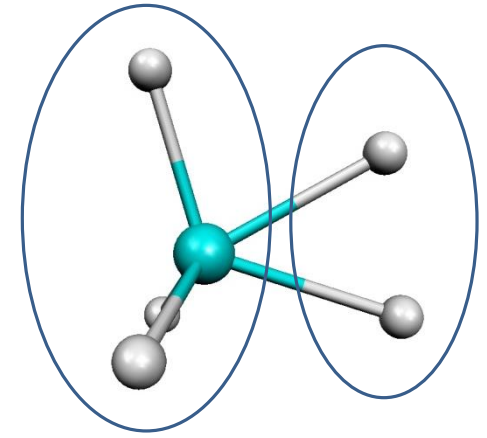


# Internuclear 1D Projections: H-H and D-D



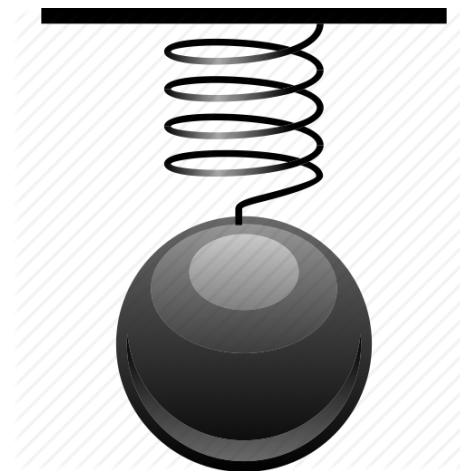
# NOT ALL FIVE POSITIONS ARE EQUAL!

The C-H bonds in the  $\text{CH}_3^+$  are stronger (higher energy, frequency) than the C-H bonds in the  $\text{H}_2$



The molecule's zero point energy is lowered more if we place a higher-mass atom in a position with a stronger C-H bond

Can we quantify how the distributions are changing as we add mass?



$$\omega = \sqrt{\frac{k}{\mu}}$$

$$\frac{1}{\sqrt{2}}\omega = \sqrt{\frac{k}{2\mu}}$$

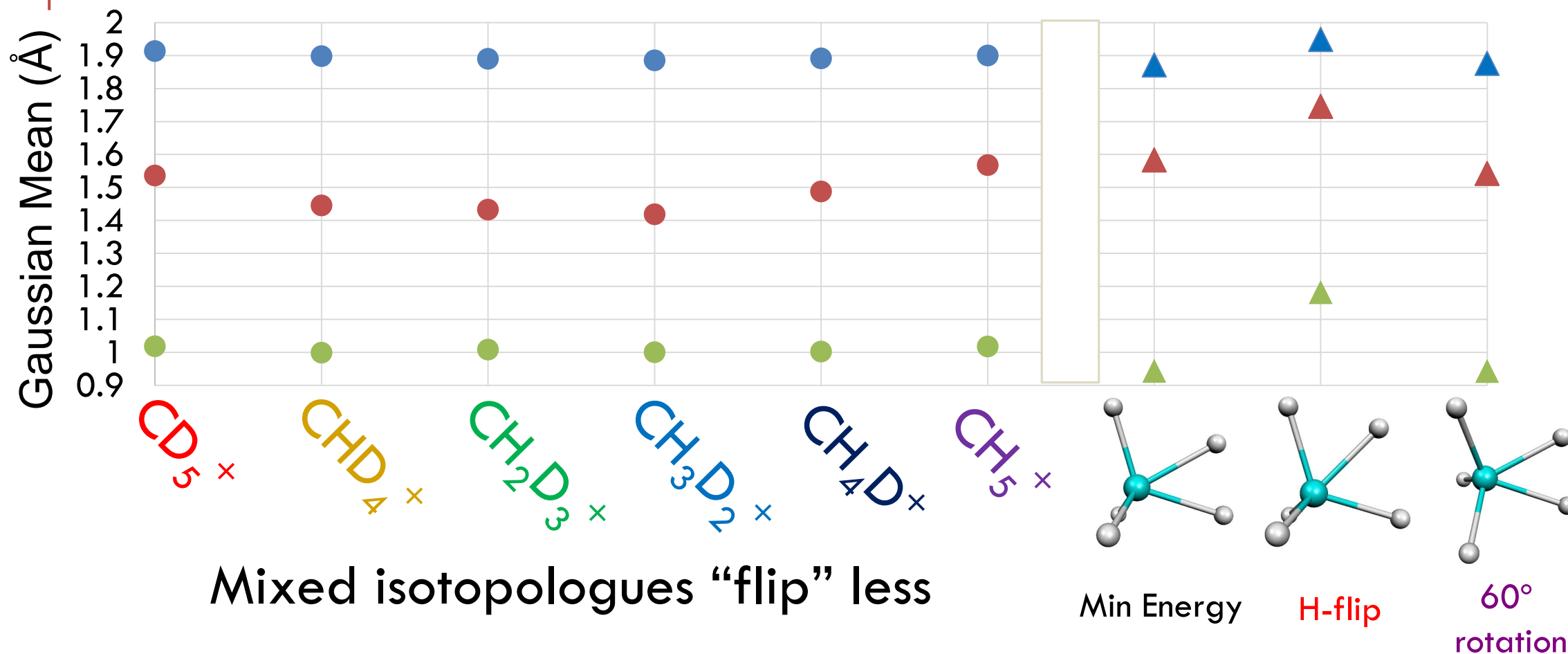
# CAN WE GET TO UNDERLYING PHYSICS BY FITTING?

$$f(x) = \sum_{i=1}^N A_i e^{\frac{-(x - c_i)^2}{2\sigma_i^2}}$$

But how many Gaussians do I need?



# TRENDS IN SUMMED DISTRIBUTIONS



# CURRENT/FUTURE WORK

Implementing DMC with 5-dimensional angular momentum

Theorized by Schmiedt, Schlemmer, and Jensen

$$\hat{H}_{rot} = \frac{B}{2} \sum_{a < b} \hat{J}_{ab}^2$$

$a, b = 1 \dots 5$

Key Future Goal: What do these states mean *physically*?

Use this new perspective to shed light on the physics behind the spectrum

## Preliminary Results

Exp. CDs (cm <sup>-1</sup> )	DMC (cm <sup>-1</sup> )	Corresponding 5D transitions
26.472	25.869	[3,0]-[1,0]
30.270	30.689	[3,1]-[1,0]
36.353	38.581	[3,2]-[1,0]

# Thank you!

Prof. Anne McCoy  
Prof. Lindsey Madison  
Dr. Meng Huang  
Victor Lee  
Ryan DiRisio  
Mark Boyer  
Mathew Joyner

