Computational Physics
Problem Set 4, September 23, 2025

Due: Monday, September 29, 2025 **by 11:59 PM**

Link to join GitHub classroom to submit homework solution: **Click here**.

Submit to the TA a link to the repository checked into your GitHub account containing a Jupyter Notebook including solutions for homework problems. The directory tree of the repository should include a directory for "homework" with subdirectories for each individual homework assignment.

You *must* label all axes of all plots, including the units if applicable.

1 Taylor Series Errors (25%)

A) Write 2 programs that calculate $f(x) = \exp(-3x)\sin(5x)$ as a product of 2 Taylor series. Do not turn the product into a double sum. In the first one, compute each term in expansions independently. In the second one, compute the terms using recursion relations. Both programs should use 32-bit floats.

B) Make a plot showing the absolute error for both programs from part A as functions of N, the number of terms used. For the true values, use numpy functions with 64-fit floats (default for Python 3). Make sure to make the range of N large enough to see the error stop decreasing.

C) Use the plots from part B to model the approximation error as $\delta_{\rm app} \simeq \alpha/N^\beta$; find values of α and β either by eye or by performing a linear regression fit of $\log\delta$. For both programs, find the optimal value for N.

2 Simpson's Rule Error (25%)

Derive the approximation error for the Simpson's Rule (Eq. 5.24 in *Newman*). Deriving this is similar to that for the Trapezoid Rule, but it does not follow exactly the same way. **You are allowed either LaTex it or to write this out on paper and scan it as a pdf to place in the repository.**

A) Define an interval of three points $\{x_{i-1}, x_i, x_{i+1}\}$ with a spacing of h in the same way that we do for the Simpson's Rule formula derivation in lecture. Expand $f(x)$ as a Taylor series centered at $x = x_i$ out to $4^{\rm th}$ order in x. Using this Taylor series, evaluate analytically the integrals $\int_{x_i}^{x_{i+1}} f(x)dx$ and $\int_{x_{i-1}}^{x_i} f(x)dx$, then add them together to derive the expression

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = 2hf_i + \frac{h^3}{3}f_i'' + \frac{h^5}{60}f_i^{(4)}\,, \qquad (1)$$

where we define $f_i^{(n)} \equiv f^{(n)}(x_i)$.

B) Using the same Taylor series for $f(x)$ that you derived in part A, evaluate both f_{i+1}, f_{i-1} and their sum in terms of h, f_i and its derivatives. In addition, approximate $f_i^{(4)}$ in terms of h,f'''_{i-1}, and f'''_{i+1}. Use these expression to write a new formula for $\int_{x_{i-1}}^{x_{i+1}} f(x)dx$ with f''_i and $f_i^{(4)}$ eliminated.

C) Sum the expression from part B over all quadrilaterals to derive the Simpson's Rule error. This part follows in a similar manner as the Trapezoid Rule error.

3 Trapezoid & Simpson Errors (25%)

A) Compute the integral

$$\int_0^1 (-4x^4 + 7x^3 - 2x^2 + 3)\, dx \tag{2}$$

both analytically and using the trapezoid rule.

B) Compute the error comparing the results computed in part A. Compare this to the error predicted from the trapezoid rule formalism. Why do they not agree perfectly?

C) Do parts A & B except this time for the Simpson's Rule.

4 The Stefan-Boltzmann Law (25%)

Exercise 5.12 in *Newman* (pg. 181).