Computational Physics

Problem Set 5, September 30, 2025

Due: Monday, October 6, 2025 **by 11:59 PM**

Link to join GitHub classroom to submit homework solution: **Click here**.

Submit to the TA a link to the repository checked into your GitHub account containing a Jupyter Notebook including solutions for homework problems. The directory tree of the repository should include a directory for "homework" with subdirectories for each individual homework assignment.

You *must* label all axes of all plots, including the units if applicable.

1 Period of an anharmonic oscillator (25%)

Exercise (5.10) on page 173 in *Newman*.

2 Vector Potential involving Elliptic Integrals (25%)

Jackson's *Classical Electrodynamics* (1988) solves for the ϕ component of the vector potential of a loop with radius a of current I in terms of elliptic integrals, given by

$$A_\phi(r,\theta) = \frac{\mu_0}{4\pi}\frac{4Ia}{\sqrt{a^2+r^2+2ar\sin\theta}}\left[\frac{(2-k^2)K(k)-2E(k)}{k^2}\right], \tag{1}$$

where

$$\begin{aligned} K(k) &= \int_0^{\pi/2}\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}} \\ E(k) &= \int_0^{\pi/2}\sqrt{1-k^2\sin^2\phi}\,d\phi \\ k^2 &= \frac{4ar\sin\theta}{a^2+r^2+2ar\sin\theta}. \end{aligned} \tag{2}$$

$K(k)$ is a complete elliptic integral of the first kind and $E(k)$ is a complete elliptic integral of the second kind. For $a=2$, $I=4$, and $\mu_0/4\pi=1$, compare and plot

A) $A_\phi(r=2.3,\theta)$ vs. θ.

B) $A_\phi(r,\theta=2\pi/3)$ vs. r.

Hint: Use Gauss-Chebyshev quadrature with $W(x)=1/\sqrt{1-x^2}$ for $-1<x<1$ to compute $K(k)$ and $E(k)$ with a large enough number of points. Don't forget to actually plot the $A_\phi(r,\theta)$ cases from parts a) and b).

3 Numerical Differentiation (25%)

Use forward-, central-, and extended-difference algorithms to differentiate the function $f(x) = e^{-3x}\cos 5x$ at $x = 0.1$, 1., and 100.
A) For all 3 methods, print out the derivative and its relative error $\mathcal{E}$ as functions of step size h. Include in the step size range the h-value where $\mathcal{E}$ equals machine precision $\epsilon_m \simeq 10^{-15}$ (use double-precision).
B) Plot $|\mathcal{E}|$ vs. h on a log-log plot and identify when approximation error dominates and likewise for round-off error. Do the trends agree with the predictions from lecture?
C) Repeat part A for the second-derivative using the 2 central difference algorithms in the second and third lines in Eq. 5.109 of *Newman*. Which is more accurate? Why?

4 Differentiating by Integrating (25%)

Exercise (5.22) on page 210 of *Newman*.