Computational Physics

Problem Set 2, September 9, 2025

Due: Monday, September 15, 2025 **by 11:59 PM**

Link to join GitHub classroom to submit homework solution: **Click here**.

Submit to the TA a link to the repository checked into your GitHub account containing a Jupyter Notebook including solutions for homework problems. The directory tree of the repository should include a directory for "homework" with subdirectories for each individual homework assignment.

You *must* label all axes of all plots, including the units if applicable.

1 2D motion (25%)

Make an animation using the *VPython* package of a 3D object undergoing parabolic motion in the Earth's gravitational field (near the surface). The formulas for parabolic motion are given by

$$\begin{aligned} x(t) &= v_0 t \cos(\theta) \\ y(t) &= h + v_0 t \sin(\theta) - \frac{1}{2} g t^2 \,, \end{aligned} \tag{1}$$

where $h > 0$ is the initial height of the object relative to the ground, v_0 is the initial velocity, θ is the launch angle relative to the horizon, and $g = 9.8$ m/s^2 is the (positive) acceleration due to gravity (at sea level). The animation should show the whole motion from launch from height h to ground, and it should stop when the object reaches the ground ($y = 0$). You can treat the object as a point particle. Show the animation for 3 cases of (h,v_0,θ).

2 Floating-point binary (20%)

Consider the 32-bit single-precision floating-point number A:

	s	e	f
Bit position	31	30 23	22 0
Value	1	0110 1010	1011 1100 0000 0000 0000 000

Note: Please complete parts A & B for this problem by hand as demonstrated in class. Explain your reasoning in the Jupyter notebook.

A) What are the values for the sign s, the exponent e, and the fractional mantissa f? Give e and f in decimal numbers with f written to as many decimal places as possible.

B) Determine the value of A in decimal, written in exponential (or scientific) notation to 4 decimal places.

3 User-defined float (30%)

For a float with N_e bits reserved for the exponent and N_f bits reserved for the mantissa, the equation for the number representation of the (normal) float is given by

$$N = (-1)^s \left[1 + \sum_{i=0}^{N_f-1} f_i \times 2^{-(N_f-i)} \right] \times 2^{e-2^{(N_e-1)}+1} . \tag{2}$$

For a subnormal number with $e = 0$, the equation changes to

$$N = (-1)^s \left[0 + \sum_{i=0}^{N_f-1} f_i \times 2^{-(N_f-i)} \right] \times 2^{-2^{(N_e-1)}+2} . \tag{3}$$

A) Consider a 32-bit float, but instead of 8 bits for the exponent and 23 bits for the mantissa, you instead have 12 bits for the exponent but only 19 bits for the mantissa. Find the largest normal number and the smallest (positive) number that can be stored.

B) Make plots showing how the common logarithms (base 10) of the largest normal number, the smallest (positive) number, and the machine precision vary given the number of bits stored in the mantissa assuming that the float is 32-bit and 1 bit is always reserved for the sign.

4 Computing Factorials (25%)

A) Exercise 4.1 in *Newman* (pg. 128).

B) Make a plot showing both $N!$ results (integers and floats) as function of N up to $N = 200$. *Hint:* To speed up the result, consider doing this recursively using $N! = N \cdot (N-1)!$.