Computational Physics

Problem Set 9, October 28, 2025

Due: Monday, November 3, 2025 **by 11:59 PM**

Link to join GitHub classroom to submit homework solution: **Click here**.

Submit to the TA a link to the repository checked into your GitHub account containing a Jupyter Notebook including solutions for homework problems. The directory tree of the repository should include a directory for "homework" with subdirectories for each individual homework assignment.

You *must* label all axes of all plots, including the units if applicable.

1 Spring problem (50%)

A) Repeat the spring analysis in the Jupyter notebook for the spring problem with a couple of changes. As before, assume the ends are bounded a meter apart, i.e. $u(x = 0, t) = u(x = 1, t) = 0$. Make the initial spatial wave function a triangle function

$$u(x, t = 0) = \begin{cases} x, & 0 < x < 0.5 \\ 1 - x, & 0.5 < x < 1 \end{cases}, \tag{1}$$

with with u and x in meters and an initial wave velocity $\dot{u}(x, t = 0) = 0$. Also, we set $q(x) = 20$ kg/m/s^2, $\rho(x) = 5$ kg/m, and $p(x) = 2x$ kg-m/s^2 for $0 < x < 1$. Compute the first 3 eigenfunctions and demonstrate the time dependence by plotting the wavefunction at multiple time steps or (for an extra 2.5% bonus!!) by making an animation.

B) Repeat part A (including 2.5% bonus for animation) except make the initial wave function $u(x, t = 0) = 0$ and the initial wave velocity and impulse with a Gaussian profile

$$\dot{u}(x, t = 0) = A \exp\left[\frac{-(x - 0.5)^2}{2\sigma^2}\right], \tag{2}$$

where $A = 0.1$ m/s and $\sigma = 0.1$ m.

2 Particle in a box: Energy Levels (50%)

For a finite square well potential with a depth of V_0 and size a, the bound energy levels E_n for a particle of mass m are determined by solving the transcendental equation

$$\tan z = \sqrt{(z_0/z)^2 - 1}\,, \tag{3}$$

where

$$\begin{aligned} z &= \frac{a}{\hbar}\sqrt{2m(E + V_0)} \\ z_0 &= \frac{a}{\hbar}\sqrt{2mV_0}\,. \end{aligned} \tag{4}$$

Write a routine to solve for all possible values of z given z_0. You can use any scipy.optimize root-solving functions in your routine. Note that there will be multiple solutions and the routine should find all of them. List all the energy levels (in units of eV) for a proton in a 1D box with length $a = 10^{-9}$ m and depth V_0 =10 meV.