Computational Physics

Problem Set 8, October 21, 2025

Due: Monday, October 27, 2025 **by 11:59 PM**

Link to join GitHub classroom to submit homework solution: **Click here**.

Submit to the TA a link to the repository checked into your GitHub account containing a Jupyter Notebook including solutions for homework problems. The directory tree of the repository should include a directory for "homework" with subdirectories for each individual homework assignment.

You *must* label all axes of all plots, including the units if applicable.

1 SVD Solver (50%)

Consider the matrix equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ where

$$\mathbf{A} = \begin{pmatrix} -5 & 2 & 0 & -2 & -3 \\ 3 & 1 & 0 & -5 & 4 \\ -7 & 2 & 1 & -1 & 1 \\ 4 & 6 & -2 & 4 & -2 \\ -5 & -2 & 3 & -15 & 11 \end{pmatrix} \tag{1}$$

A) Use SVD decomposition to show that $\mathbf{A}$ is a singular matrix. What are the rank and nullity of $\mathbf{A}$? Compute the pseudoinverse $\widetilde{\mathbf{A}}^{-1}$.

B) Use the SVD decomposition to find the basis vectors for the range of $\mathbf{A}$. Use the basis vectors to come up with 3 different values of $\mathbf{b}$ that are in the range of $\mathbf{A}$ but not parallel. Use the pseudoinverse to find the minimum solution for each value of $\mathbf{b}$ you chose.

C) Use the SVD decomposition to find the basis vectors for the null space of $\mathbf{A}$. Use these basis vectors to come up with 3 different vectors that are in the null space of $\mathbf{A}$. Add one of these vectors to each of the minimum solutions from part B and show that these modified vectors are also solutions.

D) Use the basis from part B to find a $\mathbf{b}$ not in the range of $\mathbf{A}$. Use the pseudoinverse to find the best solution for the value of $\mathbf{b}$ you chose.

2 Asymmetric quantum well (50%)

Exercise (6.9) in Newman