

## Notes

This is an abstract class and is not meant to be instantiated.

### See also:

[\*sympy.geometry.entity.GeometryEntity\*](#) (page 2194)

## Attributes

p1	
p2	
coefficients	
slope	
points	

### property bounds

Return a tuple (xmin, ymin, xmax, ymax) representing the bounding rectangle for the geometric figure.

### perpendicular\_line(*p*)

Create a new Line perpendicular to this linear entity which passes through the point *p*.

#### Parameters

**p** : Point

#### Returns

**line** : Line

## Examples

```
>>> from sympy import Point, Line
>>> p1, p2, p3 = Point(0, 0), Point(2, 3), Point(-2, 2)
>>> L = Line(p1, p2)
>>> P = L.perpendicular_line(p3); P
Line2D(Point2D(-2, 2), Point2D(-5, 4))
>>> L.is_perpendicular(P)
True
```

In 2D, the first point of the perpendicular line is the point through which was required to pass; the second point is arbitrarily chosen. To get a line that explicitly uses a point in the line, create a line from the perpendicular segment from the line to the point:

```
>>> Line(L.perpendicular_segment(p3))
Line2D(Point2D(-2, 2), Point2D(4/13, 6/13))
```

### See also:

[\*sympy.geometry.line.LinearEntity.is\\_perpendicular\*](#) (page 2221),  
[\*perpendicular\\_segment\*](#) (page 2224)

### property slope

The slope of this linear entity, or infinity if vertical.

#### Returns

**slope** : number or SymPy expression

### Examples

```
>>> from sympy import Point, Line
>>> p1, p2 = Point(0, 0), Point(3, 5)
>>> l1 = Line(p1, p2)
>>> l1.slope
5/3
```

```
>>> p3 = Point(0, 4)
>>> l2 = Line(p1, p3)
>>> l2.slope
oo
```

#### See also:

[coefficients](#) (page 2239)

**class** sympy.geometry.line.Line2D(*p1*, *pt=None*, *slope=None*, *\*\*kwargs*)

An infinite line in space 2D.

A line is declared with two distinct points or a point and slope as defined using keyword *slope*.

#### Parameters

**p1** : Point

**pt** : Point

**slope** : SymPy expression

### Examples

```
>>> from sympy import Line, Segment, Point
>>> L = Line(Point(2,3), Point(3,5))
>>> L
Line2D(Point2D(2, 3), Point2D(3, 5))
>>> L.points
(Point2D(2, 3), Point2D(3, 5))
>>> L.equation()
-2*x + y + 1
>>> L.coefficients
(-2, 1, 1)
```

Instantiate with keyword slope:

```
>>> Line(Point(0, 0), slope=0)
Line2D(Point2D(0, 0), Point2D(1, 0))
```

Instantiate with another linear object

```
>>> s = Segment((0, 0), (0, 1))
>>> Line(s).equation()
x
```

**See also:**

[sympy.geometry.point.Point](#) (page 2202)

**property coefficients**

The coefficients  $(a, b, c)$  for  $ax + by + c = 0$ .

## Examples

```
>>> from sympy import Point, Line
>>> from sympy.abc import x, y
>>> p1, p2 = Point(0, 0), Point(5, 3)
>>> l = Line(p1, p2)
>>> l.coefficients
(-3, 5, 0)
```

```
>>> p3 = Point(x, y)
>>> l2 = Line(p1, p3)
>>> l2.coefficients
(-y, x, 0)
```

**See also:**

[sympy.geometry.line.Line2D.equation](#) (page 2239)

**equation**( $x='x', y='y'$ )

The equation of the line:  $ax + by + c$ .

**Parameters**

**x** : str, optional

The name to use for the x-axis, default value is 'x'.

**y** : str, optional

The name to use for the y-axis, default value is 'y'.

**Returns**

**equation** : SymPy expression

## Examples

```
>>> from sympy import Point, Line
>>> p1, p2 = Point(1, 0), Point(5, 3)
>>> l1 = Line(p1, p2)
>>> l1.equation()
-3*x + 4*y + 3
```

**See also:**

[sympy.geometry.line.Line2D.coefficients](#) (page 2239)

**class** sympy.geometry.line.Ray2D(*p1*, *pt=None*, *angle=None*, *\*\*kwargs*)

A Ray is a semi-line in the space with a source point and a direction.

**Parameters**

**p1** : Point

The source of the Ray

**p2** : Point or radian value

This point determines the direction in which the Ray propagates. If given as an angle it is interpreted in radians with the positive direction being ccw.

**Examples**

```
>>> from sympy import Point, pi, Ray
>>> r = Ray(Point(2, 3), Point(3, 5))
>>> r
Ray2D(Point2D(2, 3), Point2D(3, 5))
>>> r.points
(Point2D(2, 3), Point2D(3, 5))
>>> r.source
Point2D(2, 3)
>>> r.xdirection
00
>>> r.ydirection
00
>>> r.slope
2
>>> Ray(Point(0, 0), angle=pi/4).slope
1
```

**See also:**

[sympy.geometry.point.Point](#) (page 2202), [Line](#) (page 2227)

**Attributes**

source	
xdirection	
ydirection	

**closing\_angle**(*r2*)

Return the angle by which *r2* must be rotated so it faces the same direction as *r1*.

**Parameters**

**r1** : Ray2D

**r2** : Ray2D

### Returns

**angle** : angle in radians (ccw angle is positive)

### Examples

```
>>> from sympy import Ray, pi
>>> r1 = Ray((0, 0), (1, 0))
>>> r2 = r1.rotate(-pi/2)
>>> angle = r1.closing_angle(r2); angle
pi/2
>>> r2.rotate(angle).direction.unit == r1.direction.unit
True
>>> r2.closing_angle(r1)
-pi/2
```

### See also:

[\*LinearEntity.angle\\_between\*](#) (page 2216)

### property **xdirection**

The x direction of the ray.

Positive infinity if the ray points in the positive x direction, negative infinity if the ray points in the negative x direction, or 0 if the ray is vertical.

### Examples

```
>>> from sympy import Point, Ray
>>> p1, p2, p3 = Point(0, 0), Point(1, 1), Point(0, -1)
>>> r1, r2 = Ray(p1, p2), Ray(p1, p3)
>>> r1.xdirection
oo
>>> r2.xdirection
0
```

### See also:

[\*ydirection\*](#) (page 2241)

### property **ydirection**

The y direction of the ray.

Positive infinity if the ray points in the positive y direction, negative infinity if the ray points in the negative y direction, or 0 if the ray is horizontal.

## Examples

```
>>> from sympy import Point, Ray
>>> p1, p2, p3 = Point(0, 0), Point(-1, -1), Point(-1, 0)
>>> r1, r2 = Ray(p1, p2), Ray(p1, p3)
>>> r1.ydirection
-oo
>>> r2.ydirection
0
```

### See also:

[xdirection](#) (page 2241)

**class** sympy.geometry.line.Segment2D(*p1*, *p2*, **\*\*kwargs**)

A line segment in 2D space.

### Parameters

**p1** : Point

**p2** : Point

## Examples

```
>>> from sympy import Point, Segment
>>> Segment((1, 0), (1, 1)) # tuples are interpreted as pts
Segment2D(Point2D(1, 0), Point2D(1, 1))
>>> s = Segment(Point(4, 3), Point(1, 1)); s
Segment2D(Point2D(4, 3), Point2D(1, 1))
>>> s.points
(Point2D(4, 3), Point2D(1, 1))
>>> s.slope
2/3
>>> s.length
sqrt(13)
>>> s.midpoint
Point2D(5/2, 2)
```

### See also:

[sympy.geometry.point.Point](#) (page 2202), [Line](#) (page 2227)

## Attributes

length	(number or SymPy expression)
midpoint	(Point)

**class** sympy.geometry.line.LinearEntity3D(*p1*, *p2*, **\*\*kwargs**)

An base class for all linear entities (line, ray and segment) in a 3-dimensional Euclidean space.

## Notes

This is a base class and is not meant to be instantiated.

## Attributes

p1	
p2	
direction_ratio	
direction_cosine	
points	

### property direction\_cosine

The normalized direction ratio of a given line in 3D.

## Examples

```
>>> from sympy import Point3D, Line3D
>>> p1, p2 = Point3D(0, 0, 0), Point3D(5, 3, 1)
>>> l = Line3D(p1, p2)
>>> l.direction_cosine
[sqrt(35)/7, 3*sqrt(35)/35, sqrt(35)/35]
>>> sum(i**2 for i in _)
1
```

### See also:

[sympy.geometry.line.Line3D.equation](#) (page 2244)

### property direction\_ratio

The direction ratio of a given line in 3D.

## Examples

```
>>> from sympy import Point3D, Line3D
>>> p1, p2 = Point3D(0, 0, 0), Point3D(5, 3, 1)
>>> l = Line3D(p1, p2)
>>> l.direction_ratio
[5, 3, 1]
```

### See also:

[sympy.geometry.line.Line3D.equation](#) (page 2244)

**class** sympy.geometry.line.**Line3D**(p1, pt=None, direction\_ratio=(), \*\*kwargs)

An infinite 3D line in space.

A line is declared with two distinct points or a point and `direction_ratio` as defined using keyword *direction\_ratio*.

### Parameters

**p1** : Point3D

**pt** : Point3D

**direction\_ratio** : list

### Examples

```
>>> from sympy import Line3D, Point3D
>>> L = Line3D(Point3D(2, 3, 4), Point3D(3, 5, 1))
>>> L
Line3D(Point3D(2, 3, 4), Point3D(3, 5, 1))
>>> L.points
(Point3D(2, 3, 4), Point3D(3, 5, 1))
```

### See also:

[sympy.geometry.point.Point3D](#) (page 2211), [sympy.geometry.line.Line](#) (page 2227), [sympy.geometry.line.Line2D](#) (page 2238)

**equation**(*x*='x', *y*='y', *z*='z', *k*=None)

Return the equations that define the line in 3D.

### Parameters

**x** : str, optional

The name to use for the x-axis, default value is 'x'.

**y** : str, optional

The name to use for the y-axis, default value is 'y'.

**z** : str, optional

The name to use for the z-axis, default value is 'z'.

**k** : str, optional

Deprecated since version 1.2: The k flag is deprecated. It does nothing.

### Returns

**equation** : Tuple of simultaneous equations

### Examples

```
>>> from sympy import Point3D, Line3D, solve
>>> from sympy.abc import x, y, z
>>> p1, p2 = Point3D(1, 0, 0), Point3D(5, 3, 0)
>>> l1 = Line3D(p1, p2)
>>> eq = l1.equation(x, y, z); eq
(-3*x + 4*y + 3, z)
>>> solve(eq.subs(z, 0), (x, y, z))
{x: 4*y/3 + 1}
```



**class** sympy.geometry.line.Ray3D(*p1*, *pt=None*, *direction\_ratio=()*, *\*\*kwargs*)

A Ray is a semi-line in the space with a source point and a direction.

#### Parameters

**p1** : Point3D

The source of the Ray

**p2** : Point or a direction vector

**direction\_ratio**: Determines the direction in which the Ray propagates.

#### Examples

```
>>> from sympy import Point3D, Ray3D
>>> r = Ray3D(Point3D(2, 3, 4), Point3D(3, 5, 0))
>>> r
Ray3D(Point3D(2, 3, 4), Point3D(3, 5, 0))
>>> r.points
(Point3D(2, 3, 4), Point3D(3, 5, 0))
>>> r.source
Point3D(2, 3, 4)
>>> r.xdirection
00
>>> r.ydirection
00
>>> r.direction_ratio
[1, 2, -4]
```

#### See also:

[sympy.geometry.point.Point3D](#) (page 2211), [Line3D](#) (page 2243)

#### Attributes

source	
xdirection	
ydirection	
zdirection	

#### property xdirection

The x direction of the ray.

Positive infinity if the ray points in the positive x direction, negative infinity if the ray points in the negative x direction, or 0 if the ray is vertical.

## Examples

```
>>> from sympy import Point3D, Ray3D
>>> p1, p2, p3 = Point3D(0, 0, 0), Point3D(1, 1, 1), Point3D(0, -1, 0)
>>> r1, r2 = Ray3D(p1, p2), Ray3D(p1, p3)
>>> r1.xdirection
oo
>>> r2.xdirection
0
```

### See also:

[ydirection](#) (page 2246)

### property ydirection

The y direction of the ray.

Positive infinity if the ray points in the positive y direction, negative infinity if the ray points in the negative y direction, or 0 if the ray is horizontal.

## Examples

```
>>> from sympy import Point3D, Ray3D
>>> p1, p2, p3 = Point3D(0, 0, 0), Point3D(-1, -1, -1), Point3D(-1, 0,
→ 0)
>>> r1, r2 = Ray3D(p1, p2), Ray3D(p1, p3)
>>> r1.ydirection
-oo
>>> r2.ydirection
0
```

### See also:

[xdirection](#) (page 2245)

### property zdirection

The z direction of the ray.

Positive infinity if the ray points in the positive z direction, negative infinity if the ray points in the negative z direction, or 0 if the ray is horizontal.

## Examples

```
>>> from sympy import Point3D, Ray3D
>>> p1, p2, p3 = Point3D(0, 0, 0), Point3D(-1, -1, -1), Point3D(-1, 0,
→ 0)
>>> r1, r2 = Ray3D(p1, p2), Ray3D(p1, p3)
>>> r1.ydirection
-oo
>>> r2.ydirection
0
>>> r2.zdirection
0
```

**See also:**

[xdirection](#) (page 2245)

**class** sympy.geometry.line.Segment3D(*p1*, *p2*, **\*\*kwargs**)

A line segment in a 3D space.

**Parameters**

**p1** : Point3D

**p2** : Point3D

**Examples**

```
>>> from sympy import Point3D, Segment3D
>>> Segment3D((1, 0, 0), (1, 1, 1)) # tuples are interpreted as pts
Segment3D(Point3D(1, 0, 0), Point3D(1, 1, 1))
>>> s = Segment3D(Point3D(4, 3, 9), Point3D(1, 1, 7)); s
Segment3D(Point3D(4, 3, 9), Point3D(1, 1, 7))
>>> s.points
(Point3D(4, 3, 9), Point3D(1, 1, 7))
>>> s.length
sqrt(17)
>>> s.midpoint
Point3D(5/2, 2, 8)
```

**See also:**

[sympy.geometry.point.Point3D](#) (page 2211), [Line3D](#) (page 2243)

**Attributes**

length	(number or SymPy expression)
midpoint	(Point3D)

**Curves**

**class** sympy.geometry.curve.Curve(*function*, *limits*)

A curve in space.

A curve is defined by parametric functions for the coordinates, a parameter and the lower and upper bounds for the parameter value.

**Parameters**

**function** : list of functions

**limits** : 3-tuple

Function parameter and lower and upper bounds.

**Raises**

**ValueError**

When *functions* are specified incorrectly. When *limits* are specified incorrectly.

## Examples

```
>>> from sympy import Curve, sin, cos, interpolate
>>> from sympy.abc import t, a
>>> C = Curve((sin(t), cos(t)), (t, 0, 2))
>>> C.functions
(sin(t), cos(t))
>>> C.limits
(t, 0, 2)
>>> C.parameter
t
>>> C = Curve((t, interpolate([1, 4, 9, 16], t)), (t, 0, 1)); C
Curve((t, t**2), (t, 0, 1))
>>> C.subs(t, 4)
Point2D(4, 16)
>>> C.arbitrary_point(a)
Point2D(a, a**2)
```

## See also:

[sympy.core.function.Function](#) (page 1050), [sympy.polys.polyfuncs.interpolate](#) (page 2426)

## Attributes

functions	
parameter	
limits	

## property ambient\_dimension

The dimension of the curve.

## Returns

int :

the dimension of curve.

## Examples

```
>>> from sympy.abc import t
>>> from sympy import Curve
>>> C = Curve((t, t**2), (t, 0, 2))
>>> C.ambient_dimension
2
```

**arbitrary\_point**(*parameter*='t')

A parameterized point on the curve.

**Parameters**

**parameter** : str or Symbol, optional

Default value is 't'. The Curve's parameter is selected with None or self.parameter otherwise the provided symbol is used.

**Returns**

Point :

Returns a point in parametric form.

**Raises**

**ValueError**

When *parameter* already appears in the functions.

**Examples**

```
>>> from sympy import Curve, Symbol
>>> from sympy.abc import s
>>> C = Curve([2*s, s**2], (s, 0, 2))
>>> C.arbitrary_point()
Point2D(2*t, t**2)
>>> C.arbitrary_point(C.parameter)
Point2D(2*s, s**2)
>>> C.arbitrary_point(None)
Point2D(2*s, s**2)
>>> C.arbitrary_point(Symbol('a'))
Point2D(2*a, a**2)
```

**See also:**

[sympy.geometry.point.Point](#) (page 2202)

**property free\_symbols**

Return a set of symbols other than the bound symbols used to parametrically define the Curve.

**Returns**

set :

Set of all non-parameterized symbols.

**Examples**

```
>>> from sympy.abc import t, a
>>> from sympy import Curve
>>> Curve((t, t**2), (t, 0, 2)).free_symbols
set()
>>> Curve((t, t**2), (t, a, 2)).free_symbols
{a}
```

## property functions

The functions specifying the curve.

### Returns

functions :

list of parameterized coordinate functions.

## Examples

```
>>> from sympy.abc import t
>>> from sympy import Curve
>>> C = Curve((t, t**2), (t, 0, 2))
>>> C.functions
(t, t**2)
```

### See also:

[parameter](#) (page 2250)

## property length

The curve length.

## Examples

```
>>> from sympy import Curve
>>> from sympy.abc import t
>>> Curve((t, t), (t, 0, 1)).length
sqrt(2)
```

## property limits

The limits for the curve.

### Returns

**limits** : tuple

Contains parameter and lower and upper limits.

## Examples

```
>>> from sympy.abc import t
>>> from sympy import Curve
>>> C = Curve([t, t**3], (t, -2, 2))
>>> C.limits
(t, -2, 2)
```

### See also:

[plot\\_interval](#) (page 2251)

## property parameter

The curve function variable.

### Returns

Symbol :

returns a bound symbol.

## Examples

```
>>> from sympy.abc import t
>>> from sympy import Curve
>>> C = Curve([t, t**2], (t, 0, 2))
>>> C.parameter
t
```

### See also:

[functions](#) (page 2249)

## plot\_interval(parameter='t')

The plot interval for the default geometric plot of the curve.

### Parameters

**parameter** : str or Symbol, optional

Default value is 't'; otherwise the provided symbol is used.

### Returns

List :

**the plot interval as below:**

[parameter, lower\_bound, upper\_bound]

## Examples

```
>>> from sympy import Curve, sin
>>> from sympy.abc import x, s
>>> Curve((x, sin(x)), (x, 1, 2)).plot_interval()
[t, 1, 2]
>>> Curve((x, sin(x)), (x, 1, 2)).plot_interval(s)
[s, 1, 2]
```

### See also:

[limits](#) (page 2250)

Returns limits of the parameter interval

## rotate(angle=0, pt=None)

This function is used to rotate a curve along given point pt at given angle(in radian).

### Parameters

**angle** :

the angle at which the curve will be rotated(in radian) in counter-clockwise direction. default value of angle is 0.

**pt** : Point

the point along which the curve will be rotated. If no point given, the curve will be rotated around origin.

### Returns

Curve :

returns a curve rotated at given angle along given point.

### Examples

```
>>> from sympy import Curve, pi
>>> from sympy.abc import x
>>> Curve((x, x), (x, 0, 1)).rotate(pi/2)
Curve((-x, x), (x, 0, 1))
```

**scale**(*x=1, y=1, pt=None*)

Override GeometryEntity.scale since Curve is not made up of Points.

### Returns

Curve :

returns scaled curve.

### Examples

```
>>> from sympy import Curve
>>> from sympy.abc import x
>>> Curve((x, x), (x, 0, 1)).scale(2)
Curve((2*x, x), (x, 0, 1))
```

**translate**(*x=0, y=0*)

Translate the Curve by (x, y).

### Returns

Curve :

returns a translated curve.

### Examples

```
>>> from sympy import Curve
>>> from sympy.abc import x
>>> Curve((x, x), (x, 0, 1)).translate(1, 2)
Curve((x + 1, x + 2), (x, 0, 1))
```



## Ellipses

**class** sympy.geometry.ellipse.**Ellipse**(*center=None, hradius=None, vradius=None, eccentricity=None, \*\*kwargs*)

An elliptical GeometryEntity.

### Parameters

**center** : Point, optional

Default value is Point(0, 0)

**hradius** : number or SymPy expression, optional

**vradius** : number or SymPy expression, optional

**eccentricity** : number or SymPy expression, optional

Two of *hradius*, *vradius* and *eccentricity* must be supplied to create an Ellipse. The third is derived from the two supplied.

### Raises

#### GeometryError

When *hradius*, *vradius* and *eccentricity* are incorrectly supplied as parameters.

#### TypeError

When *center* is not a Point.

## Notes

Constructed from a center and two radii, the first being the horizontal radius (along the x-axis) and the second being the vertical radius (along the y-axis).

When symbolic value for *hradius* and *vradius* are used, any calculation that refers to the foci or the major or minor axis will assume that the ellipse has its major radius on the x-axis. If this is not true then a manual rotation is necessary.

## Examples

```
>>> from sympy import Ellipse, Point, Rational
>>> e1 = Ellipse(Point(0, 0), 5, 1)
>>> e1.hradius, e1.vradius
(5, 1)
>>> e2 = Ellipse(Point(3, 1), hradius=3, eccentricity=Rational(4, 5))
>>> e2
Ellipse(Point2D(3, 1), 3, 9/5)
```

### See also:

[Circle](#) (page 2269)

## Attributes

center	
hradius	
vradius	
area	
circumference	
eccentricity	
periapsis	
apoapsis	
focus_distance	
foci	

### property apoapsis

The apoapsis of the ellipse.

The greatest distance between the focus and the contour.

#### Returns

**apoapsis** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.apoapsis
2*sqrt(2) + 3
```

### See also:

#### [periapsis](#) (page 2263)

Returns shortest distance between foci and contour

### **arbitrary\_point**(*parameter*='t')

A parameterized point on the ellipse.

#### Parameters

**parameter** : str, optional

Default value is 't'.

#### Returns

**arbitrary\_point** : Point

#### Raises

**ValueError**

When *parameter* already appears in the functions.

## Examples

```
>>> from sympy import Point, Ellipse
>>> e1 = Ellipse(Point(0, 0), 3, 2)
>>> e1.arbitrary_point()
Point2D(3*cos(t), 2*sin(t))
```

### See also:

[sympy.geometry.point.Point](#) (page 2202)

### property area

The area of the ellipse.

#### Returns

**area** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.area
3*pi
```

### auxiliary\_circle()

Returns a Circle whose diameter is the major axis of the ellipse.

## Examples

```
>>> from sympy import Ellipse, Point, symbols
>>> c = Point(1, 2)
>>> Ellipse(c, 8, 7).auxiliary_circle()
Circle(Point2D(1, 2), 8)
>>> a, b = symbols('a b')
>>> Ellipse(c, a, b).auxiliary_circle()
Circle(Point2D(1, 2), Max(a, b))
```

### property bounds

Return a tuple (xmin, ymin, xmax, ymax) representing the bounding rectangle for the geometric figure.

### property center

The center of the ellipse.

#### Returns

**center** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.center
Point2D(0, 0)
```

### See also:

[sympy.geometry.point.Point](#) (page 2202)

## property circumference

The circumference of the ellipse.

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.circumference
12*elliptic_e(8/9)
```

## director\_circle()

Returns a Circle consisting of all points where two perpendicular tangent lines to the ellipse cross each other.

### Returns

Circle

A director circle returned as a geometric object.

## Examples

```
>>> from sympy import Ellipse, Point, symbols
>>> c = Point(3,8)
>>> Ellipse(c, 7, 9).director_circle()
Circle(Point2D(3, 8), sqrt(130))
>>> a, b = symbols('a b')
>>> Ellipse(c, a, b).director_circle()
Circle(Point2D(3, 8), sqrt(a**2 + b**2))
```

## References

[R503]

### property `eccentricity`

The eccentricity of the ellipse.

#### Returns

**`eccentricity`** : number

## Examples

```
>>> from sympy import Point, Ellipse, sqrt
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, sqrt(2))
>>> e1.eccentricity
sqrt(7)/3
```

### `encloses_point(p)`

Return True if `p` is enclosed by (is inside of) self.

#### Parameters

**`p`** : Point

#### Returns

**`encloses_point`** : True, False or None

## Notes

Being on the border of self is considered False.

## Examples

```
>>> from sympy import Ellipse, S
>>> from sympy.abc import t
>>> e = Ellipse((0, 0), 3, 2)
>>> e.encloses_point((0, 0))
True
>>> e.encloses_point(e.arbitrary_point(t).subs(t, S.Half))
False
>>> e.encloses_point((4, 0))
False
```

### See also:

[`sympy.geometry.point.Point`](#) (page 2202)

### `equation(x='x', y='y', _slope=None)`

Returns the equation of an ellipse aligned with the `x` and `y` axes; when `slope` is given, the equation returned corresponds to an ellipse with a major axis having that slope.

#### Parameters

**`x`** : str, optional

Label for the x-axis. Default value is 'x'.

**y** : str, optional

Label for the y-axis. Default value is 'y'.

**\_slope** : Expr, optional

The slope of the major axis. Ignored when 'None'.

### Returns

**equation** : SymPy expression

### Examples

```
>>> from sympy import Point, Ellipse, pi
>>> from sympy.abc import x, y
>>> e1 = Ellipse(Point(1, 0), 3, 2)
>>> eq1 = e1.equation(x, y); eq1
y**2/4 + (x/3 - 1/3)**2 - 1
>>> eq2 = e1.equation(x, y, _slope=1); eq2
(-x + y + 1)**2/8 + (x + y - 1)**2/18 - 1
```

A point on e1 satisfies eq1. Let's use one on the x-axis:

```
>>> p1 = e1.center + Point(e1.major, 0)
>>> assert eq1.subs(x, p1.x).subs(y, p1.y) == 0
```

When rotated the same as the rotated ellipse, about the center point of the ellipse, it will satisfy the rotated ellipse's equation, too:

```
>>> r1 = p1.rotate(pi/4, e1.center)
>>> assert eq2.subs(x, r1.x).subs(y, r1.y) == 0
```

### See also:

[\*arbitrary\\_point\*](#) (page 2254)

Returns parameterized point on ellipse

### References

[R504], [R505]

**evolute**(x='x', y='y')

The equation of evolute of the ellipse.

#### Parameters

**x** : str, optional

Label for the x-axis. Default value is 'x'.

**y** : str, optional

Label for the y-axis. Default value is 'y'.

#### Returns

**equation** : SymPy expression

## Examples

```
>>> from sympy import Point, Ellipse
>>> e1 = Ellipse(Point(1, 0), 3, 2)
>>> e1.evolute()
2**(2/3)*y**(2/3) + (3*x - 3)**(2/3) - 5**(2/3)
```

### property foci

The foci of the ellipse.

#### Raises

##### ValueError

When the major and minor axis cannot be determined.

## Notes

The foci can only be calculated if the major/minor axes are known.

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.foci
(Point2D(-2*sqrt(2), 0), Point2D(2*sqrt(2), 0))
```

### See also:

[sympy.geometry.point.Point](#) (page 2202)

#### [focus\\_distance](#) (page 2259)

Returns the distance between focus and center

### property focus\_distance

The focal distance of the ellipse.

The distance between the center and one focus.

#### Returns

**focus\_distance** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.focus_distance
2*sqrt(2)
```

### See also:

[foci](#) (page 2259)

### property `hradius`

The horizontal radius of the ellipse.

#### Returns

**hradius** : number

### Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.hradius
3
```

#### See also:

[vradius](#) (page 2269), [major](#) (page 2261), [minor](#) (page 2262)

### `intersection(o)`

The intersection of this ellipse and another geometrical entity *o*.

#### Parameters

**o** : GeometryEntity

#### Returns

**intersection** : list of GeometryEntity objects

### Notes

Currently supports intersections with Point, Line, Segment, Ray, Circle and Ellipse types.

### Examples

```
>>> from sympy import Ellipse, Point, Line
>>> e = Ellipse(Point(0, 0), 5, 7)
>>> e.intersection(Point(0, 0))
[]
>>> e.intersection(Point(5, 0))
[Point2D(5, 0)]
>>> e.intersection(Line(Point(0,0), Point(0, 1)))
[Point2D(0, -7), Point2D(0, 7)]
>>> e.intersection(Line(Point(5,0), Point(5, 1)))
[Point2D(5, 0)]
>>> e.intersection(Line(Point(6,0), Point(6, 1)))
[]
>>> e = Ellipse(Point(-1, 0), 4, 3)
>>> e.intersection(Ellipse(Point(1, 0), 4, 3))
[Point2D(0, -3*sqrt(15)/4), Point2D(0, 3*sqrt(15)/4)]
>>> e.intersection(Ellipse(Point(5, 0), 4, 3))
[Point2D(2, -3*sqrt(7)/4), Point2D(2, 3*sqrt(7)/4)]
```

(continues on next page)



(continued from previous page)

```
>>> e.intersection(Ellipse(Point(100500, 0), 4, 3))
[]
>>> e.intersection(Ellipse(Point(0, 0), 3, 4))
[Point2D(3, 0), Point2D(-363/175, -48*sqrt(111)/175), Point2D(-363/
↪ 175, 48*sqrt(111)/175)]
>>> e.intersection(Ellipse(Point(-1, 0), 3, 4))
[Point2D(-17/5, -12/5), Point2D(-17/5, 12/5), Point2D(7/5, -12/5),
↪ Point2D(7/5, 12/5)]
```

#### See also:

[sympy.geometry.entity.GeometryEntity](#) (page 2194)

### is\_tangent(*o*)

Is *o* tangent to the ellipse?

#### Parameters

***o*** : GeometryEntity

An Ellipse, LinearEntity or Polygon

#### Returns

is\_tangent: boolean

True if *o* is tangent to the ellipse, False otherwise.

#### Raises

**NotImplementedError**

When the wrong type of argument is supplied.

### Examples

```
>>> from sympy import Point, Ellipse, Line
>>> p0, p1, p2 = Point(0, 0), Point(3, 0), Point(3, 3)
>>> e1 = Ellipse(p0, 3, 2)
>>> l1 = Line(p1, p2)
>>> e1.is_tangent(l1)
True
```

#### See also:

[tangent\\_lines](#) (page 2268)

### property major

Longer axis of the ellipse (if it can be determined) else hradius.

#### Returns

**major** : number or expression

## Examples

```
>>> from sympy import Point, Ellipse, Symbol
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.major
3
```

```
>>> a = Symbol('a')
>>> b = Symbol('b')
>>> Ellipse(p1, a, b).major
a
>>> Ellipse(p1, b, a).major
b
```

```
>>> m = Symbol('m')
>>> M = m + 1
>>> Ellipse(p1, m, M).major
m + 1
```

### See also:

[hradius](#) (page 2259), [vradius](#) (page 2269), [minor](#) (page 2262)

### property minor

Shorter axis of the ellipse (if it can be determined) else vradius.

#### Returns

**minor** : number or expression

## Examples

```
>>> from sympy import Point, Ellipse, Symbol
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.minor
1
```

```
>>> a = Symbol('a')
>>> b = Symbol('b')
>>> Ellipse(p1, a, b).minor
b
>>> Ellipse(p1, b, a).minor
a
```

```
>>> m = Symbol('m')
>>> M = m + 1
>>> Ellipse(p1, m, M).minor
m
```

### See also:

[hradius](#) (page 2259), [vradius](#) (page 2269), [major](#) (page 2261)

**normal\_lines**(*p*, *prec*=None)

Normal lines between *p* and the ellipse.

**Parameters**

**p** : Point

**Returns**

**normal\_lines** : list with 1, 2 or 4 Lines

**Examples**

```
>>> from sympy import Point, Ellipse
>>> e = Ellipse((0, 0), 2, 3)
>>> c = e.center
>>> e.normal_lines(c + Point(1, 0))
[Line2D(Point2D(0, 0), Point2D(1, 0))]
>>> e.normal_lines(c)
[Line2D(Point2D(0, 0), Point2D(0, 1)), Line2D(Point2D(0, 0),
↪Point2D(1, 0))]
```

Off-axis points require the solution of a quartic equation. This often leads to very large expressions that may be of little practical use. An approximate solution of *prec* digits can be obtained by passing in the desired value:

```
>>> e.normal_lines((3, 3), prec=2)
[Line2D(Point2D(-0.81, -2.7), Point2D(0.19, -1.2)),
Line2D(Point2D(1.5, -2.0), Point2D(2.5, -2.7))]
```

Whereas the above solution has an operation count of 12, the exact solution has an operation count of 2020.

**property periapsis**

The periapsis of the ellipse.

The shortest distance between the focus and the contour.

**Returns**

**periapsis** : number

**Examples**

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.periapsis
3 - 2*sqrt(2)
```

**See also:**

[\*\*apoapsis\*\*](#) (page 2254)

Returns greatest distance between focus and contour

**plot\_interval**(parameter='t')

The plot interval for the default geometric plot of the Ellipse.

**Parameters**

**parameter** : str, optional

Default value is 't'.

**Returns**

**plot\_interval** : list

[parameter, lower\_bound, upper\_bound]

**Examples**

```
>>> from sympy import Point, Ellipse
>>> e1 = Ellipse(Point(0, 0), 3, 2)
>>> e1.plot_interval()
[t, -pi, pi]
```

**polar\_second\_moment\_of\_area**()

Returns the polar second moment of area of an Ellipse

It is a constituent of the second moment of area, linked through the perpendicular axis theorem. While the planar second moment of area describes an object's resistance to deflection (bending) when subjected to a force applied to a plane parallel to the central axis, the polar second moment of area describes an object's resistance to deflection when subjected to a moment applied in a plane perpendicular to the object's central axis (i.e. parallel to the cross-section)

**Examples**

```
>>> from sympy import symbols, Circle, Ellipse
>>> c = Circle((5, 5), 4)
>>> c.polar_second_moment_of_area()
128*pi
>>> a, b = symbols('a, b')
>>> e = Ellipse((0, 0), a, b)
>>> e.polar_second_moment_of_area()
pi*a**3*b/4 + pi*a*b**3/4
```

**References**

[R506]

**random\_point**(seed=None)

A random point on the ellipse.

**Returns**

**point** : Point

## Examples

```
>>> from sympy import Point, Ellipse
>>> e1 = Ellipse(Point(0, 0), 3, 2)
>>> e1.random_point() # gives some random point
Point2D(...)
>>> p1 = e1.random_point(seed=0); p1.n(2)
Point2D(2.1, 1.4)
```

## Notes

When creating a random point, one may simply replace the parameter with a random number. When doing so, however, the random number should be made a Rational or else the point may not test as being in the ellipse:

```
>>> from sympy.abc import t
>>> from sympy import Rational
>>> arb = e1.arbitrary_point(t); arb
Point2D(3*cos(t), 2*sin(t))
>>> arb.subs(t, .1) in e1
False
>>> arb.subs(t, Rational(.1)) in e1
True
>>> arb.subs(t, Rational('.1')) in e1
True
```

### See also:

[sympy.geometry.point.Point](#) (page 2202)

**[arbitrary\\_point](#)** (page 2254)

Returns parameterized point on ellipse

**reflect**(line)

Override GeometryEntity.reflect since the radius is not a GeometryEntity.

## Examples

```
>>> from sympy import Circle, Line
>>> Circle((0, 1), 1).reflect(Line((0, 0), (1, 1)))
Circle(Point2D(1, 0), -1)
>>> from sympy import Ellipse, Line, Point
>>> Ellipse(Point(3, 4), 1, 3).reflect(Line(Point(0, -4), Point(5, 0)))
Traceback (most recent call last):
...
NotImplementedError:
General Ellipse is not supported but the equation of the reflected
Ellipse is given by the zeros of: f(x, y) = (9*x/41 + 40*y/41 +
37/41)**2 + (40*x/123 - 3*y/41 - 364/123)**2 - 1
```

## Notes

Until the general ellipse (with no axis parallel to the x-axis) is supported a `NotImplementedError` is raised and the equation whose zeros define the rotated ellipse is given.

**rotate**(*angle=0*, *pt=None*)

Rotate *angle* radians counterclockwise about Point *pt*.

Note: since the general ellipse is not supported, only rotations that are integer multiples of  $\pi/2$  are allowed.

## Examples

```
>>> from sympy import Ellipse, pi
>>> Ellipse((1, 0), 2, 1).rotate(pi/2)
Ellipse(Point2D(0, 1), 1, 2)
>>> Ellipse((1, 0), 2, 1).rotate(pi)
Ellipse(Point2D(-1, 0), 2, 1)
```

**scale**(*x=1*, *y=1*, *pt=None*)

Override `GeometryEntity.scale` since it is the major and minor axes which must be scaled and they are not `GeometryEntities`.

## Examples

```
>>> from sympy import Ellipse
>>> Ellipse((0, 0), 2, 1).scale(2, 4)
Circle(Point2D(0, 0), 4)
>>> Ellipse((0, 0), 2, 1).scale(2)
Ellipse(Point2D(0, 0), 4, 1)
```

**second\_moment\_of\_area**(*point=None*)

Returns the second moment and product moment area of an ellipse.

### Parameters

**point** : Point, two-tuple of sympifiable objects, or `None` (default=`None`)

*point* is the point about which second moment of area is to be found. If "*point=None*" it will be calculated about the axis passing through the centroid of the ellipse.

### Returns

**I<sub>xx</sub>**, **I<sub>yy</sub>**, **I<sub>xy</sub>** : number or SymPy expression

*I<sub>xx</sub>*, *I<sub>yy</sub>* are second moment of area of an ellipse. *I<sub>xy</sub>* is product moment of area of an ellipse.

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.second_moment_of_area()
(3*pi/4, 27*pi/4, 0)
```

## References

[R507]

**section\_modulus**(*point=None*)

Returns a tuple with the section modulus of an ellipse

Section modulus is a geometric property of an ellipse defined as the ratio of second moment of area to the distance of the extreme end of the ellipse from the centroidal axis.

### Parameters

**point** : Point, two-tuple of sympifyable objects, or None(default=None)

point is the point at which section modulus is to be found. If "point=None" section modulus will be calculated for the point farthest from the centroidal axis of the ellipse.

### Returns

S\_x, S\_y: numbers or SymPy expressions

S\_x is the section modulus with respect to the x-axis S\_y is the section modulus with respect to the y-axis A negative sign indicates that the section modulus is determined for a point below the centroidal axis.

## Examples

```
>>> from sympy import Symbol, Ellipse, Circle, Point2D
>>> d = Symbol('d', positive=True)
>>> c = Circle((0, 0), d/2)
>>> c.section_modulus()
(pi*d**3/32, pi*d**3/32)
>>> e = Ellipse(Point2D(0, 0), 2, 4)
>>> e.section_modulus()
(8*pi, 4*pi)
>>> e.section_modulus((2, 2))
(16*pi, 4*pi)
```

## References

[R508]

### property `semilatus_rectum`

Calculates the semi-latus rectum of the Ellipse.

Semi-latus rectum is defined as one half of the chord through a focus parallel to the conic section directrix of a conic section.

#### Returns

**`semilatus_rectum`** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.semilatus_rectum
1/3
```

#### See also:

##### [\*apoapsis\*](#) (page 2254)

Returns greatest distance between focus and contour

##### [\*periapsis\*](#) (page 2263)

The shortest distance between the focus and the contour

## References

[R509], [R510]

### `tangent_lines(p)`

Tangent lines between  $p$  and the ellipse.

If  $p$  is on the ellipse, returns the tangent line through point  $p$ . Otherwise, returns the tangent line(s) from  $p$  to the ellipse, or None if no tangent line is possible (e.g.,  $p$  inside ellipse).

#### Parameters

**`p`** : Point

#### Returns

**`tangent_lines`** : list with 1 or 2 Lines

#### Raises

**`NotImplementedError`**

Can only find tangent lines for a point,  $p$ , on the ellipse.



## Examples

```
>>> from sympy import Point, Ellipse
>>> e1 = Ellipse(Point(0, 0), 3, 2)
>>> e1.tangent_lines(Point(3, 0))
[Line2D(Point2D(3, 0), Point2D(3, -12))]
```

### See also:

[sympy.geometry.point.Point](#) (page 2202), [sympy.geometry.line.Line](#) (page 2227)

### property `vradius`

The vertical radius of the ellipse.

#### Returns

**vradius** : number

## Examples

```
>>> from sympy import Point, Ellipse
>>> p1 = Point(0, 0)
>>> e1 = Ellipse(p1, 3, 1)
>>> e1.vradius
1
```

### See also:

[hradius](#) (page 2259), [major](#) (page 2261), [minor](#) (page 2262)

**class** `sympy.geometry.ellipse.Circle(*args, **kwargs)`

A circle in space.

Constructed simply from a center and a radius, from three non-collinear points, or the equation of a circle.

#### Parameters

**center** : Point

**radius** : number or SymPy expression

**points** : sequence of three Points

**equation** : equation of a circle

#### Raises

**GeometryError**

When the given equation is not that of a circle. When trying to construct circle from incorrect parameters.

## Examples

```
>>> from sympy import Point, Circle, Eq
>>> from sympy.abc import x, y, a, b
```

A circle constructed from a center and radius:

```
>>> c1 = Circle(Point(0, 0), 5)
>>> c1.hradius, c1.vradius, c1.radius
(5, 5, 5)
```

A circle constructed from three points:

```
>>> c2 = Circle(Point(0, 0), Point(1, 1), Point(1, 0))
>>> c2.hradius, c2.vradius, c2.radius, c2.center
(sqrt(2)/2, sqrt(2)/2, sqrt(2)/2, Point2D(1/2, 1/2))
```

A circle can be constructed from an equation in the form  $a*x**2 + b*y**2 + g*x + h*y + c = 0$ , too:

```
>>> Circle(x**2 + y**2 - 25)
Circle(Point2D(0, 0), 5)
```

If the variables corresponding to  $x$  and  $y$  are named something else, their name or symbol can be supplied:

```
>>> Circle(Eq(a**2 + b**2, 25), x='a', y=b)
Circle(Point2D(0, 0), 5)
```

## See also:

[Ellipse](#) (page 2253), [sympy.geometry.point.Point](#) (page 2202)

## Attributes

radius (synonymous with hradius, vradius, major and minor)	
circumference	
equation	

## property circumference

The circumference of the circle.

## Returns

**circumference** : number or SymPy expression

## Examples

```
>>> from sympy import Point, Circle
>>> c1 = Circle(Point(3, 4), 6)
>>> c1.circumference
12*pi
```

**equation**(*x*='x', *y*='y')

The equation of the circle.

### Parameters

**x** : str or Symbol, optional

Default value is 'x'.

**y** : str or Symbol, optional

Default value is 'y'.

### Returns

**equation** : SymPy expression

## Examples

```
>>> from sympy import Point, Circle
>>> c1 = Circle(Point(0, 0), 5)
>>> c1.equation()
x**2 + y**2 - 25
```

**intersection**(*o*)

The intersection of this circle with another geometrical entity.

### Parameters

**o** : GeometryEntity

### Returns

**intersection** : list of GeometryEntities

## Examples

```
>>> from sympy import Point, Circle, Line, Ray
>>> p1, p2, p3 = Point(0, 0), Point(5, 5), Point(6, 0)
>>> p4 = Point(5, 0)
>>> c1 = Circle(p1, 5)
>>> c1.intersection(p2)
[]
>>> c1.intersection(p4)
[Point2D(5, 0)]
>>> c1.intersection(Ray(p1, p2))
[Point2D(5*sqrt(2)/2, 5*sqrt(2)/2)]
>>> c1.intersection(Line(p2, p3))
[]
```

### property radius

The radius of the circle.

#### Returns

**radius** : number or SymPy expression

### Examples

```
>>> from sympy import Point, Circle
>>> c1 = Circle(Point(3, 4), 6)
>>> c1.radius
6
```

#### See also:

[Ellipse.major](#) (page 2261), [Ellipse.minor](#) (page 2262), [Ellipse.hradius](#) (page 2259), [Ellipse.vradius](#) (page 2269)

### reflect(*line*)

Override GeometryEntity.reflect since the radius is not a GeometryEntity.

### Examples

```
>>> from sympy import Circle, Line
>>> Circle((0, 1), 1).reflect(Line((0, 0), (1, 1)))
Circle(Point2D(1, 0), -1)
```

### scale(*x=1, y=1, pt=None*)

Override GeometryEntity.scale since the radius is not a GeometryEntity.

### Examples

```
>>> from sympy import Circle
>>> Circle((0, 0), 1).scale(2, 2)
Circle(Point2D(0, 0), 2)
>>> Circle((0, 0), 1).scale(2, 4)
Ellipse(Point2D(0, 0), 2, 4)
```

### property vradius

This Ellipse property is an alias for the Circle's radius.

Whereas hradius, major and minor can use Ellipse's conventions, the vradius does not exist for a circle. It is always a positive value in order that the Circle, like Polygons, will have an area that can be positive or negative as determined by the sign of the hradius.

## Examples

```
>>> from sympy import Point, Circle
>>> c1 = Circle(Point(3, 4), 6)
>>> c1.vradius
6
```

## Polygons

**class** sympy.geometry.polygon.**Polygon**(\*args, n=0, \*\*kwargs)

A two-dimensional polygon.

A simple polygon in space. Can be constructed from a sequence of points or from a center, radius, number of sides and rotation angle.

### Parameters

**vertices** : sequence of Points

### Optional parameters

=====

**n** : If > 0, an n-sided RegularPolygon is created. See below.

Default value is 0.

### Raises

#### GeometryError

If all parameters are not Points.

## Notes

Polygons are treated as closed paths rather than 2D areas so some calculations can be be negative or positive (e.g., area) based on the orientation of the points.

Any consecutive identical points are reduced to a single point and any points collinear and between two points will be removed unless they are needed to define an explicit intersection (see examples).

A Triangle, Segment or Point will be returned when there are 3 or fewer points provided.

## Examples

```
>>> from sympy import Polygon, pi
>>> p1, p2, p3, p4, p5 = [(0, 0), (1, 0), (5, 1), (0, 1), (3, 0)]
>>> Polygon(p1, p2, p3, p4)
Polygon(Point2D(0, 0), Point2D(1, 0), Point2D(5, 1), Point2D(0, 1))
>>> Polygon(p1, p2)
Segment2D(Point2D(0, 0), Point2D(1, 0))
>>> Polygon(p1, p2, p5)
Segment2D(Point2D(0, 0), Point2D(3, 0))
```

The area of a polygon is calculated as positive when vertices are traversed in a ccw direction. When the sides of a polygon cross the area will have positive and negative contributions. The following defines a Z shape where the bottom right connects back to the top left.

```
>>> Polygon((0, 2), (2, 2), (0, 0), (2, 0)).area
0
```

When the keyword  $n$  is used to define the number of sides of the Polygon then a RegularPolygon is created and the other arguments are interpreted as center, radius and rotation. The unrotated RegularPolygon will always have a vertex at Point( $r$ , 0) where  $r$  is the radius of the circle that circumscribes the RegularPolygon. Its method *spin* can be used to increment that angle.

```
>>> p = Polygon((0,0), 1, n=3)
>>> p
RegularPolygon(Point2D(0, 0), 1, 3, 0)
>>> p.vertices[0]
Point2D(1, 0)
>>> p.args[0]
Point2D(0, 0)
>>> p.spin(pi/2)
>>> p.vertices[0]
Point2D(0, 1)
```

#### See also:

[sympy.geometry.point.Point](#) (page 2202), [sympy.geometry.line.Segment](#) (page 2233), [Triangle](#) (page 2293)

#### Attributes

area	
angles	
perimeter	
vertices	
centroid	
sides	

#### property angles

The internal angle at each vertex.

#### Returns

**angles** : dict

A dictionary where each key is a vertex and each value is the internal angle at that vertex. The vertices are represented as Points.

## Examples

```
>>> from sympy import Point, Polygon
>>> p1, p2, p3, p4 = map(Point, [(0, 0), (1, 0), (5, 1), (0, 1)])
>>> poly = Polygon(p1, p2, p3, p4)
>>> poly.angles[p1]
pi/2
>>> poly.angles[p2]
acos(-4*sqrt(17)/17)
```

### See also:

[sympy.geometry.point.Point](#) (page 2202), [sympy.geometry.line.LinearEntity.angle\\_between](#) (page 2216)

### **arbitrary\_point**(parameter='t')

A parameterized point on the polygon.

The parameter, varying from 0 to 1, assigns points to the position on the perimeter that is that fraction of the total perimeter. So the point evaluated at  $t=1/2$  would return the point from the first vertex that is  $1/2$  way around the polygon.

#### Parameters

**parameter** : str, optional

Default value is 't'.

#### Returns

**arbitrary\_point** : Point

#### Raises

**ValueError**

When *parameter* already appears in the Polygon's definition.

## Examples

```
>>> from sympy import Polygon, Symbol
>>> t = Symbol('t', real=True)
>>> tri = Polygon((0, 0), (1, 0), (1, 1))
>>> p = tri.arbitrary_point('t')
>>> perimeter = tri.perimeter
>>> s1, s2 = [s.length for s in tri.sides[:2]]
>>> p.subs(t, (s1 + s2/2)/perimeter)
Point2D(1, 1/2)
```

### See also:

[sympy.geometry.point.Point](#) (page 2202)

### **property area**

The area of the polygon.

## Notes

The area calculation can be positive or negative based on the orientation of the points. If any side of the polygon crosses any other side, there will be areas having opposite signs.

## Examples

```
>>> from sympy import Point, Polygon
>>> p1, p2, p3, p4 = map(Point, [(0, 0), (1, 0), (5, 1), (0, 1)])
>>> poly = Polygon(p1, p2, p3, p4)
>>> poly.area
3
```

In the Z shaped polygon (with the lower right connecting back to the upper left) the areas cancel out:

```
>>> Z = Polygon((0, 1), (1, 1), (0, 0), (1, 0))
>>> Z.area
0
```

In the M shaped polygon, areas do not cancel because no side crosses any other (though there is a point of contact).

```
>>> M = Polygon((0, 0), (0, 1), (2, 0), (3, 1), (3, 0))
>>> M.area
-3/2
```

### See also:

[sympy.geometry.ellipse.Ellipse.area](#) (page 2255)

### bisectors(*prec=None*)

Returns angle bisectors of a polygon. If *prec* is given then approximate the point defining the ray to that precision.

The distance between the points defining the bisector ray is 1.

## Examples

```
>>> from sympy import Polygon, Point
>>> p = Polygon(Point(0, 0), Point(2, 0), Point(1, 1), Point(0, 3))
>>> p.bisectors(2)
{Point2D(0, 0): Ray2D(Point2D(0, 0), Point2D(0.71, 0.71)),
 Point2D(0, 3): Ray2D(Point2D(0, 3), Point2D(0.23, 2.0)),
 Point2D(1, 1): Ray2D(Point2D(1, 1), Point2D(0.19, 0.42)),
 Point2D(2, 0): Ray2D(Point2D(2, 0), Point2D(1.1, 0.38))}
```

### property bounds

Return a tuple (xmin, ymin, xmax, ymax) representing the bounding rectangle for the geometric figure.