Measurement Laboratory at Home - Exercise 6

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This exercise focuses on the utilization of the UAI to measure the electrical impedance of a specific circuit or device.It builds upon concepts introduced in **Exercise 4**, where electrical impedance was explained as an extension of resistance relevant to alternating current(AC) circuits and devices. It's essential to note that impedance is frequency-specific and typically measured over a particular frequency range.

At its core, impedance represents the total opposition to AC in an electrical circuit, comprising resistance, capacitive reactance, and inductive reactance, which follows the form $Z = R + jX$, where R symbolizes resistance, and X stands for reactance. When given capacitance(C) or inductance(L), reactance can be calculated as $X_C = \frac{1}{j\omega C}$ and $X_L = j\omega L$, where the angular frequency $\omega = 2\pi f$.

Now, build the impedance measurement circuit as shown in the Figure 1. $R_t = 10k\Omega$, $C_t = 100nF$, $R_m, 1 = 470\Omega$ and $R_{m,2} = 33k\Omega$ will be used. The figure 2 and Figure 3 is the circuit on the breadboard that I placed and its drwaing.

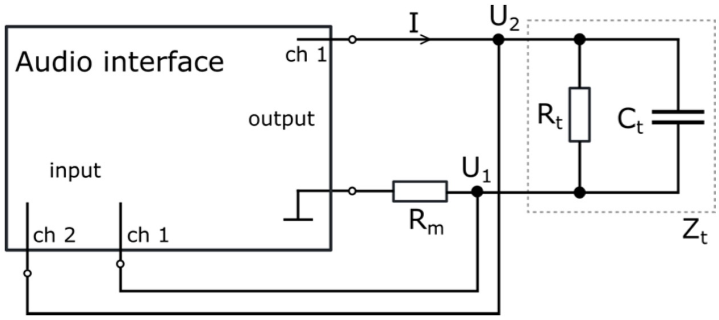

Figure 1: Schematic of the test circuit 1 for impedance measurement in the exercise sheet

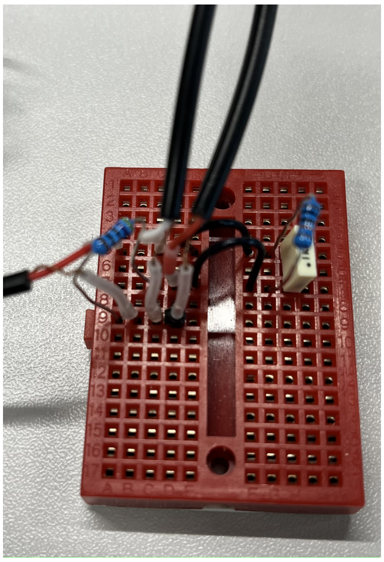

Figure 2: This Circuit is placed on the breadboard

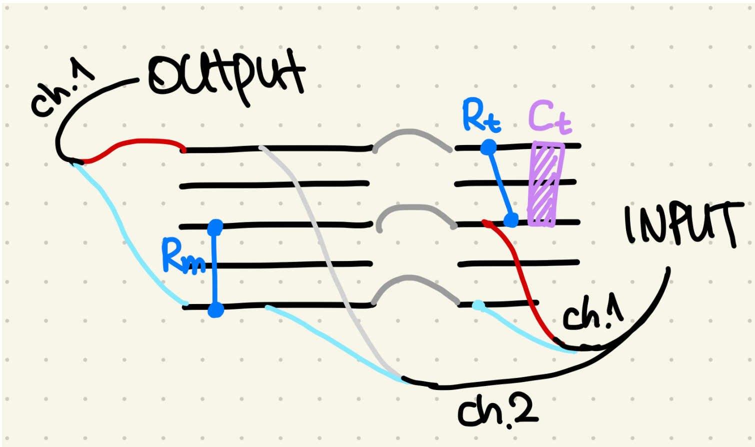

Figure 3: On the breadboard placed circuit was expressed.

1 Assignment 1

1.1 Initial settings

to measure the electrical impedance of Z_t, the MATLAB script **measure_transfer_function_multisine.m** will be used to generate a multi sine signal. And in below table 1 there is another settings of parameters.

parameters	value
frequencies(**N_freqs**)	179
f_start	55 Hz
f_stop	22 kHz
Duration(T)	5s
block size	2^{14}
t_start	0.5
t_stop	4.8
test circuit(R_m)	470 Ω

Table 1: A table with all initial parameters during Assignment 1

1.2 Calculating the averaged amplitude spectra

To calculate the Fourier Transform and plot the amplitude spectrum for $U_1(f)$ and $U_2(f)$, first it is need to process the recorded signal (U_1 and U_2) using the Fast Fourier Transform (FFT). Then the magnitude and phase of these spectra can be plotted. In the script calculates the FFT for U1 and U2 and then plots the amplitude spectrum in dB and phase spectrum in degrees. Then, replace **rec(:, 1)** and **rec(:, 2)** with the actual recorded data for U_1 and U_2 respectively.

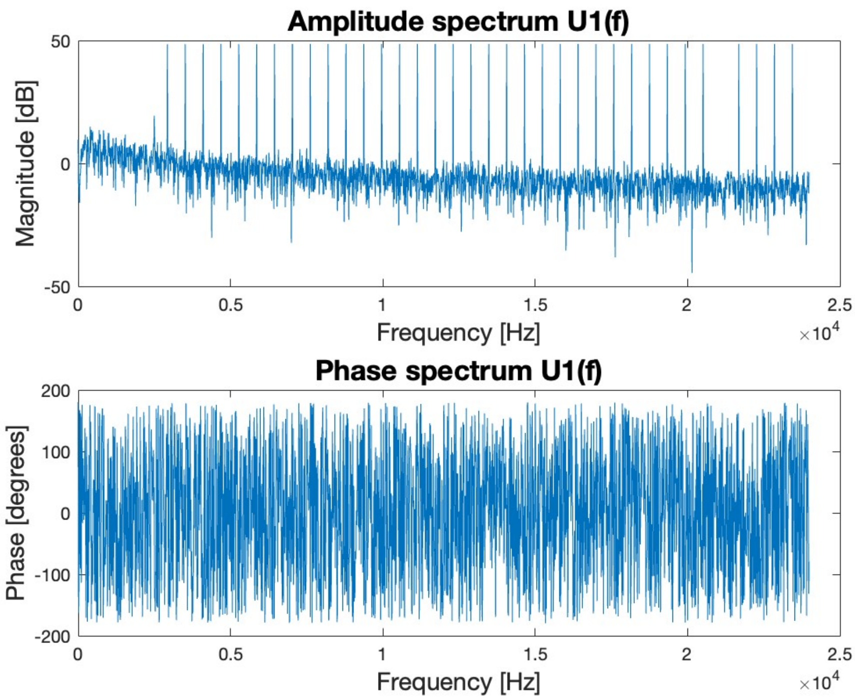

Figure 4: the averaged amplitude spectra $U_1(f)$

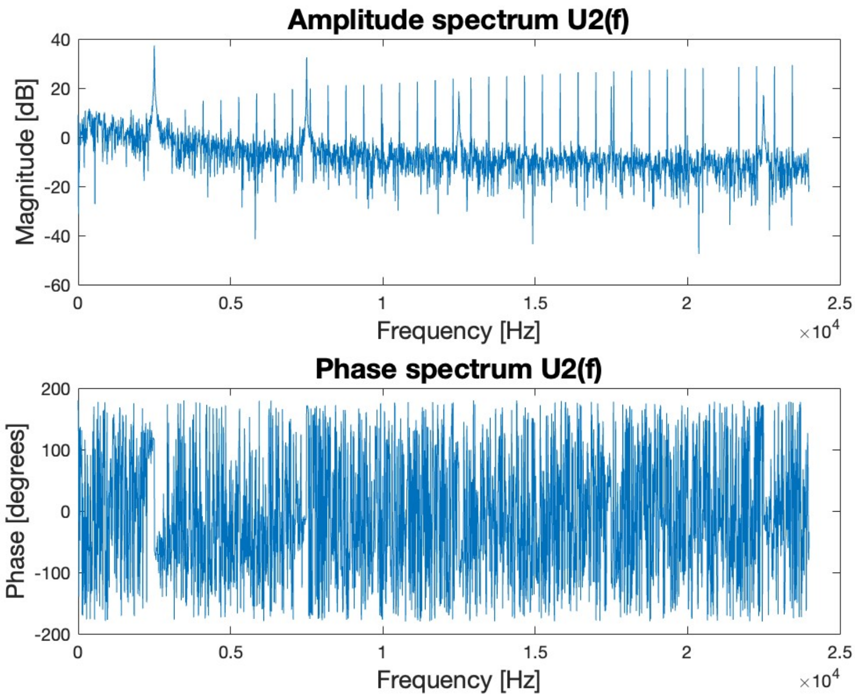

Figure 5: the averaged amplitude spectra $U_2(f)$

Figure 4 and 5 are depicted the plot of the averaged amplitude spectras.

1.3 Determine the Impedance Z_t

In this experimental setup, two voltages are measured, U_1 and U_2. U_2 is the voltage across the total impedance $Z_t + R_m$, while U_1 is the voltage across the resistor R_m.

From Ohm's law, it is trivial that, $U = I \cdot R$. Thus, we can find the current I_z through the total impedance,i.e. $Z_t + R_m$, using the measured voltage U_1 and the known resistor value R_m. And then, the voltage across U_z and the impedance of Z_t can be calculated like below.

$$I_z = \frac{U_1}{R_m} \tag{1}$$

$$U_z = U2 - U1 \tag{2}$$

$$Z_t = \frac{Uz}{Iz} \tag{3}$$

To compute these in MATLAB, we first need to find the FFT of U_1 and U_2, which I have already done in the subsection **1.2**, and then use those to find the FFT of I_z and U_z, and finally Z_t. In the figure 6 the magnitude and phase of impedance Z_t can be showned.

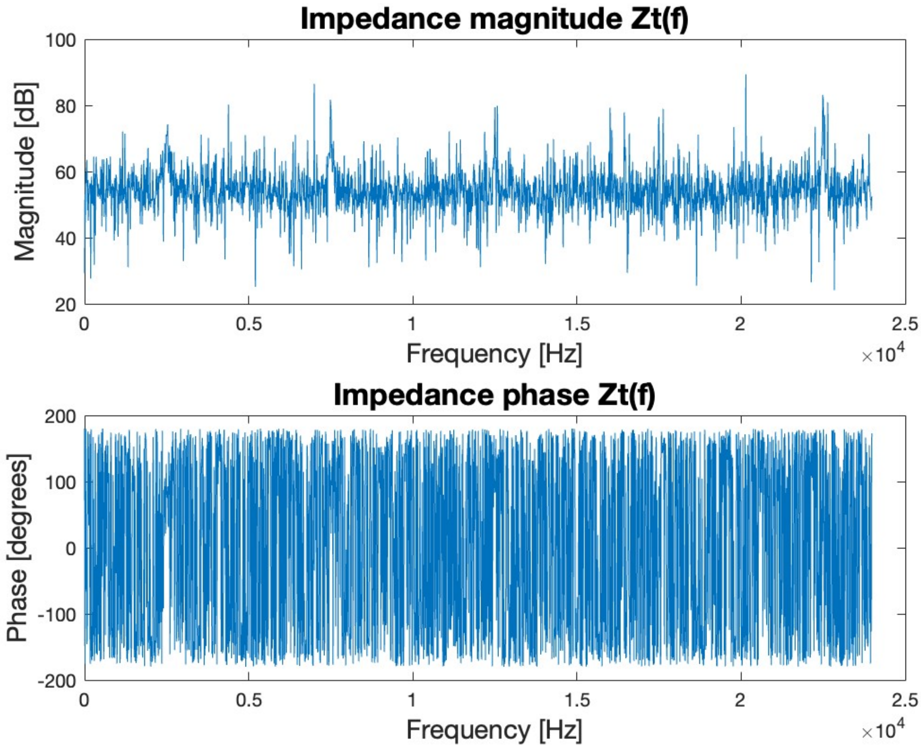

Figure 6: the magnitude and phase of the impedance Z_t in our circuit

1.4 Real parts and Imaginary parts of Impedance Z_t

The real and imaginary parts of the impedance Z_t can be plotted using the **real** and **imag** functions in MATLAB. It will create a new figure and plot the real and imaginary parts of Zt as a function of frequency. The **real**(Z_t) function extracts the real resistive part) of the impedance, and **imag**(Z_t) extracts the imaginary reactive part), in this figure 7.

The behavior of the real and imaginary parts of the impedance Z_t of a parallel RC circuit, like the one we performed, changes with frequency. In general, the behavior of these components can be understood by the properties of resistors and capacitors in an AC circuit.

The real part of the impedance, associated with resistance, remains relatively constant across the frequency spectrum in an ideal world, because the resistance value doesn't depend on the frequency. This constant value should be roughly equal to the resistance of the resistor in your circuit. However, at

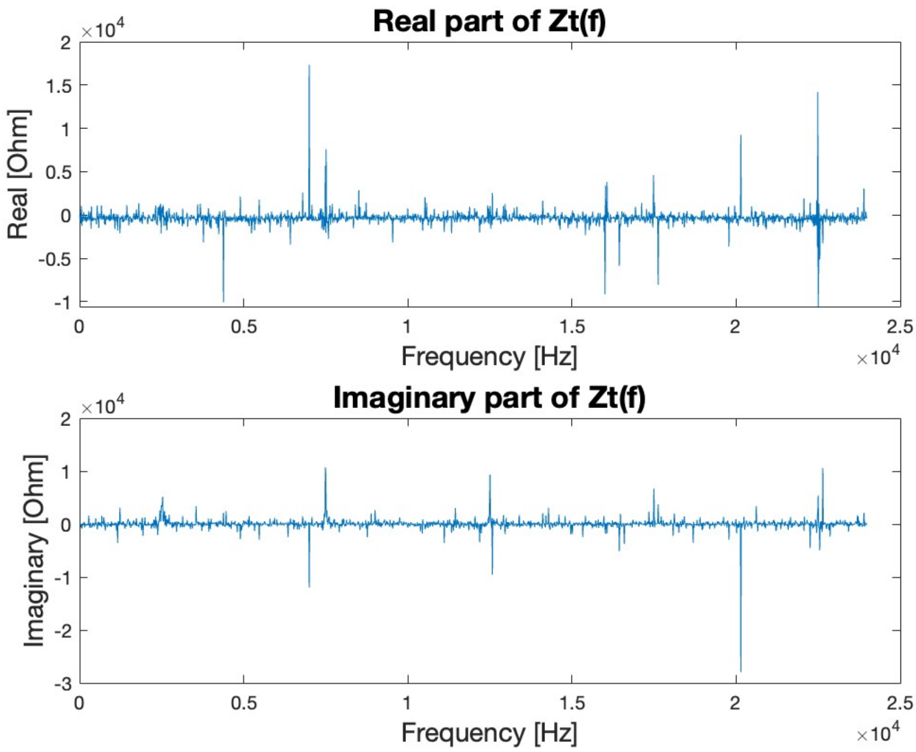

Figure 7: Plotting the real and imaginary parts of impedance Z_t as a function of frequency using the function **real** and **imag** on MATLAB

high frequencies, the effects of parasitic resistances, inductances, and capacitances can cause deviations from this ideal behavior. On the other hand, The imaginary part of the impedance, associated with reactance, changes with frequency. The reactance of the capacitor in a circuit is given by $-\frac{1}{\omega C}$. This means that as frequency increases, the magnitude of the capacitive reactance decreases (i.e, it means, less negative), and as frequency decreases, the magnitude of the capacitive reactance increases (i.e, it means, more negative).

1.5 Determine and Plot value of R_t and C_t

The impedance Z_t is a parallel connection of a resistor (R_t) and a capacitor (C_t). For such a parallel circuit, it can be more useful to work with admittances, because admittances of components in parallel simply add together. The admittance **Y** is the reciprocal of the impedance **Z**.

First, define $Y_t = \frac{1}{Z_t}$ as the total admittance, $Y_r = \frac{1}{R_t}$ as the admittance of the resistor, and $Y_c = j\omega C_t$ as the admittance of the capacitor (j is the imaginary unit, ω is the angular frequency, and C_t is the capacitance). In a parallel circuit, the total admittance is the sum of the individual admittances:

$$Y_t = Y_r + Y_c \tag{4}$$

Since $Y_t = \frac{1}{Z_t}$, Yt can be computed from the measured Z_t values. Y_t will also be a complex number, with a real part corresponding to the conductance ($G_t = Re\{Y_t\}$) and an imaginary part corresponding to the susceptance ($Bt = Im\{Y_t\}$). The conductance is the reciprocal of the resistance ($G_t = \frac{1}{R_t}$), and the susceptance is the change in admittance due to the capacitor ($B_t = \omega C_t$). Therefore, R_t and C_t can be found as below. The graph of the R_t and C_t are in the Figure 8.

$$R_t = \frac{1}{G_t} \tag{5}$$

$$C_t = \frac{B_t}{\omega} \tag{6}$$

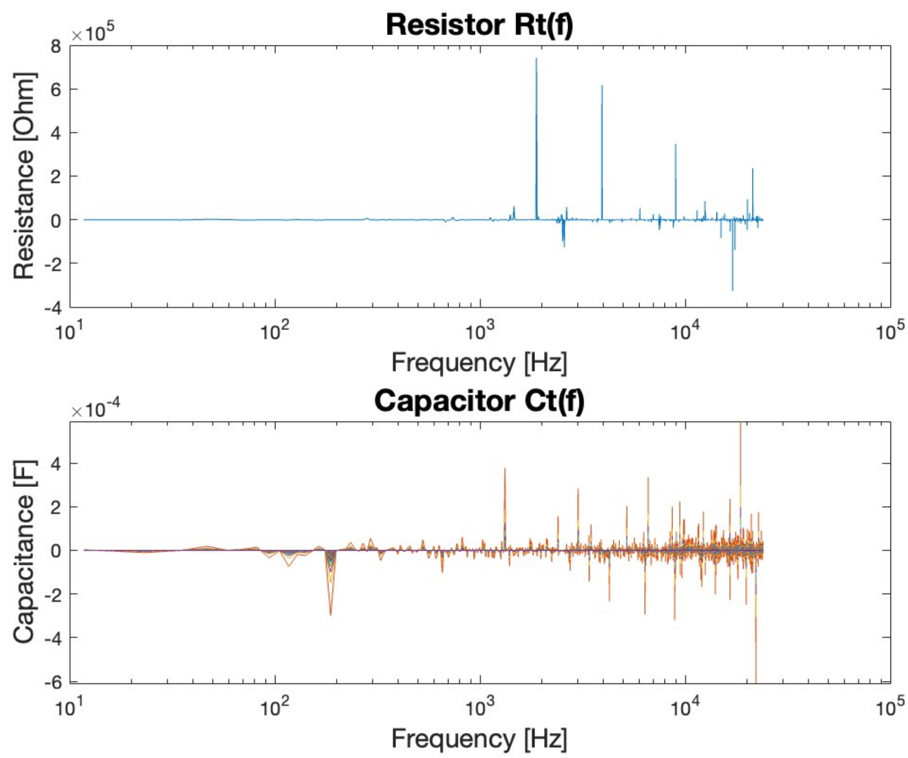

Figure 8: The values of R_t and C_t are plotted from the impedance Z_t

```
Command Window
The average value of Rt over all frequencies is 171.04 Ohm.
The average value of Ct over all frequencies is NaN F.
The average value of Ct over all frequencies is -2.09e-07 F.
The average value of Ct over all frequencies is -1.04e-07 F.
The average value of Ct over all frequencies is -6.96e-08 F.
The average value of Ct over all frequencies is -5.22e-08 F.
The average value of Ct over all frequencies is -4.18e-08 F.
The average value of Ct over all frequencies is -3.48e-08 F.
The average value of Ct over all frequencies is -2.98e-08 F.
The average value of Ct over all frequencies is -2.61e-08 F.
The average value of Ct over all frequencies is -2.32e-08 F.
The average value of Ct over all frequencies is -2.09e-08 F.
The average value of Ct over all frequencies is -1.90e-08 F.
The average value of Ct over all frequencies is -1.74e-08 F.
The average value of Ct over all frequencies is -1.61e-08 F.
The average value of Ct over all frequencies is -1.49e-08 F.
The average value of Ct over all frequencies is -1.39e-08 F.
The average value of Ct over all frequencies is -1.31e-08 F.
The average value of Ct over all frequencies is -1.23e-08 F.
The average value of Ct over all frequencies is -1.16e-08 F.
The average value of Ct over all frequencies is -1.10e-08 F.
The average value of Ct over all frequencies is -1.04e-08 F.
The average value of Ct over all frequencies is -9.95e-09 F.
The average value of Ct over all frequencies is -9.49e-09 F.
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The average value of Ct over all frequencies is -1.02e-10 F.
The average value of Ct over all frequencies is -1.02e-10 F.
The average value of Ct over all frequencies is -1.02e-10 F.
The average value of Ct over all frequencies is -1.02e-10 F.
The average value of Ct over all frequencies is -1.02e-10 F.
The average value of Ct over all frequencies is -1.02e-10 F.
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Figure 9: Some value of R_t and C_t that can be showned in Command Window on MATLAB. The average value of R_t over all frequencies is 171.04Ω and the average value of C_t is started about $-2.09 \cdot 10^{-7}F$ and eventually reaches at about $-1.02 \cdot 10^{-10}F$.

1.6 Comparing $R_m = 33k\Omega$

Now, repeat all the process again. And below there are some results and plots for $R_m = 33k\Omega$ instead of 470Ω. In below table 2 the values are compared.

Based on the results of average R_t values, the measurement with $R_m = 33k\Omega$ gives an average value that is significantly larger compared to the measurement with $R_m = 470\Omega$. However, to definitively determine which Rm value gives results closest to the actual values of R_t and C_t, the average C_t value from each measurement would be also considered and and compared both R_t and C_t to their real values.

To evaluate the accuracy of the measurement,the measured values of R_t will be compared to the actual value of $10k\Omega$, typically. The difference between the measured and actual values gives an indication of the measurement error. Also, for $R_m = 470\Omega$, the measurement error would be

values	$R_m = 470\Omega$	$R_m = 33k\Omega$
Average Value R_t	171.04 Ohm	30240.34 Ohm
first Average Value C_t	$-2.09 \cdot 10^{-7}$	$-1.10 \cdot 10^{-8}$
last Average Value C_t	$-1.02 \cdot 10^{-10}$	$-5.39 \cdot 10^{-12}$

Table 2: A table with comparing measurement Resistor $R_{m,1} = 470\Omega$ and $R_{m,2} = 33k\Omega$

$|171.04 - 10,000| = 9828.96\Omega$. On the other hands, for $R_m = 33k\Omega$, the error would be $|30240.34 - 10,000| = 20240.34\Omega$. In conclusion, the measurement with $R_m = 470\Omega$ has a smaller error and thus is closer to the actual value of R_t.

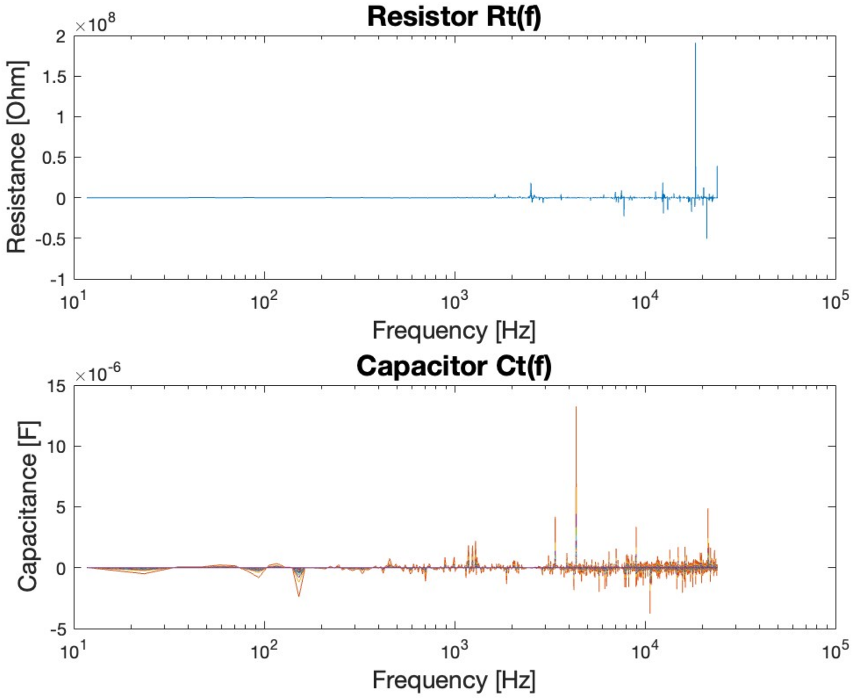
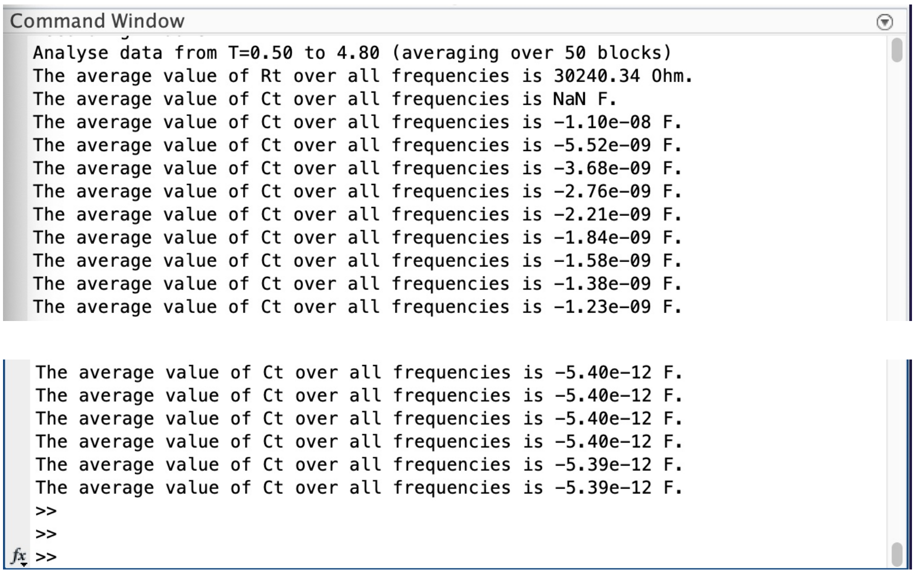


Figure 10: The values of R_t and C_t are plotted from the impedance Z_t with the measurement Resistor $R_m = 33000\Omega$.



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Command Window
Analyse data from T=0.50 to 4.80 (averaging over 50 blocks)
The average value of Rt over all frequencies is 30240.34 Ohm.
The average value of Ct over all frequencies is NaN F.
The average value of Ct over all frequencies is -1.10e-08 F.
The average value of Ct over all frequencies is -5.52e-09 F.
The average value of Ct over all frequencies is -3.68e-09 F.
The average value of Ct over all frequencies is -2.76e-09 F.
The average value of Ct over all frequencies is -2.21e-09 F.
The average value of Ct over all frequencies is -1.84e-09 F.
The average value of Ct over all frequencies is -1.58e-09 F.
The average value of Ct over all frequencies is -1.38e-09 F.
The average value of Ct over all frequencies is -1.23e-09 F.

The average value of Ct over all frequencies is -5.40e-12 F.
The average value of Ct over all frequencies is -5.40e-12 F.
The average value of Ct over all frequencies is -5.40e-12 F.
The average value of Ct over all frequencies is -5.40e-12 F.
The average value of Ct over all frequencies is -5.39e-12 F.
The average value of Ct over all frequencies is -5.39e-12 F.
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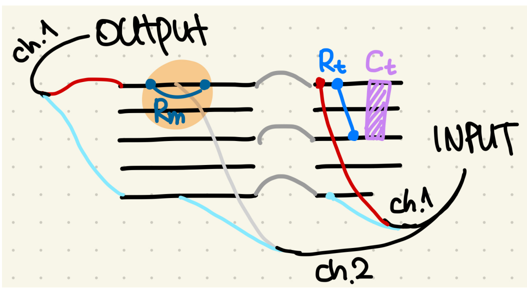
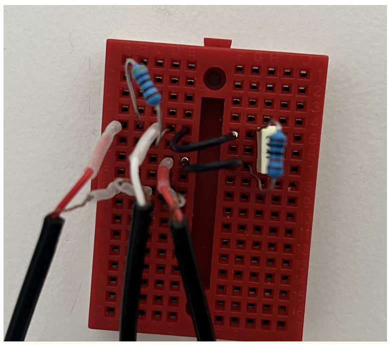
Figure 11: Some value of R_t and C_t with the measurement Resistor $R_m = 33000\Omega$ that can be showned in Command Window on MATLAB. The average value of R_t over all frequencies is 30240.34Ω and the average value of C_t is started about $-1.10 \cdot 10^{-8} F$ and eventually reaches at about $-5.39 \cdot 10^{-12} F$.

2 Assignment 2

The basic parameter settings were leaved as they are, and this time connect R_m between channel 1 of the output and channel 2 of the input on the breadboard. below Figure 12 is about the circuit for Assignment 2 on the breadboard and its simple view.



Circuit on the breadboard



Circuit on the Breadboard

Figure 12: Breadboard component connection status and its simple diagram

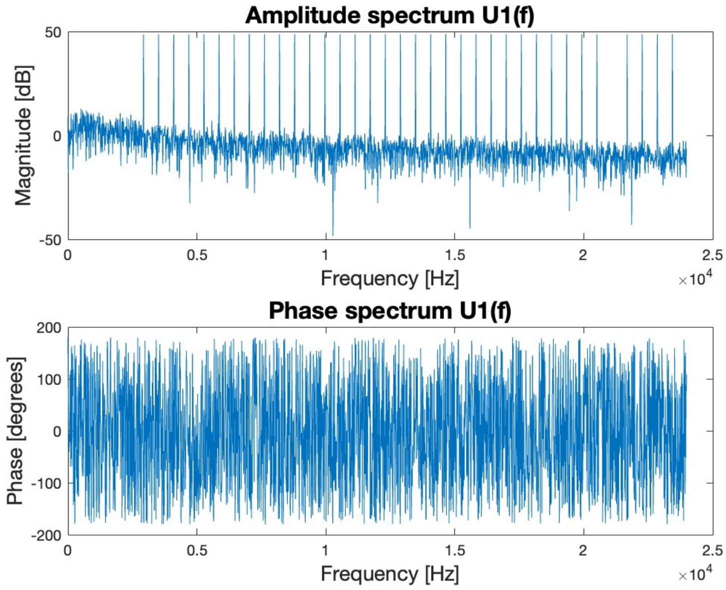


Figure 13: the averaged amplitude spectra $U_1(f)$

2.1 Calculating the averaged amplitude spectra

The measurement Resistor R_m placed as 470Ω. Figure 13 and 14 are depicted the plot of the averaged amplitude spectra.

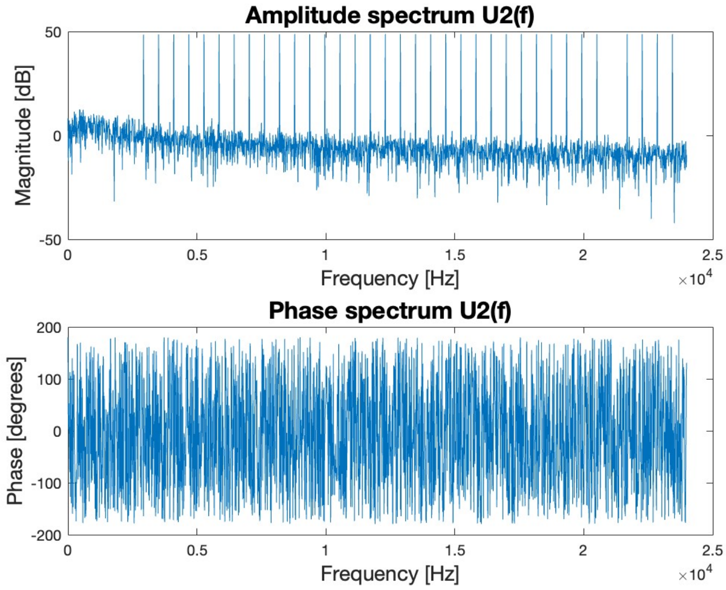


Figure 14: the averaged amplitude spectra $U_2(f)$

2.2 Determine the Impedance Z_t

Determining the impedance Z_t is as same as Assignment 1 because it was not changed. In the Figure 15 the magnitude and phase of impedance Z_t was expressed.

2.3 Real parts and Imaginary parts of Impedance Z_t

Below, there is a plot of the values of R_t and C_t from the impedance Z_t over their frequency. Figure 16 is depicted the plot of real and imaginary parts of Z_t.

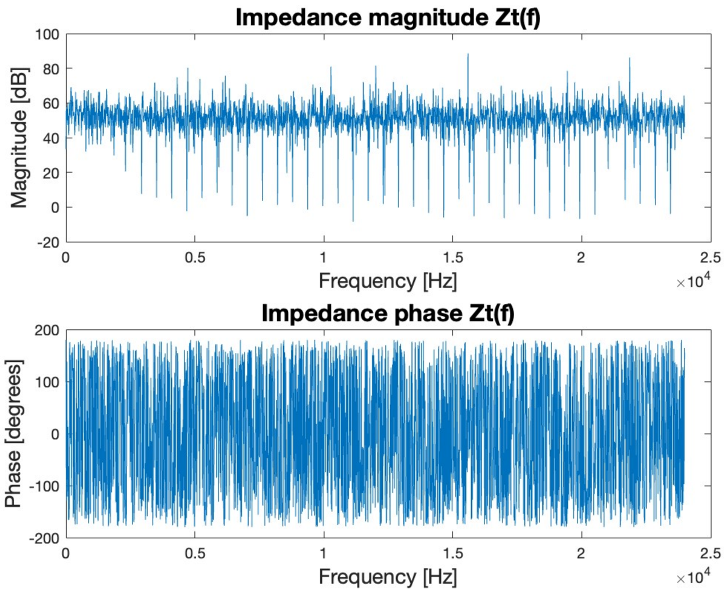


Figure 15: the magnitude and phase of the impedance Z_t in our circuit

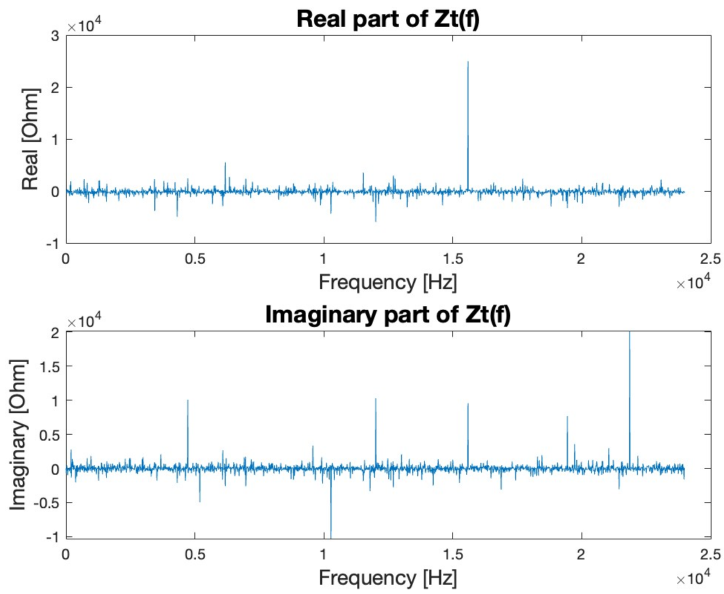


Figure 16: Plotting the real and imaginary parts of impedance Z_t as a function of frequency using the function **real** and **imag** on MATLAB

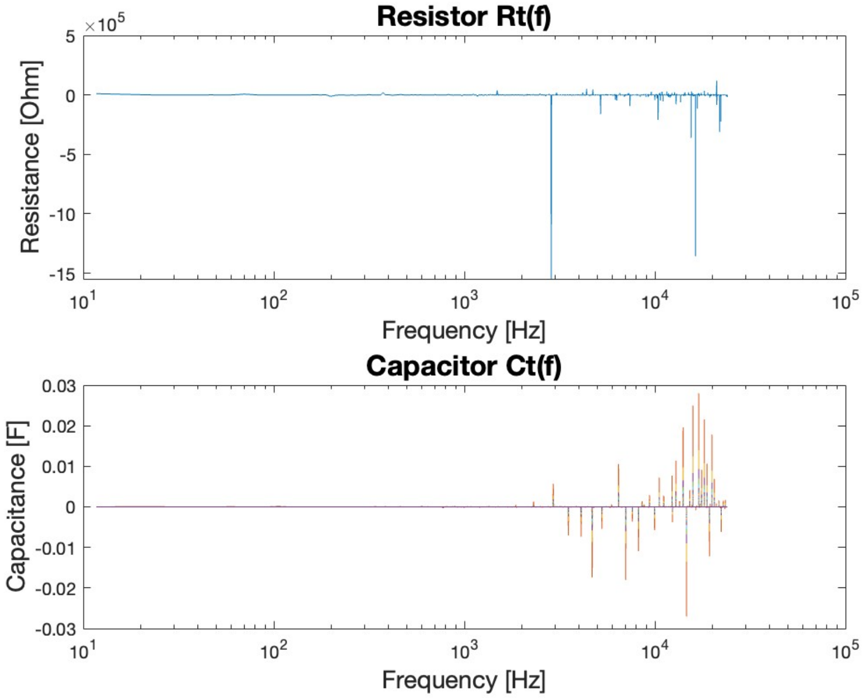


Figure 17: The values of R_t and C_t are plotted from the impedance Z_t

2.4 Determine and Plot value of R_t and C_t

In the figure 17 the values of R_t and C_t from the impedance are expressed as a function of frequency. The average value of R_t is $-2480.90\ \Omega$ on this experience. The value of C_t starts about $3.81 \cdot 10^{-5}$ and decreases and eventually reaches to about $1.86 \cdot 10^{-8}\ \Omega$. Figure 18 is described the specific value of the resistor R_t and the capacitor C_t. This results is, however, different with another measurement resistor.

Now, change the measurement resistor $R_m = 33\ k\Omega$. If we change the measurement resistor, the average results for R_t and C_t is different. From Figure 19 to Figure 20 is plot of its values and capture of values for the measurement resistor $R_m = 33k\Omega$.

```
Command Window
The average value of Rt over all frequencies is -2480.90 Ohm.
The average value of Ct over all frequencies is NaN F.
The average value of Ct over all frequencies is 3.81e-05 F.
The average value of Ct over all frequencies is 1.91e-05 F.
The average value of Ct over all frequencies is 1.27e-05 F.
The average value of Ct over all frequencies is 9.53e-06 F.
The average value of Ct over all frequencies is 7.62e-06 F.
The average value of Ct over all frequencies is 6.35e-06 F.
The average value of Ct over all frequencies is 5.44e-06 F.
The average value of Ct over all frequencies is 4.76e-06 F.
The average value of Ct over all frequencies is 4.23e-06 F.
The average value of Ct over all frequencies is 3.81e-06 F.

The average value of Ct over all frequencies is 1.87e-08 F.
The average value of Ct over all frequencies is 1.87e-08 F.
The average value of Ct over all frequencies is 1.87e-08 F.
The average value of Ct over all frequencies is 1.87e-08 F.
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The average value of Ct over all frequencies is 1.86e-08 F.
The average value of Ct over all frequencies is 1.86e-08 F.
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Figure 18: Some value of R_t and C_t for $R_m = 470\ \Omega$ that can be showned in Command Window on MATLAB. The average value of R_t over all frequencies is -2480.90Ω and the average value of C_t is started about $3.81 \cdot 10^{-5} F$ and eventually reaches at about $1.86 \cdot 10^{-8} F$.

The measured values of R_t will be compared to the actual value of $10k\Omega$ and thus, the difference between the measured and actual values gives an indication of the measurement error for $R_m = 470\Omega$ would be $|-2480.90 - 10,000| = 12480.90\Omega$. On the other hands, for $R_m = 33k\Omega$, the error would be $|-130951.51 - 10,000| = 140951.51\Omega$. In conclusion, the measurement with $R_m = 33k\Omega$ has a smaller error and thus is closer to the actual value of R_t.

2.5 Advantages and Disadvantages of each configuration

The configuration of Assignment # 1 and Assignment # 2 has some advantages and disadvantages each other.

Here is an advantages and disadvantages of #1: Assignment 1.

- Advantage: The initial configuration measures the voltage across Rm directly U_1, which can simplify calculations and reduce error. Since Z_t is directly tied to U_2, it's straightforward to derive its value.
- Disadvantage: The presence of R_m in series with the audio interface output might slightly alter the characteristics of the signal, particularly at high frequencies. This effect could be noticeable if R_m is a significant portion of the output impedance of the audio interface. Also, if the

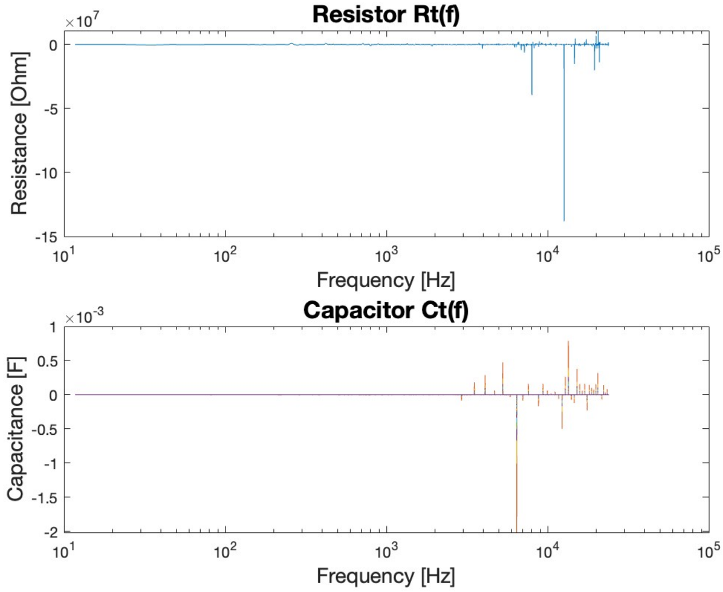
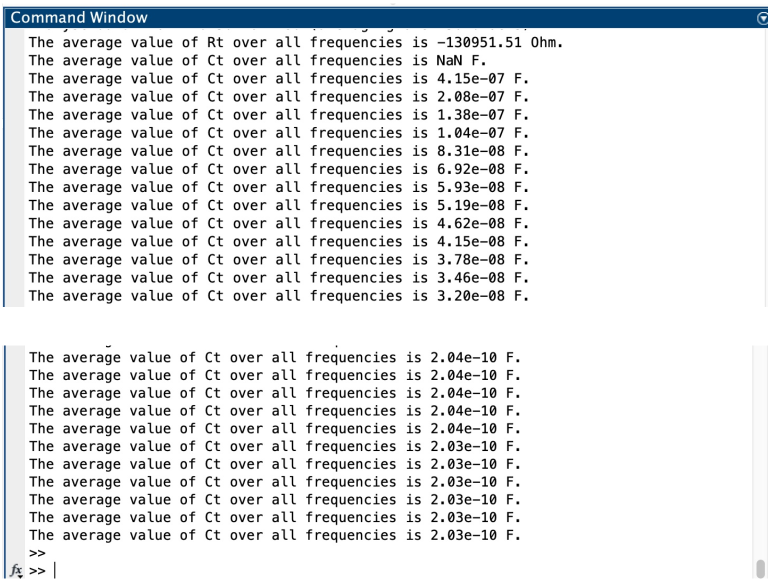


Figure 19: The values of R_t and C_t from impedance Z_t as a function of frequency, with the resistor $R_m = 33\ k\Omega$.



```
Command Window
The average value of Rt over all frequencies is -130951.51 Ohm.
The average value of Ct over all frequencies is NaN F.
The average value of Ct over all frequencies is 4.15e-07 F.
The average value of Ct over all frequencies is 2.08e-07 F.
The average value of Ct over all frequencies is 1.38e-07 F.
The average value of Ct over all frequencies is 1.04e-07 F.
The average value of Ct over all frequencies is 8.31e-08 F.
The average value of Ct over all frequencies is 6.92e-08 F.
The average value of Ct over all frequencies is 5.93e-08 F.
The average value of Ct over all frequencies is 5.19e-08 F.
The average value of Ct over all frequencies is 4.62e-08 F.
The average value of Ct over all frequencies is 4.15e-08 F.
The average value of Ct over all frequencies is 3.78e-08 F.
The average value of Ct over all frequencies is 3.46e-08 F.
The average value of Ct over all frequencies is 3.20e-08 F.

The average value of Ct over all frequencies is 2.04e-10 F.
The average value of Ct over all frequencies is 2.04e-10 F.
The average value of Ct over all frequencies is 2.04e-10 F.
The average value of Ct over all frequencies is 2.04e-10 F.
The average value of Ct over all frequencies is 2.04e-10 F.
The average value of Ct over all frequencies is 2.03e-10 F.
The average value of Ct over all frequencies is 2.03e-10 F.
The average value of Ct over all frequencies is 2.03e-10 F.
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The average value of Ct over all frequencies is 2.03e-10 F.
The average value of Ct over all frequencies is 2.03e-10 F.
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Figure 20: Some value of R_t and C_t of resistor $R_m = 33k\Omega$ that can be showned in Command Window on MATLAB. The average value of R_t over all frequencies is -130951.51 Ω and the average value of C_t is started about $4.15 \cdot 10^{-7}F$ and eventually reaches at about $2.03 \cdot 10^{-10}F$.

impedance of Z_t is quite high, the voltage across it U_2 might be too small for accurate measurement.

Now for #2: Assignment 2.

- Advantage: By moving R_m, the combined voltage across Z_t and R_m at U_1 was measured, which could potentially increase the voltage level for measurement and improve accuracy if Z_t has a high impedance. This configuration might also reduce the loading effect on the audio interface output.
- Disadvantage: The measurements become more complex as U_1 is now the voltage across both R_m and Z_t, and the voltage across R_m need to be subtracted from U_1 to obtain the voltage across Z_t.