Measurement Laboratory at Home - Exercise 3

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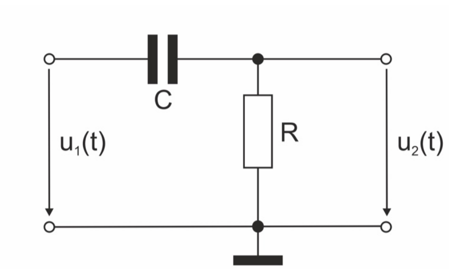
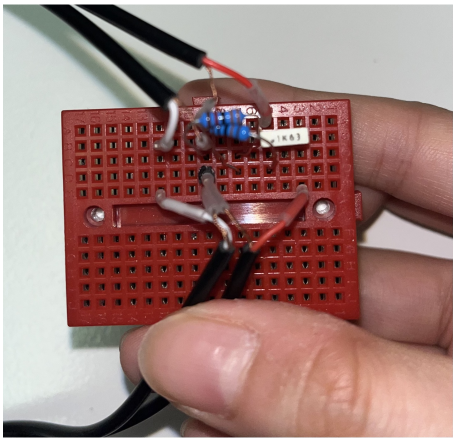
This report discusses the importance of measuring the frequency response of a system, specifically a passive RC-filter. By determining the system's transfer function, we can understand its behavior and calculate its response to various input signals. The report suggests using Kirchhoff's laws or Laplace transformation to calculate the transfer function and proposes using a UAI to measure the system's response at different frequencies.

As shown in Exercise Topic 3, assume the following RC circuit. Resistor is R = 2 kΩ and Capacitor is C = 100 nF. Assume i is the current flowing in the circuit.



Schema RC Circuit



RC Breadboard

Figure 1: Circuit Diagram and Breadboard of RC Circuit

1 Assignment 1

1.1 Calculate Theoretical Transfer Function of the RC Filter

The transfer function of a first-order RC (Resistor-Capacitor) high-pass filter can be derived using the Laplace transform. The transfer function in the frequency domain, H(f), and in the Laplace domain, H(s), are related to each other.

To express the output voltage $u_2(t)$ using the Laplace variable s, we need to take the Laplace transform of the output voltage function in the time domain. The Laplace transform of a function u(t) is denoted by U(s). In this case, we want to find $U_2(s)$, which represents the Laplace transform of the output voltage u2(t).

The voltage across the resistor is:

$$u_R(t) = R \cdot i(t), \tag{1}$$

and the voltage across the capacitor is:

$$u_C(t) = \frac{1}{C} \int i(t)\, dt \tag{2}$$

where $i(t)$ is the current flowing through the circuit.

Now i have:

$$u_1(t) = u_R(t) + u_C(t) \tag{3}$$

$$u_1(t) = R \cdot i(t) + \frac{1}{C} \int i(t)\, dt \tag{4}$$

Taking the Laplace transform of both sides.

$$U_1(s) = I(s) \left(R + \frac{1}{Cs} \right) \tag{5}$$

Then we can get the transfer function $H(s) = \frac{U_2(s)}{U_1(s)}$:

$$H(s) = \frac{U_2(s)}{U_1(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{RCs}{1 + RCs} \tag{6}$$

To get the transfer function $H(f)$ related to frequency, $s = j2\pi f$ can be replaced into the above equation, where j is the imaginary unit.

$$H(f) = H(s = j2\pi f) = \frac{1}{j2\pi f R + \frac{1}{C}} \tag{7}$$

Let substitute the given values and simplify it:

$$H(s) = \frac{(2000\,\Omega)(100 \times 10^{-9}\,\mathrm{F})s}{1 + (2000\,\Omega)(100 \times 10^{-9}\,\mathrm{F})s} \tag{8}$$

$$H(s) = \frac{200 \times 10^{-6} s}{1 + 200 \times 10^{-6} s} \tag{9}$$

To further simplify the transfer function, we can multiply the numerator and denominator by 10^6 to eliminate the decimal points:

$$H(s) = \frac{s}{5000 + s} \tag{10}$$

1.2 RC Filter

Type and Order of the Filter

This RC Filter is first ordered (passive) RC high pass filter.

First of all, the passive filter consists of only passive elements like resister and capacitor. It will not use any external power source or amplification components. This RC Filter is one type of passive filter. Therefore, it is passive RC high pass filter.

Second, first order high pass filter consists of only one capacitor. Thus, this type of filter has a transfer function of the first order. It means if you derive an equation in s-domain, the maximum power of 's' is one.

Cut-off Frequency

The cutoff frequency is defined as a frequency that creates a boundary between pass band and stop band. For a high pass filter, if the signal frequency is more than the cutoff frequency, then it will allow passing the signal. And if the signal frequency is less than the cutoff frequency, then it will attenuate the signal.

$$F_c = \frac{1}{2\pi RC} \tag{11}$$

Substitute the value:

$$F_c = \frac{1}{2\pi \cdot 2000 \cdot 10^{-7}} = \frac{\pi}{10000} \tag{12}$$

The absolute of the magnitude of the transfer function is:

$$|H(f)| = \left| \frac{1 - j\omega RC}{1 + (\omega RC)^2} \right| \tag{13}$$

To find the cutoff frequency, we need to find the value of ω at which $|H(f)|$ is equal to -3 dB or 0.707 (in magnitude).

Gain cut off frequency

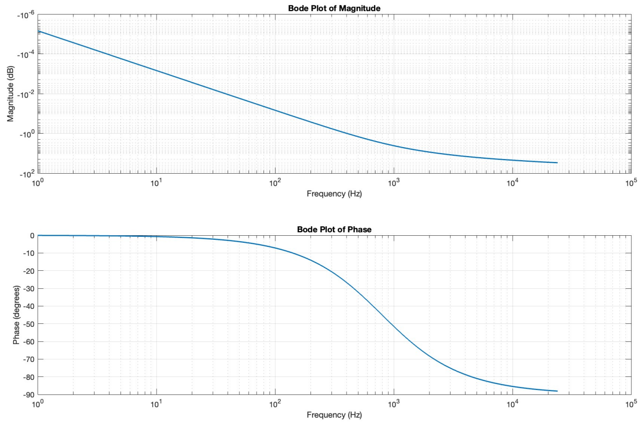
To determine the gain of the filter, we can analyze the magnitude of the transfer function at low frequencies ($\omega \to 0$). At low frequency the transfer function can be approximated as: $|H(f)| \approx 1$. Thus, the gain is approximately 1, 0 dB in low frequency. Reading from the Figure 2 bode plot, where the attenuation is 3 dB, the cut off frequency of the filter is approximately 796.18Hz

1.3 Bode Plot for RC Filter

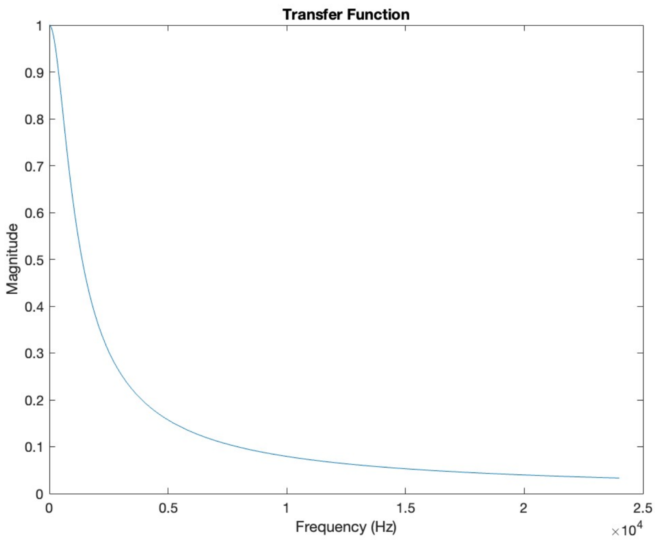
The parameters of an RC circuit defined and calculated frequency response of the filter. The result is plotted as a Bode plot, with the magnitude shown in decibels (dB) and the phase shown in degrees.

The simplified transfer Function is:

$$H(s) = \frac{s}{s + 5000} \tag{14}$$



Bode Plot for Magnitude and Phase



Transfer Function

Figure 2: Bode plot for Magnitude and Phase Transfer function of RC Circuit

2 Assignment 2

Now set the sampling rate at 48 kHz and the buffer size at 1024. To determine the impulse response of the RC filter, generate an impulse signal, apply it as the input u_1(t) to the filter and record the corresponding output, u_2(t).

2.1 Plot u_1(t) and u_2(t)

The magnitude and phase response of the filter are computed in dB and degrees, respectively. The input signal u_1(t) and output signal u_2(t) are plotted in separate subplots. This task focuses on plotting the input and output signals. The code extracts the input signal u_1(t) and output signal u_2(t) from the recorded data and plots them in separate subplots using the time vector $t_{recData}$. Figure 3 can be shown.

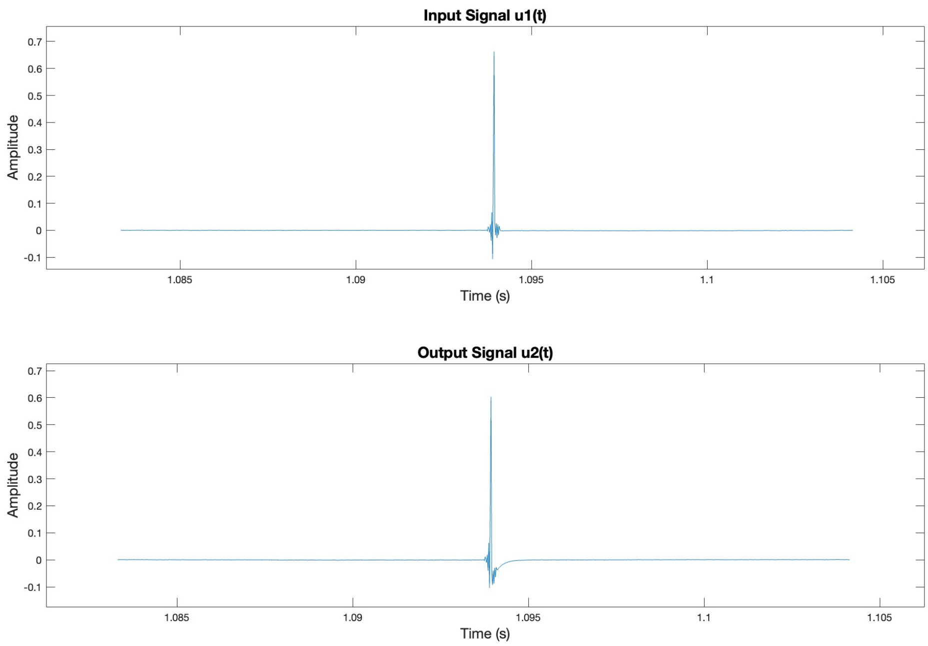


Figure 3: u_1(t) and u_2(t)

2.2 Single-Sided Amplitude Spectra for U1(f) and U2(f)

To calculate the single-sided amplitude spectra for U1(f) and U2(f) and plot them, the Fast Fourier Transform (FFT) algorithm can be used. Figure 4 can be shown.

2.3 Transfer Function H(f) and its Bode Plot

These tasks involve calculating and plotting the transfer function H(f) as well as creating a Bode plot to visualize the magnitude and phase response.

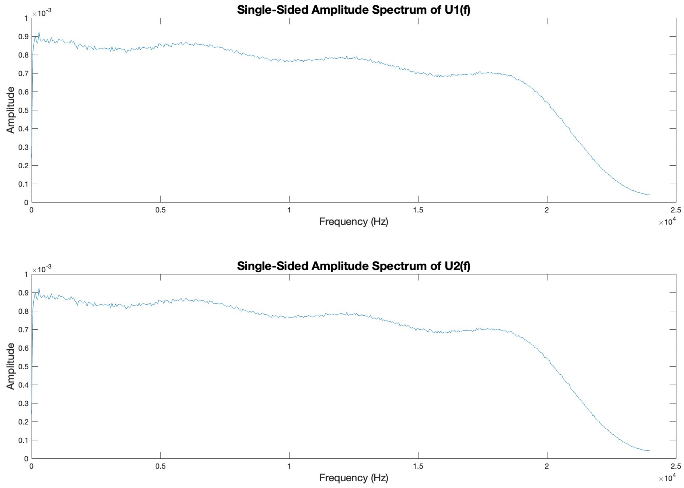


Figure 4: Single-sided amplitude spectra for U1(f) and U2(f)

Figure 5 illustrates the Bode plot based on the calcultaed Transfer Function H(f).

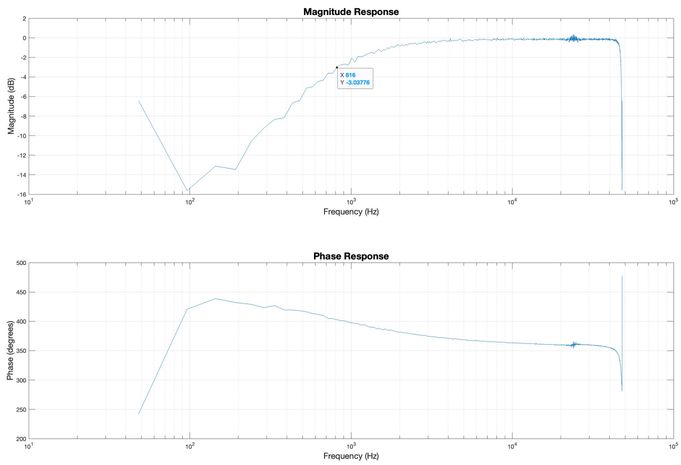


Figure 5: Bode Plot for Magnitude and Phase

3 Assignment 3: Transfer Function from White Noise

This Assignment provided discusses an alternative method for finding the transfer function of a circuit using white noise. This approach involves generating zero-mean white noise with a specified range and duration. The white noise signal is then ramped on and off using a Hanning window to ensure a smooth transition.

Generate zero-mean white noise By recording both the input u_1(t) and output u_2(t) signals simultaneously, the transfer function can be determined through spectral analysis. This method is similar to the Maximum-Length Sequence (MLS) technique, which utilizes pseudo-random binary sequences to generate signals resembling white noise in the frequency domain.

3.1 Transfer Function with Ramp and Disturbance

When calculating the transfer function, it is important to exclude the ramping portions of the signal. These ramps introduce transient effects that can distort the frequency response and affect the accuracy of the transfer function estimation. To obtain a reliable and accurate representation of the system's frequency response, it is recommended to select the portion of the signal that represents the steady-state behavior, excluding the ramping regions. This ensures that the frequency response is primarily determined by the system dynamics rather than the transient effects introduced by the ramps. Therefore for our measurement we took segmented signal it the middle which represents the steady-state behavior.

3.2 Signal Selection and Single-Sided Amplitude Spectra

Figure 6 illustrates the part of the signal selected to obtain the transfer function. Subsequently, Figure 7 depicts the calculated single-sided amplitude spectra U1(f) and U2(f).

3.3 Retrieved Transfer Function from White Noise

Figure 8 illustrates created Bode plot which shows magnitude as well as the phase of the transfer function obtained from white noise.

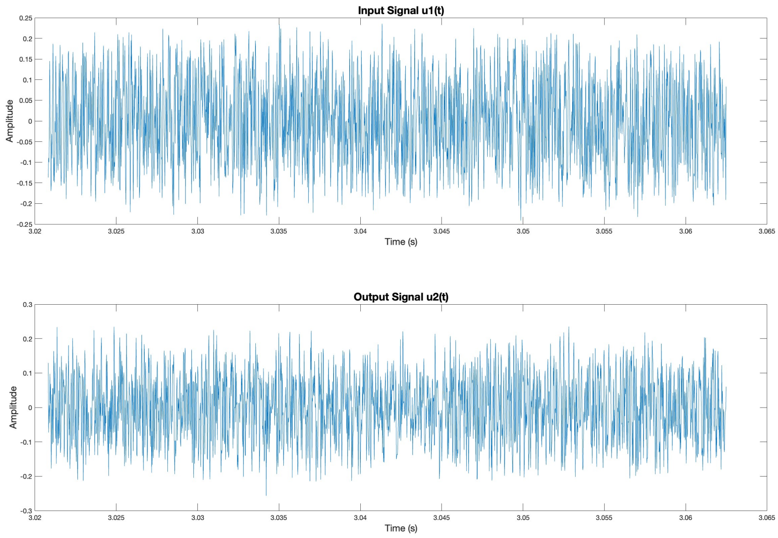


Figure 6: Part of the white noise signal selected to obtain the transfer function

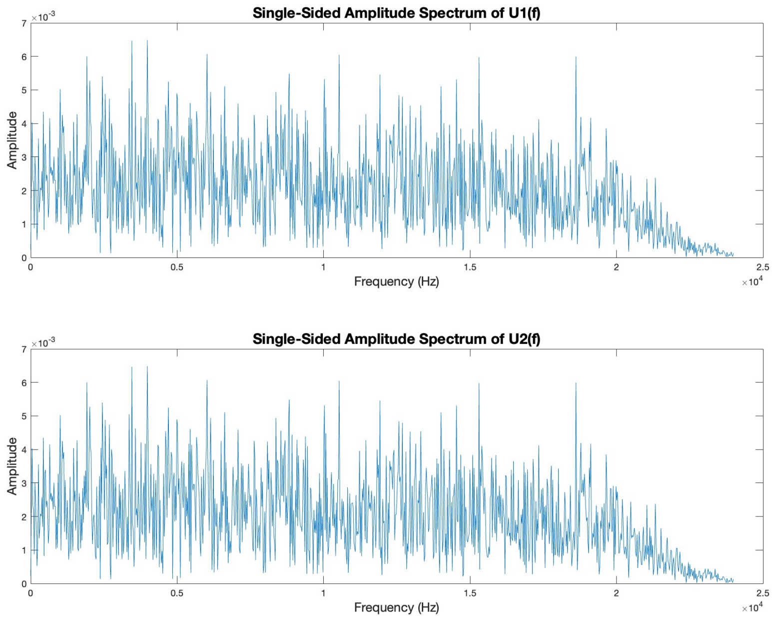


Figure 7: Single-sided amplitude spectra for U1(f) and U2(f)

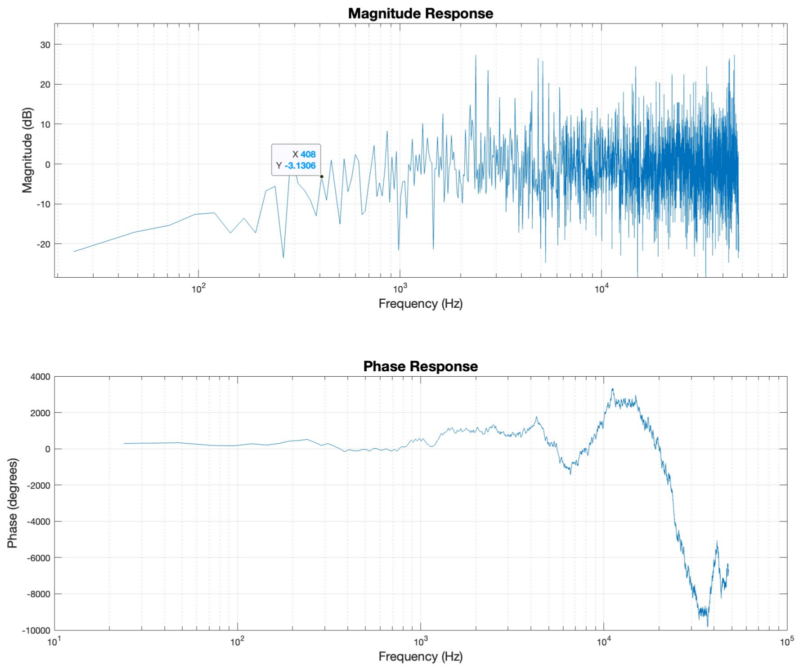


Figure 8: Bode plot obtained from transfer function based on white noise signal

4 Assignment 4 and Assignment 5

A common method to determine the transfer function of a circuit is to use a sweep signal, where a range of sine signals with constant amplitudes are applied to the circuit. It has two method to plot, linear and logarithmic.

Assignment 4 and Assignment 5 will be solved by obtaining the transfer function of the RC circuit utilizing linear sweep signal and logarithmic sweep signal as input and output signal of the system.

First, set the range for a sweep signal of frequencies from 100Hz to 24kHz. It was applied with the MATLAB function **chirp**. As above create the two channel of linear and logarithmic sweep signal.

```
#MATLAB
%  lineare-sweep signal
ls_start = 100;
ls_end = 24000;
linear_sweep = chirp(t,ls_start,T,ls_end, 'linear')';
linear_sweep(:,2) = chirp(t,ls_start,T,ls_end, 'linear')';
```

```
%  logarithmic sweep signal
ls_start = 100;
ls_end = 24000;
linear_sweep = chirp(t,ls_start,T,ls_end, 'linear')';
linear_sweep(:,2) = chirp(t,ls_start,T,ls_end, 'linear')';
```

Figure 9 presents the full recorded sweep signals, each obtained using linear and logarithmic method respectively.

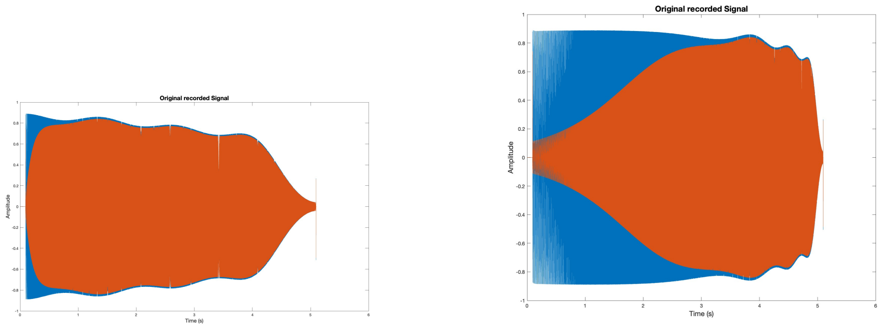


Figure 9: Original Signal of linear(left) and logarithmic(right) sweep

To be more specific, we took small part of these original signals to obtain a segmented signal as follows. The segmented signals are plotted in 10 respectively.

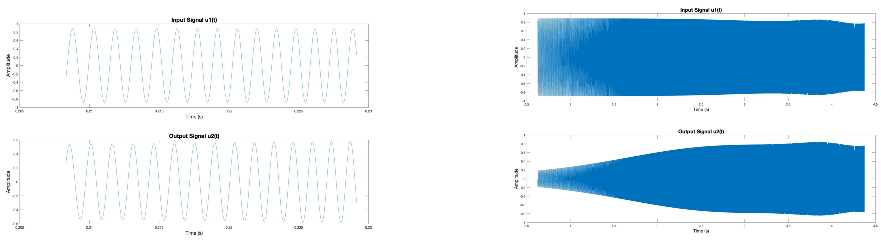


Figure 10: Segmented data of linear and logarithmic sweep signal

The magnitude spectrum $|U_1(f)|$ and $|U_2(f)|$ was calculated and plotted. And the bode plots were created. Following Figure 11 and Figure 12

Logarithmic sweeps are preferred than linear sweeps because they provide equal spacing on a logarithmic scale, cover a wide frequency range efficiently, offer higher resolution at low and high frequencies, and are effective in identifying resonances and system response peaks.

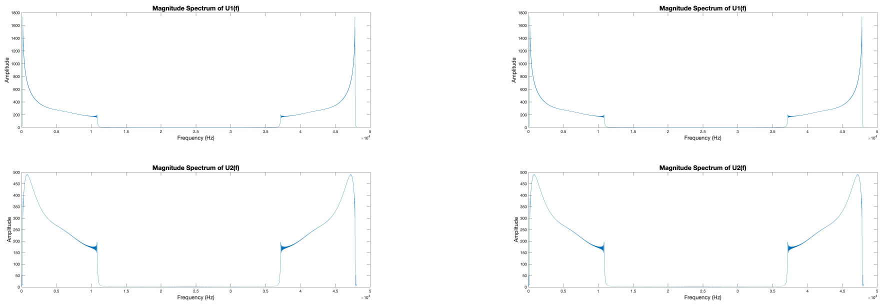


Figure 11: Magnitude spectrum of linear(left) and logarithmic(right) sweep signal

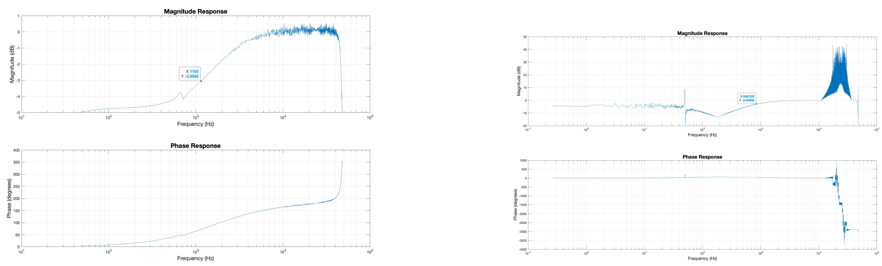
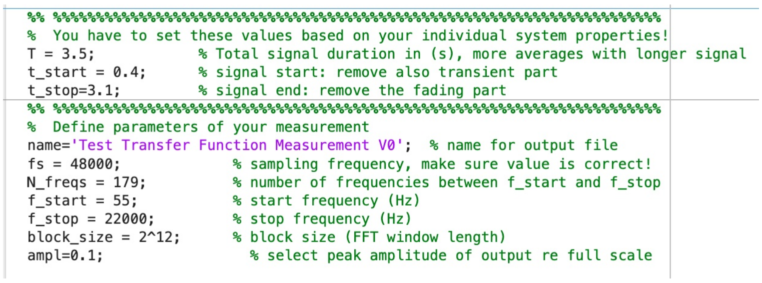


Figure 12: Bode plot of linear(left) and logarithmic(right) sweep signal

5 Assignment 6

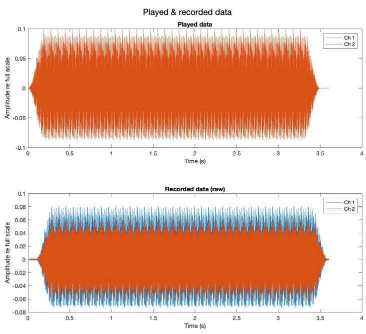
First, adjust **t_start = 0.4** and **t_stop = 0.4** for the **Duration T = 3.5**. Figure 13 is depicted.



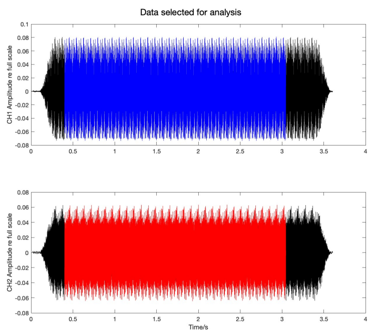
```
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  You have to set these values based on your individual system properties!
T = 3.5;            % Total signal duration in (s), more averages with longer signal
t_start = 0.4;      % signal start: remove also transient part
t_stop=3.1;         % signal end: remove the fading part
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Define parameters of your measurement
name='Test Transfer Function Measurement V0';  % name for output file
fs = 48000;             % sampling frequency, make sure value is correct!
N_freqs = 179;          % number of frequencies between f_start and f_stop
f_start = 55;           % start frequency (Hz)
f_stop = 22000;         % stop frequency (Hz)
block_size = 2^12;      % block size (FFT window length)
ampl=0.1;                 % select peak amplitude of output re full scale
```

Figure 13: MATLAB CODE

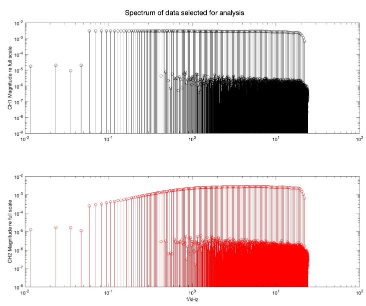
Here is all results of **measure_transfer_function_multisine. m**. Figure 14.



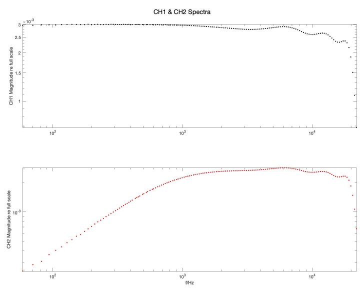
(a) Original Signal



(b) Segmented Data



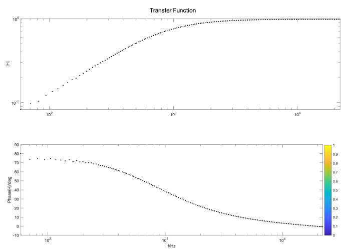
(c) Spectrum of the segmented Data



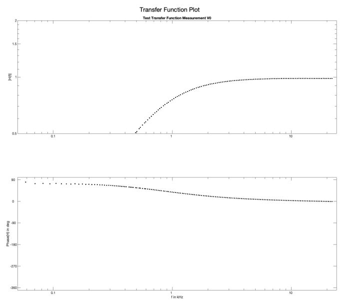
(d) Spectra of Channel 1 and 2

Figure 14: Results of **measurement_transfser_function_multisine**

The bode plot is created based on the transfer function calculated from **measurement_transfer_function_multisine.m** as below. Figure 15



Transfer Function



Plot of transfer function

Figure 15: Transfer function

6 Assignment 7

6.1 Impulse Signal

Advantages

- Simple and direct for generation and interpretation.
- Provides precise time-domain response for accurate analysis of system dynamics.

Disadvantages

- Limited frequency information in a single measurement.
- Challenging to separate system response from noise in low signal-to-noise ratio scenarios.

6.2 White Noise Signal

Advantages

- Broad frequency content allows assessment of system response across a wide spectrum.
- Statistical properties can be used for estimating system characteristics.

Disadvantages

- Transient effects introduce challenges in separating system response from noise.
- Limited frequency resolution, especially in low signal-to-noise ratio scenarios.

6.3 Frequency Sweep Signal

Advantages

- Controlled frequency variation enables targeted analysis of system behavior at different frequencies.
- Can help identify resonant frequencies and system response peaks.

Disadvantages

- Limited frequency range depending on the signal duration.
- Non-stationary signal introduces challenges in accurately capturing system response at different frequencies.